Prior User Rights: Appendix*

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R&D Expenditure Levels with Independent Projects

Discounting could easily be incorporated into this model by redefining T to represent the ratio of the value of an annuity that lasts for the lifetime of the patent to the value of a perpetuity.

A. Proof of Theorem #1

If the patent lifetime *T*, is set optimally, given α, we must have $\frac{dW}{dT} = \frac{\partial W}{\partial p}\frac{\partial p}{\partial T} + \frac{\partial W}{\partial T} = 0$, so $\frac{\partial W}{\partial p} = \frac{\partial W}{\partial T} / \frac{\partial p}{\partial T}$. The welfare impact of strengthening prior user rights is given by $\frac{dW}{d\alpha} = \frac{\partial W}{\partial p}\frac{\partial p}{\partial \alpha} + \frac{\partial W}{\partial \alpha}$. Substituting for $\partial W / \partial p$, we get $\left.\frac{dW}{d\alpha}\right|_{T=T*} = \frac{\partial p}{\partial \alpha}\frac{\partial W}{\partial T} / \frac{\partial p}{\partial T} + \frac{\partial W}{\partial \alpha}$, so $\left.\frac{dW}{d\alpha}\right|_{T=T*} > 0$ if and only if $\frac{\partial p}{\partial \alpha}\frac{\partial W}{\partial T} / \frac{\partial p}{\partial T} + \frac{\partial W}{\partial \alpha} > 0$. Since $\frac{\partial W}{\partial T} < 0$, we have $\left.\frac{dW}{d\alpha}\right|_{T=T*} > 0$ if and only if

$$\frac{\partial W}{\partial \alpha} / [-\frac{\partial W}{\partial T}] > [-\frac{\partial p}{\partial \alpha}] / \frac{\partial p}{\partial T}.$$

We now proceed to establish that this inequality is met.

* This is the Appendix to "Prior User Rights." The paper itself is available at my web site, http://faculty.haas.berkeley.edu/shapiro/prior.pdf. This Appendix is available at http://faculty.haas.berkeley.edu/shapiro/priorapp.pdf.

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The left-hand side of this inequality is easy to calculate. As noted above, $dW_B / d\alpha = W_D - W_M$,

so $\frac{\partial W}{\partial \alpha} = p^2 T(W_D - W_M)$. From the definition of $W(p,T,\alpha)$ we also get

$-\frac{\partial W}{\partial T} = p^2 (W_C - W_B) + 2p(1-p)(W_C - W_M)$. Therefore, we have

$$\frac{\partial W}{\partial \alpha} / [-\frac{\partial W}{\partial T}] = \frac{pT(W_D - W_M)}{p[W_C - W_B] + 2(1-p)[W_C - W_M]}.$$

We now look more closely at the $p(T,\alpha)$ function to obtain an expression for the right-hand side of above inequality .

Using the condition that defines the symmetric equilibrium level of *p*, we get

$\frac{\partial p}{\partial T} = \frac{(1-p)\pi_M + p\pi_B}{C''(p) + T(\pi_M - \pi_B)}$ and $-\frac{\partial p}{\partial \alpha} = \frac{pT(\pi_M / 2 - \pi_B)}{C''(p) + T(\pi_M - \pi_B)}$ so we have

$$[-\frac{\partial p}{\partial \alpha}] / \frac{\partial p}{\partial T} = \frac{pT(\frac{\pi_M}{2} - \pi_D)}{(1-p)\pi_M + p\pi_B}.$$

So, we have $\left.\frac{dW}{d\alpha}\right|_{T=T*} > 0$ if and only if

$$\frac{(W_D - W_M)}{p[W_C - W_B] + 2(1-p)[W_C - W_M]} > \frac{\frac{\pi_M}{2} - \pi_D}{(1-p)\pi_M + p\pi_B}.$$

Substituting using $W_B = (1-\alpha)W_M + \alpha W_D$ and $\pi_B = (1-\alpha)\pi_M / 2 + \alpha\pi_D$, this becomes

$$\frac{(W_D - W_M)}{p[W_C - (1-\alpha)W_M - \alpha W_D] + 2(1-p)[W_C - W_M]} > \frac{\frac{\pi_M}{2} - \pi_D}{(1-p)\pi_M + p[(1-\alpha)\pi_M / 2 + \alpha\pi_D]}.$$

Collecting terms, this becomes

$$\frac{(W_D - W_M)}{(2-p)[W_C - W_M] - \alpha p[W_D - W_M]} > \frac{\pi_M - 2\pi_D}{(2-p)\pi_M - \alpha p[\pi_M - 2\pi_D]}.$$

Inverting both sides and simplifying gives

$$\frac{W_C - W_M}{W_D - W_M} < \frac{\pi_M}{\pi_M - 2\pi_D}.$$

Inverting again and simplifying gives $\frac{2\pi_D}{\pi_M} > \frac{W_C - W_D}{W_C - W_M}$. Defining the monopoly deadweight loss as $DWL_M = W_C - W_M$ and the duopoly deadweight loss as $DWL_D = W_C - W_D$, granting stronger prior user rights raises welfare if and only if $\frac{DWL_M}{\pi_M} > \frac{DWL_D}{2\pi_D}$, as asserted in the text.

B. Ratio of Profits to Deadweight Loss

Gilbert and Shapiro (1990) show that the ratio of deadweight loss to profits rises with price is profits and welfare are both concave in output. Here we establish an alternative sufficient condition. The material in this section was developed jointly with Joseph Farrell.

Call the demand function $X(p)$. Assume that output can be produced at constant marginal cost *c*. Denote by $L(p)$ the deadweight loss if the price is *p*. [For this subsection alone, *p* denotes price, not the probability of discovery.] Denote by $\Pi(p) = (p-c)X(p)$ the total profits if price is *p*. Under what circumstances is the ratio $L(p)/\Pi(p)$ increasing in price *p* in the range $c \le p \le p^M$, where p^M is the monopoly price?

The ratio $L(p)/\Pi(p)$ is increasing in *p* if and only if $L'(p)/\Pi'(p) > L(p)/\Pi(p)$. We look at each of these ratios in turn.

By definition, $L(p) = \int_c^p [X(t) - X(p)]dt$, so $L'(p) = (p-c)[-X'(p)]$.

$\Pi'(p) = (p-c)X'(p) + X(p) = X(p) - L'(p)$. Therefore, we get

$$\frac{\Pi'(p)}{L'(p)} = \frac{X(p) - L'(p)}{L'(p)} = -1 + \frac{X(p)}{-(p-c)X'(p)} = -1 + \frac{p}{p-c}\left[\frac{X(p)}{-pX'(p)}\right] = -1 + \frac{1}{mE(p)}$$, where

$m \equiv \frac{p-c}{p}$ is the Lerner Index and $E(p) \equiv -\frac{pX'(p)}{X(p)}$ is the absolute value of the elasticity of demand. Inverting this equation, we get $\frac{L'(p)}{\Pi'(p)} = \frac{mE(p)}{1 - mE(p)}$. Assuming that $\Pi'(p) > 0$ for $p < p^M$, we know that $mE(p) < 1$ in this range; only at $p = p^M$ do we get $mE(p) = 1$.

We now look at the first-order approximations to $L'(p)/\Pi'(p)$ and $L(p)/\Pi(p)$ for values of *p* near *c*. We express these in terms of *m*, which is zero at $p = c$. Using the above calculation, we have $\frac{L'(p)}{\Pi'(p)} \approx mE(c)$ for values of *p* near *c*. . From the definition of $L(p)$, for values of *p* near *c* we get the approximation $L(p) \approx \frac{1}{2}[p-c][X(c) - X(p)] \approx \frac{1}{2}[p-c][-(p-c)X'(c)]$. Some simple algebra shows that this expression is approximately equal to $\frac{1}{2}mE(c)\Pi(p)$. Therefore, for values of *p* near *c*, we have $\frac{L(p)}{\Pi(p)} \approx \frac{1}{2}mE(c)$. We have thus shown that in the neighborhood of $p = c$, the ratio $L'(p)/\Pi'(p)$ rises with *p* twice as rapidly as does the ratio $L(p)/\Pi(p)$. Both of these ratios approach zero as $p \to c$. This reflects the fact that the deadweight loss is second-order small in $p - c$ when price is near marginal cost.

Using $\frac{L'(p)}{\Pi'(p)} = \frac{mE(p)}{1 - mE(p)}$, we know that $L'(p)/\Pi'(p)$ rises with *p* if $mE(p)$ rises with *p*, i.e. if $\left(\frac{p-c}{p}\right)E(p)$ rises with *p*. Suppose that this condition is satisfied.

Now suppose that $d[L(p)/\Pi(p)]/dp = 0$ for some value of *p*, as it must if $L(p)/\Pi(p)$ is ever to decline with *p*, since $L(p)/\Pi(p)$ is increasing with *p* near $p = c$ (and we are assuming all functions are smooth) . Call p_0 the lowest value of *p* at which $d[L(p)/\Pi(p)]/dp = 0$. So, for $p < p_0$, $L(p)/\Pi(p)$ is increasing, which we know requires that $L'(p)/\Pi'(p) > L(p)/\Pi(p)$.

We must have $L(p)/\Pi(p) = L'(p)/\Pi'(p)$ at $p = p_0$. Since $L(p)/\Pi(p)$ is locally constant with respect to *p* at $p = p_0$, and since $L'(p)/\Pi'(p)$ is increasing in *p* (by assumption), this could only happen if $L'(p)/\Pi'(p)$ were *less* than $L(p)/\Pi(p)$ for values of *p* just below p_0. But this contradicts the fact that $L'(p)/\Pi'(p) > L(p)/\Pi(p)$ for $p < p_0$. We have therefore proven:

If $(\frac{p-c}{p})E(p)$ rises with *p*, then the ratio of deadweight loss to monopoly profits also rises with *p* for prices between marginal cost and the monopoly price.

C. Uniqueness and Stability of the Symmetric Equilibrium

For ease of notation, we write $k = 1 - \frac{\pi_B}{\pi_M}$, so the first-order condition is $\frac{C'(p)}{T\pi_M} = 1 - kq$. Note that $1/2 \le k \le 1$; when $\alpha = 0$, $\pi_B = \pi_M / 2$ and $k = 1/2$, and when $\alpha = 1$, $\pi_B = \pi_D$, and $k = 1 - \pi_D / \pi_M$.

The first-order condition for the choice of *p* is given by $C'(p)/T\pi_M = 1 - kq$. The slope of the first firm's best response function is therefore given by $dp/dq = -kT\pi_M / C''(p)$. The symmetric equilibrium is stable if and only if the first firm's best-response schedule is steeper than the second firm's at that point. Since the payoffs are symmetric, this is true if and only if the absolute value of the slope of the *p* best-response curve is greater than unity at the symmetric equilibrium. So, we get stability of the symmetric equilibrium if and only if $kT\pi_M > C''(p)$ at the point where $C'(p)/T\pi_M = 1 - kp$. The necessary and sufficient condition for stability, $kT\pi_M > C''(p)$, can be written as $kpT\pi_M > pC''(p)$. From the first-order condition, we have $kpT\pi_M = T\pi_M - C'(p)$, so the stability condition can be written as $T\pi_M - C'(p) > pC''(p)$ or $T\pi_M > C'(p) + pC''(p) = C'(p)[1+E]$ where $E \equiv pC''(p)/C'(p)$ is the elasticity of the cost function with respect to the success probability. Dividing this inequality by $T\pi_M$ gives $[C'(p)/T\pi_M][1+E] < 1$. Finally, substituting using the first-order condition we get the necessary and sufficient condition for stability as $(1-kp)(1+E) < 1$.

We now provide a sufficient condition for the symmetric equilibrium to be the only equilibrium. The equation defining the symmetric equilibrium is $\frac{C'(p)}{T\pi_M} = 1 - kp$.

Suppose there were an asymmetric equilibrium with $p > q$. Then we must have $C'(p)/T\pi_M = 1 - kq$ and $C'(q)/T\pi_M = 1 - kp$. Taking ratios of these two first-order conditions, we would have $C'(p)(1-kp) = C'(q)(1-kq)$. There can be no such asymmetric equilibrium if the function $C'(p)(1-kp)$ is monotonic in *p*. This expression is decreasing in *p* if and only if $pC''(p)/C'(p) < kp/(1-kp)$, which we can write as $E(1-kp) < kp$. This is the same as the stability condition, $(1+E)(1-kp) < 1$.

To illustrate using an example, suppose that $C(p) = [\gamma p + \beta p^2/2]T\pi_M$, so $C'(p) = [\gamma + \beta p]T\pi_M$ and $C''(p) = \beta T\pi_M$. Then the symmetric equilibrium level of *p* is given by $p^* = \frac{1-\gamma}{k+\beta}$. An interior equilibrium requires that $p^* > 0$, so $\gamma < 1$, and that $p^* < 1$, so $\beta + \gamma > 1 - k$. The condition for stability is that $\beta < k$. So long as these three conditions are satisfied, we have a stable interior equilibrium.

Diversification of Research Approaches

A. Proof of Theorem #2

We are interested in exploring the welfare effects of granting stronger prior user rights. Differentiating with respect to α, we get

$$\frac{dW(x,\alpha)}{d\alpha} = \frac{\partial W(x,\alpha)}{\partial x}\frac{dx}{d\alpha} + \frac{\partial W(x,\alpha)}{\partial \alpha}.$$

As usual, the direct effect of awarding stronger prior user rights is positive, since $\partial W/\partial \alpha = B(x,y)\partial W_B/\partial \alpha = B(x,y)(W_D - W_M) > 0$. The text establishes that $dx/d\alpha > 0$, so a sufficient condition for stronger prior user rights to raise welfare is that $\partial W/\partial x > 0$ at the equilibrium.

Using the definition of *W*, we have $W(x,y,\alpha)=W_M(A(x,y)+A(y,x))+W_B B(x,y)$.

Differentiating with respect to *x*, we have $W_x(x,y,\alpha)=W_M(A_x(x,y)+A_x(y,x))+W_B B_x(x,y)$. By symmetry, $A_x(y,x)=A_y(x,y)$. So

$W_x(x,y,\alpha)=W_M(A_x(x,y)+A_y(x,y)+B_x(x,y))+(W_B-W_M)B_x(x,y)$. Evaluating this at a symmetric point where $x=y$ gives

$$W_x(x,x,\alpha)=W_M(A_x(x,x)+A_y(x,x)+B_x(x,x))+(W_B-W_M)B_x(x,x).$$

Since $A(x,y)+B(x,y)=p(x)$, we know that $A_y(x,y)+B_y(x,y)=0$. By symmetry, $B(x,y)=B(y,x)$, so $B_x(x,x)=B_y(x,x)$. Therefore we must have $A_y(x,x)+B_y(x,x)=A_y(x,x)+B_x(x,x)$. Since the left-hand side of this expression is zero, the right-hand side must also equal zero, so we get

$$W_x(x,x,\alpha)=W_M A_x(x,x)+(W_B-W_M)B_x(x,x).$$

From the condition characterizing the symmetric equilibrium, $A_x(x,x)\pi_M+B_x(x,x)\pi_B=0$. Solving this for $B_x(x,x)$, substituting, and simplifying gives

$$W_x(x,x,\alpha)=W_M A_x(x,x)[1-\frac{W_B-W_M}{W_M}\frac{\pi_M}{\pi_B}]$$

at the symmetric equilibrium. Therefore, $W_x(x,x,\alpha)>0$ at the symmetric equilibrium if and only if $\frac{\pi_B}{\pi_M}>\frac{W_B-W_M}{W_M}$.

Note: Proposition 3 in Dasgupta and Maskin (1987) provides conditions under which the market research portfolio consists of projects that are too highly correlated, so that $dx/d\alpha>0$ in my notation. However, they assume that welfare is the same whether one or both firms are successful: $W_B=W_M$ in my notation. This condition holds at $\alpha=0$, so Proposition 3 in Dasgupta and Maskin (1987), combined with the definition of prior user rights adopted in this paper, implies Corollary #2A, i.e., that some prior user rights are optimal. However, their analysis must be extended, as shown here, to study the effects of stronger prior user rights away from $\alpha = 0$.

B. Second-Order Condition and Best-Response Functions

As calculated by Dasgupta and Maskin, using my notation,

$B(x, y) = (x + y)p(x)p(y) + [1-(x + y)](p(x) + p(y))/2$ and

$A(x, y) = [1+(x + y)]p(x)/2 - [1-(x + y)]p(y)/2 - (x + y)p(x)p(y)$.

The second-order condition for the first firm is $A_{xx}\pi_M + B_{xx}\pi_B < 0$. A sufficient condition for this to hold (which is necessary if π_B is sufficiently small) is that $A_{xx} < 0$. Direct calculations show that $A_{xx}(x, y) = p'(x)[1 - p(x) - p(y)] + p''(x)[1 + (x + y)(1 - p(y))]/2$. This expression is negative if $p(x)$ and $p(y)$ are each no larger than one-half, which they must be if $p(0) \leq 1/2$. However, we could have if $p(x) + p(y) > 1$ and if $p''(x)/p'(x)$ is small. In that case, the second-order condition is not satisfied, and the first firm should increase *x* to a higher level at which the first-order condition again holds to find the optimal level of *x*, avoiding a local minimum at a lower value of *x*.

The first-order condition for the first firm is $A_x(x, y)\pi_M + B_x(x, y)\pi_B = 0$. This firm's best-response function is downward sloping if $A_{xy}(x, y)\pi_M + B_{xy}(x, y)\pi_B < 0$, which we write as $\pi_M[A_{xy}(x, y) + B_{xy}(x, y)] - B_{xy}(x, y)[\pi_M - \pi_B] < 0$. Since $A(x, y) + B(x, y) = p(x)$, $A_y(x, y) + B_y(x, y) = 0$, and $A_{xy}(x, y) + B_{xy}(x, y) = 0$ as well, so this inequality is satisfied if and only if $B_{xy}(x, y) > 0$. Since $B_{xy}(x, y) = p'(x)[p(y) - 1/2] + p'(y)[p(x) - 1/2] + (x + y)p'(x)p'(y)$, this inequality is satisfied so long as $p(x)$ and $p(y)$ are each no larger than one-half, which they must be if $p(0) \leq 1/2$.

Allocation of R&D Budgets Across Markets: Proof of Theorem #3

The welfare effect of strengthening prior user rights is given by

$$\frac{dW}{d\alpha} = \frac{\partial W}{\partial x}\frac{dx}{d\alpha} + \frac{\partial W}{\partial \alpha}.$$

As usual, we know that the $\partial W / \partial \alpha > 0$, because $\partial W_B / \partial \alpha = W_D - W_M > 0$.

We show here that each firm will shift away from the smaller market and towards the larger market as prior user rights are strengthened. Formally, we show that $\partial x / \partial \alpha < 0$. The first firm picks x to maximize $\pi(x, y, \alpha)$. Since $d\pi_B / d\alpha = \pi_D - \pi_M / 2 < 0$, $\partial x / \partial \alpha < 0$ if and only if $\pi_x(x, y, \alpha)$ rises with π_B.

Differentiating $\pi(x, y, \alpha)$ with respect to π_B gives $p(x)p(y) + \sigma[p(1-x)/\sigma][p(1-y)/\sigma]$. Differentiating this with respect to x gives $p'(x)p(y) - p'(1-x)p(1-y)/\sigma$. This is positive if and only if $[p'(x)/p'(1-x)] > [p(1-y)/p(y)]/\sigma$. We now show that this expression is positive at the symmetric equilibrium, i.e., $\frac{p'(x)}{p'(1-x)} > \frac{p(1-x)}{p(x)}\frac{1}{\sigma}$ at the symmetric equilibrium.

In a symmetric equilibrium, Cabral shows (Equation A.4) that we must have

$\frac{p'(x)}{p'(1-x)} = \frac{\pi_M - (\pi_M - \pi_B)p(1-x)/\sigma}{\pi_M - (\pi_M - \pi_B)p(x)}$. So, we are attempting to show that $\frac{\pi_M - (\pi_M - \pi_B)p(1-x)/\sigma}{\pi_M - (\pi_M - \pi_B)p(x)} > \frac{p(1-x)/\sigma}{p(x)}$. Cross-multiplying and simplifying, this is equivalent to $p(x) > p(1-x)/\sigma$, i.e., that the equilibrium probability of success is greater in the smaller market, a condition that Cabral establishes.