Networks, Markets and Inequality

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Online Appendix

B1. Other Proofs

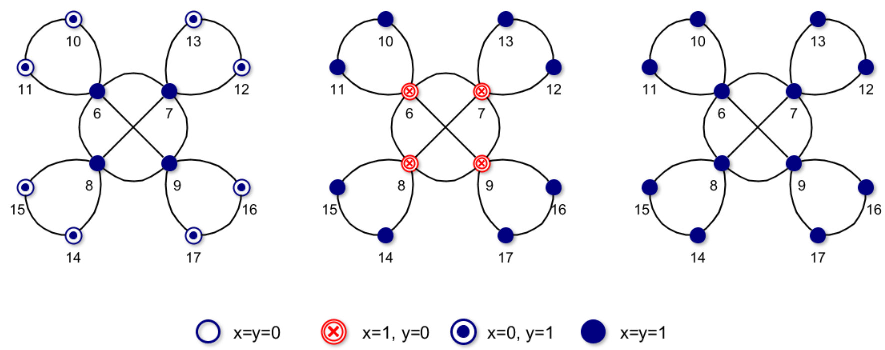
Proof of Proposition 1:

1) Substitutes. Note that $\mathcal{M}(\mathbf{g}) = 0$ whenever $\pi_y \leq 0$. When $\pi_y > 0$, however, $\mathcal{M}(\mathbf{g})$ weakly decreases in the size of $\mathbf{g}^{q_1}$ (with strong substitutes) and in individuals' degree in $\mathbf{g}^{q_2}$ (with non-strong substitutes). Since both $\mathbf{g}^{q_1}$ and $k_i(\mathbf{g}^{q_2})$ are weakly larger in a denser network, then $\mathcal{M}(\mathbf{g})$ must be weakly smaller in denser networks. Consider next the case of complements. Note that $\mathcal{M}(\mathbf{g}) = 1$ whenever $\pi_y > 0$. When $\pi_y \leq 0$, however, $\mathcal{M}(\mathbf{g})$ weakly increase in the size of $\mathbf{g}^{q_2}$ (with strong complements) and in individuals' degree in $\mathbf{g}^{q_1}$ (with non-strong complements). Since both $\mathbf{g}^{q_2}$ and $k_i(\mathbf{g}^{q_1})$ are weakly larger in a denser network, then $\mathcal{M}(\mathbf{g})$ must be weakly larger in denser networks.

2) Substitutes: From Theorems 2 and 3, we know that an individual i will choose $y_i = 1$ if and only if: one, $\pi_y > 0$; and two, she is not in the q_1-core (in the case of strong substitutes) or her degree in the q_2-core is smaller than q_3 (in the case of non-strong substitutes). Hence, it is the least connected individuals who adopt the market action. In the case of complements, an individual i will choose $y_i = 1$ if and only if: one, $\pi_y > 0$; or two, she is in the q_2-core (in the case of strong complements) or her degree within the q_1-core is larger than q_4. Hence, it is the most connected individuals who adopt the market action.

3) We prove this part for strong substitutes and complements; the proof for general complements and substitutes follows directly from this proof and is omitted.

   Substitutes. We already know that $\mathcal{M}(\mathbf{g}) = 0$ whenever $\pi_y \leq 0$. When $\pi_y > 0$, $\mathcal{M}(\mathbf{g})$ depends negatively on the size of $\mathbf{g}^{q_1}$. Hence, an increasing q_1, which has the effect of weakly *reducing* $\mathbf{g}^{q_1}$, increases $\mathcal{M}(\mathbf{g})$. From equation (6), we know that an increase in π_y increases q_1, while increasing returns to x reduce q_1. Hence, $\mathcal{M}(\mathbf{g})$ increases with π_y and decreases with the returns to x.
   Finally, an increase in the degree of complementarity between x and y can have non-monotonic effects on $\mathcal{M}(\mathbf{g})$ when x and y are not strong substitutes. The following example illustrates this possibility. Consider the



Left: $\theta = -0.3$. **Center**: $\theta = -0.24$. **Right**: $\theta = -0.05$.

FIGURE B1. MARKET PARTICIPATION: NON-MONOTONICITY IN θ

network on Figure B1, and consider again the payoffs function in Example 1. Suppose that $p_y = 0$ and $p_x = 1.5$. When $\theta = -0.3$, market participation is maximal. When θ rises to -0.24, however, individuals in the periphery find it profitable to adopt x along with y. This raises the payoffs to x for core individuals, who then switch from $a_i^* = (1,1)$ to $a_i^* = (1,0)$ (second effect). When θ falls to -0.05, the cost of adoption of y (in terms of foregone payoffs to x) is again small enough for core individuals to return to $a_i^* = (1,1)$ (first effect).

Complements. We already know that $\mathcal{M}(\mathbf{g}) = 1$ whenever $\pi_y > 0$. When $\pi_y \leq 0$, $\mathcal{M}(\mathbf{g})$ depends positively on the size of $\mathbf{g}^{q_2}$. Hence, a decrease in q_2 (which weakly increases $\mathbf{g}^{q_2}$) increases $\mathcal{M}(\mathbf{g})$. From equation (7), we know that an increasing π_y, increasing returns to x, and increasing degree of complementarity between x and y all reduce q_2. Hence, $\mathcal{M}(\mathbf{g})$ increases with π_y, the returns to x and the degree of complementarity between x and y. ■

Proof of Proposition 2: The proof to Proposition 2 for the case of substitutes is provided in the main text (Example 3). The proof of Proposition 2 in the case of complements is as follows. First note that $x_{i,0}^* \leq x_{i,1}^*$ for all $i \in N$, where $x_{i,0}^*$ ($x_{i,1}^*$) denotes i's network action before (after) the introduction of markets at the ME. Indeed, from Theorem 2, we know that $x_{i,0}^* = 1$ implies that $i \in \mathbf{g}^{q_1}$, which in turn must imply that $x_{i,1}^* = 1$.[1] It follows that $\chi_{i,0}^* \leq \chi_{i,1}^*$ for all $i \in N$. Since individual i's payoffs are weakly increasing in χ_i^*, i's payoffs must weakly increase, for all $i \in N$. ■

[1]Observe that when x and y are not strong complements, $x_{i,1}^* = 1$ if and only if $i \in \mathbf{g}^{q_1}$. When x and y are strong complements, $x_{i,1}^* = 1$ if and only if $i \in \mathbf{g}^{q_2}$; however, since $q_2 < q_1$ in the case of strong complements, then it must again be the case that $i \in \mathbf{g}^{q_1}$.

Proof of Proposition 3: We start by proving that in a regular network, the maximal equilibrium is efficient. From Theorem 2 we know that in a regular network, all individuals adopt the same strategy in the ME. The first step in the proof is the following Lemma.

Lemma 1: *In a regular network, the efficient outcome is generically symmetric.*

Proof. Suppose *a contrario* that at the efficient outcome $\mathbf{a}^*$, there exist two individuals i and j such that $a_i^* \neq a_j^*$ and $\Phi_i(a_i^*, \mathbf{a}^*_{-\mathbf{i}} \mid \mathbf{g}) \neq \Phi_j(a_j^*, \mathbf{a}^*_{-\mathbf{j}} \mid \mathbf{g})$. Assume without loss of generality that individual i is actually the best-off individual and j the worst-off. This means that $\Phi_i(a_i^*, \mathbf{a}^*_{-\mathbf{i}} \mid \mathbf{g}) > \Phi_j(a_j^*, \mathbf{a}^*_{-\mathbf{j}} \mid \mathbf{g})$, which entails that $x_i^* = 1$ and since j is the worst-off, $x_j^* = 0$.[2] Now, construct a profile $\hat{\mathbf{a}}$ with $\hat{a}_l = a_i^*$ for all $l \in N$. Note first that individual i must be weakly better off in $\hat{\mathbf{a}}$ than in $\mathbf{a}^*$ as the number of her neighbours choosing $x = 1$ has weakly grown: hence, $\Phi_i(a_i^*, \hat{\mathbf{a}}_{-\mathbf{i}}|\mathbf{g}) \geq \Phi_i(a_i^*, \mathbf{a}^*_{-\mathbf{i}}|\mathbf{g})$. Note further that individual j is necessarily strictly better off, and since the network is regular, it must also be true that $\Phi_j(\hat{a}_j, \hat{\mathbf{a}}_{-\mathbf{j}} \mid \mathbf{g}) = \Phi_i(a_i^*, \hat{\mathbf{a}}_{-\mathbf{i}} \mid \mathbf{g}) \geq \Phi_i(a_i^*, \mathbf{a}^*_{-\mathbf{i}} \mid \mathbf{g}) > \Phi_i(a_j^*, \mathbf{a}^*_{-\mathbf{j}} \mid \mathbf{g})$. All individuals in between i and j are also clearly (weakly) better-off. It follows then that $W(\mathbf{a}^*|\mathbf{g}) < W(\hat{\mathbf{a}}|\mathbf{g})$. This contradicts our hypothesis that $\mathbf{a}^*$ is the efficient outcome. This completes the proof of Lemma 1. ♣

The next step is the following Lemma.

Lemma 2: *In a regular network, the efficient outcome is an equilibrium.*

Proof. The proof is by contradiction. We start with the efficient outcome. We argue that if it is not an equilibrium then it must be inefficient. Note first from Lemma 1 that the efficient outcome is symmetric. Suppose that the efficient outcome $\mathbf{a}^*$ is not an equilibrium. Then, there must be some individual j who wants to deviate from $a_j^* = (x_j^*, y_j^*)$ to $a_j' \in \{(1 - x_j^*, y_j^*), (x_j^*, 1 - y_j^*), (1 - x_j^*, 1 - y_j^*)\}$. If j wants to deviate from a_j^* to a_j', then $\Phi_j(a_j', \mathbf{a}^*_{-j} \mid \mathbf{g}) > \Phi_j(\mathbf{a}^* \mid \mathbf{g})$. It is easy to check that if this is the case for j, then all individuals can strictly improve their payoffs by choosing a_j', which contradicts the hypothesis that $\mathbf{a}^*$ is the efficient outcome. ♣

The final step is to show that the efficient outcome is the ME. This is clearly true: if this were not true then there would exist another equilibrium that Pareto-dominates the efficient outcome, $\mathbf{a}^*$. But this would contradict the hypothesis that $\mathbf{a}^*$ is efficient. Thus the efficient outcome is a ME. As the ME is (generically) unique, the proof that the ME is efficient in regular networks is thus complete.

We complete the proof by showing via an example that the ME in non-regular networks may be inefficient. Consider the CP network introduced in Figure 3.

[2] Indeed, suppose by contradiction that $x_j = 1$. The only way i can achieve higher payoffs that j is thus if $\chi_i > \chi_j$. Since the network is regular, then there must exists an individual $k \in N_j(\mathbf{g})$ with $x_k = 0$. But then this entails that $\Phi(a_j, \mathbf{a}^*_{-\mathbf{j}}|\mathbf{g}) \neq \Phi(a_k, \mathbf{a}^*_{-\mathbf{k}}|\mathbf{g})$, which contradicts the hypothesis that j is the worst-off individual.

Suppose the payoff function is as in Example 1. Suppose first that $\theta = -0.9$, and suppose that $p_y < p_x < 1$. At the ME, periphery individuals choose $x = 0$ and $y = 1$, while core individuals choose $x = 1$ and $y = 0$, yielding us $\mathcal{W}(\mathbf{g}) = 25 - 5p_x - 5p_y$. Now construct a profile $\hat{\mathbf{a}}$, where $\hat{x}_i = 1$ and $\hat{y}_i = 0$ for all $i \in N$. The resulting aggregate welfare is $W(\hat{\mathbf{a}} \mid \mathbf{g}) = 30 - 10p_x$. Since $W(\hat{\mathbf{a}} \mid \mathbf{g}) - \mathcal{W}(\mathbf{g}) = 5 - 5p_x + 5p_y > 0$, then the ME is clearly not efficient. ■

Proof of Proposition 4:

1) Observe first that the payoffs of individuals outside the q-core, for the threshold values q_1 and q_2, are always independent of the size and density of the q-core. Thus, making the network denser and expanding the size or density of the relevant q-core leaves the payoffs of those individuals unchanged. Second, observe that the payoffs of individuals within the q-core always weakly increase in its size and density. Hence, increasing the size or density of the relevant q-core always weakly enhances aggregate welfare.

2) The proof here follows from a similar argument and from the observation that individuals inside the relevant q-core (who always choose $x = 1$) could always achieve the same payoffs as individuals outside the q-core by adopting the action of individuals outside the q-core (who always choose $x = 0$). The fact that they do not entails that they must have higher payoffs.

3) Note that π_y, the returns to x and the degree of complementarity between x and y have two effects on $\mathcal{W}(\mathbf{g})$. The first (direct) effect is the *income effect*: when payoffs increase, individuals' payoffs increase consequently. The second (indirect) effect relates closely to *q-core*: recall that aggregate welfare always depends positively on the size and density of the q-core at the threshold values q_1 and q_2, and that these values depend on the payoffs to x and y.[3] Note first that q_1 and q_2 are both decreasing in the returns to x and the degree of complementarity between x and y; hence, increasing returns to x and degree of complementarity between x and y always weakly increase the size of the q-core for any of these threshold values. Since the *income effect* and the *q- core effect* of an increase in the returns to x and/or the degree of complementarity between x and y on $\mathcal{W}(\mathbf{g})$ are both positive, then clearly $\mathcal{W}(\mathbf{g})$ increases in the returns to x and the degree of complementarity between x and y.
   Observe that if x and y are complements, then an increasing π_y always has a weakly positive q-core effect (since an increase in π_y weakly reduces q_2). In that case, the *income effect* and *the q-core effect* of an increasing π_y on

[3] Note also that the thresholds q_3 and q_4 alre depend on the payoffs to x and y. Recall however that q_3 and q_4 are thresholds values of χ_i for a individual i to adopt (or not) the market action when i has adopted the network action. As such, they do not impact the number of individuals who choose the network action. Hence, the impact of the payoffs to x and y on q_3 and q_4 is captured entirely in the income effect.

$\mathcal{W}(\mathbf{g})$ are both weakly positive, and so $\mathcal{W}(\mathbf{g})$ increases in π_y. However, when x and y are substitutes, the *income effect* and the *q-core effect* oppose each other as q_1 weakly increases with π_y. Example 3 in the main text shows that this can lead to non-monotonic effects on $\mathcal{W}(\mathbf{g})$. ■

B2. Extension: Generalized Network Exchange

In our benchmark model, described in Section I, it is assumed that the returns to network exchange depend only on the number of (direct) neighbours who choose $x = 1$. However, in principle a individual could benefit from the neighbours' of his neighbours (and so forth). This section develops a model to capture such indirect network benefits.

We shall say that individual i's returns to x increase with another j choosing $x_j = 1$ if and only if *i has access to j*, and we assume that those returns decrease with the *access distance* between i and j. For a given action profile $\mathbf{a}$, i has *access* to j if either $g_{ij} = 1$ and $x_j = 1$ or if there exists an *access path* between i and j, i.e. if there is a set of individuals $\{i_1, i_2, ...i_{j-1}, j\}$ such that $x_{i_1} = x_{i_2} = ... = x_{i_{j-1}} = x_j = 1$ and $g_{ii_1} = g_{i_1 i_2} = ... = g_{i_{j-1}j} = 1$. An access path between i and j is thus a path on which all individuals choose $x = 1$. The *access distance* between i and j, denoted by d_{ij}, is the length of the shortest access path between i and j; if there is no such access path, we assume $d_{ij} = \infty$.

To account for indirect benefits, we simply redefine $\chi_i(\mathbf{a})$, given in equation (1), as:

$$\chi_i(\mathbf{a}, \delta) = \sum_{j \in N} f(d_{ij}(\mathbf{a}), \delta) \tag{B1}$$

where f is non-decreasing in its arguments and at $f(\cdot) \in [0, 1]$. For concreteness and for ease of exposition, let us suppose that:[4]

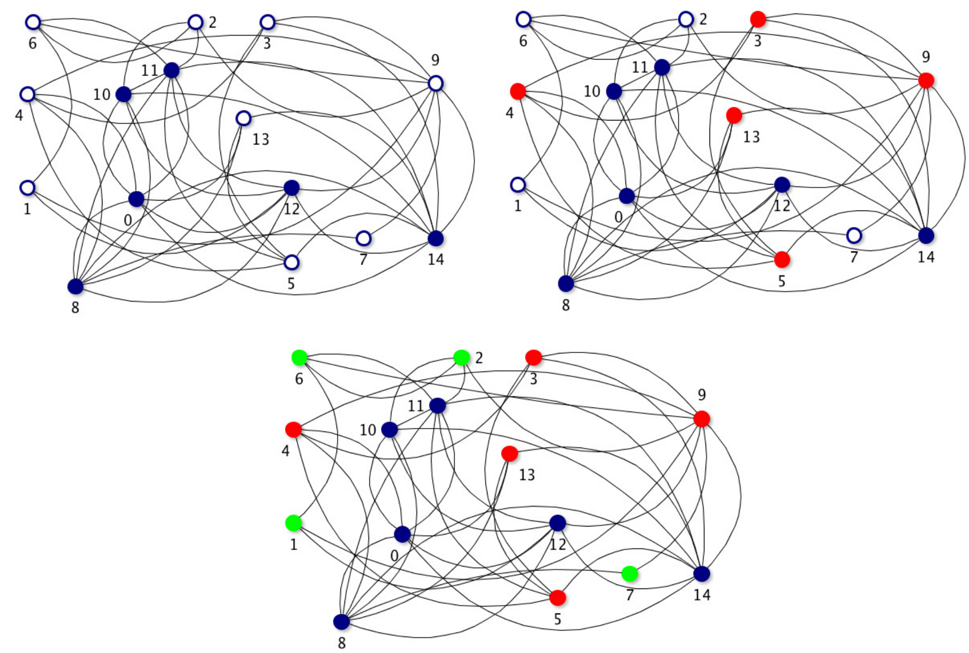
$$f(d_{ij}(\mathbf{a}), \delta) = (\delta)^{d_{ij}-1} \tag{B2}$$

The parameter $\delta \in [0, 1]$ is the *decay factor* and thus captures the extent to which distance affects payoffs. Equation (B2) tells us that when $\delta = 0$, decay over distance is maximal and we are back at the basic model: individuals do not benefit from distant neighbours. At the other extreme, when $\delta = 1$, there is no decay and so individuals benefit from distant neighbours as much as from immediate ones. In this case, our model approximates a standard formulation in the development and industrial organization literature: network benefits are defined by the size of the group (or the network component, in our case). As

[4] Our results extend to a general f where $f_{12} < 0$, and the corner conditions for $\delta = 1$ and $\delta = 0$ are satisfied; we adopt the present formulation due to its expositional simplicity.

before we assume that Assumptions 1 and 2 hold, after suitable rewording to account for equation (B1) and (B2).

Define the *q-central core* of $\mathbf{g}$, denoted by $c_q(\mathbf{g})$, as the set of individuals with at least $q-1$ decay centrality in the sub-graph formed only by the individuals in $c_q(\mathbf{g})$. Note that the definition of the q-central core is analogous to that of the q-core. The algorithm to attain $c_q(\mathbf{g})$ is analogous to Algorithm 1: instead of eliminating nodes with q or fewer links, delete nodes with q or smaller f value (or decay centrality) at each step.



Top left: $\delta = 0$. **Top right**: $\delta = 0.1$. **Bottom**: $\delta = 1$. Coloured nodes are in the 4-central core.

Figure B2. The 4-central core

We illustrate the role of δ in shaping the relevant q-central core on Figure B2 using the same network as on Figure 1. We look for the 4-central core. When $\delta = 0$, the 4-central core corresponds to the 4-core, as illustrated on Figure 1. When δ rises to 0.1, however, individuals gain access to other individuals that are not directly connected to them. These indirect connections push individuals 3, 4, 5, 9 and 13 into the 4-central core. When δ rises to 1, finally, it is *as if* all individuals were directly connected, since distant neighbours count as much as immediate ones. Hence, the maximal q-central core any individual belongs to is determined by the size of the component she lies into. Since this size is larger than 4 on Figure B2, then all individuals are in the 4-central core when $\delta = 1$.

Theorem 4: *Suppose that Assumptions 1 and 2 hold. Assume that $f(d_{ij}(\mathbf{a}), \delta)$ is given by (B2). Let $\mathbf{a}^* = (a_1^*, a_2^*, ..., a_n^*)$ be the ME.*

1) ***Strong Substitutes.*** *$a_i^* = (1,0)$ if and only if $i \in c_{q_1}(g)$. If $i \notin c_{q_1}(\mathbf{g})$, then $a_i^* = (0,0)$ in case $\pi_y \leq 0$, and $a_i^* = (0,1)$ otherwise.*

2) ***Strong Complements.*** *$a_i^* = (1,1)$ if and only if $i \in c_{q_2}(g)$. If $i \notin c_{q_2}(\mathbf{g})$, then $a_i^* = (0,0)$ in case $\pi_y \leq 0$, and $a_i^* = (0,1)$ otherwise.*

The proof to Theorem 4 is analogous to that of Theorem 2 and is omitted.

Observe that, other things being the same, the value of a neighbour switching to action x depends very much on the location of the neighbour in the network. A neighbour who creates access to many new networks members has a much larger effect than an isolated neighbour. Thus certain individuals may be important for network exchange because they connect different parts of a network. This is in the spirit of the idea of *structural holes* (Burt, 1992). To see this in the simplest possible way consider a star (or hub-spoke) network with n nodes and 1 as the centre and with $n-1$ nodes linked only to it. It is then easy to see that for a peripheral node, the value of the hub switching to x is of the order $[\delta(n-1)]$, while the value to the hub the value of a spoke node switching is only of the order δ. This difference in value between the hub and spoke individual grows with δ. The central individual spans a structural hole in the network: without her, the network would be completely fragmented. The presence of large indirect benefits clarifies the role of 'key' individuals in shaping behavior.

We illustrate further the role of structural holes with Figure B3. Suppose that two separate communities (individuals 2 to 6 and 7 to 11) get connected by a new individual, 1. Without individual 1, the largest q for which individuals 2 to 11 are in the q-central core for any δ is $4-\epsilon$, with ϵ the smallest value greater than 0. However, individual 1 connects the two communities: the benefits of indirect exchange brought by individual 1 depend on δ. When $\delta = 0.5$, for example, than the largest q such that individuals 2 to 11 are in the q-central core increases to $5.25-\epsilon$. When $\delta = 1$, then that largest q increases to $10-\epsilon$: in such case, individual 1 more than doubles the value of network exchange due to indirect payoffs!

We conclude by noting that (suitably reworded versions of) Propositions 1 to 5 hold in the extended model.

B3. Heterogeneity in returns to market action

The framework developed so far highlights the implications of individuals' heterogeneity with respect to network position. Individuals may be heterogeneous in other dimensions that affect the extent to which they can benefit from markets (e.g. human capital, initial wealth). This motivates a study of the combined effect of network heterogeneity with other types of heterogeneity. In the following

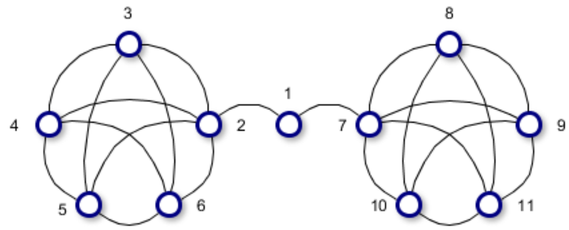


Figure B3. Generalized Network Exchange

analysis, we assume that the benefits from the market action, π_y, differ among individuals. We assume that this heterogeneity does not affect the other determinants of the payoff function (i.e. returns to x and degree of complementarity between x and y).

Let π_y^i denote individual i's payoffs from y. Note that the thresholds q_1, q_2, q_3 and q_4, given respectively by (6), (7), (8) and (9), are now heterogeneous. For each individual i, there exist unique thresholds $q_1^i \geq 0$, $q_2^i \geq 0$, $q_3^i \geq 0$, $q_4^i \geq 0$ such that for any $\chi_i > q_1^i$, $\chi_i > q_2^i$, $\chi_i > q_3^i$ and $\chi_i > q_4^i$, respectively:

$$\phi_0(\chi_i) > \max\{0, \pi_y^i\} \tag{B3}$$

$$\phi_{1,i}(\chi_i) > \max\{0, \pi_y^i\} \tag{B4}$$

$$\phi_0(\chi_i) > \phi_{1,i}(\chi_i) \tag{B5}$$

$$\phi_0(\chi_i) < \phi_{1,i}(\chi_i) \tag{B6}$$

We next adapt the definition of the q-core introduced earlier.

Definition 2: Generalised q-core. *Consider a vector $\mathbf{q} = \{q_i\}_{i \in N}$ ascribing value q_i to each individual in N. The* generalised q-core *of $\mathbf{g}$, denoted by $\mathbf{g}^{q(i)}$, is the unique largest subgraph of $\mathbf{g}$ wherein all individuals have strictly more than q_i links to other individuals in $\mathbf{g}^{q(i)}$.*

We provide a characterization of equilibrium for the cases of strong substitutes and complements. The extension to general substitutes and complements is analogous.

Theorem 5: *Suppose Assumptions 1 and 2 hold. Let $\mathbf{a}^*$ be the ME. For all $i \in N$:*

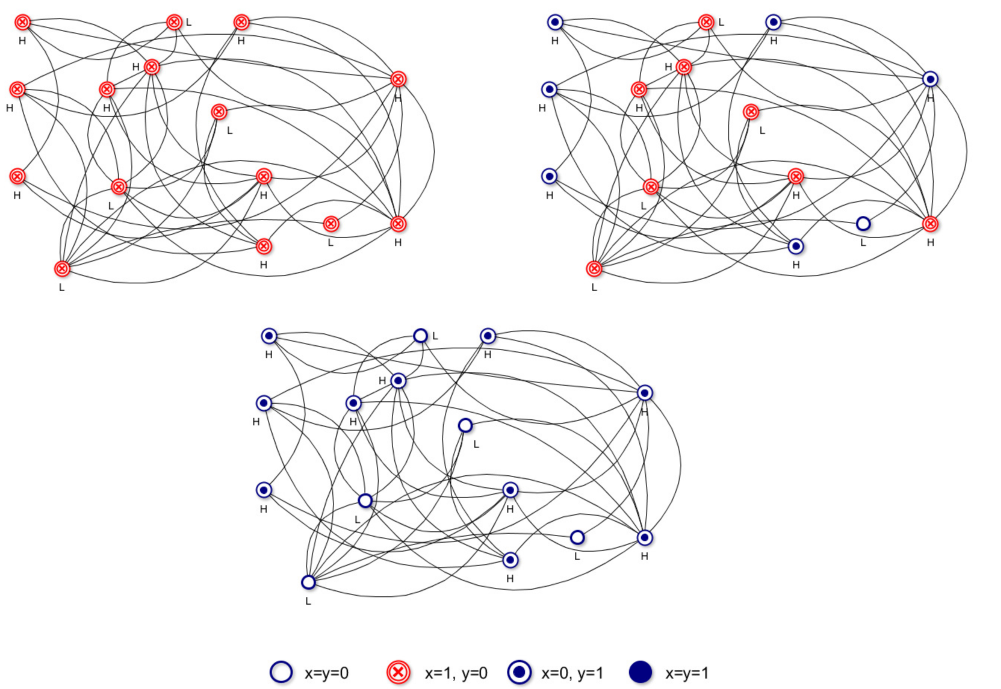
1) ***Strong Substitutes.*** *$a_i^* = (1, 0)$ if and only if $i \in \mathbf{g}^{q_1(i)}$. If $i \notin \mathbf{g}^{q_1(i)}$, then $a_i^* = (0, 0)$ in case $\pi_y^i \leq 0$, and $a_i^* = (0, 1)$ in case $\pi_y^i > 0$.*

2) ***Strong Complements.*** *$a_i^* = (1, 1)$ if and only if $i \in \mathbf{g}^{q_2(i)}$. If $i \notin \mathbf{g}^{q_2(i)}$, then $a_i^* = (0, 0)$ in case $\pi_y^i \leq 0$, and $a_i^* = (0, 1)$ in case $\pi_y^i > 0$.*

To illustrate this result, we proceed with an example close to Example 1. Suppose that the payoff function is given by:

$$\Phi_i(a_i, \mathbf{a_{-i}}|\mathbf{g}) = (1 + \theta y_i)x_i\chi_i(\mathbf{a}) - p_x x_i + \pi_y^i \tag{B7}$$

with $\theta = -0.9$. Suppose that individuals have either "high" or "low" returns from y, with $\pi_y^H = 3$ and $\pi_y^L = -1$. With these parameters, it is easy to compute that $q_1^H = 3 + p_x$ while $q_1^L = p_x$, implying that high individuals require much higher returns to x to renounce to y. Figure B4 shows the patterns of adoption of x and y for different values of p_x for the network introduced in Figure 1. Figure B4 shows that those who are the promptest to adopt y are not only poorly connected individuals, but also those who have high π_y^i (top right network). Further, with heterogeneity, it is possible for the market action to be adopted as a stand-alone by certain individuals, while others opt for $a^* = (0, 0)$ (bottom network).



Top left: $p_x = 0.5$. **Top right**: $p_x = 1.5$. **Bottom**: $p_x = 2.5$.

FIGURE B4. THE q_i-CORE IN AN ARBITRARY NETWORK, WITH $q_H = 2$ AND $q_L = 5$

Note that the results presented in Propositions 1 and 4 are robust to heterogeneity in the market action returns, π_y. The results on inequality, however, may change considerably, e.g. if π_y^i is *negatively* related to membership in the q-core. To see why, consider for example the case of substitutes. Recall that in our benchmark model, only poorly connected individuals, i.e. those out of the q_1-core, may opt for the market action. Those individuals are also the worst off, which explains why inequality always goes down with the introduction of markets in the case of substitutes. Suppose now that poorly connected individuals have high returns to y while others have no returns at all. In such case, the introduction of y may make the poorly connected individuals the best-off individuals, which may well *increase* inequality.

B4. Gini Coefficient

We discuss the effect of the introduction of y on the *Gini coefficient*. Given $\mathbf{g}$, we denote the Gini-coefficient in the ME by $\mathcal{G}(\mathbf{g})$. The following result summarizes the impact of markets on $\mathcal{G}(\mathbf{g})$.

Proposition 6: *When x and y are strong substitutes, the introduction of the market action y (weakly)* decreases $\mathcal{G}(\mathbf{g})$ *if it also (weakly) increases $\mathcal{W}(\mathbf{g})$. If x and y are not strong substitutes or if the introduction of y decreases $\mathcal{W}(\mathbf{g})$, its effect on $\mathcal{G}(\mathbf{g})$ is ambiguous. When x and y are not strong complements and there is at least one individual who adopts x without adopting y, the introduction of y (weakly) increases $\mathcal{G}(\mathbf{g})$. If they are strong complements or if all individuals who adopt x also adopt y, its effect on $\mathcal{G}(\mathbf{g})$ is ambiguous.*

Proof. We start by introducing new notation. Consider the sequence of individuals $(1, 2...j, ...n)$ such that $\Phi_j(a_j, \mathbf{a}^*_{-\mathbf{j}} \mid \mathbf{g}) \leq \Phi_{j+k}(a_{j+k}, \mathbf{a}^*_{-(\mathbf{j}+\mathbf{k})} \mid \mathbf{g})$ for all $j \in N$ and $k \in (1, n-1)$.[5] The Lorenz curve, at a given individual i under equilibrium $\mathbf{a}^*$, is given by:

$$\mathcal{L}(i \mid \mathbf{g}) = \frac{\Phi_i(a_i, \mathbf{a}^*_{-\mathbf{i}} \mid \mathbf{g}) + \sum_{j<i} \Phi_j(a_j, \mathbf{a}^*_{-\mathbf{j}} \mid \mathbf{g})}{\mathcal{W}(\mathbf{g})} \tag{B8}$$

Observe that $\mathcal{L}(0 \mid \mathbf{g}) = 0$ and $\mathcal{L}(n \mid \mathbf{g}) = 1$, by definition. We denote by $\Delta\mathcal{L}(i \mid \mathbf{g})$ the slope of the Lorenz curve at an individual i, with

$$\Delta\mathcal{L}(i \mid \mathbf{g}) = \frac{\Phi_i(a_i, \mathbf{a}^*_{-\mathbf{i}} \mid \mathbf{g})}{\mathcal{W}(\mathbf{g})} \tag{B9}$$

Given the ordering of individuals from lowest to highest (in payoffs), the Lorenz curve is increasing and convex: this means that $\Delta\mathcal{L}(j \mid \mathbf{g}) \leq \Delta\mathcal{L}(i \mid \mathbf{g})$ if and only if $j < i$ in the support. Furthermore, we say that a Lorenz curve A, $\mathcal{L}_{\mathcal{A}}(i \mid \mathbf{g})$,

[5] As multiple individuals can have the same payoffs, this sequence may not be unique.

dominates a Lorenz curve B, $\mathcal{L}_B(i \mid \mathbf{g})$, if $\mathcal{L}^A(i \mid \mathbf{g}) \geq \mathcal{L}^B(i \mid \mathbf{g})$ for all $i \in N$, with inequality strict for at least one i. Denote by $\mathcal{L}_0(i \mid \mathbf{g})$ and by $\mathcal{L}_1(i \mid \mathbf{g})$ the Lorenz curve before and after the introduction of y, respectively, and define $\mathcal{G}_0(\mathbf{g})$ and $\mathcal{G}_1(\mathbf{g})$ analogously. Observe that it is sufficient to show $\mathcal{L}_0(i \mid \mathbf{g})$ dominates $\mathcal{L}_1(i \mid \mathbf{g})$ to prove that $\mathcal{G}_0(\mathbf{g}) \leq \mathcal{G}_1(\mathbf{g})$, and vice-versa.

Substitutes. We start by proving that the introduction of the market action y when x and y are strong substitutes (weakly) *decreases* $\mathcal{G}(\mathbf{g})$ if and only if it also (weakly) increases $\mathcal{W}(\mathbf{g})$. Note that we ignore trivial cases where $\mathcal{M}(g) = 0$, wherein $\mathcal{G}(\mathbf{g})$ is obviously left unchanged by the introduction of y.

Recall that when x and y are strong substitutes and $\pi_y > 0$, individuals always prefer *either* $a = (0, 1)$ or $a = (1, 0)$. There are thus two cases possible after the introduction of y if $\mathcal{M}(\mathbf{g}) > 0$: all individuals adopt only y; and some individuals adopt x while all others adopt y. In the first case, note that all individuals have the same payoffs, and so $\mathcal{G}_1(\mathbf{g}) = 0$: clearly $\mathcal{G}_1(\mathbf{g}) \leq \mathcal{G}_0(\mathbf{g})$.

Consider now the second case. We show that either $\mathcal{L}_0(i \mid \mathbf{g})$ dominates $\mathcal{L}_1(i \mid \mathbf{g})$ or $\mathcal{L}_0(i \mid \mathbf{g}) = \mathcal{L}_1(i \mid \mathbf{g})$. We partition individuals into two groups, namely those who choose $a^*_{i,1} = (1, 0)$ and those who choose $a^*_{i,1} = (0, 1)$. Label the former group A and the latter group B. The support of $\mathcal{L}(i \mid \mathbf{g})$ can be written as $\{1, 2, ...n_B, n_B + 1, ...n\}$, and so $B = \{1, 2...n_B\}$ and $A = \{n_{B+1}, ...n\}$. Recall from Proposition 4 that $\Phi_i(a_{i,1}, \mathbf{a}^*_{-\mathbf{i},\mathbf{1}} \mid \mathbf{g}) > \Phi_j(a_{j,1}, \mathbf{a}^*_{-\mathbf{j},\mathbf{1}} \mid \mathbf{g})$ for any $i \in A$ and $j \in B$. Hence, $\mathcal{L}_1(i \mid \mathbf{g}) > \mathcal{L}_1(j \mid \mathbf{g})$ and $\Delta\mathcal{L}_1(i \mid \mathbf{g}) > \Delta\mathcal{L}_1(j \mid \mathbf{g})$ for any $i \in A$ and $j \in B$. In other words, B forms the left support of $\mathcal{L}(i \mid \mathbf{g})$, while A is its right support.

Since we know that $x^*_{i,0} \geq x^*_{i,1}$ for all $i \in N$ in the case of substitutes, we know that if an individual j chooses $a^*_{j,1} = (1, 0)$, then $a^*_{j,0} = (1, 0)$ and $\Phi_j(a_{j,1}, \mathbf{a}^*_{-\mathbf{j},\mathbf{1}} \mid \mathbf{g}) \leq \Phi_j(a_{j,0}, \mathbf{a}^*_{-\mathbf{j},\mathbf{0}} \mid \mathbf{g})$. Since $\mathcal{W}_1(\mathbf{g}) \geq \mathcal{W}_0(\mathbf{g})$, then $\Delta\mathcal{L}_1(j \mid \mathbf{g}) \leq \Delta\mathcal{L}_0(j \mid \mathbf{g})$ for any $j \in$, and:

(B10)
$$\frac{\sum_{j \in A} \Phi_j(a_{j,1}, \mathbf{a}^*_{-\mathbf{j},\mathbf{1}} \mid \mathbf{g})}{\mathcal{W}_1(\mathbf{g})} < \frac{\sum_{j \in A} \Phi_j(a_{j,1}, \mathbf{a}^*_{-\mathbf{j},\mathbf{1}} \mid \mathbf{g})}{\mathcal{W}_0(\mathbf{g})}$$

Note that $\mathcal{L}_1(n_B \mid \mathbf{g}) > \mathcal{L}_0(n_B \mid \mathbf{g})$ (since the total share of payoffs going to individuals in A falls, that going to individuals in B must rise).

It thus follows that $\mathcal{L}_1(i \mid \mathbf{g})$ dominates $\mathcal{L}_0(i \mid \mathbf{g})$ over its support segment A. To see why, suppose *a contrario* that there exists a $j \in A$ such that $\mathcal{L}_0(j \mid \mathbf{g}) > \mathcal{L}_1(j \mid \mathbf{g})$. Remark that $\mathcal{L}_0(n \mid \mathbf{g}) = \mathcal{L}_1(n \mid \mathbf{g}) = 1$ by definition. Since both $\mathcal{L}_0(i \mid \mathbf{g})$ and $\mathcal{L}_1(i \mid \mathbf{g})$ are continuous and strictly increasing and that $\mathcal{L}_1(n_B \mid \mathbf{g}) > \mathcal{L}_0(n_B \mid \mathbf{g})$, $\mathcal{L}_0(j \mid \mathbf{g}) > \mathcal{L}_1(j \mid \mathbf{g})$ entails $\mathcal{L}_0(i \mid \mathbf{g})$ and $\mathcal{L}_1(i \mid \mathbf{g})$ must cross each other at least once. Such crossing implies that there must be at least one $l \in A$ such that $\Delta\mathcal{L}_1(l \mid \mathbf{g}) > \Delta\mathcal{L}_0(l \mid \mathbf{g})$. This implies in turns $\Phi_l(a_{l,1}, \mathbf{a}^*_{-\mathbf{l},\mathbf{1}} \mid \mathbf{g}) > \Phi_l(a_{l,0}, \mathbf{a}^*_{-\mathbf{l},\mathbf{0}} \mid \mathbf{g})$, which is a contradiction.

We now show that $\mathcal{L}_1(i \mid \mathbf{g})$ must dominate $\mathcal{L}_0(i \mid \mathbf{g})$ over its support segment B too. Suppose a contrario that there exists a $j \in B$ such that $\mathcal{L}_0(j \mid \mathbf{g}) > \mathcal{L}_1(j \mid \mathbf{g})$. Recall first that $\mathcal{L}_0(i \mid \mathbf{g})$ is convex by definition. Further, since $\Phi_i(a_{i,1}, \mathbf{a}^*_{-\mathbf{i},\mathbf{1}} \mid \mathbf{g}) = \pi_y$ for all $i \in B$, then $\Delta\mathcal{L}_1(j \mid \mathbf{g})$ is the same for all $j \in B$. In other words, $\mathcal{L}_1(j \mid \mathbf{g})$ is a straight line over B. Since $\mathcal{L}_1(j \mid \mathbf{g})$ is a straight line over B, $\mathcal{L}_0(j|\mathbf{g})$ is convex, $\mathcal{L}_1(0 \mid \mathbf{g}) = \mathcal{L}_1(0 \mid \mathbf{g}) = 0$ and there exists a j such that $\mathcal{L}_0(j \mid \mathbf{g}) > \mathcal{L}_1(j \mid \mathbf{g})$, then it must be that $\mathcal{L}_1(n_B \mid \mathbf{g}) < \mathcal{L}_0(n_B \mid \mathbf{g})$. Again, this is a contradiction since the share of payoffs going to individuals in B must increase. This completes the proof.

We now show with an example that if the introduction of y decreases $\mathcal{W}(\mathbf{g})$, then $\mathcal{G}(\mathbf{g})$ can either increase or decrease. Consider the network on Figure B5, and fix $p_x = 0.75$. Before the introduction of y, the payoffs of individuals 1 to 5, 6 to 10 and 11 to 15, respectively, amount to 4.25, 3.25 and 0.25, entailing $\mathcal{W}_0(\mathbf{g}) = 38.75$ and $\mathcal{G}_0(\mathbf{g}) = 0.3441$. Now suppose that y is introduced at a price $p_y = 0.7$. The individual payoffs of individuals 1 to 5, 6 to 10 and 11 to 15, respectively, then amount to 4.25, 2.25 and 0.3. While the payoffs of individuals 11 to 15 increase in comparison to those of individuals 1 to 5, the payoffs of the latter clearly increase in proportion of $\mathcal{W}(\mathbf{g})$ due to the important fall in the payoffs of individuals 6 to 10. As a result, $\mathcal{W}_1(\mathbf{g}) = 34$ and $\mathcal{G}_1(\mathbf{g}) = 0.3873$, indicating a *rising* $\mathcal{G}(\mathbf{g})$. Now suppose that $p_y = 0.2$. Then, $\mathcal{W}_1(\mathbf{g}) = 36.5$ and $\mathcal{G}_1(\mathbf{g}) = 0.3150$, indicating a *falling* $\mathcal{G}(\mathbf{g})$ compared the situation without y.

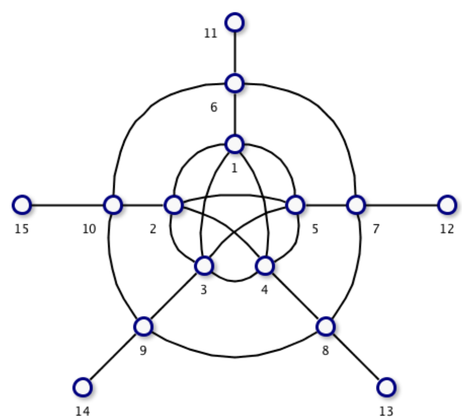


Figure B5. A three-layer society

We finally prove by construction that when x and y are not strong substitutes, then $\mathcal{G}(\mathbf{g})$ can either increase or decrease even when $\mathcal{W}(\mathbf{g})$ increases. Consider the network on Figure B6, and assume that $\theta = -0.15$ and $p_x = 0.9$, entailing that $\mathcal{W}_0(\mathbf{g}) = 55.2$ and $\mathcal{G}_0(\mathbf{g}) = .177$. Suppose first that y is introduced at $p_y = 0.5$: then, $\mathcal{W}_1(\mathbf{g}) = 55.6$ and $\mathcal{G}_0(\mathbf{g}) = .178$, showing increasing welfare and inequality. Suppose second that y is introduced at $p_y = 0.2$: then, $\mathcal{W}_1(\mathbf{g}) = 65.2$ and $\mathcal{G}_0(\mathbf{g}) = .152$, showing increasing welfare and decreasing inequality. This completes the proof for substitutes.

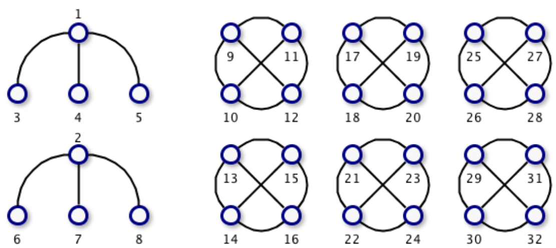


Figure B6.

Complements. We now prove that when x and y are not strong complements and there is at least one individual who adopts x without adopting y, the introduction of y (weakly) increases $\mathcal{G}(\mathbf{g})$. Recall from Lemma 3 that when x and y are not strong complements, then $x^*_{i,1} = x^*_{i,0}$ for all $i \in N$. Recall also from Proposition 2 that in such case, the introduction of y strictly increases $\mathcal{W}(\mathbf{g})$.

Note that individuals can be partitioned into three groups, namely those who choose $a^*_1 = (1,1)$, those who choose $a^*_1 = (1,0)$ and those who choose $a^*_1 = (0,0)$. Label these groups respectively A, B and C. The support of $\mathcal{L}(i \mid \mathbf{g})$ can be written as $\{1, 2, ...n_C, n_C + 1, ...n_B, n_B + 1, ...n\}$, and so $C = \{1, 2...n_C\}$, $B = \{n_C + 1, ...n_B\}$ and $A = \{n_B + 1, ...n\}$. Recall from Proposition 4 that $\Phi_i(a_{i,1}, \mathbf{a^*_{-i,1}} \mid \mathbf{g}) > \Phi_j(a_{j,1}, \mathbf{a^*_{-j,1}} \mid \mathbf{g})$ for any $i \in A$ and $j \in B$, and $\Phi_j(a_{lj1}, \mathbf{a^*_{-j,1}} \mid \mathbf{g}) > \Phi_l(a_{l,1}, \mathbf{a^*_{-l,1}} \mid \mathbf{g})$ for any $l \in C$ and $j \in B$. Our proof now proceeds in three steps.

Step 1. We first show that $\mathcal{L}_1(i \mid \mathbf{g}) = \mathcal{L}_0(i \mid \mathbf{g})$ over its support segment C. Recall first that $a^*_{i,0} = a^*_{i,1} = (0,0)$ for any $i \in C$. Hence $\Phi_i(a_{i,0}, \mathbf{a^*_{-i,0}} \mid \mathbf{g}) = \Phi_i(a_{i,1}, \mathbf{a^*_{-i,1}} \mid \mathbf{g}) = 0$ for all $i \in C$. This entails that over its segment support C, $\mathcal{L}_1(i \mid \mathbf{g}) = \mathcal{L}_0(i \mid \mathbf{g})$.

Step 2. Second, we show that $\mathcal{L}_1(i \mid \mathbf{g}) < \mathcal{L}_0(i \mid \mathbf{g})$ over its support segment B. From Lemma 3, $a^*_{i,0} = a^*_{i,1} = (1,0)$ for any $i \in B$. Hence $\Phi_i(a_{i,0}, \mathbf{a^*_{-i,0}} \mid \mathbf{g}) = \Phi_i(a_{i,1}, \mathbf{a^*_{-i,1}} \mid \mathbf{g})$ for all $i \in B$. Since $\mathcal{W}_1(\mathbf{g}) > \mathcal{W}_0(\mathbf{g})$, then clearly $\Delta\mathcal{L}_0(i \mid \mathbf{g}) > \Delta\mathcal{L}_1(i \mid \mathbf{g})$ for all $i \in B$, which implies that $\mathcal{L}_1(i \mid \mathbf{g}) < \mathcal{L}_0(i \mid \mathbf{g})$ over its support segment B.

Step 3. Lastly, we show that $\mathcal{L}_1(i \mid \mathbf{g}) < \mathcal{L}_0(i \mid \mathbf{g})$ over its support segment A. Recall again from Lemma 3 that $a^*_{i,0} = (1,0)$ and $a^*_{i,1} = (1,1)$ for any $i \in A$. The payoffs to an individual $i \in A$ before and after the introduction of y, respectively, can thus be written as follows:

(B11)
$$\Phi_i(a_{i,0}, \mathbf{a}^*_{-\mathbf{i},\mathbf{0}} \mid \mathbf{g}) = \phi_0(k_j(\mathbf{g}^{q_1}))$$

(B12)
$$\Phi_i(a_{i,1}, \mathbf{a}^*_{-\mathbf{i},\mathbf{1}} \mid \mathbf{g}) = \phi_0(k_j(\mathbf{g}^{q_1})) + \pi_y + \xi(k_j(\mathbf{g}^{q_1}))$$

Note that both $\Phi_i(a_{i,0}, \mathbf{a}^*_{-\mathbf{i},\mathbf{0}} \mid \mathbf{g})$ and $\Phi_i(a_{i,1}, \mathbf{a}^*_{-\mathbf{i},\mathbf{1}} \mid \mathbf{g})$ are increasing in $k_j(\mathbf{g}^{q_1})$. Consider next the following Claim.

Claim 1: *If $\Delta\mathcal{L}_1(i \mid \mathbf{g}) > \Delta\mathcal{L}_0(i \mid \mathbf{g})$ for one $i \in A$, then $\Delta\mathcal{L}_1(j \mid \mathbf{g}) > \Delta\mathcal{L}_0(j \mid \mathbf{g})$ for all j such that $k_j(\mathbf{g}^{q_1}) \geq k_i(\mathbf{g}^{q_1})$.*

Proof. Using equations (B9), (B11) and (B12) and rearranging terms, observe that $\Delta\mathcal{L}_1(i \mid \mathbf{g}) > \Delta\mathcal{L}_0(i \mid \mathbf{g})$ implies that:

(B13)
$$\frac{\phi_0(k_j(\mathbf{g}^{q_1})) + \pi_y + \xi(k_j(\mathbf{g}^{q_1}))}{\phi_0(k_j(\mathbf{g}^{q_1}))} > \frac{\mathcal{W}_1(\mathbf{g})}{\mathcal{W}_0(\mathbf{g})}$$

Note that the RHS of inequality (B13) is a constant and its LHS is strictly increasing in $k_j(\mathbf{g}^{q_1})$. Hence, if inequality (B13) holds for an individual $i \in A$, it must also hold for all $j \in A$ for whom $k_j(\mathbf{g}^{q_1}) > k_i(\mathbf{g}^{q_1})$. ♣

We now complete the proof by showing that $\mathcal{L}_1(i \mid \mathbf{g}) < \mathcal{L}_0(i \mid \mathbf{g})$ over A. Suppose *a contrario* that $\mathcal{L}_0(i \mid \mathbf{g})$ does not dominate $\mathcal{L}_1(i \mid \mathbf{g})$ for every $i \in A$, such that there is a $j \in A$ with $\mathcal{L}_0(i \mid \mathbf{g}) < \mathcal{L}_1(i \mid \mathbf{g})$. Since we know from Step 2 that $\mathcal{L}_0(n_B \mid \mathbf{g}) > \mathcal{L}_1(n_B \mid \mathbf{g})$, then there must exist at least one $i \in A$ with $\Phi_i(a_{i,1}, \mathbf{a}^*_{-\mathbf{i},\mathbf{1}} \mid \mathbf{g}) \leq \Phi_j(a_{j,1}, \mathbf{a}^*_{-\mathbf{j},\mathbf{1}} \mid \mathbf{g})$ such that $\Delta\mathcal{L}_1(i \mid \mathbf{g}) > \Delta\mathcal{L}_0(i \mid \mathbf{g})$. But since $\mathcal{L}_0(i \mid \mathbf{g})$ and $\mathcal{L}_1(i \mid \mathbf{g})$ are both convex and $\mathcal{L}_0(n \mid \mathbf{g}) = \mathcal{L}_1(n \mid \mathbf{g}) = 1$, then there must exist one $l \in A$ such that $\Phi_i(a_{i,1}, \mathbf{a}^*_{-\mathbf{i},\mathbf{1}} \mid \mathbf{g}) \leq \Phi_i(a_{l,1}, \mathbf{a}^*_{-\mathbf{l},\mathbf{1}} \mid \mathbf{g})$ and $\Delta\mathcal{L}_1(l \mid \mathbf{g}) < \Delta\mathcal{L}_0(l \mid \mathbf{g})$. This, however, contradicts Claim 1. This completes the proof.

Next, we show that when x and y are strong complements, then $\mathcal{G}(\mathbf{g})$ can go up or down. We proceed through an example. Consider Figure B7 and suppose that payoffs are given by Example *1*. Fix $p_y = 1.5$. Suppose first that $p_x = 7.1$. Before the introduction of y, all individuals' payoffs amount to 0 (as all individuals choose $x = 0$), and $\mathcal{W}_0(\mathbf{g}) = 0$ and $\mathcal{G}_0(\mathbf{g}) = 0$. After the introduction of y, only individuals 9 to 16 adopt both x and y, while all other individuals choose $x = y = 0$. As a result, $\mathcal{W}_1(\mathbf{g}) = 51.2$ and $\mathcal{G}_1(\mathbf{g}) = 0.5$, indicating a *rising* $\mathcal{G}(\mathbf{g})$. Now suppose that $p_x = 3.5$. Before the introduction of y, individuals 9 to 16 choose $x = 1$, while all others choose $x = 0$. As a result, $\mathcal{W}_0(\mathbf{g}) = 28$ and $\mathcal{G}_1(\mathbf{g}) = 0.5$. After the introduction of y, individuals 1 to 4 and 9 to 16 all choose $x = y = 1$, while individuals 5 to 8 stick to $x = y = 0$. As a result, $\mathcal{W}_1(\mathbf{g}) = 88$ and $\mathcal{G}_1(\mathbf{g}) = 0.4318$, indicating a *falling* $\mathcal{G}(\mathbf{g})$.

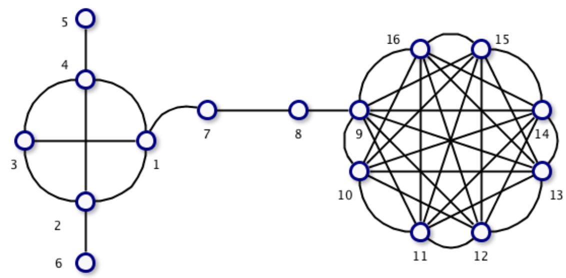


Figure B7.

Next, we show that even if x and y are not strong complements, $\mathcal{G}(\mathbf{g})$ can go up or down when there is no individual who does not adopt x without adopting y. Consider the network on Figure B8. Suppose that payoffs are given by:

$$\Pi_i(\mathbf{a}|\mathbf{g}) = x_i\chi_i(\mathbf{g}) + y_i + x_iy_i(\chi_i(\mathbf{g}))^{\frac{1}{4}} - 1.5x_i - p_yy_i \tag{B14}$$

It is easy to check that the payoff function (B14) satisfies Assumptions 1 to 3 and that x and y are not strong complements. Before the introduction of y, only individuals 5 to 15 adopt x, entailing $\mathcal{W}_0(\mathbf{g}) = 33.5$ and $\mathcal{G}_0(\mathbf{g}) = 0.3264$. Suppose first that y is introduced at $p_y = 2.2$. After the introduction of y, only those individuals choose $x = y = 1$, and $\mathcal{W}_1(\mathbf{g}) = 36.34$ and $\mathcal{G}_1(\mathbf{g}) = 0.3262$, indicating a falling $\mathcal{G}(\mathbf{g})$. Suppose second that y is introduced at $p_y = 2.3$. Again, only individuals 5 to 15 choose $x = y = 1$, and $\mathcal{W}_1(\mathbf{g}) = 35.24$ and $\mathcal{G}_1(\mathbf{g}) = 0.3280$, indicating a rising $\mathcal{G}(\mathbf{g})$.

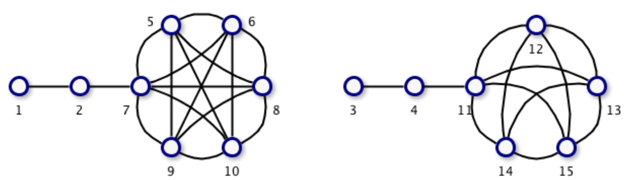


Figure B8.

Lastly, we show that when $\mathcal{M}(\mathbf{g}) = 1$, $\mathcal{G}(\mathbf{g})$ can go up or down. Consider again Figure B7 and suppose that payoffs are given by Example *1*. Fix $p_y = 0.5$. Suppose first that $p_x = 7.1$. Before the introduction of y, all individuals' payoffs amount to 0, and so $\mathcal{W}_0(\mathbf{g}) = 0$ and $\mathcal{G}_0(\mathbf{g}) = 0$. After the introduction of y, the payoffs of individuals 9 to 16 rise to 7.3 (as they now choose $x = y = 1$), while those of all other individuals rise to 0.5. As a result, $\mathcal{W}_1(\mathbf{g}) = 63.2$ and $\mathcal{G}_1(\mathbf{g}) = 0.4367$, indicating a *rising* $\mathcal{G}(\mathbf{g})$. Now let $p_x = 3.5$. Prior to introduction of y, individuals 9 to 16 choose $x = 1$, while all others choose $x = 0$. As a result, $\mathcal{W}_0(\mathbf{g}) = 28$ and $\mathcal{G}_1(\mathbf{g}) = 0.5$. After the introduction of y, the payoffs of individuals 9 to 16, 1 to 4 and 5 to 8 are respectively 11, 3 and 0.5. As a result, $\mathcal{W}_1(\mathbf{g}) = 102$ and $\mathcal{G}_1(\mathbf{g}) = 0.3873$, indicating a *falling* $\mathcal{G}(\mathbf{g})$. ■

B5. Application: Informal risk-sharing and formal insurance

Informal risk-sharing is pervasive in developing countries. There is a vast literature on the mechanisms and the limits of such insurance and it remains an active field of research in economics; recent work includes Bramoullé and Kranton (2007), Bloch, Genicot and Ray (2008), Ambrus, Mobius and Szeidl (2014), Ambrus, Chandrasekhar and Elliott (2015) and Munshi and Rosenzweig (2016). Until recently, however, relatively little attention had been given to the interaction between formal insurance markets and informal risk-sharing in networks, see e.g., Kinnan and Townsend (2012) and Mobarak and Rosenzweig (2012).[6]

We will follow Mobarak and Rosenzweig (2012) who empirically study the interaction between informal risk-sharing and the demand for index insurance. Individuals face village level rainfall shocks that are common to other households in their village, as well individual-level shocks (e.g. health shocks). Index insurance is typically issued by government/state agencies and is based on the observable level of local rainfall. Over 30 million farmers worldwide are covered by index insurance.

Mobarak and Rosenzweig (2012) begin by analyzing how (sub-caste) networks insure against village-level rainfall shocks and individual losses. They uncover a positive correlation between the size of the network within a village and the level of both types of insurance. In addition, they find that if a sub-caste network covers rainfall shocks, then its members are less likely to buy index insurance. In contrast, if a sub-caste network covers idiosyncratic risks then its members are more likely to demand index insurance. Finally, they find that households that had both individual and village levels risks covered through networks, were more likely to opt for riskier portfolios. Thus insurance cover has first order effects on risk taking and earnings.

We now map this empirical context to our model. Action x represents participation in (sub-caste) networks, while index insurance is represented by action y. A household's returns to x depend on the size of the network. In contrast, returns to y do not depend directly on other households' activity. Mobarak and Rosenzweig (2012) show that if the (sub-caste) network offers insurance on individual risks then x and y are complements; if the networks offer rainfall (or village level) cover then informal insurance and index insurance are substitutes.

Our model predicts that 'well-connected' households are more likely to adopt y, when x and y are complements; the converse is true if x and y are substitutes

[6] A notable exception is Arnott and Stiglitz (1991); they show that due to moral hazard problems a developed informal insurance system can hamper the development of formal insurance markets.

(Theorem 2 and Proposition 1). This is consistent with the findings of Mobarak and Rosenzweig (2012). We also make predictions on welfare and inequality (Propositions 2-5) that go beyond the empirical evidence presented in Mobarak and Rosenzweig (2012).

We conclude our discussion by noting that the formal model in Mobarak and Rosenzweig (2012) contains two agents engaged in mutual risk-sharing; it abstracts from social structure considerations. Our main results Theorem 2 and Proposition 1 bring out the role of q-core, and its variants, in shaping behavior. Ambrus, Mobius and Szeidl (2014) report that informal networks of risk-sharing have rich structures; to understand demand for formal insurance in different real world contexts, it is therefore important to understand the role of decay in network benefits, and thus the empirical relevance of q-cores.

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