Optimal Fiscal and Monetary Policy with Costly Wage Bargaining*

David M. Arseneau [†]
Federal Reserve Board

Sanjay K. Chugh[‡]
University of Maryland
Federal Reserve Board

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Abstract

Costly nominal wage adjustment has received renewed attention in the design of optimal policy. In this paper, we embed costly nominal wage adjustment into the modern theory of frictional labor markets to study optimal fiscal and monetary policy. Our main result is that the optimal rate of price inflation is quite volatile despite the presence of nominal wage rigidities. This finding contrasts with results obtained using standard sticky-wage models, which employ Walrasian labor markets at their core. We also find that the tax-smoothing result that lies at the heart of optimal policy prescriptions in standard Ramsey models does not carry over into our environment. Both results stem from a common source in our model. Shared rents associated with the formation of long-term employment relationships imply that the optimal policy is willing to tolerate large fluctuations in after-tax real wages that would otherwise not be tolerated in models with Walrasian labor markets. Our results demonstrate that the level at which nominal wage rigidity is modeled — whether simply layered on top of a Walrasian market or articulated in the context of an explicit relationship between workers and firms — can matter a great deal for policy recommendations.

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†E-mail address: david.m.arseneau@frb.gov.
‡E-mail address: chughs@econ.umd.edu.

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1 Introduction

The study of optimal monetary policy in the presence of nominally-rigid wages has enjoyed a resurgence of late. The typical story behind models featuring nominal wage rigidities is that wage negotiations are costly or time-consuming, which leads to infrequent adjustments. However, it is somewhat difficult to understand the idea of wage negotiations, costly or not, when the underlying model of the labor market is Walrasian, which is true of existing sticky-wage models that study optimal policy. In Walrasian markets, there are no negotiations — all transactions are simply against the anonymous market. Instead, models that feature explicit bilateral relationships between firms and workers seem to be called for in order to study the consequences of costly wage negotiations.

In this paper, we embed costly nominal wage negotiations into the modern theory of frictional labor markets to study optimal fiscal and monetary policy. Our main result is that the optimal inflation rate is quite volatile over time despite the presence of nominal wage rigidities, which stands in contrast to results obtained in environments with fundamentally Walrasian labor markets. In addition, the typical tax-smoothing incentive at the heart of optimal policy prescriptions in standard Ramsey models does not carry over into our environment: we find an optimal labor tax rate that is an order of magnitude more volatile than in a standard Ramsey model. A central message of our results, therefore, is that the level at which nominal wage rigidity is modeled — whether simply layered on top of a Walrasian market or articulated in the context of an explicit relationship between workers and firms — can matter a great deal for policy recommendations.

The cyclical properties of optimal policy in basic Ramsey monetary models are well-understood. Chari, Christiano, and Kehoe (1991) showed, in an environment with fully-flexible nominal prices and nominal wages, that under a coordinated program of fiscal and monetary policy, the Ramsey planner prefers state-contingent movements in the price level to changes in proportional taxes in response to shocks to the consolidated government budget. The Ramsey literature has recently re-examined this issue in models featuring nominally-rigid prices and wages. Schmitt-Grohe and Uribe (2004b) and Siu (2004) showed that with even a small degree of nominal rigidity in prices, optimal inflation volatility is quite small. Chugh (2006) showed that stickiness in nominal wages by itself also makes Ramsey-optimal inflation very stable over time, but in the latter the wage rigidity is introduced in an otherwise Walrasian labor market.

The contrast between our results here and those in Chugh (2006) stems from the importance the planner attaches to delivering a stable path of realized *real* wages for the economy. The key to understanding the result in Chugh (2006) is that if real wage growth is determined essentially by technological features of the economy (such as productivity) that do not fluctuate too much, then any desire to stabilize nominal wages shows up as a concern for stabilizing nominal prices. If real wages are not tied so tightly to an economy's production possibilities but instead are free to adjust without much allocative consequence, as is the case in our model here, then such an effect need not occur. In our model, which builds on the basic labor search and matching framework, wages are determined after a worker and a firm endure a costly search process. Once two parties do meet, the resulting economic rents are divided through wage negotiations. In general, there is a continuum of real wages that is acceptable for both parties to agree to consummate the match and begin production. In this sense, the real wage plays much more of a distributive, rather than a purely allocative, role. Thus, any desire to stabilize nominal wages does not immediately translate into a desire to stabilize nominal prices because the planner takes into account the fact that real wages do not critically affect allocations.

Regarding the lack of tax smoothing in our environment, the connection between the labor tax rate and the level of employment in our model is much less tight than in a standard Walrasian model. Werning (2007) recently shed new analytical insight on the quantitative finding by Chari, Christiano, and Kehoe (1991) (henceforth, CCK) that labor tax rates are virtually constant over time. However, Werning's (2007) arguments depend on a model featuring a typical Walrasian labor market — in particular, on being able to invert a standard household consumption-leisure optimality condition to solve for the Ramsey-optimal labor income tax rate. Because such a simple neoclassical relationship between employment and labor taxes does not exist in our model, one ought not to have any presumption that labor-tax-smoothing *should* arise in our model. Indeed, the rent-sharing in our environment that makes inflation stability an unimportant goal of policy is also the driving force behind the unimportance of tax smoothing. Cyclical (and large) variations in inflation and tax rates affect the distributional consequences of the real wage through what we refer to as a *dynamic bargaining power effect,* but these redistributions have little impact on real allocations.

Our primary focus is on the dynamics of optimal policy, but our model also has predictions for long-run policy. The most notable is that in the long run, the optimal inflation rate trades off three distortions. Two distortions are standard in monetary models: inefficient money holdings due to a deviation from the Friedman Rule versus resource losses stemming from nominal adjustment due to non-zero inflation. The third distortion influencing steady-state inflation in our model is inefficiencies in job creation, which positive inflation in some cases can offset. This latter policy channel is one about which Ramsey models based on Walrasian labor markets are silent; it is one that others using labor-search frameworks, notably Faia (2007) and Cooley and Quadrini (2004), have also pointed out albeit not in the context of a model studying both fiscal and monetary policy.

We articulate these ideas by incorporating two new elements into a standard model of labor search and wage bargaining. First, we assume that workers and firms negotiate over nominal wages, rather than real wages as is typically assumed in this class of models. We think it seems empirically descriptive of actual wage negotiations that bargaining occurs in terms of a nominal unit of account, but we do not claim to have any novel explanation for why this occurs. By itself, this assumption is innocuous because, as we show, bargaining in either nominal or real units has no consequence for the basic labor search model. Instead, we assume it in order to have a well-defined notion of resource costs of changing nominal wages. Once again, we do not claim we have an explanation any deeper than existing ones for why there are costs of changing nominal wages; such costs may be administrative costs of recording, reporting, and implementing a new nominal wage for an employee, for example. By pushing the notion of costly nominal wage contracting down to a more clearly-defined concept of a worker-firm pair, though, we show that monetary policy should be conducted in a very different way than predicted by sticky-nominal-wage models as typically formulated.

The idea that (after-tax) real wages may play a very different role than predicted by neoclassical models of course has a rich history in policy discussions. Barro (1977), starting from the insight that firms and workers engaged in long-term relationships have incentives to arrange rent payments among themselves to neutralize any allocative distortions, questioned the importance of nominal wage rigidities for the effects of monetary policy.[1] The labor search and bargaining framework provides a modern structure with which to think about such issues in a way that addresses the Barro (1977) critique. Indeed, our results show that costly nominal wage adjustment does not affect the basic Ramsey prescription of inflation volatility.

Our work is related to the recent literature exploring the consequence of nominal rigidities in labor search and matching environments.[2] However, our work is most closely related to Faia (2007) and Thomas (2007), both of whom study optimal monetary policy in New Keynesian models featuring labor matching frictions. In contrast, our model features fully-flexible product prices; therefore, a typical forward-looking price-Phillips curve is absent from our model. Furthermore, rather than concentrating solely on monetary policy, we conduct a traditional Ramsey exercise in which we solve an optimal public financing problem that requires specifying fiscal and monetary policy jointly. Despite obvious differences in implementation, we think the views emerging from our work and those of Faia (2007) and Thomas (2007) are complementary.

The rest of our paper is organized as follows. Section 2 builds our basic model. Section 3 presents the Ramsey problem, and Section 4 presents and discusses our main results. Section 5 summarizes and offers possible avenues for continued research. In an expanded version of our work, Arseneau

[1] More recently, Goodfriend and King (2001, p. 48-51), based partially if not explicitly on Barro's (1977) critique, conjectured that costs of adjusting nominal wages ought not to have much consequence for the dynamics of optimal inflation.

[2] For example, Blanchard and Gali (2006b), Walsh (2005), Trigari (2006), Christoffel and Linzert (2005), and Krause and Lubik (2007), to name just a few.

and Chugh (2007a), we also allow for an intensive margin of labor adjustment to demonstrate how a more standard neoclassical hours mechanism affects our results — the impression left by the results in our expanded model is largely the same as those we find here.

2 The Model

As many other recent studies have done, our model embeds the Pissarides (2000) textbook search model into a dynamic stochastic general equilibrium framework. We present in turn the composition of the representative household, the representative firm, how wages are determined, the actions of the government, and the definition of equilibrium.

2.1 Households

There is a continuum of measure one of identical households in the economy. The representative household consists of a continuum of measure one of family members. Each member of the household either works during a given time period or is unemployed and searching for a job. At time t, a measure n_t of individuals in the household are employed and a measure $1 - n_t$ are unemployed. We assume that total household income is divided evenly amongst all individuals, so each individual has the same consumption.[3]

The household's discounted lifetime utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_{1t}, c_{2t}) + \int_0^{n_t} A^i \bar{h} di + \int_{n_t}^1 v^i di \right], \tag{1}$$

where $u(c_1, c_2)$ is each family member's utility from consumption of cash goods (c_1) and credit goods (c_2), $\bar{h}$ is a fixed number of hours that an employed individual works, A^i is the utility per unit time of an employed individual i, and v^i is the utility experienced by individual i from non-work. The function u satisfies $u_j > 0$ and $u_{jj} < 0$, $j = 1, 2$. We assume symmetry in the utility amongst all employed individuals, so that $A^i = A$, as well as symmetry in the utility of non-work amongst all unemployed individuals, so that $v^i = v$. Thus, household lifetime utility can be expressed as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_{1t}, c_{2t}) + n_t A \bar{h} + (1 - n_t) v \right]. \tag{2}$$

The household does not choose how many family members work. As described below, the number of people who work is determined by a labor matching process. We also assume that each employed individual works a fixed number of hours, normalized to unity. In our expanded analysis

[3] Thus, we follow Merz (1995), Andolfatto (1996), and much of the subsequent literature in this regard by assuming full consumption insurance between employed and unemployed individuals.

in Arseneau and Chugh (2007a), we examine, among other things, the robustness of our results to endogenous adjustment of labor at the hours margin.

The household chooses state-contingent sequences of consumption of each good, nominal money holdings, and nominal bond holdings $\{c_{1t}, c_{2t}, M_t, B_t\}$, to maximize lifetime utility subject to an infinite sequence of flow budget constraints

$$M_t - M_{t-1} + B_t + R_{t-1}B_{t-1} = (1-\tau^n_{t-1})W_{t-1}n_{t-1}\bar{h} - P_{t-1}c_{1t-1} - P_{t-1}c_{2t-1} + P_{t-1}d_{t-1} \quad (3)$$

and cash-in-advance constraints

$$P_t c_{1t} \leq M_t. \quad (4)$$

M_{t-1} is the nominal money the household brings into period t, B_{t-1} is nominal bonds brought into t, W_t is the nominal wage, P_t is the price level, R_t is the gross nominally risk-free interest rate on government bonds held between t and $t+1$, τ^n_t is the tax rate on labor income, and d_t is profit income of firms received lump-sum by households. The timing of the budget and cash-in-advance constraints conforms to the timing described by CCK and used by Siu (2004) and Chugh (2006, 2007).

As is standard in this class of cash/credit models, household optimality yields the Fisher equation

$$1 = R_t E_t\left[\frac{\beta u_{1t+1}}{u_{1t}}\frac{1}{\pi_{t+1}}\right] \quad (5)$$

and an optimality condition linking the marginal rate of substitution between cash and credit goods to the nominal interest rate

$$\frac{u_{1t}}{u_{2t}} = R_t. \quad (6)$$

In a monetary equilibrium, $R_t \geq 1$, otherwise consumers could earn unbounded profits by buying money and selling bonds, thus placing an equilibrium restriction on the MRS between cash and credit goods.

2.2 Production

The production side of the economy features a representative firm that must open vacancies, which entail costs, in order to hire workers and produce. The representative firm is "large" in the sense that it operates many jobs and consequently has many individual workers attached to it through those jobs.

To be more specific, the firm requires only labor to produce its output. The firm must engage in costly search for a worker to fill each of its job openings. In each job k that will produce output, the worker and firm bargain over the pre-tax hourly nominal wage W_{kt} paid in that position. Output of job k is given by $y_{kt} = z_t f(\bar{h})$, which is subject to a common technology realization z_t. We allow for both $\bar{h} < 1$ and curvature in $f(.)$ to enhance comparability with our extended model in Arseneau and Chugh (2007a), in which we introduce labor adjustment and diminishing returns along the intensive (hours) margin. In our analysis and results here, of course, $f(\bar{h})$ is simply a constant, the choice of which we describe below.

Any two jobs k_a and k_b at the firm are identical, so from here on we suppress the second subscript and denote by W_t the nominal hourly wage in any job, and so on. Total output of the firm thus depends on the technology realization and the measure of matches n_t that produce,

$$y_t = n_t z_t f(\bar{h}). \tag{7}$$

The total nominal wage paid by the firm in any given job is $W_t\bar{h}$, and the total nominal wage bill of the firm is the sum of wages paid at all of its positions, $n_t W_t \bar{h}$.

The firm begins period t with employment stock n_t. Its future employment stock depends on its current choices as well as the random matching process. With probability $k^f(\theta)$, taken as given by the firm, a vacancy will be filled by a worker. Labor-market tightness is $\theta \equiv v/u$, and matching probabilities depend only on tightness given the Cobb-Douglas matching function we will assume.

The firm also faces a quadratic cost of adjusting nominal wages. For each of its workers, the real cost of changing nominal wages between period $t-1$ and t is

$$\frac{\psi}{2}\left(\frac{W_t}{W_{t-1}} - 1\right)^2, \tag{8}$$

where $\psi \geq 0$ governs the size of the wage adjustment cost. If $\psi = 0$, clearly there is no cost of wage adjustment. We choose a quadratic adjustment cost specification because of its simplicity and because it enhances comparability with the results in Chugh (2006), who also uses a quadratic wage adjustment cost.

Regardless of whether or not nominal wages are costly to adjust, wages are determined through bargaining, which we describe below. In the firm's profit maximization problem, the wage-setting protocol is taken as given. The firm thus chooses vacancies to post v_t and future employment stock n_{t+1} to maximize discounted nominal profits starting at date t,

$$E_t \sum_{s=0}^{\infty} \beta^s \left\{ \left(\frac{\beta\phi_{t+1+s}}{P_{t+s}}\right) \left[P_{t+s} n_{t+s} z_{t+s} f(\bar{h}) - W_{t+s} n_{t+s} \bar{h} - \gamma P_{t+s} v_{t+s} - \frac{\psi}{2}\left(\frac{W_{t+s}}{W_{t+s-1}} - 1\right)^2 n_{t+s} P_{t+s} \right] \right\}. \tag{9}$$

The representative firm discounts period-t profits using $\beta\phi_{t+1}/P_t$ because this is the value to the household of receiving a unit of nominal profit.[4] In period t, the firm's problem is thus to choose

[4]To understand this, note from the household budget constraint that period-t profits are received, in keeping with the usual timing of income receipts in a cash/credit model, in period $t+1$. The multiplier associated with the period-t household flow budget constraint is ϕ_t/P_{t-1}. Hence, the derivative of the Lagrangian of the household problem with respect to d_t is $\beta\phi_{t+1}/P_t$.

v_t and n_{t+1} to maximize (9) subject to its perceived law of motion for employment

$$n_{t+1} = (1-\rho^x)(n_t + v_t k^f(\theta_t)). \tag{10}$$

Firms incur the real cost γ for each vacancy created, and job separation occurs with exogenous fixed probability ρ^x.

The firm's first-order conditions with respect to v_t and n_{t+1} yields the job-creation condition

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left[\beta\left(\frac{\beta\phi_{t+2}}{\beta\phi_{t+1}}\right)(1-\rho^x)\left(z_{t+1}f(\bar{h}) - w_{t+1}\bar{h} - \frac{\psi}{2}\left(\pi^w_{t+1}-1\right)^2 + \frac{\gamma}{k^f(\theta_{t+1})}\right)\right], \tag{11}$$

where we have defined $\pi^w_{t+1} \equiv W_{t+1}/W_t$ as the gross nominal wage inflation rate and $w_{t+1} \equiv W_{t+1}/P_{t+1}$ is the real wage rate. The job-creation condition states that at the optimal choice, the vacancy-creation cost incurred by the firm is equated to the discounted expected value of profits from a match. Profits from a match take into account the wage cost of that match, including future nominal wage adjustment costs, as well as future marginal revenue product from the match. This condition is a free-entry condition in the creation of vacancies and is a standard equilibrium condition in a labor search and matching model. It is useful to note that in equilibrium, $(\beta\phi_{t+2})/(\beta\phi_{t+1}) = u_{2t+1}/u_{2t}$, which can be obtained from the household's optimality condition with respect to credit good consumption.

2.3 Government

The government's flow budget constraint is

$$M_t + B_t + \tau^n_{t-1}W_{t-1}n_{t-1}\bar{h} = M_{t-1} + R_{t-1}B_{t-1} + P_{t-1}g_{t-1}. \tag{12}$$

Thus, the government finances its spending through labor income taxation, issuance of nominal debt, and money creation. Note that government consumption is a credit good, following CCK, because g_{t-1} is not paid for until period t. In equilibrium, the government budget constraint can be expressed in real terms as

$$c_{1t}\pi_t + b_t\pi_t + \tau^n_{t-1}w_{t-1}n_{t-1}\bar{h} = c_{1t-1} + \frac{u_{1t-1}}{u_{2t-1}}b_{t-1} + g_{t-1}, \tag{13}$$

where $\pi_t \equiv P_t/P_{t-1}$ is the gross rate of price inflation.

2.4 Nash Wage Bargaining

As is standard in the literature, we assume that the wage paid in any given job is determined in a Nash bargain between a matched worker and firm. Thus, the wage payment divides the match surplus. Our departure from the standard Nash bargaining convention is that we assume bargaining occurs over the nominal wage payment rather than the real wage payment. With zero costs of wage adjustment, the real wage that emerges is identical to the one that emerges from bargaining directly over the real wage. The reason that nominal bargaining and real bargaining are identical if wage adjustment is costless is straightforward. A firm and worker in negotiations take the price level P as given. Bargaining over W thus pins down w; alternatively, bargaining over w pins down W. With no impediment to adjusting wages, there is no problem adjusting either w or W to achieve some desired split of the surplus, and the optimal split itself is independent of whether a real unit of account or a nominal unit of account is used in bargaining.

In addition to bargaining over nominal wages, though, we assume that nominal wage adjustment may entail a resource cost of the Rotemberg-type described in Section 2.2. Details of the solution of the Nash bargain with costly wage adjustment are given in Appendix A. Here we present only the outcome of the Nash bargain. Bargaining over the nominal wage payment yields

$$\begin{aligned}
&\frac{\omega_t}{1-\omega_t}\left[z_t f(\bar{h}) - w_t\bar{h} - \frac{\psi}{2}\left(\pi_t^w - 1\right)^2 + \frac{\gamma}{k^f(\theta_t)}\right] = \\
&\quad (1-\tau_t^n)w_t\bar{h} + \frac{A\bar{h}}{u_{2t}} - \frac{v}{u_{2t}} \\
&\quad + (1-\theta_t k^f(\theta_t))\beta E_t\left[\left(\frac{\omega_{t+1}}{1-\omega_{t+1}}\right)\left(\frac{u_{2t+1}}{u_{2t}}\right)(1-\rho^x)\left[z_{t+1}f(\bar{h}) - w_{t+1}\bar{h} - \frac{\psi}{2}\left(\pi_{t+1}^w - 1\right)^2 + \frac{\gamma}{k^f(\theta_{t+1})}\right]\right],
\end{aligned} \tag{14}$$

which characterizes the real wage w_t agreed upon in period t. In (14), ω_t is the effective bargaining power of the worker and $1-\omega_t$ is the effective bargaining power of the firm. Specifically,

$$\omega_t \equiv \frac{\eta}{\eta + (1-\eta)\Delta_t^F/\Delta_t^W}, \tag{15}$$

where Δ_t^F and Δ_t^W measure marginal changes in the value of a filled job and the value of being employed, respectively, and η is the weight given to the worker's individual surplus in Nash bargaining.[5]

Details behind (14) and (15) are provided in Appendix A, but there are three points worth mentioning here. First, effective bargaining power ω_t is related to, but may differ from, the Nash weight η. With flexible nominal wages and no labor taxation, it is straightforward to show that $\omega_t = \eta\ \forall t$ (because in that case $\Delta_t^F/\Delta_t^W = 1$). However, the presence of proportional taxes and sticky wages drives a wedge between η and ω both in the steady state and dynamically. We refer to this time-varying wedge as a *dynamic bargaining power effect* and find it useful for thinking especially about the (lack of) tax-smoothing aspect of optimal policy in our model. Second, the expected future cost of adjusting the nominal wage affects the time-t wage payment. Third, the labor tax rate appears in (14) both directly as well as through effective bargaining power. The weight ω_t depends on τ_t^n; thus the weight ω_{t+1}, which affects the time-t split of the surplus, depends on τ_{t+1}^n. Indeed, as can also be seen in our Appendix A, if wages are not at all sticky ($\psi = 0$), the

[5]Our notation surrounding the time-varying bargaining weights is adapted from Gertler and Trigari (2006).

bargaining weight varies only because of variations in the tax rate,

$$\omega_t = \frac{\eta}{\eta + (1-\eta)\frac{1}{1-\tau_t^n}}. \tag{16}$$

2.5 Matching Technology

Matches between unemployed individuals searching for jobs and firms searching to fill vacancies are formed according to a constant-returns matching technology, $m(u_t, v_t)$, where u_t is the number of searching individuals and v_t is the number of posted vacancies. A match formed in period t will produce in period $t+1$ provided it survives exogenous separation at the beginning of period $t+1$. The evolution of aggregate employment is thus given by

$$n_{t+1} = (1-\rho^x)(n_t + m(u_t, v_t)). \tag{17}$$

2.6 Private-Sector Equilibrium

The equilibrium conditions of the model are the Fisher equation (5) describing the household's optimal intertemporal choices; the household intratemporal optimality condition (6), which is standard in cash/credit models; the restriction $R_t \geq 1$, which states that the net nominal interest rate cannot be less than zero, a requirement for a monetary equilibrium; the job-creation condition arising from firm profit-maximization

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left[\left(\frac{\beta u_{2t+1}}{u_{2t}}\right)(1-\rho^x)\left(z_{t+1}f(\bar{h}) - w_{t+1}\bar{h} - \frac{\psi}{2}\left(\pi_{t+1}^w - 1\right)^2 + \frac{\gamma}{k^f(\theta_{t+1})}\right)\right], \tag{18}$$

in which the household discount factor for credit resources, $\beta u_{2t+1}/u_{2t}$, appears; the flow government budget constraint, expressed in real terms, (13) (in which we have substituted $R_{t-1} = u_{1t-1}/u_{2t-1}$ from (6) as well as the cash-in-advance constraint (4) holding with equality); the Nash wage characterized by (14); the law of motion for aggregate employment (17); the identity

$$n_t + u_t = 1 \tag{19}$$

restricting the size of the labor force to measure one; a condition relating the rate of real wage growth to nominal price inflation and nominal wage inflation

$$\frac{\pi_t^w}{\pi_t} = \frac{w_t}{w_{t-1}}; \tag{20}$$

and the resource constraint

$$c_{1t} + c_{2t} + g_t + \gamma u_t \theta_t + \frac{\psi}{2}\left(\pi_t^w - 1\right)^2 = n_t z_t f(\bar{h}). \tag{21}$$

Condition (20) is typically thought of as an identity, but is one that does not hold trivially in a model with nominally-rigid wages and thus must be included as part of the description of equilibrium;

see Chugh (2006, p. 692) for an intuitive explanation.[6] In (21), total costs of posting vacancies $\gamma u_t \theta_t$ are a resource cost for the economy, as are wage adjustment costs; in the resource constraint, we have made the substitution $v_t = u_t \theta_t$, eliminating v_t from the set of endogenous processes of the model. The private-sector equilibrium processes are thus $\{c_{1t}, c_{2t}, n_{t+1}, u_t, \theta_t, w_t, \pi_t, \pi_t^w, b_t\}$, for given processes $\{z_t, g_t, \tau_t^n, R_t\}$.

3 Ramsey Problem in Basic Model

The problem of the Ramsey planner is to raise exogenous revenue for the government through labor income taxes and money creation in such a way that maximizes the welfare of the representative household, subject to the equilibrium conditions of the economy. In period zero, the Ramsey planner commits to a policy rule. Because of the complexity of the model, we cast the Ramsey problem as one of choosing both allocation and policy variables rather than in the pure primal form often used in the literature, in which it is just allocations that are chosen directly by the Ramsey planner. The Ramsey problem is to choose $\{c_{1t}, c_{2t}, n_{t+1}, u_t, \theta_t, w_t, \pi_t, \pi_t^w, b_t, \tau_t^n\}$ to maximize (2) subject to (5), (13), (14), (17), (18), (19), (20), and (21) and taking as given exogenous processes $\{z_t, g_t\}$. In principle, we must also impose the inequality condition

$$u_1(c_{1t}, c_{2t}) - u_2(c_{1t}, c_{2t}) \geq 1 \tag{22}$$

as a constraint on the Ramsey problem. This inequality constraint ensures (in terms of allocations — refer to condition (6)) that the zero-lower-bound on the nominal interest rate is not violated. We thus refer to constraint (22) as the ZLB constraint. The ZLB constraint in general is an occasionally-binding constraint.

Because our model likely is too complex, given current technology, to solve using global approximation methods that would be able to properly handle occasionally-binding constraints, for our dynamic results we drop the ZLB constraint and then check whether it is ever violated during simulations. As we discuss when we present our parameterization in Section 4.1, using this approach raises an issue for one aspect of our model calibration. Finally, throughout, we assume that the first-order conditions of the Ramsey problem are necessary and sufficient and that all allocations are interior.

[6] In models with sticky nominal wages, condition (20) can be thought of as a law of motion for real wages. For example, Erceg, Henderson, and Levin (2000) and Schmitt-Grohe and Uribe (2005) also impose such a constraint in their models of optimal policy with sticky nominal wages. Briefly, the monetary forces acting on the nominal price-inflation rate and the wage-setting forces (in our model, wage bargaining) acting on the nominal wage-inflation rate in general will not be consistent with the equilibrium evolution of real wages, hence the need to explicitly include this condition as a description of equilibrium.

4 Quantitative Results

We characterize the Ramsey steady-state of our model numerically. In computing the deterministic steady state, the ZLB poses no problem because we can numerically solve the fully-nonlinear Ramsey first-order conditions. Before turning to our results, we describe how we parameterize the model. Because a number of our steady-state results have a close analog in the optimal capital taxation results of Arseneau and Chugh (2006), we adopt, where possible, their calibration to enhance comparability.

4.1 Model Parameterization

We assume that the instantaneous utility function over cash and credit goods is

$$u\left(c_{1t}, c_{2t}\right) = \frac{\left\{\left[(1-\kappa)c_{1t}^{\phi} + \kappa c_{2t}^{\phi}\right]^{1/\phi}\right\}^{1-\sigma} - 1}{1-\sigma}, \tag{23}$$

with, as is typical in cash/credit models, a CES aggregator over cash and credit goods. For the aggregator, we adopt the calibration used by Siu (2004) and Chugh (2006, 2007) and set $\kappa = 0.62$ and $\phi = 0.79$. The time unit of the model is meant to be a quarter, so we set the subjective discount factor to $\beta = 0.99$, yielding an annual real interest rate of about four percent. We set the curvature parameter with respect to consumption to $\sigma = 1$, consistent with many macro models.

Our timing assumptions are such that production in a period occurs after the realization of separations. Following the convention in the literature, we suppose that the unemployment rate is measured *before* the realization of separations. We set the quarterly probability of separation at $\rho^x = 0.10$, consistent with Shimer (2005). Thus, letting n denote the steady-state level of employment, $n(1-\rho^x)^{-1}$ is the employment rate, and $1 - n(1-\rho^x)^{-1}$ is the steady-state unemployment rate.

The match-level production function in general displays diminishing returns in labor,

$$f(\bar{h}) = \bar{h}^{\alpha}, \tag{24}$$

and we set the fixed number of hours a given individual works to $\bar{h} = 0.35$, making our baseline model comparable to our expanded model in Arseneau and Chugh (2007a). In the richer model, we allow intensive labor adjustment and calibrate utility parameters so that steady-state hours are $h = 0.35$. Thus, we set $\bar{h} = 0.35$ here. Regarding curvature, we choose $\alpha = 0.70$, a conventional value in DSGE models.

Also as in much of the literature, the matching technology is Cobb-Douglas,

$$m(u_t, v_t) = \psi^m u_t^{\xi_u} v_t^{1-\xi_u}, \tag{25}$$

with the elasticity of matches with respect to the number of unemployed set to $\xi_u = 0.40$, following Blanchard and Diamond (1989), and ψ^m a calibrating parameter that can be interpreted as a measure of matching efficiency.

We normalize the utility of non-work to $v = 0$. With this normalization, our choice of a specific value of A is guided by Shimer (2005), who calibrates his model so that unemployed individuals receive, in the form of unemployment benefits, about 40 percent of the wages of employed individuals. With his linear utility assumption, unemployed individuals are therefore 40 percent as well off as employed persons. Our model differs from Shimer's (2005) primarily in that we assume full consumption insurance, but also in that we have curvature in utility. Thus, in the context of our model, we interpret Shimer's (2005) calibration to mean that unemployed individuals must receive 2.5 times more consumption of both cash goods and credit goods (in steady-state) than employed individuals in order for the total utility of the two types of individuals to be equalized. That is, we set A such that in steady-state

$$u\left(2.5\bar{c_1}, 2.5\bar{c_2}\right) + v = u\left(\bar{c_1}, \bar{c_2}\right) + A\bar{h}, \tag{26}$$

where $\bar{c_j}$ denotes steady-state consumption, $j = 1, 2$. The resulting value is $A = 2.6$, but we point out that our qualitative results do not depend on the exact value of A.

Regarding the Nash bargaining weight η, we focus on the case $\eta = \xi_u = 0.40$ so that the usual Hosios (1990) parameterization for efficiency in job creation is satisfied. The Nash bargaining weight being a relatively esoteric parameter, it is hard to say whether such a parameterization is empirically-justified. Nonetheless, it is a parameterization of interest because many results in the quantitative labor search literature are obtained assuming it. We thus present our primary steady-state and dynamic results using $\eta = 0.40$, although we do explore some sensitivity.

We adopt Chugh's (2006) calibration strategy for the cost-adjustment parameter ψ and consider four different values for our main results: $\psi = 0$ (flexible wages), $\psi = 1.98$ (nominal wages sticky for two quarters on average), $\psi = 5.88$ (nominal wages sticky for three quarters on average), and $\psi = 9.61$ (nominal wages sticky for four quarters on average). We recognize that Chugh's (2006) mapping of duration of wage-stickiness to the cost-adjustment parameter may need to be modified because we have a fundamentally different model, but we think it is a useful starting point and allows us to demonstrate our main points. We leave an empirical investigation of a "wage Phillips curve" in the presence of labor search frictions to future work.

Finally, the exogenous productivity and government spending shocks follow AR(1) processes in logs,

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_t^z, \tag{27}$$

$$\ln g_t = (1 - \rho_g) \ln \bar{g} + \rho_g \ln g_{t-1} + \epsilon_t^g, \tag{28}$$

where $\bar{g}$ denotes the steady-state level of government spending, which we calibrate in our baseline model to constitute 18 percent of steady-state output in the Ramsey allocation. The resulting value is $\bar{g} = 0.07$. The innovations ϵ_t^z and ϵ_t^g are distributed $N(0, \sigma_{\epsilon^z}^2)$ and $N(0, \sigma_{\epsilon^g}^2)$, respectively, and are independent of each other. We choose parameters $\rho_z = 0.95$, $\rho_g = 0.97$, $\sigma_{\epsilon^z} = 0.006$, and $\sigma_{\epsilon^g} = 0.03$, consistent with the RBC literature and CCK. Also regarding policy, we assume that the steady-state government debt-to-GDP ratio (at an annual frequency) is 0.4, in line with evidence for the U.S. economy and with the calibrations of Schmitt-Grohe and Uribe (2004b) and Siu (2004).

4.2 Ramsey Steady State

We begin by analyzing the long-run Ramsey equilibrium. Table 1 presents steady-state allocations and policy variables under the Ramsey plan for various values of ψ, as well as the socially-efficient allocations. Starting with the benchmark case in which wage adjustment is costless, $\psi = 0$, the top row of the table shows that optimal policy in the long run features the Friedman Rule of a zero net nominal interest rate, leaving government expenditure to be financed completely via the labor income tax. This policy prescription echoes that from any standard Ramsey model for essentially identical reasons. In absence of wage adjustment costs, the optimal policy mix trades off the wedge in households' consumption-leisure margin due to labor income taxation against the monetary distortion due to the inflation tax. The tradeoff is resolved completely in favor of eliminating the monetary distortion, hence the optimality of the Friedman Rule, just as in CCK.

A comparison of the Ramsey steady state in absence of adjustment costs to the socially-efficient allocation (shown in the last row of Table 1) reveals an interesting feature of our model. Despite the fact that the Hosios parameterization ($\eta = \xi_u$) is in place in our model, there is inefficient job creation in the Ramsey equilibrium. While this may seem puzzling in light of the well-known Hosios (1990) result, the source of the inefficiency can be traced to the wedge between the Nash bargaining weight η and effective bargaining power ω, which in our model ultimately governs the (after-tax) share of the labor surplus accruing to workers. This wedge, which is induced by the labor income tax, is described by condition (16). Because $\tau^n > 0$ in the Ramsey equilibrium, we thus have $\omega < \eta = \xi_u$, and the match surplus is not divided in the Hosios-efficient manner despite the typical Hosios parameterization being in place. Because workers' effective share of labor surpluses is too low from the point of view of social efficiency, job creation by firms is inefficiently high. Arseneau and Chugh (2006), who focus on optimal capital taxation, find similar distortionary effects of labor income taxation on job creation.

Moving to the case of costly wage adjustment, $\psi > 0$, the Friedman Rule ceases to be optimal, as shown in Figure 1. In the face of nominal rigidities, the optimal inflation rate trades off the usual monetary distortion (which, in isolation, calls for the Friedman Rule) against distortions stemming directly from the nominal rigidity (which, in isolation, calls for zero inflation). For even very small costs of wage adjustment, this tension is resolved largely in favor of minimizing distortions arising from nominal rigidities, putting the optimal long-run inflation rate in the neighborhood of price stability, a result again consistent with standard Ramsey result. Thus, one may think that labor search and matching frictions in and of themselves change standard long-run Ramsey prescriptions very little.

This conclusion would be premature, however, because a unique aspect of our results emerges as ψ gets sufficiently large. For large enough costs of nominal wage adjustment (in our calibration, between two and three quarters of wage rigidity on average), the optimal rate of inflation rises *above* zero, rising asymptotically to about 0.6 percent in Figure 1. Expression (14) (the details of which are provided in Appendix A) shows that, with $\psi > 0$, long-run inflation interacts with the labor income tax in driving a wedge between ω and η. Thus, in addition to the standard monetary distortion and the distortion stemming from costs of nominal adjustment, the optimal inflation rate must now also take into consideration this novel policy-induced wedge in the job-creation margin.

With three distortions being weighed against each other, the long-run inflation tax thus indirectly influences and is influenced by labor-market outcomes in our model. A natural conjecture is that positive inflation is standing in for a tax instrument that operates more directly on the labor market. In the expanded version of our work, Arseneau and Chugh (2007a), we verify this by introducing a direct proportional tax on vacancy creation by firms. With this additional instrument, we find that the optimal vacancy tax is positive, and the standard tradeoff only between minimizing the monetary distortion and minimizing the sticky-wage distortion is reinstated (and again resolved overwhelmingly in favor of minimizing the latter). That the inflation tax can be used as a proxy for a missing instrument is well-known in the Ramsey literature. Cooley and Quadrini (2004) and Faia (2007), for example, also obtain this result, albeit in models abstracting from distortionary taxation.

In terms of long-run policy, then, a crucial difference between our results and those of Faia (2007) is the source of any long-run deviation from Hosios efficiency. Our results differ due to the wedge in our model between effective bargaining power and the Nash bargaining weight, an issue that does not arise in Faia (2007). The Hosios parameterization being a very special case, it is thus also useful to know how the Ramsey steady state varies with changes in η. Figure 2 displays some comparative statics of Ramsey policy variables along this dimension under the assumption of zero costs of wage adjustment ($\psi = 0$). As the left column of Figure 2 reveals, optimal policy can be quite sensitive to η, which is why we limit attention in the right column of Figure 2 to a quite narrow range, $\eta \in [0.35, 0.45]$, around our baseline calibration $\eta = 0.40$. Particularly useful to note is that the net nominal interest rate rises above zero for $\eta > \xi_u = 0.40$, which will be helpful in understanding an issue that arises in our dynamic results.

4.3 Ramsey Dynamics

To study dynamics, we approximate our model by linearizing in levels the Ramsey first-order conditions for time $t > 0$ around the non-stochastic steady-state of these conditions. Our numerical method is our own implementation of the perturbation algorithm described by Schmitt-Grohe and Uribe (2004c). As in Khan, King, and Wolman (2003) and others, we assume that the initial state of the economy is the asymptotic Ramsey steady state.[7] Throughout, we assume, as is common in the literature, that the first-order conditions of the Ramsey problem are necessary and sufficient and that all allocations are interior. We also point out that because we assume full commitment on the part of the Ramsey planner, the use of state-contingent inflation is not a manifestation of time-inconsistent policy.[8] The "surprise" in surprise inflation is due solely to the unpredictable components of government spending and technology and not due to a retreat on past promises.

4.3.1 Computational Issues

As we mentioned above, we drop the ZLB constraint when computing first-order-accurate equilibrium decision rules. There are two main issues that arise by doing so, one conceptual and one technical. The conceptual issue that arises is that the true equilibrium decision rules of the economy of course do take into consideration that the ZLB *sometimes* (i.e., for some region of the state space) will bind, whereas decision rules computed by ignoring the restriction of course do not factor in this risk.[9] The technical issue that arises is that for an economy sufficiently close to the zero lower bound on average, even business-cycle magnitude shocks would be expected to cause the economy to pierce the ZLB, thus technically rendering the equilibrium a non-monetary one.

Our strategy, which is a commonly-employed one, is to drop the ZLB constraint and then check in our simulated economies how often the ZLB is violated. Although this does not address the

[7] Doing so achieves analysis of policy dynamics from what is commonly referred to as the "timeless" perspective, which captures the idea that the optimal policy has already been operational for a long time. The analysis in Thomas (2007), for example, is also from the timeless perspective, as are all Ramsey monetary models descending from Lucas and Stokey (1983) and CCK of which we are aware.

[8] The problem of time-inconsistency of policy would itself be an interesting one to study in this model because, even though our model does not include capital, employment is a pre-determined stock variable in any given period. Hence, one might imagine that the Ramsey planner may find it optimal to tax the initial employment stock, which a very large labor tax rate may be able to achieve. We intentionally sidestep this issue by assuming, as is common in Ramsey analysis, the existence of a sufficiently-strong commitment mechanism. Our adoption of the timeless perspective of course does not obviate thinking about time-consistency issues.

[9] By framing the issue at hand in terms of the *risk* that the ZLB may bind, the problem can be thought of as the appropriateness of invoking certainty equivalence.

problem of ignoring the risk associated with the mere presence of the ZLB constraint, we found that the ZLB was violated only in simulations of the flexible-wage version ($\psi = 0$) of our economy and never in any of our cases with $\psi > 0$. For our flexible-wage model, the ZLB was violated 48 percent of the time, certainly not negligible. We can remedy this, however, by slightly increasing the Nash bargaining weight from our baseline $\eta = 0.40$ to $\eta = 0.44$; doing so makes the optimal nominal interest rate slightly positive assuming $\psi = 0$ (i.e., the Friedman Rule is no longer optimal, as can be seen from Figure 2). Solving and simulating the costless-wage-adjustment version of our model with this alternative setting for η, we find that the ZLB is never violated dynamically and the cyclical properties of policy and quantity variables are virtually identical to those presented in Table 2. Thus, conceding that we cannot handle the *risk* that the ZLB may bind in our computed decision rules, the interpretation that emerges from our results does not hinge on properly handling the ZLB constraint. To avoid yet another fundamental distortion in our model, though, we chose to report results for just the $\eta = 0.40$ case.[10] In any case, the ZLB is never violated during simulations of any of our models with $\psi > 0$.

With these caveats in mind and our first-order accurate equilibrium decision rules computed in this way, we conduct 5000 simulations, each 100 periods long. To make the comparisons meaningful as we vary ψ, the same realizations for government spending shocks and productivity shocks are used across versions of our model. We limit the length of each repetition because it turns out the Ramsey equilibrium features a near-unit root in real government debt and thus we must prevent the model from wandering too far from initial conditions. For each simulation, we then compute first and second moments and report the medians of these moments across the 5000 simulations. By averaging over so many short-length simulations, we are likely obtaining a fairly accurate description of model dynamics even if a handful of simulations drift far away from the steady state.

4.3.2 Policy Dynamics

Table 2 presents simulation-based moments for the key policy variables of our model for various degrees of nominal wage rigidity. The top panel of Table 2 shows that if nominal wages are costless to adjust, the average level of inflation is near the Friedman deflation (consistent with our steady-state results) and price inflation volatility, at about 4.5 percent annualized, is quite high. The high volatility of inflation, well-known in Ramsey models since CCK, is due to the fact that (large) state-contingent variations in inflation render nominally risk-free debt payments state-contingent

[10] A very similar issue arises in Cooley and Quadrini (2004). When studying the dynamics of their model, in order to ensure that the simulations are bounded away from the zero lower bound, they alter the Nash bargaining weight from the Hosios condition as well as introduce a second source of inefficiency. They report (p. 188), however, that the introduction of these features just induce a level shift of variables without altering the basic cyclical properties of the model; the same is true in our model.

in real terms, thereby financing a large portion of innovations to the government budget in a non-distortionary way. Thus, search and matching frictions in the labor market in and of themselves do nothing to overturn this benchmark result. With flexible wages, nominal wage inflation is also quite volatile. Coupled with volatile price inflation, the path for the real wage turns out to be relatively stable, with a standard deviation of about 1 percent, much lower than the volatility of output, which has a standard deviation of about 1.8 percent.[11]

If nominal wages are instead costly to adjust, nominal wage inflation is near zero with very low variability, as the second, third, and fourth panels of Table 2 show. However — and this is the central result of our study — optimal *price* inflation volatility remains quite high in the presence of costs of nominal wage rigidity. With two or three quarters of nominally-rigid wages on average, we find that price inflation volatility is still around five percent, little changed from the fully-flexible case. With four quarters of wage stickiness, inflation volatility is actually higher than in the fully-flexible case. These results are directly opposite the results in Chugh (2006), who finds that even just two quarters of nominal wage rigidity (modeled through an identical Rotemberg-adjustment-cost specification) lowers price inflation volatility by an order of magnitude.

Intuitively, the reason behind low and stable nominal wage inflation if $\psi > 0$ is easy to understand: the Ramsey planner largely eliminates the direct resource cost associated with changes in nominal wages. Absent direct resource costs stemming from nominal price changes, however, the tradeoff facing the Ramsey planner in setting a state-contingent price inflation rate is the welfare loss due to any induced volatility in the *real* wage versus the welfare gain due to the usual CCK-type of shock absorption afforded by state-contingent inflation. In our model, the tradeoff is quantitatively resolved in favor of high inflation volatility. An attendant consequence is that real wages become volatile. As Table 2 shows, real wage volatility indeed rises as ψ rises, by about 50 percent moving from the flexible-wage case to the case of three quarters of wage rigidity.

Volatility in real wages, however, is not very undesirable from the Ramsey point of view in our model because the real wage plays a largely distributive role. In a basic labor search and matching framework, which is what we employ in our model, the real wage divides the economic rents generated by the formation of a match between a worker and a firm. Real wages are not the critical signal governing formation of matches and hence production.[12] As such, the Ramsey planner, concerned with allocations and not distributions, is not compelled to "stabilize" real wages over time. This is in direct contrast to the mechanism underlying the results in Chugh (2006): there, real wage volatility is undesirable because the labor market is standard neoclassical, meaning that

[11] These standard deviations in percentage terms are simply equal to the raw standard deviations presented in Table 2 divided by the means. We could have equivalently computed the standard deviation of the logged variables.

[12] Rather, as is well-understood in search and matching models, it is aggregate market tightness — our variable θ — that is typically the most important variable governing efficiency.

the real wage *is* the key allocational signal. Thus, in Chugh (2006), costs of nominal wage changes translate into a desire for stabilizing nominal prices out of a concern for inducing a real wage path close to the efficient one. In our model, loosely speaking, there is no notion of an efficient path for real wages. Thus emerges a central conclusion of our study: it clearly matters for prescriptions regarding optimal inflation in what type of underlying environment — a Walrasian labor market or a labor market with fundamental frictions — nominal wage rigidity is modeled.

Another important dimension along which policy dynamics differ sharply between our model and a basic Ramsey model is tax-rate dynamics. As Table 2 shows, our model displays tax-rate variability that is an order of magnitude larger than in a basic Ramsey model, regardless of whether or not nominal wages are costly to adjust.[13] Our intuition regarding why tax-rate variability is not as undesirable in our model as in a basic Ramsey model stems from a *dynamic bargaining power effect* that links fluctuations in τ_t^n to fluctuations in workers' effective bargaining power ω_t. It is easiest to understand the mechanism for the case $\psi = 0$. Recall from expression (16) how ω_t and τ_t^n are linked dynamically: in any period in which τ_t^n is high, ω_t is low, and this relationship is linear. The top left panel of Figure 3, which scatters dynamic realizations of ω_t and τ_t^n from a representative simulation of our model, confirms this. Thus, variations in the labor tax rate cause variations in parties' effective bargaining shares, but this has only distributive consequences, not allocative consequences. The basic intuition behind tax-rate variability in our model is therefore the same as the basic intuition behind inflation variability. With $\psi > 0$, high tax-rate variability remains intact, and, as shown in Figure 3, the cyclical correlation between τ_t^n and ω_t is still strongly negative.

Because the labor search model is so well-suited to thinking about issues regarding unemployment, one may wonder whether a Phillips Curve arises in our model. Figure 4 shows a negative relationship between cyclical inflation rates and cylical unemployment rates does arise if wages are flexible. However, this Phillips relation is not a feature of optimal policy with sticky nominal wages.

Finally, we do not report our model's predictions regarding the volatility of unemployment, vacancies, and labor market tightness, a topic that has received much attention since Shimer (2005) and Hall (2005). We provide a more thorough analysis of this aspect of our model in Arseneau and Chugh (2007a), but the upshot is that, although the volatility of all three variables increase slightly as the costs of nominal wage adjustment increase, our model fails to make progress on this research front, but that was not our intention in any case. A more sweeping read of our results is that taking into account optimal policy setting generally speaking may not offer any breakthroughs on this issue.

[13]Tax rate volatility does not arise because our overall model is excessively volatile: as we noted above and as can be seen in Table 2, the coefficient of variation of total output is about 1.8 percent, in line with empirical evidence and with basic Ramsey models.

To summarize our results on optimal stabilization policy, neither inflation variability nor tax-rate variability creates quantitatively-important distortions in our model because the variations in realized (after-tax) real wages that they induce are largely isolated from determination of quantities. Our results suggest that if the realized real wage did affect allocations more directly than they do in the model we have developed, then the optimal degree of price inflation volatility may fall as the cost of nominal wage adjustment rises. In the expanded version of our work, Arseneau and Chugh (2007a), we pursue this idea by introducing an intensive margin of labor adjustment that potentially is affected by the realized real wage in a similar manner as in a standard neoclassical model. The broad result is that our inflation volatility result is robust to the introduction of an intensive margin, although the precise quantitative results can depend on the details of the intensive-determination mechanism.

5 Conclusion

The goal of our work here was to explore the implications of nominally-rigid wages on optimal policy in a model featuring explicit bilateral relationships between workers and firms. The results turn out to be quite different than in models with nominal rigidities in wages modeled in otherwise-Walrasian labor markets. In our model, realized real wages play primarily a distributive role and are not as critical for efficiency as they are in a labor market with standard neoclassical underpinnings. Thus, although unanticipated fluctuations in inflation and the labor income tax rate cause unanticipated fluctuations in (after-tax) real wages, job formation and production are largely unaffected. Our results give quantitative voice to the conjecture, based on Barro's (1979) critique, that sticky nominal wages ought not to have much consequence for optimal monetary policy because firms and workers engaged in ongoing relationships have the proper incentives to neutralize any allocative effects.

This paper is also part of a larger project studying the policy implications of deep-rooted, non-Walrasian frictions in goods markets, money markets, and labor markets. A central focus of this larger project has been to think about what sorts of departures from typical Walrasian frameworks make consumer price inflation stability an important goal of policy, but along the way we have uncovered other aspects of policy not evident in standard models. In this paper, we characterized optimal policy when labor markets are non-Walrasian but goods markets and money markets are standard. Aruoba and Chugh (2006) characterized optimal policy when money markets are non-Walrasian but labor markets and goods markets are standard. Arseneau and Chugh (2007b) characterize optimal policy when goods markets are non-Walrasian but labor markets and money markets are standard. We now turn to studying optimal policy when multiple markets feature fundamental trading frictions.

Type of allocation	Wage Rigidity	$R-1$	$\pi-1$	τ^n	gdp	N	v	θ
Ramsey	None	0	-3.9404	0.2350	0.4195	0.8748	0.1649	1.3167
Ramsey	Two quarters	4.0649	-0.0357	0.2343	0.4195	0.8748	0.1649	1.3166
Ramsey	Three quarters	4.6049	0.4831	0.2343	0.4196	0.8749	0.1650	1.3195
Ramsey	Four quarters	4.7203	0.5939	0.2343	0.4197	0.8751	0.1652	1.3224
Social Planner	—	0	-3.9404	0	0.4160	0.8674	0.1564	1.1791

Table 1: Steady-state policy and allocations. Inflation rate and interest rate reported in annualized percentage points. For Social Planning problem, implied policy variables constructed residually using equilibrium conditions.

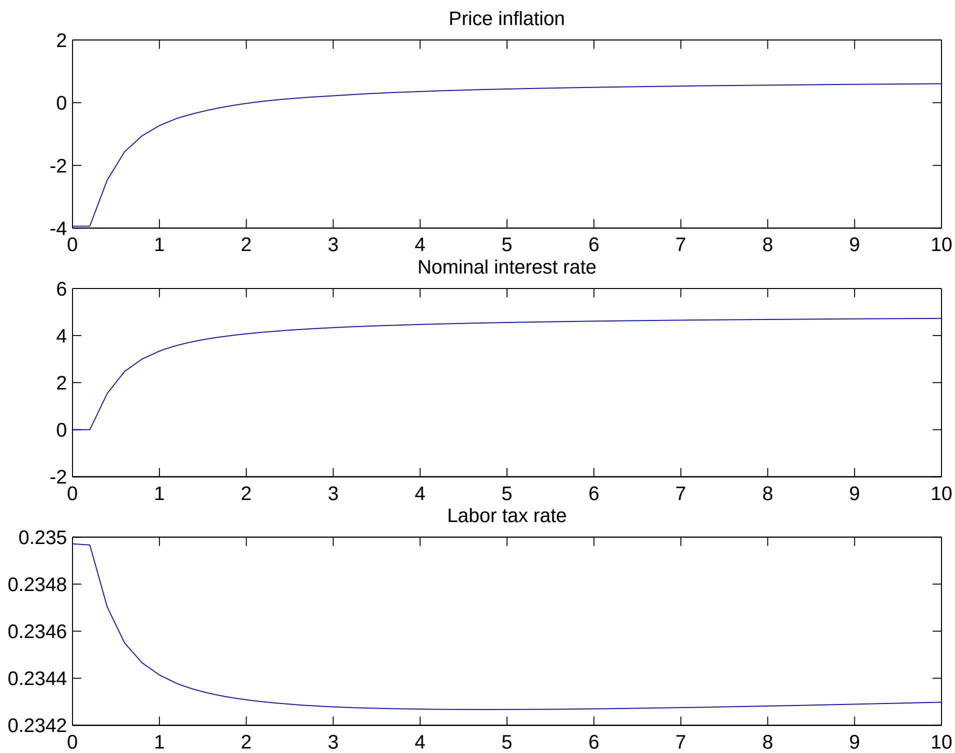

Figure 1: Steady-state Ramsey policy variables as a function of nominal wage adjustment cost parameter ψ. Inflation rate and nominal interest rate expressed in annualized percentage points.

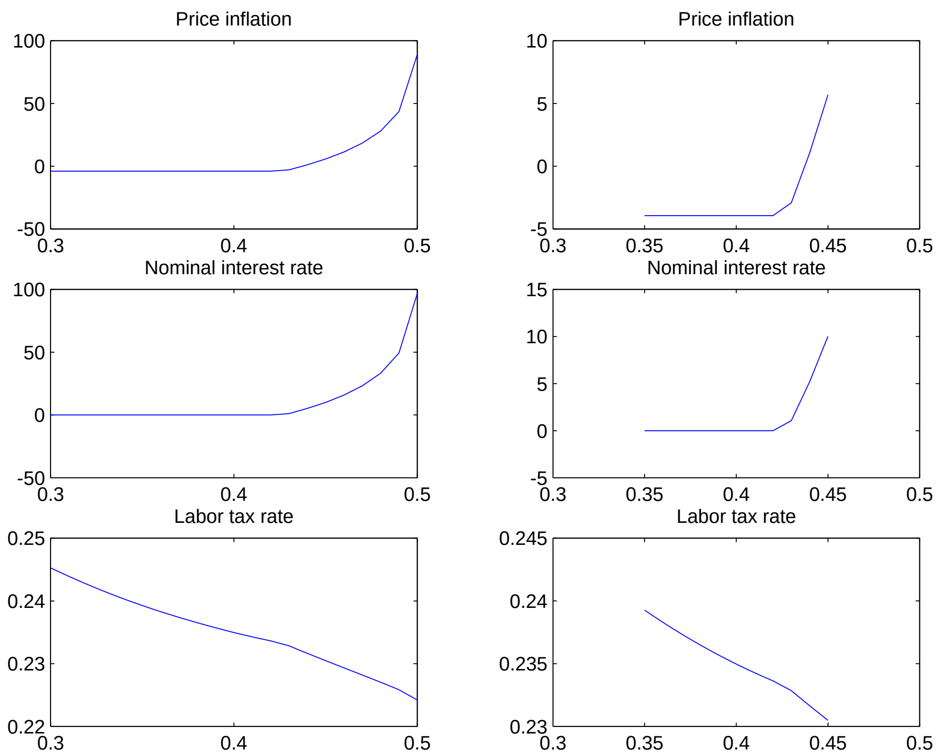

Figure 2: Steady-state Ramsey policy variables as a function of Nash bargaining weight η, with $\psi = 0$ fixed. Inflation rate and nominal interest rate expressed in annualized percentage points. Left column displays results for $\eta \in [0.30, 0.40]$, right column displays results for $\eta \in [0.35, 0.45]$.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, gdp)	Corr(x, z)	Corr(x, g)
			Flexible wages			
τ^n	0.2346	0.0120	0.9793	0.1490	0.1172	0.0677
$\pi - 1$	-3.9173	4.5430	0.6556	0.1590	0.1155	-0.0431
$\pi^w - 1$	-3.9084	4.6534	0.8749	0.8521	0.8353	-0.0149
$^*R - 1$	0.0189	0.1182	0.9932	0.3543	0.3324	-0.1652
gdp	0.4194	0.0078	0.9135	1.0000	0.9969	-0.0421
w	0.9954	0.0096	0.9473	0.9722	0.9727	0.0200
ω	0.3379	0.0049	0.9793	-0.1490	-0.1172	-0.0677
			Two quarters of nominal wage stickiness			
τ^n	0.2341	0.0225	0.9588	0.4916	0.3983	-0.0007
$\pi - 1$	0.0002	4.3283	0.1789	-0.5752	-0.6307	0.0168
$\pi^w - 1$	-0.0787	0.5914	0.9023	-0.1199	-0.1605	0.1259
$R - 1$	4.1206	0.2291	0.9604	-0.7690	-0.7474	0.0813
gdp	0.4194	0.0082	0.9201	1.0000	0.9930	-0.0423
w	0.9954	0.0133	0.6583	0.8578	0.9032	-0.0026
ω	0.3383	0.0105	0.9426	-0.3946	-0.3018	-0.0304
			Three quarters of nominal wage stickiness			
τ^n	0.2337	0.0221	0.9840	0.4649	0.3737	-0.0051
$\pi - 1$	0.5735	5.5750	0.0166	-0.4523	-0.5104	0.0185
$\pi^w - 1$	0.4544	0.5261	0.7514	-0.1762	-0.2374	0.1857
$R - 1$	4.6827	0.2367	0.9538	-0.7464	-0.7180	0.0830
gdp	0.4195	0.0082	0.9193	1.0000	0.9926	-0.0427
w	0.9951	0.0155	0.5203	0.7422	0.8005	-0.0061
ω	0.3381	0.0135	0.9555	-0.2138	-0.1213	-0.0908
			Four quarters of nominal wage stickiness			
τ^n	0.2337	0.0308	0.4773	0.3202	0.2734	-0.0109
$\pi - 1$	1.1630	12.9058	-0.1151	-0.1881	-0.2417	0.0280
$\pi^w - 1$	0.5670	0.8526	0.4385	-0.1215	-0.2031	0.1746
$R - 1$	4.8418	0.3111	0.9158	-0.5682	-0.5171	0.0497
gdp	0.4195	0.0081	0.9182	1.0000	0.9906	-0.0405
w	0.9948	0.0308	0.2897	0.3613	0.4398	-0.0344
ω	0.3379	0.0206	0.9197	-0.0642	0.0407	-0.1497

Table 2: Simulation-based moments. π, π^w, and R reported in annualized percentage points. Asterisk denotes zero-lower-bound is violated during simulations.

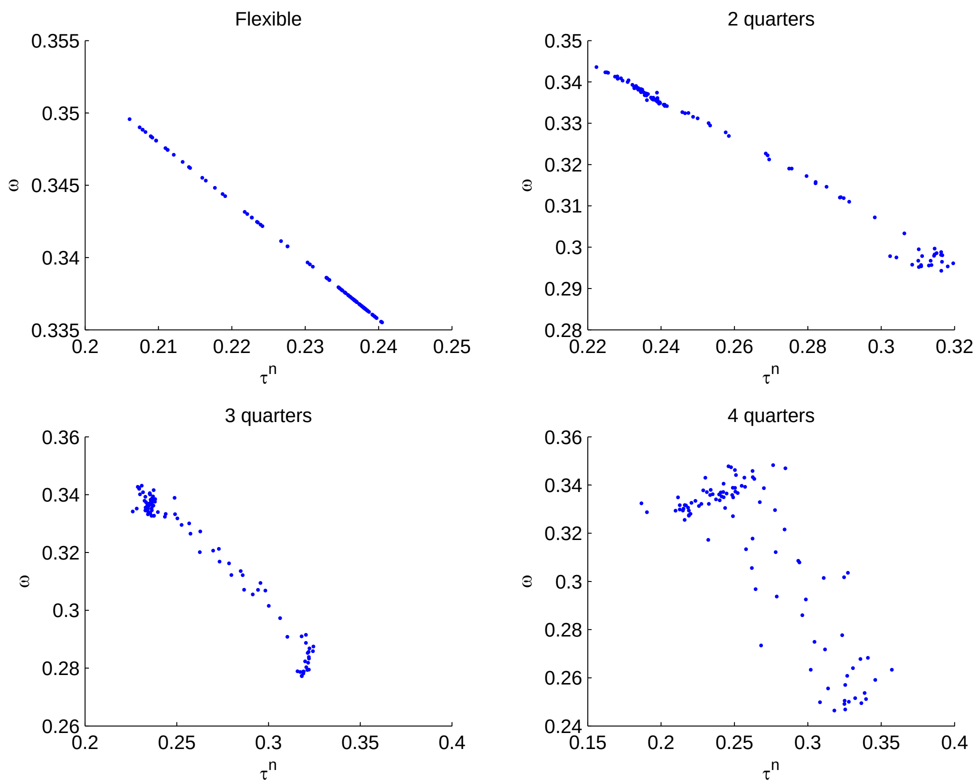

Figure 3: Dynamic relationship between worker's effective bargaining power (ω) and labor tax rate under the Ramsey policy for various degrees of nominal wage rigidity.

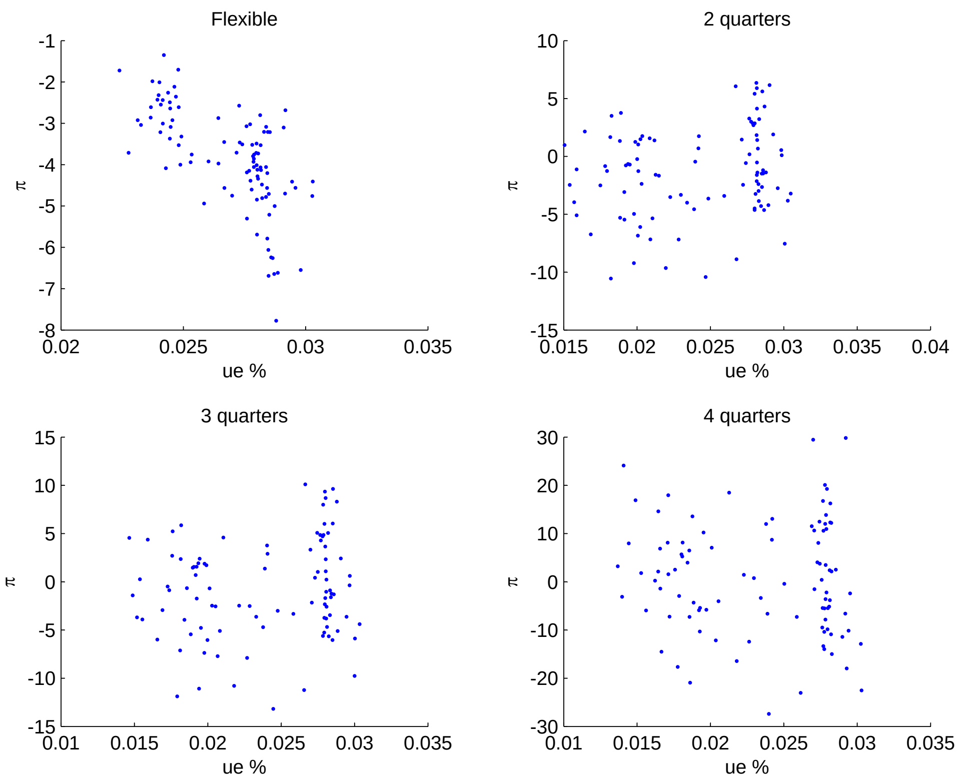

Figure 4: Realizations of inflation and unemployment rate under the Ramsey policy for various degrees of nominal wage rigidity.

A Nash Bargaining Over Wages

Here we derive the Nash-bargaining solution between an individual worker and the firm in the model without an intensive margin. For notational simplicity, we omit the conditional expectations operator E_t where it is understood. Individuals' and firms' asset values are defined in nominal terms. The marginal (nominal) value to the household of an individual who works is

$$\mathbf{W_t} = (1-\tau_t^n)W_t\bar{h} + \frac{P_t A\bar{h}}{u_{2t}} + \beta E_t\left[\left(\frac{u_{2t+1}}{u_{2t}}\right)\left(\frac{P_t}{P_{t+1}}\right)((1-\rho^x)\mathbf{W_{t+1}} + \rho^x \mathbf{U_{t+1}})\right]. \tag{29}$$

The marginal (nominal) value to the household of an individual who is unemployed and searching is

$$\mathbf{U_t} = \frac{P_t v}{u_{2t}} + \beta E_t\left[\left(\frac{u_{2t+1}}{u_{2t}}\right)\left(\frac{P_t}{P_{t+1}}\right)\left(\theta_t k^f(\theta_t)(1-\rho^x)\mathbf{W_{t+1}} + (1-\theta_t k^f(\theta_t)(1-\rho^x))\mathbf{U_{t+1}}\right)\right]. \tag{30}$$

Note that because these asset values are defined as nominal, the nominal discount factor, which involves P_t/P_{t+1}, appears. The value to a firm of a filled job is

$$\mathbf{J_t} = P_t z_t f(\bar{h}) - W_t\bar{h} - \frac{\psi}{2}\left(\frac{W_t}{W_{t-1}} - 1\right)^2 P_t + \beta E_t\left[\left(\frac{u_{2t+1}}{u_{2t}}\right)\left(\frac{P_t}{P_{t+1}}\right)(1-\rho^x)\mathbf{J_{t+1}}\right], \tag{31}$$

where $\frac{\psi}{2}\left(\frac{W_t}{W_{t-1}} - 1\right)^2$ is a Rotemberg-type resource cost of nominal wage adjustment.

Bargaining occurs every period over W_t. The firm and worker maximize the Nash product

$$(\mathbf{W_t} - \mathbf{U_t})^\eta \mathbf{J_t}^{1-\eta}, \tag{32}$$

where $\eta \in (0,1)$ is the fixed weight given to the worker's individual surplus. The first-order condition of the Nash product with respect to W_t is

$$\eta(\mathbf{W_t} - \mathbf{U_t})^{\eta-1}\left(\frac{\partial \mathbf{W_t}}{\partial W_t} - \frac{\partial \mathbf{U_t}}{\partial W_t}\right)\mathbf{J_t}^{1-\eta} + (1-\eta)(\mathbf{W_t} - \mathbf{U_t})^\eta \mathbf{J_t}^{-\eta}\frac{\partial \mathbf{J_t}}{\partial W_t} = 0. \tag{33}$$

We have $\frac{\partial \mathbf{W_t}}{\partial W_t} = (1-\tau_t^n)\bar{h}$, $\frac{\partial \mathbf{U_t}}{\partial W_t} = 0$, and

$$\frac{\partial \mathbf{J_t}}{\partial W_t} = -\bar{h} - \psi\left(\pi_t^w - 1\right)\frac{\pi_t}{w_{t-1}} + \psi(1-\rho^x)\beta E_t\left[\frac{u_{2t+1}}{u_{2t}}\frac{P_t}{P_{t+1}}\left(\pi_{t+1}^w - 1\right)\pi_{t+1}^w\frac{\pi_{t+1}}{w_t}\right]. \tag{34}$$

In computing the latter, we defined $\pi_t^w \equiv W_t/W_{t-1}$ as the gross rate of nominal wage inflation between t and $t+1$ and of course had to take into account that W_t affects $\mathbf{J_{t+1}}$ through the adjustment cost function. A wage Phillips Curve is essentially subsumed inside $\partial\mathbf{J_t}/\partial W_t$.

Define

$$\Delta_t^W \equiv -\left(\frac{\partial \mathbf{W_t}}{\partial W_t} - \frac{\partial \mathbf{U_t}}{\partial W_t}\right), \tag{35}$$

$$\Delta_t^F \equiv \frac{\partial \mathbf{J_t}}{\partial W_t}, \tag{36}$$

and

$$\omega_t \equiv \frac{\eta}{\eta + (1-\eta)\Delta_t^F/\Delta_t^W}. \tag{37}$$

The latter means

$$1-\omega_t = \frac{(1-\eta)\Delta_t^F/\Delta_t^W}{\eta + (1-\eta)\Delta_t^F/\Delta_t^W}. \tag{38}$$

With these definitions, we can write the Nash sharing rule as

$$(1-\omega_t)(\mathbf{W_t} - \mathbf{U_t}) = \omega_t \mathbf{J_t}, \tag{39}$$

which is a generalization of the usual Nash sharing rule.

Using the Bellman equation for the value of a match along with the job-creation condition, $\mathbf{J_t} = P_t f(\bar{h}) - W_t\bar{h} + \frac{P_t\gamma}{k^f(\theta_t)} - \frac{\psi}{2}(\pi_t^w - 1)^2$. Using this as well as the values $\mathbf{W_t}$ and $\mathbf{U_t}$, we can, after some tedious algebra, express the outcome of the Nash bargain as

$$\begin{aligned}
&\frac{\omega_t}{1-\omega_t}\left[P_t z_t f(\bar{h}) - W_t\bar{h} - \frac{\psi}{2}\left(\pi_t^w - 1\right)P_t + \frac{P_t\gamma}{k^f(\theta_t)}\right] = \\
&\quad (1-\tau_t^n)W_t\bar{h} + \frac{P_t A\bar{h}}{u_{2t}} - \frac{P_t v}{u_{2t}} + \\
&\quad (1-\theta_t k^f(\theta_t))\beta E_t\left[\left(\frac{\omega_{t+1}}{1-\omega_{t+1}}\right)\Xi_{t+1|t}(1-\rho^x)\left[P_{t+1}z_{t+1}f(\bar{h}) - W_{t+1}\bar{h} - \frac{\psi}{2}\left(\pi_{t+1}^w - 1\right)P_{t+1} + \frac{P_{t+1}\gamma}{k^f(\theta_{t+1})}\right]\right],
\end{aligned}$$

where $\Xi_{t+1|t} \equiv (u_{2t+1}/u_{2t})/(P_t/P_{t+1})$. Next, divide through by P_t and define the real wage as $w_t \equiv W_t/P_t$ to write the outcome of bargaining as

$$\begin{aligned}
&\frac{\omega_t}{1-\omega_t}\left[z_t f(\bar{h}) - w_t\bar{h} - \frac{\psi}{2}\left(\pi_t^w - 1\right) + \frac{\gamma}{k^f(\theta_t)}\right] = \\
&\quad (1-\tau_t^n)w_t\bar{h} + \frac{A\bar{h}}{u_{2t}} - \frac{v}{u_{2t}} + \\
&\quad (1-\theta_t k^f(\theta_t))\beta E_t\left[\left(\frac{\omega_{t+1}}{1-\omega_{t+1}}\right)\left(\frac{u_{2t+1}}{u_{2t}}\right)(1-\rho^x)\left[z_{t+1}f(\bar{h}) - w_{t+1}\bar{h} - \frac{\psi}{2}\left(\pi_{t+1}^w - 1\right) + \frac{\gamma}{k^f(\theta_{t+1})}\right]\right],
\end{aligned}$$

which is expression (14) in the text.

Note that if $\psi = 0$, then $\partial\mathbf{J_t}/\partial W_t = -\bar{h}$, hence

$$\omega_t = \frac{\eta}{\eta + (1-\eta)\frac{1}{1-\tau_t^n}}, \tag{40}$$

so that it is only fluctuations in the labor tax rate that drive fluctuations between ω_t and η.

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