Identity-Based Organizations

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Supplementary Appendix

Proof of Proposition 1. Firstly, $i \in I_\theta$ will join $k = 0$ if and only if:

$$\pi_{\theta 0} - x_i^2 - c > \pi_{\theta 0}, \tag{1}$$

which clearly does not hold for any θ and pair $x_i \geq 0$ and $c > 0$. Therefore, $M_0 = \emptyset$. This establishes part (ii).

Agent $i \in I_\theta$ will join $k = 1$ if and only if:

$$\pi_{\theta 1}\overline{x}_1 + \pi_{\theta 0}(1 - \overline{x}_1) - x_i^2 - c > \pi_{\theta 0}. \tag{2}$$

Clearly, this cannot hold for $\theta = 0$, since $\pi_{00} > \pi_{01}$. Hence all $i \in I_0$ remain unaffiliated, establishing part (i).

To establish part (iv), suppose for the moment that $x_i = s_1$ in equilibrium. Substituting this into equation (1) of the main paper, inequality (2) above holds for $i \in I_1$ if and only if:

$$c < (\pi_{11} - \pi_{10})s_1 - s_1^2 = \tau s_1 - s_1^2 \equiv \bar{c}. \tag{3}$$

Therefore, $|M_1| = |I_1|F(\bar{c})$. By the assumptions on F, $|M_1| \in (0, 1)$ if and only if $0 < s_1 < \tau$. Hence one can restrict attention to $s_1 \in (0, \tau)$, because the organization's objective function X_1 equals zero otherwise.

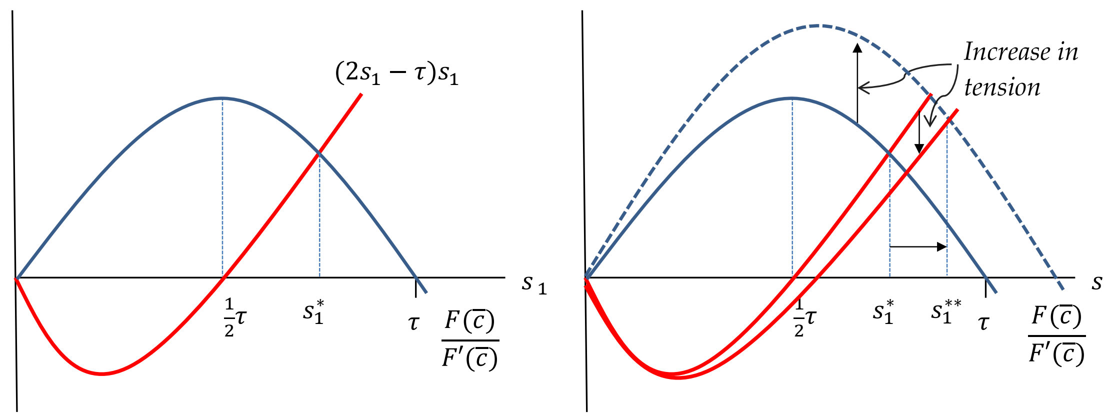
Thus, the organization's problem is:

$$max_{s_1}\ |I_1|F\big(\bar{c}(s_1)\big)s_1, \tag{4}$$

subject to $0 < s_1 < \tau$. The first-order condition for an interior optimum is:

$$\frac{F\big(\bar{c}(s_1)\big)}{F'\big(\bar{c}(s_1)\big)} = (2s_1 - \tau)s_1. \tag{5}$$

Consider the LHS of (5). Recall that F is twice differentiable and strictly log-concave, so the LHS is continuous and strictly increasing in $\bar{c}$. From (3), on $[0, \frac{1}{2}\tau)$, $\bar{c}(s_1)$ is continuous and strictly increasing in s_1. On $(\frac{1}{2}\tau, \tau)$, $\bar{c}(s_1)$ is continuous and strictly decreasing. Therefore, the LHS is continuous, strictly increasing in s_1 on $\big[0, \frac{1}{2}\tau\big)$ and strictly decreasing in s_2 on $\big(\frac{1}{2}\tau, \tau\big)$.



(a) Equilibrium

(b) Comparative Static

Figure 1: Equilibrium strictness s_1^* is strictly decreasing in tension τ.

In addition, since $F(0) = 0$ and $F'(0) > 0$, the LHS equals zero for $s_1 \in \{0, \tau\}$ and is positive for $s_1 \in (0, \tau)$.

The RHS of (5) is nonpositive for $0 \leq s_1 \leq \frac{1}{2}\tau$ and positive and strictly increasing in s_1 for $s_1 > \frac{1}{2}\tau$.

Therefore, the two curves intersect at some unique value $s_1^* \in (\frac{1}{2}\tau, \tau)$. The solution is depicted in figure 1(a). Clearly, the second-order condition for a maximum holds at s_1^*.

Finally, let us establish part (iii). Suppose that $x_i > s_1$ in equilibrium. Differentiating equation (1) of the main paper with respect to x_i yields the first-order condition

$$\frac{\pi_{\theta 1} - \pi_{\theta 0}}{|M_1|} - 2x_i = 0,$$

and the unconstrained optimizer

$$x_i = \frac{\tau}{2|M_1|}, \tag{6}$$

for all $i \in M_1$. We have already established that the optimal symmetric participation profile from organization 1's perspective involves $x_i = s_1^* > \tau/2$, which is greater than or equal to (6). Hence $x_i^* = s_1^*$ for all $i \in M_1$. □

Proof of Proposition 2. An increase in τ causes the LHS of (5) to shift up and the RHS to shift down. This implies that s_1^* is strictly increasing in τ, as depicted in figure 1(b).

Finally, consider total participation, $X_1^*(s_1^*)$. By the envelope theorem:

$$\begin{aligned} \frac{dX_1^*(s_1^*)}{d\tau} &= \frac{\partial X_1^*(s_1^*)}{\partial \tau} \\ &= F'\big(\bar{c}(s_1^*)\big)\frac{\partial \bar{c}(s_1^*)}{\partial \tau} \\ &= F'\big(\bar{c}(s_1^*)\big)s_1^* > 0. \qquad \square \end{aligned} \tag{7}$$