Web Appendix
Product versus Process: Innovation Strategies of Multi-Product Firms

By Lisandra Flach and Michael Irlacher*

In this Web Appendix, we provide a model extension where we introduce an exchange rate into our main framework. We show that an exchange rate devaluation provides incentives for exporting firms to increase investments into product and process innovation. Since a devaluation of the home country's currency reduces consumer prices abroad, the devaluation acts like an increase in the foreign market size and allows exporting firms to exploit economies of scale in both innovation activities.

I. Model Extension

In this Web Appendix, we maintain the main structure of our baseline model, however we introduce an exchange rate to show that the effect of a devaluation qualitatively works as an increase in the market size. We consider a domestic multi-product firm (MPF) in country H exporting to a foreign country F. The nominal exchange rate ε between H and F is expressed in country H's currency per country F's currency. Therefore, a devaluation of H's currency implies an increase in ε. Given this definition, the consumer price of variety i in country F's currency is $p_F\left(i\right) = \frac{p(i)}{\varepsilon}$.

Consumer Demand. — Consumers in country F have the quadratic preferences defined in the main text, whereas demand for product i is given by

$$y(i) = \frac{a - \frac{p(i)}{\varepsilon} - b'eY}{b'(1-e)}. \tag{1}$$

Profit Maximization. — The exporting MPF simultaneously chooses optimal scale $y(i)$ and process innovation $k(i)$ per product as well as optimal product scope δ. A firm located in H receives a price $p\left(i\right) = \varepsilon p_F\left(i\right)$ in H's currency. Profits are given

* Flach: University of Munich, CESifo and CEPR, Ludwigstrasse 28, 80539 Munich, Germany, (e-mail: lisandra.flach@econ.lmu.de). Irlacher: University of Munich, Ludwigstrasse 28, 80539 Munich, Germany, (e-mail:michael.irlacher@econ.lmu.de).

by:

(2)
$$\pi = \int_0^\delta \left[\varepsilon p_F(i) - c - c_1 i + 2\left(1-\theta(e)\right)k(i)^{0.5} + 2\theta(e)K\right] y(i)di - \int_0^\delta r_k k(i)\,di - \delta r_\delta.$$

Optimal Scale:. — Maximizing profits in Eq. (2) with respect to scale $y(i)$ considering demand in (1) leads to optimal scale for variety i:

$$y(i) = \frac{a - \frac{\left(c + c_1 i - 2(1-\theta(e))k(i)^{0.5} - 2\theta(e)K\right)}{\varepsilon} - 2b'eY}{2b'(1-e)}. \tag{3}$$

Integrating over the latter expression gives total firm scale:

$$Y = \frac{\delta a - \frac{1}{\varepsilon}\left(\delta c + c_1\frac{\delta^2}{2} - 2\left(1-\theta(e)+\theta(e)\delta\right)K\right)}{2b'(1-e+e\delta)}. \tag{4}$$

Optimal Process Innovation:. — Maximizing profits in Eq. (2) with respect to optimal investments in process innovation leads to

$$k(i) = \left(\frac{\left(1-\theta(e)\right)y(i) + \theta(e)Y}{r_k}\right)^2, \tag{5}$$

whereas integrating over the latter expression gives total investments in process innovation:

$$K \equiv \int_0^\delta k(i)^{0.5}\,di = \frac{\left(1-\theta(e)+\theta(e)\delta\right)}{r_k}Y \tag{6}$$

Optimal Product Innovation:. — The first order condition for scope is given by:

$$\frac{\partial\pi}{\partial\delta} = \left(\varepsilon p_F(\delta) - c(\delta)\right)y(\delta) + \int_0^\delta \frac{\partial\left(\varepsilon p_F(i) - c(i)\right)}{\partial\delta}y(i)\,di - r_k k(\delta) - r_\delta = 0. \tag{7}$$

Using the solution strategy from the main paper, we solve for

$$y(\delta) = \sqrt{\frac{r_k k(\delta) + r_\delta - 2\theta(e)k(\delta)^{0.5}Y}{\varepsilon b'(1-e)}}. \tag{8}$$

Analogous to Condition 1 in the main paper, we assume that the following parameter restriction is fulfilled throughout our analysis

$$\varepsilon b' r_k > \frac{2\theta(e)\left((1-\theta(e))\,y(\delta)+\theta(e)\,Y\right)}{ey(\delta)}. \tag{9}$$

II. Comparative Statics

After we have established the baseline theoretical framework, we show the effects of a devaluation of H's currency on optimal firm behavior. To derive our results, we follow the solution path from the paper and express the equilibrium equations in terms of Y and δ only. Since we are only interested in changes in the parameter ε, we do not assume a specific functional form for spillovers $\theta(e)$.

Combining Eqs. (4) and (6), we derive total firm scale

$$Y = \frac{\delta\left(\varepsilon a - c - \frac{c_1\delta}{2}\right)}{2\left(\varepsilon b'(1-e+e\delta) - \frac{(1-\theta(e)+\theta(e)\delta)^2}{r_k}\right)}. \tag{10}$$

Following the mathematical steps which are described in more detail in the main paper, we derive the explicit expression for product scope:

$$\delta = \frac{\varepsilon a - c - 2\sqrt{\left(\varepsilon b'(1-e) - \frac{(1-\theta(e))^2}{r_k}\right)\left(r_\delta - \frac{\theta(e)^2 Y^2}{r_k}\right)} - 2\left(\varepsilon b' e - \left(\frac{2\theta(e)(1-\theta(e))}{r_k}\right)\right) Y}{\left(c_1 - 2\frac{\theta(e)^2 Y}{r_k}\right)}. \tag{11}$$

Totally differentiating Eqs. (10) and (11) and writing results in matrix notation gives:

$$\begin{bmatrix} 2\left(\varepsilon \frac{b}{L}(1-e+e\delta) - \frac{(1-\theta(e)+\theta(e)\delta)^2}{r_k}\right) Y & -2\left(\left(\varepsilon b'(1-e) - \frac{(1-\theta(e))^2}{r_k}\right) y(\delta) + \delta \frac{\theta(e)^2}{r_k} Y\right)\delta \\ 2\left(\varepsilon b' e - \theta(e)\frac{2(1-\theta(e))+\theta(e)\delta}{r_k} - \frac{\theta(e)^2 Y}{y(\delta) r_k}\right) Y & \left(c_1 - 2\frac{\theta(e)^2 Y}{r_k}\right)\delta \end{bmatrix}$$

$$\times \begin{bmatrix} d\ln Y \\ d\ln\delta \end{bmatrix} = \begin{bmatrix} (a\delta - 2b'(1-e+e\delta)Y) \\ (a - y(\delta)\, b'(1-e)) \end{bmatrix} \varepsilon d\ln\varepsilon. \tag{12}$$

The determinant Δ of the system is always positive. The condition stated in Eq. (9) ensures that $\Delta > 0$ (the proof works analogous to the proof in the Appendix of the main paper). The fact that $\Delta > 0$ ensures a unique and stable equilibrium.

Effect of ε Firm Scale Y:. —

$$\frac{d\ln Y}{d\ln\varepsilon}=\frac{1}{\Delta}\begin{vmatrix}\left(a\delta-2b'(1-e+e\delta)Y\right)\varepsilon & -2\left(\left(\varepsilon b'(1-e)-\frac{(1-\theta(e))^2}{r_k}\right)y\left(\delta\right)+\delta\frac{\theta(e)^2}{r_k}Y\right)\delta\\ \left(a-y\left(\delta\right)b'(1-e)\right)\varepsilon & \left(c_1-2\frac{\theta(e)^2Y}{r_k}\right)\delta\end{vmatrix}>0 \tag{13}$$

To proof the later result, we compute the matrix $\Delta_{Y\varepsilon}$ as:

$$\Delta_{Y\varepsilon}=\delta\left\{\begin{array}{c} a\delta c_1+2\left(\varepsilon b'(1-e)-\frac{(1-\theta(e))^2}{r_k}\right)y\left(\delta\right)\left(a-y\left(\delta\right)b'(1-e)\right)\\ +2b'(1-e)\frac{\theta(e)^2Y}{r_k}\left(Y-\delta y\left(\delta\right)\right)+2b'\left((1-e+2e\delta)\frac{\theta(e)^2Y^2}{r_k}-(1-e+e\delta)c_1\right)\end{array}\right\}>0, \tag{14}$$

whereas each additive component of the expression is positive.

Effect of ε on Optimal Scope δ:. —

$$\frac{d\ln\delta}{d\ln\varepsilon}=\frac{1}{\Delta}\begin{vmatrix}2\left(\varepsilon\frac{b}{L}(1-e+e\delta)-\frac{(1-\theta(e)+\theta(e)\delta)^2}{r_k}\right)Y & \left(a\delta-2b'(1-e+e\delta)Y\right)\varepsilon\\ 2\left(\varepsilon b'e-\theta\left(e\right)\frac{2(1-\theta(e))+\theta(e)\delta}{r_k}-\frac{\theta(e)^2Y}{y(\delta)r_k}\right)Y & \left(a-y\left(\delta\right)b'(1-e)\right)\varepsilon\end{vmatrix}>0 \tag{15}$$

To proof the later result, we compute the matrix $\Delta_{\delta\varepsilon}$ as:

$$\Delta_{\delta\varepsilon}=2\varepsilon Y\left\{\begin{array}{c}\frac{b'(1-e)\left(a\varepsilon(Y-\delta y(\delta))+\left(c+\frac{c_1\delta}{2}\right)\delta y(\delta)\right)}{2Y}+a\left(\frac{\theta(e)^2Y\delta}{y(\delta)r_k}+\frac{\varepsilon b'(1-e)}{2}-\frac{(1-\theta(e))^2}{r_k}\right)\\ +2b'(1-e+e\delta)Y\left(\varepsilon b'e-\theta\left(e\right)\frac{2(1-\theta(e))+\theta(e)\delta}{r_k}-\frac{\theta(e)^2Y}{y(\delta)r_k}\right)\end{array}\right\}>0, \tag{16}$$

whereas each additive component of the expression is positive.

Effect of ε on Optimal Scope δ:. — After having determined the effects of a change in the exchange rate ε on scale Y and scope δ, identifying the effect on process innovation K is straightforward. Totally differentiating Eq. (6) with respect to ε yields the following results:

$$r_kK\frac{d\ln K}{d\ln\varepsilon}=\left(1-\theta\left(e\right)+\theta\left(e\right)\delta\right)Y\frac{d\ln Y}{d\ln\varepsilon}+\theta\left(e\right)\delta Y\frac{d\ln\delta}{d\ln\varepsilon}>0 \tag{17}$$

In this Web Appendix, we have shown that our result in Proposition 1 in the main paper is consistent to the analysis of an exchange rate devaluation.

PROPOSITION 1: *A devaluation of country H's currency (higher ε) increases total scale Y and induces firms to invest more in both product δ and process innovation K, i.e. $\frac{d\ln Y}{d\ln\varepsilon}>0$, $\frac{d\ln\delta}{d\ln\varepsilon}>0$, and $\frac{d\ln K}{d\ln\varepsilon}>0$. For exporters, a devaluation means improved access to foreign markets since products become cheaper. Qualitatively a devaluation acts like an increase in the foreign market size.*