# Online Appendix for The Role of Startups in Structural Transformation

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This document presents additional details related to the analysis for the paper "The Role of Startups in Structural Transformation". We first show how we extend the dynamic decomposition framework in Pugsley and Şahin (2014) to correctly account for the joint dynamics of age and sector. In the second section, we provide results for a complete set of sectors.

## EXTENDING THE FRAMEWORK

### A1. Conditional distribution of employment by age given sector j

This extends the dynamic decomposition from Pugsley and Şahin (2014). Let  $\vec{E}_t^j$  be a  $3 \times 1$  vector of employment by age group k for sector  $j \in \{1, \ldots, J\}$ . Let  $S_t^j$  be a scalar of startup employment in sector j. Let  $P_t^j$  be a  $3 \times 3$  transition matrix across age groups for sector j

$$P_{jt} = \begin{bmatrix} 0 & x_{yt}^{j}(1+n_{yt}^{j}) & 0\\ 0 & q_{yt-1}^{j}x_{yt}^{j}(1+n_{yt}^{j}) & (1-q_{mt-1}^{j})x_{mt}^{j}(1+n_{mt}^{j})\\ 0 & 0 & x_{mt}^{j}(1+n_{mt}^{j}) \end{bmatrix}$$

where  $q_{t-1}^{j}$  is the fraction of the year t-1 young cohort that remains young in year t. This can be easily extended to include additional age groups with an appropriate choice of  $q_{t-1}^{k}$  for each additional age group. The law of motion for sector j is

$$\vec{E}_t^j = P_t^{j'} \vec{E}_{t-1}^j + S_t^j (1,0,0)'.$$

# A2. Joint distribution of age and sector

Let  $\vec{E}_t$  be a 3 \* J length vector of employment by age group  $k \in \{S, Y, M\}$  and sector  $j \in \{1, \ldots, J\}$ , switching first over age group and then over sector. Let  $\vec{S}_t$  be a J length vector of startup employment and define a J length vector of startup employment shares  $\vec{s}_t \equiv \vec{S}_t/S_t$  where  $S_t$  is aggregate startup employment. Let  $P_t$  be a  $3 * J \times 3 * J$  block diagonal transition matrix with sector specific transition matrices  $P_t^j$  along its diagonal. The law of motion for the joint distribution is

$$ec{E}_t = P'_t ec{E}_{t-1} + S_t ec{s}_t \otimes (1,0,0)'$$
.  
A3. Aggregating by sector

Let H be an operator that converts the joint distribution over age and sector to a distribution over sector by summing the 3 age groups for each sector

$$H \equiv I_J \otimes (1,1,1)$$

Applying to the law of motion

$$\begin{aligned} H\vec{E}_t &= HP'_t\vec{E}_{t-1} + S_tH\vec{s}_t \otimes (1,0,0)' \\ &= HP'_t\vec{E}_{t-1} + S_t\vec{s}_t \end{aligned}$$

or in shares

$$H\vec{e}_t = \frac{HP_t'}{1+g_t}\vec{e}_{t-1} + \frac{S_t}{E_t}\vec{s}_t,$$

where  $1 + g_t = E_t / E_{t-1}$ 

#### Additional results

This section presents results for a complete set of sectors. In figure B1, we show each sector's share of overall employment (solid lines) and shares that would have arisen had the age-specific growth and survival rates been flat at their sample averages. Overall, these plots illustrate that changes in conditional life cycle dynamics do not matter much for employment reallocation.

Next, we look at finer categories of sectors and compute, for each 4-digit NAICS sector, the maximum difference between that sector's employment and the employment share that would have arisen had the age-specific growth and survival rates remained flat at their sample averages. The left panel of figure B2 shows the histograms of these share differences, whereas the right panel shows the same for log differences. These plots further confirm the result that changes in conditional life cycle dynamics matter very little for employment reallocation.

Lastly, we quantify the relative roles of startups and incumbents in driving structural transformation for 8 broad sectors. The results are presented in table B1.

|                      | Incumbent | Entry Margin |              | Overall Change (pp) |                    |
|----------------------|-----------|--------------|--------------|---------------------|--------------------|
|                      | Lifecycle | Initial      | Time-varying | Actual              | No Startup Deficit |
| Mining               | -178.2    | -37.5        | 315.7        | -0.1                | -0.1               |
| Construction         | 128.4     | -87.0        | 58.6         | -1.8                | -1.7               |
| Manufacturing        | 33.6      | 53.2         | 13.3         | -11.1               | -12.0              |
| Trans., Comm., Util. | -218.3    | 204.2        | 114.1        | -0.3                | -0.6               |
| Wholesale Trade      | -12.0     | 42.2         | 69.8         | -1.1                | -1.3               |
| Retail Trade         | -289.8    | 526.4        | -136.6       | 0.7                 | 1.3                |
| FIRE                 | 3.9       | 134.6        | -38.5        | -0.7                | -0.9               |
| Services             | 47.4      | 19.7         | 32.9         | 14.5                | 15.3               |

Table B1—: Actual and without startup deficit employment shares

*Note:* This table presents the fraction of the change in employment share that is explained by the incumbent and entry margins for different sectors as well as the actual change in employment shares and the change in the absence of the startup deficit. Source: U.S. Census Bureau Business Dynamics Statistics and authors' calculations.

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## REFERENCES

Pugsley, Benjamin W, and Ayşegül Şahin. 2014. "Grown-up business cycles." FRB of New York Staff Report, , (707).



*Notes:* This figure depicts each sector's share of overall employment (solid lines) for 1987 to 2012. The dashed lines show sector employment shares that would have arisen had the age-specific growth and survival rates been flat at their sample averages. Source: U.S. Census Bureau Longitudinal Business Database and authors' calculations.

Figure B1. : Employment Shares by Industry (LBD)



*Notes:* This figure depicts the maximum difference between NAICS4-level employment shares and the employment share that would have arisen had the age-specific growth and survival rates been flat at their sample averages. The left panel shows changes in the employment share while the right panel shows log changes. Source: U.S. Census Bureau Longitudinal Business Database and authors' calculations.

Figure B2. : Employment Shares by Industry (LBD)



*Notes:* This figure shows the effects of the entry and incumbent margins over the period 1987-2013. The solid black lines show the evolution of sector employment shares under the assumption that conditional life cycle dynamics are the same across sectors and the startup employment is allocated according to sectors' overall employment shares. The blue dashed lines highlight the contribution evolution of sector shares by allowing the conditional life cycle dynamics to vary across sectors. The green dashed lines show the evolution of sector shares if, in addition to incumbent life cycle effects, the startups were allocated according to the empirical allocation at the beginning of the sample period. Finally, the red dashed lines allow for the startup shares to vary over time as they do in the data. Source: U.S. Census Bureau Longitudinal Business Database and authors' calculations.

Figure B3. : Decomposition of sectoral reallocation