Online Appendix for
The Great Escape? A Quantitative Evaluation of the Fed's Liquidity Facilities

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A Data Appendix

A.1 Construction of the Liquidity Share

The liquidity share in the model is defined as $LS_t = \frac{B_{t+1}}{B_{t+1}+P_t q_t K_{t+1}}$ (equation (24)). The two quantities in the definition of the liquidity share are the dollar value of the amount of U.S. government liabilities B_{t+1} (by assumption, the empirical counterpart of the liquid assets in the model) and of net claims on private assets (capital) $P_t q_t K_{t+1}$, respectively.

Recall that in the model, as in the actual economy, households hold claims on the capital held by other households (the N^O_{t+1} and N^I_{t+1} terms mentioned in the discussion of the household's balance sheet). The term $P_t q_t K_{t+1}$, however, measures the net amount of these claims – that is, the value of capital in the economy. We therefore consolidate the balance sheet of households, the non-corporate and the corporate sectors to obtain the market value of aggregate capital. For households, we sum real estate (B.100 line 3), equipment and software of non-profit organizations (B.100 line 6), and consumer durables (B.100 line 7). For the non-corporate sector, we sum real estate (B.103 line 3), equipment and software (B.103 line 6) and inventories (B.103 line 9). For the corporate sector, we obtain the market value of the capital stock by summing the market value of equity (B.102 line 35) and liabilities (B.102 line 21) net of financial assets (B.102 line 6). We then subtract from the market value of capital for the private sector the government credit market instruments (B.106 line 5), TARP (B.106 line 10), and trade receivables (B.106 line 11).

Our measure of liquid assets B_{t+1} consists of all liabilities of the federal government – that is, Treasury securities (L.106 line 17) net of holdings by the monetary authority (L.106 line 12) and the budget agency (L.209 line 20) plus reserves (L.108 line 26), vault cash (L.108 line 27) and currency (L.108 line 28) net of remittances to the federal government (L.108 line 29).

Three qualifications are in order. First, no data are available for the physical capital stock of the financial sector. Second, not all of the assets in the flow of funds are evaluated at market value. Specifically, the capital stock of households (consumer durable goods) and non-corporate firms (equipment and software owned by non-profit organizations) are measured at replacement cost. Last, in our calculations we do not net out liquid and illiquid assets held by the rest of the world. Even if we do, however, the numbers are not very different, since the rest of the world, on net, holds both liquid (government liabilities) and illiquid (private sector liabilities) assets in roughly the same proportion. The liquidity share calculated excluding the foreign sector averages 10.56%, as opposed to 12.64%, over the sample period and exhibits very similar dynamics.

A.2 Liquidity Spreads

We collect daily data on a number of spreads that the literature has identified as having to do mostly with liquidity, broadly defined:

- The Refcorp/Treasury spread for various maturities, which Longstaff (2004) suggests is mostly (if not entirely) due to liquidity as Refcorp bonds are effectively guaranteed by the U.S. government, and are subject to the same taxation.[42] As in Longstaff (2004), we measure the spread by taking the differences between the constant maturity .50, 1-, 2-, 3-, 4-, 5-, 7-, 10-, and 20-year points on the Bloomberg fair value curves for Refcorp and Treasury zero-coupon bonds.[43] The Bloomberg mnemonics are 'C091[X]Y Index' and 'C079[X]Y Index', respectively, where [X] represents the maturity. We collect daily data from 4/16/1991 to 9/06/2014.[44]
- Fleckenstein et al. (2014) provide ample evidence of what they call the "TIPS-Treasury bond puzzle," that is, of differences in prices between Treasury bonds of various maturities and inflation-swapped Treasury Inflation-Protected Securities (TIPS) issues exactly replicating the cash flows of the Treasury bond of the same maturities. Specifically,

[42]Refcorp bonds differ from most other agency bonds in that their principal is fully collateralized by Treasury bonds and full payment of coupons is guaranteed by the Treasury under the provisions of the Financial Institutions Reform, Recovery, and Enforcement Act of 1989.

[43]We do not use the yield on the 30-year bond as it has a limited sample, and on the 3-month bill as our model is quarterly.

[44]Specifically, the Bloomberg description states: `C091[X]Y Index (BFV USD US REFCO Strips Yield [X]). C079[X]Y Index (BFV USD US Treasury Strips). The indices are composite yields derived from BVAL -priced bonds. Quote type: yield /mid. The index ... are the zero coupon yields derived by stripping the par coupon curve.` We use the Bloomberg default setting (`PX LAST`), indicating that the underlying security prices correspond to the mid point between the bid and ask values for the last transaction in each day.

they find that the price of a Treasury bond and an inflation-swapped TIPS issue exactly replicating the cash flows of the Treasury bond can differ by more than $20 per $100 notional – a difference that, they argue, is orders of magnitude larger than the transaction costs of executing the arbitrage strategy. We therefore collect TIPS-Treasury spreads, which we measure by taking the differences between the constant maturity 5-, 7-, 10-, 20-, and 30-year points on the Bloomberg fair value curves for TIPS and Treasury zero-coupon bonds, and adjusting the former using the inflation swap spreads for the same maturities. The Bloomberg mnemonics are 'H15X[X]YR Index', 'H15T[X]Y Index', and 'USSWIT[X] Curncy' respectively for TIPS, nominal Treasuries, and inflation swaps, where X represents the maturity. We collect daily data from 7/21/2004 to 12/31/2014.[45]

- The CDS-Bond basis spread is the difference between the yield on corporate bonds whose credit risk is hedged using a credit default swap (CDS) and a Treasury security of equivalent maturity. Bai and Collin-Dufresne (2013) find that measures of funding liquidity (i.e., the Libor-OIS, and the repo-Tbill spreads; see Garleanu and Pedersen (2011)) are the main drivers of the CDS-Bond basis. Similarly, Longstaff et al. (2005) find that the non-default component of corporate spreads (essentially, the CDS-Bond basis) is strongly related to measures of bond-specific illiquidity as well as to macroeconomic measures of bond market liquidity. We obtain indices of "Par Equivalent" CDS-Bond basis spreads from the JP Morgan database for portfolios of corporate bonds of rating AA, A, and BBB (the mnemonics are 'High Grade AA CDS-Bond Basis', 'High Grade A CDS-Bond Basis', and 'High Grade BBB CDS-Bond Basis'). We do not know the exact maturity of the underlying contracts in each index, however five-year maturity CDS contracts are the most prevalent (Choi and Shachar (2013)). We collect daily data from 9/5/2006 to 9/8/2014.

- A commonly used measure of market liquidity is the spread between the most recently

[45]Specifically, the Bloomberg description states: `H15X[X]YR Index (Federal Reserve US H.15 TII Constant Maturity [X]). Yields on Treasury inflation protected securities (TIPS) adjusted to constant maturities. Index ... are the par return on the zero coupon yields. H15T[X]Y Index (US Treasury Yield Curve Rate T Note Constant Maturity [X]). Yields on actively traded non-inflation -indexed issues adjusted to constant maturities. The index ... are the zero coupon yields derived by stripping the par coupon curve. USSWIT[X] Curncy (USD Infl Zero Coupon [X]) Inflation swap quoted as the zero coupon fixed rate leg necessary to build a par swap against a leg on zero coupon CPI appreciation on CPURNSA Index [CPI-U NSA]. Quoted from various contributors with standard defaults of LAG (3 months) and interpolation.` We use the Bloomberg default setting (`PX LAST`), indicating that the underlying security prices correspond to the mid point between the bid and ask values for the last transaction in each day.

issued and older Treasury bonds of the same maturity, called the on-the-run/off-the-run or the bond/old-bond spread. Krishnamurthy (2002) finds that the bond/old-bond spread is highly correlated with the three-month commercial paper (CP) Treasury Bills spread. We use a measure of the 10-year on-the-run/off-the-run spread constructed by the FRBNY Research department for a recent BIS report (Study Group-Committee on the Global Financial System (2014)), which is based on the difference between yields of the 10-year on-the-run Treasury and a synthetic counterpart. The FRBNY data are available from 11/4/2005 to 2/12/2014.

- Krishnamurthy and Vissing-Jorgensen (2012) argue that the Aaa-Treasury spread is primarily driven by liquidity given the low default rate on Aaa bonds. We collect daily data from `FRED`, the same sources of Krishnamurthy and Vissing-Jorgensen (2012), from 10/01/1993 to 12/31/2014 (the mnemonics are 'DAAA' and 'DGS20' for the and the 20-year Treasury, respectively).

Given that our model is quarterly we chose not to include very short term products (3-months) to our cross-section of spreads. In robustness analysis (not shown) we added the 3-month Refcorp spread and the spread between the 3-month safe CP and Treasury-Bill used in Krishnamurthy and Vissing-Jorgensen (2012) and found the time series of CY_t to be virtually the same as that shown in Figure 3.[46]

[46] We collect daily data from `FRED` from 01/02/1997 to 12/31/2014. The mnemonics are 'DCPN3M' and 'DTB3' for the safe Commercial Paper and the 3-month Treasury bill, respectively.

B Additional Model Details and Derivations

B.1 Final and Intermediate Goods Producers

Competitive final-goods producers combine intermediate goods Y_{it}, where $i \in [0,1]$ indexes intermediate-goods-producing firms, to sell a homogeneous final good Y_t according to the technology

$$Y_t = \left[\int_0^1 Y_{it}^{\frac{1}{1+\lambda_p}} \, di \right]^{1+\lambda_p}, \tag{A-1}$$

where $\lambda_p > 0$. Their demand for the generic i^{th} intermediate good is

$$Y_{it} = \left[\frac{P_{it}}{P_t} \right]^{-\frac{1+\lambda_p}{\lambda_p}} Y_t, \tag{A-2}$$

where P_{it} is the nominal price of good i. The zero profit condition for competitive final goods producers implies that the aggregate price level is

$$P_t = \left[\int_0^1 P_{it}^{-\frac{1}{\lambda_p}} \, di \right]^{-\lambda_p}. \tag{A-3}$$

The intermediate goods firm i uses K_{it} units of capital and H_{it} units of composite labor to produce output Y_{it} according to the production technology

$$Y_{it} = A_t {K_{it}}^{\gamma} {H_{it}}^{1-\gamma} - \Gamma, \tag{A-4}$$

where $\gamma \in (0,1)$ is share of capital, $\Gamma > 0$ is fixed cost of production, and A_t is an aggregate productivity shock. Intermediate-goods firms operate in monopolistic competition and set prices on a staggered basis (Calvo (1983)) taking the real wage $\frac{W_t}{P_t}$ and the rental rate of capital r_t^k as given. With probability $1 - \zeta_p$, the firm resets its price, while with the complementary probability the price remains fixed. In the event of a price change at time t, the firm chooses the price $\tilde{P}_{it}$ to maximize the present discounted value of profits ($D_{is} = P_{is}Y_{is} - w_s H_{is} - r_s^K K_{is} - \Gamma$, $s \geq t$) conditional on not changing prices in the future subject to the demand for its own good (A-2). We assume that the profit is zero in the deterministic steady state.[47]

[47] We choose the fixed cost of production so that the free entry in the long-run leads to a steady state in which exactly a unit mass of intermediate goods producer continues production. In the short-run, there is no entry nor exit so that the profit can be positive or negative.

B.2 Labor Agencies and Wage Setting

Competitive labor agencies combine j-specific labor inputs into a homogeneous composite H_t according to

$$H_t = \left[\left(\frac{1}{1-\varkappa} \right)^{\frac{\lambda_w}{1+\lambda_w}} \int_{\varkappa}^{1} H_t\left(j\right)^{\frac{1}{1+\lambda_w}} dj \right]^{1+\lambda_w}, \tag{A-5}$$

where $\lambda_w > 0$.[48] Firms hire the labor input from the labor agencies at the wage W_t, which in turn remunerate the household for the labor actually provided. The zero profit condition for labor agencies implies that

$$W_t H_t = \int_{\varkappa}^{1} W_t(j) H_t(j) dj. \tag{A-6}$$

The demand for the j^{th} labor input is

$$H_t\left(j\right) = \frac{1}{1-\varkappa} \left[\frac{W_t\left(j\right)}{W_t} \right]^{-\frac{1+\lambda_w}{\lambda_w}} H_t, \tag{A-7}$$

where $W_t\left(j\right)$ is the wage specific to type j and W_t is the aggregate wage index that comes out of the zero profit condition for labor agencies

$$W_t = \left[\frac{1}{1-\varkappa} \int_{\varkappa}^{1} W_t(j)^{-\frac{1}{\lambda_w}} dj \right]^{-\lambda_w}. \tag{A-8}$$

Labor unions representing workers of type j set wages on a staggered basis, taking as given the demand for their specific labor input (Erceg et al. (2000)). In each period, with probability $1-\zeta_w$, a union is able to reset the wage $W_t(j)$, while with the complementary probability the wage remains fixed. Workers are committed to supply whatever amount of labor is demanded at that wage. In the event of a wage change at time t, unions choose the wage $\tilde{W}_t\left(j\right)$ to minimize the present discounted value of the disutility from work conditional on not changing the wage in the future subject to (A-7).[49]

B.3 Capital-Goods Producers

Capital-goods producers are perfectly competitive. These firms transform consumption goods into investment goods. Their problem consists of choosing the amount of investment goods

[48] We add constant $(1-\varkappa)^{-1}$ to the labor composite ($A-5$) so that it is equal to the average labor used under symmetry. Because there is no entry of new types of labor, it only simplifies the notation without changing the substance.

[49] Although each household supplies many types of labor, it is difficult for unions (which represent many households) to cooperate. Thus, each union is monopolistically competitive, taking the wages of the other unions as given.

produced I_t to maximize the profits

$$D_t^I = \left\{ p_t^I - \left[1 + S\left(\frac{I_t}{I} \right) \right] \right\} I_t, \tag{A-9}$$

taking the price of investment goods p_t^I as given. The price of investment goods differs from the price of consumption goods because of the adjustment cost function, which depends on the deviations of actual investment from its steady-state value I. We assume that, when evaluated in steady state, the adjustment cost function and its first derivative are zero ($S(1) = S'(1) = 0$), while its second derivative is positive ($S''(I_t/I) > 0$) globally.

B.4 Derivation of Liquidity Constraint

The household's balance sheet (excluding human capital) is given in Table in Section 2.1 in the text. The existence of two financial frictions constrains the evolution of both equity issued and others' equity. The entrepreneur cannot issue new equity more than a fraction θ of the investment undertaken in the current period plus a fraction $\phi_t^I \in (0, 1)$ of the undepreciated capital stock previously not mortgaged ($K_t - N_t^I$). Therefore, equity issued evolves according to

$$N_{t+1}^I(j) \leq (1-\delta)N_t^I + \theta I_t(j) + (1-\delta)\phi_t^I(K_t - N_t^I). \tag{A-10}$$

Similarly, the entrepreneur cannot sell more than a fraction ϕ_t^O of holdings of the others' equity remained. Therefore, others' equity evolves according to

$$N_{t+1}^O(j) \geq (1-\delta)N_t^O - (1-\delta)\phi_t^O N_t^O. \tag{A-11}$$

The key assumption that allows us to derive a single constraint on the evolution of net equity ($N_t \equiv N_t^O + K_t - N_t^I$) is that the "resaleability" parameters are the same, that is $\phi_t^I = \phi_t^O = \phi_t$. Then two constraints (A-10) and (A-11) yield (5) in the text.

B.5 Optimality conditions

B.5.1 Household's Optimality Conditions

Because each entrepreneur must satisfy the financing constraints on equity holdings (5), bond holdings (6) and non-negativity constraint of consumption, the aggregate investment of the representative household must satisfy:

$$I_t \equiv \int_0^\chi I_t(j)dj \leq \varkappa \frac{\left[R_t^k + (1-\delta)\, q_t\phi_t\right] N_t + \frac{R_{t-1}B_t}{P_t} - \tau_t}{p_t^I - \theta q_t}. \tag{A-12}$$

As explained in the text, we separate the wage setting from the consumption, investment and portfolio decision. The household chooses C_t, I_t, N_{t+1} and B_{t+1} to maximize the utility (2) subject to the budget constraint (15) and the financing constraint of investment (A-12). Let ξ_t and η_t be the Lagrange multipliers attached to (15) and (A-12). The first order conditions for consumption, investment, equity and government bond are respectively

$$C_t^{-\sigma} = \xi_t, \tag{A-13}$$

$$\xi_t(q_t - p_t^I) = \eta_t, \tag{A-14}$$

$$q_t\xi_t = \beta\mathbb{E}_t\left\{\xi_{t+1}[R_{t+1}^k + (1-\delta)q_{t+1}] + \eta_{t+1}\frac{\varkappa[R_{t+1}^k + (1-\delta)\phi_{t+1}q_{t+1}]}{p_{t+1}^I - \theta q_{t+1}}\right\}, \tag{A-15}$$

$$\xi_t = \beta\mathbb{E}_t\left[\frac{R_t}{\pi_{t+1}}\left(\xi_{t+1} + \eta_{t+1}\frac{\varkappa}{p_{t+1}^I - \theta_{t+1}q_{t+1}}\right)\right]. \tag{A-16}$$

We focus on equilibria in which financing constraint on investment is sufficiently tight so that the equity price is bigger than its installation cost, i.e. $q_t > p_t^I$ in the neighborhood of the steady state equilibrium. This condition is also always satisfied in our simulations outside the steady state. Therefore, the Lagrange multiplier η_t on the financing constraint on investment equation (A-12) is always positive. This implies that each entrepreneur satisfy the financing constraints on equity holdings (5) bond holdings (6) with equality and his/her consumption is zero $C_t(j) = 0$ for $j \in [0, \chi)$. Also (A-12) holds with equality, or we have (14) in the text. Substituting the Lagrange multipliers from (A-13) and (A-14) into (A-15) and (A-16) gives the Euler equations for bond and equity that characterize the household portfolio decisions (16) and (17). We first define the premium of liquidity from relaxing the investment constraint as

$$\Lambda_t = \varkappa\frac{q_t - p_t^I}{p_t^I - \theta q_t}. \tag{A-17}$$

The convenience yield in our model is defined as the expected value of the premium of liquidity of the next period as

$$CY_t = \mathbb{E}_t\left(\Lambda_{t+1}\right). \tag{A-18}$$

The Euler equations (16, 17) become

$$C_t{}^{-\sigma} = \beta\mathbb{E}_t\left[C_{t+1}{}^{-\sigma}\frac{R_t}{\pi_{t+1}}(1 + \Lambda_{t+1})\right] \tag{A-19}$$

$$C_t{}^{-\sigma} = \beta\mathbb{E}_t\left\{C_{t+1}{}^{-\sigma}\frac{R_{t+1}^k + (1-\delta)q_{t+1}}{q_t}\left[1 + \Lambda_{t+1}\frac{R_{t+1}^k + \phi_{t+1}(1-\delta)q_{t+1}}{R_{t+1}^k + (1-\delta)q_{t+1}}\right]\right\}. \tag{A-20}$$

Let us L_{t+1} be the real value of liquid assets at the end of period

$$L_{t+1} \equiv \frac{B_{t+1}}{P_t}. \tag{A-21}$$

Together with the expression for dividends, aggregate investment (14) can be rewritten as

$$I_t = \varkappa \frac{\left[R_t^k + (1-\delta)\, q_t \phi_t\right] N_t + \frac{R_{t-1} L_t}{\pi_t} - \tau_t}{p_t^I - \theta q_t} \tag{A-22}$$

B.5.2 Wage Setting Decision

Competitive labor agencies chooses $H_t(j)$ to maximize their profits

$$W_t H_t - \int_\chi^1 W_t(j) H_t(j) dj$$

subject to (A-5), taking wages $W_t(j)$ as given. The first order condition determines the demand for the j^{th} labor input (A-7), where $W_t(j)$ is the wage specific to type j and W_t is the aggregate wage index that comes out of the zero profit condition for labor agencies (A-8).

Labor unions representing suppliers of type-j labor set wages on a staggered basis, taking as given the demand for their specific labor input. In each period, with probability $1-\zeta_w$, a union is able to reset the wage $W_t(j)$, while with the complementary probability the wage remains fixed. Household are committed to supply whatever labor is demanded at that wage. In the event of a wage change at time t, unions choose the wage $\tilde{W}_t(j)$ to maximize

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\zeta_w)^{s-t} \left[\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{\omega}{1+\nu} \int_\chi^1 H_s(j)^{1+\nu} dj\right]$$

subject to (15) and (A-7) with $W_{t+s}(j) = \tilde{W}_t(j), \forall s \geq 0$.

The first order condition for this problem is

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\zeta_w)^{s-t} C_s^{-\sigma} \left[\frac{\tilde{W}_t(j)}{P_s} - (1+\lambda_w) \frac{\omega H_s(j)^\nu}{C_s^{-\sigma}}\right] H_s(j) = 0.$$

All unions face an identical problem. We focus on a symmetric equilibrium in which all unions choose the same wage $\tilde{W}_t(j) = \tilde{W}_t$. Let $w_t \equiv W_t/P_t$ denote the real wage. The first order condition for optimal wage setting becomes

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\zeta_w)^{s-t} C_s^{-\sigma} \left\{\frac{\tilde{w}_t}{\pi_{t,s}} - (1+\lambda_w) \frac{\omega \left[\left(\frac{\tilde{w}_t}{\pi_{t,s} w_s}\right)^{-\frac{1+\lambda_w}{\lambda_w}} H_s\right]^\nu}{C_s^{-\sigma}}\right\} \left(\frac{\tilde{w}_t}{\pi_{t,s} w_s}\right)^{-\frac{1+\lambda_w}{\lambda_w}} H_s = 0, \tag{A-23}$$

where $\pi_{t,s} = P_s/P_t$.

By the law of large numbers, the probability of changing the wage corresponds to the fraction of types who actually do change their wage. Consequently, from expression (A-8), the real wage evolves according to

$$w_t^{-\frac{1}{\lambda_w}} = (1-\zeta_w)\tilde{w}_t^{-\frac{1}{\lambda_w}} + \zeta_w \left(\frac{w_{t-1}}{\pi_t}\right)^{-\frac{1}{\lambda_w}}. \tag{A-24}$$

Defining the wage inflation as $\pi_t^w = W_t/W_{t-1}$ and using (A-24), (A-23) becomes

$$\left(\frac{1-\zeta_w {\pi_t^w}^{\frac{1}{\lambda_f}}}{1-\zeta_w}\right)^{-\lambda_w+(1+\lambda_w)\nu} = \frac{X_{1t}^w}{X_{2t}^w}, \tag{A-25}$$

where X_{1t}^w and X_{2t}^w are the expected present value of marginal disutility of work and real marginal wage revenue as

$$X_{1t}^w = \frac{\omega}{(1-\varkappa)^\nu} {H_t}^{1+\nu} + \beta\zeta_w \mathbb{E}_t \left({\pi_{t+1}^w}^{\frac{(1+\lambda w)(1+\nu)}{\lambda_w}} X_{1t+1}^w\right) \tag{A-26}$$

$$X_{2t}^w = \frac{1}{1+\lambda_w} {C_t}^{-\sigma} w_t H_t + \beta\zeta_w \mathbb{E}_t \left({\pi_{t+1}^w}^{\frac{1}{\lambda_w}} X_{2t+1}^w\right). \tag{A-27}$$

B.5.3 Final and Intermediate Goods Producers

Competitive final goods producers choose $Y_t(i)$ to maximize profits

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di,$$

where $P_t(i)$ is the price of the i^{th} variety, subject to (A-1). The solution to the profit maximization problem yields the demand for the generic i^{th} intermediate good (A-2). The zero profit condition for competitive final goods producers implies that the aggregate price level is (A-3).

Monopolistically competitive intermediate goods producers hire labor from households and rent capital from entrepreneurs to produce intermediate goods according to the production technology (A-4) and subject to the demand condition (A-2). We solve the problem for intermediate goods producers in two steps. First, we solve for the optimal amount of inputs (capital and labor) demanded. For this purpose, intermediate goods producers minimize costs

$$r_t^k K_{it} + w_t H_{it}$$

subject to (A-4). Let mc_{it} be the Lagrange multiplier on the constraint, the real marginal cost. The first order condition implies that the capital-labor ratio at the firm level is independent of firm-specific variables as

$$\frac{K_{it}}{H_{it}} = \frac{K_t}{H_t} = \frac{\gamma}{1-\gamma}\frac{w_t}{r_t^k}. \tag{A-28}$$

Then the marginal cost is independent of firm-specific variables as

$$mc_{it} = mc_t = \frac{1}{A_t}\left(\frac{r_t^k}{\gamma}\right)^{\gamma}\left(\frac{w_t}{1-\gamma}\right)^{1-\gamma}. \tag{A-29}$$

The second step consists of characterizing the optimal price setting decision in the event that firm i can adjust its price. Recall that this adjustment occurs in each period with probability $1-\zeta_p$, independent of previous history. If a firm can reset its price, it chooses $\tilde{P}_t(i)$ to maximize

$$\mathbb{E}_t \sum_{s=t}^{\infty}(\beta\zeta_p)^{s-t}C_s^{-\sigma}\left[\frac{\tilde{P}_t(i)}{P_s} - mc_s\right]Y_s(i),$$

subject to (A-2). The first order condition for this problem is

$$\mathbb{E}_t \sum_{s=t}^{\infty}(\beta\zeta_p)^{s-t}C_s^{-\sigma}\left[\frac{\tilde{P}_t(i)}{P_s} - (1+\lambda_f)mc_s\right]Y_s(i) = 0.$$

All intermediate goods producers face an identical problems. As for the wage setting decision, we focus on a symmetric equilibrium in which all firms choose the same price $\tilde{P}_t(i) = \tilde{P}_t$. Let $\tilde{p}_t \equiv \tilde{P}_t/P_t$ denote the optimal relative price. The first order condition for optimal price setting becomes

$$\mathbb{E}_t \sum_{s=t}^{\infty}(\beta\zeta_p)^{s-t}C_s^{-\sigma}\left[\frac{\tilde{p}_t}{\pi_{t,s}} - (1+\lambda_f)mc_s\right]\left(\frac{\tilde{p}_t}{\pi_{t,s}}\right)^{-\frac{1+\lambda_f}{\lambda_f}} Y_s = 0. \tag{A-30}$$

By the law of large numbers, the probability of changing the price coincides with the fraction of firms who actually do change the price in equilibrium. Therefore, from expression (A-3), inflation depends on the optimal reset price according to

$$1 = (1-\zeta_p)\tilde{p}_t^{-\frac{1}{\lambda_f}} + \zeta_p\left(\frac{1}{\pi_t}\right)^{-\frac{1}{\lambda_f}} \tag{A-31}$$

Using (A-31), the price setting rule (A-30) becomes

$$\left(\frac{1-\zeta_p\pi_t^{\frac{1}{\lambda_f}}}{1-\zeta_p}\right)^{-\lambda_f} = \frac{X_{1t}^p}{X_{2t}^p}, \tag{A-32}$$

where X_{1t}^p and X_{2t}^p are expected present value of real marginal cost and real marginal revenue as

$$X_{1t}^p = C_t^{-\sigma}Y_t mc_t + \beta\zeta_p\mathbb{E}_t\left(\pi_{t+1}^{\frac{1+\lambda_f}{\lambda_f}}X_{1t+1}^p\right) \tag{A-33}$$

$$X_{2t}^p = \frac{1}{1+\lambda_f}C_t^{-\sigma}Y_t + \beta\zeta_p\mathbb{E}_t\left(\pi_{t+1}^{\frac{1}{\lambda_f}}X_{2t+1}^p\right) \tag{A-34}$$

The evolution of real wage is given by

$$\frac{w_t}{w_{t-1}} = \frac{\pi_t^w}{\pi_t}. \tag{A-35}$$

The fact that the capital-output ratio is independent of firm-specific factors implies that we can obtain an aggregate production function

$$A_t K_t^{\gamma} H_t^{1-\gamma} - \Gamma = \int_0^1 Y_t(i) di = \sum_{s=0}^{\infty} \zeta_p (1-\zeta_p)^{t-s} \left(\frac{\widetilde{p}_{t-s}}{\pi_{t-s,t}}\right)^{-\frac{1+\lambda_f}{\lambda_f}} Y_t,$$

where $K_t \equiv \int_0^1 K_{it} di$ and $H_t \equiv \int_0^1 H_{it} di$. Defining the effect of price dispersion as

$$\Delta_t = \sum_{s=0}^{\infty} \zeta_p (1-\zeta_p)^{t-s} \left(\frac{\widetilde{p}_{t-s}}{\pi_{t-s,t}}\right)^{-\frac{1+\lambda_f}{\lambda_f}},$$

the aggregate production function becomes

$$A_t {K_t}^{\gamma} {H_t}^{1-\gamma} - \Gamma = \Delta_t Y_t. \tag{A-36}$$

Using (A-31), we can define Δ_t recursively as

$$\Delta_t = \zeta_p \Delta_{t-1} \pi_t^{\frac{1+\lambda_f}{\lambda_f}} + (1-\zeta_p) \left(\frac{1-\zeta_p \pi_t^{\frac{1}{\lambda_f}}}{1-\zeta_p}\right)^{1+\lambda_f}. \tag{A-37}$$

B.5.4 Capital Producers

Capital producers transform consumption into investment goods and operate in a competitive national market. Their problem consists of choosing the amount of investment goods produced I_t to maximize (A-9) taking the price of investment goods p_t^I as given. The first order condition for this problem is

$$p_t^I = 1 + S\left(\frac{I_t}{I}\right) + S'\left(\frac{I_t}{I}\right)\frac{I_t}{I}. \tag{A-38}$$

B.5.5 Dividend of Equity

The dividend per unit of equity is the sum of rental rate of capital and the profits of intermediate goods producers and capital goods producers per unit of capital as

$$R_t^k = r_t^k + \frac{Y_t - w_t H_t - r_t^k K_t + p_t^I I_t - I_t \left[1 + S\left(\frac{I_t}{I}\right)\right]}{K_t} \tag{A-39}$$

B.5.6 Government budget

Using the expression of real value of liquidity, government budget constraint and tax rule (22,23) can be written as

$$q_t N_{t+1}^g + \frac{R_{t-1} L_t}{\pi_t} = \tau_t + [R_t^k + (1-\delta) q_t] N_t^g + L_{t+1}. \tag{A-40}$$

$$\tau_t - \tau = \psi_\tau \left(\frac{R_{t-1} L_t}{\pi_t} - \frac{RL}{\pi} - q_t N_t^g \right) \tag{A-41}$$

B.6 Market-Clearing and Equilibrium

The market-clearing conditions for composite labor and capital use are

$$H_t = \int_0^1 H_{it} di$$

and

$$K_t = \int_0^1 K_{it} di.$$

The aggregate capital stock evolves according to

$$K_{t+1} = (1-\delta) K_t + I_t, \tag{A-42}$$

and capital stock is owned by either households or government as

$$K_{t+1} = N_{t+1} + N_{t+1}^g. \tag{A-43}$$

Finally, the aggregate resource constraint requires that

$$Y_t = C_t + \left[1 + S\left(\frac{I_t}{I} \right) \right] I_t. \tag{A-44}$$

The total factor productivity and resaleability (A_t, ϕ_t) follow an exogenous Markov process. In addition to these, we have five endogenous state variables of $(K_t,\ N_t^g,\ R_{t-1}L_t,\ w_{t-1},\ \Delta_{t-1})$ - aggregate capital stock, government ownership of capital, a real liquidity measure, the real wage rate and the effect of price dispersion from the previous period. The recursive competitive equilibrium is given by nine quantities $(C_t,\ I_t,\ H_t,\ Y_t,\ \tau_t,\ K_{t+1},\ N_{t+1},\ N_{t+1}^g,\ L_{t+1})$, and fifteen prices $(R_t,\ q_t,\ p_t^I,\ w_t,\ r_t^k,\ R_t^k,\ mc_t, \Lambda_t,\ \pi_t, \pi_t^w, X_{1t}^p,\ X_{2t}^p,\ X_{1t}^w,\ X_{2t}^w,\ \Delta_t)$ as a function of the state variables $(K_t, N_t^g, R_{t-1}L_t, w_{t-1}, \Delta_{t-1}, A_t, \phi_t)$ which satisfies the twenty four equilibrium conditions (20, 21, A-42, A-43, A-44, A-17, A-19, A-20, A-22) (A-25 - A-29), (A-32 - A-41). Once all the market clearing condition and the government budget constraints are satisfied, the household budget constraint (15) is satisfied by Walras' Law.

Additionally, we define:

$$R_t^q = \mathbb{E}_t \left[\frac{R_{t+1}^k + (1-\delta) q_{t+1}}{q_t} \right] : \text{Expected rete of return on equity.} \tag{A-45}$$

B.6.1 Steady state

We consider a steady state economy in which there is no change in the total factor productivity, resaleability, the nominal price level, and the endogenous quantities and prices. Condition (A-28) at steady state implies

$$\frac{K}{H} = \frac{\gamma}{1-\gamma} \frac{w}{r^k}. \tag{A-46}$$

In steady state all firms charge the same price, hence $\tilde{p} = 1$ and the real marginal cost is equal to the inverse of the markup

$$mc = \frac{1}{A} \left(\frac{r^k}{\gamma} \right)^{\gamma} \left(\frac{w}{1-\gamma} \right)^{1-\gamma} = \frac{1}{1+\lambda_p}. \tag{A-47}$$

We also choose the fixed cost of production so that the profit equals zero in the steady state as

$$Y = mc \cdot (Y + \Gamma) \tag{A-48}$$

Incorporating these three equations into the steady state version of the production function (A-36) yields a relation between the capital-output ratio and the rental rate of capital

$$\frac{Y}{K} = \frac{r^k}{\gamma}. \tag{A-49}$$

Because the ratio between capital and hours is a function of the capital-output ratio (from the production function), equation (A-47) also yields an expression for the real wage as a function of the rental rate

$$w = (1-\gamma) \left(\frac{A}{1+\lambda_f} \right)^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{r^k} \right)^{\frac{\gamma}{1-\gamma}}. \tag{A-50}$$

In steady state, the real wage is equal to a markup over the marginal rate of substitution between labor and consumption

$$w = (1+\lambda_w) \frac{\omega H^{\nu}}{C^{-\sigma}}. \tag{A-51}$$

From the steady state version of (16), we can solve for the steady state real interest rate ($r \equiv R/\pi = R$) as a function of q

$$\beta^{-1} = r \left(1 + \varkappa \frac{q-1}{1-\theta q} \right) \tag{A-52}$$

where we used the fact that in steady state $p^I = 1$ because $(S(1) = S'(1) = 0$ (from A-38). This condition implies that the liquid asset has a return that is less that β^{-1} as long as $1 < q < 1/\theta$.

Steady state tax obtain from (A-40)

$$\tau = (r-1)L. \tag{A-53}$$

In steady state, zero profit condition implies

$$R^k = r^k.$$

Then condition (A-22) implies

$$I = \varkappa \frac{[R^k + (1-\delta)\phi q]K + rL - \tau}{1-\theta q} = \varkappa \frac{[r^k + (1-\delta)\phi q]K + L}{1-\theta q}, \tag{A-54}$$

where we used (A-53) to eliminate transfers and the fact that in steady state $K = N$ since by assumption $N^g = 0$. Steady state investment is simply equal to depreciated steady state capital

$$\frac{I}{K} = \delta. \tag{A-55}$$

Combining (A-54) with (A-55), we obtain

$$\delta(1-\theta q) = \varkappa \left[r^k + (1-\delta)\phi q + \frac{L}{K} \right]. \tag{A-56}$$

Using the steady state capital output ratio (A-49), we obtain a relationship between r^k and q

$$\delta - [\delta\theta + \varkappa(1-\delta)\phi]q = \varkappa \left(1 + \frac{1}{\gamma}\frac{L}{Y}\right) r^k, \tag{A-57}$$

where L/Y is ratio of liquid assets to GDP that we take as exogenous in our calibration.

Another relationship between r^k and q obtains from the steady state version of (17)

$$\beta^{-1} = \frac{r^k + (1-\delta)q}{q}\left(1 + \varkappa\frac{q-1}{1-\theta q}\right) - \frac{\varkappa(1-\delta)(1-\phi)(q-1)}{1-\theta q} \tag{A-58}$$

where $(r^k + \lambda q)/q$ is the steady state return on equity. As long as $\phi < 1$, the return on equity is larger than the steady state return on the liquid assets by

$$\frac{\varkappa(1-\delta)(1-\phi)(q-1)}{(1-\theta q)\left(1 + \varkappa\frac{q-1}{1-\theta q}\right)}.$$

We can insert the solution for r^k from (A-57) into (A-58) and solve for q. Once we have q and r^k, r can be obtained from (A-52), w from (A-50), K/Y from (A-49), K/H from (A-46), I/K from (A-55) and C/Y from the resource constraint. Finally economy size Y is determined to satisfy (A-51). The size of fixed cost Γ is chosen so that zero profit condition is satisfied with exactly the unit mass of intermediate goods producers in (A-48).

B.7 Zero-Coupon Bonds Returns

While the paper only considers one-period perfectly liquid securities and illiquid stocks, our empirical analysis in Section 3.2 describing the calibration of the liquidity shock considers the spread between illiquid and liquid long term securities, most of which are zero-coupon bonds. Therefore in this section we derive the spreads for zero-coupon bonds with varying degree of liquidity ϕ^j. As mentioned in the paper, we assume that the net supply of these bonds is zero so that the equilibrium condition does not change from our model.

We will show that, for short term bonds the convenience yield in our model CY_t approximately equals the spread between perfectly illiquid and perfectly liquid assets $\overline{CY}_t$. For long term bonds this is not the case: intuitively, the spread in this case is proportional to the average of expected future convenience yields, as shown in expression (A-70) below. The last part of the section also shows that under the assumption that the ϕ_t^j have a common and an idiosyncratic component, the yield spreads have a factor structure where the common factor is proportional to the convenience yield.

Let $P_t^{(T,j)}$ be the price of a long-term zero coupon bond j with maturity T which pays \$1 at date t+T for sure. Euler equation is given by

$$P_t^{(T,\,j)} = \mathbb{E}_t\left[\frac{m_{t+1}}{\pi_{t+1}}(1+\phi_{t+1}^j\Lambda_{t+1})P_{t+1}^{(T\,-\,1,\,j)}\right], \tag{A-59}$$

where $P_{t+1}^{(T\,-\,1,\,j)}$ is the price of this bond at t+1 (as it becomes bond with maturity T-1), and $m_{t+1} = \beta\,(c_{t+1}/c_t)^{-\sigma}$ is marginal rate of substitution.

Iterating this equation forward we obtain:

$$P_t^{(T,\,j)} = \mathbb{E}_t\left[\prod_{s=1}^{T}\frac{m_{t+s}}{\pi_{t+s}}(1+\phi_{t+s}^j\Lambda_{t+s})\right]. \tag{A-60}$$

The nominal gross yield to maturity $\text{nytm}_t^{(T,\,j)}$ and the price are related as $(\text{nytm}_t^{(T,\,j)})^T = 1/P_t^{(T,\,j)}$. We can then rewrite the Euler condition for the long term bonds as

$$1 = \texttt{nytm}_t^{(T,\,j)}\left(\mathbb{E}_t\left[\prod_{s=1}^{T}\frac{m_{t+s}}{\pi_{t+s}}(1+\phi_{t+s}^j\Lambda_{t+s})\right]\right)^{1/T}. \tag{A-61}$$

Let $j = l$ denote the bond that is always perfectly liquid, i.e., $\phi_t^l = 1$ for all t, and let $j = 0$ denote the bond that is always totally illiquid, i.e., $\phi_t^0 = 0$ for all t. Then we have

$$1 = \texttt{nytm}_t^{(T,\,l)}\left(\mathbb{E}_t\left[\prod_{s=1}^{T}\frac{m_{t+s}}{\pi_{t+s}}(1+\Lambda_{t+s})\right]\right)^{1/T} \quad \text{and} \tag{A-62}$$

$$1 = \mathtt{nytm}_t^{(T,\,0)} \left(\mathbb{E}_t \left[\prod_{s=1}^{T} \frac{m_{t+s}}{\pi_{t+s}} \right] \right)^{1/T}. \tag{A-63}$$

Here we assume entrepreneurs cannot use the return on the totally illiquid bond for funding investment even at the maturity date.

At steady state these conditions imply:

$$1 = \mathtt{nytm}^{(T,\,j)} \frac{\beta}{\pi}(1 + \phi^j CY) = \text{nytm}^{(T,\,l)} \frac{\beta}{\pi}(1 + CY)$$

or

$$\mathtt{ytm}^{(T,\,j)} - \mathtt{ytm}^{(T,\,l)} = \frac{\mathtt{nytm}^{(T,\,j)}}{\pi} - \frac{\mathtt{nytm}^{(T,\,j)}}{\pi} = \beta^{-1} \frac{CY}{1 + CY} \frac{1 - \phi^j}{1 + \phi^j CY}, \tag{A-64}$$

where ϕ^j is liquidity of the private bonds in the steady state. From the spread before the crisis (which we consider the deterministic steady state), we can estimate ϕ^j as

$$1 - \phi^j = \frac{1 + CY}{CY} \frac{\beta(\mathtt{ytm}^{(T,\,j)} - \mathtt{ytm}^{(T,\,l)})(1 + CY)}{1 + \beta(\mathtt{ytm}^{\,(T,\,j)} - \mathtt{ytm}^{(T,\,l)})(1 + CY)}. \tag{A-65}$$

This is the equation (25) in the text. We also use (A-62, A-63) to get

$$\overline{CY} \equiv \mathtt{ytm}^{(T,\,0)} - \mathtt{ytm}^{(T,\,l)} = \beta^{-1} \frac{CY}{1 + CY} \simeq CY. \tag{A-66}$$

In the steady state, the convenience yield approximately equals the yield spread between totally illiquid and perfectly liquid bonds.

In practice our data on yields are percent annualized net nominal returns $YTM^{(T,\,j)}$ and $YTM^{(T,\,l)}$. We compute the spread in real terms $(ytm^{(T,\,j)} - ytm^{(T,\,l)})$ as $\exp\{(YTM^{\,(T,\,j)} - \bar{\pi})/400\} - \exp\{(YTM^{(T,\,l)} - \bar{\pi})/400\}$ where $\bar{\pi}$ is net inflation, percent annualized. We compute the steady state convenience yield as the difference between annualized gross returns of perfectly illiquid and perfectly liquid bonds, in percent. Of course CY and $\overline{CY}$ are not the same, but in practice they are very close. When computing ϕ^j in Table A-2 we use expression (A-65).

Outside the steady state, for discount bond of one-period maturity, we can use (A-62, A-63) to obtain by ignoring the covariance terms as

$$\begin{aligned} 1 &= \mathtt{nytm}_t^{(1,\,0)} \mathbb{E}_t \left(\frac{m_{t+1}}{\pi_{t+1}} \right) \\ &\simeq \mathtt{nytm}_t^{(1,\,0)} \mathbb{E}_t \left(m_{t+1} \right) \mathbb{E}_t \left(\frac{1}{\pi_{t+1}} \right) \end{aligned}$$

$$\begin{aligned} 1 &= \mathtt{nytm}_t^{(1,\,l)} \mathbb{E}_t \left[\frac{m_{t+1}}{\pi_{t+1}} (1 + \Lambda_{t+1}) \right] \\ &\simeq \mathtt{nytm}_t^{(1,\,l)} \mathbb{E}_t \left(m_{t+1} \right) \mathbb{E}_t \left(\frac{1}{\pi_{t+1}} \right) (1 + CY_t) \end{aligned}$$

Thus we learn

$$\overline{CY}_t = \left[\texttt{nytm}_t^{(1,0)} - \texttt{nytm}^{(1,l)}\right] \mathbb{E}_t \left(\frac{1}{\pi_{t+1}}\right) \simeq \frac{1}{\mathbb{E}_t\left(m_{t+1}\right)} \frac{CY_t}{1+CY_t} \simeq CY_t. \tag{A-67}$$

Thus, even outside the steady state, the convenience yield of our model CY_t defined in (A-18) approximately equals the yield spread between perfectly illiquid and perfectly liquid one-period bonds, both of which do not have default risk.

When we use the zero coupon bond with longer maturity, such simple relationship no longer holds. Applying the log-linearization of approximation to (A-61) and (A-62) by denoting $\widehat{x}_t \equiv \ln x_t - \ln x \simeq (x_t - x)/x$ as approximately proportional deviation of x_t from the steady state, we get:

$$\begin{aligned}
0 \;=\; & \widehat{\texttt{nytm}}_t^{(T,j)} + \mathbb{E}_t[\frac{1}{T}\sum_{s=1}^{T}\hat{m}_{t+s}] - \mathbb{E}_t[\frac{1}{T}\sum_{s=1}^{T}\hat{\pi}_{t+s}] \\
& + \frac{\phi^j CY}{1+\phi^j CY}\left\{\mathbb{E}_t[\frac{1}{T}\sum_{s=1}^{T}\hat{\phi}^j_{t+s}] + \mathbb{E}_t[\frac{1}{T}\sum_{s=1}^{T}\widehat{\Lambda}_{t+s}]\right\} \\
=\; & \widehat{\texttt{nytm}}_t^{(T,j)} + \mathbb{E}_t[\frac{1}{T}\sum_{s=1}^{T}\hat{m}_{t+s}] - \mathbb{E}_t[\frac{1}{T}\sum_{s=1}^{T}\hat{\pi}_{t+s}] \\
& + \left[1 - \beta \mathrm{ytm}^{(T,j)}\right]\left\{\mathbb{E}_t[\frac{1}{T}\sum_{s=1}^{T}\hat{\phi}_{t+s}] + \mathbb{E}_t[\frac{1}{T}\sum_{s=1}^{T}\widetilde{\phi}^j_{t+s}] + \mathbb{E}_t[\frac{1}{T}\sum_{s=1}^{T}\widehat{\Lambda}_{t+s}]\right\},
\end{aligned}$$

where we define $\widetilde{\phi}^j_{t+s} = \hat{\phi}^j_{t+s} - \hat{\phi}_{t+s}$ as the idiosyncratic shock to the resaleability of bond j and use (A-64) for the last step. Similarly we get

$$0 = \widehat{\texttt{nytm}}_t^{(T,l)} + \mathbb{E}_t[\frac{1}{T}\sum_{s=1}^{T}\hat{m}_{t+s}] - \mathbb{E}_t[\frac{1}{T}\sum_{s=1}^{T}\hat{\pi}_{t+s}] + [1 - \beta\texttt{ytm}^{(T,l)}]\mathbb{E}_t[\frac{1}{T}\sum_{s=1}^{T}\Lambda_{t+s}].$$

Taking the difference between the two, we obtain

$$\begin{aligned}
\widehat{\texttt{nytm}}_t^{(T,j)} - \widehat{\texttt{nytm}}_t^{(T,l)} =& \beta\left[\mathrm{ytm}^{(T,j)} - \mathrm{ytm}^{(T,l)}\right]\mathbb{E}_t[\frac{1}{T}\sum_{s=0}^{T-1}\widehat{CY}_{t+s}] \\
& - [1 - \beta\mathrm{ytm}^{(T,j)}]\mathbb{E}_t[\frac{1}{T}\sum_{s=1}^{T}\hat{\phi}_{t+s}] - [1 - \beta\mathrm{ytm}^{(T,j)}]\mathbb{E}_t[\frac{1}{T}\sum_{s=1}^{T}\widetilde{\phi}_{t+s}],
\end{aligned}$$

using $\widehat{CY}_t = \mathbb{E}_t\left(\widehat{\Lambda}_{t+1}\right)$.

If we assume that $\widehat{CY}_t$ is approximately proportional to $\hat{\phi}_t$ as argued in footnote (31) and we assume both $\hat{\phi}_t$ and $\widehat{CY}_t$ follow an AR(1) process with autoregressive coefficient ρ_ϕ we obtain

$$\widehat{\texttt{nytm}}_t^{(T,j)} - \widehat{\texttt{nytm}}_t^{(T,l)} = \left\{\beta\left[\text{ytm}^{(T,j)} - \text{ytm}^{(T,l)}\right] - [1-\beta\text{ytm}^{(T,j)}]\rho_\phi\left(\frac{\hat{\phi}_t}{\widehat{CY}_t}\right)\right\}\frac{1-\rho_\phi^T}{(1-\rho_\phi)T}\widehat{CY}_t - [1-\beta\text{ytm}^{(T,j)}]\mathbb{E}_t[\frac{1}{T}\sum_{s=1}^{T}\widetilde{\phi}_{t+s}]. \quad \text{(A-68)}$$

This equation shows the real yield spreads between many pairs of zero coupon bonds with the same payoff and different liquidity tend to comove with the convenience yield except for the term reflecting the idiosyncratic shocks to the resaleability of each bond. Using $\beta\text{ytm}^{(T,j)} = \frac{1}{1+\phi^j CY}$ and $\beta\text{ytm}^{(T,l)} = \frac{1}{1+CY}$ and ignoring the terms which are higher order than that is proportional to the convenience yield, obtain

$$\begin{aligned}\texttt{nytm}_t^{(T,j)} - \texttt{nytm}_t^{(T,l)} - [\texttt{nytm}^{(T,j)} - \texttt{nytm}^{(T,l)}] \\ \simeq \frac{1-\phi^j-\phi^j(1+CY)\rho_\phi\left(\frac{\hat{\phi}_t}{\widehat{CY}_t}\right)}{1+\phi^j CY}\frac{1-\rho_\phi^T}{(1-\rho_\phi)T}[CY_t - CY] \\ - \frac{\phi^j CY(1+CY)}{1+\phi^j CY}\mathbb{E}_t[\frac{1}{T}\sum_{s=1}^{T}\widetilde{\phi}_{t+s}]. \quad \text{(A-69)}\end{aligned}$$

(Here we also approximate $\pi/[\beta(1+CY)^2] \simeq 1$ by assuming the length of period is short and that inflation rate is not too high). This is the base of our dynamic factor formula used in Section 3.2. Note that term $(\hat{\phi}_t/\widehat{CY}_t)$ is negative in our model when shocks to resaleability are important.

For the special case of totally illiquid bond, we have

$$\texttt{nytm}_t^{(T,0)} - \texttt{nytm}_t^{(T,l)} - [\texttt{nytm}^{(T,0)} - \texttt{nytm}^{(T,l)}] \simeq \frac{1-\rho_\phi^T}{(1-\rho_\phi)T}[CY_t - CY]. \quad \text{(A-70)}$$

Thus to the extent the shock is not permanent so that $\frac{1-\rho_\phi^T}{(1-\rho_\phi)T} < 1$, the change of the yield spread between totally illiquid and perfectly liquid long-term bonds in the left hand side underestimates the change of the convenience yield $CY_t - CY$ of our model. Our approximations ignore potentially very important covariance terms, associated with risks. Examining asset price implications of liquidity constraints and liquidity shocks which takes into account risks is a topic of future research.

C Impulse Responses to Other Shocks

In this section, we document the response of macroeconomic variables to standard shocks studied in the literature: productivity, monetary policy, and government spending. All three shocks follow a stationary autoregressive process

$$\begin{aligned} A_t &= (1-\rho_A)A + \rho_A A_{t-1} + \varepsilon_{At} \\ \iota_t &= \rho_\iota \iota_{t-1} + \varepsilon_{\iota t} \\ G_t &= (1-\rho_G)G + \rho_G G_{t-1} + \varepsilon_{Gt} \end{aligned}$$

The introduction of productivity and monetary policy shocks in the model is straightforward. The former affects the production function (A-4) while the latter enters the monetary policy rule (20). Both shocks, however, do not change the steady state. Conversely, the government spending shock requires some amendments to the steady state. The resource constraint now becomes

$$Y_t = C_t + \left[1 + S\left(\frac{I_t}{I}\right)\right] + G_t,$$

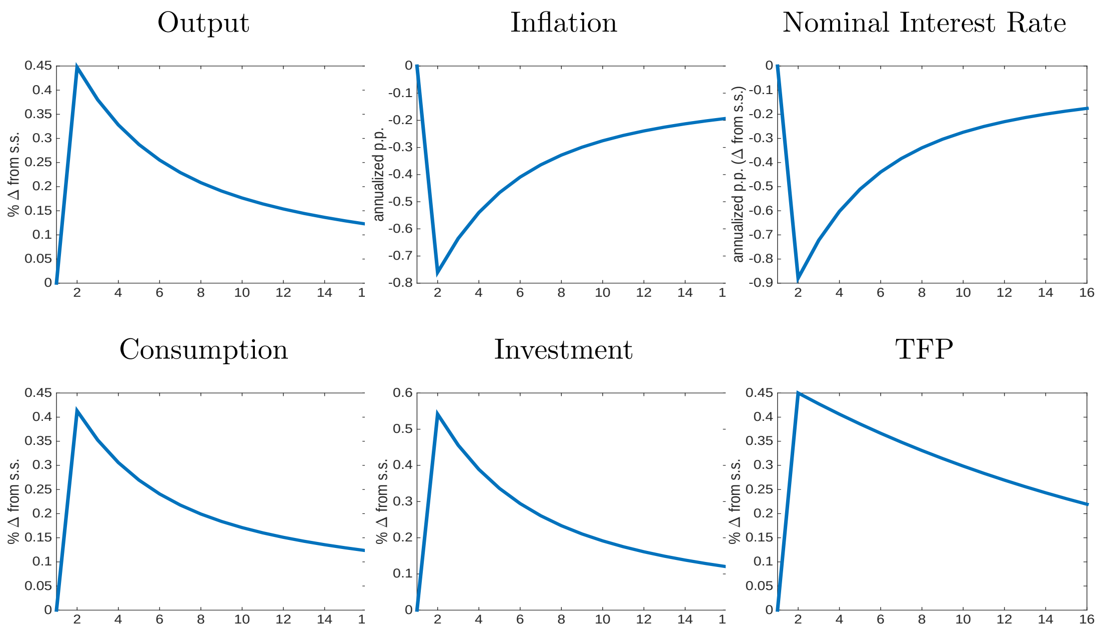
where G_t is the level of government spending. The government budget constraint becomes

$$q_t N^g_{t+1} + \frac{R_{t-1}L_t}{\Pi_t} = \tau_t - G_t + [R^K_t + (1-\delta)q_t]N^g_t + L_{t+1}.$$

We calibrate the ratio of government spending to GDP to 21%, which corresponds to the postwar US data. In the KM model, the way government spending is financed is not neutral. If an increase in government spending is financed primarily via debt issuance, liquidity increases. To keep our results comparable with the literature, we study the response to a government spending shock assuming the fiscal authority keeps the real value of liquid assets constant ($B_t/P_t = b$).[50]

For the productivity and government spending shock, we calibrate the size of the shocks (0.45 and 0.52%) and the persistence parameters ($\rho_A = 0.95$ and $\rho_G = 0.97$) to the posterior mode in Smets and Wouters (2007). The size of the monetary policy shock is 25 basis points annualized, and we assume a high persistence parameter ($\rho_\iota = 0.8$) to compensate for the absence of interest rate smoothing in the monetary policy rule.[51]

Figure A-1: Response of macroeconomic variables to a productivity shock.



Notes: Impulse response function to an increase in productivity.

C.1 Productivity Shocks

In response to a persistent increase in productivity (see Figure A-1), all quantities (output, consumption and investment) increase as in a standard New Keynesian model. The higher level of productivity, however, reduces the firms' marginal cost, thus inducing disinflationary pressures. As a reaction, the central bank cuts the nominal interest rate.

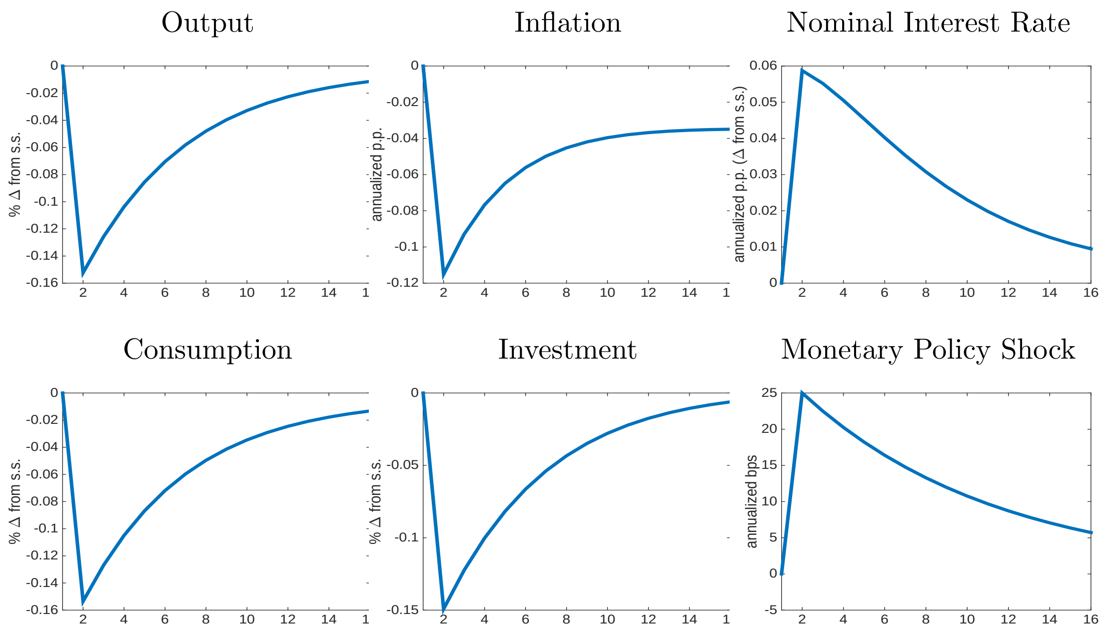
C.2 Monetary Policy Shocks

In response to a persistent increase in nominal interest rates (see Figure A-2), aggregate demand falls, and so does inflation. The systematic component of the monetary policy rule accommodates the downturn, so in equilibrium the nominal interest rate actually increases less than the original shock.

[50] In a model similar to ours, Kara and Sin (Forthcoming) analyze how the government spending multiplier changes depending on the government financing decisions.

[51] This value is consistent with the posterior mode for the interest rate smoothing parameter in Smets and Wouters (2007).

Figure A-2: Response of macroeconomic variables to a monetary policy shock.

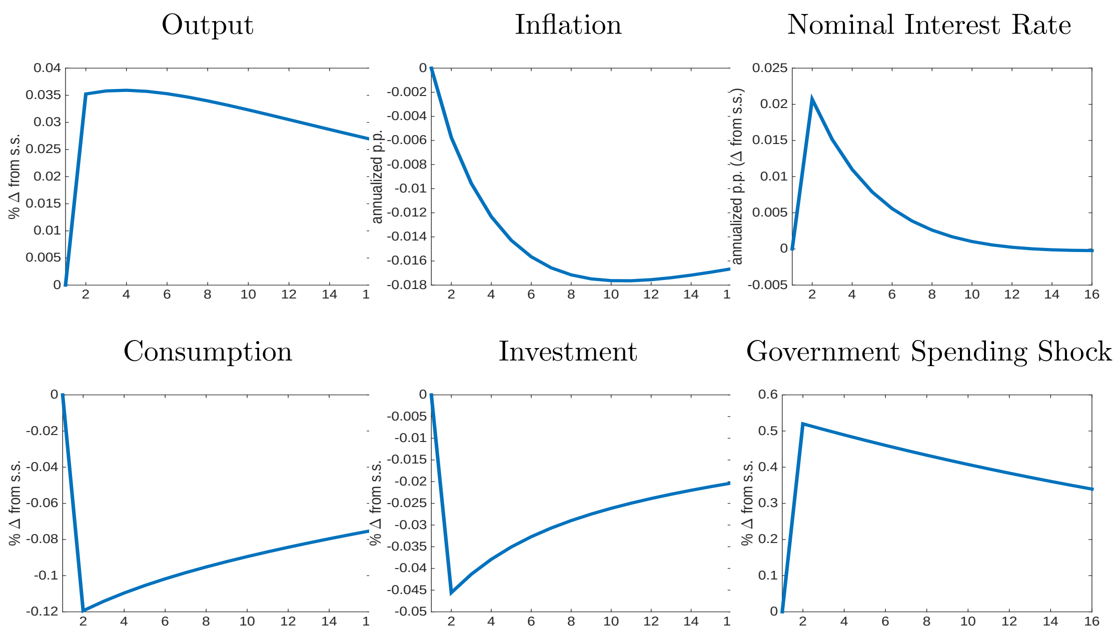


Notes: Impulse response function to an increase in the nominal interest rate.

C.3 Government Spending Shocks

The increase in government spending (see Figure A-3) raises output but crowds out private demand, so consumption and investment fall. The demand shock leads the central bank to increase the interest rate. In equilibrium, inflation falls, but the effect is small and the result depends on the high degree of persistence of the shock. With lower persistence, inflation would rise, at least on impact.

Figure A-3: Response of macroeconomic variables to a government spending shock.



Notes: Impulse response function to an increase in government spending.

D Robustness

In this section, we discuss the robustness of our quantitative results to changes of a selected number of key parameters: (i) the degree of price and wage rigidities ζ_p and ζ_w; (ii) the coefficient of relative risk aversion σ; and (iii) the adjustment cost parameter $S''(1)$. We keep the shock fixed to the baseline case but adjust the feedback coefficient in the policy rule for private asset purchases ψ_k so that the intervention remains 10% of GDP. Table A-1 summarizes the results.

The first column reports the impact response to the liquidity shock of output and its components, the inflation rate, the nominal value of the capital stock (all in percentage deviations from steady state), and the convenience yield (in annualized basis points) in the baseline case. In the next five columns, we vary one parameter at a time and report the same statistics.

Column (2) shows the implications of having a higher degree of price and wage rigidity relative to the baseline calibration, equal to 0.85, implying that firms (unions) reset prices (wages) every six and a half quarters. A lower frequency of price changes induces a smaller drop in inflation relative to the baseline. Consequently, the real interest rate rises less, and the fall in output is less pronounced. Because the financial frictions affect investment largely independently of nominal rigidities, the effect on investment is still large, while the effect on consumption is proportionally smaller. Overall, the composition of output is still comparable to the data, with a much larger fall in investment than in consumption. The fall in the price of capital is smaller on account of the lower deflation. Column (3) considers the opposite experiment, that is, a lower degree of price and wage rigidity relative to the baseline calibration, equal to 0.66, implying that firms (unions) reset prices (wages) every three quarters. Not surprisingly, the direction of the change relative the baseline is the opposite of what just discussed for the case of higher price and wage rigidities. In this case, the fall in investment and consumption (and hence output) is closer to the data, but the model overestimates the effect on inflation, which now drops by almost five percent. More flexible prices make the intervention more powerful when the ZLB is binding, because the real interest rate increases by more when prices are more flexible.

Column (4) of Table A-1 reports the results of increasing the coefficient of relative risk aversion to 2. The fall in output is a bit smaller than in the baseline calibration. The intertemporal elasticity of substitution (the inverse of the coefficient of relative risk aversion) is now smaller, so consumption is less sensitive to the increase in the real rate that results from the combination of the zero lower bound and the deflationary pressures. Conversely, the fall in investment is roughly unchanged. Overall, this case is not very different from the baseline.

The last two columns of Table A-1 show the results of decreasing (column 5) and increasing (column 6) adjustment costs. Here, the differences with the baseline calibration depend primarily on the relative response of investment and the value of equity. With lower adjustment costs, investment falls more, dragging down output, but the smaller decline in the value of equity falls partially tempers this effect. In this case, however, the increase in convenience yield is less than observed in the data. The opposite occurs with larger adjustment cost, as investment falls less than in the baseline, while the value of equity drops more and the convenience yield rises more significantly.

The last two rows report the response of output and inflation in the absence of intervention for the same parameters. The overall message is that the effect of the liquidity injection is roughly stable across parameterizations.

Table A-1: Robustness

	Baseline	*Alternative Parameterizations*				
	(1)	(2)	(3)	(4)	(5)	(6)
Nominal Rigidities (ζ_p, ζ_w)	0.75	0.85	0.66			
Risk Aversion (σ)	1			2		
Adjustment Costs $(S''(1))$	0.75				0.5	1
Period 1 response with intervention						
Output	-4.41	-3.89	-5.28	-4.22	-5.14	-3.99
Consumption	-1.32	-0.86	-2.09	-0.99	-0.76	-1.71
Investment	-14.17	-13.37	-15.45	-14.41	-19.16	-11.18
Inflation	-2.51	-0.79	-4.90	-2.66	-2.48	-2.52
Value of Capital	-1.80	-0.85	-3.21	-1.99	-0.75	-2.51
Convenience Yield	180	181	176	179	166	190
Period 1 response without intervention						
Output	-5.78	-4.62	-7.46	-5.23	-6.36	- 5.40
Inflation	-3.50	-0.99	-7.49	-3.57	-3.38	-3.54

Note: The first three rows of the table show the alternative parameters that we consider. The next six rows report the first-period response of output, consumption, investment, inflation, the nominal value of capital, and the convenience yield for the baseline calibration with intervention and for alternative parameterizations of nominal rigidities, risk aversion, and adjustment costs. The last two rows report the first-period response of output and inflation without intervention for the same parameters.

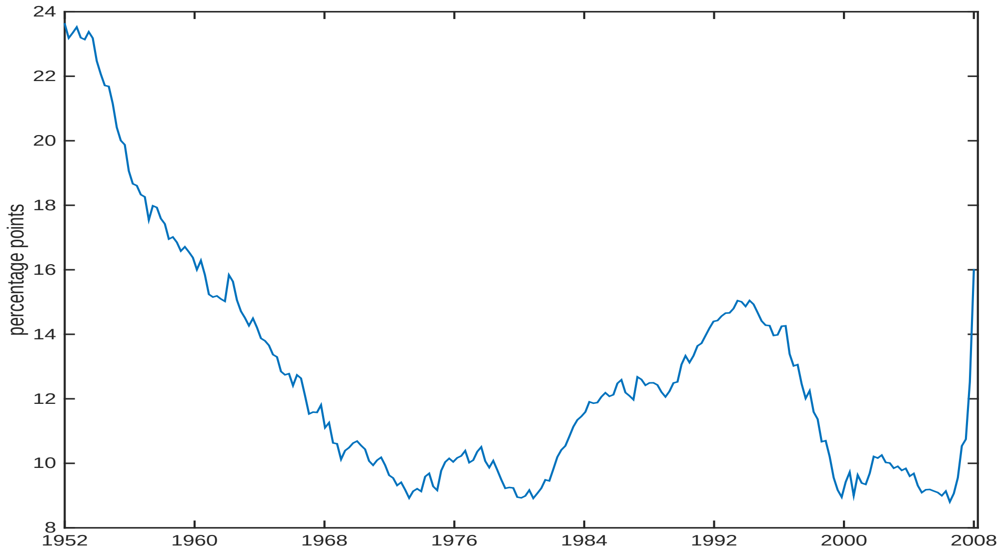
E Additional Tables and Figures

Table A-2: Average Returns and Implied ϕ_j

2004/ 7/21–2007/ 6/29 CY: 0.46			2008/10/ 1–2008/12/31 CY: 3.42		
	ϕ	spread		ϕ	spread
1Y Refcorp	*1.150*	-0.07	20Y TIPS	*0.806*	0.65
6M Refcorp	*1.067*	-0.03	On-Off	*0.795*	0.69
2Y Refcorp	*0.990*	0.00	20Y Refcorp	*0.747*	0.85
AA CDS-Bond Basis	*0.985*	0.01	2Y Refcorp	*0.720*	0.94
A CDS-Bond Basis	*0.945*	0.02	5Y Refcorp	*0.702*	1.01
3Y Refcorp	*0.920*	0.04	7Y Refcorp	*0.701*	1.01
On-Off	*0.889*	0.05	10Y Refcorp	*0.692*	1.04
5Y Refcorp	*0.854*	0.07	1Y Refcorp	*0.690*	1.05
BBB CDS-Bond Basis	*0.851*	0.07	10Y TIPS	*0.682*	1.07
4Y Refcorp	*0.822*	0.08	3Y Refcorp	*0.679*	1.08
7Y Refcorp	*0.779*	0.10	4Y Refcorp	*0.665*	1.13
10Y Refcorp	*0.671*	0.15	5Y TIPS	*0.654*	1.17
20Y Refcorp	*0.659*	0.15	6M Refcorp	*0.612*	1.31
5Y TIPS	*0.371*	0.28	7Y TIPS	*0.586*	1.40
7Y TIPS	*0.311*	0.31	AA CDS-Bond Basis	*0.548*	1.53
20Y TIPS	*0.298*	0.31	AAA	*0.448*	1.86
10Y TIPS	*0.219*	0.35	A CDS-Bond Basis	*0.282*	2.43
AAA	*-0.294*	0.58	BBB CDS-Bond Basis	*0.000*	3.39

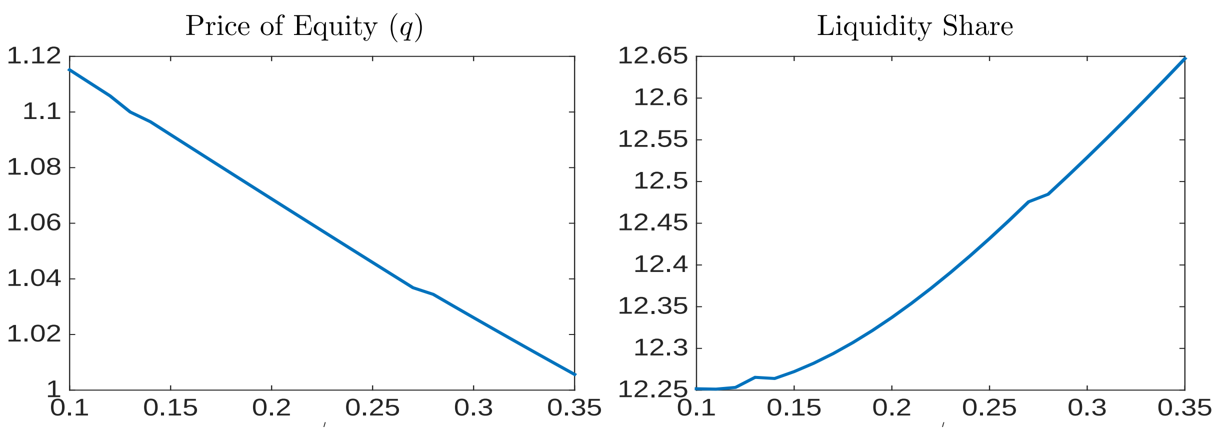
Notes: The two panels show the average spread for the securities listed above (see Appendix A.2 for a description) for the 2004/ 7/21–2007/ 6/29 (left) and 2008/10/ 1–2008/12/31 (right) periods, as well as the implied ϕ^j computed according to formula (25).

Figure A-4: The Liquidity Share in the Data



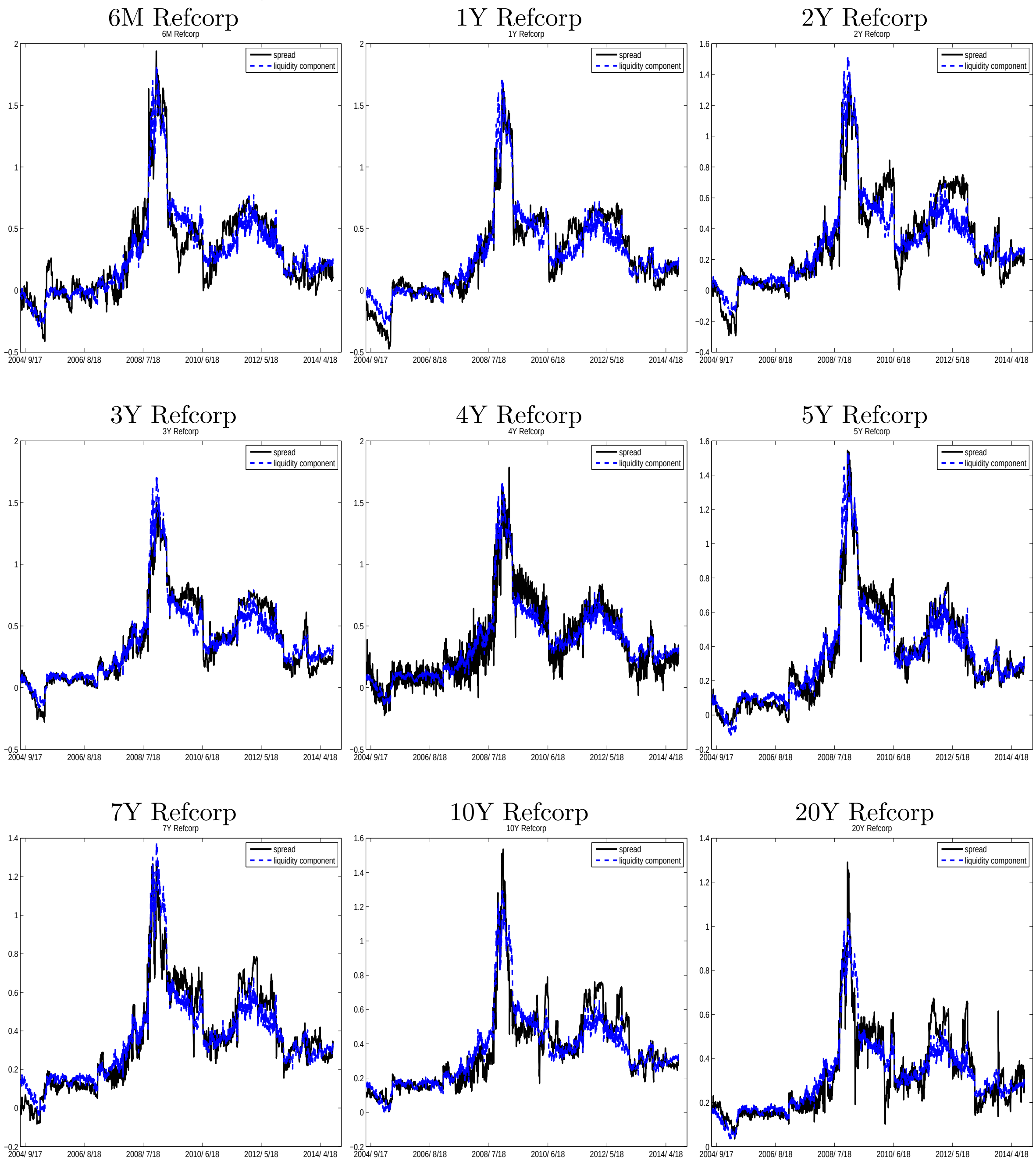
Notes: The figure plots the evolution of the liquidity share, defined as the ratio of government liabilities (liquid assets) to total assets in the U.S. economy, over the sample period 1953Q1:2008Q3.

Figure A-5: Steady State as a Function of ϕ



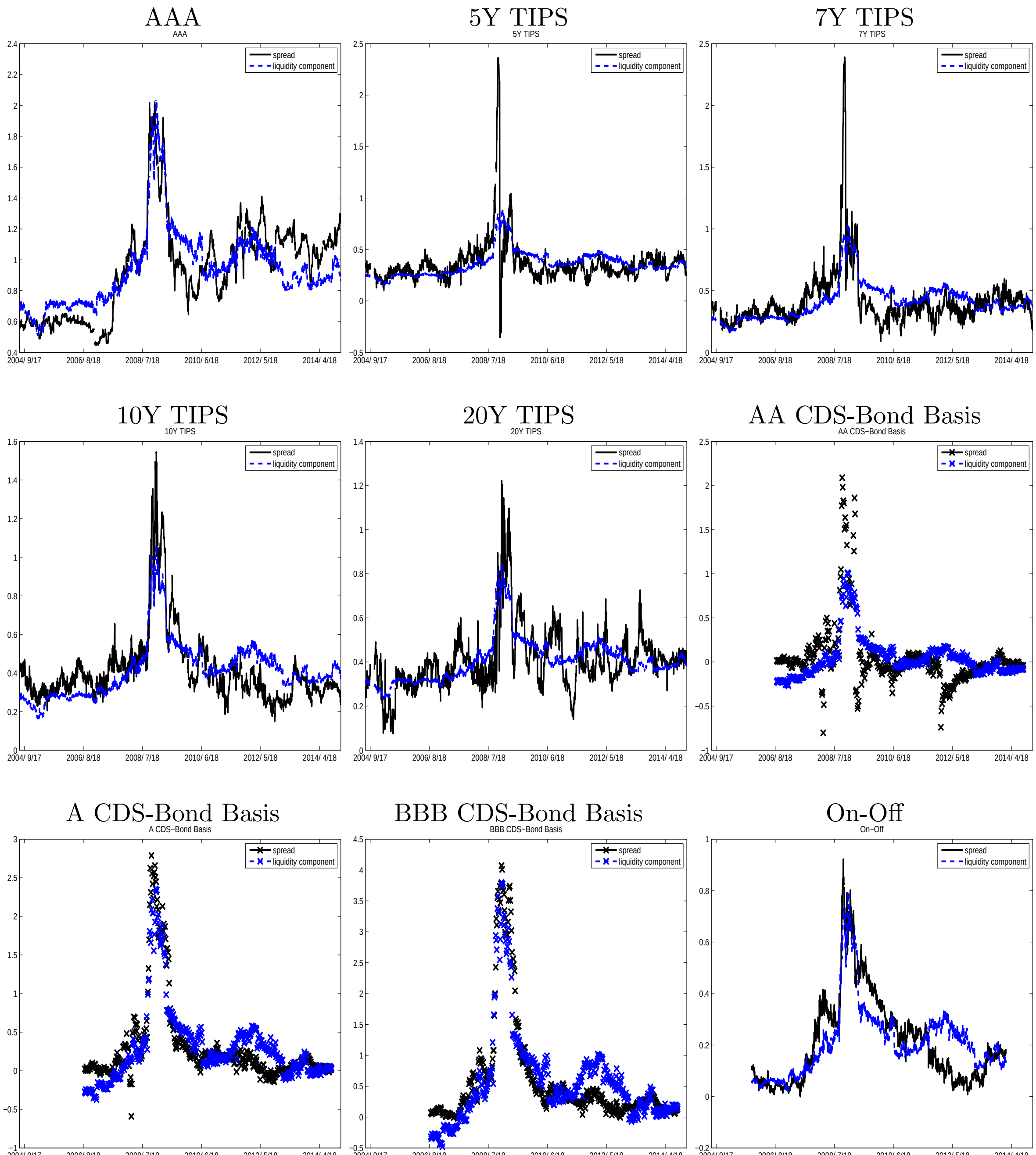
Notes: The figure plots the steady-state price of equity (left panel) and liquidity share (right panel) as a function of the steady-state value of the resaleability parameter ϕ.

Figure A-6: Liquidity Spreads and Common Factor



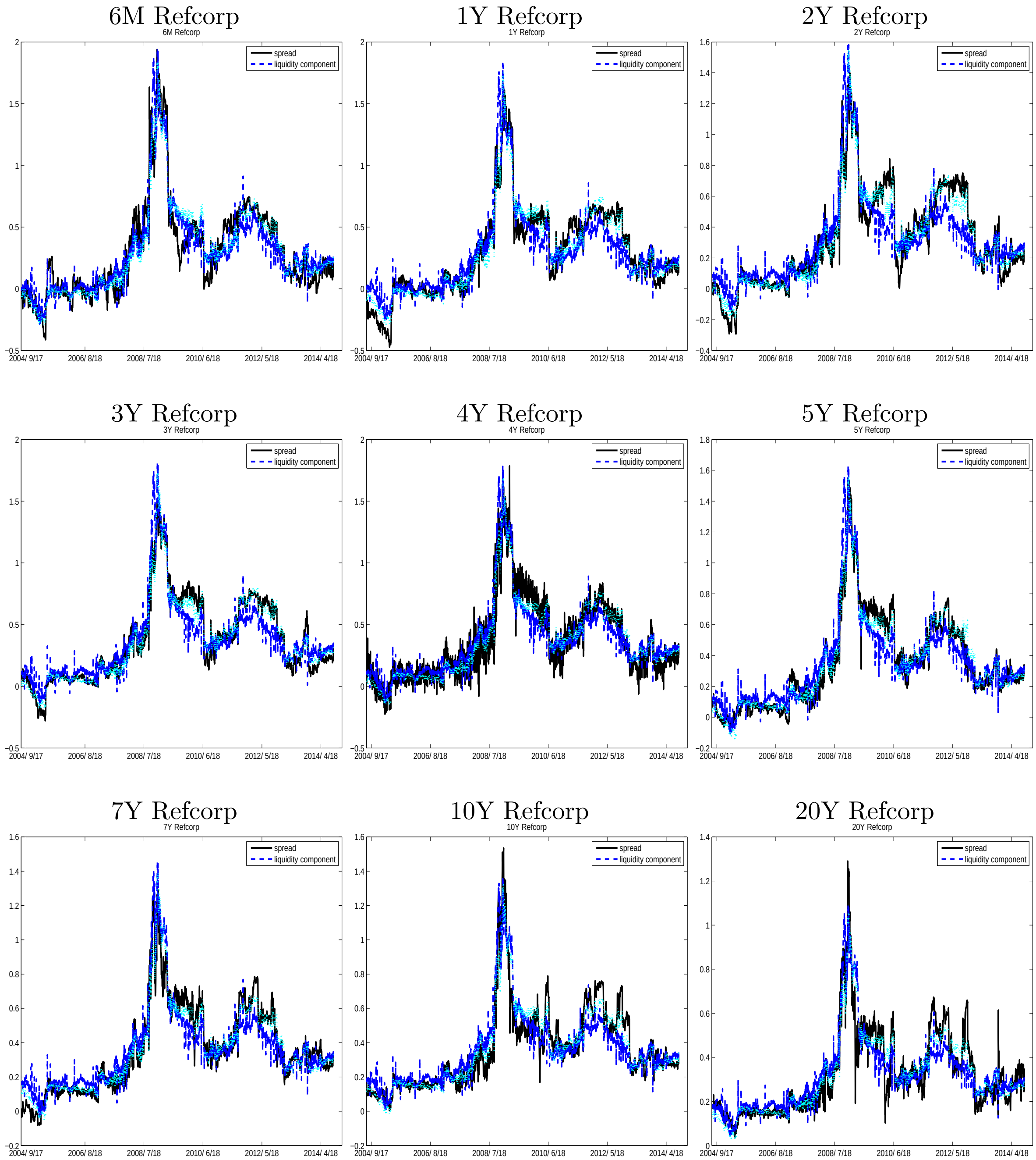
Notes: The figure shows the daily time series (black) of each spread as well as its projection on the first principal component and a constant (blue).

Figure A-7: Liquidity Spreads and Common Factor – Continued



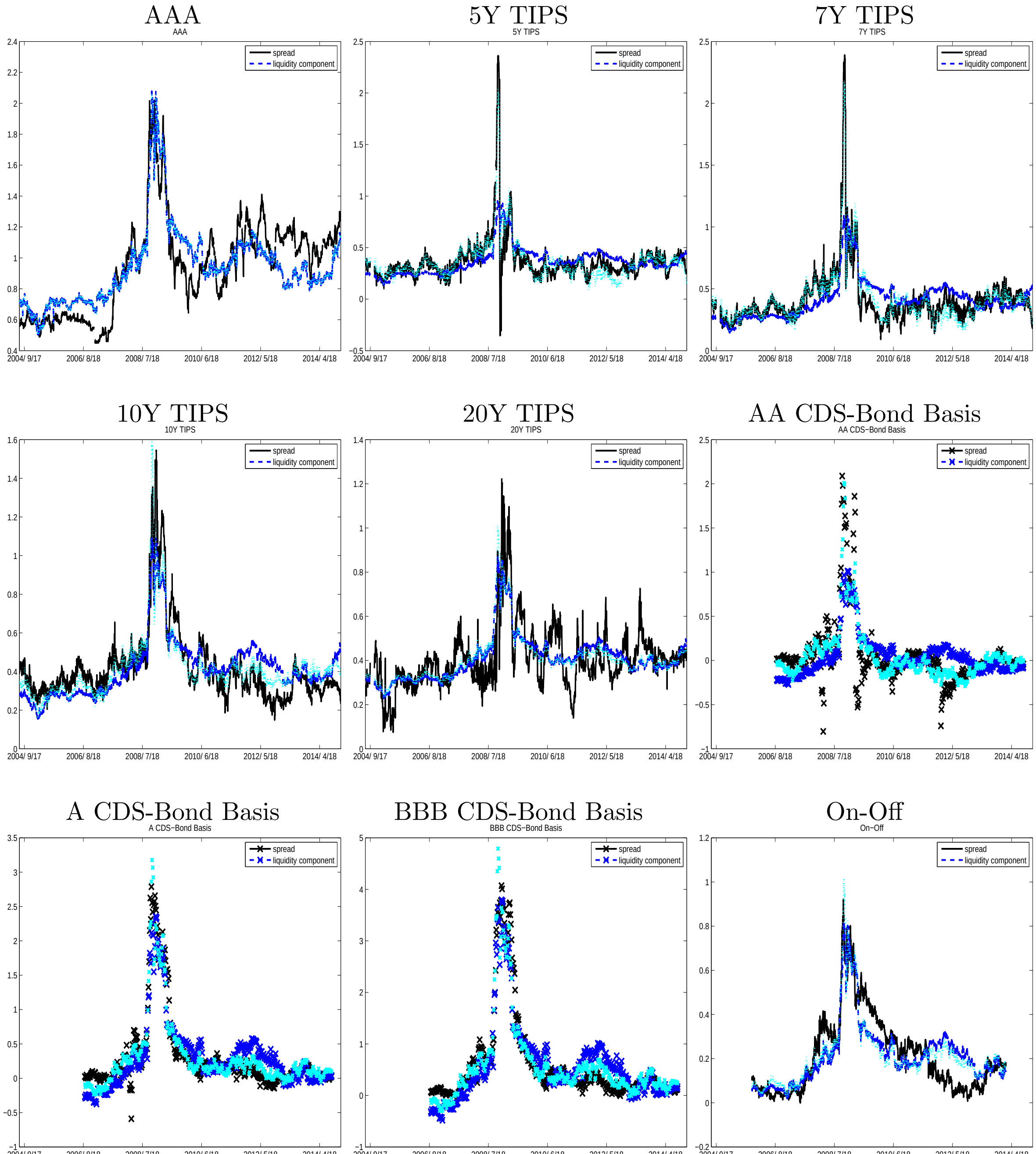
Notes: The figure shows the daily time series (black) of each spread as well as its projection on the first principal component and a constant (blue).

Figure A-8: Liquidity Spreads and Common Factors – Two Factor Model



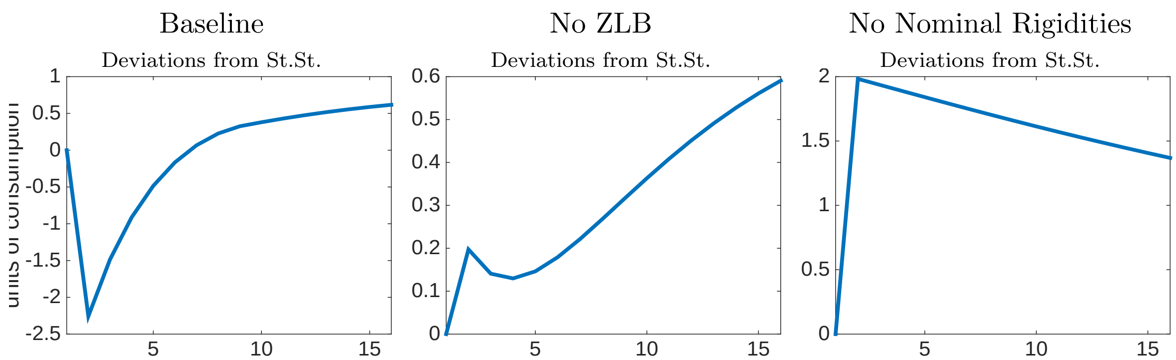
Notes: The figure shows the daily time series (black) of each spread as well as its projection on the first principal component and a constant (dark blue) and on the first two principal components and a constant (light blue).

Figure A-9: Liquidity Spreads and Common Factors – Two Factor Model – Continued



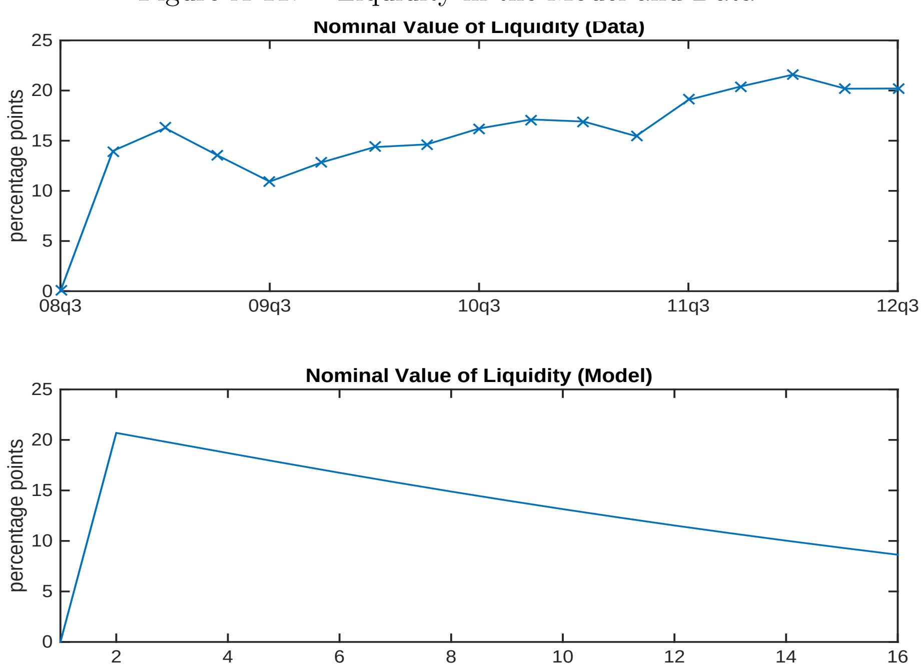
Notes: The figure shows the daily time series (black) of each spread as well as its projection on the first principal component and a constant (dark blue) and on the first two principal components and a constant (light blue).

Figure A-10: Response of q (the Relative Price of Capital in terms of Consumption) to the Baseline Liquidity Shock



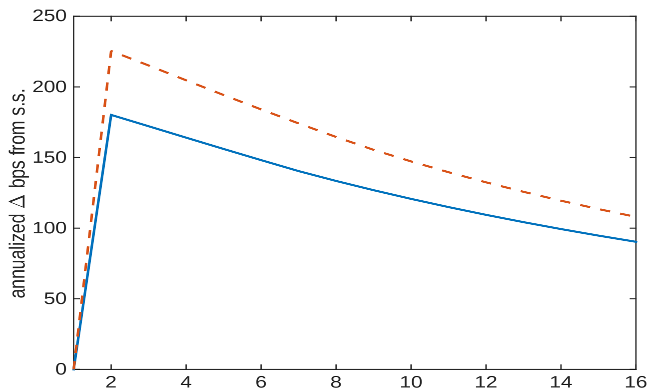
Notes: The figure plots the response of the value of capital to the calibrated liquidity shock in the baseline scenario (left panel), without zero lower bound (middle panel), and without nominal rigidities (right panel).

Figure A-11: Liquidity in the Model and Data



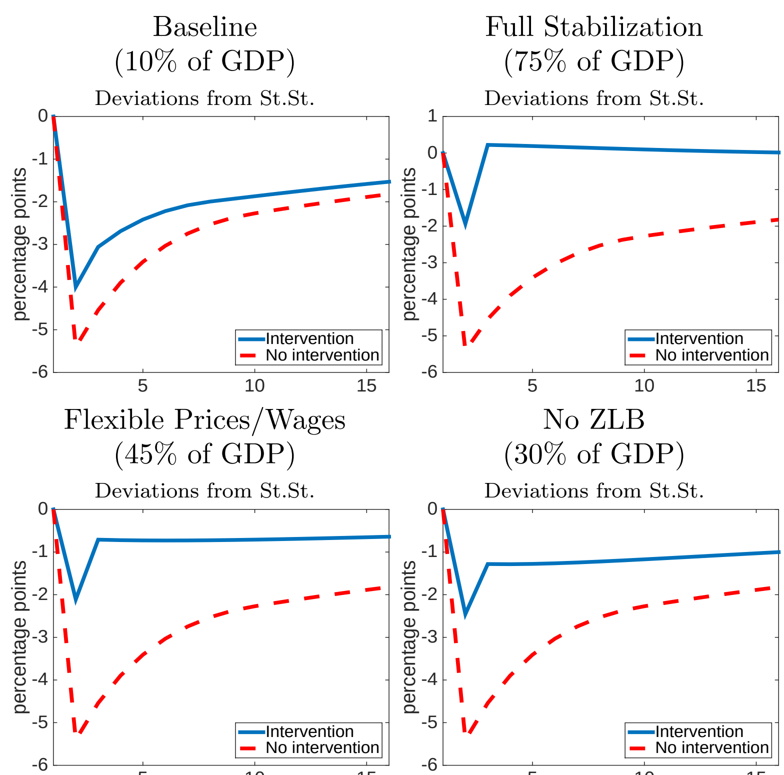
Notes: The top panel plots the evolution of nominal liquidity in the data, as defined in computing the liquidity share. The data are detrended and normalized to zero in 2008Q3. The bottom panel plots the response of nominal liquidity in the model.

Figure A-12: Response of the Convenience Yield with and without Intervention.



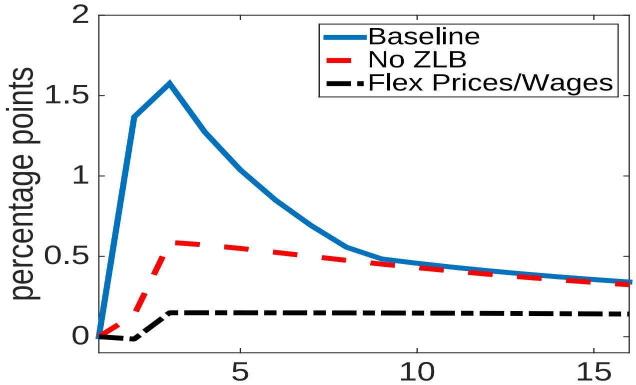
Notes: The continuous blue line reports the response of the convenience yield to the liquidity shock with intervention. The dashed red line reports the response of the convenience yield to the liquidity shock in the absence of intervention.

Figure A-13: Response of Output Under Baseline (10% of GDP), Full Stabilization (75% of GDP), Flex Prices/Wages Replication (45% of GDP), and No ZLB Replication (30% of GDP) Intervention.



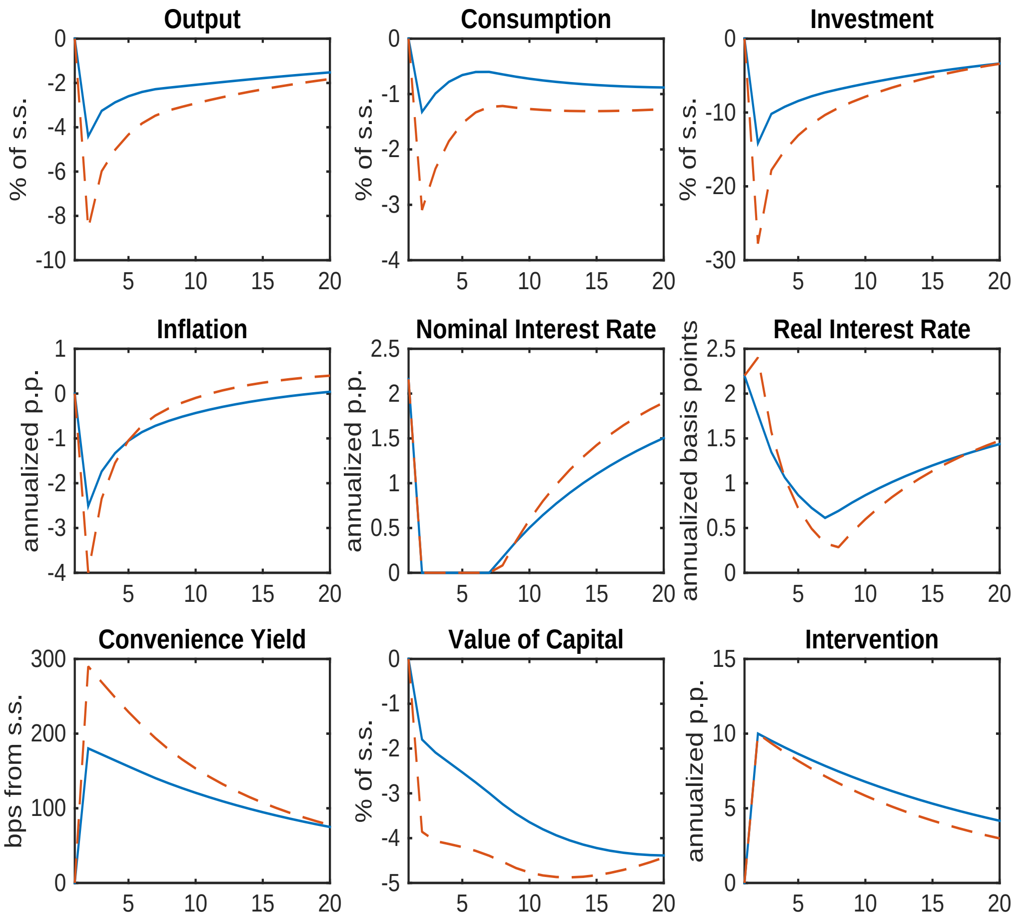
Notes: The figure plots the response of output to the calibrated liquidity shock with (solid blue) and without (dashed red) intervention in four cases. The top-left panel is the baseline scenario. In the top-right panel, we calibrate the initial intervention to fully stabilize output from the second period of the crisis onward. In the bottom-left panel, we calibrate the initial intervention to approximate the path of output without nominal rigidities under the baseline intervention from the second period of the crisis onward. In the bottom-right panel, we calibrate the initial intervention to match the fall in output in the first period in the absence of the ZLB.

Figure A-14: The Effect of the Liquidity Facilities on Output in the Baseline Case, Without the ZLB, and Without Nominal Rigidities



Notes: The figure shows the difference between counterfactual and actual response of output in the baseline scenario (solid blue), without the zero lower bound (dashed red), and without nominal rigidities (dashed-dotted black).

Figure A-15: The Effect of a Large Liquidity Shock



Notes: The figure reports the response of output (top-left), consumption (top-middle), investment (top-right), inflation (center-left), nominal interest rate (center-middle), real interest rate (center-right), convenience yield (bottom-left), value of capital (bottom-middle), and government purchases of private assets (bottom-right) in the baseline case (solid blue) and in response to a shock calibrated to an increase in the convenience yield of 290 basis points annualized (dashed red) with intervention.