Prices and Money after Interest Rate Shocks with Endogenous Market Segmentation

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Abstract

I obtain a slow response of prices and money and a decrease in the quantity of money after interest rate shocks. I study two shocks: a permanent and a temporary increase in the interest rate. Prices, nominal and real balances adapt slowly to the shocks. I obtain the short and long run behavior of prices and money in line with the empirical evidence with the same model. I calibrate the model with U.S. data. Agents decide the time to exchange bonds for money: markets are endogenously segmented. The model with fixed segmentation is not able to generate decreasing nominal and real balances after the shocks. The framework is a general equilibrium Baumol-Tobin model with focus on the transition.

JEL classification: E3, E4, E5.

Keywords: price level, money demand, interest rate shocks, monetary policy, transfer costs, endogenous market segmentation.

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1. INTRODUCTION

I obtain a slow response of prices and money and a decrease in the quantity of money after monetary shocks. The key aspect of the model is endogenous market segmentation: because of a transfer cost, agents decide when to participate in open market operations and only a fraction of agents participates in each period. Several empirical studies report a slow response of money and prices after monetary shocks. For example, Cochrane (1994), Christiano, Eichenbaum and Evans (1999), and Uhlig (2005)[1]. I offer a monetary model to explain these facts.

I study two different shocks: a permanent and a temporary increase of one percentage point in the nominal interest rate. The main contribution is to obtain persistent real effects after monetary shocks using a general equilibrium model with the following two characteristics. (1) In the short run, slow responses of prices and money and a decrease in the quantity of money. (2) In the long run, price level and money growth rates equal to the steady state inflation rate, and a decrease in real balances for the permanent shock. These two characteristics of the transition are in accordance with the empirical evidence on the short and long run behavior of prices and money.

Endogenous segmentation has substantial effects on the response of prices and money compared to models with fixed transfer times. One important difference is on the long run effects of interest rates on real balances. If the interval between transfers is fixed, long-run real balances are *increasing* in interest rates (Romer, 1986)[2]. With endogenous transfer times, in contrast, real balances are *decreasing* in the interest rate. After a permanent increase in the interest rate, the present model predicts a

[1] Further evidence are in Leeper et al. (1996), Christiano et al. (1996, 2005), King and Watson (1996), Bernanke et al. (2005), and the references therein. The literature on the empirical evidence of monetary shocks with similar findings is extensive.

[2] Romer studies a case with log utility and zero intertemporal discount. But this result is not particular to a specific version of the model. In a more general model, Silva (2007) finds that real balances are decreasing in interest rates only for large elasticities of intertemporal substitution if the transfer intervals are fixed.

delayed response of nominal and real balances followed by a long-run decrease in real balances whereas there is a long-run increase in real balances, close to zero response, with fixed transfer times. The temporary shock also implies a delayed decrease in nominal and real balances. Another difference is that the effects after the shocks last longer with endogenous transfer times than with fixed transfer times. There are other differences in the response of money and prices in the short and long run. I discuss the differences in detail in section 4.

Agents have different money holdings. When there is a shock, agents with little balances visit the bond market earlier and transfer money taking into account the new interest rate path. Agents with more balances take longer to make their first transfer after the shock. This generates the lagged responses of prices and money[3]. Grossman and Weiss (1983) and Rotemberg (1984) proposed the first models with market segmentation to study the liquidity effect. In these models, agents participate in open market operations in fixed transfer times[4].

This paper is most closely related to Alvarez, Atkeson and Kehoe (2002) and Alvarez, Atkeson and Edmond (2003), henceforth AAK and AAE. In AAK, there is a fixed cost of going to the asset market and making a transfer. Agents use the transfer to purchase goods, it alleviates the cash in advance constraint. The cash constraint, however, is assumed to bind in every period and so velocity is constant. In AAE, the cash constraint is not assumed to hold in every period – agents can maintain some of their balances for the future – and so velocity varies after monetary shocks. When there are short run fluctuations, the heterogeneity of money holdings complicates the problem and the model is simplified in AAE by fixing the transfer times. The current

[3] Christiano, Eichenbaum and Evans (1996) and Vissing-Jorgensen (2002) provide evidence that households take time to adjust their portfolios after a shock.

[4] Fuerst (1992), Lucas (1990), Alvarez and Atkeson (1997), Alvarez, Lucas and Weber (2001), Alvarez, Atkeson and Edmond (2003), and Occhino (2004) have further contributions with models of exogenous market segmentation. Williamson (2006) combines market segmentation with search. I use limited participation and market segmentation as synonyms.

paper can be understood as an extension of AAK by letting agents maintain their balances over longer periods or of AAE by endogenizing the transfer times[5].

I endogenize the transfer times but I simplify the economy in a different way: I study the transition given the new interest rate paths and remove all other shocks to the economy. The government unexpectedly announces the new interest rate paths from an initial steady state. One advantage of this assumption is to focus on the effects of endogenous segmentation. Short-run shocks affect the consumption pattern with endogenous and fixed segmentation and this can hide the effects caused by the change in the transfer times. Without the short-run shocks, we can concentrate on the effects of endogenous segmentation.

The framework is a general equilibrium version of Baumol (1952) and Tobin (1956). I use the framework in Silva (2007) and focus on the transition. The model has infinitely-lived agents, positive intertemporal discount, optimal transfer times and optimal consumption pattern within holding periods[6]. The transfer cost is in goods. This is important for the convergence of prices and money. I let prices and the real interest rate change during the transition[7].

I structure the paper in five sections: introduction, the model, steady state, transition after the shocks, and conclusions. All proofs are in the appendix.

[5] This paper is also related to Khan and Thomas (2007). I let agents decide the interval between transfers while Khan and Thomas let agents decide to access the bond market according to a random draw of the transfer cost. Another difference is that I study the effects on prices and money after interest rate shocks, while Khan and Thomas study the effects on prices and interest rates after changes in the money growth rate.

[6] Jovanovic (1982) and Romer (1986) focus on the steady state. Jovanovic assumes constant consumption within transfer periods. The difficulty of allowing optimal transfer times is in the relation between aggregate variables and individual behavior (Caplin and Leahy, 1991 and 1997).

[7] Romer (1987), Fusselman and Grossman (1989), Heathcote (1998) and Chiu (2005) have models with transfer cost in utility terms. Romer and Heathcote have overlapping generations and obtain convergence as old generations are removed from the economy. Romer fixes interest rates and prices to keep the real interest rate constant. Chiu studies the steady state and analyzes small shocks in which agents do not change their transfer times. Heathcote has a discrete-time version of Romer. This discretization of time, however, implies steady state real interest rates increasing with inflation and zero real money balances for a wide range of inflation rates.

2. THE MODEL

The model is a general equilibrium version of the Baumol (1952) and Tobin (1956) inventory-theoretic model of money demand: agents need to use money to purchase goods and there is a transfer cost whenever agents sell interest-bearing bonds for money. Silva (2007) discusses the model in more detail and focus on the steady state. In this paper, I use the initial distribution of money holdings and study the transition. I briefly state the model and then concentrate on the transition[8].

There is a continuum of agents. Agents are infinitely lived and discount the future at the rate $\rho > 0$. They incur a transfer cost when they sell bonds for money. Agents choose consumption at each time, $c(t)$, and the time of each transfer, T_j. Let $N_j \geq 0$ $j = 1, 2, ...$ denote the interval between transfers. Hence, the time of each transfer is $T_j = \sum_{s=1}^{j} N_j$, $T_0 \equiv 0$. Time is continuous[9].

Agents have preferences

$$U(c) = \sum_{j=0}^{\infty} \int_{T_j}^{T_{j+1}} e^{-\rho t} \log c(t)\, dt. \tag{1}$$

The transfer cost does not enter in the utility function. The logarithmic utility is not essential for the results[10].

There is a brokerage account and a bank account, as in AAE. The brokerage account contains bonds used in the asset market and the bank account contains money used for goods purchases. Each agents produces Y units of a single and nonstorable good in every period. They sell the production in the goods market and deposit the proceeds

[8] The main differences from previous general equilibrium versions of the Baumol-Tobin model, such as the seminal models of Jovanovic (1982) and Romer (1986), are that agents are infinitely-lived, smooth consumption within holding periods, have positive intertemporal discount, and pay the transfer cost in goods. The model is adapted from Grossman (1987), which has proportional transfer costs within holding periods and constant transfer intervals.

[9] This is a simplifying assumption. It allows us to ignore integer constraints on T_j.

[10] Silva (2007) discusses the model with general constant relative risk aversion utility.

in the brokerage account at each time. Agents cannot use the proceeds from goods sales in the same period[11]. Production is assumed constant. This is not a restrictive assumption. It allows us to isolate the effects of market segmentation on the behavior of prices and money after interest rate shocks. It also simplifies the computation of the transition. Denote the price level by $P(t)$ and inflation by $\pi(t)$.

Agents have to pay a fee γY, $\gamma > 0$, in order to transfer resources from the brokerage account to the bank account. The transfer cost is the crucial assumption of the paper. It generates a nondegenerate distribution of money holdings across agents, a slow response of prices after monetary shocks and a propagation mechanism. The transfer cost proportional to income is a technical assumption. Consumption and money demand in the steady state will be linear in income with this assumption. A value $\gamma = 1$ means that the transfer cost is equal to one working day per transfer[12].

The initial resources in the bank account are M_0. These balances can be used promptly for goods transactions. The initial resources in the brokerage account, W_0, are equal to the initial bond holdings distributed by the government, B_0, plus the present value of production,

$$W_0 = B_0 + \int_0^{\infty} Q(t) P(t) Y dt, \tag{2}$$

where $Q(t)$ is the value at time zero of one dollar to be received at time t. Each agent is identified by the pair (M_0, W_0). There is a given initial distribution F of M_0 and W_0.

Agents need to use money in order to buy goods. Therefore, they face the cash in

[11] An agent can be viewed as a family composed of two types of individuals, a worker and a shopper, as in Lucas (1990).

[12] In a different setting, Khan and Thomas (2007) introduce stochastic transfer costs, as in Dotsey, King and Wolman (1999).

advance constraint

$$\dot{M}\left(t, M_0, W_0\right) = -P\left(t\right) c\left(t, M_0, W_0\right), \, t \neq T_1, T_2, ... \tag{3}$$

where $M\left(t, M_0, W_0\right)$ and $c\left(t, M_0, W_0\right)$ denote money balances and consumption at time t of agent $\left(M_0, W_0\right)$. The given balances M_0 are not necessarily the optimal amount for consumption between 0 and T_1. Therefore, agents are allowed to transfer $K \geq 0$ from the bank account to the brokerage account at the first transfer time T_1[13]. After T_1, with positive interest rates, agents transfer the exact amount of money necessary to consume during the holding period.

Agents decide consumption and the optimal transfer times at time zero, given the price level and the interest rate paths. Therefore, the individual maximization problem is to maximize (1), subject to

$$\sum_{j=1}^{\infty} Q\left(T_j\right) \int_{T_j}^{T_{j+1}} P\left(t\right) c\left(t\right) dt + \sum_{j=1}^{\infty} Q\left(T_j\right) P\left(T_j\right) \gamma Y \leq W_0 + Q\left(T_1\right) K, \tag{4}$$

and

$$\int_0^{T_1} P\left(t\right) c\left(t\right) + K \leq M_0 \tag{5}$$

plus the non-negativity constraints for $c\left(t\right)$, N_j, and K. I remove the reference to $\left(M_0, W_0\right)$ of these variables to simplify notation when it does not lead to ambiguity.

The constraint (4) states that the present value of all money transfers and the payment of the transfer fee is equal to the present value of deposits in the brokerage account. It uses the cash-in-advance constraint and the fact that money holdings are exhausted at the end of each holding period. The constraint (5) states that

[13] K is the quantity of money not used in $[0, T_1)$ and transferred to the brokerage account at T_1. $K > 0$ if M_0 is higher than the value otherwise chosen by the agent.

consumption until the first transfer and any unspent balance K must be financed from the initial money holdings M_0.

The first order condition with respect to consumption within a holding period for agent (M_0, W_0) is

$$e^{-\rho t} u' \left(c \left(t, M_0, W_0 \right) \right) = P \left(t \right) \lambda \left(M_0, W_0 \right) Q \left(T_j \right), \quad T_j < t < T_{j+1},\ j = 1, 2, ..., \tag{6}$$

where $\lambda (M_0, W_0)$ is the Lagrange multiplier of (4). Consumption within holding periods is decreasing if inflation is greater than or equal to $-\rho$. Agents concentrate consumption in the beginning of a holding period to avoid losing resources for inflation. See the appendix for the full characterization of the first order conditions.

Denote $c^+ (t)$ and $c^- (t)$ as consumption respectively in the beginning and in the end of a holding period. Combining the first order conditions with respect to the time T_j and consumption yields

$$\gamma Y \left[r \left(T_j \right) - \pi \left(T_j \right) \right] + r \left(T_j \right) \int_{T_j}^{T_{j+1}} \frac{P \left(t \right) c \left(t \right)}{P \left(T_j \right)} dt = c^+ \left(T_j \right) \left[u \left(c^+ \left(T_j \right) \right) - u \left(c^- \left(T_j \right) \right) \right], \tag{7}$$

for $j \geq 2$, where $r (t)$ is the nominal interest rate at time t.

The left hand side of (7) is the marginal gain of delaying the transfer and the right hand side is the marginal loss. The marginal gain is given by postponing the transfer and decreasing real balances for purchases from T_j to T_{j+1}. It also takes into account the net effect of increasing the period from T_{j-1} to T_j and decreasing the period from T_j to T_{j+1}. But this net effect is equal to zero with log utility. The marginal loss on the right hand side is the net effect in utility of the change in the length of holding periods T_{j-1}, T_j and T_j, T_{j+1}. From the first order conditions, $c (t, M_0, W_0; Y)$ and $T_j (M_0, W_0; Y)$ are homogeneous of degree one and degree zero in (M_0, W_0, Y).

If money, bonds and production are multiplied by the same factor, then all agents maintain their transfer times and multiply consumption by this factor.

The initial quantity of bonds held by the public, B_0^S, is financed by the government via seigniorage. The government budget constraint is

$$B_0^S = \int_0^\infty Q(t) P(t) \frac{\dot{M}^S(t)}{P(t)} dt, \tag{8}$$

where $M^S(t)$ denotes the money supply at time t. The government injects money with bond exchanges in the asset market, that is, with open market operations. The present value of these operations is reflected by the term B_0^S.[14]

The market clearing conditions for money and bonds are $\int M(t, M_0, W_0) dF(M_0, W_0) = M^S(t)$ and $\int B_0(M_0, W_0) dF(M_0, W_0) = B_0^S$. The market clearing condition for goods must take into account the transfer cost. Denote the set of agents making a transfer during $[t, t+\delta)$ by $A(t,\delta) = \{(M_0, W_0) : T_j(M_0, W_0) \in [t, t+\delta)\}$, for a certain $j \geq 1$. The number of transfers from t to $t+\delta$ is given by the measure of $A(t,\delta)$. The market clearing condition for goods is therefore

$$\int c(t, M_0, W_0) dF(M_0, W_0) + \gamma Y \lim_{\delta \to 0} \int_{A(t,\delta)} \frac{1}{\delta} dF(M_0, W_0) = Y. \tag{9}$$

The second term in the left-hand side are the resources directed to transfers at t.

Equilibrium is defined as prices $P(t)$, $Q(t)$, demands $c(t, M_0, W_0)$, and interval between transfers $N_j(M_0, W_0)$ such that (i) $c(t, M_0, W_0)$ and $N_j(M_0, W_0)$ solve the maximization problem of each agent (M_0, W_0), (ii) the government budget constraint holds, and (iii) the market clearing conditions for goods, money and bonds hold.

[14] Taxes and government purchases are not in the government budget contraint to abstract from fiscal policy. They would not change results insofar as they are set exogenously.

3. STEADY STATE: MONEY HOLDINGS AND CALIBRATION

I obtain the steady state interval between transfers N and the distribution of money holdings across agents in this section. I aggregate money holdings and use it to calibrate the transfer cost γ. The higher the value of γ, the higher N and the money-income ratio (smaller velocity). We need to write the distribution of money holdings before the shocks because agents react differently to shocks according to their money holdings. In the next section, the shocks hit the economy initially in the steady state.

In the steady state, inflation and interest rate are constant and the interval between transfers is the same for all agents: $N_j\left(M_0, W_0\right) = N$ for all $\left(M_0, W_0\right)$ and $j \geq 2$. Consumption, money and bond holdings are different across agents but they have the same pattern within holding periods. As agents have the same present value of production, the same number of agents must exchange bonds for money at each time to imply constant aggregate money holdings and consumption. Therefore, the distribution F of $\left(M_0, W_0\right)$ is such that $T_1\left(M_0, W_0\right)$ is uniformly distributed along $[0, N)$[15]. Write $N\left(r, \gamma, \rho\right)$ to stress the dependence of the interval between transfers to the parameters of the model.

Consumption within holding periods is given by $c\left(t\right) = c_0 e^{-r\left(t-T_j\right)}$, $T_j \leq t < T_{j+1}$, by its first order condition, where c_0 denotes consumption just after a transfer. The market clearing condition for goods implies

$$\frac{1}{N\left(r,\gamma,\rho\right)} \int_0^{N\left(r,\gamma,\rho\right)} c_0 e^{-rx} dx + \frac{\gamma Y}{N\left(r,\gamma,\rho\right)} = Y. \tag{10}$$

[15] We can have constant aggregate consumption with different consumption patterns by adjusting F. For example, F could be higher for those consumers with low consumption patterns. It is natural for the steady state, however, to require uniform consumption patterns.

The value of c_0 as a function of $N(r, \gamma, \rho)$ is then

$$c_0(N) = Y\left(1 - \frac{\gamma}{N(r,\gamma,\rho)}\right)\frac{rN(r,\gamma,\rho)}{1 - e^{-rN(r,\gamma,\rho)}}. \tag{11}$$

Consumption is homogeneous of degree one in income[16]. We need $NY > \gamma Y$ to have $c_0 > 0$: production during holding periods must be greater than the cost to obtain money for the same period. This in fact the case in equilibrium.

We obtain the optimal transfer interval with the first order conditions (6) and (7). Proposition 1 writes the value of N given the interest rate, transfer cost, and intertemporal discount, and establishes existence and uniqueness of N. Proposition 1 also states that N is decreasing in the interest rate and increasing in the transfer cost.

Proposition 1. The optimal interval between transfers in the steady state, $N(r, \gamma, \rho)$, is given by the positive root of the equation

$$rN - \frac{r}{\rho}\left(1 - e^{-\rho N}\right) = \rho\gamma\left[\frac{c_0(N)}{Y}\right]^{-1} \tag{12}$$

where $c_0(N)$ is given by (11). $N(r,\gamma,\rho)$, exists and is unique for all positive r, ρ and γ. Moreover, $\frac{\partial N}{\partial r} < 0$ and $\frac{\partial N}{\partial \gamma} > 0$[17].

The value of N implied by equation (12) is not far from the square-root rule. We obtain $N \approx \sqrt{\frac{2\gamma}{r}}$ with a second-order Taylor expansion of $e^{-\rho N}$ and e^{rN}, as rN and ρN are close to zero[18].

[16] Consumption can be less than Y during the holding period with transfer cost in goods. With transfer cost in utility terms, the term $1 - \gamma/N$ vanishes and c_0 is always greater than Y.

[17] Proposition 1 do not depend on logarithmic utility. Silva (2007) proves these properties for general constant relative risk aversion utility. We also have that $\lim_{\gamma \to 0} N = 0$, $\frac{\partial N}{\partial \rho} > 0$ and $NY > \gamma Y$.

[18] For example, $rN = 0.02$ and $\rho N = 0.01$ with $r = 4\%$ p.a., $\rho = 3\%$ p.a. and $\gamma = 1.79$, see the calibration of γ below. Jovanovic (1982) also obtains the square-root formula with an approximation in his model. Lucas (2000) obtains the square-root formula with the McCallum-Goodfriend transactions technology.

Define the functions $M_0(n)$ and $W_0(n)$ as the initial amounts of deposits in the bank and brokerage accounts such that an agent with $M_0(n)$ and $W_0(n)$ transfers resources at $t = n,\ n+N,\ n+2N$ and so on. $M_0(n)$ is equal to steady state consumption spending in the interval $[0, n)$. On the other hand, $W_0(n)$ is equal to the present value of future transfer amounts and transfer fees.

Proposition 2. The values of initial money holdings, $M_0(n)$, and initial wealth in the brokerage account, $W_0(n)$, such that agent $(M_0(n), W_0(n))$ chooses $T_1 = n$ are

$$M_0(n) = P_0 c_0(N) e^{rn} e^{-rN} \frac{1-e^{-\rho n}}{\rho}$$

and

$$W_0(n) = \frac{e^{-\rho n}}{1-e^{-\rho N}} \left(P_0 c_0(N) \frac{1-e^{-\rho N}}{\rho} + P_0 \gamma Y \right),$$

for $n \in [0, N)$, where $c_0(N)$ is given by (11).

With proposition 2, we can index agents by the time of the first transfer, $n \in [0, N)$. $M_0(n)$ is increasing in n, agents with more initial money holdings make the first transfer later. Analogously, $W_0(n)$ is decreasing in n, agents with less initial bond holdings make the first transfer later.

Individual money demand at time t is equal to spending from t until the next transfer $T_j(n)$. With propositions 1 and 2, and $c(t) = c_0 e^{-r(t-T_j)}$, we obtain individual money demands for each agent n. Aggregate money demand is then given by the sum of individual money demands. It can be shown (Silva, 2007) that the steady state real money demand, $m = M(t)/P(t)$, is given by

$$m(r, Y, \gamma, \rho) = \frac{c_0(N; r, Y, \gamma, \rho)}{\rho} \left[\frac{1-e^{-rN}}{rN} - e^{-\rho N} \frac{1-e^{-(r-\rho)N}}{(r-\rho)N} \right], \tag{13}$$

where $c_0(N; r, Y, \rho, \gamma)$ is given by (11) and $N(r, \gamma, \rho)$ is given by proposition 1.

The income elasticity of the money demand in (13) is equal to one, as c_0 is homogeneous of degree one in Y. More important, the real money demand is decreasing in the interest rate, with interest elasticity close to $-1/2$. If the interval between transfers is not allowed to change, that is, if N is fixed in the formula above, then the real money demand is *increasing* in the interest rate, with elasticity close to zero[19]. This is especially relevant to study the effects of a permanent increase in the interest rate, as studied in section 4 in one of the shocks. The present model, with endogenous segmentation, predicts a long-run decrease in the real money demand. A model with fixed segmentation (fixed N), in contrast, would predict a long-run increase in the real money demand. A negative interest elasticity of money demand is a well established empirical fact. See, for example, Meltzer (1963) and Lucas (1988). I relate (13) to a long run real money demand, as the interest rate in (13) refers to a steady state. We obtain P_0 with m and the initial supply of money M_0^S.

Calibration

I calibrate the model with U.S. annual data for 1900 to 1997 and (13). I use annual data and an extensive period because (13) refers to a long run money demand. Also, I use the model to predict the effects of monetary policy changes when the economy is initially in the steady state. Therefore, the initial behavior in the model economy should approximate the data for the long run. I use a similar dataset as in Lucas (2000). In particular, I use M1 for the monetary aggregate.

There are only two parameters to calibrate in the model: ρ and γ. I set the intertemporal discount so that inflation is equal to zero when $r = 3$ percent per year. Therefore, $\rho = 3$ percent per year. For the transfer cost, I set γ so that the theoretical money-income ratio given by (13) passes through the geometric mean of the data[20].

[19] This result is not particular to this version of the Baumol-Tobin model. Romer (1986) shows that the money demand is *increasing* in the interest rate if the interval between transfer is unresponsive to the interest rate. Silva (2007) finds the interest elasticity is negative and close to $-1/2$ with fixed N only with high elasticity of intertemporal substitution.

[20] Lucas (2000) also chooses $r = 3$ as the interest rate associated to zero inflation and calibrates

That is, under the average interest rate during the period, 3.6 percent, the model matches the average velocity of 3.9, or average money-income ratio of 0.257, during the same period. This implies $\gamma = 1.79$. I discuss this value for γ in detail below.

Figure (1) shows the steady-state equilibrium values of the money-income ratio m/Y given by equation (13) and the calibrated parameters along with the data. Income in the model, Y, is assumed constant and normalized to one. Even though the model abstracts from several details, the fit apparent in figure (1) is surprisingly good. In particular, the model is able to predict the low velocity of the 1940's period, with low interest rates, and the high velocity of the 1980's, with high interest rates. Note that the increase in velocity from 2 to 7 from 1945 to 1981 in the U.S. is not a puzzle in the model: velocity increased with the interest rate. What is more difficult to explain is the relatively high velocity in the 90's, and the relatively low velocity in the beginning of the century, with interest rates between 3 and 6 percent. Nevertheless, the model is able to predict the general pattern of the money-income ratio. Lucas (2000) argues that a demand for real balances with constant interest elasticity of $-1/2$, as in the present model predicts, has a good fit to U.S. data. An increase in γ implies a parallel upward shift on the curve in figure (1). That is, γ has very little effect on the interest elasticity, it only affects velocity.

A transfer cost parameter $\gamma = 1.79$ means that the average agent pays the equivalent of roughly 1.8 working days per transfer. If the interest rate is equal to 4 percent, the calibration implies $N = 181$ days or about 2 transfers per year from high-yielding assets to money. These are not ATM withdrawals, which simply transfer resources from checking accounts to currency but do not change M1. With 5 working days per week and 52 weeks per year, the average agent devotes around 1.38 percent of the total working time to financial transfers. According to OECD data, U.S. workers worked around $1,900$ hours per year on average from 1950 to 1997. Therefore, the

the money demand so that it passes through the geometric mean of the data.

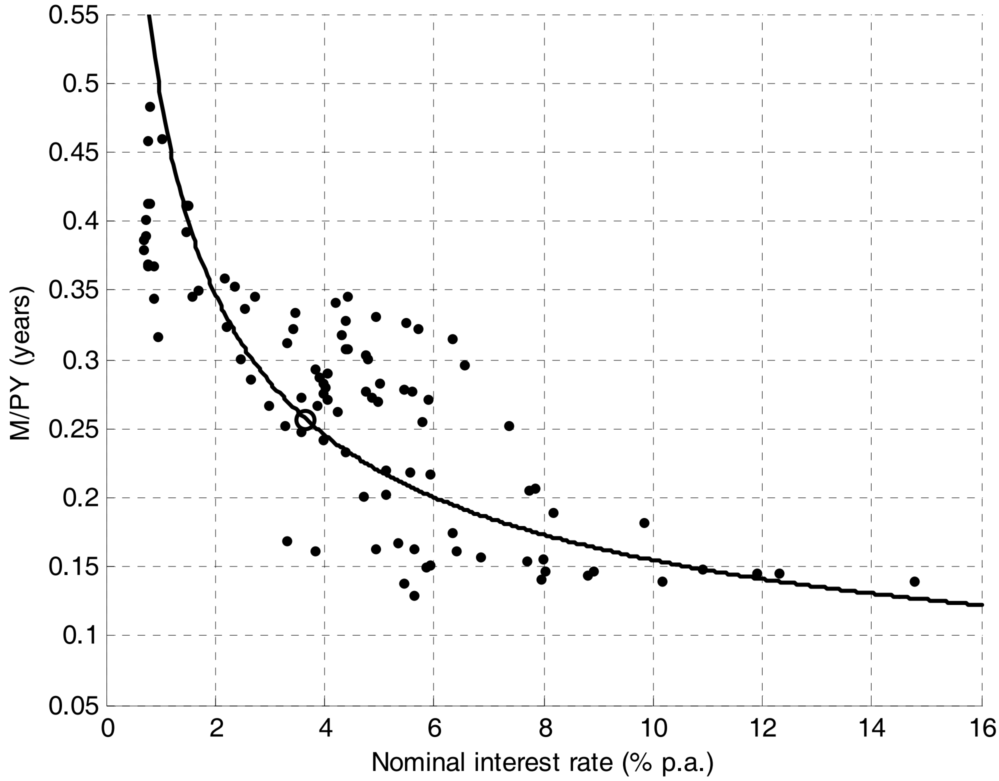

FIG. 1. Money-income ratio in the steady state. The data points are for the U.S. economy in the period 1900-1997, the monetary aggregate is M1. The circle o marks the geometric mean of the data.

model estimates on average about 30 minutes per week devoted to financial transactions when inflation is equal to one percent per year. In a very different setting, the calibration in Lucas (2000) implies that agents devote 1 percent of their working time to transfers when the interest rate is equal to 4 percent per year[21], close to the number found here. The present paper also predicts 1 percent of total working time for transfers, and 22 minutes per week, if we consider 360 days working days per year instead of $5 \times 52 = 260$ working days.

[21] Lucas (2000), p. 267: $k = 400 \Rightarrow s\,(0.04) = 0.01$.

The values of 22 to 30 minutes per week for financial transactions seem reasonable and put in perspective the calibration of the model. Although the number of transfers per year seems small, this number is consistent with the empirical findings in Vissing-Jorgensen (2002) and it is common in the literature related to this paper. For example, the calibrations in AAE imply holding periods from 1.5 to 3 years. In Khan and Thomas (2007), the calibrations imply average holding periods from 1.2 to 2.4 years and maximum holding periods from 1.5 to 2.4 years[22]. The transfer costs in the present paper and in Khan and Thomas are not fully comparable, as in Khan and Thomas there is a distribution of transfer costs and each agent has a different transfer cost in each period. In particular, agents can avoid making a transfer when they have a high-cost draw. However, the first calibration in Khan and Thomas implies that the average ex-ante transfer cost corresponds to 2.6 percent of total production in a year and the present paper implies a transfer cost that corresponds to 1 percent of total production in a year[23].

The present model and the models above have in common that the main concern is to model the macroeconomic behavior of prices and money and not the microeconomic behavior of consumers. In order to model the microeconomic behavior of consumers, the structure of the model would have to account for the money demand of firms, heterogeneous endowments, and other features. These values are not trivial. For example, according to Bover and Watson (2005), M1 held by firms in the non-financial

[22] The calibrations in these papers use a monetary aggregate close to M2. As these papers also use velocity for the calibration, a calibration with M1 would decrease the holding periods. AAE assume in some simulations that a fraction of income is paid directly to the bank account.

[23] Khan and Thomas set the transfer cost between 0 and 0.25 uniformly distributed in the first calibration (the calibration with smaller transfer costs) and obtain an average interval of 4.82 quarters. This implies an average cost of 0.25/2 per transfer and 0.83 transfers per year. As the model is in quarters, and production is normalized to one in this calibration, this corresponds to $0.25/2 \times 0.83/4 = 0.026$ of total yearly production. Those are ex-ante costs. As agents with high-cost draws avoid making a transfer, the effective cost payment is smaller, but the calibration requires a cost between 0 and 0.25. The present paper has 1.8 working days per transfer and 2 transfers per year. This implies $1.8 \times 2/360 = 0.01$ of total yearly production.

sector is over 62 percent in the U.S. in 2000. This paper, and the literature in which it is included, simplify all of these features in one type of agents. The agent in these models has to be viewed as a summary of all participants in the economy[24]. As the fit in figure (1) reveals, this assumption is enough to generate a money demand able to reproduce the general pattern given by the data. In section 4, we will see that the model also succeeds in reproducing important features of the effects of monetary shocks, given the empirical evidence in Christiano et al. (1999) and in other studies.

4. INTEREST RATE SHOCKS

Suppose that the nominal interest rate has been constant for a long time and suddenly changes. What will be the effects on prices, money and real balances?

I study two interest rate shocks. In the first, the interest rate increases permanently from 3 to 4 percent per year. In the second, the interest rate increases to 4 percent and gradually returns to 3 percent per year. I call the two changes a permanent and a temporary shock to the interest rate respectively. Hence, I interpret a monetary policy shock as a permanent or temporary increase in the nominal interest rate.

I change the interest rate rather than the money supply for two reasons: to be closer to the policy of Central Banks and to simplify the analysis. First, Central Banks usually track the interest rate in their daily operations rather than the quantity of money (Woodford, 2003). Implicitly, they assume that the quantity of money changes according to the interest rate[25]. Second, for a technical reason, there is the need to decrease the dimensionality of the problem. With this procedure, instead of working with two equilibrium prices, $P(t)$ and $r(t)$, and two market clearing conditions, for goods and money, we have to find only one equilibrium price, $P(t)$, with the

[24] See also the discussion in Edmond and Weill (forthcoming). See Alvarez and Lippi (2007) for a Baumol-Tobin model focused on individual household behavior.

[25] Christiano, Eichenbaum and Evans (2005) and Grossman (1987) also assume that the monetary authority adjusts the quantity of money according to the interest rate.

market clearing condition for goods. With $P(t)$ and the path for the interest rate, we obtain the money supply using the market clearing for money. The problem would be intractable if we had to find the price level together with the interest rate with two market clearing conditions and endogenous transfer times.

As there have been no shocks for a long time, assume that the economy is initially in the steady state. Change the analytical structure of the problem in the following way, with the objective of approximating a situation in which the economy is initially in the steady state and the interest rate changes unexpectedly. Suppose the existence of two possible states. In state 1 the government sets the nominal interest rate at r_1 for all periods. In state 2, the government sets the path of the nominal interest rate at $r(t)$ for each time, not necessarily constant. The realization of the state occurs at time zero. Agents trade bonds contingent on the realization of the states. Money is not contingent on the state.

Agents use M_0 from time zero until the first transfer. We now have two budget constraints from time zero until the first transfer in problem (1), (4) and (5). One for each price level path in the respective state. On the other hand, we need only one present value budget constraint after the first transfer, as agents use contingent bonds to transfer resources between states. Finally, agents now maximize the expected value of utility weighted by the probabilities of each state[26].

The initial cross section of money holdings is close to the one with only one state if the probability of the shock is small. Therefore, proceed in the following way to obtain optimal consumption and transfer times. First, calculate money and bond holdings such that the economy is in the steady state under the interest rate r_1. Money holdings are given by $M_0(n)$ in proposition 2, where $n \in [0, N)$ and N is the holding period under r_1. Second, calculate the new optimal individual consumption and transfer times given the interest rate path $r(t)$ and initial money holdings $M_0(n)$.

[26] See appendix for the analytical statement of the problem.

Using this procedure, consumption at time t and transfer times $T_j \equiv N_1 + ... + N_j$ for each agent n are

$$c(t,n) = \frac{e^{-\rho t}}{\lambda Q(T_j(n)) P(t)},\ t \in (T_j(n), T_{j+1}(n)),\ j \geq 1, \tag{14}$$

and

$$[R(T_j) - R(T_{j-1})] - \frac{\gamma Y [r(T_j) - \pi(T_j)]}{c^+(T_j(n))} = r(T_j) \frac{1 - e^{-\rho N_{j+1}}}{\rho},\ j \geq 2, \tag{15}$$

where $Q(t) = e^{-R(t)}$ is the bond price, $R(t) \equiv \exp\left(-\int_0^t r(s)\,ds\right)$, $\pi(t)$ is the inflation rate, $c^+(T_j(n)) = e^{-\rho T_j(n)} / [\lambda Q(T_j(n)) P(T_j(n))]$ is consumption just after the jth transfer, and λ is the Lagrange multiplier of the budget constraint after the first transfer[27]. Equation (15) states how the transfer interval N_j relates to the next transfer interval N_{j+1} during the transition. The second term in equation (15) appears because the transfer cost is paid in goods. It relates the price level with the decision of transfer times and it allows for convergence after the shock[28].

Different monetary shocks are described by the path of the interest rate after the shock. I assume that the economy is initially in equilibrium with a constant nominal interest rate equal to 3 percent per year. This implies zero inflation before the shock as the intertemporal rate of discount is equal to 3 percent per year[29]. For the permanent shock, the monetary authority sets $r(t) = 4$ percent per year. For the temporary shock, the monetary authority sets the interest rate at 4 percent per year at time zero and then reduces it towards 3 percent per year at a constant rate. The process is, therefore, $r(t) = r_1 + (r_2 - r_1) e^{-\eta t}$ where $r_1 = 3$, $r_2 = 4$ and η is the persistency

[27] The equation for the first transfer time $T_1 (= N_1)$ is in the appendix.

[28] This term disappears in models with transfer cost in the utility function.

[29] The initial state is arbitrary. I choose an initial steady state with zero inflation to facilitate the interpretation of the effects of the shocks and the comparison of the final and initial states. An initial state with positive inflation would not change the qualitative aspects of the model.

of the shock. I follow AAE and choose η to approximate the response of the nominal interest rate to a shock similar to the response shown in Christiano et al. (1999) and Uhlig (2005)[30].

I proceed numerically in order to obtain the equilibrium path of the price level and the other equilibrium values. For each agent, we have a system of equations in the form $h(N_j, N_{j+1}) = 0$. In order to have a finite system, I assume that after the Jth transfer each agent chooses $N_{J+1} = N'$, where N' is the steady state transfer interval under the new interest rate. The value of J should be large to approximate the solution. We then have a system of J equations and J unknowns $N_1, ..., N_J$ for each agent. See appendix for the detailed description of the algorithm.

The procedure to find the price level during the transition is the following. (i) Start with a guess for the price level during the transition. (ii) Calculate the transfer times and consumption for each agent. (iii) Check the market clearing condition. (iv) If the difference between demand and supply is smaller than a preestablished value for every t, stop. If not, change $P(t)$ and repeat steps (i)-(iii).

The results of the simulations for the permanent and the temporary shocks are in figures (2) and (3)[31]. The liquidity effect is common to both shocks. In the short run, the price level adapts slowly, and money and real balances decrease. After six months, there is an overshooting in the price level for both the permanent and the temporary shocks, followed by dampened oscillations towards the new steady state. The figures make reference to the money-income ratio $M/(PY)$ – the inverse of velocity. The behavior of this ratio is equal to the behavior of real balances in this model, as Y is exogenous and constant, normalized to one.

Figures (2) and (3) also have the results for the case with fixed transfer times

[30] For the time in days, $\eta = \frac{-12 \log 0.87}{365}$.

[31] The results of the daily values show oscillations with decreasing amplitude as time evolves. The interval between transfers for each agent approaches gradually the interval in the new steady state. In order to focus on the main results of the simulations, figures (2) and (3) show the annual means of money and money-income ratio.

for comparison. In order to find the equilibrium paths with fixed transfer periods, I use the same model as described above but fix the transfer times. That is, I let agents optimize within holdings periods but do not let them change the transfer times. I fix the transfer interval equal to its value in the initial steady state. Hence, the two economies behave in the same way in the first steady state, with the same transfer intervals. They are different only after the shocks. With $r = 3$ percent, both economies have an initial value $N = 209$ days. With the permanent shock to $r = 4$ percent, the model agents in the economy with endogenous transfer times gradually decrease N to 181 days while they maintain the initial N in the economy with fixed transfer times. With the temporary shock, the agents in the economy with endogenous transfer times temporarily decrease N to 195 days on average but later return to the initial value of 209 days, as r returns to 3 percent[32]. The economy with fixed transfer times maintains the initial N during the transition. Different from the case with endogenous transfer times, we can obtain analytical formulas for the transition with fixed transfer times. The formulas for the transition in this case are in the appendix. Note that a Baumol-Tobin model with fixed holding periods is similar to AAE and to Grossman and Weiss (1983). The difference is that the model is now in continuous time and that, different from AAE, I removed the short-run variations in the interest rate. The results with endogenous and fixed N are very different. In particular a model with fixed transfer times is not able to generate decreasing nominal and real balances after an interest rate shock when we remove the short-run variations[33].

[32] In the simulations, I discretize the interval $[0, N)$ in units of 0.10 days, and say that agent n makes a transfer at day t if $t \leq T_j(n) < t + 1$. With the permanent shock, the average number of transfers per day increases from 10 to 11.6.

[33] The dynamics in AAE shows a decrease in the price level and money while the present model with fixed N shows a sluggish response but no decrease. Although AAE and this version of the model have fixed N, they are different in some aspects. Some differences are the following. First, the dynamics presented here is the result of a single shock from an economy initially in the steady state with no shocks while the dynamics in AAE is the impulse-response function obtained from a stochastic system. The present paper removes all shocks with the exception of the change in the path of interest rate at time zero. Second, the dynamics presented here with fixed N was obtained analytically while the dynamics in AAE was obtained numerically, from the log-linearized system.

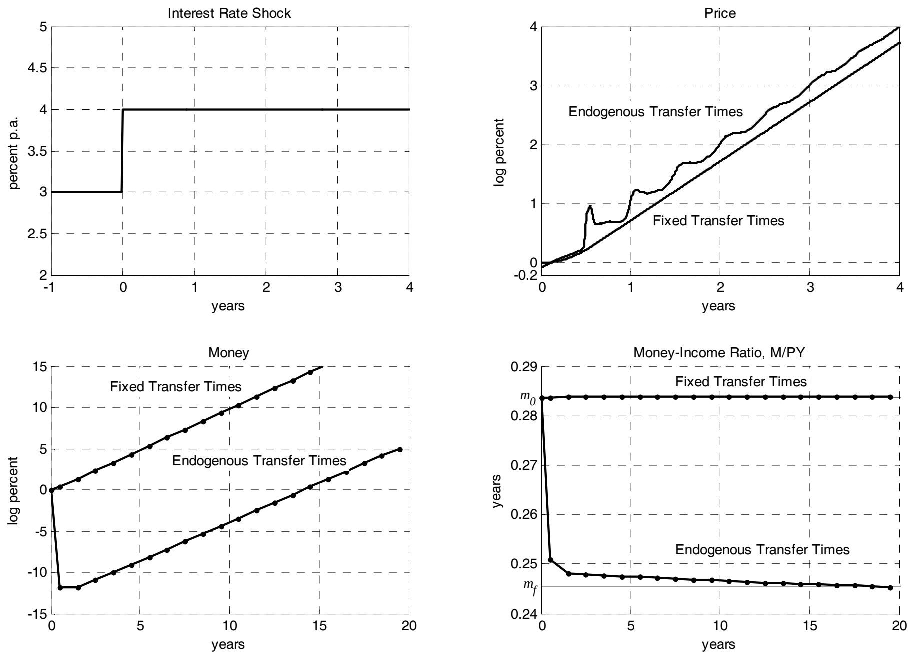

FIG. 2. Results of a permanent interest rate shock from 3 to 4 percent per year. m_0 and m_f: steady states before and after the shock. Log percent from the values before the shock. Annual means of money and money-income ratio, centered on July 1st, the first point is the value before the shock.

Permanent shock

For the permanent shock, money decreases about 11 percent during the first two years. The price level increases at a rate lower than its long-run growth rate for the first six months. Real money decreases slowly towards its new steady state. After one year, it decreases about 12 percent from its initial level and is 2 percent higher than the new steady state level. The behavior of money after the shock is compatible with the estimations in Christiano et al. (1999): money decreases for two quarters

Third, the price level in AAE was assumed constant at the moment of the shock while I assume a constant Lagrange multiplier across the two states, derived from the fact that agents can trade bonds contingent on the states. I also calculated the dynamics with a constant price in the moment of the shock, as in AAE, and I obtained a similar path for prices as presented here.

after a contractionary monetary policy. In the long run, real balances decrease, as the interest rate is higher, and prices and money grow at the inflation rate, one percent per year, equal to the difference between nominal and real interest rates. After the initial stickiness in the price level, there is a sharp overshooting as agents initially synchronize the timing of their response to the shock. This effect is common to both shocks, I analyze it in more detail below.

In the present model, real balances *decrease* after a permanent increase in the nominal interest rate. With fixed transfer times, long-run real balances are approximately constant or *increase*[34]. The key to economize on money is to decrease holding periods when inflation is high. With fixed transfer times, real balances are approximately constant after the shock (they increase 0.1 percent). Agents change their consumption pattern within holding periods, but this is not enough to decrease real balances. A negative long-run relation between interest rates and real balances is a well-known empirical fact. The present model generates this fact. The steady states with fixed and endogenous transfer times are similar if the transfer intervals are the same. But the transition after the shock is very different.

Another difference from models with constant transfer times is that, in these models, the effects of a permanent change in the interest rate last for only one holding period, 209 days with N fixed. I find more persistent effects. This is a surprising result because the real effects could vanish as agents adjusted their transfer times. The changes in the transfer times work as a propagation mechanism.

Temporary shock

For the temporary interest-rate increase, nominal and real balances decrease 5 percent during the first year. The price level increases towards its new steady state level in the long run, 0.6 percent higher than its initial value. It does not jump to a

[34] This result is mentioned by Romer (1986). Grossman (1987) also mentions that the interest elasticity is close to zero with fixed transfer periods.

higher value as we would have in the usual CIA model[35]. The price level just after the temporary shock behaves within the ranges of the empirical estimation in Uhlig (2005). In the long run, however, the model predicts an increase in the price level[36]. With the gradual decrease in the interest rate, agents will be eventually willing to hold more real balances. As a result, prices increase.

As the nominal interest rate returns to its initial value, equal to the real interest rate, the inflation rate returns to zero. Hence, in the long run, prices and money are constant and real balances return to their initial level. Nominal money is higher than its initial level because there is inflation during the transition. As real balances return to its initial level, there must be an increase in the quantity of money to offset the increase in prices. Prices and nominal money both increase 0.6 percent in the long run.

The price level falls for both shocks and returns to its initial level only after around 30 days. During the first quarter, the price level is close to constant: the difference between the geometric mean of the price level during the first quarter and the initial price level is only 0.01 percent for the permanent shock and 0.02 percent for the temporary shock. A researcher with access to the average price level during this period would probably conclude that the price level is sticky after the shocks.

The reason for the effects in the short run and the dampened oscillations is the different behavior of agents according to their initial balances. The transfer cost makes agents economize in the use of money to avoid making a transfer too soon. Agents with more balances will not increase their consumption rate within holding periods as they would without the transfer cost. Agents with little balances make

[35] Note that this is the response of an increase in the interest rate, not a temporary contraction of the money supply. Grossman and Weiss (1983) and other models of market segmentation study the response after money supply shocks. Grossman (1987), with fixed segmentation (but with a proportional transfer cost for transfers within periods), also study changes after an increase in the interest-rate and find a long-run increase in the price level.

[36] Note that Uhlig (2005) restricts the behavior of the price level: he considers only dynamics with a decreasing price level.

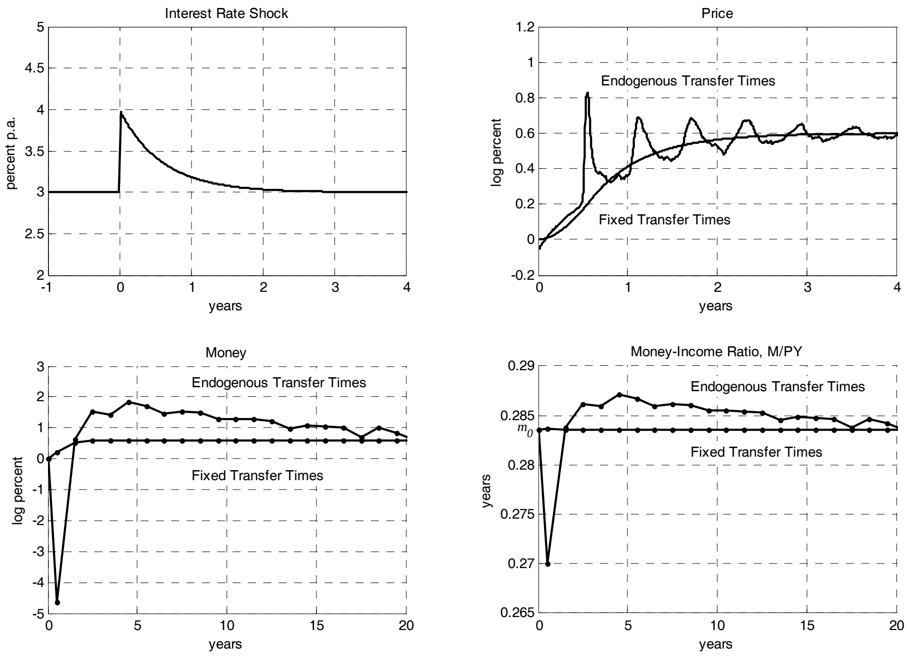

FIG. 3. Results of a temporary interest rate shock from 3 to 4 percent per year. See caption in figure (2) for notes and definitions.

a transfer sooner and can readjust their balances and consumption patterns. They consume at a faster rate because they decrease the interval between transfers to reflect the higher interest rate. Initially, the number of agents that have made a transfer after the shock is relative small. Consequently, prices do not change instantaneously with the change in the nominal interest rate.

After about six months, two groups of agents with different consumption patterns meet. The first group is composed of agents who had little balances and were about to make a transfer when the shock hit the economy. They are now making their second transfer. The second group is composed of those who had substantial money holdings at the time of the shock and have not made a transfer since that time. When the two groups meet, there is a fast increase in the price level because the first group consumes at a faster rate and are now making the second transfer. The new steady state interval between transfers for 4 percent interest rate is 181 days, as stated above, approximately six months. The temporary fluctuations in the price level at intervals of six months reflect these large groups of households synchronizing the timing of their response to the shock.

Prices become smooth because agents pay the transfer cost in goods. The synchronization of transfers is temporary as prices increase at these dates. Hence, agents change their transfer times to periods in which prices are lower. This behavior eventually makes the number of transfers per day constant and the economy converges to the new steady state. The redistribution of transfers is slow. The economy experiences changes in the price level five years after the shock[37]. With transfer cost in utility terms, the economy lacks this price incentive: the price level disappears from the first order conditions and, in contrast, prices and money do not converge.

In the long run, with an increase in the interest rate, agents have more real balances than they would like to have. As a consequence, each agent tries to spend more than with a lower interest rate. As total demand cannot be higher than total output, and output is constant, the price level increases. This explanation for the effects of an interest rate increase is also in Friedman (1969).

Although the short and long run implications of the model are compatible with the empirical evidence, as claimed above, the sharp fluctuations in the price level, temporarily higher than the steady state changes, are not. The model focus on only one mechanism of convergence, the change in the transfer periods, and abstracts from several other elements present in the actual economies. With the comparison of the same version of the model with fixed N, we can understand what is the role

[37]A different calibration would make the transition faster, but the qualitative aspects would not change. For example, if we consider only the period after 1980, with higher velocity, γ decreases to 1.18. This implies smaller intervals $N = 170$ days under $r = 3$ percent and $N = 147$ days under 4 percent. The oscillation would occur at intervals of five months, decaying over time.

of endogenous segmentation. It is beyond the objectives of this paper to predict all price and money movements with this single modification. We would need to add other elements to the model such as different types of agents, endogenous production, and other kinds of shocks[38].

Friedman (1969) studies the effects of a once-and-for-all change in the quantity of money and of a continuous increase in the quantity of money. I relate the two exercises with the temporary and permanent increase in the interest rate. The reason is that the final effect of a temporary increase in the interest rate is a once-and-for-all increase in the quantity of money, and the final effect of a permanent increase is a continuous increase in the quantity of money. The present model gives an analytical justification for the effects of the shocks. The advantage is that now we can quantify the changes in prices and money, and we can give a meaning to what we understand by the short and long run. With the calibration of section 3, most of the effects of the policy shocks occur within the first six months, and the long run stands for the behavior after two years, as prices and money are close to their new steady state values[39].

5. CONCLUSIONS

I introduce a monetary model to calculate the effects of interest rate shocks on prices and money. The only departure from the cash in advance model is a transfer cost whenever agents exchange bonds for money. I study two shocks: a permanent and a temporary increase in the nominal interest rate. The implications of the model are in accordance with the empirical evidence on the short and long run behavior of

[38] Khan and Thomas (2007) obtain smaller price fluctuations with stochastic transfer costs and stochastic changes in the growth rate of money, together with contingent bonds on the transfer cost and on the aggregate state.

[39] We can also be more specific about the predictions. For example, the path of the price level after a permanent change is close to the price level C in figure 4 of Friedman (1969). Friedman also predicts the overshooting in the price level.

prices and money. The price level slowly adjusts to the new steady state. Nominal and real balances decrease after the shocks and slowly adapt to the shocks.

Agents optimally adjust their transfer intervals. This assumption changes results in important ways. First, nominal and real balances decrease after the shocks. Second, real balances eventually decrease with a permanent interest rate increase. Third, the effects of the shocks last longer. The model with fixed transfer periods does not generate these facts. In particular, with fixed transfer periods, money does not decrease after the shocks, and real balances increase when the interest rate increases. The models with endogenous and fixed transfer periods are similar in the steady state, but they have very different behavior after the shocks.

The key parameter obtained with the calibration is the transfer cost value. The data implies a high transfer cost and, in turn, a large support for the distribution of money holdings. This increases the persistence of the shock. We obtain convergence because the transfer cost is paid in goods. Agents avoid making transfers when prices are high as the transfer cost is higher during these periods. The variation in the price level makes the number of transfers in each day converge. The movement of agents to rearrange their transfer intervals works as a propagation mechanism of the interest rate shock.

The model does not have several features present in the actual economies to focus on the mechanism of endogenous segmentation. This procedure emphasizes the role of endogenous segmentation but produces volatile prices just after the shocks. As an advantage, the comparison of the transition in the cases with fixed and endogenous transfers highlights how endogenous segmentation alone is able to explain the observed movements of prices and money.

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APPENDIX A - FIRST ORDER CONDITIONS AND PROOFS

The Lagrangian of the problem in (1), (4) and (5) is $\mathcal{L} = \sum_{j=0}^{\infty} \int_{T_j}^{T_{j+1}} e^{-\rho t} u(c(t))\, dt + \lambda(M_0, W_0)\left[W_0 + Q(T_1) K(M_0, W_0) - \sum_{j=1}^{\infty} Q(T_j) \int_{T_j}^{T_{j+1}} P(t) c(t, M_0, W_0)\, dt - \sum_{j=1}^{\infty} Q(T_j) P(T_j) Y\gamma\right] + \mu(M_0, W_0)\left[M_0 - \int_0^{T_1} P(t) c(t, M_0, W_0)\, dt - K(M_0, W_0)\right]$, where $T_j = T_j(M_0, W_0)$.

The first order conditions with logarithmic utility are – see Silva (2007) for general constant relative risk aversion –

$c(t, M_0, W_0)$: for $t \in [T_j, T_{j+1}]$, $j = 1, 2, ...$

$$\begin{aligned} P(t) c(t, M_0, W_0) &= e^{-\rho t} / [\lambda(M_0, W_0) Q(T_j)], \text{ for } t \in (T_j, T_{j+1}) \\ P(T_j) c^{+}(T_j, M_0, W_0) &= e^{-\rho T_j} / [\lambda(M_0, W_0) Q(T_j)], \\ P(T_{j+1}) c^{-}(T_{j+1}, M_0, W_0) &= e^{-\rho T_{j+1}} / [\lambda(M_0, W_0) Q(T_j)]; \end{aligned}$$

and, for $t \in [0, T_1]$,

$$\begin{aligned} P(t) c(t, M_0, W_0) &= e^{-\rho t} / \mu(M_0, W_0), \quad t \in (0, T_1), \\ P(0) c^{+}(0, M_0, W_0) &= 1/\mu(M_0, W_0), \\ P(T_1) c^{-}(T_1, M_0, W_0) &= e^{-\rho T_1} / \mu(M_0, W_0). \end{aligned}$$

T_1 :

$$\begin{aligned} &e^{-\rho T_1} \log c^{-}(T_1) - e^{-\rho T_1} \log c^{+}(T_1) = \lambda \left[\dot{Q}(T_1) \int_{T_1}^{T_2} P(t) c(t)\, dt - \dot{Q}(T_1) K \right. \\ &\quad \left. - Q(T_1) P(T_1) c^{+}(T_1)\right] + \mu P(T_1) c^{-}(T_1) + \lambda Y \gamma \left[\dot{Q}(T_1) P(T_1) + Q(T_1) \dot{P}(T_1)\right]; \end{aligned}$$

T_j, $j = 1, 2, ...$:

$$\begin{aligned} e^{-\rho T_j} \log c^{-}(T_j) - e^{-\rho T_j} \log c^{+}(T_j) = \lambda &\left[\dot{Q}(T_j) \int_{T_j}^{T_{j+1}} P(t) c(t)\, dt \right. \\ &\left. - Q(T_j) P(T_j) c^{+}(T_j) + Q(T_{j-1}) P(T_j) c^{-}(T_j)\right] \\ &+ \lambda \gamma Y \left[\dot{Q}(T_j) P(T_j) + Q(T_j) \dot{P}(T_j)\right]. \end{aligned}$$

K : $Q(T_1) \lambda(M_0, W_0) - \mu(W_0, M_0) \leq 0$ ($= 0$ if $K > 0$); and the budget constraints.

Using the budget constraint and the first order condition with respect to consumption, we have $\lambda\left(M_0, W_0\right)=\frac{e^{-\rho T_1}}{\rho} \times\left[W_0+Q\left(T_1\right) K-\gamma Y \sum_{j=1}^{\infty} Q\left(T_j\right) P\left(T_j\right)\right]^{-1}$. The value of λ in the steady state is $\lambda=\dfrac{1}{P_0 c_0}$. Working analogously for $\mu\left(M_0, W_0\right)$ with the budget constraint for $0 \leq t<T_1$, we obtain $\mu\left(M_0, W_0\right)=\frac{1}{M_0-K} \frac{1-e^{-\rho T_1}}{\rho}$.

Proposition 1. *Proof.* The first order condition with respect to T_j, $j \geq 2$, implies

$$\begin{aligned}\frac{1}{\lambda} \frac{e^{-\rho T_j}}{P\left(T_j\right) Q\left(T_j\right)} \log \frac{c^{+}\left(T_j\right)}{c^{-}\left(T_j\right)}= & r\left(T_j\right) \int_{T_j}^{T_{j+1}} \frac{P(t) c(t)}{P\left(T_j\right)} d t \\ & +c^{+}\left(T_j\right)-\frac{Q\left(T_{j-1}\right)}{Q\left(T_j\right)} c^{-}\left(T_j\right)+\gamma Y\left[r\left(T_j\right)-\pi\left(T_j\right)\right] .\end{aligned}$$

With the first order conditions for $c(t)$, this expression simplifies to

$$c^{+}\left(T_j\right) \log \frac{Q\left(T_{j-1}\right)}{Q\left(T_j\right)}=r\left(T_j\right) \int_{T_j}^{T_{j+1}} \frac{P(t) c(t)}{P\left(T_j\right)} d t+\gamma Y\left[r\left(T_j\right)-\pi\left(T_j\right)\right] . \tag{16}$$

In the steady state, $r(t)=r$, $\pi(t)=\pi$ and $r=\rho+\pi$. Moreover, $Q\left(T_{j-1}\right) / Q\left(T_j\right)= e^{r T_j-r T_{j-1}}=e^{r N_j}$, $N_j=N$ and $c^{+}\left(T_j\right)=c_0$. So, (16) simplifies to

$$c_0 r N=r \int_{T_j}^{T_{j+1}} \frac{P(t) c(t)}{P\left(T_j\right)} d t+\rho \gamma Y .$$

Using $c(t)=c_0 e^{-r\left(t-T_j\right)}$ and $P(t)=P_0 e^{\pi t}$ yields

$$c_0 r N=r c_0 e^{\rho T_j} \int_{T_j}^{T_{j+1}} e^{-\rho t} d t+\rho \gamma Y .$$

Solving the integral and rearranging yields the result in the body of the text.

For existence and uniqueness, define the functions $a, b, G:(\gamma,+\infty) \rightarrow R$ by

$$a(N)=\left(\frac{1-e^{-r N}}{r N}\right)\left(1-\frac{\gamma}{N}\right)^{-1},$$

$$b(N)=r N-\frac{r}{\rho}\left(1-e^{-\rho N}\right),$$

and

$$G(N)=b(N)-\rho \gamma a(N) .$$

$a(N) = Y/c_0(N)$. The optimal interval N^* is such that $G(N^*) = 0$.

$\lim_{N\to\gamma_+} a(N) = +\infty$ and $\lim_{N\to\gamma_+} b(N)$ is bounded. Therefore, $\lim_{N\to\gamma_+} G(N) = -\infty$. Also, $\lim_{N\to\infty} a(N) = 0$. Intuitively, for a given value of r, if N increases then most of the consumption happens in the beginning of a holding period and c_0 is very large. Moreover, $b'(N) > 0$, and $a'(N) = \dfrac{1}{(N-\gamma)^2}\dfrac{1}{re^{rN}}\left(1 + rN - e^{rN} - \gamma re^{rN}\right) < 0$ because $e^{rN} > 1 + Nr$. Hence, $G'(N) = b'(N) - \rho\gamma a'(N) > 0$.

Even though G is increasing, it can be the case that $\lim_{N\to+\infty} G(N) < 0$. This possibility is ruled out because $\lim_{N\to\infty} b'(N) = r$. Therefore, there exists an N^* such that $G(N^*) = 0$. As G is increasing, N^* is unique.

For $\frac{\partial N}{\partial r} = -\frac{\partial G(r;N)/\partial r}{G'(N)}$. We know that $G'(N) > 0$. On the other hand, $a_r(r;N) = \frac{1+rN-e^{rN}}{e^{rN}r^2N}\left(1 - \frac{\gamma}{N}\right)^{-1} < 0$ and $b_r(r;N) = N\left(1 - \frac{1-e^{-N\rho}}{N\rho}\right) > 0$. Hence, $G_r(r;N) = b_r(r;N) - \rho\gamma a_r(r;N) > 0$ and $\partial N/\partial r < 0$.

For $\frac{\partial N}{\partial \gamma} = -\frac{\partial G(r;N)/\partial \gamma}{G'(N)}$. $G_\gamma(\gamma;N) = -\rho a(N) - \rho\gamma a_\gamma(N)$, $a_\gamma(N) = \frac{1-e^{-rN}}{rN}\frac{N}{(N-\gamma)^2} > 0$. Thus, $G_\gamma(\gamma;N) < 0$ and $\partial N/\partial\gamma = -G_\gamma(\gamma;N^*)/G'(N^*) > 0$.■

Proposition 2. *Proof.* $M_0(n)$ allows agents to consume exactly at the steady state rate in the interval $[0,n)$. This value is such that $M(n) = \int_0^n P(t)\,c(t)\,dt$. $c(0)$ is not necessarily equal to the level of consumption just after a transfer, c_0. This is only true for the agent $n = 0$. We know that $c^-(n) = c_0e^{-rN}$, for all $n \in [0,N)$, and that $\dot{c}/c = -r$. Solving this differential equation yields $c(x,n) = c_0e^{rn}e^{-rN}e^{-rx}$, $0 \le x < n$. Therefore, we obtain the value of $M(n)$ solving the integral $M(n) = \int_0^n P_0e^{\pi t}c_0e^{rn}e^{-rN}e^{-rt}dt$.

For $W_0(n)$. First, the value of money needed in each holding period is given by

$$M_j = \int_{n+(j-1)N}^{n+jN} P(t)\,c_0e^{-r(t-T_j)}dt,$$

$j = 1, 2, \ldots$ and $T_j = n + (j-1)N$. So, $M_j = P_0c_0e^{\pi n}\frac{1-e^{-\rho N}}{\rho}e^{\pi(j-1)N} \equiv \bar{M}e^{\pi(j-1)N}$.

The value at $t = n$ of these transfers is $A_M \equiv \bar{M}\frac{1}{1-e^{-\rho N}}$. For the transfer cost, we have $TC_j = \gamma YP(n + (j-1)N) = P_0\gamma Ye^{\pi(n+(j-1)N)}$, $j \ge 1$. Working analogously, $A_{TC} \equiv P_0\gamma Ye^{\pi n}\frac{1}{1-e^{-\rho N}}$. Finally, the value of $W(n)$ is given by

$$W(n) = e^{-rn}A_M + e^{-rn}A_{TC}.\blacksquare$$

APPENDIX B - EQUATIONS FOR SECTION 4

The maximization problem of each agent is

$$\max \quad \theta \sum_{j=0}^{\infty} \int_{T_j(1)}^{T_{j+1}(1)} e^{-\rho t} u(c(t,1))\, dt + (1-\theta) \sum_{j=0}^{\infty} \int_{T_j(2)}^{T_{j+1}(2)} e^{-\rho t} u(c(t,2))\, dt$$

subject to

$$\sum_{s=1,2} \left[\sum_{j=1}^{\infty} Q(T_j(s), s) \int_{T_j(s)}^{T_{j+1}(s)} P(t,s)\, c(t,s)\, dt \right.$$
$$\left. + \sum_{j=1}^{\infty} Q(T_j(s), s)\, P(T_j(s), s)\, \gamma Y \right] = W_0 + \sum_{s=1,2} Q(T_1(s), s)\, K(s),$$

$$\int_0^{T_1(s)} P(t,s)\, c(t,s)\, dt + K(s) = M_0,\ s = 1, 2,$$

where $W_0 \equiv B_0 + \sum_{s=1,2} \int_0^{\infty} Q(t,s)\, Y P(t,s)\, dt$ denotes deposits in the brokerage account, and $s = 1, 2$ denotes the two possible states.

The first order conditions with respect to $c(t)$ are analogous to the ones described in appendix A. The first order conditions with respect to $c(t)$ and T_j in the state 2 imply, for $j \geq 2$,

$$c^+(T_j)\,[R(T_j) - R(T_{j-1})] - \gamma Y\,[r(T_j) - \pi(T_j)] = r(T_j) \int_{T_j}^{T_{j+1}} \frac{P(t)\, c(t)}{P(T_j)} dt,$$

where $c^+(T_j) = \left[\lambda e^{\rho T_j} Q(T_j)\, P(T_j)\right]^{-1}$. For T_1, the first order conditions imply

$$c^+(T_1)\, R(T_1) - \gamma Y\,[r(T_1) - \pi(T_1)] - \log \frac{\lambda}{\mu} + \frac{r(T_1)\, K}{P(T_1)} = r(T_1) \int_{T_1}^{T_2} \frac{P(t)\, c(t)}{P(T_1)} dt.$$

APPENDIX C - DATA

I am using a similar data set as the one used in Lucas (2000).

GDP

From 1900 to 1928 it is from the Bureau of the Census (1975), *Historical Statistics of the United States: Colonial Times to 1970.* Series F1, Nominal GDP. From 1929 to 2000 it is from NIPA, Tables 1.1.5, 1.1.6.

Interest Rate

The nominal interest rate is the short commercial paper rate. From 1900 to 1975 it is from Friedman and Schwartz (1982), *Monetary trends in the United States and the United Kingdom: their relation to income, prices and interest rates, 1875-1975*, Chicago: University of Chicago Press, Table 4.8, column 6, p. 122, "Interest Rate, Annual Percentage, Short-Term, Commercial Paper Rate". From 1976 to 1997 it is from the *Economic Report of the President*, Table B-73 "Bond Yields and Interest rates". In Friedman and Schwartz, the data are for commercial paper 60 to 90 days before 1924, and 4 to 6 months thereafter. In the Economic Report of the President, the data are for commercial paper 4 to 6 months before 1980, and 6 months thereafter.

Money

From 1900 to 1913, it is from the Bureau of the Census (1960), *Historical Statistics of the United States: colonial times to 1957*, Series X-267, "demand deposits adjusted plus currency outside banks". From 1914 to 1958 it is from Friedman and Schwartz (1963), *A Monetary History of the United States, 1867-1960,* December of each year, seasonally adjusted. For M1, I used column 7, sum of currency and demand deposits. From 1959 to 1997 it is from the Federal Reserve Bank of St. Louis, FRED Database, series M1SL, December of each year, seasonally adjusted.

APPENDIX D - ALGORITHM

The objective is to find the price path $P(t)$ after the announcement of the interest rate $r(t)$ from time zero and on. The economy is initially in the steady state with the nominal interest rate r_1 equal to 3 percent per year.

We need first to describe the situation in the steady state before the shock. With the values of r_1, γ and ρ we find N in the initial steady state with proposition 1. The values of production Y and money supply before the shock M_0^S are normalized to 1. The price level before the shock is given by $P_0 = M_0^S/m$, where m are real balances. With N, we index agents by $n \in [0, N)$ and find the initial values of money that should be given to each agent in order to be in the steady state, $M_0(n)$, by proposition 2.

With the announcement of the new policy $r(t)$, agents decide how much to consume at each time $c(t, n)$ and when to make a transfer $T_j(n)$. For the logarithmic case, the optimal transfer times for agent n is given by the system of equations

$$R(T_1) - \lambda Q(T_1) \frac{\gamma Y P(T_1)}{e^{-\rho T_1}} [r(T_1) - \pi(T_1)] - \log \frac{\lambda}{\mu(n)} + r(T_1) \lambda \frac{e^{-R(T_1)}}{e^{-\rho T_1}} K = r(T_1) \frac{1 - e^{-\rho N_2}}{\rho}, \quad (17)$$

$$\left[R\left(T_j\right)-R\left(T_{j-1}\right)\right]-\lambda Q\left(T_j\right)\frac{\gamma Y P\left(T_j\right)}{e^{-\rho T_j}}\left[r\left(T_j\right)-\pi\left(T_j\right)\right]=r\left(T_j\right)\frac{1-e^{-\rho N_{j+1}}}{\rho}, \quad (18)$$

for $j \geq 2$. If $K=0$ then $\mu(n)=\frac{1-e^{-\rho T_1(n)}}{\rho M_0(n)}$ and equation (17) simplifies to

$$R\left(T_1\right)-\frac{e^{-R(T_1)}}{e^{-\rho T_1}}\gamma Y\frac{P\left(T_1\right)}{P_0 c_0}\left[r\left(T_1\right)-\pi\left(T_1\right)\right]-\log\frac{\lambda}{\mu}=r\left(T_1\right)\frac{1-e^{-\rho N_2}}{\rho},$$

we have to check the condition $\mu(n)>Q\left(T_1\right)\lambda$ in this case. If $K>0$ then $\mu/\lambda = Q\left(T_1\right)$. As stated in the body of the text, the value of λ is the value before the shock, $\lambda=1/\left(P_0 c_0\right)$. Given the optimal transfer times, we find optimal consumption for agent n by

$$c(t)=\frac{e^{-\rho t}}{\lambda P(t) Q\left(T_j\right)},\ c^{+}\left(T_j\right)=\frac{e^{-\rho T_j}}{\lambda P\left(T_j\right) Q\left(T_j\right)},\ c^{-}\left(T_{j+1}\right)=\frac{e^{-\rho T_{j+1}}}{\lambda P\left(T_{j+1}\right) Q\left(T_j\right)}, \quad (19)$$

for $T_j(n) \leq t \leq T_{j+1}(n)$, $j \geq 1$ and

$$c(t)=\frac{e^{-\rho t}}{\mu P(t)},\ c\left(0^{+}\right)=\frac{1}{\mu P\left(0^{+}\right)},\ c^{-}\left(T_1\right)=\frac{e^{-\rho T_1}}{\mu P\left(T_1\right)}, \quad (20)$$

for $0 \leq t \leq T_1(n)$. We have $Q(t)=e^{-R(t)}$, $R(t)=\int_0^t r(s)\,ds$. For the permanent shock, $r(t)=r_2$ and $R(t)=r_2 t$. For the temporary shock, $r(t)=r_1+\left(r_2-r_1\right)e^{-\eta t}$ and $R(t)=r_1 t-\frac{r_2-r_1}{\eta}e^{-\eta t}+\frac{r_2-r_1}{\eta}$.

In order to have a finite system with (18) and (17), I assume that agents choose the new steady state interval after a long period under the new interest rate. That is, $N_{J+1}=N'$ where N' is the steady state under the new interest rate. We have then a system of J equations in N_1, ..., N_J for each agent. We can solve this system for a given price path $P(t)$. In the simulations, $J=40$ which implies $N_{41}=N'$ in about 20 years after the shock.

In order to solve the system for each n, the interval $[0,N)$ is discretized as $\{n_1, n_2,\ldots, n_{\max}\}$ where $n_1=0$ and $n_{\max}$ is smaller than N but sufficiently close. In the simulations, the number of agents is such that $n_{i+1}-n_i$ is equal to 0.10 day. This implies $2,094$ agents for the parameters used.

We update the price path with the market clearing condition for goods. We have to sum consumption at time t for each agent and total resources used for transfers at time t to find aggregate demand. In equilibrium, we have,

$$\frac{1}{n_{\max}}\sum_n c(t,n;P)+\frac{1}{n_{\max}}\gamma Y\times \textit{Number of Transfers}\,(t;P)=Y,$$

for each time t, where P stands for the path of the price level. The consumption values are given by equations (19) and (20). The left-hand side of this equation is equal to the aggregate demand and the right-hand side is equal to the aggregate supply. The number of transfers at t is calculated summing the agents with $T_j(n)$ such that $t \leq T_j(n) < t+1$, that is, the unit of time is one day. Several agents make a transfer at each day. The left-hand side is divided by $n_{\max}$ because the density of agents is uniform over $[0, N)$. If demand is higher than supply at time t then increase $P(t)$ and recalculate the optimal transfer intervals for the new price. If demand is lower than supply at time t, decrease $P(t)$.

For money demand. Given the values of individual spending $P(t)\,c(t)$ implied by the first order conditions, individual money demand is $M(t,n) = e^{R(T_j)}\,(e^{-\rho t} - e^{-\rho T_{j+1}})/(\lambda\rho)$, for $T_j \leq t < T_{j+1}$, $j \geq 1$, and $M(t,n) = \left(e^{-\rho t} - e^{-\rho T_1}\right)/(\mu(n)\,\rho)$ for $0 \leq t < T_1$. With the values of T_j, we obtain individual money demand for each agent. We then aggregate over agents to find aggregate money demand at time t.

The program used for the code is Matlab. The initial guess for the price path is $P(t) = P_0 e^{\pi t}$ where π is inflation in the new steady state. Several other simulations were performed with different numbers of intervals (different J's), number of agents, transfer costs, and initial guesses for prices. These changes do not affect the qualitative behavior of the price level or of the other equilibrium variables.

APPENDIX E - TRANSITION WITH FIXED TIME INTERVALS

This section shows the calculations for the transition with fixed time intervals in figures (3) and (1). These calculations are done to compare to the transition in this paper with endogenous transfer periods. The utility maximization problem is similar to the one with endogenous transfers. The difference is that now agents optimize within their transfer intervals but they cannot change the moments of the transfers.

Agents are indexed by $n \in [0, N)$ for a fixed holding period N, equal to the value in the initial steady state for the economy with endogenous transfers. The first order conditions for consumption are the same as in the problem with endogenous transfer periods. They are $c(t,n) = e^{-\rho t}/(\mu(n)\,P(t))$ for $0 < t < T_1$ and $c(t,n) = e^{-\rho t}/(\lambda Q(T_j)\,P(t))$, for $T_j < t < T_{j+1}$. The initial money holdings are given by proposition 2. They are such that $T_1(n) = n$. The initial quantity of money is $M_0^S = 1$, the price level before the shock is $P_0 = M_0^S/m$, where m is real money demand before the shock, the interest rate before the shock is $r_1 = \rho$. We have $\lambda = 1/(P_0 c_0)$. The interest rate after the shock is equal to $r(t) = r_2$ for the permanent shock and equal to $r(t) = r_1 + (r_2 - r_1)\,e^{-\eta t}$ for the temporary shock.

For an arbitrary $t > jN$, $t < (j+1)\,N$, agents will be in their jth or $(j+1)$th holding period, $j \geq 0$. Taking this into account, aggregate consumption for $t > N$ is

$$C(t) = \frac{1}{N}\int_0^{t-jN} \frac{e^{-\rho t}}{\lambda Q(T_{j+1}) P(t)} dn + \frac{1}{N}\int_{t-jN}^{N} \frac{e^{-\rho t}}{\lambda Q(T_j) P(t)} dn.$$

This yields

$$C(t) = \frac{1}{N}\int_0^{N} \frac{e^{-\rho t} e^{R(t-x)}}{\lambda P(t)} dx.$$

Analogously, for $0 \leq t < N$, aggregate consumption is

$$C(t) = \frac{1}{N}\int_0^{t} \frac{e^{-\rho t}}{\lambda Q(T_1) P(t)} dn + \frac{1}{N}\int_{t}^{N} \frac{e^{-\rho t}}{\mu(n) P(t)} dn.$$

With $K = 0$, $\mu(n) = \frac{1-e^{-\rho T_1(n)}}{\rho M_0(n)}$. With the values of $M_0(n)$ in proposition 2, $\mu(n) = \frac{1}{P_0 c_0 e^{r_1 n} e^{-r_1 N}}$. The condition for $K = 0$ is verified for the permanent and for the temporary shocks. Therefore, aggregate consumption for $0 \leq t < N$ is

$$C(t) = \frac{1}{N}\int_0^{t} \frac{e^{-\rho t} e^{R(t-x)}}{\lambda P(t)} dx + \frac{e^{-\rho t} P_0 c_0 e^{-r_1 N}}{N P(t)} \frac{e^{r_1 N} - e^{r_1 t}}{r_1}.$$

The market clearing condition is given by $C(t) + \gamma Y \dfrac{1}{N} = Y$ for all $t \geq 0$. As $T_j(n)$ are fixed, there is always the same number of agents making transfers at each time. The transfer cost γ is included in order to have results the most comparable as possible with the model with endogenous N. It does not affect the paths in the figures as they are shown as a percentage of the value before the shock. It does not affect the result that real balances increases with the interest rate for a fixed N.

We can isolate the price level at each time with the market clearing condition and the interest rate path.

For the permanent shock, we obtain

$$P(t) = e^{(r_2-\rho)t} \frac{P_0 c_0}{Y} \left(1 - \frac{\gamma}{N}\right)^{-1} \frac{1 - e^{-r_2 N}}{N r_2},$$

for $t \geq N$ and

$$P(t) = e^{(r_2-\rho)t} \frac{P_0 c_0}{Y} \left(1 - \frac{\gamma}{N}\right)^{-1} \left(\frac{1 - e^{-r_2 t}}{N r_2} + e^{-r_2 t} \frac{1 - e^{-r_1 (N-t)}}{N r_1}\right),$$

for $0 \leq t < N$. Note that inflation is constant for $t \geq N$. An economy with fixed transfer intervals N reaches the steady state in exactly N periods after the new steady state interest rate is set. Also, $P(0)$ is equal to the price before the shock, P_0, and $P(t)$ is continuous at $t = N$.

For the temporary shock, we obtain

$$P(t) = \frac{P_0 c_0}{Y}\left(1 - \frac{\gamma}{N}\right)^{-1} \frac{e^{\frac{r_2 - r_1}{\eta}}}{N} \int_0^N e^{-r_1 x} e^{-\frac{r_2 - r_1}{\eta} e^{-\eta t} e^{\eta x}} dx,$$

for $t \geq N$ and

$$P(t) = \frac{P_0 c_0}{Y}\left(1 - \frac{\gamma}{N}\right)^{-1} \left(\frac{e^{\frac{r_2 - r_1}{\eta}}}{N} \int_0^t e^{-r_1 x} e^{-\frac{r_2 - r_1}{\eta} e^{-\eta t} e^{\eta x}} dx + e^{-\rho t} \frac{1 - e^{-r_1 (N-t)}}{N r_1}\right),$$

for $0 \leq t < N$. $P(t)$ is continuous at $t = N$ and $P(0)$ is equal to the price before the shock, P_0.

For the money demand, first use the first order conditions to obtain individual spending for agent n: $P(t)\,c(t,n) = e^{R(T_j)} e^{-\rho t}/\lambda$, $t \in [T_j, T_{j+1})$, $j \geq 1$. Thus, individual money demand at $T_j \leq t < T_{j+1}$ is $M(t,n) = e^{R(T_j)} \int_t^{T_{j+1}} e^{-\rho t}/\lambda dt$. Analogously, individual money demand at $0 \leq t < T_1$ is $M(t,n) = \int_t^{T_{j+1}} e^{-\rho t}/\mu(n)\,dt$. For $t > jN$, agents will be in their jth or $(j+1)$th holding period, $j \geq 1$. Therefore, aggregate money demand is

$$M(t) = \frac{1}{N} \int_0^{t-jN} e^{R(T_{j+1})} \int_t^{T_{j+2}} \frac{e^{-\rho t}}{\lambda} dt dn + \frac{1}{N} \int_{t-jN}^{N} e^{R(T_j)} \int_t^{T_{j+1}} \frac{e^{-\rho t}}{\lambda} dt dn.$$

As the transfer timings are fixed, $T_j \equiv n + (j-1)N$. With a change of variables and simplification, we obtain

$$M(t) = \frac{1}{N} \int_0^N e^{R(t-x)} \frac{e^{-\rho t} - e^{-\rho(t-x+N)}}{\lambda \rho} dx.$$

For $0 \leq t < N$, working analogously,

$$M(t) = \frac{1}{N} \int_0^t \frac{e^{R(n)}}{\lambda} \frac{e^{-\rho t} - e^{-\rho(n+N)}}{\rho} dn + \frac{1}{N} \int_t^N \frac{1}{\mu(n)} \frac{e^{-\rho t} - e^{-\rho n}}{\rho} dn,$$

where $\mu(n) = 1/\left(P_0 c_0 e^{r_1 n} e^{-r_1 N}\right)$. Note that if $r(s) = r_1$ we have the same steady state formulas for the money demand.

Substituting the interest rate path and solving the integrals above, we obtain the nominal money demands as follows.

For the permanent shock, the nominal money demand is

$$M(t) = \frac{P_0 c_0}{\rho} e^{(r_2-\rho)t} \left(\frac{1 - e^{-r_2 N}}{N r_2} - e^{-\rho N} \frac{1 - e^{-(r_2-\rho)N}}{N(r_2 - \rho)} \right)$$

for $t \geq N$, and

$$M(t) = \frac{P_0 c_0}{\rho} e^{(r_2-\rho)t} \left(\frac{1 - e^{-r_2 t}}{N r_2} - e^{-\rho N} \frac{1 - e^{-(r_2-\rho)t}}{N(r_2 - \rho)} \right) + \frac{P_0 c_0}{\rho} \left(\frac{e^{-r_1 t} - e^{-r_1 N}}{N r_1} - e^{-\rho N} \frac{N - t}{N} \right)$$

for $0 \leq t < N$.

For the temporary shock, the nominal money demand is

$$M(t) = \frac{e^{\frac{r_2 - r_1}{\eta}}}{N} \frac{P_0 c_0}{\rho} \int_0^N e^{-\frac{r_2 - r_1}{\eta} e^{-\eta t} e^{\eta x}} \left(e^{-r_1 x} - e^{-\rho N} \right) dx$$

for $t \geq N$, and

$$M(t) = \frac{P_0 c_0}{\rho} \frac{e^{\frac{r_2 - r_1}{\eta}}}{N} \int_0^t e^{-\frac{r_2 - r_1}{\eta} e^{-\eta t} e^{\eta x}} \left(e^{-r_1 x} - e^{-\rho N} \right) dx + \frac{P_0 c_0}{\rho} \left(\frac{e^{-r_1 t} - e^{-r_1 N}}{N r_1} - e^{-\rho N} \frac{N - t}{N} \right)$$

for $0 \leq t < N$.

Real balances are obtained by dividing $M(t)$ by the price level.

In the simulations, the interval between transfer is set to the value before the shock for the economy with endogenous decisions. The economies with endogenous and fixed transfer periods are similar in the steady state if they have the same interval between transfer. The transition after the shock, however, is very different.