

Online Appendix:

Asymmetric Reciprocity and the Cyclical Behaviour of Wages, Effort and Job Creation

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A Mathematical Proofs

Proof of Lemma 1. To aid the characterisation of $\Omega(W_t, Z_t)$, consider first the following envelope conditions for the value function, recalling that $R_{t+1} = W_t$:

$$J_R(W_t, Z_{t+1}) = \begin{cases} -Z_{t+1}\eta W_t^{-\gamma} & \text{if } W_{t+1} > R_{t+1} \\ -1 + \frac{\delta[1-s]}{\exp(\Pi)}\Omega(W_t, Z_{t+1}) & \text{if } W_{t+1} = R_{t+1} \\ -Z_{t+1}\lambda\eta W_t^{-\gamma} & \text{if } W_{t+1} < R_{t+1}. \end{cases} \quad (1)$$

By taking the derivative of $J(W_t, Z_{t+1}) dF(Z_{t+1}|Z_t)$, as given by equation (18), with respect to its first argument, it can be shown that Ω takes the following form (in which $\psi \equiv \frac{\delta[1-s]}{\exp(\Pi)}$):

$$\begin{aligned} \Omega(W_t, Z_t) = & - \int_0^{Z^l(W_t)} Z_{t+1}\lambda\eta W_t^{-\gamma} dF(Z_{t+1}|Z_t) - \int_{Z^l(W_t)}^{Z^u(W_t)} 1 dF(Z_{t+1}|Z_t) \\ & - \int_{Z^u(W_t)}^{\infty} Z_{t+1}\eta W_t^{-\gamma} dF(Z_{t+1}|Z_t) + \psi \int_{Z^l(W_t)}^{Z^u(W_t)} \Omega(W_t, Z_{t+1}) dF(Z_{t+1}|Z_t), \end{aligned} \quad (2)$$

where the derivatives with respect to the integral limits cancel each other out due to the continuity of the value function and application of the Leibniz rule (see Elsby, 2009; Dickson and Fongoni, 2019). Define (2) as $(T\Omega)(W_t, Z_t)$ in the remainder of the proof. It can be verified that Blackwell's sufficient conditions for a contraction—i.e. monotonicity and discounting—are satisfied, implying that T is a contraction mapping. To do so, the analysis is restricted to a subset of (W_t, Z_t) around the optimum, and hence it is assumed that both the state and control spaces are compact, which implies that Ω is bounded, and that the operator T maps the space of bounded functions into itself. Denote this space by $\mathcal{B}(W_t, Z_t)$. Monotonicity requires that for given $\Omega, \hat{\Omega} \in \mathcal{B}(W_t, Z_t)$ and $\Omega(W_t, Z_t) \leq \hat{\Omega}(W_t, Z_t)$, $(T\Omega)(W_t, Z_t) \leq (T\hat{\Omega})(W_t, Z_t)$. This can be verified by noting

that

$$\begin{aligned} & (T\Omega)(W_t, Z_t) - (T\hat{\Omega})(W_t, Z_t) \Rightarrow \\ & \Rightarrow \psi \int_{Z^l(W_t)}^{Z^u(W_t)} \Omega(W_t, Z_{t+1}) dF(Z_{t+1}|Z_t) - \psi \int_{Z^l(W_t)}^{Z^u(W_t)} \hat{\Omega}(W_t, Z_{t+1}) dF(Z_{t+1}|Z_t) \leq 0. \end{aligned}$$

Hence, T is monotonic in Ω . Discounting requires that there exists some $\beta \in (0, 1)$ such that $[T(\Omega + a)](W_t, Z_t) \leq (T\Omega)(W_t, Z_t) + \beta a$. This can be verified by noting that

$$\begin{aligned} [T(\Omega + a)](W_t, Z_t) &= (T\Omega)(W_t, Z_t) + \psi a \int_{Z^l(W_t)}^{Z^u(W_t)} dF(Z_{t+1}|Z_t) \\ &\leq (T\Omega)(W_t, Z_t) + \psi a; \end{aligned}$$

since $\psi \in (0, 1)$. Hence, T is a contraction with modulus ψ . Finally, it follows from (2), which maps the space of strictly negative functions into itself, that Ω is strictly negative. \blacksquare

Proof of Proposition 1. This proof proceeds as follows. First, it will be shown that by imposing an additional, though innocuous, assumption on the state and control spaces, there exists a unique solution to the functional equation characterising the firm's wage setting problem. Then, using the method developed by Elsby (2009), a solution for the form of $Z^u(R_t)$ and $Z^l(R_t)$ will be derived. Finally, the firm's wage setting policy will be characterised.

Preliminaries. Denote the state space of Z by \mathcal{Z} . By definition, the state and control spaces \mathcal{Z} , \mathcal{R} and \mathcal{W} are all convex subsets of \mathbb{R}_+ . Throughout the proof it is assumed that \mathcal{Z} , \mathcal{R} and \mathcal{W} are also compact, with upper bounds sufficiently large as to ensure that the solutions to the firm's problem are interior. Given this premise, it is possible to establish that the firm's instantaneous payoff is both bounded and continuous in its domain. This, together with the fact that $\psi \equiv \frac{\delta[1-s]}{\exp(\Pi)} \in (0, 1)$, implies that the operator T defined as

$$(TJ)(R) \equiv \max_{W_t \in \mathcal{W}} \left\{ Z_t e(W_t, R_t, \lambda) - W_t + \psi \int J(W_t, Z_{t+1}) dF(Z_{t+1}|Z_t) \right\},$$

which maps the space of continuous and bounded functions into itself, is a contraction with a unique fixed point. Hence, there exists a unique solution to the functional equation given by (5) and at least one optimal wage policy exists (see, for instance, Theorem 9.6, p. 263 of Stokey and Lucas (1989)).

Threshold functions. First, note that following Assumption 1, the p.d.f. and c.d.f. of

Z_{t+1} are respectively given by

$$f(Z_{t+1}|Z_t) = \frac{1}{Z_{t+1}} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{[\ln Z_{t+1} - \hat{Z}]^2}{2\sigma^2}\right); \quad F(Z_{t+1}|Z_t) = \Phi\left(\frac{\ln Z_{t+1} - \hat{Z}}{\sigma}\right);$$

where $\hat{Z} = \ln Z_t + \Pi$ is the mean and σ the standard deviation of $\ln Z_{t+1}$; Φ is the c.d.f. of the standard normal. Next, consider the following set of equalities, which stem from the application of the theory of partial expectations to a log-normally distributed random variable Z_{t+1} (\bar{Z} and \underline{Z} are given thresholds):

$$\begin{aligned} \int_0^{\underline{Z}} Z_{t+1} dF(Z_{t+1}|Z_t) &= \exp\left(\hat{Z} + \frac{1}{2}\sigma^2\right) \Phi\left(\frac{\ln \underline{Z} - \hat{Z} - \sigma^2}{\sigma}\right); \\ \int_{\underline{Z}}^{\bar{Z}} Z_{t+1} dF(Z_{t+1}|Z_t) &= \exp\left(\hat{Z} + \frac{1}{2}\sigma^2\right) \left[\Phi\left(\frac{\ln \bar{Z} - \hat{Z} - \sigma^2}{\sigma}\right) - \Phi\left(\frac{\ln \underline{Z} - \hat{Z} - \sigma^2}{\sigma}\right) \right]; \\ \int_{\bar{Z}}^{\infty} Z_{t+1} dF(Z_{t+1}|Z_t) &= \exp\left(\hat{Z} + \frac{1}{2}\sigma^2\right) \left[1 - \Phi\left(\frac{\ln \bar{Z} - \hat{Z} - \sigma^2}{\sigma}\right) \right]. \end{aligned}$$

To proceed, consider the expression for Ω as given by (2). It is conjectured that

$$\Omega(W_t, Z_t) = \omega(Z_t W_t^{-\gamma}),$$

and that

$$Z^u(W_t) = \bar{U} W_t^\gamma; \quad Z^l(W_t) = \bar{L} W_t^\gamma;$$

with \bar{U} and \bar{L} being constants. That is, Ω is homogenous of degree zero in Z_t and W_t^γ , and the thresholds are log-linear functions of their argument. Using these conjectures, and the results on partial expectations of a log-normally distributed random variable stated above, it is possible to rewrite (2) as

$$\begin{aligned} \Omega(W_t, Z_t) &= -\lambda\eta \exp\left(\Pi + \frac{1}{2}\sigma^2\right) \Phi\left(\frac{\ln \bar{L} - \ln(Z_t W_t^{-\gamma}) - \Pi - \sigma^2}{\sigma}\right) Z_t W_t^{-\gamma} \\ &\quad - \left[\Phi\left(\frac{\ln \bar{U} - \ln(Z_t W_t^{-\gamma}) - \Pi}{\sigma}\right) - \Phi\left(\frac{\ln \bar{L} - \ln(Z_t W_t^{-\gamma}) - \Pi}{\sigma}\right) \right] \\ &\quad - \eta \exp\left(\Pi + \frac{1}{2}\sigma^2\right) \left[1 - \Phi\left(\frac{\ln \bar{U} - \ln(Z_t W_t^{-\gamma}) - \Pi - \sigma^2}{\sigma}\right) \right] Z_t W_t^{-\gamma} \\ &\quad + \psi \int_{\bar{L} W_t^\gamma}^{\bar{U} W_t^\gamma} \omega(Z_{t+1} W_t^{-\gamma}) dF(Z_{t+1}|Z_t). \quad (3) \end{aligned}$$

where, after a change of variable, the last term can be expressed as

$$\psi \int_{\frac{\bar{L}}{Z_t W_t^{-\gamma}}}^{\frac{\bar{U}}{Z_t W_t^{-\gamma}}} \omega \left(\frac{Z_{t+1}}{Z_t} Z_t W_t^{-\gamma} \right) d\tilde{F} \left(\frac{Z_{t+1}}{Z_t} \right),$$

in which \tilde{F} is the distribution of Z_{t+1}/Z_t which is i.i.d. due to Assumption 1. These considerations establish that $\Omega(W_t, Z_t)$ is a function of $Z_t W_t^{-\gamma}$ and confirm the conjecture that $\Omega(W_t, Z_t) = \omega(Z_t W_t^{-\gamma})$. Finally, to confirm the conjectured form of the thresholds, it is possible to use the results just obtained to rewrite conditions (10) and (11)—implicitly defining the thresholds—as

$$\bar{U}\eta - 1 + \psi\omega(\bar{U}) = 0; \quad (4)$$

$$\bar{L}\lambda\eta - 1 + \psi\omega(\bar{L}) = 0; \quad (5)$$

which are functions of a constant and can be solved numerically for \bar{U} and \bar{L} (see Appendix C) for a given configuration of the model parameters $\{\delta, \sigma, \eta, \lambda, \Pi\}$.

Wage setting policy. For a given R_t , the wage policy depends on the realisation of Z_t in relation to the two thresholds $Z^u(R_t)$ and $Z^l(R_t)$ characterised by (10) and (11) respectively. If $Z_t > Z^u(R_t)$, the wage set by the firm will exceed R_t and will be the solution to the first-order condition (9) in which $e_W(W_t, R_t, \lambda) = \eta W_t^{-\gamma}$. Using the results established above, this solution is given by $\tilde{W}(R_t, Z_t) = Z^{u-1}(Z_t) = [Z_t/\bar{U}]^{\frac{1}{\gamma}}$. If $Z_t < Z^l(R_t)$, the wage will be below R_t and will be the solution to the first-order condition (9) in which $e_W(W_t, R_t, \lambda) = \lambda\eta W_t^{-\gamma}$, that is, $\tilde{W}(R_t, Z_t) = Z^{l-1}(Z_t) = [Z_t/\bar{L}]^{\frac{1}{\gamma}}$. If $Z_t \in [Z^l(R_t), Z^u(R_t)]$, the wage will be such that $\tilde{W}(R_t, Z_t) = R_t$. ■

Proof of Proposition 2. Existence and uniqueness of an equilibrium path are straightforward to verify after noticing that for any given initial $R_0 \in \mathcal{R}$, $Z_0 \in \mathcal{Z}$, P_0 and Π , in each period there exists a unique $R_n = R_0$ for all t . This implies a unique optimal wage paid to new hires $\tilde{W}_{nt} = \tilde{W}(R_n, Z_t)$, as established in Proposition 1, with real counterpart given by $\tilde{w}_{nt} = \tilde{w}(r_n, z_t)$, where $r_n = r_0 = R_0/P_0$ and $z_t = Z_t/P_t$, as given by equation (16). New hires will exert effort e_{nt} , which is a function of the wage they are paid, in relation to their given reference wage. An analogous logic applies to incumbent workers, since in each period there exists a unique $R_{it} = \tilde{W}_{t-1}$, due to Assumption 2, which implies a unique optimal wage paid to incumbents $\tilde{W}_{it} = \tilde{W}(\tilde{W}_{t-1}, Z_t)$ as established in Proposition 1, and real wage $\tilde{w}_{it} = \tilde{w}(\tilde{w}_{t-1}, z_t)$ as given by equation (16). Incumbents will exert effort e_{it} , which is a function of the wage they are paid in relation to the wage they were paid in the previous period. Hence, in each period there exists a unique $J(r_n, z_t)$ determining the equilibrium value of labour market tightness $\tilde{\theta}_t = \tilde{\theta}(r_n, z_t)$. Finally, notice that since $J(r_n, z_t)$ is strictly decreasing in r_n and $\tilde{\theta}_t$ is

strictly increasing in J , it follows that $\tilde{\theta}_t$ is also strictly decreasing in r_n . Since u_t is strictly decreasing in $\tilde{\theta}_t$, it follows that u_t is strictly increasing in r_n . ■

Proof of Proposition 3. Existence and uniqueness of a steady-state equilibrium follow from the results established in Proposition 2 and the properties of firms' optimal wage setting policy. First, notice that any given $r_n = r_0$ is a steady-state reference wage for new hires and that since the analysis is restricted for values of r_n that are 'not too high', such that $z > z^u(r_n)$, the optimal steady-state wage paid to new hires will be the solution to the firms' first-order condition for the case of $w > r$. The steady-state analog to the first-order condition characterised by (9) is given by ($\psi \equiv \delta[1 - s]$)

$$z\eta w^{-\gamma} - 1 + \psi\Omega(w) = 0 \quad \forall w > r,$$

where the envelope condition is

$$\Omega(w) \equiv J_r(w) = -z\eta w^{-\gamma}.$$

Following the logic implemented in the Proof of Proposition 1, it is then straightforward to show that for a given r_n , $z^u(r_n) = \frac{1}{\eta[1-\psi]}r_n^\gamma$ and that since $z > z^u(r_n)$, the optimal steady-state wage paid to new hires is $\tilde{w}_n = \{z\eta[1-\psi]\}^{\frac{1}{\gamma}} > r_n$, implying that new hires exert $e_n > \bar{e}$ in the steady state. Next, consider incumbent workers and notice that any $r_i = \tilde{w}(r_i)$ is a steady state. By virtue of the first-order condition just derived, any r_0 will be such that $z > z^u(r_0)$, which implies that the optimal wage paid to incumbents is given by $\tilde{w}_i = \tilde{w}(r_0) = \{z\eta[1-\psi]\}^{\frac{1}{\gamma}}$. Moreover, due to Assumption 2, it follows that $r_i = \tilde{w}(r_0)$, which implies that for all $t > 0$, $z^u(r_i) = z$ and $\tilde{w}_i = \tilde{w}(r_i) = r_i = \{z\eta[1-\psi]\}^{\frac{1}{\gamma}}$, which is a steady state in which incumbent workers exert $e_i = \bar{e}$.

Hence, $\tilde{w}_n = \tilde{w}_i$, and the steady-state value of a new job is given by

$$J(r_n) = ze_n - \tilde{w}_n + \psi J(\tilde{w}_n);$$

the steady-state value of a job with an incumbent is given by

$$\begin{aligned} J(r_i) &= ze_i - \tilde{w}_n + \psi J(r_i) \\ J(\tilde{w}_n) &= ze_i - \tilde{w}_n + \psi J(\tilde{w}_n) \\ &= \frac{ze_i - \tilde{w}_n}{1 - \psi}, \end{aligned}$$

where the second equality follows from the fact that $r_i = \tilde{w}_i = \tilde{w}_n$. Substituting $J(\tilde{w}_n)$ into the expression for $J(r_n)$, and subsequently substituting this out from the steady-state

analog of the job creation condition (17), yields:

$$\tilde{\theta} = \tilde{\theta}(r_n) = \left\langle \frac{\bar{m}}{\kappa} \left\{ z e_n - \tilde{w}_n + \frac{\psi}{1-\psi} [z e_i - \tilde{w}_n] \right\} \right\rangle^{\frac{1}{\alpha}},$$

which is equivalent to the expression in (22) ■

Proof of Proposition 4. Consider the first-order and envelope condition for the firm's optimal wage policy in the steady state, as derived in the Proof of Proposition 3 $z\eta\tilde{w}_n^{-\gamma} - 1 - \psi z\eta\tilde{w}_n^{-\gamma} = 0$. This can be rearranged as

$$z\eta\tilde{w}_n^{-\gamma} = \frac{1}{1-\psi}.$$

Next, from the definition of the elasticity of effort with respect to the wage ($\varepsilon_{e_n, w} = \frac{de_n}{d\tilde{w}_n} \frac{\tilde{w}_n}{e_n}$), it is possible to substitute for $e_n \varepsilon_{e_n, w} = \eta\tilde{w}_n^{-\gamma} \tilde{w}_n$ in the equation for the steady-state elasticity of market tightness given by (23). Then, after collecting $\varepsilon_{w, z} \tilde{w}_n$ as the common factor, the second and third term in the numerator of (23) can be rearranged as

$$\varepsilon_{w, z} \tilde{w}_n \left[z\eta\tilde{w}_n^{-\gamma} - \frac{1}{1-\psi} \right],$$

where the term inside the square brackets is zero by virtue of the first-order condition derived above. ■

Proof of Proposition 5. Consider the expression for $\varepsilon_{\tilde{\theta}, z}$ as given by (25). From the definition of the elasticity of effort $\varepsilon_{e_n, w} \equiv e_w \frac{\tilde{w}_n}{e_n}$, it is possible to express $\tilde{w}_n = \frac{e_n}{e_w} \varepsilon_{e_n, w}$ and $e_n = e_w \frac{\tilde{w}_n}{\varepsilon_{e_n, w}} = \frac{\eta\tilde{w}_n^{1-\gamma}}{\varepsilon_{e_n, w}}$, since $e_w = \eta\tilde{w}_n^{-\gamma}$. Hence it follows that $e_n = \eta \frac{e_n^{1-\gamma}}{e_w^{1-\gamma}} \frac{\varepsilon_{e_n, w}^{1-\gamma}}{\varepsilon_{e_n, w}}$. Substituting these into the equation for $\varepsilon_{\tilde{\theta}, z}$ yields:

$$\varepsilon_{\tilde{\theta}, z} = \frac{1}{\alpha} \frac{[1-\psi] z \eta \frac{e_n^{1-\gamma}}{e_w^{1-\gamma}} \frac{\varepsilon_{e_n, w}^{1-\gamma}}{\varepsilon_{e_n, w}} + \psi z \bar{e}}{[1-\psi] z \eta \frac{e_n^{1-\gamma}}{e_w^{1-\gamma}} \frac{\varepsilon_{e_n, w}^{1-\gamma}}{\varepsilon_{e_n, w}} + \psi z \bar{e} - \frac{e_n}{e_w} \varepsilon_{e_n, w}}.$$

Next, denote the numerator of the second factor of the expression above by $N(\varepsilon_{e_n, w})$ and the respective denominator by $D(\varepsilon_{e_n, w})$ and notice that $D(\varepsilon_{e_n, w}) = N(\varepsilon_{e_n, w}) - \frac{e_n}{e_w} \varepsilon_{e_n, w}$. Hence, differentiating $\varepsilon_{\tilde{\theta}, z}$ with respect to $\varepsilon_{e_n, w}$ yields:

$$\frac{\partial \varepsilon_{\tilde{\theta}, z}}{\partial \varepsilon_{e_n, w}} = \frac{1}{\alpha} \frac{1 - N'(\varepsilon_{e_n, w}) \frac{e_n}{e_w} \varepsilon_{e_n, w} + N(\varepsilon_{e_n, w}) \frac{e_n}{e_w}}{D(\varepsilon_{e_n, w})^2} > 0,$$

which is positive, since $N'(\varepsilon_{e_n, w}) = -\gamma \frac{e_n^{1-\gamma}}{\varepsilon_{e_n, w}^2} < 0$. Hence, all else equal, and in particular, for any given $\gamma > 0$, $\varepsilon_{\theta, z}$ is increasing in $\varepsilon_{e_n, w}$. ■

B Derivation of the Workers' Effort Function

This section closely follows Dickson and Fongoni (2019). An employed worker's instantaneous payoff in each period is additively separable and takes the following form:

$$\hat{u}(e, W, R) = \hat{v}(W) - d(e) + M(e, W, R),$$

where \hat{v} is strictly increasing and concave and captures the worker's evaluation of the wage; d is strictly convex with $d'(0) < 0$ and represents the worker's intrinsic psychological net cost of productive activity; and $M(e, W, R) \equiv e\mu(\hat{v}(W) - \hat{v}(R))$ is the 'morale function' that depends on the worker's evaluation of the wage in relation to the reference wage, where μ a piecewise-linear gain-loss function that exhibits loss aversion in the spirit of Kahneman and Tversky (1979) and Tversky and Kahneman (1991). The morale function captures the psychological cost/benefit of productive effort associated with the worker's perception of fairness. If the wage exceeds the reference wage (it is perceived as a gift) the worker gains some additional benefit of productive effort and an increase in effort (a gift to the firm) will increase utility. If the wage falls short of the reference wage (it is perceived as unfair) there is a psychological cost of productive effort and a reduction in effort (an 'unkind' action towards the firm) increases utility. As such, the morale function implies the worker's payoff exhibits reciprocity. This paper considers the following functional forms: $\hat{v}(W) = \frac{W^{1-\gamma}}{1-\gamma}$, $\gamma \in (0, 1)$; $d(e) = \frac{e^2}{2} - be$, $b \in \mathbb{R}_{++}$; and $\mu(x) = \eta x$ if $x \geq 0$, and $\mu(x) = \lambda \eta x$ if $x < 0$, with $\eta \in \mathbb{R}_+$, and $\lambda \geq 1$. For a given reference wage R and wage W , the worker will choose the level of effort e that maximises utility. The necessary and sufficient first-order condition for optimal effort is

$$-e + b + \mu\left(\frac{W^{1-\gamma}}{1-\gamma} - \frac{R^{1-\gamma}}{1-\gamma}\right) = 0,$$

which yields an explicit solution, the form of which is equivalent to the effort function (4) assumed in the main body of the paper where $\bar{e} \equiv b$.

C Computational Approach

As noted in Section II, solving for the equilibrium requires finding the optimal wage and effort levels of employed workers for any given r_t and z_t , which can then be used to solve for the firms' value of a new job $J(r_{nt}, z_t)$ and for the equilibrium value of labour market tightness $\tilde{\theta}(r_n, z_t)$.

Using the results established by Proposition 1, the optimal wage can be found by

solving numerically for the threshold coefficients \bar{U} and \bar{L} . This is done by constructing an algorithm that recursively solves for Ω in (3) and uses conditions (4) and (5) to update the thresholds after each iteration.¹ To do so, it turns out to be convenient to define $x_t \equiv Z_t W_t^{-\gamma}$ and to rewrite the expression for Ω in (3) as

$$\begin{aligned} \omega(x_t) = & -\lambda\eta \exp\left(\Pi + \frac{1}{2}\sigma^2\right) \Phi\left(\frac{\ln \bar{L} - \ln x_t - \Pi - \sigma^2}{\sigma}\right) x_t \\ & - \left[\Phi\left(\frac{\ln \bar{U} - \ln x_t - \Pi}{\sigma}\right) - \Phi\left(\frac{\ln \bar{L} - \ln x_t - \Pi}{\sigma}\right) \right] \\ & - \eta \exp\left(\Pi + \frac{1}{2}\sigma^2\right) \left[1 - \Phi\left(\frac{\ln \bar{U} - \ln x_t - \Pi - \sigma^2}{\sigma}\right) \right] x_t \\ & + \psi \int_{\bar{L}/x_t}^{\bar{U}/x_t} \omega(x_{t+1}) d\tilde{F}\left(\frac{x_{t+1}}{x_t}\right); \quad (6) \end{aligned}$$

where $x_{t+1} \equiv Z_{t+1} W_t^{-\gamma}$ since $W_{t+1} = W_t$ when $x_{t+1} \in (\bar{L}, \bar{U})$; and \tilde{F} is the distribution of Z_{t+1}/Z_t which is log-normal and i.i.d. (see Assumption 1), with mean $\exp(\Pi)$ and standard deviation σ , when $x_{t+1} \in (\bar{L}, \bar{U})$.

The next step requires finding the solution to the firms' value function for any given r_n and z_t , as given by

$$J(r_n, z_t) = \pi(z_t, r_n) + \delta[1 - s] \int J(\tilde{w}_{nt}, z_{t+1}) dF(z_{t+1}|z_t); \quad (7)$$

where π denotes firms' instantaneous payoff function in each period, in which $\tilde{w}_{nt} = \tilde{w}(z_t, r_n) > r_n$. Crucially, to solve for $J(r_n, z_t)$ it is first necessary to solve for $J(\tilde{w}_{nt}, z_{t+1})$, the value of a job with an incumbent worker, which takes the following recursive form in each period (for any given $r_{it} = \tilde{w}_{t-1} \frac{1}{\exp(\Pi)}$ and z_t):

$$\begin{aligned} J(\tilde{w}_{t-1}, z_t)^{-;=;+} = & \pi(z_t, \tilde{w}_{t-1})^{-;=;+} + \delta[1 - s] \left\{ \int_0^{z^l(\tilde{w}_t)} J(\tilde{w}_t, z_{t+1})^- dF(z_{t+1}|z_t) \right. \\ & \left. + \int_{z^l(\tilde{w}_t)}^{z^u(\tilde{w}_t)} J(\tilde{w}_t, z_{t+1})^- dF(z_{t+1}|z_t) + \int_{z^u(\tilde{w}_t)}^{\infty} J(\tilde{w}_t, z_{t+1})^+ dF(z_{t+1}|z_t) \right\}; \quad (8) \end{aligned}$$

where $\tilde{w}_t = \tilde{w}(z_t, \tilde{w}_{t-1})$ is given by (16) and the superscripts $-;=;+$ stand for the case in which $w < r; w = r; w > r$ respectively.

Although it has been established that there exist a unique solution of the above functional equation, it is not straightforward to find its numerical value for the following reasons. First, the value function $J(\tilde{w}_{t-1}, z_t)$ is kinked, the kinks being at the points in which $z_t = z^u(\tilde{w}_{t-1})$ and $z_t = z^l(\tilde{w}_{t-1})$, and therefore in which $\tilde{w}_t = \tilde{w}_{t-1} \frac{1}{\exp(\Pi)}$. This implies that solving for (8) requires to solve simultaneously for three value functions: $J(\tilde{w}_t, z_{t+1})^-$,

¹The author thanks Mike Elsby for suggesting this approach.

$J(\tilde{w}_t, z_{t+1})^-$ and $J(\tilde{w}_t, z_{t+1})^+$. Second, for a given z_t , the probability that z_{t+1} falls below, within, or above the range of rigidity $[z^l(\tilde{w}_t), z^u(\tilde{w}_t)]$ in $t + 1$, depends on \tilde{w}_t , which is endogenous and dependent on \tilde{w}_{t-1} and z_t . Hence, for a given stochastic recursive sequence characterising the dynamics of z_t , the transition probabilities determining whether the realised value of z_{t+1} falls below, within, or above the range $[z^l(\tilde{w}_t), z^u(\tilde{w}_t)]$ are endogenous and state dependent. This is why knowledge of the optimal wage policy substantially aids the computational approach, as it enables to characterise the mapping between the state in t , (\tilde{w}_{t-1}, z_t) , and the relationship between the state in $t + 1$, (\tilde{w}_t, z_{t+1}) , and the thresholds in $t + 1$, $z_{t+1}^l = z^l(\tilde{w}(\tilde{w}_{t-1}, z_t))$ and $z_{t+1}^u = z^u(\tilde{w}(\tilde{w}_{t-1}, z_t))$.

These considerations—along with the recursive nature of the problem—imply that with knowledge of the stochastic process for z_t and of the optimal wage policy $\tilde{w}(\tilde{w}_{t-1}, z_t)$, it is possible to construct a transition matrix which captures the transition probabilities between J^- , $J^=$ and J^+ for a given z_t and \tilde{w}_{t-1} . Once this is done, it is possible to apply a value function iteration algorithm to find the numerical value of a job with an incumbent worker, which can subsequently be used to calculate the expected continuation value of a job with an incumbent worker, and therefore to find the numerical value of a job with a new hire. For this procedure, the stochastic process characterizing the dynamics of z_t is approximated by a highly persistent AR(1) process using Rouwenhorst (1995) method, with moments consistent with the ones used in the numerical simulations (that is, a correlation coefficient of 0.99 and a standard deviation of the i.i.d. shocks of 0.009). The approach described above can be carried out for any given initial r_n and z_t , enabling to construct a discrete mapping between the state (r_n, z_t) and J . Linear interpolation is then used to generate a continuous mapping between z_t and J for a given r_n , and therefore between z_t and $\tilde{\theta}$.

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