The Realization Effect: Risk-Taking After Realized Versus Paper Losses
Appendix: For Online Publication

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1 Bracketing and Realization

To set up the basic framework with no prior outcomes, allow preferences to satisfy the standard assumptions of Cumulative Prospect Theory (Tversky and Kahneman, 1992). Let $(x^1, p^1; ...x^n, p^n)$ represent a lottery with n possible outcomes, where x^i is the outcome with objective probability p^i, $x^i > x^j$ iff $i > j$, and $\sum_{i=1}^{n} p^i = 1$.

The decision maker (DM) evaluates the lottery as $\sum_{i=1}^{n} \pi^i \ V(x^i|r)$ relative to reference point $r \in \mathbb{R}$, where π^i and $V(x^i|r)$ are the decision weight and output of the value function V, respectively, used to evaluate outcome x^i. Let $V(x^i|r)$ satisfy the standard assumptions of a prospect theory value function, which is differentiable everywhere except a kink at r,

$$V(x^i|r) = \begin{cases} v(x^i - r) & if\ x \geq r \\ -\lambda v(-(x^i - r)) & if\ x < r \end{cases} \tag{1}$$

where $V(r|r) = 0$, v is concave, and $\lambda > 1$ implies loss aversion. Note that this implies that the value function $V(x^i|r)$ is concave for gains ($x^i \geq r$) and convex for losses ($x^i < r$).

Under CPT, the probability weighting function $w : [0, 1] \to [0, 1]$ transforms objective probabilities into decision weights π. The weighting function w is assumed to satisfy the reflection property, assigning the same weight to a given gain-probability as to a given loss-probability (Prelec, 1998; Tversky and Kahneman, 1992). Unlike in the original prospect theory (Kahneman and Tversky, 1979), in CPT probability weighting is rank-dependent, such that

$$\pi_i = \begin{cases} w(p^i + ... + p^n) - w(p^{i+1} + ... + p^n) & if\ x^i \geq r \\ w(p^1 + ... + p^i) - w(p^1 + ... + p^{i-1}) & if\ x^i < r \end{cases} \tag{2}$$

with $w(0) = 0$ and $w(1) = 1$. Weighting functions proposed in the literature are typically non-linear, S-shaped transformations of objective probabilities reflecting the observed overweighting of small probabilities and the underweighting of large probabilities in empirical studies. For example, Prelec (1998) proposes the exponential, one-parameter weighting function $w(p) = exp(-(-ln\ p)^{\alpha})$. Tversky and Kahneman (1992) consider the one-parameter function

$$w(p) = \frac{p^{\delta}}{(p^{\delta} + (1-p)^{\delta})^{\frac{1}{\delta}}} \tag{3}$$

Note that there are several important differences between CPT preferences and those of Expected Utility Theory. First, the DM derives utility from gains and losses relative to the reference point rather than final wealth. Second, rather than being differentiable everywhere, the value function is kinked at the reference point such that losses loom larger than gains. This is captured by the parameter $\lambda > 1$, which implies that the absolute value of a loss relative to the reference point has a greater impact on utility than an equivalent gain; a greater λ corresponds to greater loss aversion. Third, the value function $V(x|r)$ is concave for gains ($x \geq r$) and convex over losses ($x < r$); the DM is risk-averse in the gain domain but risk-*seeking* in the domain of losses. Finally, under CPT the DM evaluates prospects using the probability weighting function $w(\cdot)$, which transforms objective probabilities into decision weights π assigned to each of the lottery's possible outcomes x.

To examine the differential effects of realized versus paper outcomes on risk-taking, consider the case where prior outcomes of past accepted lotteries may have occurred. Begin by defining the role of realization in determining whether outcomes are evaluated jointly within the same mental account – integration – or separately in different mental accounts. Consider a dynamic choice problem with $t \in \mathscr{T} = \{1, ..., T-1\}$, $T \in \mathbb{N}$. Let $\{R_t\}_{t=2}^{T}$ be the stochastic process indicating whether realization of an outcome – transfer of money lost or won – occurs at time t, $R_t : s \times \mathscr{T} \to \{0, 1\}$, with $R_t = 1$ corresponding to realization and $R_t = 0$ corresponding to no realization, and $s = max\{\tau_n\ s.t.\ \tau_n \leq t\}$. Take $\{\tau_n\}$ be a sequence of hitting times ($R_{\tau_n} = 1$) such that $\tau_1 = inf\{t \in \mathscr{T}\ s.t.\ R_t = 1\}$ and $\tau_n = inf\{t \in \{\tau_{n-1} + 1, ..., T\}\ s.t.\ R_t = 1\}$ for $n \geq 2$.[1]

Let there be three periods, $T = 3$. The DM is offered a mixed, positively skewed lottery

[1] For example, if $t = 5$ and outcomes had been realized in $t = 3, 5$, then $R_t = \{R_2 = 0, R_3 = 1, R_4 = 0, R_5 = 1\}$ and $\{\tau_n\} = \{\tau_1 = 3, \tau_2 = 5\}$.

between two outcomes $L = (x^g, p; x^l, 1-p)$ in the first of two periods, where $p < .5$, $x^g > 0 > x^l$ and $x^g > |x^l|$. If the DM accepts L in the first period, he learns the outcome and proceeds to the next period to make the same choice over L again; if he rejects, no further lotteries are offered.[2] The value function in (1) is amended to allow for the effect of prior outcomes such that at time t,

$$V(x^i|r, \sum_s^{t-1} L_s) = \begin{cases} v(x^i + \sum_s^{t-1} L_s - r) & if\ x^i + \sum_s^{t-1} L_s \geq r \\ -\lambda v(-(x^i + \sum_s^{t-1} L_s - r)) & if\ x^i + \sum_s^{t-1} L_s < r \end{cases} \tag{4}$$

where $L_s \in \{x^g, x^l\}$ denotes the outcome of the lottery that was accepted in period s. A note on the timing notation: the realization of a prospect evaluated at time t occurs at the start of the next period $t+1$: $R_{t+1} = 1$ if the outcome is realized or $R_{t+1} = 0$ if it is not.

The expression in (4) presents the distinction between paper and realized outcomes. In period t the DM evaluates the prospective risky choice L jointly with the preceding sequence of outcomes within the same bracket if no prior outcome in that sequence had been realized ($\forall\ n \in \{\tau+1, ..., t\}$: $R_n = 0$). The realization of a prior outcome (e.g. $R_t = 1$) closes the bracket containing the preceding sequence, and the prospect L is evaluated separately. In the last period, the accumulated paper outcomes are always realized, $R_3 = 1$. As in Shefrin and Statman (1985) and Barberis and Xiong (2012), the DM derives ex-post utility only from realized outcomes.[3] In the first period $t = 1$, the DM evaluates prospective choices relative to a clean slate, $\sum_s^0 L_s = 0$. Note that R_1 is not part of the sequence because in the first period there are no prior outcomes.

[2]Since the paper examines risk-taking after prior losses, this assumption was made for simplicity to focus on the cases where the DM experiences a prior loss. In the myopic case (Section 1.1), since the DM does not consider the second lottery when deciding on the first, relaxing the assumption does not affect the results. In the non-myopic case (Section 1.2), relaxing the assumption introduces the potential strategies of rejecting the lottery in both periods or rejecting the lottery in the first period and accepting it in the second. The valuations of both strategies are equivalent to strategies already considered. Specifically, the strategy of rejecting the lottery in both periods corresponds to rejecting the lottery in the first period, and the strategy of rejecting the lottery in the first period and accepting it in the second corresponds to the strategy of accepting the lottery in the first period and rejecting it in the second regardless of the outcome.

[3]The results also hold if the utility derived from a realized outcome is greater than from a paper one. Shefrin and Statman (1985) make the point that realizing losses at the closing of a mental account hurts more than the equivalent paper loss within a mental account. Similarly, Thaler (1999) writes “one clear intuition is that a realized loss is more painful than a paper loss.”

1.1 Myopic Case

If the DM does not consider the second gamble when making the first choice, in each period he evaluates a prospect with two possible outcomes. As such, CPT coincides with the original version of prospect theory of (Kahneman and Tversky, 1979), where $\pi^g = w(p)$ and $\pi^l = w(1-p)$. Without assuming a particular functional form, let $w(p) + w(1-p) \leq 1$, a condition met by all weighting functions proposed in the literature, e.g. Prelec (1998), and follow Barberis (2012) and Tversky and Kahneman (1992) in taking the reference point r to be the status quo, $(r = 0)$. Expression (4) can be rewritten as

$$V(x^i|0, \sum_s^{t-1} L_s) = \begin{cases} v(x^i + \sum_s^{t-1} L_s) & if\ x^i + \sum_s^{t-1} L_s \geq 0 \\ -\lambda v(-(x^i + \sum_s^{t-1} L_s)) & if\ x^i + \sum_s^{t-1} L_s < 0 \end{cases} \tag{5}$$

In the first period, the DM evaluates accepting the lottery or rejecting it and retaining the status quo. The DM accepts the lottery if

$$0 < w(p)v(x^g) - \lambda w(1-p)v(-x^l). \tag{6}$$

To analyze the effects of a prior loss on subsequent risk taking, assume that the DM chooses to accept the first lottery. He then has a loss, $L_1 = x^l$, that is not realized $(R_2 = 0)$ – a paper loss. The DM is then offered a second lottery. Since all outcomes will be realized at the beginning of the final period $T = 3$, now the DM compares the valuation of accepting the second lottery, $w(p)v(x^g + x^l) - \lambda w(1-p)v(-2x^l)$, which allows him to avoid realizing the prior loss, to rejecting the lottery and realizing the loss with certainty, $-\lambda v(-x^l)$. The DM accepts the lottery if

$$0 < w(p)v(x^g + x^l) - \lambda w(1-p)v(-2x^l) + \lambda v(-x^l). \tag{7}$$

Now suppose that the DM accepts the first lottery and has a realized loss, $(R_2 = 1)$. In the second period, he is offered the same lottery again. The prior realized loss is not integrated with the prospect and the DM accepts the gamble if (6) is met.

The first prediction compares risk taking after a paper loss to risk taking after a realized loss.

Prediction 1. *A loss averse DM who experienced a prior paper loss,* $(R_2 = 0)$*, will be more willing to accept the lottery than one who experienced a prior realized loss,* $(R_2 = 1)$*.*

For Prediction 1 to hold, it is necessary to demonstrate that if the DM accepts L after a realized loss, even when indifferent, he would always be willing to accept L after a paper loss. Particularly, that the DM's valuation of accepting the lottery (relative to rejecting it) is greater after a paper loss than after a realized loss. Prediction 1 holds if

$$\lambda > \frac{w(p)(v(x^g) - v(x^g + x^l))}{w(1-p)(v(-x^l) - v(-2x^l)) + v(-x^l)} \tag{8}$$

for any level of loss aversion $\lambda > 1$.

Proof. First, note that replacing $v(-2x^l)$ with $2v(-x^l)$ and rearranging terms, by subadditivity of concave function of v (since $v(0) = 0$), if

$$\lambda > \left[\frac{w(p)}{1 - w(1-p)}\right]\left[\frac{v(x^g) - v(x^g + x^l)}{v(-x^l)}\right]. \tag{9}$$

holds, then (8) holds as well. Since $w(p) + w(1-p) \leq 1$ then $\frac{w(p)}{1-w(1-p)} \leq 1$ as well. Given that $x^g > 0 > x^l$ and $x^g > |x^l|$, $v(x^g) - v(x^g + x^l) \leq v(-x^l)$ by the subadditivity of concave function v, such that $\frac{v(x^g)-v(x^g+x^l)}{v(-x^l)} \leq 1$. Since the right hand side of (9) is (weakly) less than 1, it follows that (9) holds for $\lambda > 1$.

□

The second prediction compares risk taking after a paper loss to risk taking before the loss.

Prediction 2. *A loss averse DM who experienced a prior paper loss,* $(R_2 = 0)$*, will be more willing to accept the lottery than before experiencing the loss.*

Since the same condition (6) specifies if the DM accepts the lottery both before a loss and after a realized loss, if Prediction 1 holds, Prediction 2 holds as well.

The third prediction compares risk taking after a realized loss to risk taking before the loss.

Prediction 3. *A loss averse DM who experienced a prior realized loss,* $(R_2 = 1)$*, will be less willing to accept the lottery than before experiencing the loss.*

As noted in Section 3 of the paper, for Prediction 3 to hold in the myopic case, such that the DM takes on less risk after a realized loss not only relative to a paper loss, but relative to before the loss, more structure is needed. A number of different mechanisms discussed in the literature will produce such an effect when applied to realized losses, including, *sensitization* (Barberis, Huang and Santos, 2001; Thaler and Johnson, 1990), a diminished capacity for dealing with bad "news" (Koszegi and Rabin, 2009; Linville and Fischer, 1991; Pagel, 2012), the increased salience of the potential downside of risk (Bordalo, Gennaioli and Shleifer, 2012), or a change in mood (Loewenstein, 1996). Since (6) specifies the DM's willingness to accept the lottery both before and after a realized loss, it is straightforward to show that any of these factors that produce a greater distaste for losses after a realized loss lead to Prediction 3.

As an example of incorporating sensitization into the framework, apply the structure of Barberis *et al.* (2001) to realized losses by allowing loss aversion λ_t to depend on prior *realized* outcomes represented by z_t, such that $\lambda(z_t) = \lambda + k(z_t - 1)$, with $k \geq 0$.[4] Note that Barberis *et al.* (2001) apply this structure to all losses. Finally, let

$$z_t = 1 - R_t(\sum_{u}^{s-1} L_u). \tag{10}$$

where $u = \tau_{n-1}$. In turn, the DM appears more loss averse after a prior realized loss ($R_t = 1$) but not after a prior paper loss ($R_t = 0$), with a larger realized loss sensitizing him more than a smaller one. To show that Prediction 3 holds in the myopic case, it is straightforward to demonstrate that after a realized loss, ($R_2 = 1$), the DM's valuation of the lottery, $w(p)v(x^g) - \lambda(z_2)w(1-p)v(-x^l)$, is lower than before the loss, $w(p)v(x^g) - \lambda(z_1)w(1-p)v(-x^l)$.

Proof. The condition $w(p)v(x^g) - \lambda(z_2)w(1-p)v(-x^l) < w(p)v(x^g) - \lambda(z_1)w(1-p)v(-x^l)$ can be rewritten as $1 < \frac{\lambda(z_2)}{\lambda(z_1)}$. Before a prior outcome, $\lambda(z_1) = \lambda - k(1-1) = \lambda$. Suppose the DM accepted the first lottery and suffered a realized loss, $R_2 = 1$. Now, $\lambda(z_2) = \lambda - kx^l$. Since $x^l < 0$, $1 < \frac{\lambda - kx^l}{\lambda}$ for any $k > 0$. □

1.2 Non-Myopic Case

As noted in Section 3, if the second lottery is anticipated and the DM formulates a strategy which is a fully contingent plan of risk taking prior to playing the first, the decision problem changes significantly. Barberis (2012) examines such a decision problem with paper outcomes, demonstrating the difficulty of finding an analytical solution given non-linear probability weighting. While Section 3 presented general conditions for the predictions in the non-myopic case, here I incorporate realization and build on the framework of Barberis (2012), using similar simulation techniques to outline the conditions under which the proposed predictions hold. Note that Prediction 1 regarding the differential effect of realized

[4]Note that setting $k = 0$ reduces the model to the standard form.

versus paper losses on risk taking holds under the same general conditions in the myopic and non-myopic case.

For the analysis that follows, take the value function

$$V(x^i|0, \sum_s^{t-1} L_s) = \begin{cases} (x^i + \sum_s^{t-1} L_s)^\alpha & if \ x^i + \sum_s^{t-1} L_s \geq 0 \\ -\lambda(-(x^i + \sum_s^{t-1} L_s))^\alpha & if \ x^i + \sum_s^{t-1} L_s < 0 \end{cases} \tag{11}$$

with curvature parameter $\alpha \leq 1$, and let w correspond to the weighting function in (3) proposed by Tversky and Kahneman (1992). As in the experiments, let $L = (6, \frac{1}{6}; -1, \frac{5}{6})$.

To set up the decision problem where the DM maximizes over the set of all possible ex-ante strategies, it is helpful to think of choices over lotteries as a binomial tree. Figure 1(a) illustrates a sequence of these choices as nodes in the tree for the case when $T = 3$ and the lottery is accepted in every round. Each node in the tree is described by a pair of numbers (t, j), where t corresponds to the period and j, ranging from 1 to t, corresponds to the distance of the node from the top of the column for a given t. For example, the top most node for $t = 2$ would have $j = 1$, $(2, 1)$; the bottom most node would have $j = 2$, $(2, 2)$. The horizontal position of a node corresponds to the period when the DM makes the choice to reject or accept the lottery, while the vertical position corresponds to the outcomes of lotteries prior to this choice. For example, the node (2,1) corresponds to a choice in period $t = 2$ where the the DM accepted the lottery in the previous period and won; alternatively, (2,2) corresponds to a choice in $t = 2$ where the DM accepted the lottery in the previous period and lost. The terminal nodes in $t = 3$ list the accumulated earnings and the respective ex-ante probabilities of getting to that node, e.g. a DM at (3,1) would earn \$12 from winning twice and has a $\frac{1}{36}$ chance of arriving at this node.

In the first period, the DM makes a decision of whether to accept or reject L. He makes this decision as part of his ex-ante optimal risk-taking strategy. The strategy is a mapping from each node (t, j) in the tree to one of two choices: accept ($A_{(t,j)}$) or reject ($R_{(t,j)}$). The first corresponds to accepting the lottery at (t, j), the second corresponds to rejecting the lottery at that node. Choosing to reject the lottery corresponds to realizing the accumulated paper outcomes since no additional lotteries will be offered.

Following the notation of Barberis (2012), let $S_{t,j}$ be the set of all strategy mappings available starting at node (t, j). Note that this set grows rapidly as T increases. For every strategy $s \in S_{(t,j)}$, let the random variable $\tilde{L}_s$ correspond to the lottery generated by strategy s where the outcomes are accumulated earnings at each node where earnings are realized. Table 1 below lists all strategies $s \in S_{(1,1)}$ available in the first period and the corresponding

Table A1. Strategies $s \in S_{(1,1)}$

s	$\tilde{L}_s$
$R_{(1,1)}$	(0, 1)
$A_{(1,1)}A_{(2,1)}R_{(2,2)}$	$(12, \frac{1}{36}; 5, \frac{5}{36}; -1, \frac{30}{36})$
$A_{(1,1)}R_{(2,1)}A_{(2,2)}$	$(6, \frac{6}{36}; 5, \frac{5}{36}; -2, \frac{25}{36})$
$A_{(1,1)}R_{(2,1)}R_{(2,2)}$	$(6, \frac{1}{6}; -1, \frac{5}{6})$
$A_{(1,1)}A_{(2,1)}A_{(2,2)}$	$(6, \frac{1}{36}; 5, \frac{10}{36}; -2, \frac{25}{36})$

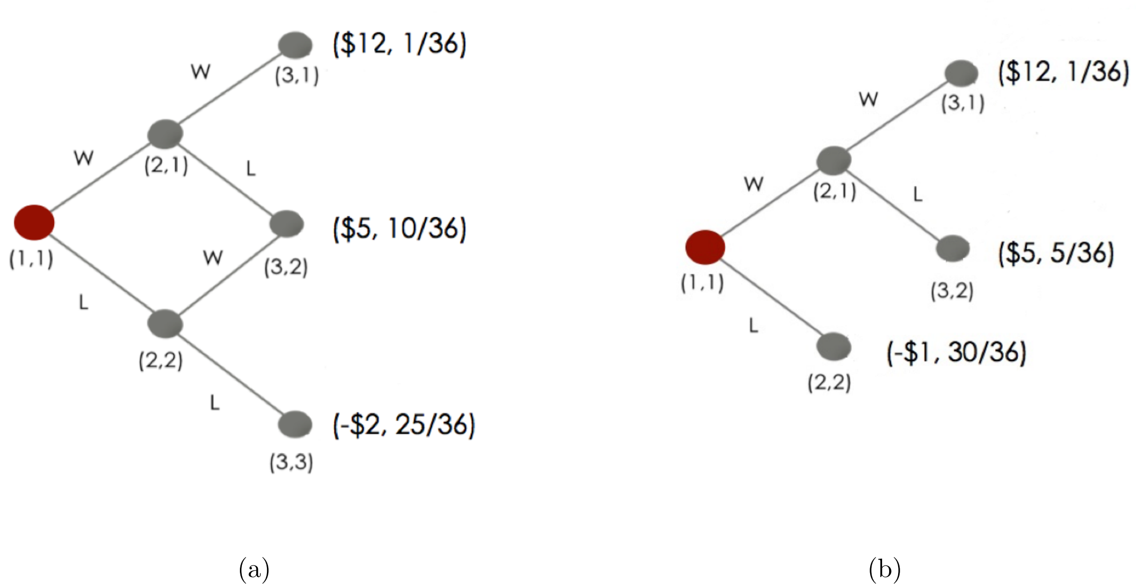


Figure 1. Strategies $s \in S_{(1,1)}$

lotteries $\tilde{L}_s$. For example, the strategy to accept the lottery in each period regardless of the prior outcome is denoted as $s = A_{(1,1)}A_{(2,1)}A_{(2,2)}$ and corresponds to the sequence in Figure 1(a); the "loss-exit" plan of rejecting the lottery whenever accumulated earnings fall below 0 and to accept otherwise is denoted as $s = A_{(1,1)}A_{(2,1)}R_{(2,2)}$ and corresponds to the sequence in Figure 1(b). Alternatively, the strategy to reject the first lottery is denoted as $s = R_{(1,1)}$ and corresponds to choosing the status quo and deriving 0 utility.

In $t = 1$, the DM evaluates what strategy s yields the maximum value by solving

$$\max_{s \in S_{(1,1)}} U(\tilde{L}_s|0, 0). \tag{12}$$

where $U(\tilde{L}_s|0,0) = \sum_{i=1}^{n} \pi^i \; V(x^i|0,0)$ is the CPT valuation of lottery $\tilde{L}_s$ with n outcomes generated by strategy s with respect to a status quo reference point and no prior paper outcomes. Let s^* denote the optimal strategy that maximizes expression (12). The optimal strategy involves accepting the first lottery iff $U(\tilde{L}_{s^*}|0,0) > 0$ in the first round. As in prior work on CPT in dynamic settings (Barberis, 2012; Ebert and Strack, 2014), I assume that the DM makes a choice believing he will follow the optimal strategy and is naive about any dynamic inconsistency in his preferences, i.e. the DM does not consider the possibility that he may deviate from s^*.

As in Barberis (2012), simulations of the decision problem are used to characterize the conditions under which a given strategy $s \in S_{(1,1)}$ solves (12). Results are presented for parameter values typically considered in the literature, $\lambda \in [1, 3.5]$ and $\delta \in [0.1, 1]$; for purposes of illustrating the results and given the relatively small stakes considered, v is taken to be approximately linear, $\alpha = 1$. To solve (12), the valuation $U(\tilde{L}_s|0,0)$ is computed for each of the five strategies in Table 1; the strategy with the highest valuation is taken to be the optimal strategy s^*.

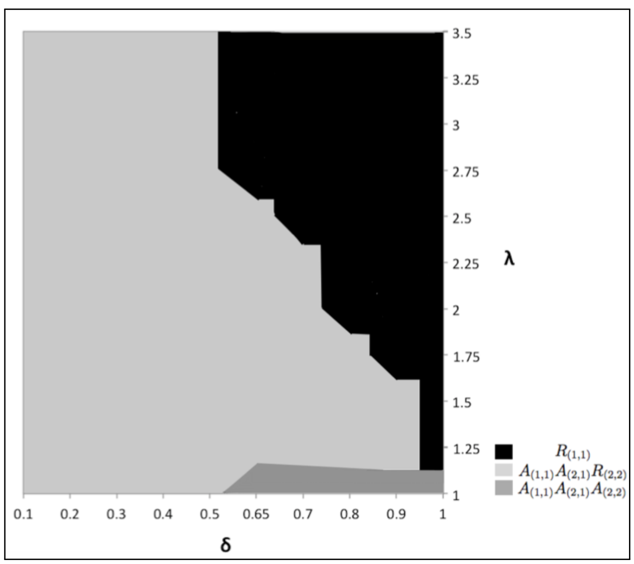


FIGURE 2. s^* for each (δ, λ).

First consider the case where any potential realization in the second period is not anticipated and hence does not affect the valuation of strategies in the first period. Figure 2 displays the DM's optimal strategy for each parameter pair (δ, λ). As noted in Section 3 and discussed above, if the DM accepts the first lottery, the optimal strategy is the loss-exit plan, $s^* = A_{(1,1)}A_{(2,1)}R_{(2,2)}$, under the vast majority of conditions including structural estimates from prior work including the median estimates from Tversky and Kahneman (1992),

$(\delta, \lambda) = (0.65, 2.25)$, which are used as the benchmark in Barberis (2012), the median estimates from Abdellaoui (2000), Tanaka, Camerer and Nguyen (2010), and Gonzalez and Wu (1999).[5] The "loss-exit" plan maximizes the possible upside while minimizing the downside and generates a more positively skewed lottery $\tilde{L}_s$ than the single lottery L. Probability weighting makes accepting the first lottery as part of the "loss-exit" plan more attractive than the other, less skewed strategies including the single play of the lottery ($s = A_{(1,1)}R_{(2,1)}R_{(2,2)}$). As can be seen in Figure 2, the plan is more likely to be optimal when probability weighting is more pronounced (smaller δ) and at lower levels of loss aversion (smaller λ).

To analyze the effects of a prior loss on risk-taking, suppose the DM accepts the first lottery with the aim of following the "loss-exit" plan. If the DM suffers a paper loss, now at node $(2, 2)$, he faces the choice between accepting or rejecting the second lottery and evaluates the utility generated by the set of strategies available in $t = 2$. The DM accepts the second lottery if condition (7) is met. Alternatively, if the loss is realized, the lottery is accepted if condition (6) is met. As in the myopic case, Prediction 1 does not require numerical methods and holds generally for any level of loss aversion, $\lambda > 1$, i.e. if condition (8) is met.

Prediction 2 states that the DM should be more willing to accept the lottery after a paper loss than before the loss. As outlined in Section 3 of the paper, this prediction holds if the DM's valuation of accepting the second lottery (relative to rejecting it) after a paper loss is higher than the valuation of accepting the first lottery as part of the "loss-exit" plan,

$$\lambda > \frac{w(p^2)[v(2x^g) - v(x^g + x^l)]}{w(1-p)[v(-x^l) - v(-2x^l)] + v(-x^l)} \tag{13}$$

Figure 3(a) displays when this condition holds for each parameter pair (δ, λ). Conditional on accepting the lottery in the first period as part of the "loss-exit" plan, the DM values accepting the lottery more after a paper loss than before the loss for the majority of preference parameters, including the estimates from prior work, e.g. Tversky and Kahneman (1992). Analogous to the results of Barberis (2012), the DM displays *dynamic inconsistency* after a paper loss for a broad set of conditions: rather than rejecting the second lottery after a loss as planned, he not only accepts the lottery but would accept an even *riskier* gamble. After a paper loss, the available strategies generate a less skewed distribution of accumulated earnings than prevailed in the first period; the DM compares accepting the second lottery to experiencing the sure loss if the lottery is rejected. Since taking the risk allows the DM to avoid the negative realization, loss aversion leads him to accept greater risk than originally

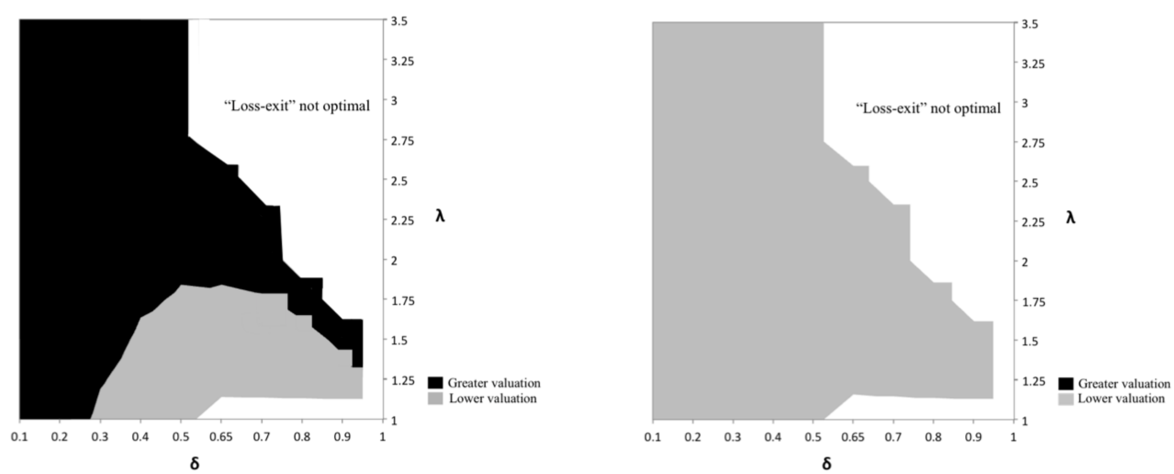
[5] Strategies $A_{(1,1)}R_{(2,1)}A_{(2,2)}$ and $A_{(1,1)}R_{(2,1)}R_{(2,2)}$ are dominated by the three strategies in Figure 2.

planned. As shown in Figure 3(a), as λ increases, the DM values the second lottery more after a paper loss than before a paper loss.

For Prediction 3, the relative valuation of the second lottery after a realized loss must be lower than the valuation of accepting the first lottery as part of the "loss-exit" plan, which is met if

$$0 < w(p^2)v(2x^g) + [w(p) - w(p^2)]v(x^g + x^l) - w(p)v(x^g). \tag{14}$$

Figure 3(b) shows that conditional on accepting the first lottery as part of the "loss exit" plan, after a realized loss the DM values the second lottery *less* than before the loss for all preference parameters considered. After a realized loss, the available strategies generate less skewed lotteries than before the loss. Unlike after a paper loss, accepting the lottery no longer affords the possibility of avoiding a sure negative realization. In turn, the DM is less willing to accept it.



(a) (b)

FIGURE 3. Relative valuation of second lottery after paper loss (a) versus realized loss (b)

The last prediction compares the likelihood of the DM deviating from his ex-ante optimal strategy to take on more risk after a loss that was realized versus one that was not. Since the "loss-exit" plan is the only undominated strategy from which the DM *can* deviate to take on greater risk (and the one the DM is most likely to find optimal conditional on accepting the first lottery), for Prediction 4 assume that the DM accepts the first lottery with an optimal strategy of, $s^* = A_{(1,1)}A_{(2,1)}R_{(2,2)}$.

Prediction 4. *A loss averse is less likely to deviate from his ex-ante optimal strategy after a realized loss* ($R_2 = 1$) *than after a paper loss* ($R_2 = 0$).

Proof. The DM deviates from s^* to accept the lottery after a paper loss if expression (7) holds. The DM deviates from his optimal strategy to accept the lottery after a realized loss if expression (6) holds. Since expression (8) holds for all $\lambda > 1$, expression (6) is less likely to hold than expression (7) for any level of loss aversion. □

Now consider the case where the DM fully takes into account a realization in the second period. The conditions for Prediction 2 to hold are unchanged since it does not involve realization in the second period. If outcomes are realized in the second period, strategies in the first period can no longer generate a more skewed distribution over realized outcomes than strategies in the second period, and probability weighting does not lead to dynamic inconsistency. In turn, the decision problem is analogous to the myopic case in that no available strategy can generate a distribution over realized outcomes that is more skewed than the lottery L. Prediction 1 follows the same logic as before: after a paper loss the DM accepts the lottery if (7) holds while after a realized loss he accepts the lottery if (6) holds; he is more willing to accept the lottery after a paper loss than a realized one if (8) holds, which it does for $\lambda > 1$. As in the myopic case, for Prediction 3 to hold additional structure, e.g. sensitization, is needed. Prediction 4 follows from the same steps as above.

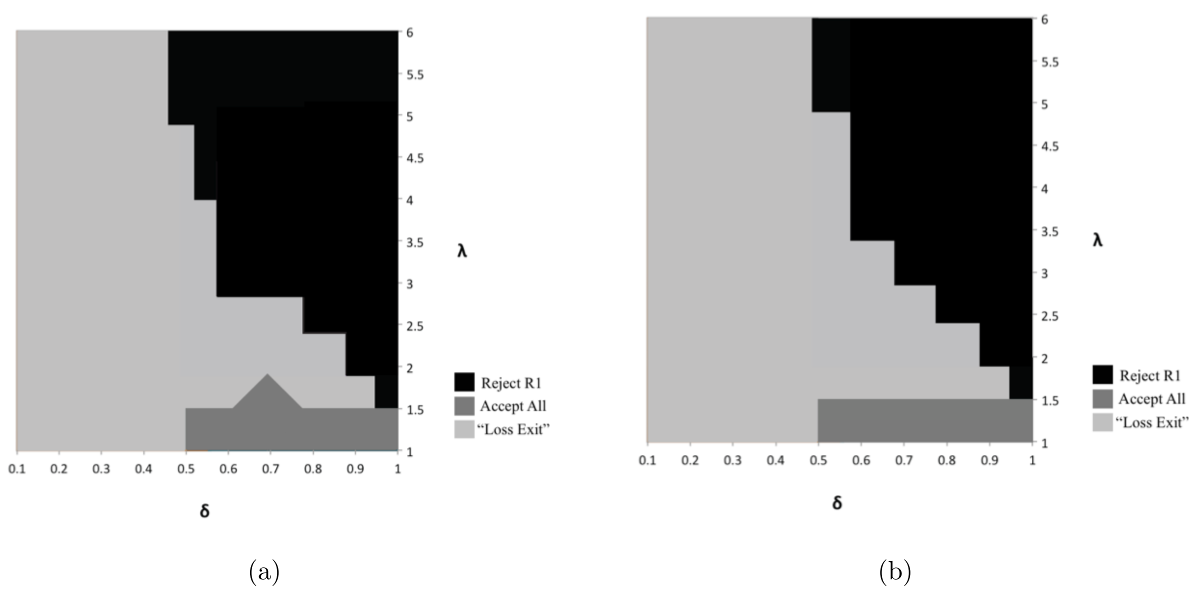


FIGURE 4. Relative valuation of second lottery after paper loss (a) versus realized loss (b)

Note that for $T = 3$, the set of optimal strategies in the first period differs depending on whether or not the DM fully anticipates realization in the second period. Particularly, the set of optimal strategies is reduced to either $s_1^* = A_{(1,1)}A_{(2,1)}A_{(2,2)}$ or $s_1^* = R_{(1,1)}$; the DM still accepts the first lottery under a broad set of conditions, but he does so with the aim of following the non-contingent plan of accepting the lottery in both periods (and does not deviate from it regardless of the prior outcome). However, as the number of periods before realization grows, the set of ex-ante optimal strategies when realization is anticipated converges to the case when it is not. Figure 4 shows the first period optimal strategy s^* for each (δ, λ) when $T = 4$ and realization in $T = 3$ is either anticipated, Figure 4(a), or not, Figure 4(b). The figures look similar and the optimal strategies overlap for the majority of conditions. For example, in both cases the "loss-exit" plan is optimal for the set of parameters estimated in prior work, e.g. Tversky and Kahneman (1992). Further increasing the number of periods prior to realization leads to progressively greater overlap between ex-ante optimal plans and behavior before realization in the case where realization is anticipated and the case where it is not.[6]

1.3 Expectations as a referent

I now outline the predictions for the myopic case in Appendix Section 1.1 when the reference point is defined in terms of stochastic expectations. To do so I follow the framework and terminology of Koszegi and Rabin (2006), henceforth KR, who present a model of expectation-based, reference-dependent preferences.

As in KR, let

$$u(x^i | r, \sum_{s}^{t-1} L_s) = m(x^i + \sum_{s}^{t-1} L_s) + \mu(m(x^i + \sum_{s}^{t-1} L_s) - m(r)). \tag{15}$$

correspond to the overall utility evaluated relative to a reference point r, where $m(\cdot)$ represents "consumption utility" typically studied in economics and $\mu(\cdot)$ correspond to the gain-loss utility satisfying the properties of the prospect theory value function. Over the relatively small stakes considered, following KR let $m(\cdot)$ be linear $m(x^i) = x^i$ and $\mu(\cdot)$ be a piece-wise linear,

$$\mu(c) = \begin{cases} c & if\ c \geq 0 \\ \lambda c & if\ c < 0 \end{cases} \tag{16}$$

[6]Note that for the experiments in Section 4 of the paper, in the Realized condition realization occurs after three periods.

Both the consumption outcome x and referent r can be stochastic. Let the referent r be drawn according to probability measure G and the consumption outcome x be drawn according to probability measure F. The individual evaluates expected utility of F given each possible realization of the reference point drawn from G:

$$U(F|G) = \int\int u(x|r, \sum_{s}^{t-1} L_s)dG(r)dF(x). \tag{17}$$

Sprenger (2014) applied KR to a static setting with binary lotteries and no prior outcomes. Following his analysis, take G and F to be binomial distributions characterized by probability values p and q, respectively.

To demonstrate the first three predictions within this framework, suppose the DM chooses between the expected value of accepting the lottery L and being endowed with it, or rejecting the lottery. In turn, the expected value of accepting the lottery corresponds to the DM's valuation with respect to a stochastic referent.

In period t, the DM evaluates accepting L as:

$$\begin{aligned} U(L|L, \sum_{s}^{t-1} L_s) &= p^2 u(x^g|x^g, \sum_{s}^{t-1} L_s) + (1-p)^2 u(x^l|x^l, \sum_{s}^{t-1} L_s) \\ &+ p(1-p)u(x^g|x^l, \sum_{s}^{t-1} L_s) + p(1-p)u(x^l|x^g, \sum_{s}^{t-1} L_s). \end{aligned} \tag{18}$$

The four terms in (18) corresponds to the following: getting x^g as the outcome while expecting x^g, getting x^l while expecting x^l, getting x^g while expecting x^l, and getting x^l while expecting x^g, with both the referent and the outcome being integrated with unrealized prior outcomes, $\sum_{s}^{t-1} L_s$. In $t = 1$ the DM compares the utility from accepting the lottery, $U(L|L, 0)$, or rejecting it, $U(0|0, 0) = 0$. He accepts the lottery if

$$p^2 x^g + (1-p)^2 x^l + p(1-p)(2x^g + \lambda(x^l - x^g)) > 0. \tag{19}$$

To analyze behavior after a loss, suppose the DM accepted the lottery and has a paper loss. In $t = 2$ is offered the second lottery, and he compares the value of accepting it, $U(L|L, x^l)$, to rejecting it. The DM accepts the second lottery if

$$\begin{aligned} &p^2(x^g + x^l) + (1-p)^2(2x^l) + p(1-p)(2x^g + 2x^l + \lambda(x^l - x^g)) \\ &> x^l + \lambda(x^l - (px^g + (1-p)x^l)). \end{aligned} \tag{20}$$

The left hand side corresponds to the valuation of the second lottery while integrating the prior loss, $U(L|L, x^l)$. The right hand side corresponds to rejecting the second lottery: since risk in the first lottery was resolved with an outcome of x^l, the gain-loss utility of rejecting the second lottery is the outcome minus the referent, which is equal to the sum of the probability that the DM expected a gain, px^g, and the probability that the DM expected a loss, $(1-p)x^l$.

First, I show that Prediction 2 holds. For the second prediction to hold, it is necessary to demonstrate that the DM's valuation of accepting the lottery (relative to rejecting it) is greater after a paper loss than before the loss. Combining expressions (19) and (20), Prediction 2 holds if

$$\begin{aligned} &p^2(x^g+x^l)+(1-p)^2(2x^l)+p(1-p)(2x^g+2x^l+\lambda(x^l-x^g))-x^l-\lambda(x^l-(px^g+(1-p)x^l)) \\ &\quad > p^2x^g+(1-p)^2x^l+p(1-p)(2x^g+\lambda(x^l-x^g)). \end{aligned} \tag{21}$$

Proof. Rearranging terms, (21) can be expressed as

$$p^2x^l + (1-p)^2(x^l) + p(1-p)(2x^l) - x^l - \lambda(x^l - (px^g + (1-p)x^l)) > 0. \tag{22}$$

Further rearranging terms yields

$$x^l[p^2 + (1-p)^2 + 2p(1-p)] - x^l(1+\lambda) + \lambda(px^g + (1-p)x^l) > 0. \tag{23}$$

Since $[p^2 + (1-p)^2 + 2p(1-p)] = 1$ for all p, this yields

$$\lambda(px^g + (1-p)x^l - x^l) > 0. \tag{24}$$

Since $x^l < 0$, (24) holds for all $\lambda > 1$.

□

Since the same condition (19) specifies the DM's willingness to accept the lottery both before a loss and after a realized loss, if Prediction 2 holds, Prediction 1 holds as well.

Demonstrating Prediction 3 follows the same logic as in the myopic case with a status quo referent. Suppose the DM is sensitized after prior realized losses as in (10). Prediction 3 holds if the DM values accepting the first lottery, the left-hand side of (19), more than accepting the second lottery after a realized loss,

$$\begin{aligned} &p^2x^g + (1-p)^2x^l + p(1-p)(2x^g + \lambda(z_1)(x^l - x^g)) \\ &\quad > p^2x^g + (1-p)^2x^l + p(1-p)(2x^g + \lambda(z_2)(x^l - x^g)). \end{aligned} \tag{25}$$

Condition (25) can be rewritten as $1 < \frac{\lambda(z_2)}{\lambda(z_1)}$. Before a prior outcome, $\lambda(z_1) = \lambda - k(1 - 1) = \lambda$. If the DM suffered a realized loss, $R_2 = 1$, then $\lambda(z_2) = \lambda - kx^l$. Since $x^l < 0$, $1 < \frac{\lambda - kx^l}{\lambda}$ for any $k > 0$.

2 Tables

Table A2.
Investment Change after Loss at End of Previous Round: Study 1

Treatment	**Round 2**	**Round 3**	**Round 4**
Realized	$-0.07	$-0.03	$-0.15
	(.08)	(.08)	(.06)
Paper	$-0.07	$-0.09	$+0.23
	(.06)	(.04)	(.10)
Paper S	$-0.07	$+0.01	$+0.16
	(.08)	(.07)	(.06)

† Standard errors in parentheses.

Table A3.
Investment Change after Loss at End of Previous Round: Study 2

Treatment	**Round 2**	**Round 3**	**Round 4**
Transfer	$-0.07	$+.01	$-0.14
	(.04)	(.05)	(.07)
Realized	$-0.13	$+.01	$-0.28
	(.07)	(.09)	(.14)
Paper S	$-0.07	$-.04	$+0.19
	(.09)	(.09)	(.07)
Interupt	$-0.09	$-0.08	$+0.20
	(.08)	(.07)	(.08)

† Standard errors in parentheses.

TABLE A4.
Investment Change after Loss at End of Previous Round: Study 3

Treatment	Round 2	Round 3	Round 4
Realized	$-0.12	$+0.05	$-0.27
	(.09)	(.12)	(.12)
Paper	$-0.09	$-0.08	$+0.29
	(.08)	(.11)	(.10)
Flexible	$-0.10	$+0.16	$+0.33
	(.06)	(.12)	(.11)

† Standard errors in parentheses.

TABLE A5.
Investment Change after Loss at End of Previous Round: Online Studies

Treatment	Round 2	Round 3	Round 4
Paper (Study 2)	$-0.00	$-0.02	$+0.02
	(.01)	(.01)	(.01)
Realized (Study 2)	$+0.01	$-0.00	$-0.03
	(.01)	(.01)	(.02)
Paper (Study 3)	$-0.01	$-0.01	$+0.03
	(.01)	(.01)	(.01)
Realized (Study 3)	$-0.01	$-0.00	$-0.02
	(.01)	(.01)	(.01)

† Standard errors in parentheses.

Table A6.
Total Investment in Risk: Study 1

Treatment	Round 1	Round 2	Round 3	Round 4
Realized	\$0.82	\$0.72	\$0.76	\$0.63
	(.08)	(.09)	(.09)	(.08)
Paper	\$0.85	\$0.78	\$0.71	\$1.01
	(.09)	(.09)	(.10)	(.11)
Paper S	\$0.82	\$0.75	\$0.76	\$0.94
	(.08)	(.08)	(.10)	(.11)

† Standard errors in parentheses.

Table A7.
Total Investment in Risk: Study 2

Treatment	Round 1	Round 2	Round 3	Round 4
Transfer	\$0.87	\$0.77	\$0.83	\$0.81
	(.08)	(.09)	(.10)	(.09)
Realized	\$0.98	\$0.89	\$0.89	\$0.73
	(.07)	(.07)	(.08)	(.07)
Paper S	\$0.92	\$0.84	\$0.83	\$1.00
	(.09)	(.10)	(.11)	(.10)
Interrupt	\$0.86	\$0.76	\$0.77	\$0.92
	(.07)	(.08)	(.08)	(.09)

† Standard errors in parentheses.

Table A8.
Total Investment in Risk: Study 3

Treatment	Round 1	Round 2	Round 3	Round 4
Realized	$0.99	$0.96	$0.94	$0.74
	(.09)	(.10)	(.11)	(.10)
Paper	$1.01	$0.94	$0.83	$1.16
	(.09)	(.10)	(.09)	(.11)
Flexible	$1.04	$0.96	$1.06	$1.24
	(.08)	(.10)	(.10)	(.10)

† Standard errors in parentheses.

Table A9.
Total Investment in Risk: Online Studies

Treatment	Round 1	Round 2	Round 3	Round 4
Paper (Study 2)	$0.10	$0.12	$0.09	$0.12
	(.01)	(.01)	(.01)	(.01)
Realized (Study 2)	$0.11	$0.11	$0.12	$0.10
	(.01)	(.01)	(.01)	(.01)
Paper (Study 3)	$0.10	$0.10	$0.10	$0.12
	(.01)	(.01)	(.01)	(.01)
Realized (Study 3)	$0.12	$0.11	$0.11	$0.09
	(.01)	(.01)	(.01)	(.01)

† Standard errors in parentheses.

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