Online Appendix for "Aggregate and Intergenerational Implications of School Closures: A Quantitative Assessment"

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A Calibration Details

Most calibration targets are based on samples from the 2003-2017 waves of the ATUS, combined with the Current Population Survey (Yum 2023). Table A1 reports the estimation results that are used to compute the educational gradients in parental time investments. The sample is restricted to households who have any number of children and aged between 21 and 55 (inclusive), as in Guryan et al. (2008). The three periods in the model ($j = 3, 4, 5$) correspond to the youngest children's age bands: ages 0-4, ages 5-9, and ages 10-14, respectively. The coefficient on the dummy college variable, divided by the corresponding average, captures the educational gradient while controlling for parents' sex, age, and marital status. We note that the college coefficients are quite stable regardless of control variables, in line with the evidence in Guryan et al. (2008).

Table A1: Education gradients in parental time investments

	$j = 3$	$j = 4$	$j = 5$
College-educated	1.342	.561	.416
	(.133)	(.109)	(.091)
Sex	-2.62	-1.51	-1.20
	(.123)	(.101)	(.083)
Age	-.041	.016	.023
	(.009)	(.007)	(.006)
Married	-.911	-.318	-.102
	(.085)	(.064)	(.053)
R^2	.023	.014	.017
Average x	6.43	3.78	2.06

Notes: Numbers in parentheses are standard errors. The dependent variable is parental time investments (weekly hours). These estimates are from Yum (2023).

Table A2 reports the gross growth rates of human capital by age and education. These are

Table A2: Gross growth rates of human capital by age and education

$j =$	1	2	3	4	5	6	7	8
$\gamma_{j,1}$	1.231	1.052	1.017	1.004	0.998	0.995	0.994	0.994
$\gamma_{j,2}$	1.317	1.152	1.101	1.063	1.032	1.004	0.975	0.942

Notes: The reported values are based on the estimates from the PSID samples in Rupert and Zanella (2015).

Table A3: Parameter values for progressive taxation

	τ_j	λ_j
$j = 1, 2$	.1106	.8177
$j = 3, ..., 6$	.1585	.9408
$j = 7, 8, 9$	.1080	.8740

Notes: The reported values are based on the estimates in Holter et al. (2019).

computed based on the estimates from the PSID samples in Rupert and Zanella (2015).

Table A3 reports the estimates of τ_j and λ_j in labor taxation by age, obtained from Holter et al. (2019). We use the estimates for single households for $j = 1, 2$, and the estimates for married households for the later periods (either with a child for $j = 3, ..., 6$ or without children for $j = 7, 8, 9$).

B Aggregation order in a Nested CES Technology and Parental Time Responses

In this section, we illustrate the implication of a different order of aggregation in a nested CES technology for parental time responses following school closures. Recall that our baseline technology aggregates parental time and monetary investments, which are then aggregated with public investment (Fuchs-Schündeln et al. 2022; Yum 2023). As discussed in Jones and Manuelli (1999), one could consider an alternative order. Here, we consider a specification where parental monetary investment e and public investment g are aggregated first, and then the composite monetary investment is aggregated with parental time investment x (e.g., Daruich 2022). We will argue that our baseline specification is preferred to the alternative specification, based on a set of different empirical evidence jointly.

First, we note that empirical studies find that parental time increased during the periods of Covid-19 induced school closures. For example, Andrew et al. (2020) find that during the lockdown that involved school closures in 2020, parents in the UK spent much more time with children (including active childcare), as compared to a normal weekday in 2014-15. Our quantitative model can replicate such positive responses in parental time across all children's age groups (see e.g., Figure 3).

To provide guidance on how the model can replicate this pattern, we consider a simplified model framework. This simple framework is particularly useful because it can not only illustrate the implications of different CES aggregations but also isolate the role of the two relevant elasticities of substitution: (i) one between public and private investments and (ii) the other one between parental time and monetary investments.

Consider a household's optimization problem:

$$\max_{c,x,e} \left\{ \log c - bx + \eta \log h' \right\}$$

subject to

$$c + e = m$$

$$h' = \left\{ \left(x^{\zeta} + e^{\zeta} \right)^{\frac{\psi}{\zeta}} + g^{\psi} \right\}^{\frac{1}{\psi}}, \tag{A1}$$

where the human capital investment function (A1) features a nested CES technology with the aggregation order used in our quantitative model.[1] The structure is simple yet is similar to the quantitative model: b denotes the disutility of investing time, η captures altruism, m denotes disposable income, $\psi \leq 1$ shapes the elasticity of substitution between parental monetary investment and public investment, and $\zeta \leq 1$ governs the elasticity of substitution between parental time and the aggregated monetary investments.

The optimal parental time x is a function of g. The top panel of Figure A1 shows how the optimal parental time investment responds with respect to a 10% decline in g with different values of ζ under our baseline CES aggregation. The bottom panel of Figure A1 shows the results when we replace (A1) with the alternative order of aggregation described above:

$$h' = \left\{ x^{\zeta} + \left(e^{\psi} + g^{\psi} \right)^{\frac{\zeta}{\psi}} \right\}^{\frac{1}{\zeta}}. \tag{A2}$$

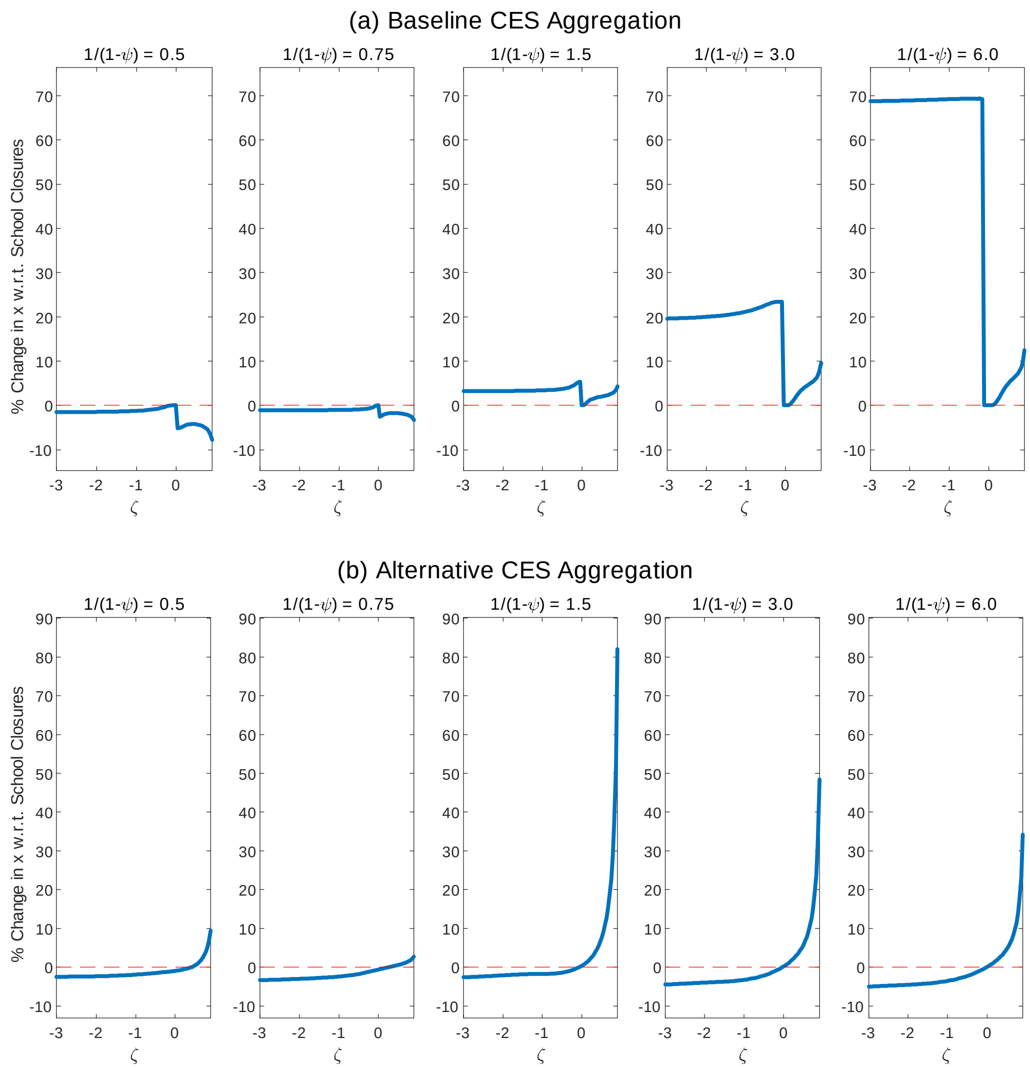
Overall, the figure shows that it is in principle possible to generate positive responses in parental time under both our baseline specification (Panel (a)) and the alternative specification (Panel (b)). Importantly, the figure also conveys information about the range of parameter values necessary for this result. Specifically, under our baseline specification, the model generates positive parental time responses as long as $\psi > 0$ or $1/(1-\psi) > 1$. By contrast, under the alternative specification, the positive response is only possible when ζ is sufficiently high (greater than zero).[2]

We then point to (limited) empirical evidence on these two elasticities of substitution in the

[1] We abstract from share parameters to focus on our key message about the order of aggregation.

[2] This result can be shown analytically in this simple model.

Figure A1: CES aggregation order and parental time responses to school closures



Note: The top panel shows percent changes in optimal x with respect to a 10% decline in g with a CES aggregation order (A1), which aggregates parental time and monetary investments first in line with the one used in our quantitative model. The bottom panel shows the counterparts with a CES aggregation order which aggregates parental monetary investment and public investment first (A2). We plot these effects for the three different values of ψ, as in the main text. The figures are based on the following values of parameters: $b = 1, \eta = 0.3, m = 10$ and $g = 1$.

literature. The first is about the elasticity of substitution between public and private education, shaped by ψ. As we highlighted in the main text, a standard assumption in the literature is perfect substitutability. Our baseline calibration resorts to the estimate from Kotera and Seshadri (2017), which implies that they are still quite substitutable (though not perfectly). In accordance with the literature, all the values we consider in the main text (i.e., $\psi > 0$) are chosen so that the elasticity of substitution is high (i.e., larger than one). The figure shows that parental time responses would remain positive with such values (from the third to fifth graphs). They can become negative only when the elasticity of substitution is empirically implausible (i.e., $\psi < 0$).

On the other hand, the alternative specification is not able to generate positive time responses if $\zeta < 0$ (i.e., parental time and monetary investments are complementary to each other). In our model calibration, ζ is indeed quite negative when children are young as in Yum (2023). Moreover, Caucutt, Lochner, Mullins, and Park (2020) find strong complementarity between parental time and monetary investments using the sample of relatively young children (aged between 0 and 12). Therefore, the alternative specification would have difficulties in generating positive parental time responses to school closures with the value of ζ in line with such empirical evidence, unlike our baseline specification.

C Input Normalization in CES Technology

In our model, we divide inputs by their corresponding means in CES production functions. This normalization helps us to achieve computational stability in our overlapping generations model by keeping human capital distributions within certain ranges while varying parameters related to the elasticity of substitution. The key source of the issue is the scale effects of changing the elasticity of substitution parameter in CES production functions.

To illustrate this, consider a CES production function:

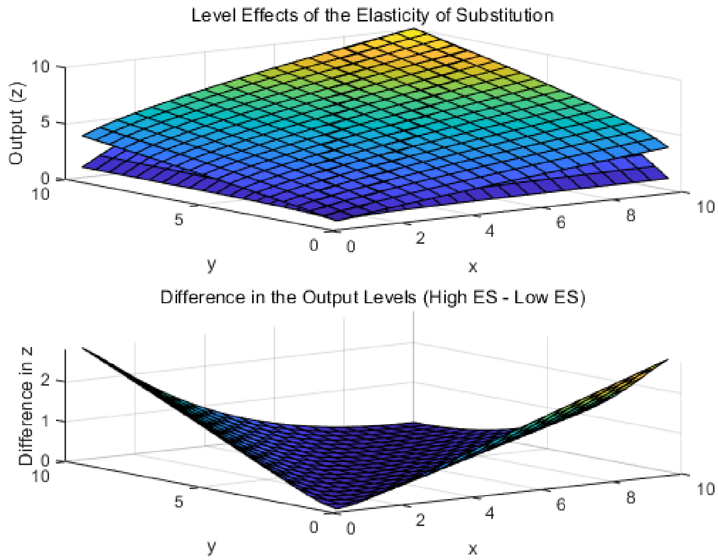
$$z = \left(0.5x^{\psi} + 0.5y^{\psi}\right)^{\frac{1}{\psi}} \tag{A3}$$

where $\psi \leq 1$, which determines the elasticity of substitution: $1/(1-\psi)$. The top panel of Figure A2 plots this function for two different values of ψ, -1 (below) and 0.5 (top), so that the corresponding elasticity of substitution becomes 0.5 and 2.0, respectively. It is clear to see that a higher ψ (or a higher elasticity of substitution) has positive level effects on the output especially when two input values (x and y) differ from each other.

To see its implications for output distributions more clearly, we simulate a sample of 100,000 where two inputs are randomly drawn from two normal distributions: $x \sim N(20, 1)$ and $y \sim N(10, 1)$. Then, we generate z according to (A3) again with two different values of $\psi : -1$ and 0.5. Figure A3 shows that the implied distribution of z is shifted to the right with its mean being 9.5% higher with the higher elasticity of substitution.

We also generate z according to the same technology with the two values of ψ *after* we divide

Figure A2: CES production functions with different elasticities of substitution



Note: The top panel shows the output level implied by the CES technology (A3) with a high elasticity of substitution (2.0, top) or a low elasticity of substitution (0.5, below). The bottom panel shows their difference in the output levels.

each input by its corresponding mean:

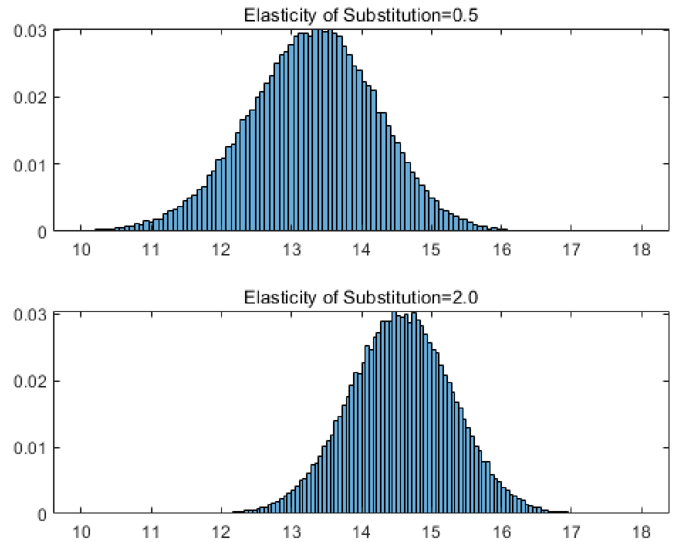
$$z = \left(0.5\left(x/\bar{x}\right)^{\psi} + 0.5\left(y/\bar{y}\right)^{\psi}\right)^{\frac{1}{\psi}} \tag{A4}$$

Figure A4 shows that the level effects are much mitigated: the mean difference is now very low at around 0.2%.

D Partial (Stochastic) Closures

We also consider additional experiments based on partial school closures. Specifically, we assume that school closures are still unexpected but there is another dimension of uncertainty: half of the agents still experience full closures, but the other half experience a school closure of limited intensity. This within-period variation could capture additional closures due to local outbreaks of COVID-19 cases even after re-opening nationwide. This could also capture the variability of effectiveness of online substitute teaching by schools. The results reported below are based on a partial intensity of 50%. As shown in Figure A5, and Tables A4, A5 and A6, our findings suggest that the main findings are generalizable in terms of the relationship between average school closure length and the corresponding aggregate effects. But they also suggest that partial closures induce additional variations that happen within each cohort, as shown in the bottom two panels of Tables A5 and A6.

Figure A3: Distributions of CES outputs with different elasticities of substitution



Note: The distribution of outputs implied by the CES technology (A3) is shown in the top panel for a low elasticity of substitution (0.5) and in the bottom panel for a high elasticity of substitution (2.0).

E Determinants of the Relative Demand of Private to Public Education Investments

Motivated by Jones and Manuelli (1999), we present a simple model to demonstrate how the relative demand of private to public inputs for human capital formation can be shaped by different forces, which include substitutability between the two inputs in the human capital production function.

Specifically, the representative household with a child faces the following optimization problem:

$$\max_{c,e,g} \left\{ \log c + \eta \log h' \right\} \tag{A5}$$

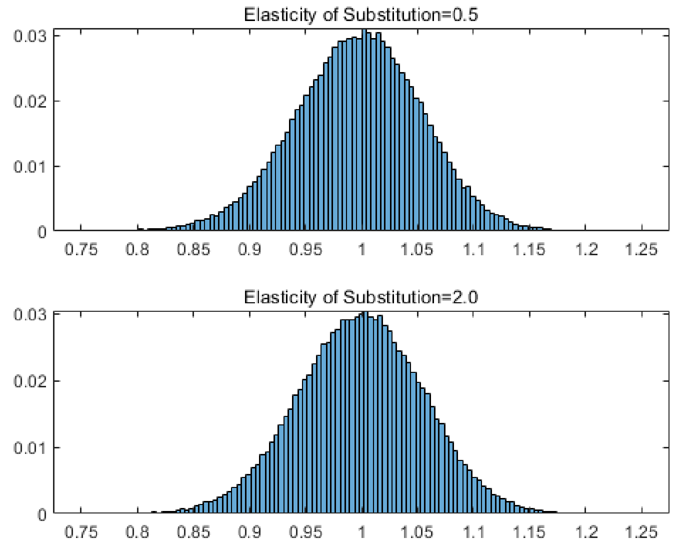
such that

$$c + (1-s)e = w - T \tag{A6}$$

$$h' = \left(\theta \left(\varsigma_e e \right)^{\psi} + (1-\theta) \left(\varsigma_g g \right)^{\psi} \right)^{\frac{1}{\psi}} \tag{A7}$$

where c is consumption, η captures the degree of altruism associated with the child's human capital h', s is a subsidy rate for private human capital investment e, T is a lump-sum tax to finance the education subsidies and public education, and w is income. As shown in (A7), child human capital h' is shaped by two inputs—private investments e and public investments g—with a CES aggregator with an elasticity of substitution given by $1/(1-\psi)$. Each input in the production function is

Figure A4: Distributions of CES outputs with different elasticities of substitution after input normalizations



Note: The distribution of outputs implied by the CES technology after input normalizations (A4) is shown in the top panel for a low elasticity of substitution (0.5) and in the bottom panel for a high elasticity of substitution (2.0).

allowed to have different shares governed by $\theta \in [0, 1]$ and different productivity levels, ς_e and ς_g. To make the illustration cleaner, we assume that both inputs are equally priced in the absence of subsidies.

We note that our goal is to analytically derive a mapping from the parameters related to human capital technology to the relative demand e/g by the representative household in a parsimonious way.[3] Therefore, we take the other government policies such as s and T as given. In the real world, various forms of subsidies to private education for children exist (e.g., income tax credits and childcare subsidies), set out by various factors (e.g., political reasons) other than optimal policy concerns.

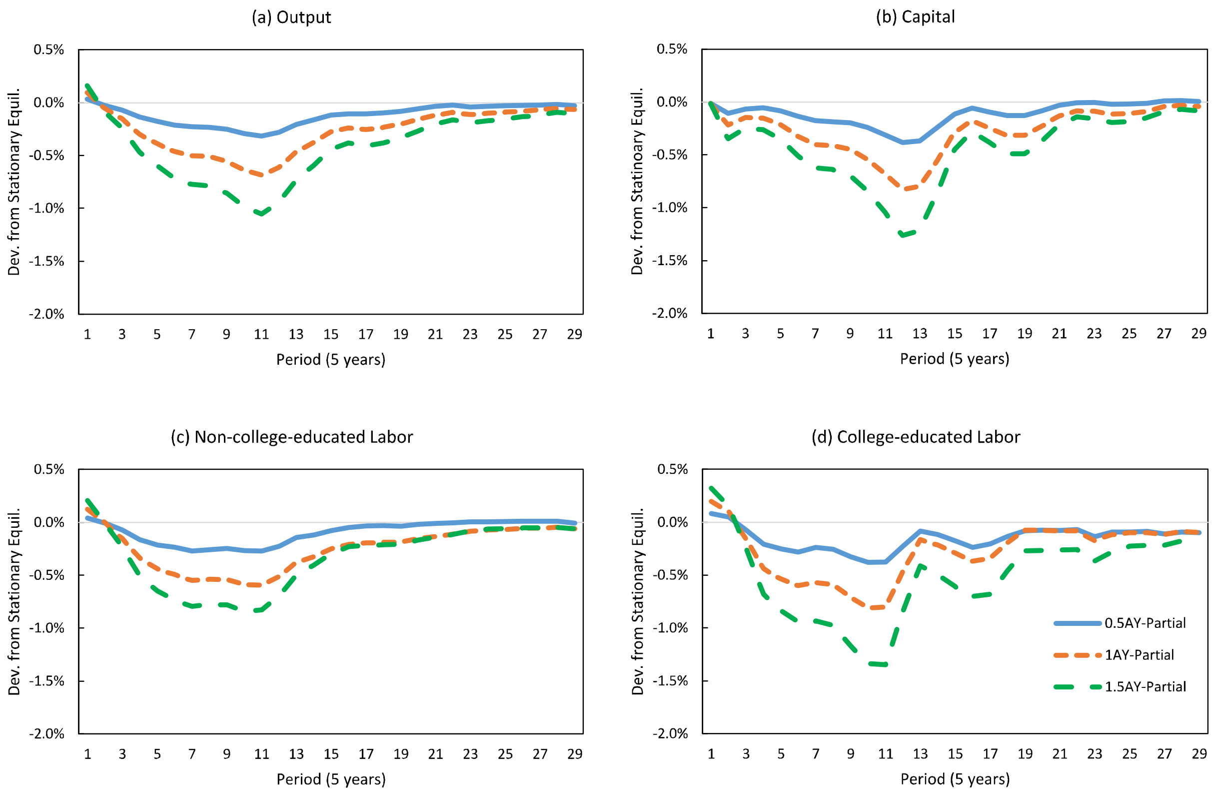
The first-order conditions are then given by:

$$[e] : \frac{-(1-s)}{w - T - (1-s)e} + \frac{\eta\psi\theta\left(\varsigma_e e\right)^{\psi-1}\varsigma_e}{\psi\left(\theta\left(\varsigma_e e\right)^{\psi} + (1-\theta)\left(\varsigma_g g\right)^{\psi}\right)} = 0,$$

$$[g] : \frac{-1}{w - T - (1-s)e} + \frac{\eta\psi\left(1-\theta\right)\left(\varsigma_g g\right)^{\psi-1}\varsigma_g}{\psi\left(\theta\left(\varsigma_e e\right)^{\psi} + (1-\theta)\left(\varsigma_g g\right)^{\psi}\right)} = 0.$$

[3] Therefore, our exercise herein differs from the Ramsey problem that seeks optimal tax/subsidy system, which is very interesting but is analytically less tractable.

Figure A5: Evolution of macroeconomic aggregates: Partial closures



Note: A half of agents experience full closures whereas the other agents experience partial closures, the intensity of which is given by 50%.

Combining these two, we obtain

$$1 - s = \left(\frac{\theta}{1-\theta}\right)\left(\frac{e}{g}\right)^{\psi-1}\left(\frac{\varsigma_e}{\varsigma_g}\right)^{\psi} \Rightarrow \frac{e}{g} = \left(\frac{\theta}{1-\theta}\right)^{\frac{1}{1-\psi}}\left(\frac{1}{1-s}\right)^{\frac{1}{1-\psi}}\left(\frac{\varsigma_e}{\varsigma_g}\right)^{\frac{\psi}{1-\psi}}.$$

This equation tells us that the relative demand of e to g can be shaped by three different forces (and their interactions). First, we begin with the effect of ψ or the elasticity of substitution $1/(1-\psi)$. The above equation implies that the effect of ψ on the ratio, e/g, would interact with the other human capital technology primitives. Specifically, it is positive if $\theta\varsigma_e > (1-s)(1-\theta)\varsigma_g$. This condition is more likely to be satisfied if there are private education subsidies ($s > 0$) or private investments are relatively more important than public investments (i.e., a higher θ or a higher ratio of ς_e/ς_g). This implies that the representative agent would prefer to invest more through private education e instead of public education g if these two inputs are more substitutable in an economy where private education is subsidized or human capital technology puts more weight on private

Table A4: Distributional changes over time: Partial closures

	Steady state	Time (1 period: 5 years) 1	2	3	4	5
		% change rel. to no school closure				
Closure length: 0.5 AY						
Gini income	.338	0.0	-0.0	0.1	0.2	0.1
Bottom 20% inc (%)	8.1	-0.0	0.0	-0.1	-0.1	-0.0
Share of college (%)	33.6	-0.0	0.0	-0.1	-0.1	-0.1
Closure length: 1 AY						
Gini income	.338	-0.0	-0.1	0.2	0.4	0.2
Bottom 20% inc (%)	8.1	-0.0	0.0	-0.2	-0.2	-0.1
Share of college (%)	33.6	-0.0	0.0	-0.1	-0.3	-0.2
Closure length: 1.5 AY						
Gini income	.338	-0.0	-0.1	0.3	0.5	0.4
Bottom 20% inc (%)	8.1	-0.0	0.1	-0.3	-0.4	-0.1
Share of college (%)	33.6	-0.0	0.1	-0.2	-0.5	-0.3

investments relative to public investments.[4]

The above equation suggests that this relative demand can also be affected by the other human capital technology primitives. Specifically, the relative demand, e/g, increases with θ, which captures the relative share of private investments in the technology. The effect of school closures with a higher θ should be weaker in terms of the adverse aggregate effects because the direct effect of school closure is weaker (due to a lower weight on g) and also because parents' compensatory investment is more effective (due to a higher weight on e). At the same time, it is going to have a quantitatively stronger mobility consequence because parental responses have greater influences. In the end, the aggregate and mobility consequences between countries with a high e/g and those with a low e/g could appear similar when we use the share parameter instead of the substitutability between public and private education investments, despite the underlying mechanism being different.

Interestingly, the relative demand increases with ς_e/ς_g, the ratio of productivity levels between private and public investments, provided that $\frac{\psi}{1-\psi} > 0$. This means that a higher productivity of private investments relative to public investments would increase the relative demand only if substitutability between the two inputs is strong enough (i.e., $\psi > 0$ or the elasticity of substitution being greater than one).

[4] In other words, a higher elasticity of substitution can play a role of amplifying the relative demand of private to public education in a society where private investment is relatively more important than public investment.

Table A5: Effects on intergenerational mobility of lifetime income: Partial closures

	IGE			Rank cor.			Upward Mobility		
Steady state	.413			.392			6.8%		
Closure length	**% change** rel. to no school closure, by cohort								
	C1	C2	C3	C1	C2	C3	C1	C2	C3
	All children								
0.5 AY	0.1	1.6	1.8	0.1	1.4	1.6	-0.3	-2.5	-3.0
1.0 AY	0.2	3.3	3.7	0.2	2.9	3.3	-0.4	-5.2	-6.0
1.5 AY	0.3	5.1	5.7	0.2	4.5	5.2	-0.7	-7.8	-8.9
	Children who experienced full closure								
0.5 AY	0.1	2.2	2.4	0.1	2.0	2.3	-0.3	-3.4	-4.1
1.0 AY	0.3	4.5	5.0	0.2	4.2	4.7	-0.6	-7.1	-8.3
1.5 AY	0.5	7.0	7.8	0.3	6.4	7.3	-1.0	-11.3	-12.3
	Children who experienced 50% closure								
0.5 AY	0.1	1.1	1.2	0.0	0.9	1.0	-0.0	-1.5	-2.1
1.0 AY	0.1	2.1	2.4	0.1	1.8	2.0	-0.1	-2.9	-3.9
1.5 AY	0.2	3.3	3.6	0.1	2.7	3.1	-0.4	-4.7	-5.7

F Models with Different Elasticities of Substitution between Private and Public Investments

The baseline model in the main text is calibrated with $\psi = 2/3$. We now report the calibration tables for the economies with a higher value ($\psi = 5/6$) in Table A7 and with a lower value ($\psi = 1/3$) in Table A8. We also report the key experiment results from the model with $\psi = 1/3$ in Figure A6, and Tables A9 and A10.

G Additional Figures and Tables

As can be seen in Figure A7, while the variance of log wage in the model-generated data does not feature kinks, the variance of log earnings (and income) shows a non-monotonic pattern in the model-generated data. As the variance of log earnings which are based on both wage and hours worked begins to display different trends before and after children becomes independent, the kinks should be driven by different labor supply behaviors depending on whether parents additionally face endogenous parental investment decisions or not.

Table A6: Effects on inequality and loss of lifetime income: Partial closures

	Lifetime income Gini			Lifetime income Average			Fraction of College-educated		
Steady state	.282			4.2 (rel. to Y_s)			.336		
Closure length	**% change** rel. to no school closure, by cohort								
	C1	C2	C3	C1	C2	C3	C1	C2	C3
	All children								
0.5 AY	0.0	0.2	0.3	-0.0	-1.2	-1.2	0.7	-0.7	-0.9
1.0 AY	0.1	0.5	0.5	-0.0	-2.4	-2.5	1.2	-1.6	-2.1
1.5 AY	0.1	0.7	0.8	-0.1	-3.7	-3.8	1.5	-2.7	-3.5
	Children who experienced full closure								
0.5 AY	0.0	0.3	0.3	-0.1	-1.6	-1.7	0.7	-1.3	-1.6
1.0 AY	0.0	0.6	0.7	-0.1	-3.3	-3.4	1.1	-2.9	-3.4
1.5 AY	0.1	1.0	1.0	-0.2	-5.0	-5.1	1.4	-4.7	-5.5
	Children who experienced 50% closure								
0.5 AY	0.0	0.1	0.1	0.0	-0.8	-0.8	0.8	-0.1	-0.3
1.0 AY	0.0	0.3	0.3	0.0	-1.5	-1.5	1.3	-0.4	-0.8
1.5 AY	0.1	0.5	0.5	0.0	-2.3	-2.5	1.7	-0.8	-1.4

Note: Y_s denotes steady-state output per capita.

Table A7: Internally calibrated parameters and target statistics for the alternative model economy with a higher elasticity of substitution between public and parental investments

Parameter		Target statistics	Data	Model
$\psi = 5/6$		(*elasticity of substitution = 6*)		
β	.939	Equilibrium real interest rate (annualized)	.04	.04
b	6.76	Mean hours of work in $j = 3, ..., 9$	.287	.299
φ	.490	Mean hours of work in $j = 3, 4, 5$	.299	.290
η	.283	Ratio of inter-vivos transfers over total savings	.30	.364
θ_3^x	.819	Mean parental time investments in $j = 3$	.061	.062
θ_4^x	.158	Mean parental time investments in $j = 4$	.036	.036
θ_5^x	.126	Mean parental time investments in $j = 5$	.020	.020
θ_3^p	.517	Rank corr. of parental income & child earnings	.282	.294
θ_3^I	.597	Mean parental monetary investments in $j = 3$	.056	.056
θ_4^I	.665	Mean parental monetary investments in $j = 4$	.136	.130
θ_5^I	.397	Mean parental monetary investments in $j = 5$	.160	.157
ζ_3	-1.75	Educational gradients in parental time in $j = 3$ (%)	20.9	18.6
ζ_4	0.54	Educational gradients in parental time in $j = 4$ (%)	14.8	15.0
ζ_5	0.55	Educational gradients in parental time in $j = 5$ (%)	20.2	21.0
ν	.546	Fraction with a college degree (%)	34.2	34.2
μ_ξ	.226	Average college expenses/GDP per-capita	.140	.140
δ_ξ	.600	Observed college wage gap (%)	75.0	68.1
ρ_ϕ	.011	Intergenerational corr. of percentile-rank income	.341	.398
σ_ϕ	.445	Gini wage	.37	.340
σ_z	.148	Slope of variance of log wage from $j = 2$ to $j = 8$	.18	.184
$\underline{a}$	$-.070$	Average unsecured debt rel. to annual disposable income	.010	.010

Table A8: Internally calibrated parameters and target statistics for the alternative model economy with a lower elasticity of substitution between public and parental investments

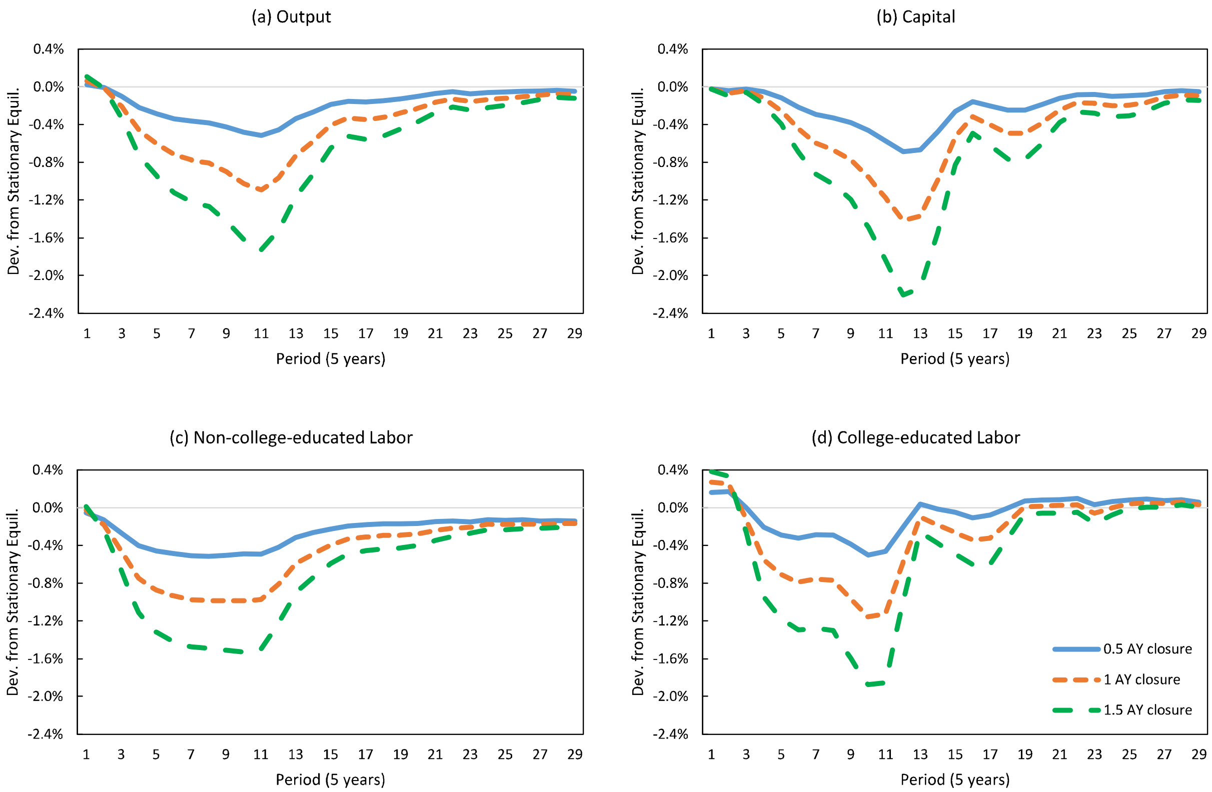
Parameter		Target statistics	Data	Model
$\psi = 1/3$		(*elasticity of substitution = 1.5*)		
β	.942	Equilibrium real interest rate (annualized)	.04	.04
b	6.72	Mean hours of work in $j = 3, ..., 9$	.287	.302
φ	.408	Mean hours of work in $j = 3, 4, 5$	.299	.292
η	.252	Ratio of inter-vivos transfers over total savings	.30	.342
θ_3^x	.869	Mean parental time investments in $j = 3$	.061	.058
θ_4^x	.115	Mean parental time investments in $j = 4$	.036	.036
θ_5^x	.049	Mean parental time investments in $j = 5$	.020	.020
θ_3^p	.621	Rank corr. of parental income & child earnings	.282	.268
θ_3^I	.672	Mean parental monetary investments in $j = 3$	.056	.056
θ_4^I	.684	Mean parental monetary investments in $j = 4$	.136	.125
θ_5^I	.398	Mean parental monetary investments in $j = 5$	.160	.150
ζ_3	-2.43	Educational gradients in parental time in $j = 3$ (%)	20.9	19.8
ζ_4	-0.19	Educational gradients in parental time in $j = 4$ (%)	14.8	14.0
ζ_5	-0.32	Educational gradients in parental time in $j = 5$ (%)	20.2	19.6
ν	.542	Fraction with a college degree (%)	34.2	35.0
μ_ξ	.227	Average college expenses/GDP per-capita	.140	.140
δ_ξ	.624	Observed college wage gap (%)	75.0	67.6
ρ_ϕ	.112	Intergenerational corr. of percentile-rank income	.341	.368
σ_ϕ	.487	Gini wage	.37	.343
σ_z	.148	Slope of variance of log wage from $j = 2$ to $j = 8$	.18	.185
$\underline{a}$	$-.068$	Average unsecured debt rel. to annual disposable income	.010	.010

Table A9: Effects on intergenerational mobility of lifetime income with a lower elasticity of substitution between public and parental investments

$\psi = 1/3$		IGE			Rank cor.			Upward Mobility	
Steady state		.394			.375			6.9%	
				% change rel. to no school closure, by cohort					
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
0.5 AY	0.1	1.0	1.2	0.1	1.0	1.2	-0.3	-1.3	-1.6
1.0 AY	0.3	2.2	2.5	0.2	2.1	2.5	-0.6	-2.5	-3.1
1.5 AY	0.4	3.5	3.9	0.2	3.4	3.9	-0.8	-4.0	-5.0

Note: The elasticity of substitution between private and public investments is equal to 1.5.

Figure A6: Evolution of macroeconomic aggregates with a lower elasticity of substitution between public and parental investments



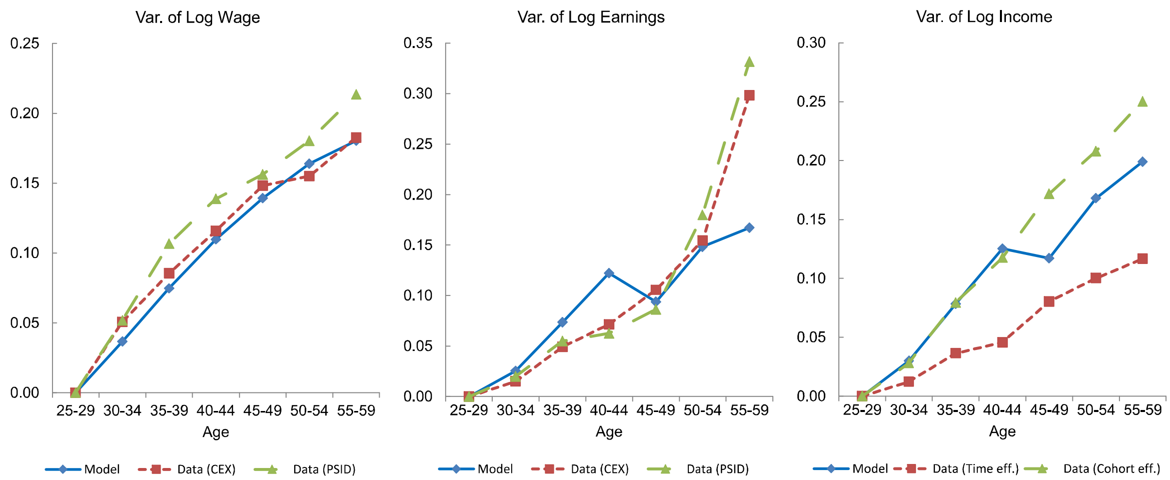
Note: The elasticity of substitution between private and public investments is equal to 1.5.

Table A10: Effects on inequality and loss of lifetime income with a lower elasticity of substitution between public and parental investments

$\psi = 1/3$	Lifetime income						Fraction of		
	Gini			Average			College-educated		
Steady state	.284			4.2 (rel. to Y_s)			.346		
	% change rel. to no school closure, by cohort								
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
0.5 AY	0.0	0.0	0.1	-0.1	-2.1	-2.1	0.6	-1.3	-1.5
1.0 AY	0.0	0.1	0.1	-0.2	-4.3	-4.3	1.3	-2.7	-3.1
1.5 AY	0.1	0.2	0.1	-0.3	-6.7	-6.8	2.1	-4.3	-4.9

Note: The elasticity of substitution between private and public investments is equal to 1.5. Y_s denotes steady-state output per capita.

Figure A7: Inequality over the life cycle



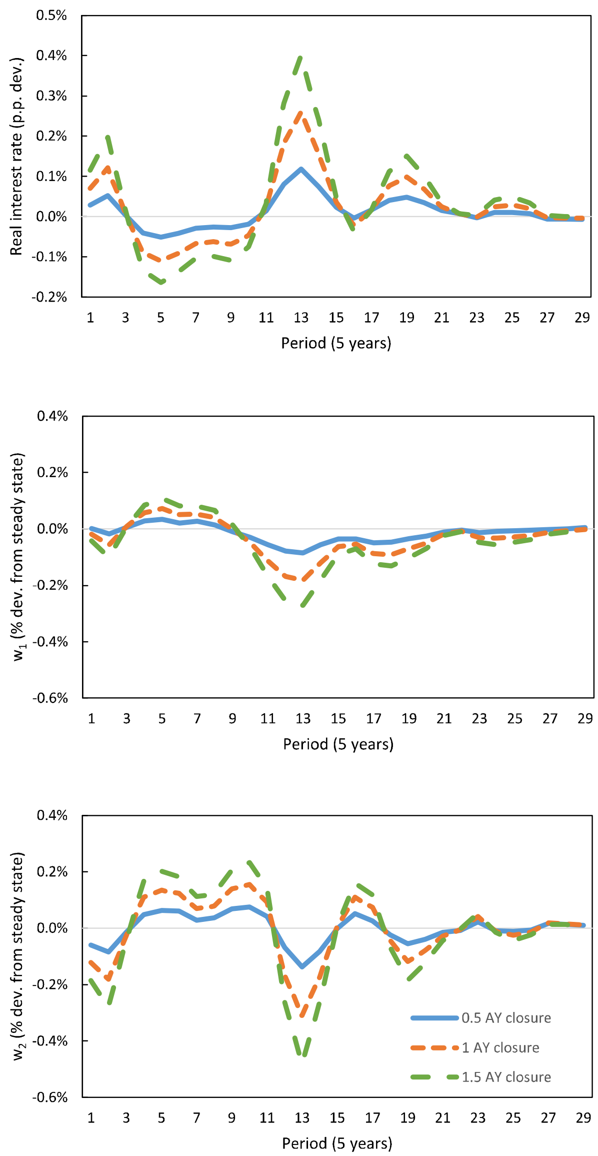
Note: The left figure shows the variance of log wage by age relative to age 25-29. The middle figure shows the variance of log earnings by age relative to age 25–29. The right figure plots the variance of log income by age relative to age 25–29. US data is from Heathcote et al. (2010).

Table A11: School closure effects on different cohorts with a very long closure

	IGE			Rank cor.			Upward Mobility		
Steady state	.413			.392			6.8%		
				% change rel. to no school closure, by cohort					
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
1 AY	0.3	4.5	5.0	0.2	4.1	4.6	-0.6	-6.8	-8.3
4 AY	0.9	22.6	26.1	0.4	19.6	23.0	-2.5	-32.6	-37.7
	Lifetime income: Gini index			Lifetime income: Average			Fraction of College-educated		
Steady state	.282			4.2 (rel. to Y_s)			.336		
				% change rel. to no school closure, by cohort					
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
1 AY	0.1	0.6	0.7	-0.1	-3.3	-3.4	1.4	-2.5	-3.2
4 AY	0.4	3.5	3.6	-0.6	-14.5	-15.4	6.4	-13.2	-16.7

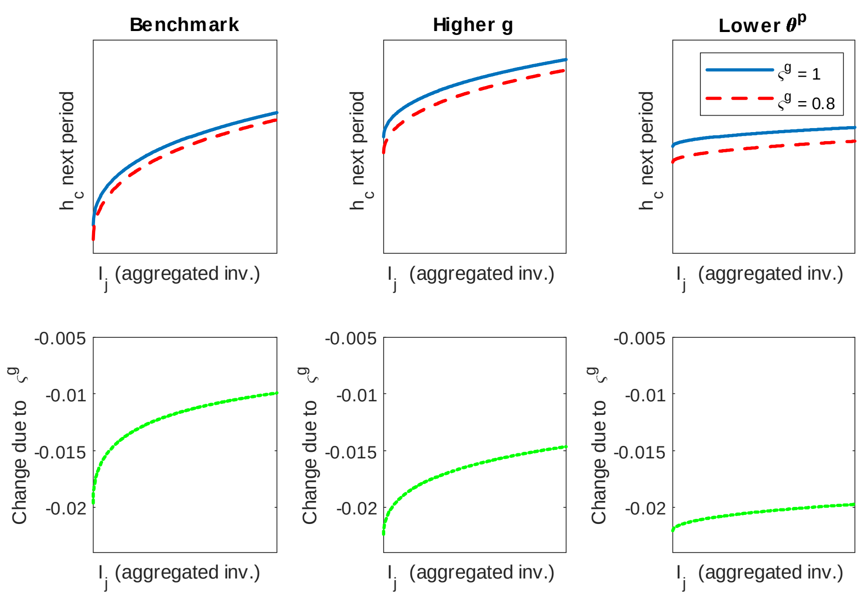
Note: Y_s denotes steady-state output per capita.

Figure A8: Evolution of equilibrium prices in the baseline model



Note: The top panel shows the equilibrium interests over the transition. The middle panel shows the equilibrium wages for non-college workers, and the bottom panel shows the equilibrium wages for college-educated workers over the transition

Figure A9: Illustration of direct effects of school closures on skill formation



Note: The figures visualize how children's human capital output h_c' is related to parental investments I_j aggregated from time and money depending on the presence of school closures (i.e., $\varsigma^g = 1$ or $\varsigma^g = 0.8 < 1$). Note that because parental investments are largely shaped by income, I_j can be interpreted as the parental socioeconomic status (SES). The middle panel raises the size of g and the right panel increases the relative importance of public schooling (with a lower θ^p).

Figure A10: Evolution of macroeconomic aggregates with a very long closure

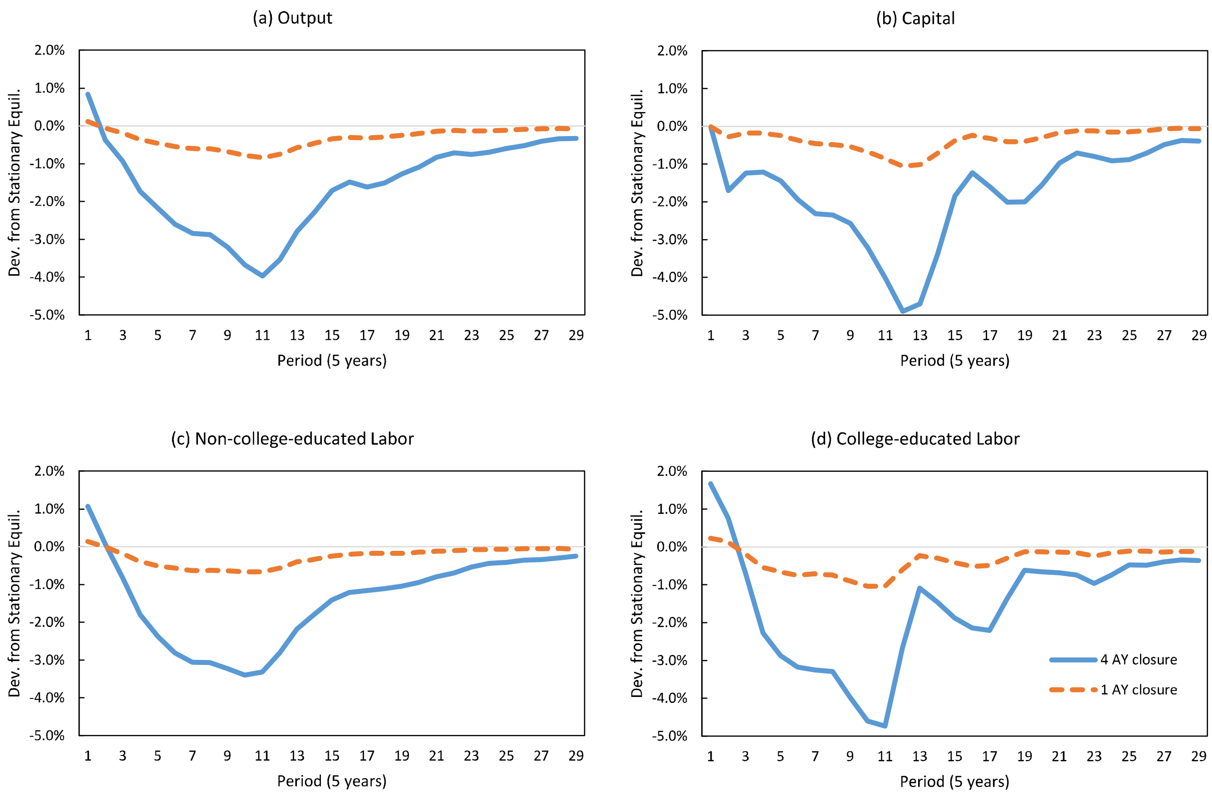
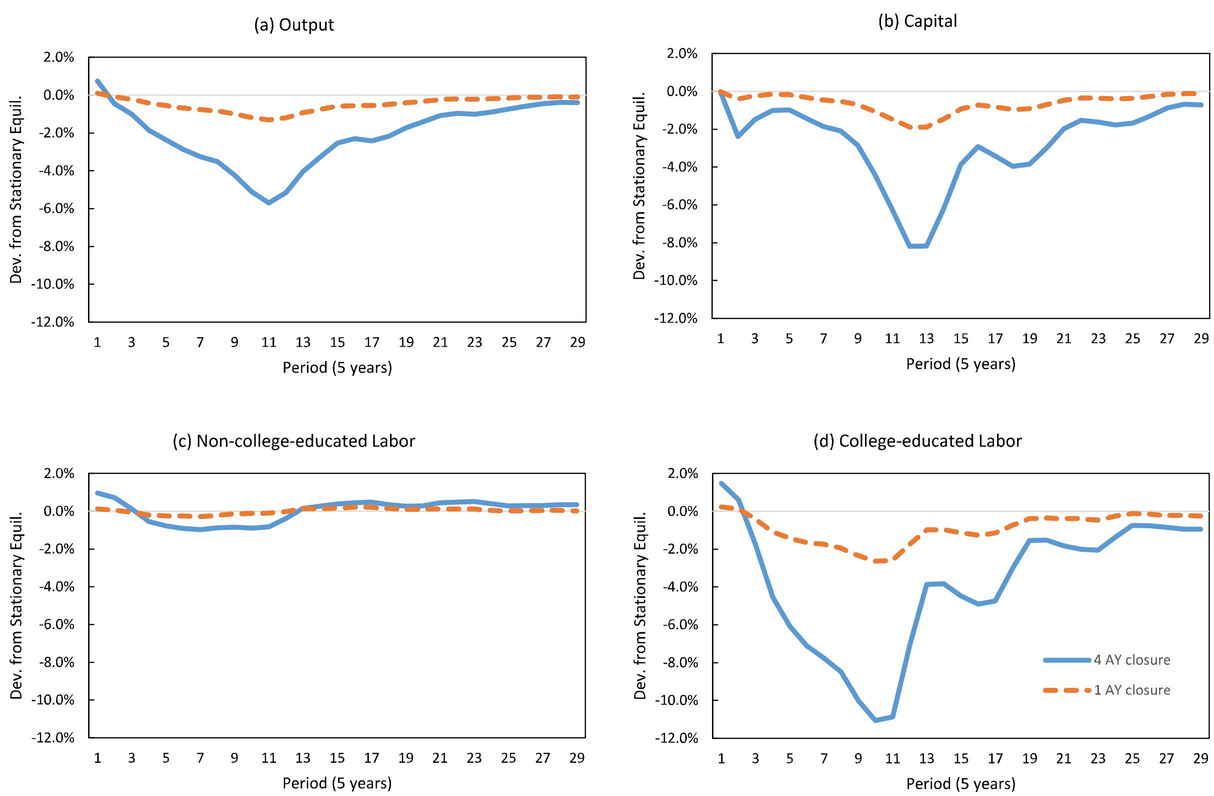


Figure A11: Evolution of macroeconomic aggregates with a very long closure: no general equilibrium effects



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