

# Web Appendix to: Firm-Level Dispersion in Productivity: Is the Devil in the Details?

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## A The plant's problem

### A.1 Product demand and general input demand functions

We specify product demand following Bartelsman, Haltiwanger and Scarpetta (2013) and add to their model by explicitly modeling the idiosyncratic demand component. Firms face a downward sloping demand schedule in a differentiated products environment. The final good is a CES aggregator of intermediate goods produced by individual firms. The final goods sector is perfectly competitive with the only inputs coming from intermediate goods:  $Y_t = N_t^{(\rho-1)/\rho} \left( \sum_i \left( Q_{it} \xi_i^{1/\rho} \right)^\rho \right)^{1/\rho}$ , implies the following inverse demand for the product of plant  $i$  in period  $t$ :<sup>1</sup>

$$P_{it} = P_t \left( \bar{Q}_t / Q_{it} \right)^{1-\rho} \xi_i, \quad (1)$$

where  $P_t$  and  $P_{it}$  denote aggregate and plant-level product prices, and  $\bar{Q}_t$  and  $Q_{it}$  denote aggregate and plant-level product quantities, respectively. We assume plant-specific shocks to residual demand are captured by  $\xi_i$ . Assuming a Cobb-Douglas production function  $Q_{it} = A_{it} \Pi_j X_{ijt}^{\alpha_j}$ , where  $X_{ijt}$  denotes input  $j$  of plant  $i$  and  $A_{it}$  denotes TFPQ, firm revenues and marginal revenue products of input factors can be written as

$$\begin{aligned} P_{it} Q_{it} &= P_t \xi_i \bar{Q}_t^{1-\rho} Q_{it}^{\rho-1} Q_{it} = P_t \xi_i \bar{Q}_t^{1-\rho} Q_{it}^\rho, \\ \frac{\partial P_{it} Q_{it}}{\partial X_{ijt}} &= P_t \xi_i \bar{Q}_t^{1-\rho} \rho Q_{it}^{\rho-1} \frac{\partial Q_{it}}{\partial X_{ijt}}. \end{aligned}$$

In the presence of distortions, the first order condition for profit maximization implies:

$$\rho \alpha_j \frac{P_{it} Q_{it}}{X_{ijt}} = (1 + \tau_{ij}) w_j, \quad (2)$$

which is to be modified in the presence of output distortions, denoted by  $(1 - \tau_{Q_i}) A_{it}$ , as

$$\rho \alpha_j \frac{P_{it} Q_{it}}{X_{ijt}} = w_j \frac{1 + \tau_{ij}}{1 - \tau_{Q_i}} = w_k \kappa_j, \quad (3)$$

where the combined effect of input and output distortions are denoted by  $(1 + \tau_{ij}) / (1 - \tau_{Q_i}) = \kappa_j$ . Equation (3) has the same interpretation as equations (10)-(11) in Hsieh and Klenow (2009) (hereafter HK), except that the effects of all distortions are accounted for in this case. The after-tax marginal revenue products (MRP) of production factors are equalized across firms, and equal their price  $w_j$ . Therefore before-tax MRP-s must be higher for firms that face higher input distortions and can be lower where firms benefit from subsidies.

MRP <sub>$j$</sub>  can be rewritten using the following expression  $\frac{\partial Q_{it}}{\partial X_{ijt}} = \alpha_j X_{ijt}^{\alpha_j-1} A_{it} \Pi_{s \neq j} X_{ist}^{\alpha_s} = \alpha_j X_{ijt}^{\alpha_j-1} F_{ijt}$ , where  $F_{ijt} = A_{it} \Pi_{s \neq j} X_{ist}^{\alpha_s}$  denotes the part of production not attributed by  $X_j$ .

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<sup>1</sup>In the simpler case without idiosyncratic shocks, the CES aggregator defined above implies  $\frac{\partial Y_t}{\partial Q_{it}} = N^{(\rho-1)/\rho} (1/\rho) (\sum_i Q_{it}^\rho)^{(1-\rho)/\rho} \rho Q_{it}^{\rho-1}$ ,  $\forall i$ . The optimum product allocation satisfies  $MRS_{ji} = \frac{P_i}{P_j}$ . Since  $MRS_{ji} = \left( \frac{\partial Y}{\partial Q_i} \right) / \left( \frac{\partial Y}{\partial Q_j} \right) = \left( \frac{Q_i}{Q_j} \right)^{\rho-1}$ , we have that  $\left( \frac{Q_i}{Q_j} \right)^{\rho-1} = \frac{P_i}{P_j}$ . Choosing product  $j=1$  as the numeraire,  $P_j=1$  implying  $Y_{it}^{\rho-1} = P_{it}$ , and choosing the average as the numeraire, we have that  $(Q_{it}/\bar{Q}_t)^{\rho-1} = P_{it}/P_t$ , which immediately implies the inverse demand function in equation (1).

In this notation, the demand for input  $j$  can be written as

$$X_{ijt}^* = \left[ \frac{P_t \xi_i \bar{Q}_t^{1-\rho} \alpha_j}{(1 + \tau_{ij}) w_j} \rho F_{ijt}^\rho \right]^{1/(1-\alpha_j \rho)}. \quad (4)$$

Equation (4) is a useful starting point because it can be used to determine the capital-labor ratio without actually solving it. We derive this ratio in the remainder of this section. We can rewrite the expression for labor and capital demand as

$$\begin{aligned} L_{it}^* &= \left[ \frac{P_t \xi_i \bar{Q}_t^{1-\rho} \alpha_L}{(1 + \tau_{iL}) w_L} \rho F_{i,AME}^\rho \right]^{1/(1-\alpha_L \rho)} K_{it}^{\alpha_K \rho / (1-\alpha_L \rho)}, \\ K_{it}^* &= \left[ \frac{P_t \xi_i \bar{Q}_t^{1-\rho} \alpha_K}{(1 + \tau_{iK}) w_K} \rho F_{i,AME}^\rho \right]^{1/(1-\alpha_K \rho)} L_{it}^{\alpha_L \rho / (1-\alpha_K \rho)}. \end{aligned}$$

where  $F_{i,AME} \equiv A_{it} M_{it}^{\alpha_M} E_{it}^{\alpha_E}$ .

Solving for  $L_{it}^*$  and  $K_{it}^*$  yields

$$L_{it}^* = [\rho P_t \xi_i \bar{Q}_t^{1-\rho} F_{i,AME}^\rho]^{-\frac{1}{1-\rho(\alpha_K + \alpha_L)}} \left( \frac{\alpha_L}{w_L(1 + \tau_{iL})} \right)^{\frac{1-\alpha_K \rho}{1-\rho(\alpha_K + \alpha_L)}} \left( \frac{\alpha_K}{w_K(1 + \tau_{iK})} \right)^{\frac{\alpha_K \rho}{1-\rho(\alpha_K + \alpha_L)}} \quad (5)$$

$$K_{it}^* = [\rho P_t \xi_i \bar{Q}_t^{1-\rho} F_{i,AME}^\rho]^{-\frac{1}{1-\rho(\alpha_K + \alpha_L)}} \left( \frac{\alpha_L}{w_L(1 + \tau_{iL})} \right)^{\frac{\alpha_L \rho}{1-\rho(\alpha_K + \alpha_L)}} \left( \frac{\alpha_K}{w_K(1 + \tau_{iK})} \right)^{\frac{1-\alpha_L \rho}{1-\rho(\alpha_K + \alpha_L)}}, \quad (6)$$

implying that the capital-labor ratio can be written as:

$$\begin{aligned} \frac{K_{it}}{L_{it}} &= \left( \frac{\alpha_L}{w_L(1 + \tau_{iL})} \right)^{\frac{\rho(\alpha_L + \alpha_K) - 1}{1-\rho(\alpha_K + \alpha_L)}} \left( \frac{\alpha_K}{w_K(1 + \tau_{iK})} \right)^{\frac{1-\rho(\alpha_L + \alpha_K)}{1-\rho(\alpha_K + \alpha_L)}} \\ &= \frac{w_L(1 + \tau_{iL})}{\alpha_L} \frac{\alpha_K}{w_K(1 + \tau_{iK})} = \frac{\alpha_K}{\alpha_L} \frac{w_L}{w_K} \frac{1 + \tau_{iL}}{1 + \tau_{iK}}, \end{aligned} \quad (7)$$

exactly as in HK, except that we allow for distortions for both capital and labor. Adding output distortions does not affect these results. The relative optimal allocations for energy and intermediate inputs can be obtained along the exact same logic. The analogous ratios are given by:

$$\frac{E_{it}}{L_{it}} = \frac{\alpha_E}{\alpha_L} \frac{w_L}{w_E} \frac{\tau_{iL}}{\tau_{iE}} \quad (8)$$

$$\frac{M_{it}}{L_{it}} = \frac{\alpha_M}{\alpha_L} \frac{w_L}{w_M} \frac{\tau_{iL}}{\tau_{iM}}. \quad (9)$$

## B Input demand, output, prices and TFPR

### B.1 Input demand

We derive key equations assuming that, in addition to capital and labor, plants use intermediate inputs and energy in the production process. The calculations leave returns to scale as a free parameter. Let us write out equation (4) for four production factors, and redefine  $\tau_{iJ} \equiv 1 + \tau_{iJ}$

for expositional purposes:

$$\begin{aligned}
K_{it}^* &= (P_t \xi_i \bar{Q}_t^{1-\rho} \rho)^{1/(1-\alpha_K \rho)} \left( \frac{\alpha_K}{w_K \tau_{iK}} \right)^{1/(1-\alpha_K \rho)} (A_i L_i^{\alpha_L} E_i^{\alpha_E} M_i^{\alpha_M})^{\frac{\rho}{(1-\alpha_K \rho)}} \\
L_{it}^* &= (P_t \xi_i \bar{Q}_t^{1-\rho} \rho)^{1/(1-\alpha_L \rho)} \left( \frac{\alpha_L}{w_L \tau_{iL}} \right)^{1/(1-\alpha_L \rho)} (A_i K_i^{\alpha_K} E_i^{\alpha_E} M_i^{\alpha_M})^{\frac{\rho}{(1-\alpha_L \rho)}} \\
E_{it}^* &= (P_t \xi_i \bar{Q}_t^{1-\rho} \rho)^{1/(1-\alpha_E \rho)} \left( \frac{\alpha_E}{w_E \tau_{iE}} \right)^{1/(1-\alpha_E \rho)} (A_i K_i^{\alpha_K} L_i^{\alpha_L} M_i^{\alpha_M})^{\frac{\rho}{(1-\alpha_E \rho)}} \\
M_{it}^* &= (P_t \xi_i \bar{Q}_t^{1-\rho} \rho)^{1/(1-\alpha_M \rho)} \left( \frac{\alpha_M}{w_M \tau_{iM}} \right)^{1/(1-\alpha_M \rho)} (A_i K_i^{\alpha_K} L_i^{\alpha_L} E_i^{\alpha_E})^{\frac{\rho}{(1-\alpha_M \rho)}}.
\end{aligned}$$

One way to solve the system above is using the per-unit-of-labor representation of inputs in (7)-(9) to eliminate the other inputs from the respective equation above. Consider labor demand as an example. We can substitute out  $K_{it}$ ,  $E_{it}$  and  $M_{it}$  using these ratios and write labor demand as a function of  $L_{it}$ , aggregate variables,  $A_i$  and demand/production parameters. Denoting returns to scale by  $\gamma$  so that  $\alpha_K + \alpha_E + \alpha_M = \gamma - \alpha_L$ , and after rearranging, the closed-form solution for  $L_{it}^*$  is given by

$$L_{it}^* = [\rho P_t \xi_i \bar{Q}_t^{1-\rho} A_i^\rho]^{\frac{1}{1-\rho\gamma}} \left( \frac{\alpha_L}{w_L \tau_{iL}} \right)^{\frac{1-\rho(\gamma-\alpha_L)}{1-\rho\gamma}} \left( \frac{\alpha_K}{w_K \tau_{iK}} \right)^{\frac{\rho\alpha_K}{1-\rho\gamma}} \left( \frac{\alpha_E}{w_E \tau_{iE}} \right)^{\frac{\rho\alpha_E}{1-\rho\gamma}} \left( \frac{\alpha_M}{w_M \tau_{iM}} \right)^{\frac{\rho\alpha_M}{1-\rho\gamma}}. \quad (10)$$

The demand functions for the other production factors are obtained by multiplying (10) by  $\frac{\alpha_j w_L \tau_{iL}}{\alpha_L w_j \tau_{ij}} = \left( \frac{\alpha_L}{w_L \tau_{iL}} \right)^{-1} \frac{\alpha_j}{w_j \tau_{ij}}$ , which implies that the exponent of  $\left( \frac{\alpha_L}{w_L \tau_{iL}} \right)$  in (10) decreases by 1 and the exponent of the ratio including the  $j$ -th factor elasticity increases by 1. We have the following formulas for the other factor demand functions

$$K_{it}^* = [\rho P_t \xi_i \bar{Q}_t^{1-\rho} A_i^\rho]^{\frac{1}{1-\rho\gamma}} \left( \frac{\alpha_L}{w_L \tau_{iL}} \right)^{\frac{\rho\alpha_L}{1-\rho\gamma}} \left( \frac{\alpha_K}{w_K \tau_{iK}} \right)^{\frac{1-\rho(\gamma-\alpha_K)}{1-\rho\gamma}} \left( \frac{\alpha_E}{w_E \tau_{iE}} \right)^{\frac{\rho\alpha_E}{1-\rho\gamma}} \left( \frac{\alpha_M}{w_M \tau_{iM}} \right)^{\frac{\rho\alpha_M}{1-\rho\gamma}} \quad (11)$$

$$E_{it}^* = [\rho P_t \xi_i \bar{Q}_t^{1-\rho} A_i^\rho]^{\frac{1}{1-\rho\gamma}} \left( \frac{\alpha_L}{w_L \tau_{iL}} \right)^{\frac{\rho\alpha_L}{1-\rho\gamma}} \left( \frac{\alpha_K}{w_K \tau_{iK}} \right)^{\frac{\rho\alpha_K}{1-\rho\gamma}} \left( \frac{\alpha_E}{w_E \tau_{iE}} \right)^{\frac{1-\rho(\gamma-\alpha_E)}{1-\rho\gamma}} \left( \frac{\alpha_M}{w_M \tau_{iM}} \right)^{\frac{\rho\alpha_M}{1-\rho\gamma}} \quad (12)$$

$$M_{it}^* = [\rho P_t \xi_i \bar{Q}_t^{1-\rho} A_i^\rho]^{\frac{1}{1-\rho\gamma}} \left( \frac{\alpha_L}{w_L \tau_{iL}} \right)^{\frac{\rho\alpha_L}{1-\rho\gamma}} \left( \frac{\alpha_K}{w_K \tau_{iK}} \right)^{\frac{\rho\alpha_K}{1-\rho\gamma}} \left( \frac{\alpha_E}{w_E \tau_{iE}} \right)^{\frac{\rho\alpha_E}{1-\rho\gamma}} \left( \frac{\alpha_M}{w_M \tau_{iM}} \right)^{\frac{1-\rho(\gamma-\alpha_M)}{1-\rho\gamma}}. \quad (13)$$

## B.2 Output and prices

The factors' contribution to output is calculated using equations (10)-(13):

$$\begin{aligned}
(L_{it}^*)^{\alpha_L} &= D^{\frac{\alpha_L}{1-\rho\gamma}} A_i^{\frac{\rho\alpha_L}{1-\rho\gamma}} \left( \frac{\alpha_L}{w_L \tau_{iL}} \right)^{\alpha_L \left( \frac{\rho\alpha_L}{1-\rho\gamma} + 1 \right)} \left( \frac{\alpha_K}{w_K \tau_{iK}} \right)^{\frac{\rho\alpha_K \alpha_L}{1-\rho\gamma}} \left( \frac{\alpha_E}{w_E \tau_{iE}} \right)^{\frac{\rho\alpha_E \alpha_L}{1-\rho\gamma}} \left( \frac{\alpha_M \alpha_L}{w_M \tau_{iM}} \right)^{\frac{\rho\alpha_M \alpha_L}{1-\rho\gamma}} \\
(K_{it}^*)^{\alpha_K} &= D^{\frac{\alpha_K}{1-\rho\gamma}} A_i^{\frac{\rho\alpha_K}{1-\rho\gamma}} \left( \frac{\alpha_L}{w_L \tau_{iL}} \right)^{\frac{\rho\alpha_L \alpha_K}{1-\rho\gamma}} \left( \frac{\alpha_K}{w_K \tau_{iK}} \right)^{\alpha_K \left( \frac{\rho\alpha_K}{1-\rho\gamma} + 1 \right)} \left( \frac{\alpha_E}{w_E \tau_{iE}} \right)^{\frac{\rho\alpha_E \alpha_K}{1-\rho\gamma}} \left( \frac{\alpha_M}{w_M \tau_{iM}} \right)^{\frac{\rho\alpha_M \alpha_K}{1-\rho\gamma}},
\end{aligned}$$

and similarly for the other inputs, where  $D$  includes demand-related variables for the sake of exposition. Collecting exponents for terms in  $Q_i^* = A_i \Pi_j^J (X_j^*)^{\alpha_j}$ <sup>2</sup> implies we can write equilibrium output as

$$Q_{it}^* = D^{\frac{\gamma}{1-\rho\gamma}} A_i^{\frac{1}{1-\rho\gamma}} \left( \frac{\alpha_L}{w_L \tau_{iL}} \right)^{\frac{\alpha_L}{1-\rho\gamma}} \left( \frac{\alpha_K}{w_K \tau_{iK}} \right)^{\frac{\alpha_K}{1-\rho\gamma}} \left( \frac{\alpha_E}{w_E \tau_{iE}} \right)^{\frac{\alpha_E}{1-\rho\gamma}} \left( \frac{\alpha_M}{w_M \tau_{iM}} \right)^{\frac{\alpha_M}{1-\rho\gamma}}, \quad (14)$$

and equilibrium prices are given by

$$P_{it}^* = (P_t \xi_i \bar{Q}_t^{1-\rho})^{\frac{\gamma(\rho-1)}{1-\rho\gamma} + 1} \left[ \rho^\gamma A_i \left( \frac{\alpha_L}{w_L \tau_{iL}} \right)^{\alpha_L} \left( \frac{\alpha_K}{w_K \tau_{iK}} \right)^{\alpha_K} \left( \frac{\alpha_E}{w_E \tau_{iE}} \right)^{\alpha_E} \left( \frac{\alpha_M}{w_M \tau_{iM}} \right)^{\alpha_M} \right]^{\frac{\rho-1}{1-\rho\gamma}}. \quad (15)$$

Equation (15) is a generalization of equation (6) in HK to include more production factors and idiosyncratic demand shocks.

### B.3 TFPR and its dispersion

The output and price equations (14)-(15) can be used to determine the revenue function:

$$P_{it}^* Q_{it}^* = (P_t \xi_i \bar{Q}_t^{1-\rho})^{\frac{1}{1-\rho\gamma}} \rho^{\frac{\rho\gamma}{1-\rho\gamma}} A_i^{\frac{\rho}{1-\rho\gamma}} \left( \frac{\alpha_L}{w_L \tau_{iL}} \right)^{\frac{\rho\alpha_L}{1-\rho\gamma}} \left( \frac{\alpha_K}{w_K \tau_{iK}} \right)^{\frac{\rho\alpha_K}{1-\rho\gamma}} \left( \frac{\alpha_E}{w_E \tau_{iE}} \right)^{\frac{\rho\alpha_E}{1-\rho\gamma}} \left( \frac{\alpha_M}{w_M \tau_{iM}} \right)^{\frac{\rho\alpha_M}{1-\rho\gamma}}. \quad (16)$$

By definition, TFPR is revenue per unit of input index, where the input index is a weighted average of production factors. Combining (14), (15) and the input index yields the following expression for TFPR:

$$\text{TFPR}_{it} = \left[ (P_t \xi_i \bar{Q}_t^{1-\rho})^{1-\gamma} \rho^{\gamma(\rho-1)} A_i^{\rho(1-\gamma)} \right]^{\frac{1}{1-\rho\gamma}} \times \left( \frac{\alpha_L}{w_L \tau_{iL}} \right)^{\frac{\alpha_L(\rho-1)}{1-\rho\gamma}} \left( \frac{\alpha_K}{w_K \tau_{iK}} \right)^{\frac{\alpha_K(\rho-1)}{1-\rho\gamma}} \left( \frac{\alpha_E}{w_E \tau_{iE}} \right)^{\frac{\alpha_E(\rho-1)}{1-\rho\gamma}} \left( \frac{\alpha_M}{w_M \tau_{iM}} \right)^{\frac{\alpha_M(\rho-1)}{1-\rho\gamma}}, \quad (17)$$

The expression collapses to the case discussed in HK if we impose CRS ( $\gamma=1$ ), abstract from intermediate inputs and energy, and set  $\tau_{iL} = 0$ .<sup>3</sup> In this special case, high revenue-productivity is a signal that distortions affect the allocation of production factors such that input demands are smaller relative to the undistorted optimum (and so is output).

Equation (17) implies that any quantile-based dispersion measure of TFPR will be a function of distance measures of distortions, TFPQ (denoted by  $A$ ) and demand shocks. Specifically, define dispersion in TFPR by  $\delta(\text{tfpr})_i^u = q(\text{tfpr})_i^u - q(\text{tfpr})_i^l$ , where  $u$  and  $l$  denote the chosen quantiles, typically 75-25 or 90-10. The log-difference of (17) between the  $u$ -th and  $l$ -th quantiles is affected by the variation in  $A_i$ ,  $\xi_i$  and  $\tau_{ij}$  because industry-level variables cancel. Taking logs in (17) and calculating the log difference yields

$$\begin{aligned} \delta_t(\text{tfpr})_i^u &= \frac{1-\gamma}{1-\rho\gamma} \delta_t(\ln \xi)_i^u + \frac{\rho(1-\gamma)}{1-\rho\gamma} \delta_t(A)_i^u + \frac{1-\rho}{1-\rho\gamma} \sum_{j=1}^J \alpha_j \delta(\tilde{\kappa}_{jt})_i^u \\ &= \frac{1-\gamma}{1-\rho\gamma} (\delta_t(\ln \xi)_i^u + \rho \delta_t(A)_i^u) + \frac{1-\rho}{1-\rho\gamma} \sum_{j=1}^J \alpha_j \delta(\tilde{\kappa}_{jt})_i^u \end{aligned} \quad (18)$$

<sup>2</sup>Note that one is added to the exponent of  $A_i$  since the objective is to obtain an expression for  $Q_i^*$ .

<sup>3</sup>See footnote 10 in HK.

where  $\delta(y)_l^u$  denotes the distance between the  $u$ -th and  $l$ -th quantile in the log-TFPR distribution, measured in terms of variable  $y$ . For example,  $\delta(A)_{25}^{75}$  is calculated as the difference between the log-TFPQ levels of the plants at the 75<sup>th</sup> and 25<sup>th</sup> percentiles of the log-TFPR distribution. Equation (18) makes it explicit that under CRS, distortions are the only source of variation and therefore the difference between the  $u$ -th and  $l$ -th percentile of distortions also gives log-TFPR dispersion. The difference relative to the two-factor case discussed in HK is that under non-CRS technology, demand shocks and TFPQ shocks also contribute to dispersion in TFPR.

## C TFPR and revenue residuals

Under the assumption of iso-elastic product demand, we can recover factor elasticities ( $\widehat{\alpha}_j$ ) from revenue elasticities ( $\widehat{\beta}_j$ ). This approach has often been used in the literature, see for example Klette and Griliches (1996), Melitz (2000), Martin (2008), De Loecker (2011), Bartelsman, Haltiwanger and Scarpetta (2013). Allowing for idiosyncratic shocks in residual demand, see equation (1), generates cross-plant variation in demand. Since  $P_i Q_i = P Q^{1-\rho} Q_i^{\rho-1} \xi_i Q_i = P Q^{1-\rho} Q_i^{\rho} \xi_i$ , plant-level revenues can be written as

$$p_i + q_i = \rho q_i + (1 - \rho)q + p + \ln \xi_i = \rho \left( \sum_{j=1}^J \alpha_j x_i^j + a_i \right) + (1 - \rho)q + p + \ln \xi_i, \quad (19)$$

where the coefficients  $\widehat{\beta}_j = \widehat{\rho} \widehat{\alpha}_j$  and  $\widehat{\beta}_q = 1 - \widehat{\rho}$  are jointly estimated. In other words, the coefficient of  $q$  determines the demand elasticity parameter  $\widehat{\rho} = 1 - \widehat{\beta}_q$ , and factor elasticities are determined by  $\widehat{\alpha}_j = \widehat{\beta}_j / \widehat{\rho}$ .

The residual from equation (19) is given by

$$p_i + q_i - \rho \sum_{j=1}^J \alpha_j x_i^j - (1 - \rho)q = \rho a_i + p + \ln \xi_i,$$

which says that the residual from a regression of revenues on inputs and a measure of aggregate demand includes the effects of physical TFP, demand shocks and aggregate prices.<sup>4</sup> Formally,

$$\begin{aligned} \widehat{\text{res}}_i &= p_i + q_i - \sum_{j=1}^J \widehat{\beta}_j x_i^j - \widehat{\beta}_R q \\ &= p_i + q_i - \widehat{\rho} \sum_{j=1}^J \widehat{\alpha}_j x_i^j - (1 - \widehat{\rho})q = \widehat{\rho} a_i + \widehat{p} + \widehat{\ln} \xi_i. \end{aligned} \quad (20)$$

Given an estimate for  $\alpha_j$  we can measure TFPR as

$$\widehat{\text{tfpr}}_i = p_i + q_i - \sum_{j=1}^J \widehat{\alpha}_j x_i^j, \quad (21)$$

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<sup>4</sup>Using the same logic, we can express  $\text{tfpr}_i^{cs}$  as  $\text{tfpr}_i^{cs} = p + (1 - \rho)q + \ln \xi_i + \rho q_i - \sum_j \widehat{\alpha}_j x_{ij}$ , which shows that  $\text{tfpr}_i^{cs} \neq \text{tfpr}_i^{rr}$ .

where  $\hat{\alpha}_j = \hat{\beta}_j / \hat{\rho}$ . implying we can use equation (21) to calculate  $\delta_t(\text{tfpr})_l^u$  on the left hand side of (18). The expression in brackets within the first term on the left hand side of (18) is by definition the dispersion in the revenue residual, or  $\widehat{\text{res}}_i$ , implying we can write (18) as

$$\begin{aligned} \delta_t(\text{tfpr})_l^u &= \frac{1-\gamma}{1-\rho\gamma} \delta(\text{res})_l^u + \frac{1-\rho}{1-\rho\gamma} \sum_{j=1}^J \alpha_j \delta(\tilde{\kappa}_{jt})_l^u \\ \sum_{j=1}^J \alpha_j \delta(\tilde{\kappa}_{jt})_l^u &= \frac{1-\rho\gamma}{1-\rho} \delta_t(\text{tfpr})_l^u - \frac{1-\gamma}{1-\rho} \delta(\text{res})_l^u. \end{aligned} \quad (22)$$

This equation shows the basic logic of our exploratory empirical analysis, which can be summarized as follows. Using estimates of revenue elasticities and demand elasticities, we calculate factor elasticities and TFPR. Next,  $\delta_t(\widehat{\text{tfpr}})_l^u$  and  $\delta_t(\widehat{\text{res}})_l^u$  are calculated. These estimates ( $\hat{\gamma}$  and  $\hat{\rho}$ ) together uniquely determine the dispersion of composite distortions that are consistent with equation (22).

## D Empirical results

This section presents descriptive statistics on estimates of returns to scale, price elasticity, and various dispersion measures. In order to recover these estimates, we jointly estimate revenue elasticities and the price elasticity parameter, as outlined in Appendix C and described in more detail in Klette and Griliches (1996).

Table 1 shows summary statistics of industry-level moments of estimated coefficients and dispersion measures. We first calculate each measure for each industry and the table reports summary statistics of those industry-level measures. We focus on results obtained by the method described in Olley and Pakes (1996) because these are consistent estimates. While we report OLS results for comparison, we note that OLS estimates are biased because factor inputs and unobserved productivity are endogenous. The main conclusion is that the properties of revenue residual dispersion and distortion dispersion are very similar. The results also highlight that revenue elasticities together with iso-elastic demand may imply extreme returns to scale estimates in a nontrivial number of industries. We note that Klette and Griliches (1996), who we follow in our approach, also found industries where some of the revenue function estimates are extreme, see the negative capital elasticities in table II in their paper. Repeating these calculations in a restricted industry where returns to scale estimates are not extreme, yielded the same conclusions with smaller standard errors. Focusing on industries where the demand elasticity parameter is less than 1 and all estimated revenue elasticities are positive, as these are consistent with our assumptions about demand, the returns to scale estimates are by construction closer to 1 and show smaller variation.

Table 2 shows the average of the within-industry correlation measures between key variables. We first calculate the industry-level correlations using the plant-level measures within each industry and then in the table we report the average industry correlations. The main findings can be summarized as follows. First,  $\text{tfpr}$  and  $\text{tfpr}^{rr}$  are positively correlated but the correlation is less than 1. The same pattern holds for  $\text{tfpr}$  and distortions: they are also correlated but not perfectly. Third,  $\text{tfpr}^{rr}$  is correlated with distortions.<sup>5</sup> Panel B shows correlations based on a restricted set of industries, where  $\hat{\rho}$  and  $\hat{\gamma}$  are not extreme. We believe applying such

<sup>5</sup>We note that instead of computing the correlations for each industry first and then taking averages, we have considered controlling for industry and year effects in the plant-level data and then computing correlations using pooled data across all plants regardless of industry. This exercise yields similar qualitative patterns

restrictions is useful because our crude approach to model demand may not provide a good fit in all industries. One such sign is that  $\hat{\rho}$  is larger than 1. If  $\hat{\rho}$  is poorly estimated, it will effect  $\hat{\gamma}$ . Similar arguments can be made about estimates of revenue elasticities and returns to scale. The number of industries in the restricted set depends on the estimation method: 24 of them are included under OLS and 25 under OP. The results are qualitatively similar but quantitatively stronger than in the full sample.

It is illustrative to look at the correlation between TFPR and distortions, denoted by  $r(\hat{\kappa}, \widehat{tfpr})$ , on an industry-by-industry basis. This is the main variable of interest in the context of the findings in HK. Figures (1(a)-1(b)) plot  $r(\hat{\kappa}, \widehat{tfpr})$  against  $\hat{\gamma}$  for two estimators. They suggest that around CRS  $r(\hat{\kappa}, \widehat{tfpr}) \approx 1$ , as we would expect based on HK. However,  $r(\hat{\kappa}, \widehat{tfpr})$  decreases quadratically in the distance between  $\hat{\gamma}$  and 1, which emphasizes the empirical importance of CRS in the context of the findings in HK.

We note that these results should be interpreted with caution because the identification and both demand and production function parameters is based on a relatively crude approach. Ideally, one would use data on plant level prices and quantities. Instead, we are using industry-level variation to identify the demand parameter which in turn implies our converting revenue elasticities into factor elasticities should be viewed with caution. But even with these limitations, our illustrative example yields conclusions that are similar to those implied by the estimates of Foster et al. (2015). To facilitate comparison with the latter, we include in the results comparisons with the moments from Foster et al. (2015). We find that  $tfpr_0^{rr}$  from Foster et al. (2015) is highly correlated with  $tfpr^{rr}$  from this analysis where we seek to identify separately the demand parameters as well as the revenue elasticities. We also find that both of these measures are highly correlated with  $tfpr^{cs}$  from Foster et al. (2015). The inference that emerges from both of these approaches is that estimates of distortions are highly correlated with estimates of fundamentals (i.e., demand and technology shocks). As we discussed in the main paper, a more reasonable interpretation for the U.S. is that measures of distortions actually reflect adjustment frictions so that plants with high current realizations of fundamentals have not yet adjusted yet so they have high estimates of tfpr.

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but somewhat lower correlations. This is not surprising since this approach may imply we pool plants from heterogeneous industries, which reduces correlations. This pooling method also gives a higher weight to industries with more observations. The latter may be industries with especially high within industry heterogeneity across plants.



Table 1: Cross-industry moments of the estimated demand parameter ( $\rho$ ), returns to scale ( $\gamma$ ), and dispersion measures:  $\text{tfpr}$ ,  $\text{tfpr}^{rr}$ ,  $\text{tfpr}^{cs}$  and distortions.

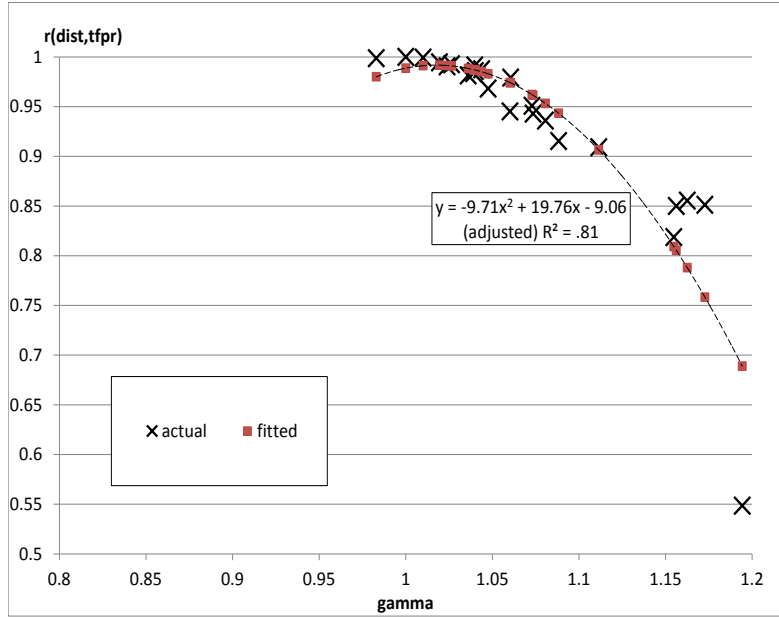
	$\rho$	$\gamma$	$\delta_{\text{tfpr}}$	$\delta_{\text{tfpr}^{rr}}$	$\delta_{\kappa}$
OLS					
mean	0.91	1.2	0.62	0.24	0.25
sd	0.19	0.51	1.01	0.09	0.11
OP					
mean	0.95	1.09	0.53	0.29	0.27
sd	0.16	0.51	1.08	0.08	0.09
$\text{tfpr}^{cs}$					
mean	.	1	0.31	.	0.31
sd	.	1	0.11	.	0.11

Dispersion measures for the revenue residual and distortions were calculated using the averages of appropriate variables within percentiles of the log-TFPR distribution. The top panel shows ordinary least squares estimates, the middle panel is based on the estimation method described in Olley and Pakes (1996), and the bottom panel is based on cost-shares. Note that the average dispersion in the revenue residual is similar but not identical to those reported in Foster et al. (2015). These statistics are unweighted while the ones in Foster et al. (2015) are weighted using the number of plant-year observations in an industry.

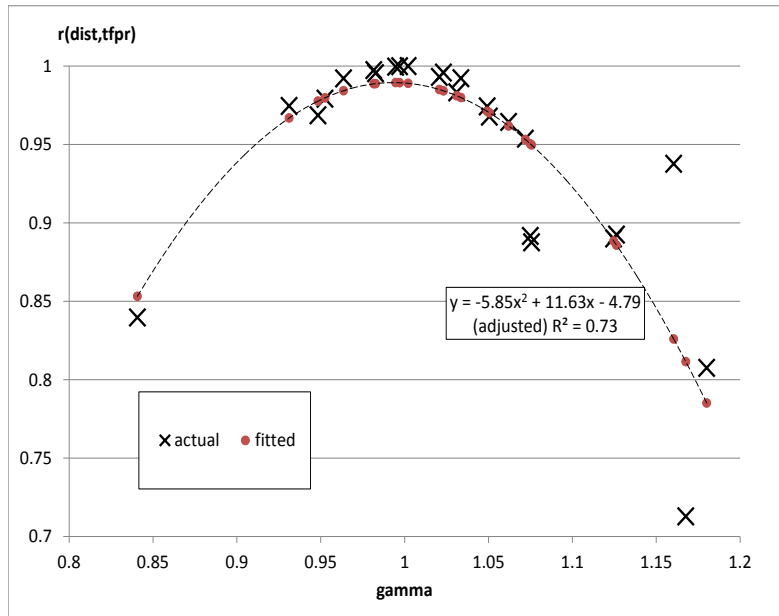
Table 2: Cross-industry averages of within-industry correlations of terms underlying the dispersion measures in equation (22).

Panel A: 50 industries											
	OLS						OP				
	$\text{tfpr}^{rr}$	$\text{tfpr}$	dist	$\text{tfpr}^{cs}$	$\text{tfpr}_0^{rr}$		$\text{tfpr}^{rr}$	$\text{tfpr}$	dist	$\text{tfpr}^{cs}$	$\text{tfpr}_0^{rr}$
$\text{tfpr}^{rr}$	1					$\text{tfpr}^{rr}$	1				
$\text{tfpr}$	0.83	1				$\text{tfpr}$	0.9	1			
dist	1	0.81	1			dist	0.96	0.88	1		
$\text{tfpr}^{cs}$	0.92	0.77	0.92	1		$\text{tfpr}^{cs}$	0.88	0.81	0.92	1	
$\text{tfpr}_0^{rr}$	0.87	0.66	0.87	0.88	1	$\text{tfpr}_0^{rr}$	0.85	0.76	0.82	0.75	1
Panel B: Industries with $\hat{\beta}_j < 0$ , $1 < \hat{\rho}$ and $1.2 < \hat{\gamma}$ excluded											
	OLS						OP				
	$\text{tfpr}^{rr}$	$\text{tfpr}$	dist	$\text{tfpr}^{cs}$	$\text{tfpr}_0^{rr}$		$\text{tfpr}^{rr}$	$\text{tfpr}$	dist	$\text{tfpr}^{cs}$	$\text{tfpr}_0^{rr}$
$\text{tfpr}^{rr}$	1					$\text{tfpr}^{rr}$	1				
$\text{tfpr}$	0.94	1				$\text{tfpr}$	0.93	1			
dist	1	0.93	1			dist	0.97	0.94	1		
$\text{tfpr}^{cs}$	0.92	0.89	0.92	1		$\text{tfpr}^{cs}$	0.9	0.89	0.94	1	
$\text{tfpr}_0^{rr}$	0.9	0.8	0.89	0.92	1	$\text{tfpr}_0^{rr}$	0.89	0.81	0.84	0.77	1

$\text{tfpr}^{rr}$  and  $\text{tfpr}$  respectively denote: the revenue residual and the estimate of  $\text{tfpr}$ , see equations (20) and (21).  $\text{dist}$  denotes the composite of distortions, i.e. the variable underlying the left hand side of equation (22).  $\text{tfpr}^{cs}$  and  $\text{tfpr}_0^{rr}$  denote the cost-share based residual (growth accounting) and the regression based revenue residual used in Foster et al. (2015).



(a) OLS



(b) OP

Figure 1: Relationship between returns to scale and the correlation between TFPR and distortions ( $r(\text{tfpr}, \text{dist})$ ). Unit of observation: industry. Industries with  $\hat{\rho} > 1$ ,  $\hat{\beta}_j < 0$  and  $\hat{\gamma} > 1.2$  are excluded from analysis.

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