Online Appendix for "Sequential Market, Market Power and Arbitrage"

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A Derivation of Equilibrium Strategies

A.1 Equilibrium without Arbitrage

Consider the case in which there is no arbitrage. At the second stage, the monopolist sets

$$\mathrm{p}_2(p_1) = \frac{p_1 + c}{2}, \tag{A.1}$$

$$\mathrm{q}_2(p_1) = b_2 \frac{p_1 - c}{2}. \tag{A.2}$$

From these expressions one can already see that, if the monopolist is a net seller in the first stage and $p_1 \geq c$,, then p_2 will be at most p_1.

At the first stage, optimal strategies imply,

$$p_1^* = \frac{2A + 2b_1 c - b_2 c}{4b_1 - b_2}, \tag{A.3}$$

$$q_1^* = \frac{(2b_1 - b_2)(A - b_1 c)}{4b_1 - b_2}, \tag{A.4}$$

$$p_2^* = \frac{A + 3b_1 c - b_2 c}{4b_1 - b_2}, \tag{A.5}$$

$$q_2^* = b_2 \frac{A - b_1 c}{4b_1 - b_2}. \tag{A.6}$$

Link to Result 1. We can use these expressions to show the results in Result 1. From the above expressions, one can see that the monopolist will be adjusting its quantity upwards in the second market as long as $A > b_1 c$, which is a necessary condition for q_1^* to be positive. Under the assumption that the monopolist is a net seller, it also implies that $p_1^* > p_2^*$, as $2A - 2b_1 c > A + 3b_1 c$. The forward premium is given by, $p_1^* - p_2^* = \frac{A - b_1 c}{4b_1 - b_2}$. The premium is increasing in A, decreasing in b_1 and increasing in b_2, showing the first and second part of Result 1. Looking at the special case of $b_1 = b_2$, the expressions of q_1^* and q_2^* simplify, and $q_1^* = q_2^*$. This implies that, if the forward and real-time market have the same elasticity, then the monopolist will sell the same amount of quantity in both markets. If $b_1 > b_2$ and the monopolist is a net seller (i.e., $A - b_1 c > 0$), $q_1^* - q_2^* = \frac{2(b_1 - b_2)(A - b_1 c)}{4b_1 - b_2} > 0$. This shows the third and fourth part of Result 1.

A.2 Equilibrium with Arbitrage

Now consider the case in which there is a competitive arbitrageur that can choose a quantity s to arbitrage between markets. We consider a Nash equilibrium in which the arbitrageur takes the actions of the monopolist as given, and the monopolist takes the actions of the arbitrageur as given.

Under the modified demands presented in (5) and (6), optimal strategies at the second stage imply,

$$\mathrm{p}_2(p_1,s) = \frac{p_1+c}{2} + \frac{s}{2b_2}, \tag{A.7}$$

$$\mathrm{q}_2(p_1,s) = b_2\frac{p_1-c}{2} + \frac{s}{2}. \tag{A.8}$$

At the first stage, optimal strategies imply, for a given level of arbitrage s,

$$\mathrm{p}_1(s) = \frac{2A+2b_1c-b_2c-s}{4b_1-b_2}, \tag{A.9}$$

$$\mathrm{q}_1(s) = \frac{(2b_1-b_2)(A-b_1c)-(3b_1-b_2)s}{4b_1-b_2}, \tag{A.10}$$

$$\mathrm{p}_2(s) = \frac{A+3b_1c-b_2c+\frac{2b_1-b_2}{b_2}s}{4b_1-b_2}, \tag{A.11}$$

$$\mathrm{q}_2(s) = \frac{Ab_2-b_1b_2c+(2b_1-b_2)s}{4b_1-b_2}. \tag{A.12}$$

The arbitrage level is given by the non-arbitrage condition $\mathrm{p}_2(p_1,s) = p_1$. Setting p_2 equal to p_1 in equation (A.7), we obtain

$$\mathrm{s}(p_1) = (p_1-c)b_2. \tag{A.13}$$

Using this equilibrium condition in expressions (A.9)-(A.12), we obtain

$$p_1^{**} = \frac{A+b_1c}{2b_1}, \tag{A.14}$$

$$q_1^{**} = (b_1-b_2)\frac{A-b_1c}{2b_1}, \tag{A.15}$$

$$p_2^{**} = \frac{A+b_1c}{2b_1}, \tag{A.16}$$

$$q_2^{**} = b_2\frac{A-b_1c}{2b_1}, \tag{A.17}$$

$$s^{**} = \frac{b_2(A-b_1c)}{2b_1}. \tag{A.18}$$

Link to Result 2. From $\mathrm{q}_1(s)$ and $\mathrm{q}_2(s)$ it is clear that quantities in the first market are decreasing in s and quantities in the second market are increasing in s. Comparing p_1^{**} to p_1^* and p_2^*, one can check that p_1^{**} is smaller than p_1^* as long as $A - b_1c > 0$, whereas p_2^* is lower than $p_1^{**} = p_2^{**}$. In particular, $p_1^{**} - p_1^* = -b_2\frac{A-b_1c}{8b_1^2-2b_1b_2} < 0$, and $p_2^{**} - p_2^* = \frac{(2b_1-b_2)(A-b_1c)}{2b_1(4b_1-b_2)} > 0$. The monopolist reacts to the arbitrage by lowering total quantity, and $q_1^{**}+q_2^{**} > q_1^*+q_2^*$. In particular, $(q_1^{**}+q_2^{**})-(q_1^*+q_2^*) = -b_2\frac{A-b_1c}{8b_1-2b_2} > 0$, which completes the results.

A.3 Equilibrium with Limited Arbitrage

Now we include the restriction that $s \leq K$, i.e., there are some institutional constraints that limit the amount of arbitrage. As explained in the main text, the justification for such restrictions can be physical (power plants cannot arbitrage more than their total capacity), or regulatory. Taking the equilibrium value of s^{**} in the case with unlimited arbitrage, this implies that the constraint will be binding as long as,

$$K < \frac{(A - b_1 c)b_2}{2b_1}, \tag{A.19}$$

in which case $s = K$. Otherwise, the equilibrium features full arbitrage and $s = s^{**}$. Whenever the constraint is binding, the equilibrium becomes,

$$\overline{p}_1^{**} = \frac{2 + bc - K}{3b}, \tag{A.20}$$

$$\overline{q}_1^{**} = A - K, \tag{A.21}$$

$$\overline{p}_2^{**} = \frac{b_2(3b_1 - b_2)c + (2b_1 - b_2)K - A(b_1 - b_2) - b_1^2 c}{b_2(3b_1 - b_2)}, \tag{A.22}$$

$$\overline{q}_2^{**} = \frac{(2b_1 - b_2)K - A(b_1 - b_2) - b_1^2 c}{3b_1 - b_2}, \tag{A.23}$$

$$\overline{s}^{**} = K. \tag{A.24}$$

Link to Result 3. From the equations describing the capacity constrained equilibrium, one can see that, if K is binding, $\overline{p}_1^{**} > \overline{p}_2^{**}$, as $\overline{p}_1^{**} - \overline{p}_2^{**} = \frac{Ab_2 - b_1 b_2 c - 2b_1 K}{4b_1 b_2 - b_2^2} > 0$, whenever the constraint is binding. Trivially, the tighter the constraint K, the more often this will happen. From the constraint itself expressed in expression (A.19), we can also see that it is more likely to bind when A is larger. Taking derivatives with respect to b_1 and b_2, it is easy to check that the constraint is more likely to bind when b_1 is smaller and b_2 is larger.

A.4 Equilibrium with Strategic Arbitrage

We consider the case in which there is a single arbitrageur. Therefore, it is not optimal for the arbitrageur to close price differences, but rather to close them in an optimal way that maximizes its profits. We calculate the Cournot equilibrium between the monopolist producer (q_1, q_2) and the monopolist arbitrageur (s). The profit of the arbitrageur is given by,

$$\Pi^a = (\mathrm{p}_1(q_1, s) - \mathrm{p}_2(q_1, s))s,$$

where q_1 is taken as given and p_2 is implicitly defined by the equilibrium price in the second stage as a function of the first stage choices.

In the presence of strategic arbitrage (monopolist), the equilibrium becomes,

$$p_1^a = \frac{4Ab_1 + 2Ab_2 + 4b_1^2c + b_1b_2c - b_2^2c}{8b_1^2 + 3b_1b_2 - b_2^2}, \tag{A.25}$$

$$q_1^a = \frac{\left(4b_1^2 - b_1b_2 - b_2^2\right)(A - b_1c)}{8b_1^2 + 3b_1b_2 - b_2^2}, \tag{A.26}$$

$$p_2^a = \frac{3Ab_1 + Ab_2 + 5b_1^2c + 2b_1b_2c - b_2^2c}{8b_1^2 + 3b_1b_2 - b_2^2}, \tag{A.27}$$

$$q_2^a = \frac{b_2(3b_1 + b_2)(A - b_1c)}{8b_1^2 + 3b_1b_2 - b_2^2}, \tag{A.28}$$

$$s^a = \frac{2b_1b_2(A - b_1c)}{8b_1^2 + 3b_1b_2 - b_2^2}. \tag{A.29}$$

Link to Result 4. The price difference is given by $p_1^a - p_2^a = \frac{(b_1+b_2)(A-b_1c)}{8b_1^2+3b_1b_2-b_2^2} > 0$. Therefore, as in Result 1, the price premium is increasing in A and b_2, and decreasing in b_1. One can also see that price differences are smaller than in the case where no arbitrage is present, i.e., $p_1^a - p_2^a < p_1^* - p_2^*$.

A.5 Equilibrium with Wind Farms

Assume now that the strategic arbitrageur is producing q^w units of wind, which are exogenously given. The profit of the arbitrageur becomes,

$$\Pi^w = (\mathrm{p}_1(q_1, q^w, s) - \mathrm{p}_2(q_1, q^w, s))s + \mathrm{p}_1(q_1, q^w, s)q^w,$$

where prices are now also affected by wind production.

The wind farmer has now a smaller interest to arbitrage, as arbitraging reduces the price received by wind production. Note that this formulation still allows the arbitrageur to set $s < 0$, in which case the wind farmer would be withholding output from the first market. In equilibrium,

$$p_1^w = \frac{4Ab_1 + 2Ab_2 + 4b_1^2c + b_1b_2c - b_2^2c - 4b_1q^w}{8b_1^2 + 3b_1b_2 - b_2^2}, \tag{A.30}$$

$$q_1^w = \frac{(4b_1^2 - b_1b_2 - b_2^2)(A - b_1c) + 2(2b_1^2 + 5b_1b_2 - b_2^2)q^w}{8b_1^2 + 3b_1b_2 - b_2^2}, \tag{A.31}$$

$$p_2^w = \frac{3Ab_1 + Ab_2 + 5b_1^2c + 2b_1b_2c - b_2^2c - 7b_1q^w + b_2q^w}{8b_1^2 + 3b_1b_2 - b_2^2}, \tag{A.32}$$

$$q_2^w = \frac{b_2(3b_1 + b_2)(A - b_1c) + b_2q^w(b_2 - 7b_1)}{8b_1^2 + 3b_1b_2 - b_2^2}, \tag{A.33}$$

$$s^w = \frac{2b_2(Ab_1 - b_1^2c - 5b_1q^w + b_2q^w)}{8b_1^2 + 3b_1b_2 - b_2^2}. \tag{A.34}$$

Link to Result 5. The price premium is still positive, as $p_1^w - p_2^w = \frac{(b_1+b_2)(A-b_1c)+(3b_1-b_2)q^w}{8b_1^2+3b_1b_2-b_2^2} > 0$. The price premium increases with A and q^w, and decreases with b_1. The premium increases with b_2 as long

Table A.1: Comparison Across Equilibria when $b_1 = b_2 = b$

	No Arbitrage	Full Arbitrage	Limited	Strategic	Strategic Wind
p_1	$\frac{2A+bc}{3b}$	$\frac{A+bc}{2b}$	$\frac{2A-K+bc}{3b}$	$\frac{3A+2bc}{5b}$	$\frac{3A+2bc-2q^w}{5b}$
p_2	$\frac{A+2bc}{3b}$	$\frac{A+bc}{2b}$	$\frac{A-2K+bc}{3b}$	$\frac{2A+3bc}{5b}$	$\frac{2A+3bc-3q^w}{5b}$
$p_1 - p_2$	$\frac{A-bc}{3b}$	0	$\frac{A-bc-2K}{3b}$	$\frac{A-bc}{5b}$	$\frac{A-bc+q^w}{5b}$
q_1	$\frac{1}{3}(A-bc)$	0	$\frac{1}{3}(A-bc-2K)$	$\frac{1}{5}(A-bc)$	$\frac{1}{5}(A-bc+q^w)$
q_2	$\frac{1}{3}(A-bc)$	$\frac{1}{2}(A-bc)$	$\frac{1}{3}(A-bc+K)$	$\frac{2}{5}(A-bc)$	$\frac{2}{5}(A-bc-\frac{3}{2}q^w)$
q_1+q_2	$\frac{2}{3}(A-bc)$	$\frac{1}{2}(A-bc)$	$\frac{2}{3}(A-bc-\frac{K}{2})$	$\frac{3}{5}(A-bc)$	$\frac{3}{5}(A-bc-\frac{2}{3}q^w)$
s	-	$\frac{1}{2}(A-bc)$	$\frac{1}{2}(3K-2A-bc)$	$\frac{1}{5}(A-bc)$	$\frac{1}{5}(A-bc-4q^w)$

Notes: Limited arbitrage case for the case in which the arbitrage capacity is binding, i.e., $K < \frac{1}{2}(A-bc)$.

as wind production is small enough, i.e., $q^w < \tilde{q}^w \equiv \frac{\left(5b_1^2+2b_1b_2+b_2^2\right)(A-b_1c)}{17b_1^2-6b_1b_2+b_2^2}$. Wind farm arbitrages price differences as long as it is small enough, i.e., as long as $s^w > 0$, which implies $q^w < \underline{q}^w \equiv \frac{b_1A-b_1c)}{5b_1-b_2}$. Otherwise, the farm will no longer arbitrage price differences, and behave as an oligopolistic producer instead, with an incentives to drive the premium up. The monopolist will contribute to the price premium as long as $q_2^w > 0$, which implies $q^w < \overline{q}^w \equiv \frac{(3b_1+b_2)(A-b_1c)}{7b_1-b_2}$. One can check that $\overline{q}^w - \underline{q}^w = \frac{\left(8b_1^2+3b_1b_2-b_2^2\right)(A-b_1c)}{35b_1^2-12b_1b_2+b_2^2} > 0$. One can also check that $\overline{q}^w - \tilde{q}^w = \frac{2(b_1-b_2)\left(8b_1^2+3b_1b_2-b_2^2\right)(A-b_1c)}{(7b_1-b_2)\left(17b_1^2-6b_1b_2+b_2^2\right)} > 0$, and $\tilde{q}^w - \underline{q}^w = \frac{(b_1+b_2)\left(8b_1^2+3b_1b_2-b_2^2\right)(A-b_1c)}{(5b_1-b_2)\left(17b_1^2-6b_1b_2+b_2^2\right)} > 0$.

A.6 Comparison for special case, $b_1 = b_2$

To gain some intuition on the comparative statics between regimes, it is useful to consider the simplified expressions for the case in which $b_1 = b_2 = b$. Table A.1 presents equilibrium prices and quantities for each of the cases considered. The table is useful to confirm some of the basic predictions of the model. First, one confirms that $p_1 > p_2$ for all equilibria considered, except for the case of full arbitrage, in which case $p_1 = p_2$. One can also see that, whenever positive, the premium is increasing in A, decreasing in b and increasing in q^w.

From the table, the price premium is largest in the absence of arbitrage, as long as q^w is sufficiently small. One can also see that the strategic arbitrageur reduces the price premium compared to the case of no arbitrage, but also that a strategic arbitrageur with wind production will have a lesser incentive to arbitrage. In this simplified example, the wind arbitrageur will have an incentive to arbitrage as long as $q^w < \frac{1}{4}(A-bc)$, i.e., as long as the wind farm is sufficiently small. As a point of comparison, the monopolist total production is $\frac{2}{3}(A-bc)$ in the case of no arbitrage and $\frac{1}{2}(A-bc)$ in the case of full arbitrage.

B Computational details

B.1 Last stage: Capacity-constrained Cournot

We use a mixed integer solver to find the solution to the capacity-constrained Cournot equilibrium. The first order conditions can be expressed as a complementarity problem (Bushnell et al., 2008). We use an equivalent mixed-integer representation, and represent the first-order conditions as a set of constraints.

Assume market demand is $Q = A - bp$ in the day-ahead market. We observe Q, b and p in the data, and back out A to infer the intercept.[1] As in Bushnell et al. (2008), we model the marginal cost curve in piece-wise linear segments. For a given firm $i = 1, \ldots, N$, segment $j = 1, \ldots, J$, and quantity q $c_{ij}(q) = \alpha_{ij} + \beta_{ij} q$. Each segment has a maximum capacity $\overline{q}_{ij}$. Marginal costs are constructed so that the cost curve is continuous across segments, i.e. $\alpha_{ij} + \beta_{ij}\overline{q}_{ij} = \alpha_{ij+1}$. The model can also accommodate non-continuous, weakly increasing steps.

Define $\underline{\mathbf{u}}$ and $\overline{\mathbf{u}}$ a vector of dummies of length $N \times J$ that specifies whether a given step in the marginal cost curve is used at all ($q_{ij} > 0$), and whether it is used at full capacity ($q_{ij} = \overline{q}_{ij}$), respectively. Define $\psi_{ij} \geq 0$ as the shadow value when $\overline{u}_{ij}$ is binding. The equilibrium solves for the optimal vectors $\underline{\mathbf{u}}, \overline{\mathbf{u}}, \psi$, and $\mathbf{q}$. In addition to the range conditions, the equilibrium conditions using a mixed integer formulation are as follows:

$$\begin{aligned}
&\text{[FOC 1]} && P - \sum_J q_{ij}/b - \alpha_{ij} - \beta_{ij} q_{ij} - \psi_{ij} \leq 0 && \forall i, j, && \text{(B.1)}\\
&\text{[FOC 2]} && P - \sum_J q_{ij}/b - \alpha_{ij} - \beta_{ij} q_{ij} - \psi_{ij} \geq M \underline{u}_{ij} - M && \forall i, j, && \text{(B.2)}\\
&\text{[Complementarity]} && \psi_{ij} - M \overline{u}_{ij} \leq 0 && \forall i, j, && \text{(B.3)}\\
&\text{[Definition } \underline{\mathbf{u}}\text{]} && q_{ij} - \overline{q}_i \underline{u}_{ij} \leq 0 && \forall i, j, && \text{(B.4)}\\
&\text{[Definition } \overline{\mathbf{u}}\text{]} && \overline{q}_i \overline{u}_{ij} - q_{ij} \leq 0 && \forall i, j, && \text{(B.5)}\\
&\text{[Sorting 1]} && \overline{u}_{ij} - \underline{u}_{ij} \leq 0 && \forall i, j, && \text{(B.6)}\\
&\text{[Sorting 2]} && \underline{u}_{ij} - \underline{u}_{ij-1} \leq 0 && \forall i, j = 2 \ldots J, && \text{(B.7)}\\
&\text{[Sorting 3]} && \overline{u}_{ij} - \overline{u}_{ij-1} \leq 0 && \forall i, j = 2 \ldots J, && \text{(B.8)}
\end{aligned}$$

where P is implicitly defined as $P \equiv A/b - \sum_{N,J} q_{ij}/b$, and M is a large value, e.g., $M = 10^6$.

The first condition establishes that marginal revenue is below or equal marginal cost. The second condition establishes that the marginal revenue equals marginal cost whenever a given step is used to produce. The third condition (Complementarity) establishes that the shadow value will only be positive if the step is binding, as it is the shadow value for capacity. This ensures that if a step is used to produce at an interior range, the FOC will be satisfied with equality and the shadow value will be equal to zero. The rest of the equations are used to define the auxiliary integer variables $\underline{\mathbf{u}}$ and $\overline{\mathbf{u}}$, as well as to establish the merit order in the supply curve.

[1] The intercept is not directly interpretable. It is a way to ensure that our local approximation to demand is in the right range. Alternatively, the model can be adapted to have a full representation of the demand curve using a piece-wise linear approximation.

We use a mixed-integer solver (CPLEX) to find a solution to the first-order conditions.

Link to the dynamic model The equations here are defined broadly for a capacity-constrained equilibrium. However, in our setting, the capacity-constrained equilibrium is the second stage of a dynamic game. Two key variables play a role: $\mathbf{Q}_1$ and s. $\mathbf{Q}_1$ represents the vector of committed quantities by each firm in the first stage. s determines the amount of arbitrage in the first stage. All these variables are pre-determined at this stage. $\mathbf{Q}_1$ affects the first order conditions as follows:

$$\text{[FOC 1 Dynamic]} \qquad P - \sum_J q_{ij}/b + Q_{i1}/b - \alpha_{ij} - \beta_{ij} q_{ij} - \psi_{ij} \leq 0 \qquad \forall i, j, \tag{B.9}$$

$$\text{[FOC 2 Dynamic]} \qquad P - \sum_J q_{ij}/b + Q_{i1}/b - \alpha_{ij} - \beta_{ij} q_{ij} - \psi_{ij} \geq M \underline{u}_{ij} - M \qquad \forall i, j, \tag{B.10}$$

i.e., it reduces the incentives of the firm to put markups, for $Q_{i1} > 0$..

The amount of arbitrage affects the equilibrium price, which is now defined as $P \equiv (A + s)/b - \sum_{N,J} q_{ij}/b$, as the arbitrageurs buy back their commitments in the second stage, increasing the effective demand. In the simulations, we also allow for exogenous cost shocks to demand, so that $P \equiv (A + s + \epsilon)/b - \sum_{N,J} q_{ij}/b$.

Finally, it is important to clarify how we accommodate for a different b in the second market. We calibrate the residual demand in the second market to go through the same point as the residual demand at the equilibrium price from the first market, absent any arbitrage. Therefore, we set A_2 such that $A_2 - b_2 p_1 = Q_1$. As explained above, A_2 is not directly interpretable, but it provides a convenient computational formulation to model local changes around the residual demand curve.

From the equilibrium price and quantities, we can compute the profit of each firm,

$$\Pi_{i2} = P \cdot \left(\sum_J q_{ij} - Q_{i1} \right) - \sum_J \left(\alpha_{ij} + \beta_{ij} \frac{q_{ij}}{2} \right) q_{ij}$$

Impact of counterfactuals on last stage The main effect of the different counterfactuals is on the amount of arbitrage s. In the no arbitrage case, $s = 0$. In the wind arbitrage (baseline case), $s^w = 0.20 q^w$. In the strategic arbitrage case, $s = s^m$, where s is given by the solution in the first stage where the arbitrageur mazimizes profits. Finally, the full arbitrage case sets $s = s^{**}$, such that $p_1 = E[p_2]$, and is also determined in the first stage. Importantly, for the purposes of the last stage, s is sunk and given by the first stage.

B.2 First stage: Gauss-Seidel iteration

The pseudo-code in Algorithm 1 describes the iteration procedure, which is a standard Gauss-Seidel procedure that iteratively calculates the best response of each firm until no firm finds a profitable deviation. To define the profit of the firm when computing a best-response, we consider the case in which there is uncertainty being realized between the forward and the real-time market. Therefore, it is an expected profit over several realizations of uncertainty.

Algorithm 1 First stage iteration

```
procedure COURNOTDYNAMIC
    guess ← zeros(N,1)
    crit ← 1000.0
    iter ← 1
    while iter < maxiter & crit > tol do
        oldguess ← guess
        for n = 1 : N do
            guess(i) ← argmax_{q_i} Σ_ε Π_i(q_i, guess_{-i}, s, ε)
        end for
        crit ← || guess − oldguess ||
        iter ← iter + 1
    end while
end procedure
```

Define a firm's profit as,

$$\Pi_i(q_i, q_{-i}, s, \epsilon) = p_1(q_i, q_{-i}, s) + \Pi^*_{2i}(q_i, q_{-i}, s, \epsilon),$$

where $\Pi^*_{2i}(q_i, q_{-i}, s)$ is the equilibrium profit in the second stage when q_i, q_{-i}, and s are played in the first stage. The differences across counterfactuals come from the amount of arbitrage. As explained above, $s = 0$ in the case of no arbitrage, and $s = 0.20q^w$ for the case of wind arbitrage. The strategic arbitrage case and the full arbitrage case need to solve endogenously for the amount of arbitrage. In those cases, the algorithm is expanded to also compute the best response for the arbitrageur (who maximizes profits in the strategic case, and equalizes prices in the full arbitrage case). This is implemented adding a fifth firm to the iteration procedure, who is either maximizing arbitrage profits or equalizing prices, taking the actions of the other players as given. The vector *guess* in the algorithm is modified to be of size $N + 1$. The algorithm stops when both firm quantities and arbitrage have converged.[2]

C Estimation details

In this section, we detail how we estimate the different parameters that are part of the model.

C.1 Dominant Firms

Unit-level characteristics, such as generation capacity, type of fuel, thermal rates, age, and location are publicly available for power plant units in the Iberian electricity market.[3] In addition to the unit-level characteristics, we collect daily fuel cost data such as the price of natural gas, oil, and coal from Bloomberg.[4]

[2]We have examined the properties of the algorithm, and we have found that the algorithm converges smoothly in few iterations (typically less than 10). We have also examined the possibility of multiple equilibria both at the second stage and the first stage using some new tools that we are concurrently developing (Reguant, 2015), and we have not found evidence of multiple equilibria.

[3]Thermal parameters for the Spanish power are obtained from the Ministry of Industry. Parameters for newly constructed combined-cycle plants are based on industry standards.

[4]For coal units, we use the MCIS Index, for fuel units we use the F.O.1% CIF NWE prices, and for gas units we use EEX prices.

For each unit, we calculate the marginal cost of production using the unit-level characteristics, daily fuel costs, and 3) parameters from an engineering model that provides the relationship between the marginal cost, plant characteristics, and fuel costs. Finally, we obtain CO2 emissions prices and emissions rates at the unit level.[5] We add emissions costs to the unit level marginal costs, following Fabra and Reguant (2014). This calculation provides daily unit-level marginal costs. We use this information to obtain daily firm-level marginal cost curves for the dominant firms—Iberdrola, Endesa, EDP, and Gas Natural.

This marginal cost curve can deviate from the actual cost curve relevant for our structural estimation for a few reasons. First, marginal costs are likely to be observed with error due to a variety of factors (measurement error in the thermal rates, the presence of transportation or other transaction costs due to long term contracts, etc.).[6] Therefore, our measures of marginal cost should be interpreted as an estimate that reflects average marginal costs for these technologies, but not necessarily an exact precise measurement of marginal cost for a given unit and/or date. To the extent that marginal costs are not systematically off, this enables us to capture firm-behavior reasonably in our simulations. Second, firms may not use some plants because of maintenance and other reasons. We identify units that are not available at a given date, and exclude these plants from that daily marginal cost curve. Together with these adjustments, we can obtain a daily firm-level marginal cost curve that characterizes the firm's actual marginal cost curve given the assumptions in our calculation.

To finalize the data construction for dominant firms, we also gather data on bilateral contracts. We observe all bilateral contracts tied to units of operation. Firms are required to report their bilateral contracts to the market operator. In the auction, they act as zero-price bids that shift the supply (or price-cap bids that shift the demand). By construction, supply and demand bilateral contracts cancel each other. Bilateral contracts are not given priority in the congestion market, and therefore cannot be used strategically to ensure access to the network. Absent congestion, firms typically produce the amount of power specified by their bilateral contracts. In the simulations, we treat bilateral contracts as a financial position, but we obtain similar results if we impose that firms always produce at least enough electricity to cover their bilateral contracts. In our simulations, we find that the constraint is binding less than 1% of the times.

In the simulations, we simulate firms' choices for their portfolio of nuclear and thermal plants. We take into account the bilateral contracts associated with these plants only. We have also experimented with alternative definitions of forward positions. In particular, we considered including observed production minus bilateral contracts from other units owned by those plants. We found similar qualitative results using this alternative bilateral definition, although it appeared to overstate the degree of market power, leading to larger welfare losses from market power and arbitrage. We believe there are two why such definition of bilateral contracts might overstate market power. First, we do not have financial contracts which can become more of an issue as we add more and more production, and second, such infra marginal quantity assumes that output from other technologies is sunk, but in practice firm choose it endogenously at the market. In any case, qualitative results are similar, although the estimates of market power and inefficiencies from arbitrage are larger, and the predicted price premia is larger than in the actual data due to the increased market power.

[5]We obtained EU-ETS permit prices from http://www.eea.europa.eu/data-and-maps/figures/eua-future-prices-200520132011: Last accessed on September 7, 2015.

[6]See Fabra and Reguant (2014) for evidence on measurement error for these cost estimates.

C.2 Residual Demand

Our bidding data provide unit-level demand and supply bids for each market and for each hour of production. We aggregate unit-level bids to obtain firm-level bids. We then identify fringe firms' supply bids that are accepted in the market. To obtain residual demand, we subtract fringe firms' accepted supply bids from aggregate demand. Note that we do not need to estimate residual demand in this process because we observe firm-level demand and supply bids. The obtained residual demand is usually a downward step function because bidding prices are discontinuous. In addition, the residual demand curve is often nonlinear, which makes the slope of residual demand vary by the price. We use two methods to calculate the slope of residual demand around the market clearing price. The first approach is to fit a quadratic function to the residual demand curve and obtain a local slope at the market clearing price. The second approach is to fit linear splines with knots at 0, 10, 20, 30, 40, 50, 60, 70, 90 Euro/MWh to the residual demand curve.

We use the first approach for our main regression results and simulations. In terms of regression results, the two approaches produce very similar results, as the local slopes that we obtain using the two methods are highly correlated. In terms of simulation results, we find the quadratic approximation to produce simulation results that better fit the actual data. The reason is that the linear spline approximation does not perform as well is that the slopes, even at 10 Euro intervals, can be quite local in nature and often rely only on a limited number of bids to be estimated. Because we use a linear approximation around the equilibrium price, we find that such slopes can produce quite volatile prices and poor fit.[7] The quadratic fit, on the contrary, provides a smoother fit that can be extrapolated with a linear approximation in a more stable manner.

To approximate the uncertainty in the demand intercept (A), we obtain data from total scheduled demand and renewable power from the market operator from 2007 to 2012. We prefer to use data from the market operator, because the goal is to capture the uncertainty faced by the firms between the day-ahead and the real-time market. To estimate the residual uncertainty faced by the firms, we estimate a predictive model of changes in demand and renewable power between the day-ahead and the intra-day market, as some of the shifts in demand are endogenous to the model, e.g., renewable power has predictable changes in their offered quantity as shown in this paper. The dependent variable is the change in demand between the day-ahead and intra-day market, net of renewable power changes. In the model, we include controls for the hour interacted with day of the week, hour interacted with month, day of the week interacted with month, hour interacted with publicly available demand and wind forecasts, and day of the week interacted with demand and wind forecasts. In some specifications, we also include lagged changes in the scheduled net demand from the same hour on previous days.

The residual uncertainty that results from the above procedure, appears to be normally distributed, but suffers from serious outliers due to the nature of demand and renewable forecasting.[8] By removing the top and bottom 1% of predictions, we find that the normal approximation used in the simulations fits very well the data.[9] The fact that the residual error is normal looking is reassuring, as it should resemble white

[7]For example, if the local slope at low prices happens to be very inelastic, the model might predict extremely high or extremely low negative prices. However, in reality the inverse residual demand tends to zero as quantity grows, as the price is capped. The quadratic fit will capture such flattening of the inverse residual demand curve more parsimoniously.

[8]Whereas the histogram appears to be bell-shaped, severe outliers bring the skewness to 26 and the kurtosis to 950.

[9]After removing outliers, we find low levels of skewness, and kurtosis that are only slightly above 3.

noise if it is unpredictable to the firms. We find that such procedure leads to residual uncertainty with a standard deviation between 300 and 500 MWh, depending on the specification and hour considered. We set the standard deviation in the simulations to 350 MWh, but our results are similar for alternative values of the standard deviation.[10]

D Additional Figures and Tables

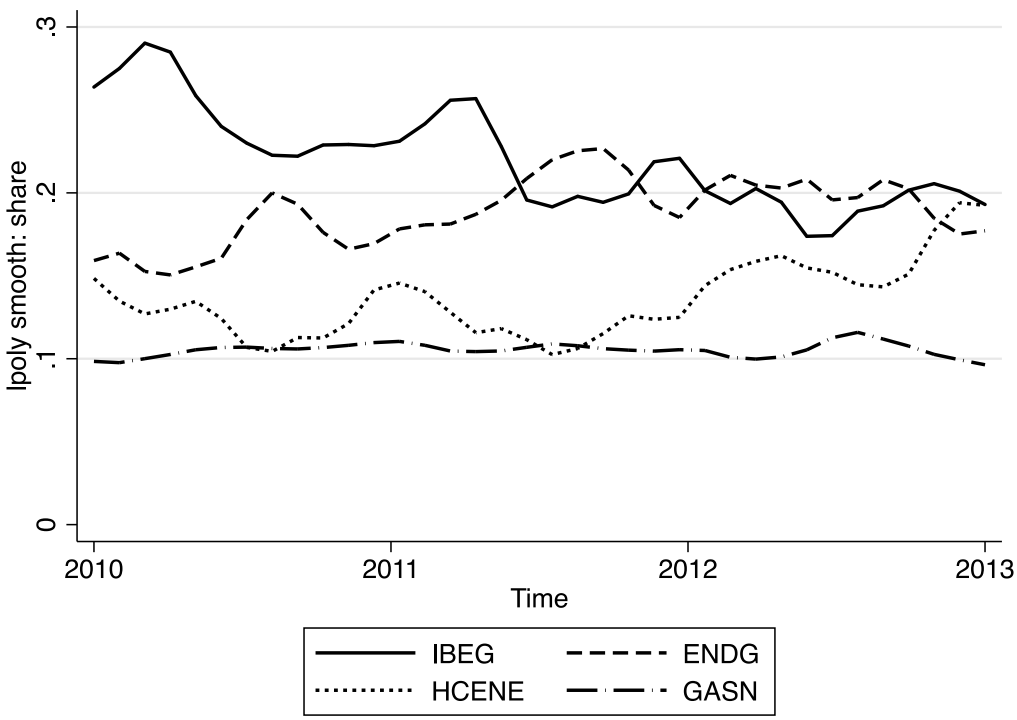


Figure D.1: Market Share of the Four Biggest Producers Over Time

Note: This figure shows the evolution of market share by the four biggest producers. As one can see, there are some fluctuations over time, which are driven by seasonality in electricity demand and hydro power, as well as changes in input costs, given that each firm has a different composition of power plants.

[10] Alternative standard deviations will affect the dispersion of prices and premiums, but we have not found the simulations to be extremely sensitive to such dispersion. The main quantitative and qualitative results are similar to those reported in the paper.

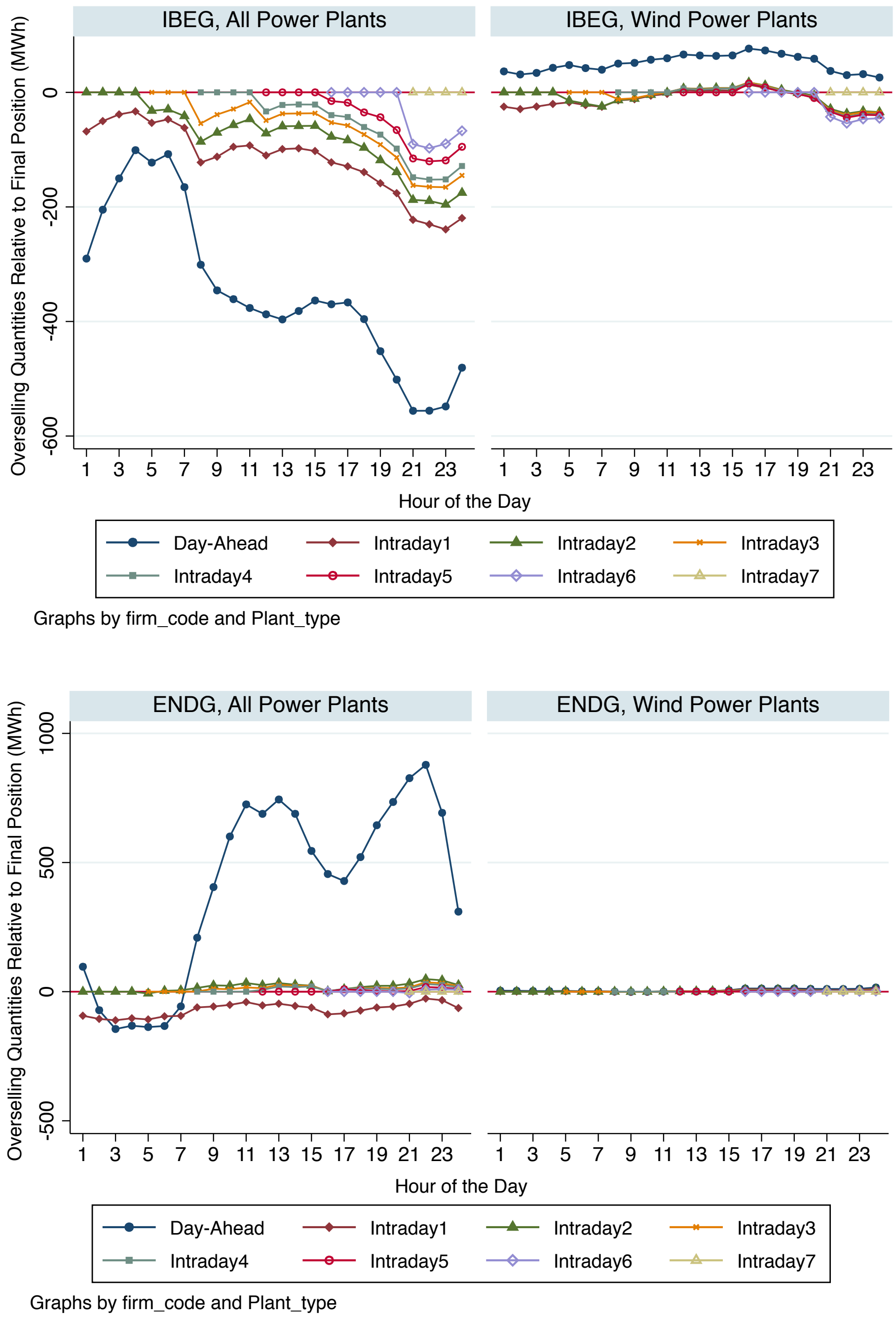
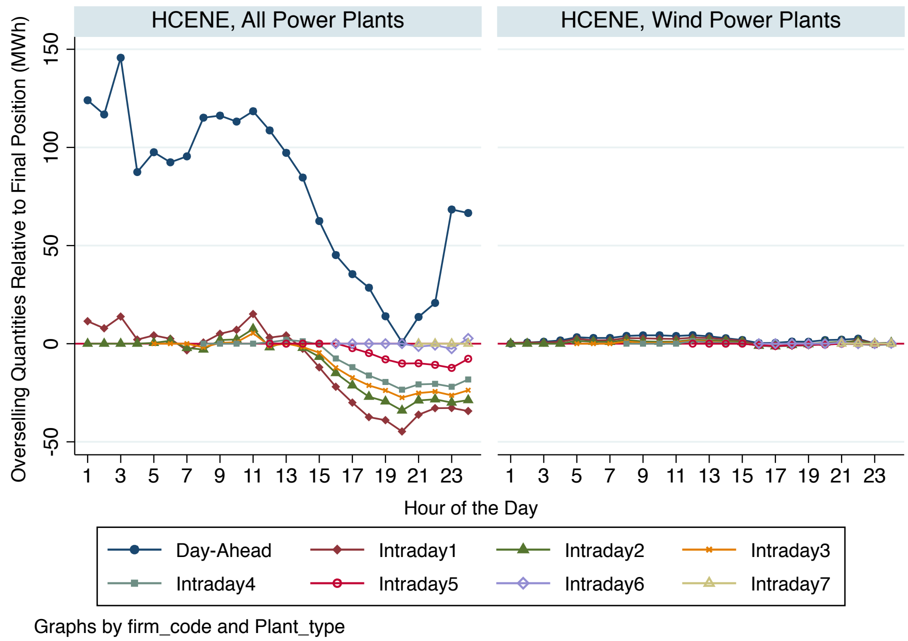


Figure D.2: Overselling and Underselling Relative to Final Positions (in MWh) by Each Dominant Firm

Note: This figure shows average changes in a firm position between a given market and a firm's final commitment. Positive values imply that a firm is promising more production than it actually delivers after all markets close.



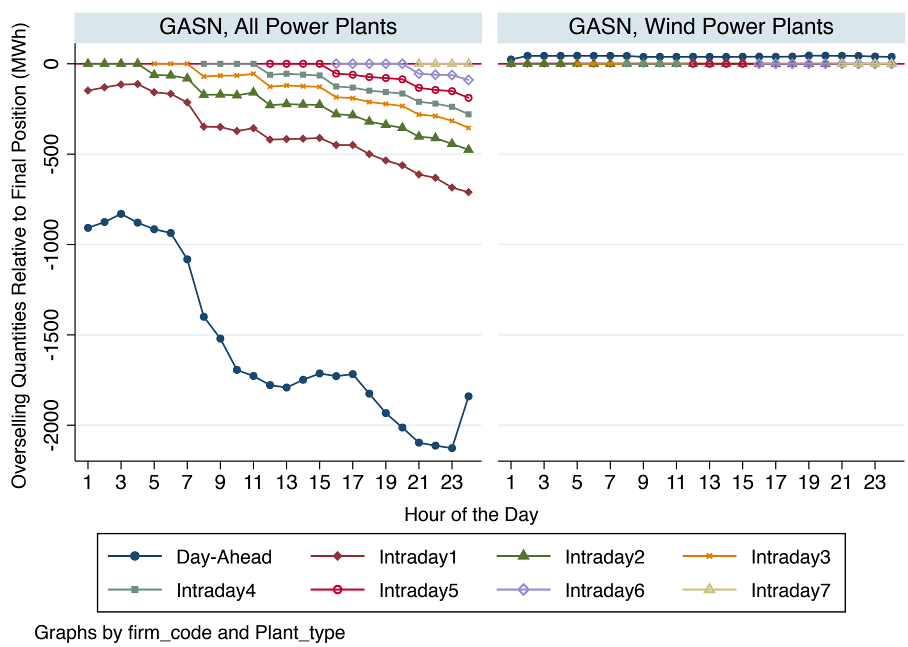
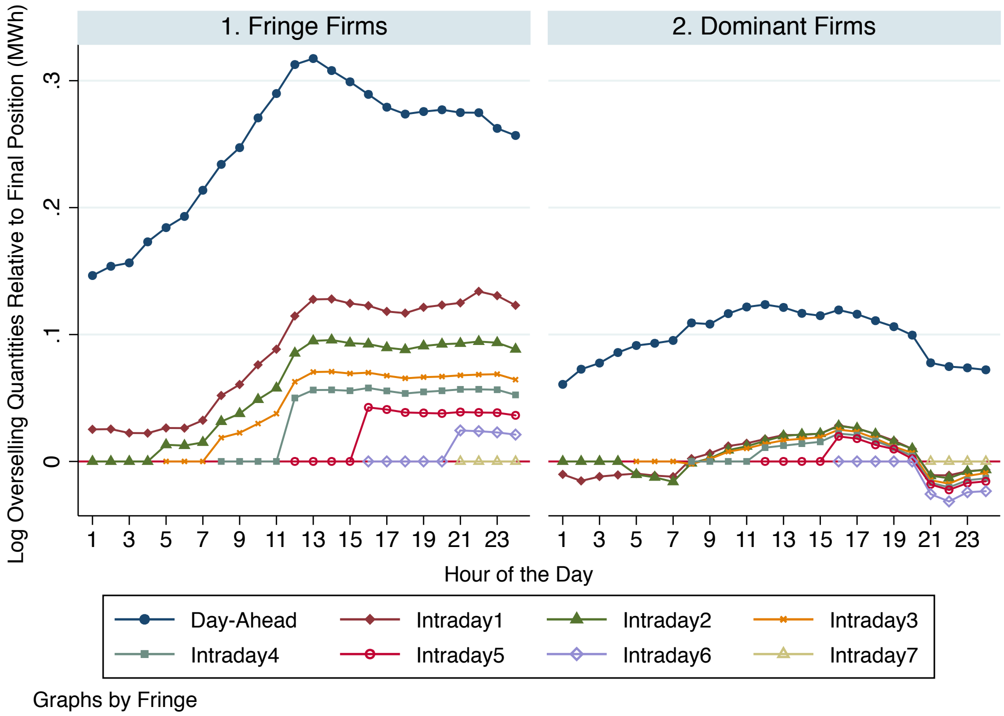


Figure D.3: Overselling and Underselling Relative to Final Positions (in MWh) by Each Dominant Firm

Note: This figure shows average changes in a firm position between a given market and a firm's final commitment. Positive values imply that a firm is promising more production than it actually delivers after all markets close.

Panel A: Wind Farms



Panel B: All Power Plants

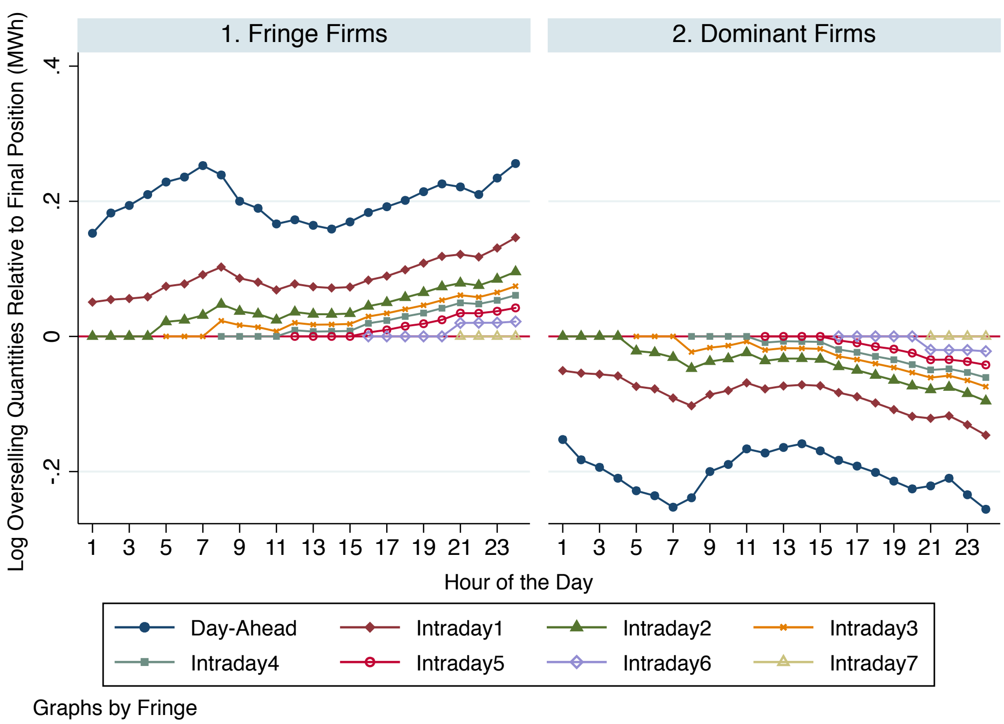


Figure D.4: Systematic Overselling and Underselling in Forward-Markets Relative to Final Positions (in Log)

Note: This figure shows average changes in fringe and dominant positions between a given market and their final commitment. Positive values imply that a group is promising more production than it actually delivers after all markets close.

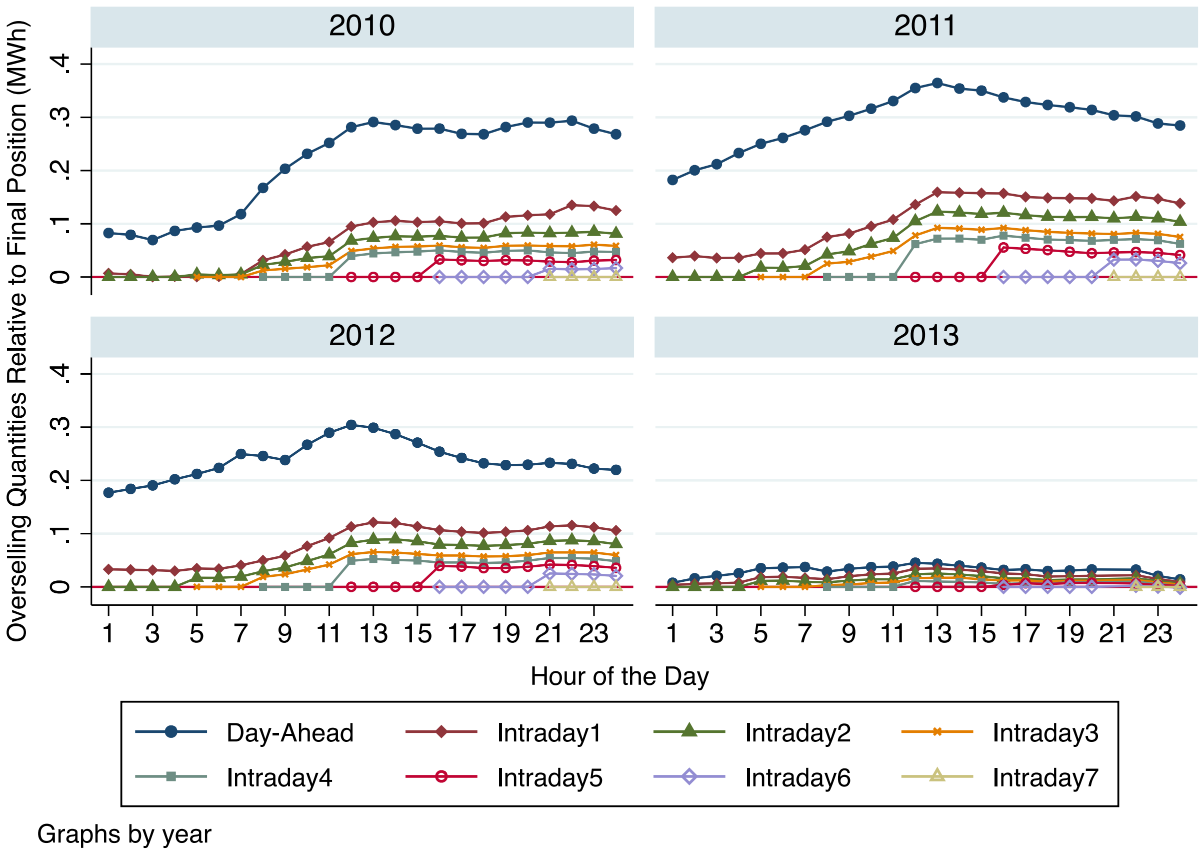


Figure D.5: By Calendar Year: Overselling in Forward-Markets by Fringe Wind Farms (in Log)

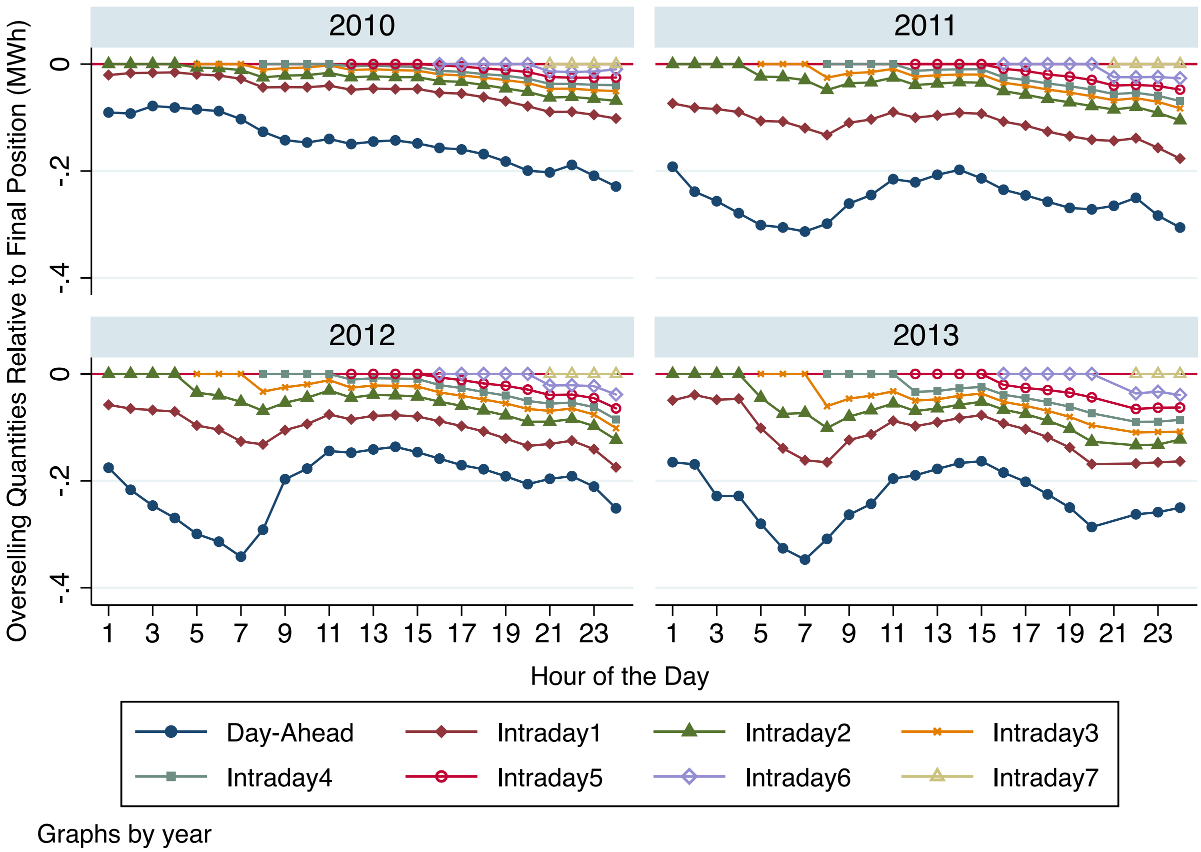


Figure D.6: By Calendar Year: Withholding in Forward-Markets by Dominant Firms (in Log)

Table D.1: Systematic Day-Ahead Price Premium

	pDA vs. pI1	pDA vs. pI2	pDA vs. pI3	pDA vs. pI4	pDA vs. pI5	pDA vs. pI6	pDA vs. pI7
Hour 1	0.00	0.00					
	[-1.01,2.21]	[-1.99,4.33]					
Hour 2	0.00	0.02					
	[-1.19,2.62]	[-2.05,4.49]					
Hour 3	0.00	0.00					
	[-1.44,2.51]	[-3.40,3.77]					
Hour 4	0.00	0.00					
	[-0.53,3.39]	[-3.00,4.65]					
Hour 5	0.10	0.00	0.00				
	[-0.11,3.23]	[-2.60,3.00]	[-3.52,4.11]				
Hour 6	0.00	0.00	0.00				
	[-0.52,3.10]	[-2.56,3.00]	[-3.64,3.51]				
Hour 7	0.50	0.00	0.00				
	[-0.01,3.18]	[-1.10,3.03]	[-2.00,3.95]				
Hour 8	0.55	0.00	0.00	0.00			
	[0.00,3.00]	[-0.90,2.92]	[-1.37,2.64]	[-2.70,4.17]			
Hour 9	0.00	0.00	0.00	0.00			
	[-0.48,2.05]	[-1.27,2.42]	[-1.30,2.56]	[-2.38,4.54]			
Hour 10	0.00	0.00	0.00	0.03			
	[-0.54,1.82]	[-1.08,2.27]	[-1.12,2.63]	[-2.00,5.37]			
Hour 11	0.00	0.00	0.00	0.80			
	[-0.76,1.73]	[-1.00,2.32]	[-1.00,2.81]	[-1.88,7.47]			
Hour 12	0.00	0.01	0.00	0.03	0.50		
	[-0.61,1.66]	[-0.76,2.28]	[-0.85,2.64]	[-1.00,3.44]	[-1.75,6.82]		
Hour 13	0.00	0.04	0.10	0.24	0.83		
	[-0.68,1.78]	[-0.82,2.48]	[-0.68,2.89]	[-0.97,3.53]	[-1.82,7.00]		
Hour 14	0.00	0.10	0.10	0.45	1.00		
	[-0.70,1.72]	[-0.62,2.34]	[-0.72,2.65]	[-0.97,3.64]	[-1.57,6.62]		
Hour 15	0.08	0.10	0.12	0.44	1.00		
	[-0.72,1.72]	[-0.59,2.29]	[-0.73,2.71]	[-0.73,3.56]	[-1.21,6.75]		
Hour 16	0.10	0.31	0.07	0.51	0.15	1.05	
	[-0.50,1.82]	[-0.46,2.37]	[-0.44,2.51]	[-0.55,3.34]	[-0.93,3.05]	[-1.70,7.29]	
Hour 17	0.47	0.50	0.29	0.90	0.33	1.12	
	[-0.15,2.00]	[-0.13,2.68]	[-0.21,2.82]	[-0.49,3.75]	[-0.60,3.41]	[-1.41,7.05]	
Hour 18	0.53	0.65	0.45	1.00	0.50	0.86	
	[-0.10,2.00]	[-0.19,2.68]	[-0.26,3.00]	[-0.34,3.96]	[-0.69,3.66]	[-1.67,6.50]	
Hour 19	0.75	1.00	0.74	1.00	0.50	0.45	
	[-0.01,2.35]	[-0.07,3.08]	[-0.09,3.11]	[-0.20,4.13]	[-0.75,3.72]	[-2.00,6.01]	
Hour 20	1.00	1.08	1.10	1.18	0.76	0.40	
	[0.00,2.90]	[0.00,3.85]	[-0.05,4.15]	[-0.20,4.86]	[-0.78,5.00]	[-2.12,6.78]	
Hour 21	1.06	1.43	1.39	1.50	1.12	0.52	0.54
	[0.00,3.00]	[0.00,4.18]	[0.00,4.70]	[-0.01,5.40]	[-0.33,5.00]	[-0.85,5.40]	[-1.83,8.13]
Hour 22	1.55	1.64	1.80	2.04	1.62	1.00	1.13
	[0.00,3.71]	[0.00,5.00]	[0.00,5.35]	[0.00,6.12]	[-0.10,5.43]	[-0.51,6.34]	[-1.62,8.64]
Hour 23	1.00	1.22	1.13	1.56	1.00	0.56	0.21
	[0.00,2.65]	[0.00,3.53]	[0.00,4.00]	[0.00,4.98]	[-0.27,4.23]	[-0.59,4.99]	[-1.57,6.71]
Hour 24	1.09	1.31	1.40	1.79	1.69	1.25	1.45
	[0.00,2.50]	[0.00,3.26]	[0.00,3.76]	[0.00,4.79]	[0.00,4.23]	[0.00,5.00]	[-0.40,7.01]

Note: This table shows the 25th, 50th, and 75th percentiles of the day-ahead price premium for each market by hours. pDA is the day-ahead price and pI1,...,pI7 are the prices in the first,...,seventh intra-day markets. We show the 25th and 75th percentiles in brackets below the 50th percentile. The distributions show that the day-ahead price tends to be larger than the prices in other markets, particularly during later hours of the day.

Table D.2: Day-ahead Price Premium, Demand Forecast, and Slope of Residual Demand

Dependent Variable: Day-Ahead Price Premium (EUR/MWh)					
	(1)	(2)	(3)	(4)	(5)
Demand Forecast (Log)	3.50	3.68	2.64	2.56	0.72
	(0.73)	(0.76)	(0.74)	(0.73)	(0.99)
Slope of Residual Demand in Day-Ahead Market (Log)		-4.19	-5.13	-7.82	-14.09
		(0.58)	(0.61)	(0.66)	(2.15)
Slope of Residual Demand in Intra-Day Market (Log)			1.86	2.35	6.27
			(0.24)	(0.25)	(1.28)
Wind Forecast (Log)				1.42	2.52
				(0.15)	(0.38)
Observations	26145	26143	26142	26142	26090
IV	No	No	No	No	Yes

Note: This table shows the estimation results of equation (7). The dependent variable is the day-ahead price premium in EUR/MWh. The standard errors are clustered at the week of sample. For the IV regression, we use average daily temperature, maximum daily temperature, minimum daily temperature, hourly temperature, dew points, and humidity interacted with the hour of the day to instrument the slopes of the residual demand for the day-ahead market and the intra-day market.

Table D.3: Heterogeneity in Arbitrage Among Fringe Wind Farms: Large v.s. Small Bidders

Panel A: Fringe wind bidders with capacity less than 300 MW

Year	Mean	25th percentile	50th percentile	75th percentile
2010	0.032	-0.002	0.009	0.047
2011	0.026	-0.002	0.010	0.055
2012	0.037	-0.002	0.046	0.072
2013	0.015	-0.006	0.004	0.027

Panel B: Fringe wind bidders with capacity more than 300 MW

Year	Mean	25th percentile	50th percentile	75th percentile
2010	0.066	0.005	0.057	0.118
2011	0.095	0.020	0.080	0.160
2012	0.073	0.008	0.059	0.129
2013	0.002	-0.041	0.002	0.048

Note: For each bidder, we calculate the log deviation between day-ahead sales and final output by: $\Delta \ln q_{jht,\text{DA}} = \ln q_{jht,\text{DA}} - \ln q_{jht,\text{FI}}$ for firm j, hour h, day t. Then, we calculate the mean, 25th, 50th, and 75th percentiles of $\Delta \ln q_{jht,\text{DA}}$ for each firm by year. There are 46 bidders that manage bids for fringe wind farms, 7 of which have total wind capacity more than 300 MW. We define these bidders as "large bidders" and other bidders as "small bidders." For the large bidders and the small bidders, we calculate the average of each statistic by year. For example, the top-left cell in Panel A shows that the average of the mean of $\Delta \ln q_{jht,\text{DA}}$ for the small bidders in 2010 is 0.032. The table shows three findings. First, both small and large bidders show systematic overselling in 2010, 2011, and 2012. Second, both types of bidders do not show systematic overselling in 2013, which is the effect of the policy change discussed in the paper. Third, the large bidders oversell more strongly than small sellers.

Table D.4: Welfare Comparison for 8am

	p_1 (E/MWh)	p_2 (E/MWh)	Premium (E/MWh)	Q_1 (GWh)	$Q_1 + Q_2$ (GWh)	Dominant Profit (000 E/h)	Δ Ineff. from FB (000 E/h)	Δ Cons. Cost from FB (000 E/h)
First best (b_1)	-	36.4	-	-	13.6	51.8	-	-
Spot only (b_1)	-	43.5	-	-	11.5	97.5	13.4	200.1
Case $b_2 = b_1$								
No arbitrage	42.3	37.5	4.9	11.8	13.2	97.0	1.1	166.9
Str. arbitrage	41.9	38.0	3.9	10.9	13.1	94.8	1.4	153.7
Wind 20%	42.0	37.8	4.2	11.0	13.1	92.6	1.2	156.8
Full Arbitrage	40.0	40.0	0.0	7.2	12.5	81.0	3.7	102.0
Case $b_2 < b_1$								
No arbitrage	41.2	36.6	4.6	12.2	12.4	89.4	4.4	137.1
Str. arbitrage	41.1	38.0	3.1	11.8	12.4	88.8	4.7	133.3
Wind 20%	41.0	39.4	1.6	11.3	12.3	87.9	4.9	130.8
Full Arbitrage	40.8	40.8	0.0	10.9	12.3	86.5	5.4	125.4
Original Data	41.7	40.2	1.5	11.3	13.4	-	-	-

Note: Welfare comparisons at 8am during January 2010 to December 2011. Profits and costs represent average *hourly* costs. Profits are the sum of net profits across the four dominant firms. Total costs include both dominant firms production costs and fringe production costs.

Table D.5: Welfare Comparison for noon

	p_1 (E/MWh)	p_2 (E/MWh)	Premium (E/MWh)	Q_1 (GWh)	$Q_1 + Q_2$ (GWh)	Dominant Profit (000 E/h)	Δ Ineff. from FB (000 E/h)	Δ Cons. Cost from FB (000 E/h)
First best (b_1)	-	38.6	-	-	15.6	61.5	-	-
Spot only (b_1)	-	46.7	-	-	13.1	124.1	17.3	272.7
Case $b_2 = b_1$								
No arbitrage	45.4	39.9	5.5	13.4	15.2	122.9	1.3	228.2
Str. arbitrage	44.9	40.5	4.4	12.2	15.0	119.9	1.7	210.3
Wind 20%	45.1	40.2	4.9	12.7	15.1	118.2	1.4	217.8
Full Arbitrage	42.8	42.8	0.0	7.8	14.3	101.6	4.8	141.9
Case $b_2 < b_1$								
No arbitrage	44.3	39.1	5.2	13.8	14.1	113.0	5.9	190.1
Str. arbitrage	44.1	40.7	3.4	13.4	14.1	112.1	6.3	185.0
Wind 20%	44.1	41.8	2.3	13.0	14.0	111.2	6.4	183.5
Full Arbitrage	43.8	43.8	0.0	12.5	14.0	109.3	7.1	175.5
Original Data	46.2	45.7	0.5	12.4	13.9	-	-	-

Note: Welfare comparisons at noon during January 2010 to December 2011. Profits and costs represent average *hourly* costs. Profits are the sum of net profits across the four dominant firms. Total costs include both dominant firms production costs and fringe production costs.

Table D.6: Welfare Comparison for 6pm

	p_1 (E/MWh)	p_2 (E/MWh)	Premium (E/MWh)	Q_1 (GWh)	$Q_1 + Q_2$ (GWh)	Dominant Profit (000 E/h)	Δ Ineff. from FB (000 E/h)	Δ Cons. Cost from FB (000 E/h)
First best (b_1)	-	37.9	-	-	15.1	57.7	-	-
Spot only (b_1)	-	45.5	-	-	12.7	115.0	16.0	244.2
Case $b_2 = b_1$								
No arbitrage	44.2	39.0	5.2	13.1	14.8	114.1	1.1	204.3
Str. arbitrage	43.7	39.6	4.1	11.9	14.6	111.3	1.5	188.1
Wind 20%	43.9	39.4	4.5	12.2	14.6	108.8	1.4	192.9
Full Arbitrage	41.8	41.8	0.0	7.7	13.9	94.4	4.4	125.9
Case $b_2 < b_1$								
No arbitrage	43.1	38.3	4.9	13.5	13.8	105.0	5.3	169.9
Str. arbitrage	43.0	39.8	3.2	13.0	13.7	104.1	5.7	165.0
Wind 20%	42.9	41.0	1.9	12.6	13.7	102.9	5.8	162.4
Full Arbitrage	42.7	42.7	0.0	12.1	13.6	101.2	6.5	155.2
Original Data	44.5	43.5	1.0	12.2	13.8	-	-	-

Note: Welfare comparisons at 6pm during January 2010 to December 2011. Profits and costs represent average *hourly* costs. Profits are the sum of net profits across the four dominant firms. Total costs include both dominant firms production costs and fringe production costs.

Table D.7: Welfare Comparison for 9pm

	p_1 (E/MWh)	p_2 (E/MWh)	Premium (E/MWh)	Q_1 (GWh)	$Q_1 + Q_2$ (GWh)	Dominant Profit (000 E/h)	Δ Ineff. from FB (000 E/h)	Δ Cons. Cost from FB (000 E/h)
First best (b_1)	-	40.0	-	-	16.9	70.8	-	-
Spot only (b_1)	-	50.1	-	-	14.0	156.3	22.1	344.1
Case $b_2 = b_1$								
No arbitrage	48.5	41.6	6.9	14.5	16.4	153.9	1.6	287.3
Str. arbitrage	47.8	42.4	5.4	13.1	16.2	149.8	2.1	264.7
Wind 20%	48.0	42.1	5.9	13.6	16.3	146.1	1.9	272.9
Full Arbitrage	45.3	45.4	0.0	8.2	15.4	125.7	6.3	181.7
Case $b_2 < b_1$								
No arbitrage	47.3	40.8	6.4	14.9	15.2	142.0	7.6	246.8
Str. arbitrage	47.0	42.8	4.2	14.3	15.2	140.6	8.1	239.2
Wind 20%	46.9	44.1	2.9	13.9	15.1	138.2	8.4	236.2
Full Arbitrage	46.6	46.6	0.0	13.2	15.0	136.2	9.3	224.8
Original Data	51.8	49.7	2.1	12.6	14.0	-	-	-

Note: Welfare comparisons at 9pm during January 2010 to December 2011. Profits and costs represent average *hourly* costs. Profits are the sum of net profits across the four dominant firms. Total costs include both dominant firms production costs and fringe production costs.