

Monetary Policy According to HANK

Greg Kaplan, Benjamin Moll, and Giovanni L. Violante

Online Appendix

A Proofs and Additional Details for Section 2

This Appendix spells out in more detail the simple RANK and TANK models in Section 2 and proves the results stated there.

A.1 Details for Section 2.1

A representative household has preferences over utility from consumption C_t discounted at rate $\rho \geq 0$

$$\int_0^\infty e^{-\rho t} U(C_t) dt, \quad U(C) = \frac{C^{1-\gamma}}{1-\gamma}, \quad \gamma > 0. \quad (\text{A.1})$$

There is a representative firm that produces output using only labor according to the production function $Y = N$. Both the wage and final goods price are perfectly rigid and normalized to one. The household commits to supplying any amount of labor demanded at the prevailing wage so that its labor income equals Y_t in every instant. The household receives (pays) lump-sum government transfers (taxes) $\{T_t\}_{t \geq 0}$ and can borrow and save in a riskless government bond at rate r_t . Its initial bond holdings are B_0 . The household's budget constraint in present-value form is

$$\int_0^\infty e^{-\int_0^t r_s ds} C_t dt = \int_0^\infty e^{-\int_0^t r_s ds} (Y_t + T_t) dt + B_0. \quad (\text{A.2})$$

The government sets the path of taxes/transfers in a way that satisfies its budget constraint

$$\int_0^\infty e^{-\int_0^t r_s ds} T_t dt + B_0 = 0. \quad (\text{A.3})$$

As described in Section 2, the monetary authority sets an exogenous time path for real rates $\{r_t\}_{t \geq 0}$.

An equilibrium in this economy is a time path for income $\{Y_t\}_{t \geq 0}$ such that (i) the household maximizes (A.1) subject to (A.2) taking as given $\{r_t, Y_t, T_t\}_{t \geq 0}$, (ii) the government budget constraint (A.3) holds, and (iii) the goods market clears

$$C_t(\{r_t, Y_t, T_t\}_{t \geq 0}) = Y_t, \quad (\text{A.4})$$

where $C_t(\{r_t, Y_t, T_t\}_{t \geq 0})$ is the optimal consumption function for the household.

The overall effect of a change in the path of interest rates on consumption is determined from only two conditions. First, household optimization implies that the time path of consumption satisfies the Euler equation $\dot{C}_t/C_t = \frac{1}{\gamma}(r_t - \rho)$. Second, by assumption, consumption returns back to its steady state level $C_t \rightarrow \bar{C} = \bar{Y}$ as $t \rightarrow \infty$. Therefore, we have

$$C_t = \bar{C} \exp\left(-\frac{1}{\gamma} \int_t^\infty (r_s - \rho) ds\right) \Leftrightarrow d \log C_t = -\frac{1}{\gamma} \int_t^\infty dr_s ds. \quad (\text{A.5})$$

A.2 Proof of Proposition 1

The proof covers both the case $B_0 = 0$ as in Proposition 1 and the case $B_0 > 0$ as in (7). A key virtue of the simple model we consider is that it admits a closed-form solution for the household's optimal consumption function.

Lemma A.1 *For any time paths $\{r_t, Y_t, T_t\}_{t \geq 0}$, initial consumption is given by*

$$C_0(\{r_t, Y_t, T_t\}_{t \geq 0}) = \frac{1}{\chi} \left(\int_0^\infty e^{-\int_0^t r_s ds} (Y_t + T_t) dt + B_0 \right), \quad (\text{A.6})$$

$$\chi = \int_0^\infty e^{-\frac{\gamma-1}{\gamma} \int_0^t r_s ds - \frac{1}{\gamma} \rho t} dt. \quad (\text{A.7})$$

The derivatives of the consumption function evaluated at $(r_t, Y_t, T_t) = (\rho, \bar{Y}, \bar{T})$ are.⁶²

$$\frac{\partial C_0}{\partial r_t} = -\frac{1}{\gamma} \bar{Y} e^{-\rho t} + \rho B_0 e^{-\rho t} \quad \frac{\partial C_0}{\partial Y_t} = \frac{\partial C_0}{\partial T_t} = \rho e^{-\rho t}. \quad (\text{A.8})$$

Proof of Lemma A.1 Integrating the Euler equation forward in time, we have

$$\log C_t - \log C_0 = \frac{1}{\gamma} \int_0^t (r_s - \rho) ds \Rightarrow C_t = C_0 \exp\left(\frac{1}{\gamma} \int_0^t (r_s - \rho) ds\right)$$

Substituting into the budget constraint (A.2):

$$C_0 \int_0^\infty e^{-\int_0^t r_s ds + \frac{1}{\gamma} \int_0^t (r_s - \rho) ds} dt = \int_0^\infty e^{-\int_0^\tau r_s ds} (Y_\tau + T_\tau) d\tau + B_0,$$

or, equivalently, (A.6) with χ defined in (A.7).

⁶²In our continuous-time model the interest rate r_t and income Y_t are *functions* of time. Strictly speaking, the consumption function $C_0(\{r_t, Y_t, T_t\}_{t \geq 0})$ is therefore a functional (i.e. a ‘‘function of a function’’). The derivatives $\partial C_0/\partial r_t$, $\partial C_0/\partial Y_t$ and $\partial C_0/\partial T_t$ are therefore so-called functional derivatives rather than partial derivatives.

Next, consider the derivatives $\partial C_0/\partial r_t$, $\partial C_0/\partial Y_t$ and $\partial C_0/\partial T_t$. Differentiating C_0 in (A.6) with respect to Y_t yields $\partial C_0/\partial Y_t = \frac{1}{\chi} e^{-\int_0^t r_s ds}$. Evaluating at the steady state, we have

$$\frac{\partial C_0}{\partial Y_t} = \rho e^{-\rho t}. \quad (\text{A.9})$$

The derivative with respect to T_t is clearly identical.

Next consider $\partial C_0/\partial r_t$. Write (A.6) as

$$\begin{aligned} C_0 &= \frac{1}{\chi} (Y^{PDV} + T^{PDV} + B_0), \\ Y^{PDV} &= \int_0^\infty e^{-\int_0^\tau r_s ds} Y_\tau d\tau, \quad T^{PDV} = \int_0^\infty e^{-\int_0^\tau r_s ds} T_\tau d\tau. \end{aligned} \quad (\text{A.10})$$

We have

$$\frac{\partial C_0}{\partial r_t} = \frac{1}{\chi} \left(\frac{\partial Y^{PDV}}{\partial r_t} + \frac{\partial T^{PDV}}{\partial r_t} \right) - \frac{1}{\chi^2} \frac{\partial \chi}{\partial r_t} (Y^{PDV} + T^{PDV} + B_0). \quad (\text{A.11})$$

We calculate the different components in turn. From (A.10)

$$\frac{\partial Y^{PDV}}{\partial r_t} = \frac{\partial}{\partial r_t} \int_0^\infty e^{-\int_0^\tau r_s ds} Y_\tau d\tau = \frac{\partial}{\partial r_t} \int_t^\infty e^{-\int_0^\tau r_s ds} Y_\tau d\tau \quad (\text{A.12})$$

where we used that $e^{-\int_0^\tau r_s ds} Y_\tau$ does not depend on r_t for $\tau < t$. Next, note that for $\tau > t$

$$\frac{\partial}{\partial r_t} e^{-\int_0^\tau r_s ds} = -e^{-\int_0^\tau r_s ds} \frac{\partial}{\partial r_t} \int_0^\tau r_s ds = -e^{-\int_0^\tau r_s ds}$$

where the second equality uses $\frac{\partial}{\partial r_t} \int_0^\tau r_s ds = 1$ for $t < \tau$. Substituting into (A.12), we have

$$\frac{\partial Y^{PDV}}{\partial r_t} = - \int_t^\infty e^{-\int_0^\tau r_s ds} Y_\tau d\tau.$$

Similarly

$$\frac{\partial T^{PDV}}{\partial r_t} = - \int_t^\infty e^{-\int_0^\tau r_s ds} T_\tau d\tau, \quad (\text{A.13})$$

and

$$\frac{\partial \chi}{\partial r_t} = \frac{\partial}{\partial r_t} \int_t^\infty e^{-\frac{\gamma-1}{\gamma} \int_0^\tau r_s ds - \frac{1}{\gamma} \rho \tau} d\tau = -\frac{\gamma-1}{\gamma} \int_t^\infty e^{-\frac{\gamma-1}{\gamma} \int_0^\tau r_s ds - \frac{1}{\gamma} \rho \tau} d\tau.$$

Plugging these into (A.11)

$$\frac{\partial C_0}{\partial r_t} = -\frac{1}{\chi} \int_t^\infty e^{-\int_0^\tau r_s ds} (Y_\tau + T_\tau) d\tau + \frac{1}{\chi^2} \frac{\gamma-1}{\gamma} \int_t^\infty e^{-\frac{\gamma-1}{\gamma} \int_0^\tau r_s ds - \frac{1}{\gamma} \rho \tau} d\tau (Y^{PDV} + T^{PDV} + B_0).$$

Evaluating at the steady state and using $\bar{\chi} = 1/\rho$, $Y^{PDV} = \bar{Y}/\rho$, $T^{PDV} = \bar{T}/\rho$ and $\int_t^\infty e^{-\rho\tau} d\tau = e^{-\rho t}/\rho$:

$$\frac{\partial C_0}{\partial r_t} = -(\bar{Y} + \bar{T})e^{-\rho t} + \frac{\gamma - 1}{\gamma}e^{-\rho t} (\bar{Y} + \bar{T} + \rho B_0). \quad (\text{A.14})$$

The government budget constraint is $T^{PDV} + B_0 = 0$, so that in steady state $\bar{T} = -\rho B_0$ and hence (A.14) reduces to the expression in (A.8). \square

Conclusion of Proof Plugging (A.8) into (6), we have

$$dC_0 = \left(-\frac{1}{\gamma}\bar{Y} + \rho B_0\right) \int_0^\infty e^{-\rho t} dr_t dt + \rho \int_0^\infty e^{-\rho t} dY_t dt + \rho \int_0^\infty e^{-\rho t} dT_t dt. \quad (\text{A.15})$$

It remains to characterize dY_t and dT_t and to plug in. First, from (A.5) in equilibrium

$$d \log Y_t = -\frac{1}{\gamma} \int_t^\infty dr_s ds. \quad (\text{A.16})$$

Next, totally differentiate the government budget constraint

$$\int_0^\infty \frac{\partial}{\partial r_t} \left(\int_0^\infty e^{-\int_0^\tau r_s ds} T_\tau d\tau \right) dr_t dt + \int_0^\infty e^{-\int_0^\tau r_s ds} dT_\tau d\tau = 0.$$

Using (A.13) and evaluating at the steady state $-\frac{1}{\rho} \int_0^\infty \bar{T} e^{-\rho t} dr_t dt + \int_0^\infty e^{-\rho t} dT_\tau d\tau$. Using that $\bar{T} = -\rho B_0$,

$$\int_0^\infty e^{-\rho t} dT_\tau d\tau = -B_0 \int_0^\infty e^{-\rho t} dr_t dt \quad (\text{A.17})$$

Plugging (A.16) and (A.17) into (A.15), we have

$$d \log C_0 = \underbrace{\left(-\frac{1}{\gamma} + \rho \frac{B_0}{\bar{Y}}\right) \int_0^\infty e^{-\rho t} dr_t dt}_{\text{direct response to } r} - \underbrace{\frac{\rho}{\gamma} \int_0^\infty e^{-\rho t} \int_t^\infty dr_s ds dt}_{\text{indirect effects due to } Y} - \underbrace{\rho \frac{B_0}{\bar{Y}} \int_0^\infty e^{-\rho t} dr_t dt}_{\text{indirect effects due to } T}. \quad (\text{A.18})$$

Equation (4) in Proposition 1 is the special case with $B_0 = 0$.

To see that this decomposition is additive, consider the second term in (A.18) and

integrate by parts:

$$\begin{aligned}
\frac{\rho}{\gamma} \int_0^\infty e^{-\rho t} \int_t^\infty dr_s ds dt &= -\frac{\rho}{\gamma} \int_t^\infty e^{-\rho s} ds \int_t^\infty dr_s ds \Big|_0^\infty - \frac{\rho}{\gamma} \int_0^\infty \int_t^\infty e^{-\rho s} ds dr_t dt \\
&= -\frac{\rho}{\gamma} \frac{1}{\rho} e^{-\rho t} \int_t^\infty dr_s ds \Big|_0^\infty - \frac{\rho}{\gamma} \frac{1}{\rho} \int_0^\infty e^{-\rho t} dr_t dt \\
&= \frac{1}{\gamma} \int_0^\infty dr_s ds - \frac{1}{\gamma} \int_0^\infty e^{-\rho t} dr_t dt.
\end{aligned}$$

Therefore it is easy to see that the first, second and third terms in (A.18) sum to $-\frac{1}{\gamma} \int_0^\infty dr_s ds$. \square

Remark: The fact that second term in (4) scales with $1/\gamma$ —and therefore the result that with $B_0 = 0$ the split between direct and indirect effects is independent of $1/\gamma$ —is an equilibrium outcome. In particular, without imposing equilibrium, the decomposition with $B_0 = 0$ (4) is

$$d \log C_0 = \underbrace{-\frac{1}{\gamma} \int_0^\infty e^{-\rho t} dr_t dt}_{\text{direct response to } r} + \underbrace{\rho \int_0^\infty e^{-\rho t} d \log Y_t dt}_{\text{GE effects due to } Y}.$$

But in equilibrium $d \log Y_t = -\frac{1}{\gamma} \int_t^\infty dr_s ds$ which scales with $1/\gamma$. Also see footnote 8.

Derivation of (5): In the special case (1), we have $dr_t = e^{-\eta t} dr_0$. Hence $\int_0^\infty e^{-\rho t} dr_t dt = \int_0^\infty e^{-(\rho+\eta)t} dt dr_0 = \frac{1}{\rho+\eta} dr_0$. Similarly $\int_0^\infty e^{-\rho t} \int_t^\infty dr_s ds dt = \int_0^\infty e^{-\rho t} \int_t^\infty e^{-\eta s} ds dt dr_0 = \frac{1}{\eta} \int_0^\infty e^{-(\rho+\eta)t} dt dr_0 = \frac{1}{\eta} \frac{1}{\rho+\eta} dr_0$. Plugging these into (4) yields (5).

A.3 Details for Section 2.2

In the environment described in Section 2.2, aggregate consumption is given by

$$C_t = \Lambda C_t^{sp} + (1 - \Lambda) C_t^{sa}. \tag{A.19}$$

Savers face the present-value budget constraint

$$\int_0^\infty e^{-\int_0^t r_s ds} C_t^{sa} dt = \int_0^\infty e^{-\int_0^t r_s ds} (Y_t + T_t^{sa}) dt + B_0^{sa},$$

The government budget constraint is

$$\int_0^\infty e^{-\int_0^t r_s ds} (\Lambda T_t^{sp} + (1 - \Lambda) T_t^{sa}) dt + B_0 = 0, \quad (\text{A.20})$$

where B_t is government debt. The market clearing condition for government debt is

$$B_t = (1 - \Lambda) B_t^{sa}. \quad (\text{A.21})$$

We additionally assume that the economy starts at a steady state in which $C_t^{sp} = C_t^{sa} = \bar{C} = \bar{Y}$ (and hence $\bar{T}^{sp} = 0$). As before, we also assume that the economy ends up in the same steady state (and hence in particular $T_t^{sp} \rightarrow \bar{T}^{sp} = 0$ as $t \rightarrow \infty$).

We now show how to derive the results of Section 2.2. First, consider the overall effect of interest rate changes on aggregate consumption. As before, the consumption response of savers is given by $C_t^{sa} = \bar{C} \exp\left(-\frac{1}{\gamma} \int_t^\infty (r_s - \rho) ds\right)$. From (A.19) and because spender consumption equals $C_t^{sp} = Y_t + T_t^{sp}$, therefore

$$C_t = \Lambda(Y_t + T_t^{sp}) + (1 - \Lambda)\bar{C} \exp\left(-\frac{1}{\gamma} \int_t^\infty (r_s - \rho) ds\right).$$

Using that in equilibrium $C_t = Y_t$:

$$C_t = \frac{\Lambda}{1 - \Lambda} T_t^{sp} + \bar{C} \exp\left(-\frac{1}{\gamma} \int_t^\infty (r_s - \rho) ds\right) \quad (\text{A.22})$$

We first show how (9) is derived. The government budget constraint (A.20) can be written in flow terms as $\dot{B}_t = r_t B_t + \Lambda T_t^{sp} + (1 - \Lambda) T_t^{sa}$. Under the assumption that the government keeps debt constant at its initial level, $B_t = B_0$, we need

$$\Lambda(T_t^{sp} - \bar{T}^{sp}) + (1 - \Lambda)(T_t^{sa} - \bar{T}^{sa}) + (r_t - \rho)B_0 = 0$$

Alternatively, denoting by Λ^T the fraction of income gains that is rebated to spenders and using the assumption that $\bar{T}^{sp} = 0$:

$$\Lambda T_t^{sp} = -\Lambda^T (r_t - \rho) B_0$$

Substituting into (A.22) and totally differentiating

$$d \log C_t = -\frac{\Lambda^T}{1 - \Lambda} \frac{B_0}{\bar{Y}} dr_t - \frac{1}{\gamma} \int_t^\infty dr_s ds \quad (\text{A.23})$$

Equation (9) is obtained by specializing to the interest rate time path (1). When $B_0 = 0$, the total response of aggregate consumption and income in this simple TANK model is therefore identical to that in the RANK version above.

Finally, we show how equation (8) is derived. Because the savers in our TANK model solve the same problem as the representative agent in the RANK model above, their consumption satisfies the analogue of (A.15):

$$dC_0^{sa} = \left(-\frac{1}{\gamma} \bar{Y} + \rho B_0^{sa} \right) \int_0^\infty e^{-\rho t} dr_t dt + \rho \int_0^\infty e^{-\rho t} dY_t dt + \rho \int_0^\infty e^{-\rho t} dT_t^{sa} dt$$

From (A.23) and using $Y_t = C_t$ and (A.20), their income satisfies $d \log Y_t = -\Lambda^T \frac{B_0^{sa}}{\bar{Y}} dr_t - \frac{1}{\gamma} \int_t^\infty dr_s ds$. Since spenders receive a fraction Λ^T of the government's income gains from expansionary monetary policy, savers receive the rest and hence $(1 - \Lambda) T_t^{sa} = -(1 - \Lambda^T)(r_t - \rho) B_0$ or from (A.20) $T_t^{sa} = -(1 - \Lambda^T)(r_t - \rho) B_0^{sa}$ and hence $dT_t^{sa} = -(1 - \Lambda^T) B_0^{sa} dr_t$. Therefore

$$\int_0^\infty e^{-\rho t} dT_t^{sa} dt = -(1 - \Lambda^T) B_0^{sa} \int_0^\infty e^{-\rho t} dr_t dt$$

Substituting these expressions into the one for saver consumption:

$$\begin{aligned} d \log C_0^{sa} &= \left(-\frac{1}{\gamma} + \rho \frac{B_0^{sa}}{\bar{Y}} \right) \int_0^\infty e^{-\rho t} dr_t dt - \rho \int_0^\infty e^{-\rho t} \left(\frac{1}{\gamma} \int_t^\infty dr_s ds + \Lambda^T \frac{B_0^{sa}}{\bar{Y}} dr_t \right) dt \\ &\quad - \rho (1 - \Lambda^T) \frac{B_0^{sa}}{\bar{Y}} \int_0^\infty e^{-\rho t} dr_t dt \end{aligned}$$

Next, characterize spenders' consumption response

$$\begin{aligned} \frac{dC_0^{sp}}{C_0} &= \frac{dY_0 + dT_0^{sp}}{Y_0} = -\frac{\Lambda^T}{1 - \Lambda} \frac{B_0}{\bar{Y}} dr_0 - \frac{1}{\gamma} \int_0^\infty dr_t dt - \frac{\Lambda^T}{\Lambda} \frac{B_0}{\bar{Y}} dr_0 \\ &= -\frac{\Lambda^T}{\Lambda(1 - \Lambda)} \frac{B_0}{\bar{Y}} dr_0 - \frac{1}{\gamma} \int_0^\infty dr_t dt \end{aligned}$$

From (A.19) $d \log C_0 = (1 - \Lambda) d \log C_0^{sp} + \Lambda \frac{dY_0 + dT_0^{sp}}{Y_0}$. Therefore, the analogue of Propo-

sition 1 is

$$\begin{aligned}
d \log C_0 &= \left(-\frac{1-\Lambda}{\gamma} + \rho \frac{B_0}{\bar{Y}} \right) \int_0^\infty e^{-\rho t} dr_t dt \\
&\quad - \frac{\rho(1-\Lambda)}{\gamma} \int_0^\infty e^{-\rho t} \int_t^\infty dr_s ds dt - \rho \Lambda^T \frac{B_0}{\bar{Y}} \int_0^\infty e^{-\rho t} dr_t dt - \frac{\Lambda \Lambda^T}{1-\Lambda} \frac{B_0}{\bar{Y}} dr_0 \\
&\quad - \rho(1-\Lambda^T) \frac{B_0}{\bar{Y}} \int_0^\infty e^{-\rho t} dr_t dt - \frac{\Lambda}{\gamma} \int_0^\infty dr_t dt - \Lambda^T \frac{B_0}{\bar{Y}} dr_0
\end{aligned} \tag{A.24}$$

The first line is the direct response to r , the second line are indirect effects due to Y , and the third line are indirect effects due to T . An instructive special case is the one without government debt, $B_t = 0$ for all t . In that case

$$d \log C_0 = \underbrace{-\frac{1-\Lambda}{\gamma} \int_0^\infty e^{-\rho t} dr_t dt}_{\text{direct response to } r} - \underbrace{\frac{\rho(1-\Lambda)}{\gamma} \int_0^\infty e^{-\rho t} \int_t^\infty dr_s ds dt - \frac{\Lambda}{\gamma} \int_t^\infty dr_s ds}_{\text{indirect effects due to } Y}.$$

Equation (8) then follows from the fact that in the special case (1), $dr_t = e^{-\eta t} dr_0$.

For completeness, we also derive the split between direct and indirect effects for our analytic example $dr_t = e^{-\eta t} dr_0$ in the case with both hand-to-mouth agents $\Lambda > 0$ and government debt $B_0 > 0$. Collecting some of the indirect effects on the second and third lines of (A.24) and specializing to $dr_t = e^{-\eta t} dr_0$, we have

$$-\frac{d \log C_0}{dr_0} = \frac{1}{\gamma \eta} \left[\underbrace{\frac{\eta}{\rho + \eta} \left(1 - \Lambda - \rho \gamma \frac{B_0}{\bar{Y}} \right)}_{\text{direct response to } r} + (1-\Lambda) \frac{\rho}{\rho + \eta} + \Lambda + \frac{\eta}{\rho + \eta} \rho \gamma \frac{B_0}{\bar{Y}} \right] + \frac{\Lambda^T}{1-\Lambda} \frac{B_0}{\bar{Y}}. \tag{A.25}$$

A.4 Details on Medium-Scale DSGE Model (Section 2.3)

The Smets-Wouters model is a typical medium-scale DSGE RANK model with a variety of shocks and frictions. The introduction of Smets and Wouters (2007) provides a useful overview and a detailed description of the model can be found in the paper's online Appendix.⁶³ We here only outline the ingredients of the model that are important for the purpose of our decomposition exercise (reported in Table 1) as well as some details on the implementation of this exercise.

An important difference relative to the stylized model of Section 2.1 is that the

⁶³Available at https://www.aeaweb.org/aer/data/june07/20041254_app.pdf

representative household's utility function features external habit formation:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1 - \sigma_c} (C_t(j) - hC_{t-1})^{1 - \sigma_c} \exp \left(\frac{\sigma_c - 1}{1 + \sigma_l} L_t(j)^{1 + \sigma_l} \right) \quad (\text{A.26})$$

where $C_t(j)$ is consumption of one of a continuum of individual households and C_t is aggregate consumption (in equilibrium the two are equal). The parameter $h \in [0, 1]$ disciplines the degree of external habit formation. As mentioned in the main text, the model also features investment with investment adjustment costs and capital utilization, as well as partially sticky prices and wages.

Our starting point for the decomposition are the impulse response functions (IRFs) to an expansionary monetary policy shock in a log-linearized, estimated version of the model. We set each of the model's parameters to the mode of the corresponding posterior distribution (see Table 1 in [Smets and Wouters \(2007\)](#) for the parameter values). The IRFs are computed in Dynare using an updated version of the replication file of the published paper.⁶⁴ For our purposes, the relevant IRFs are the sequences $\{C_t, R_t, Y_t, I_t, G_t, UC_t, L_t\}_{t=0}^{\infty}$ for consumption C_t , interest rates R_t , labor income Y_t , investment I_t , government spending G_t , capital utilization costs $UC_t = a(Z_t)K_{t-1}$ and labour supply L_t . We further denote consumption at the initial steady state by \bar{C} .

Given these IRFs, we decompose the overall consumption response to an expansionary monetary policy shock into direct and indirect effects as follows. Suppressing j -indices for individual households, the budget constraint of households is

$$C_t + \frac{B_t}{R_t P_t} + T_t \leq \frac{B_{t-1}}{P_t} + M_t \quad (\text{A.27})$$

$$M_t = \frac{W_t^h L_t}{P_t} + \frac{R_t^h K_{t-1} Z_t}{P_t} - a(Z_t) K_{t-1} + \frac{Div_t}{P_t} + \frac{\Pi_t}{P_t} - I_t \quad (\text{A.28})$$

where the reader should refer to the online Appendix of [Smets and Wouters \(2007\)](#) for an explanation of each term (the budget constraint is their equation (9)).⁶⁵ In present-value form

$$\sum_{t=0}^{\infty} \frac{1}{\prod_{k=0}^{t-1} \tilde{R}_k} C_t = \sum_{t=0}^{\infty} \frac{1}{\prod_{k=0}^{t-1} \tilde{R}_k} (M_t - T_t)$$

where $\tilde{R}_t = \frac{R_t}{\Pi_t}$ denotes the real interest rate. Households maximize (A.26) subject to

⁶⁴ Available at <http://www.dynare.org/phpBB3/viewtopic.php?f=1&t=3750>.

⁶⁵ Note that Smets and Wouters' budget constraint features some typos: it does not include dividends from firm ownership Π_t and there is a "minus" in front of T_t suggesting it is a transfer even though it enters as a tax in the government budget constraint (equation (24) in their online Appendix).

this budget constraint. For any price sequences, initial consumption C_0 then satisfies:

$$\begin{aligned}
C_0 &= \frac{1}{\chi} \left(X + \frac{B_{-1}}{P_0} + \sum_{t=0}^{\infty} \frac{1}{\prod_{k=0}^{t-1} \tilde{R}_k} (M_t + T_t) \right) \\
\chi &= \sum_{t=0}^{\infty} \frac{1}{\prod_{k=1}^{t-1} \tilde{R}_k} \left(\sum_{k=0}^t x_{t-k} \left(\frac{h}{g} \right)^k \right) \\
X &= \sum_{t=0}^{\infty} \frac{1}{\prod_{k=1}^{t-1} \tilde{R}_k} \sum_{k=0}^{t-1} x_{t-k} \left(\frac{h}{g} \right)^{k+1} \bar{C} \\
x_s &= \left(\bar{\beta}^s \prod_{k=0}^{s-1} \tilde{R}_k \right)^{1/\sigma_c} \exp \left(\frac{\sigma_c - 1}{\sigma_c(1 + \sigma_l)} (L_s - L_0) \right)
\end{aligned} \tag{A.29}$$

where $\bar{\beta} = \frac{\beta}{g^{\sigma_c}}$ and g is the gross growth rate of the economy. The direct effect of consumption to interest rate changes is then computed from (A.29) by feeding in the equilibrium sequence of real interest rates $\{\tilde{R}_t\}_{t=0}^{\infty}$ while holding $\{M_t, T_t, L_t\}_{t=0}^{\infty}$ at their steady state values. When computing this direct effect in practice, we simplify the right-hand side of (A.29) further taking advantage of the fact that most terms are independent of the sequence of real interest rates $\{\tilde{R}_t\}_{t=0}^{\infty}$. In particular, in equilibrium, profits and labor union dividends are $\Pi_t = P_t Y_t - W_t L_t - R_t^h Z_t K_{t-1}$ and $Div_t = (W_t - W_t^h) L_t$ and therefore, substituting into (A.28)

$$M_t = Y_t - a(Z_t)K_{t-1} - I_t. \tag{A.30}$$

Further, the government budget constraint in present-value form is

$$\sum_{t=0}^{\infty} \frac{1}{\prod_{k=0}^t \tilde{R}_k} T_t = \sum_{t=0}^{\infty} \frac{1}{\prod_{k=0}^t \tilde{R}_k} G_t. \tag{A.31}$$

Substituting (A.30) and (A.31) into (A.29), we have

$$C_0 = \frac{1}{\chi} (X + Y^{PDV} - I^{PDV} - G^{PDV} - UC^{PDV}) \tag{A.32}$$

where Y^{PDV} , I^{PDV} , G^{PDV} and UC^{PDV} are the present values of $\{Y_t, I_t, G_t, UC_t\}_{t=0}^{\infty}$ discounted at $\{\tilde{R}_t\}_{t=0}^{\infty}$.

Note that although the series $\{C_t, \tilde{R}_t, Y_t, I_t, G_t, UC_t, L_t\}_{t=0}^{\infty}$ are generated using a log-linearized approximation around the trend, our decomposition uses a non-linear solution. In particular, both the overall and direct elasticities of consumption to interest rate changes in Table 1 are computed using the exact non-linear Euler equation but

evaluated at the equilibrium prices from the linearized models – see the formula (A.32). The fraction due to direct effects is the ratio of this direct elasticity to the overall elasticity, with both numerator and denominator computed in this non-linear fashion. In our baseline exercise in Table 1 this fraction equals 99 percent. For small shocks the overall elasticity of consumption computed with the exact formula is very close to the elasticity computed using the linearized output from Dynare. For larger shocks, the two can differ somewhat. We have also recomputed the share of direct effects as the ratio of the direct elasticity computed in a non-linear fashion and the overall elasticity computed in a linear fashion. For the baseline exercise in Table 1, this yields a share of direct effects of 91%.

As already stated in the main text, our main result is that – at the estimated parameter values of Smets and Wouters (2007) – the direct effect amounts for 99 percent of the total response of initial consumption to an expansionary monetary policy shock. We have conducted a number of robustness checks with respect to various parameter values, and in particular with respect to the habit formation parameter h . The results are robust. In the case without habit formation $h = 0$, 95.1 percent of the overall effect are due to direct intertemporal substitution effects. Finally, note that a difference between (A.26) and the specification of preferences in textbook versions of the New Keynesian model is the non-separability between consumption and labor supply. We have conducted an analogous decomposition exercise with a separable version of (A.26). The decomposition is hardly affected.

A.5 Details on the Two-Asset RANK and TANK Models

A.5.1 Model

We begin by outlining the two-agent, spender-saver version of the model (TANK). The representative agent is a special case with the fraction of spenders equal to zero. The model is written and solved in discrete time.

Households. A fraction Λ of households are spenders indexed by “sp” and a fraction $1 - \Lambda$ are savers indexed by “sa”.

Savers. Savers derive utility from consuming c_t^{sa} and have disutility from supplying labor ℓ_t^{sa} . Savers are able to borrow and save in a liquid government bond at rate r_t^b . They also have access to an illiquid asset a_t with rate of return r_t^a . Assets of type a are illiquid in the sense that households need to pay a cost for depositing into or withdrawing from their illiquid account. Let d_t denote the deposit decision and $\chi(d_t)$

the cost of depositing d_t . The saver's problem in its sequential formulation is therefore given by

$$\max_{\{c_t^{sa}, \ell_t^{sa}, d_t, b_{t+1}, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t^{sa}, \ell_t^{sa})$$

$$\text{S.t. } c_t^{sa} + b_{t+1} + d_t + \chi(d_t, a_t) = (1 - \tau)(w_t \ell_t^{sa} + \Gamma_t^{sa}) + T_t^{sa} + (1 + r_t^b) b_t \quad (\lambda)$$

$$a_{t+1} = (1 + r_t^a) a_t + d_t \quad (\eta)$$

where

$$u(c, \ell) = \log c - \varphi \frac{\ell^{1+\nu}}{1+\nu}$$

$$\chi(d) = \chi_1 |d|^{\chi_2}, \quad \chi_1 > 0, \chi_2 > 1.$$

The first-order conditions for the consumer's problem can be written as

$$1 = \left\{ \lambda_{t,t+1} (1 + r_{t+1}^b) \right\} \quad (\text{A.33})$$

$$\eta_t = \left\{ \lambda_{t,t+1} \left[\eta_{t+1} (1 + r_{t+1}^a) \right] \right\} \quad (\text{A.34})$$

$$\eta_t = 1 + \text{sign}(d_t) \times \tilde{\chi} |d_t|^{\chi_2-1}, \quad \tilde{\chi} = \chi_1 \chi_2 \quad (\text{A.35})$$

$$\varphi (\ell_t^{sa})^\nu c_t^{sa} = (1 - \tau) w_t \quad (\text{A.36})$$

where

$$\lambda_{t,t+1} := \frac{\lambda_{t+1}}{\lambda_t} = \beta \left(\frac{c_{t+1}^{sa}}{c_t^{sa}} \right)^{-1}. \quad (\text{A.37})$$

Note that, by combining (A.33) and (A.34), one obtains that in steady state $r_b = r_a$.

Spenders. Spenders are hand-to-mouth, i.e. consume their labor income every period. Their only margin of adjustment is labor supply ℓ^{sp} . The spender's problem is

$$\max_{c_t^{sp}, \ell_t^{sp}} u(c_t^{sp}, \ell_t^{sp}) \quad \text{s.t.}$$

$$c_t^{sp} = (1 - \tau)(w_t \ell_t^{sp} + \Gamma_t^{sp}) + T_t^{sp}$$

with first-order conditions

$$c_t^{sp} = (1 - \tau)(w_t \ell_t^{sp} + \Gamma_t^{sp}) + T_t^{sp} \quad (\text{A.38})$$

$$w_t = \frac{\varphi}{1 - \tau} (\ell_t^{sp})^\nu c_t^{sp}. \quad (\text{A.39})$$

Firms. There is a continuum of intermediate-goods monopolistic firms, each producing a variety j using a constant returns to scale production function

$$y_t(j) = k_t(j)^\alpha n_t(j)^{1-\alpha}. \quad (\text{A.40})$$

Each intermediate producer chooses its price $p_t(j)$ and inputs $k_t(j), n_t(j)$ to maximize

$$\frac{p_t(j)}{P_t} y_t(j) - w_t n_t(j) - r_t^k k_t(j) - \Theta \left(\frac{p_t(j)}{p_{t-1}(j)} \right) \quad (\text{A.41})$$

taking into account that the demand for its product depends on the price $p_t(j)$ charged. The function $\Theta(\cdot)$ is a quadratic adjustment cost for the price change and is expressed as a fraction of final good output Y_t

$$\Theta_t \left(\frac{p_t}{p_{t-1}} \right) = \frac{\theta}{2} \left(\frac{p_t}{p_{t-1}} - 1 \right)^2 Y_t.$$

We divide the problem of the firm in two parts. First, the cost minimization problem of producing y units of variety j delivers the following optimality conditions

$$w_t = (1 - \alpha) m_t \frac{y}{n_t(j)} \quad (\text{A.42})$$

$$r_t^k = \alpha m_t \frac{y}{k_t(j)} \quad (\text{A.43})$$

where marginal cost m_t is the same across firms

$$m_t = \left(\frac{r_t^k}{\alpha} \right)^\alpha \left(\frac{w_t}{1 - \alpha} \right)^{1-\alpha}.$$

Since all firms face the same marginal cost, we drop the j subscript from now onwards. Taking cost minimization decisions as given, each intermediate producer chooses

$\{p_t\}_{t=0}^{\infty}$ to maximize discounted profits

$$\max_{\{p_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\lambda_t \eta_t) \left\{ \left(\frac{p_t}{P_t} - m_t \right) y_t - \Theta_t \left(\frac{p_t}{p_{t-1}} \right) \right\} \quad (\text{A.44})$$

$$\text{s.t. } y_t = \left(\frac{p_t}{P_t} \right)^{-\varepsilon} Y_t \quad (\text{A.45})$$

where the discount factor used by the firm reflects that dividends will accrue to the illiquid account of *savers*. In a symmetric equilibrium, all firms will choose the same price, which will be also the aggregate price P_t . That gives rise to the following Phillips curve relating aggregate inflation $\pi_t = \frac{P_t}{P_{t-1}} - 1$ and marginal costs

$$1 - \theta \pi_t (1 + \pi_t) + \theta \left(\frac{\lambda_{t+1} \eta_{t+1}}{\lambda_t \eta_t} \right) \pi_{t+1} (1 + \pi_{t+1}) \frac{Y_{t+1}}{Y_t} = (1 - m_t) \varepsilon. \quad (\text{A.46})$$

Note that, from equation (A.34), effectively firm discount at rate r_a , which is also the discounting that appears in the Phillips curve, exactly as in our HANK model.

Moreover, since in equilibrium all firms choose the same price, they all produce the same quantity and hire the same amount of input on factor markets. Hence we can aggregate production function of each firm to get

$$y_t(j) = k_t(j)^\alpha n_t(j)^{1-\alpha} \Rightarrow Y_t = K_t^\alpha N_t^{1-\alpha}.$$

Finally, profits are then given by

$$\Pi_t = Y_t \left((1 - m_t) - \frac{\theta}{2} \pi^2 \right). \quad (\text{A.47})$$

Illiquid Assets. As in HANK, illiquid assets a_t consist of both capital holdings (k_t^{sa}) and equity claims (s_t) to a fraction ω of profits. Since there is no aggregate uncertainty, no arbitrage dictates that the return to capital must be equal to the return on equity. We denote this return by r_t^a

$$r_t^a \equiv \frac{\omega \Pi_t + (q_t - q_{t-1})}{q_{t-1}} = r_t^k - \delta \quad (\text{A.48})$$

which restricts how asset prices q_t evolves over time:

$$q_t = \frac{1}{1 + r_{t+1}^a} \left(\omega \Pi_{t+1} + q_{t+1} \right).$$

In the event of an unexpected shock, however, the realized returns between capital and shares *do not need to be equalized* at the moment of impact. What no-arbitrage pricing requires, in such a circumstance, is for the stock price to jump so as to make the return from holding shares the same as the return from holding capital from that period onwards. Realized returns at impact, though, need not to be equalized, since asset positions are pre-determined. Hence, it is useful to write the law of motion of illiquid assets by keeping track of portfolio composition

$$\begin{aligned}
a_{t+1} &\equiv k_{t+1}^{sa} + s_{t+1}q_t \\
&= (1 + r_t^a)a_t + d_t \\
&= (1 + r_t^k - \delta)k_t^{sa} + s_t(\omega\Pi_t + q_t) + d_t.
\end{aligned} \tag{A.49}$$

By combining (A.48) and (A.49), it is easy to see that, in steady-state, savers withdraw from the illiquid account an amount $d = -r^a a$.

As in HANK, the remaining fraction $(1 - \omega)$ of profits are distributed to households (both spenders and savers) as a direct transfer Γ_t to agents liquid budget constraint (since there is no difference in productivity between the two groups, they are distributed lump-sum):

$$\Gamma_t = (1 - \omega)\Pi_t, \quad \Gamma_t^{sp} = \Gamma_t^{sa} = \Gamma_t. \tag{A.50}$$

We set $\omega = \alpha$ so as to neutralize the role of countercyclical profits as explained in the main text.

Government. The government issues bonds denoted by B^g , with the convention that negative values denote government debt. Its budget constraint is therefore given by

$$B_{t+1}^g = (1 + r_t^b)B_t^g + \tau(w_t N_t + \Gamma_t) - T_t - G_t \tag{A.51}$$

with government transfers T_t given by

$$T_t = \Lambda T_t^{sp} - (1 - \Lambda)T_t^{sa} \tag{A.52}$$

$$\Lambda^T T_t = \Lambda T_t^{sp} \tag{A.53}$$

Note that we allow for $\Lambda^T \neq \Lambda$, i.e. spenders may receive bigger or smaller share of transfers than their population share.

Monetary authority. Monetary policy follows a Taylor rule for the nominal interest rate

$$i_t = \bar{r}_t^b + \phi\pi_t + \epsilon_t, \quad \epsilon_{t+1} = \rho\epsilon_t + u_{t+1}, \quad u_{t+1} \sim \mathcal{N}(0, \sigma_\epsilon^2) \quad (\text{A.54})$$

Given inflation and the nominal interest rate, the real return realized on liquid assets held by savers is given by

$$1 + r_t^b = \frac{1 + i_{t-1}}{1 + \pi_t}. \quad (\text{A.55})$$

Equilibrium. To close the model, we state market clearing conditions

$$0 = (1 - \Lambda)b_{t+1}^{sa} + B_{t+1}^g \quad (\text{A.56})$$

$$1 = (1 - \Lambda)s_{t+1} \quad (\text{A.57})$$

$$K_{t+1} = (1 - \Lambda)k_{t+1}^{sa} \quad (\text{A.58})$$

$$I_t = K_{t+1} - (1 - \delta)K_t \quad (\text{A.59})$$

$$Y_t = C_t + I_t + G_t + (1 - \Lambda)\chi_t + \Theta_t \quad (\text{A.60})$$

where

$$C_t = \Lambda c_t^{sp} + (1 - \Lambda)c_t^{sa} \quad (\text{A.61})$$

$$N_t = \Lambda \ell_t^{sp} + (1 - \Lambda)\ell_t^{sa}. \quad (\text{A.62})$$

A.5.2 Parameterization

Some parameter values are set exactly as in the simple TANK models of Section 2. In particular, we set $\Lambda = \Lambda^T = 0.3$, risk aversion γ to 1, and the discount rate to 5% annually. In this model, the liquid real rate also equals 5% in steady state.

Other parameters (Frisch elasticity, transaction cost, demand elasticity, price adjustment cost, share of profits paid as dividends, government policy parameters, and Taylor rule coefficient) are set as in HANK. Only two parameters are calibrated internally. The disutility of labor is set so that on average 1/3 of the time endowment is spent working, and the depreciation rate is set to match the same illiquid wealth to GDP ratio as in HANK (13, quarterly). Table A.1 summarizes the parameterization.

A.5.3 Simulations and decompositions

We use Dynare to solve for the model's steady state, its transitional dynamics and the decompositions. The model is solved globally (i.e. without local linearization) from

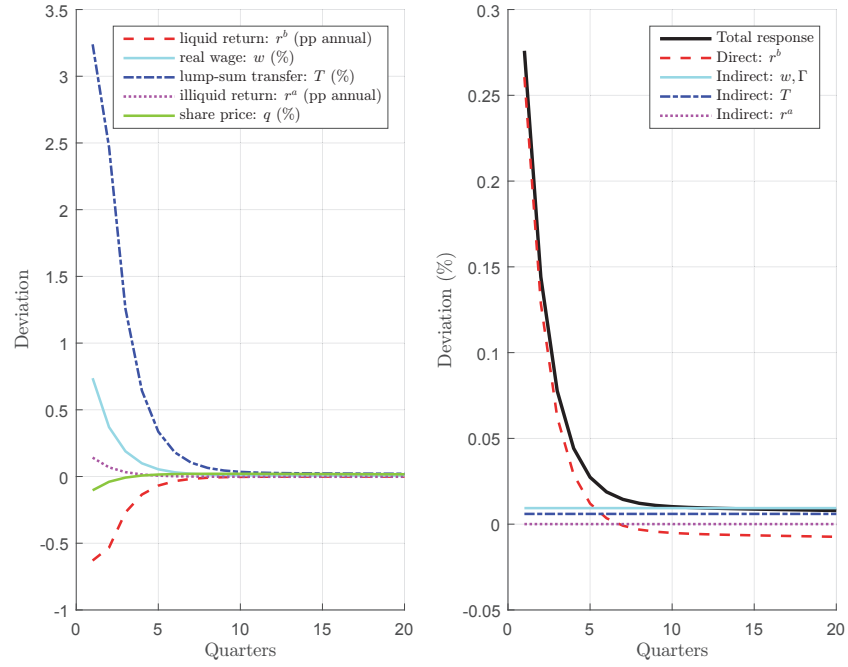
Table A.1: Parameter values (period length is one quarter)

<i>Preferences</i>		
Fraction of HtM	Λ	0.30
Discount factor	β	1.0125^{-1}
Risk aversion	γ	1.00
Disutility of labor	φ	
Frisch elasticity	$1/\nu$	1.00
<i>Deposit costs</i>		
Scaling	χ_1	0.956
Curvature	χ_2	1.402
<i>Production</i>		
Demand elasticity	ε	10
Capital share	α	0.33
Depreciation rate	δ	0.017
Price adjustment cost	θ	100
Share of profit paid as liquid dividends	ω	0.33
<i>Government policy</i>		
Proportional labor tax	τ	0.25
Lump sum transfer (rel GDP)	T	0.06
Fraction transfer to HtM	Λ^T	0.30
Govt debt/annual GDP	$ B^g /(4Y)$	0.23
<i>Monetary Policy</i>		
Steady state real liquid return (pa)	r^b	0.05
Taylor rule coefficient	ϕ	1.25
Shock size at impact	u_0	0.0025
Shock persistence	ρ	0.50

the equilibrium system of nonlinear equations. We always analyze monetary shocks of the same size as in HANK, i.e., 25 basis points, with a quarterly persistence of 0.5. It is worth emphasizing that $\eta = 0.5$ is the correct choice also for discrete time, if we wish to compare across models. To see this, consider that the cumulative deviation of the interest rate path from $t = 0$ is $\int_0^\infty (r_s - \rho) ds = \int_0^\infty \exp(-\eta s) ds = 1/\eta$. In discrete time the cumulative deviation is $\sum_{t=0}^\infty \rho^t = 1/(1 - \rho)$. Thus, a proper comparison with a continuous time model where $\eta = 0.5$ requires setting $\rho = 0.5$.

Figure A.1 reports the IRFs and decomposition in RANK and TANK for the baseline fiscal policy scenario (T-adjust case). It is the counterpart of Figure 4 for HANK. The elasticities and share of direct effects for the two-asset RANK and TANK reported in Table 1 are obtained from these experiments.

RANK C



TANK C

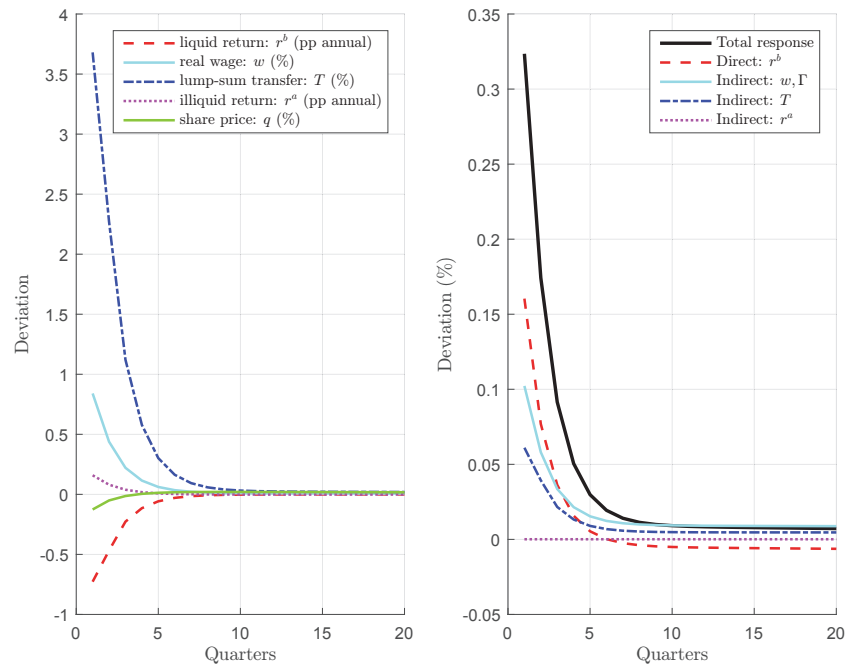


Figure A.1: Impulse Response Functions and Decompositions in RANK and TANK

B Additional Details on the HANK Model

B.1 HJB and Kolmogorov Forward Equations for Household's Problem

We here present the households' HJB equation, and the Kolmogorov forward equation for the evolution of the cross-sectional distribution μ . We focus on the stationary versions of these equations under the assumption that the logarithm of income $y_{it} = \log z_{it}$ follows a “jump-drift process”

$$dy_{it} = -\beta y_{it} dt + dJ_{it}.$$

Jumps arrive at a Poisson arrival rate λ . Conditional on a jump, a new log-earnings state y'_{it} is drawn from a normal distribution with mean zero and variance σ^2 , $y'_{it} \sim \mathcal{N}(0, \sigma^2)$. The stationary version of households' HJB equation is then given by

$$\begin{aligned} (\rho + \zeta)V(a, b, y) = & \max_{c, \ell, d} u(c, \ell) + V_b(a, b, y) [(1 - \tau)we^y \ell + r^b(b)b + T - d - \chi(d, a) - c] \\ & + V_a(a, b, y)(r^a a + d) \\ & + V_y(a, b, y)(-\beta y) + \lambda \int_{-\infty}^{\infty} (V(a, b, x) - V(a, b, y)) \phi(x) dx \end{aligned} \quad (\text{B.63})$$

where ϕ is the density of a normal distribution with variance σ^2 .

Similarly, the evolution of the joint distribution of liquid wealth, illiquid wealth and income can be described by means of a Kolmogorov forward equation. To this end, denote by $g(a, b, y, t)$ the density function corresponding to the distribution $\mu_t(a, b, z)$, but in terms of log productivity $y = \log z$. Furthermore, denote by $s^b(a, b, y)$ and $s^a(a, b, y)$ the optimal liquid and illiquid asset saving policy functions, i.e. the optimal drifts in the HJB equation (B.63). Then the stationary density satisfies the Kolmogorov forward equation

$$\begin{aligned} 0 = & -\partial_a(s^a(a, b, y)g(a, b, y)) - \partial_b(s^b(a, b, y)g(a, b, y)) \\ & - \partial_y(-\beta y g(a, b, y)) - \lambda g(a, b, y) + \lambda \phi(y) \int_{-\infty}^{\infty} g(a, b, x) dx \\ & - \zeta g(a, b, y) + \zeta \delta(a - a_0) \delta(b - b_0) g^*(y), \end{aligned} \quad (\text{B.64})$$

where δ is the Dirac delta function, (a_0, b_0) are starting assets and income and $g^*(y)$ is the stationary distribution of y . [Achdou et al. \(2017\)](#) explain in detail how to solve (B.63) and (B.64), including how to handle the state constraints, using a finite

difference method.

B.2 Proof of Lemma 1 (Derivation of Phillips Curve)

The firm's problem in recursive form is

$$r^a(t)J(p, t) = \max_{\pi} \left(\frac{p}{P(t)} - m(t) \right) \left(\frac{p}{P(t)} \right)^{-\varepsilon} Y(t) - \frac{\theta}{2} \pi^2 Y(t) + J_p(p, t) p \pi + J_t(p, t)$$

where $J(p, t)$ is the real value of a firm with price p . The first order and envelope conditions for the firm are

$$J_p(p, t) p = \theta \pi Y$$

$$(r^a - \pi) J_p(p, t) = - \left(\frac{p}{P} - m \right) \varepsilon \left(\frac{p}{P} \right)^{-\varepsilon-1} \frac{Y}{P} + \left(\frac{p}{P} \right)^{-\varepsilon} \frac{Y}{P} + J_{pp}(p, t) p \pi + J_{tp}(p, t).$$

In a symmetric equilibrium we will have $p = P$, and hence

$$J_p(p, t) = \frac{\theta \pi Y}{p} \tag{B.65}$$

$$(r^a - \pi) J_p(p, t) = - (1 - m) \varepsilon \frac{Y}{p} + \frac{Y}{p} + J_{pp}(p, t) p \pi + J_{tp}(p, t). \tag{B.66}$$

Differentiating (B.65) with respect to time gives

$$J_{pp}(p, t) \dot{p} + J_{pt}(p, t) = \frac{\theta Y \dot{\pi}}{p} + \frac{\theta \dot{Y} \pi}{p} - \frac{\theta Y \dot{p}}{p^2}.$$

Substituting into the envelope condition (B.66) and dividing by $\theta Y/p$ gives

$$\left(r^a - \frac{\dot{Y}}{Y} \right) \pi = \frac{1}{\theta} (-(1 - m) \varepsilon + 1) + \dot{\pi}.$$

Rearranging, we obtain equation (19) in the main text. \square

B.3 Computation of Marginal Propensities to Consume

In a continuous-time one-asset model, Achdou et al. (2017) define the notion of an MPC that is directly comparable to the empirical evidence. This Appendix generalizes their definition to our two-asset environment.

Definition 1 The *Marginal Propensity to Consume over a period τ* for an individual

with state vector (a, b, z) is given by

$$\text{MPC}_\tau(a, b, z) = \frac{\partial C_\tau(a, b, z)}{\partial b}, \quad \text{where} \quad (\text{B.67})$$

$$C_\tau(a, b, z) = \mathbb{E} \left[\int_0^\tau c(a_t, b_t, z_t) dt \mid a_0 = a, b_0 = b, z_0 = z \right]. \quad (\text{B.68})$$

Similarly, the *fraction consumed out of x additional units of liquid wealth over a period τ* is given by

$$\text{MPC}_\tau^x(a, b, z) = \frac{C_\tau(a, b + x, z) - C_\tau(a, b, z)}{x}. \quad (\text{B.69})$$

The conditional expectation $C_\tau(a, b, z)$ in (B.68) and, therefore, the MPCs in Definition 1 can be conveniently computed using the Feynman-Kac formula. This formula establishes a link between conditional expectations of stochastic processes and solutions to partial differential equations. Applying the formula, we have $C_\tau(a, b, z) = \Gamma(a, b, y, 0)$, with $y = \log z$, where $\Gamma(a, b, y, t)$ satisfies the partial differential equation

$$\begin{aligned} 0 = & c(a, b, y) + \Gamma_b(a, b, y, t)s^b(a, b, y) + \Gamma_a(a, b, y, t)s^a(a, b, y) \\ & + \Gamma_y(a, b, y, t)(-\beta y) + \lambda \int_{-\infty}^{\infty} [\Gamma(a, b, x, t) - \Gamma(a, b, y, t)] \phi(x) dx + \Gamma_t(a, b, y, t) \end{aligned}$$

on $[0, \infty) \times [\underline{b}, \infty) \times [y_{\min}, y_{\max}] \times (0, \tau)$, with terminal condition $\Gamma(a, b, y, \tau) = 0$, and where c, s^b and s^a are the consumption and saving policy functions that solve (B.63).

B.4 Extension with firms' profits allocated to both a and b

Under this extension, a fraction ω of aggregate profits is paid into the illiquid accounts proportionately to the shares owned by each household and the remaining $1 - \omega$ fraction is paid in liquid form to every individual i as a lump-sum rescaled by household productivity, i.e., $\pi_t^b(z_{it}) = \frac{z_{it}}{\bar{z}}(1 - \omega)\Pi_t$ where \bar{z} is average productivity. As explained in the main text, we interpret $\pi_t^b(z_t)$ as bonuses and commissions and $w_t z_t \ell_t + \pi_t^b(z_t)$ as total compensation. Labor income taxes are levied on total compensation.

Therefore, omitting the subscript i to ease notation, a household's holdings of liquid assets b_t evolve according to

$$\dot{b}_t = (1 - \tau_t) [w_t z_t \ell_t + \pi_t^b(z_t)] + r_t^b(b_t) b_t + T_t - d_t - \chi(d_t, a_t) - c_t \quad (\text{B.70})$$

The dynamics of illiquid assets are still given by (11).

To solve their optimization problem, households take also as given $\Pi_t^b = \frac{(1-\omega)}{\bar{z}}\Pi_t$, the rescaled fraction of aggregate profits that is paid out proportionally to individual

productivity. It is useful to define $W_t = (w_t, \Pi_t^b)$, the vector of aggregates that characterizes worker’s compensation. Then, in the vector Γ_t , w_t should be replaced by W_t . Similarly, in our decomposition, the term capturing the indirect effects from changes in labor income induced by the monetary shock —the third term of equation (30)— becomes:

$$\int_0^\infty \left(\frac{\partial C_0}{\partial W_t} \right)' dW_t dt = \int_0^\infty \left(\frac{\partial C_0}{\partial w_t} dw_t + \frac{\partial C_0}{\partial \Pi_t^b} d\Pi_t^b \right) dt. \quad (\text{B.71})$$

Finally, the arbitrage condition between shares of the intermediate producers and capital and the government budget constraint become

$$\frac{\omega \Pi_t + \dot{q}_t}{q_t} = r_t^k - \delta. \quad (\text{B.72})$$

and the government budget constraint reads

$$\dot{B}_t^g + G_t + T_t = \tau_t \int (w_t z \ell_t(a, b, z) + \pi_t^b(z)) d\mu_t + r_t^b B_t^g \quad (\text{B.73})$$

C Details on 2004 SCF and FoF data

Our starting point is the balance sheet for U.S. households in 2004 (Flow of Funds Tables B.100, and B100e for the value of market equity). An abridged version of this table that aggregates minor categories into major groups of assets and liabilities is reproduced in Table C.1 (columns labelled FoF).

The columns labelled SCF in Table C.1 report the corresponding magnitudes, for each asset class, when we aggregate across all households in the 2004 Survey of Consumer Finances (SCF). The comparison between these two data sources is, in many respects, reassuring. For example, aggregate net worth is \$43B in the FoF and \$49B in the SCF, and the FoF ranking (and order of magnitude) of each of these major categories is preserved by the SCF data.⁶⁶ Nevertheless, well known discrepancies exist across the two data sources.⁶⁷

On the liabilities side, credit card debt in FoF data is roughly half as large as in SCF data. The reason is that SCF measures outstanding consumer debt, whereas the FoF measures consumer credit, which includes current balances, whether or not they

⁶⁶This is remarkable, since the underlying data sources are entirely different. The SCF is a household survey. The macro-level estimates of U.S. household sector net worth in the FoF are obtained as a residual with respect to all the other sectors of the economy, whose assets and liabilities are measured based on administrative data derived from aggregate government reports, regulatory filings as well as data obtained from private vendors and agencies such as the Bureau of Economic Analysis (BEA), the Census Bureau, and the Internal Revenue Service (IRS).

⁶⁷For systematic comparisons, see Antoniewicz (2000) and Henriques and Hsu (2013).

Assets				Liabilities			
	FoF	SCF	Liquid		FoF	SCF	Liquid
Real estate	21,000	27,700	N	Mortgage Debt	7,600	8,500	N
Consumer Durables	4,100	2,700	N	Nonrev. Cons. Credit	1,400	1,200	N
				Revolving Cons. Credit	800	400	Y
Deposits	5,800	2,800	Y				
Treasury Bonds	700	200	Y				
Corporate Bonds	900	500	Y				
Corporate Equity	12,600	14,200	N				
Equity in Noncorp. Bus.	7,300	11,100	N				
Total	52,400	59,200		Total	9,800	10,100	

Table C.1: Balance sheet of US households for the year 2004.

Sources: Flow of Funds (FoF) and Survey of Consumer Finances (SCF). Values are in Billions of 2004 US\$. Y/N stands for Yes/No in the categorization of that asset class as liquid.

get paid in full. Thus, the SCF estimate seems more appropriate, given that a negative value of b in the model means the household is a net borrower.

On the asset side, real estate wealth in the SCF is 30 pct higher than in the FoF. The SCF collects self-reported values that reflects respondents' subjective valuations, whereas the FoF combines self-reported house values, from the American Housing Survey (AHS) with national housing price index from CoreLogic and net investment from the BEA. However, during the house-price boom, AHS owner-reported values were deemed unreliable and a lot more weight was put on actual house price indexes, an indication that SCF values of owner-occupied housing may be artificially inflated by households' optimistic expectations.

The valuation of private equity wealth is also much higher in the SCF, by a factor exceeding 1.5. Once again, the FoF estimates appear more reliable, as it relies on administrative intermediary sources such as SEC filings of private financial businesses (security brokers and dealers) and IRS data on business income reported on tax returns, whereas, as with owner-occupied housing, the SCF asks non-corporate business owners how much they believe their business would sell for today.⁶⁸

Finally, deposits and bonds are more than twice as large in the FoF.⁶⁹ Antoniewicz (2000) and Henriques and Hsu (2013) attribute this discrepancy to the fact that the FoF "household sector" also includes churches, charitable organizations and personal trusts (that are more likely to hold wealth in safe instruments) and hedge-funds (that

⁶⁸According to Henriques and Hsu (2013), another reason why the SCF data on private business values is problematic is the combination of a very skewed distribution and the small sample size of the survey that make the aggregate value obtained in the SCF very volatile.

⁶⁹The SCF does not contain questions on household currency holdings, but SCF data summarized above contain an imputation for cash. See Kaplan and Violante (2014) for details.

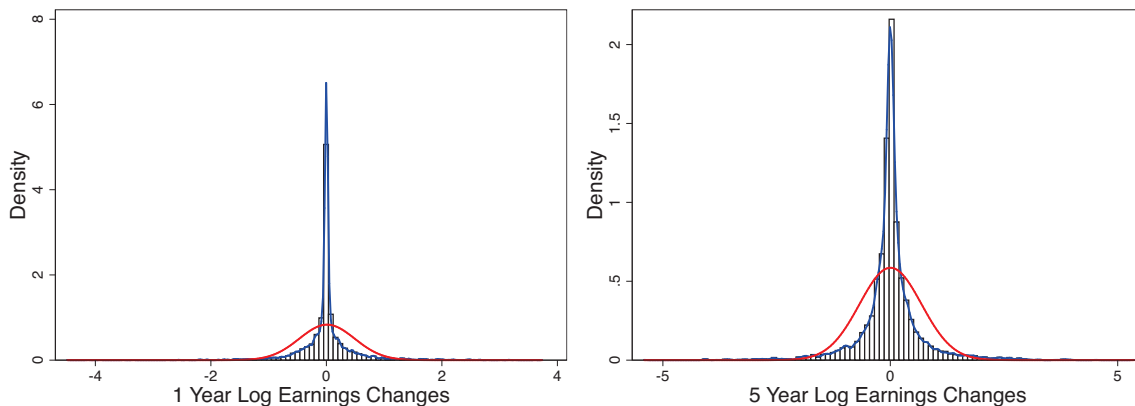


Figure D.1: Growth Rate Distribution of Estimated Earnings Process

may hold large amount of cash to timely exploit market-arbitrage opportunities).

D Further Details on Calibration

D.1 Earnings Process

Figure D.1 displays histograms of one- and five-year log earnings changes generated by our estimated earnings process (33)-(34), overlaid with Normal distributions with the same means and variances. The leptokurtosis of annual income changes is clearly evident from these figures. For a comparison with the analogous figures from SSA male earnings data, we refer readers to Figure 1 in Guvenen et al. (2015).

In order to translate the estimated earnings processes (33)-(34) to a form that can be used in the households' consumption-saving problem (B.63), we take the following steps.

First, we approximate the estimated continuous-time continuous-state processes with continuous-time discrete-state processes. For each of the two components ($j = 1, 2$) we construct a grid for z_j . We use 11 grid points for the persistent component and 3 grid points for the transitory component. We then construct the associated continuous time transition matrix based on a finite difference approximation of the processes in (33)-(34), evaluated at the estimated parameters. We choose the grid widths and spacing so that the annual moments produced by simulating the combined discrete-state process are as close as possible to the annual earnings moments from the combined continuous-state process. These moments are reported in Table D.1. The Lorenz curves for the ergodic distributions associated with the continuous and discretized process are shown by the black dashed line and the green dash-dot line in Figure D.2, respectively. The two Lorenz curves are very close, as are the moments of

Moment	Data	Model	Model
		Estimated	Discretized
Variance: annual log earns	0.70	0.70	0.74
Variance: 1yr change	0.23	0.23	0.21
Variance: 5yr change	0.46	0.46	0.49
Kurtosis: 1yr change	17.8	16.5	15.5
Kurtosis: 5yr change	11.6	12.1	13.2
Frac 1yr change < 10%	0.54	0.56	0.63
Frac 1yr change < 20%	0.71	0.67	0.71
Frac 1yr change < 50%	0.86	0.85	0.83

Table D.1: Earnings Process Estimation Fit

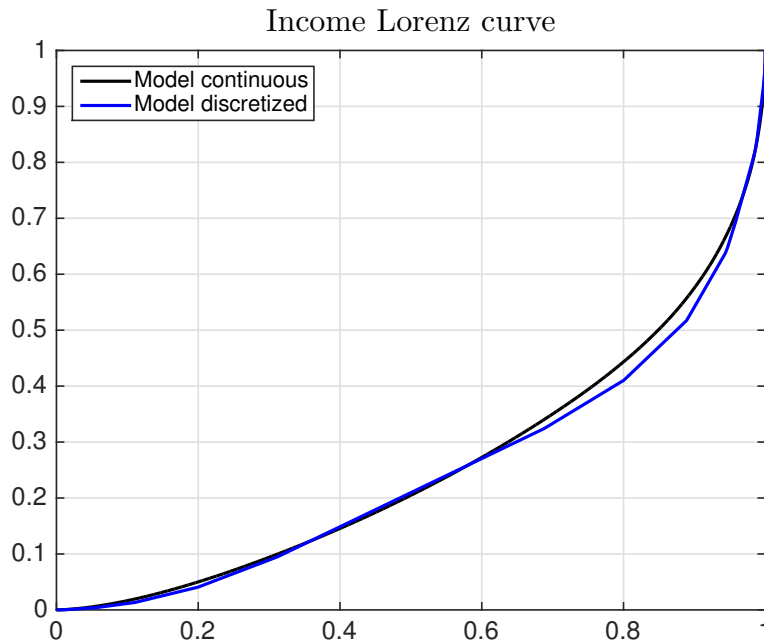


Figure D.2: Earnings Lorenz Curve

earnings changes, which demonstrates that the discrete approximations are accurate.

Second, since earnings in the model are determined by both idiosyncratic productivity z_{it} and endogenous labor supply decisions ℓ_{it} , we make an adjustment to the productivity grid so that the resulting cross-sectional distribution of earnings $y_{it} = w_t z_{it} \ell_{it}$ is as dispersed as in the data. This adjustment is necessary because with our preference specification, optimal labor supply decisions ℓ_{it} are positively related to individual productivity z_{it} . Hence earnings inequality in the model with labor supply is larger than productivity inequality. To bring earnings inequality in line with the data we shrink the log productivity grid by a factor $1 + \zeta \frac{1}{\nu}$, where $\frac{1}{\nu}$ is the Frisch elasticity of labor supply. We set the constant ζ equal to 0.85, which generates a standard deviation

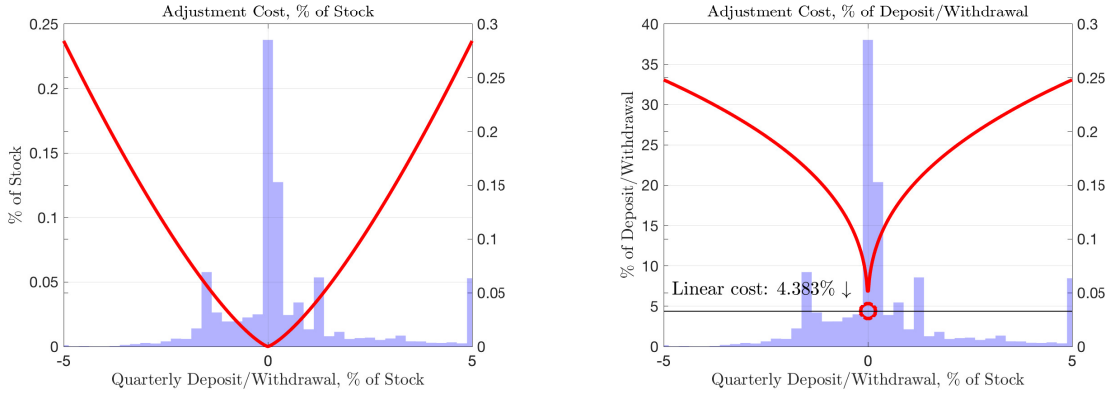


Figure D.3: Calibrated Adjustment Cost Function

of log earnings in the model $\log y_{it}$ equal to the standard deviation of log household earnings in the data. To estimate the standard deviation of log household earnings implied by the SSA data (which we cannot observe directly), we rescale the standard deviation of log male earnings in the SSA data by the ratio of the standard deviation of log household earnings to the standard deviation of log male earnings in the Panel Study of Income Dynamics from 2002 to 2006.

The red dash-dot line in Figure D.2 shows the Lorenz curve for the productivity distribution once it has been re-scaled in this way. Note that it is less dispersed than the raw productivity process (green dash-dot line). The blue solid line shows the Lorenz curve for gross labor income, taking into account the optimal labor supply decisions of households. Note that gross labor income is more dispersed than the adjusted productivity process (because of the substitution effect), but is less dispersed than the raw productivity process (because of the distinction between household earnings and individual male earnings).

D.2 Adjustment Cost Function

The calibrated transaction cost function is shown in Figure D.3. Consider first panel (a). The horizontal axis shows the quarterly transaction expressed as a fraction of a household's existing stock of illiquid assets, d/a . The vertical axis shows the cost of withdrawing or depositing this amount in a single quarter expressed as a fraction of the stock of illiquid assets, $\chi(d, a)/a$. For values of a above the threshold \underline{a} , this function does not depend on the level of illiquid assets. From (14) for $a > \underline{a}$, $\chi(d, a)/a = \chi_0|d/a| + \chi_1|d/a|^2$. The light-blue histogram displays the stationary distribution of adjustments d/a . Roughly 20% percent of households are inactive and neither deposit nor withdraw. Of the remaining households, some deposit and some withdraw. The

histogram shows that, on average, households in the stationary distribution withdraw, taking advantage of the fact that the income generated by the high return of illiquid assets replenishes the illiquid account.

Panel (b) provides an alternative view of the adjustment cost function. The horizontal axis shows the quarterly transaction expressed as a fraction of illiquid assets, d/a , as in panel (a). The vertical axis shows the cost of withdrawing or depositing expressed as a fraction of the amount being transacted, $\chi(d, a)/d$, i.e. the “fee” for each transaction. For values of a above the threshold \underline{a} , this function also does not depend on the level of illiquid assets. From (14) for $a > \underline{a}$, $\chi(d, a)/d = |d/a| + \chi_1 |d/a|^{\chi_2 - 1}$. The overlaid histogram is the same as in panel (a).

These two panels illustrate that, for the most common transaction sizes, the cost is at most 25 percent of the value of the transaction, or at most 0.05 percent of the stock of illiquid wealth.

E Additional Monetary Policy Experiments

In this appendix we report results on the overall effectiveness of monetary policy and its decomposition between direct and indirect effects, when we vary the key parameters that govern the “heterogenous agent block” of the model. These features include the borrowing limit, the cost of borrowing and the parameters of the adjustment cost function. Unlike the robustness experiments conducted in the main text, changing these parameters affects the level and distribution of wealth in the steady state. Hence to maintain comparability across experiments, in each case we re-calibrate the discount rate ρ so as to keep the mean of the illiquid asset distribution (and hence the equilibrium $\frac{K}{Y}$ ratio, wage rate w , interest rates (r, r^a) , and output Y constant. The distribution of illiquid wealth, as well as the mean and distribution of liquid wealth, necessarily differ across the experiments, hence we report these features of the alternative economies alongside the results of the monetary policy shock

Table E.1 reports robustness analyses on the borrowing environment —the tightness of the borrowing limit and the wedge between the interest rates on borrowing and saving in the liquid assets. Reasonable changes in these features of the model have a significant effect on the level and distribution of liquid wealth holdings, but none of the main findings about the size and decomposition of the monetary policy shock are affected by these changes.

Table E.2 reports robustness analyses on the adjustment cost function. As with the borrowing environment, changes in the adjustment cost function affect the level and distribution of liquid wealth holdings but none of the main findings about the size and decomposition of the monetary policy shock are affected.

	Baseline	Loose Borrow Limit $\underline{b} = 4 \times Y^{qu}$	Low Borrow Costs $\kappa = 4\% \text{ pa}$	High Borrow Costs $\kappa = 8\% \text{ pa}$	Very High Borrow Costs $\kappa = 20\% \text{ pa}$
	(1)	(2)	(3)	(4)	(5)
Mean b (rel. to GDP)	0.23	0.21	0.22	0.24	0.25
Frac with $b = 0, a = 0$	0.10	0.10	0.07	0.12	0.09
Frac with $b = 0, a > 0$	0.19	0.17	0.19	0.18	0.15
Frac with $b < 0$	0.15	0.17	0.25	0.09	0.04
Quarterly \$500 MPC	0.16	0.16	0.14	0.16	0.15
Change in r^b (pp)	-0.28 %	-0.29 %	-0.28 %	-0.27 %	-0.27 %
Elasticity of Y	-3.96	-3.75	-3.72	-4.17	-4.17
Elasticity of I	-9.43	-9.05	-8.94	-9.89	-9.94
Elasticity of C	-2.93	-2.81	-2.75	-3.08	-3.08
Partial Eq. Elast. of C	-0.55	-0.64	-0.62	-0.52	-0.52
Component of change in C due to:					
Direct effect: r^b	19 %	23 %	23 %	17 %	17 %
Indirect effect: w	51 %	50 %	49 %	53 %	53 %
Indirect effect: T	32 %	29 %	30 %	33 %	33 %
Indirect effect: $r^a q$	-2 %	-1 %	-2 %	-2 %	-2 %

Table E.1: Robustness: borrowing environment

Notes: Average responses over the first year. Column (1) is the baseline specification. Column (2) loosens the borrowing limit from 1 times quarterly GDP to 4 times quarterly GDP. Column (3) lowers the wedge between the liquid borrowing and liquid savings rates from 6%pa to 4%pa. Column (4) raises the wedge between the liquid borrowing and liquid savings rates from 6%pa to 8%pa. Column (5) raises the wedge between the liquid borrowing and liquid savings rates from 6%pa to 20%pa. All experiments re-calibrate the discount rate ρ so that mean illiquid assets relative to GDP is held constant.

	Baseline	No Linear Cost $\chi_0 = 0$	High Linear Cost $\chi_0 = 0.10$	Low Convex Cost $\chi_2 = 0.10$	High Convex Cost $\chi_2 = 1$
	(1)	(2)	(3)	(4)	(5)
Mean b (rel. to GDP)	0.23	0.22	0.26	0.44	0.11
Frac with $b = 0, a = 0$	0.10	0.09	0.12	0.22	0.01
Frac with $b = 0, a > 0$	0.19	0.19	0.18	0.05	0.19
Frac with $b < 0$	0.15	0.15	0.14	0.12	0.13
Quarterly \$500 MPC	0.16	0.16	0.15	0.11	0.14
Change in r^b (pp)	-0.28 %	-0.28 %	-0.27 %	-0.27 %	-0.26 %
Elasticity of Y	-3.96	-3.99	-3.94	-3.64	-5.52
Elasticity of I	-9.43	-9.68	-9.20	-7.36	-18.64
Elasticity of C	-2.93	-2.88	-3.02	-3.47	-2.59
Partial Eq. Elast. of C	-0.55	-0.54	-0.55	-0.56	-0.43
Component of change in C due to:					
Direct effect: r^b	19 %	19 %	18 %	16 %	17 %
Indirect effect: w	51 %	52 %	51 %	43 %	62 %
Indirect effect: T	32 %	31 %	33 %	42 %	26 %
Indirect effect: $r^a q$	-1 %	-2 %	-2 %	0 %	-4 %

Table E.2: Robustness: adjustment cost function

Notes: Average responses over the first year. Column (1) is the baseline specification. Column (2) sets the linear component of the adjustment cost function to zero. Column (3) increases the linear component of adjustment cost function from 4.4% to 10%. Column (4) reduces the exponent on the convex component of the adjustment cost function from 0.4 to 0.1. Column (5) increases the exponent on the convex component of the adjustment cost function from 0.4 to 1. All experiments re-calibrate the discount rate ρ so that mean illiquid assets relative to GDP is held constant.

F Decomposition of Direct Elasticity

We here only lay out the relevant result and discuss its interpretation. The derivations as well as some additional results can be found in [Kaplan et al. \(2017\)](#).

The time-zero direct consumption response $d \log c_0(a, b, z)$ to small interest rate deviations $\{dr_t^b\}_{t \geq 0}$ of an individual with asset portfolio (a, b) and labor income z can be split into substitution and income effects as follows:

$$d \log c_0(a, b, z) = \underbrace{-\frac{1}{\gamma} \mathbb{E}_0 \left[\int_0^\tau e^{-\int_0^t \varrho_s ds} M_t dr_t^b dt \right]}_{\text{substitution effect}} + \underbrace{\mathbb{E}_0 \left[\int_0^\tau e^{-\int_0^t \varrho_s ds} M_t \left(\frac{\partial_b c_t}{c_t} \right) b_t dr_t^b dt \right]}_{\text{income effect}}$$

where $\varrho_t := \partial_b c_t + (1 + \chi_d(d_t, a_t)) \partial_b d_t - \partial_a d_t$ and $M_t := \frac{u'(c_t)}{u'(c_0)} e^{-\int_0^t (\rho - r_s^b) ds}$

(F.74)

and where $\tau := \inf\{t \geq 0 | \varrho_t = \infty\}$ is the stopping time at which the effective discount rate ϱ_t becomes infinite (which happens for example if liquid wealth reaches zero or the borrowing constraint \underline{b} because $\partial_b c_t$ becomes infinite). Here the expectations are over sample paths of (a_t, b_t, z_t) starting from $(a_0, b_0, z_0) = (a, b, z)$. The term $\partial_b c_t$ is short-hand notation for $\partial_b c_t(a_t, b_t, z_t)$, i.e. the (instantaneous) marginal propensity to consume (MPC) out of liquid wealth, and similarly for $\partial_b d_t$ and $\partial_a d_t$.

The first term in (F.74) is the substitution effect, which is negative and scales with the intertemporal elasticity of substitution (IES) $1/\gamma$. The second term is the income effect, which depends on the time paths of both liquid wealth b_t and the (instantaneous) MPC $\partial_b c_t$.

To better understand (F.74) it is useful to focus on the substitution effect and consider the corresponding formula in a one-asset model. In this special case, deposits are zero ($d_t \equiv 0$) so that the effective discount rate simplifies to $\varrho_t = \partial_b c_t$. Because with uninsurable risk $\partial_b c_t$ is always weakly higher than ρ , the MPC in RANK, the substitution effect is a dampened version of its counterpart in RANK.⁷⁰ The first term in (F.74) is therefore a natural dynamic generalization of Auclert's expression for the substitution effect in response to a transitory one-period interest rate change which he writes as $-\text{IES} \times (1 - \text{MPC})$. It also captures in a transparent fashion the intuition of [McKay et al. \(2016\)](#) that a high likelihood of being constrained in the future is equivalent to a shorter planning horizon.

⁷⁰Specifically, in RANK the substitution effect can be written as $-\frac{1}{\gamma} \int_0^\infty e^{-\rho t} \mathbb{E}_0(M_t) dr_t^b dt$. In the one asset model, it can be written as $-\frac{1}{\gamma} \int_0^\infty \left[\mathbb{E}_0(e^{-\int_0^t \partial_b c_s ds}) \mathbb{E}_0(M_t) + \text{cov}(e^{-\int_0^t \partial_b c_s ds}, M_t) \right] dr_t^b dt$. The substitution effect is higher in RANK because $\partial_b c_t \geq \rho$ and the covariance term is negative by concavity of the consumption function.

In our two-asset model, the effective discount rate ρ_t differs from that in a one-asset model by the term $(1 + \chi_d(d_t, a_t))\partial_b d_t - \partial_a d_t$. In our computations this term is always strictly positive and hence the effective discount rate is strictly larger than that in a one-asset model, which further dampens the intertemporal substitution effect. This additional effect is linked to portfolio rebalancing between liquid and illiquid asset. In a one-asset model a fall in the liquid rate is an incentive for households to reduce savings and consume more. In contrast, in a two-asset model, households also have the option to shift funds from their liquid to illiquid accounts. If the gap between illiquid and liquid rates becomes sufficiently large, then households respond to the fall in r^b by rebalancing their portfolios rather than by increasing their consumption. Intuitively, the fact that individuals may rebalance their portfolios in the future further shortens their effective time horizon.

The contribution of portfolio reallocation at different points of the liquid wealth distribution in Figure 6(a) in the main text, i.e. the solid pink line, is defined as the difference between the sum of income and substitution effects in the two-asset model and the analogous sum computed ‘as in’ a one-asset model by setting the term $(1 + \chi_d(d_t, a_t))\partial_b d_t - \partial_a d_t$ to zero.

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