Online Appendix for

Information Spillovers in Asset Markets with Correlated Values

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This supplemental appendix considers two extensions of the model analyzed in Asriyan, Fuchs and Green (2016). First, we consider the case in which asset values are perfectly correlated and demonstrate that there is continuity in the limit as the correlation goes to one. Second, we extend our results to the case in which the degree of post-trade transparency is intermediate. We characterize the set of equilibria as it depends on the degree of transparency and demonstrate several welfare comparative statics.

1 Perfect Correlation

The following result shows that the set of equilibria are continuous in the limit as $\lambda \to 1$.

Proposition 1.1 (Perfect Correlation) *The set of equilibria with perfect correlation are generically equivalent, in terms of welfare and trading probability σ, to the limit equilibria with imperfect correlation as $\lambda \to 1$.*

Proof. When correlation is perfect, we may need to specify off-equilibrium beliefs. In particular, suppose that the equilibrium specifies that low type trades w.p.1 in the first period, and that one of the low type sellers deviates and trades in the second period. In this case, buyers can put any probability $\pi^{off} \in [0,1]$ to the remaining seller being a low type. Hence, the continuation value of the low type is the same as before with the exception that λ is set to 1 and buyers' posterior following bad news has the property: $\pi_i(b;\sigma,\sigma)|_{\sigma=1} = \pi^{off}$. There are two sets of equilibria to consider depending on whether the low type seller plays a pure strategy of trading immediately in the first period or a mixed trading strategy. By the same reasoning as before, a symmetric equilibrium with no trade is not possible.

Consider first the equilibria where the low type seller trades probabilistically. There is no off-equilibrium path to consider in this case. Therefore, using the fact that the probabilities of bad news $\{\rho_\theta(b)\}_{\theta=L,H}$ and the posteriors $\{\pi_i(z)\}_{z=b,g}$ are left-continuous in λ at $\lambda = 1$, it is straight-forward to show that these equilibria are the limits of the low and medium trade equilibria in Theorem 1 (proof available upon request).

Consider now the equilibrium in which the low type seller trades w.p.1. in the first period. In this equilibrium, the off-equilibrium beliefs have a bite. In that case, the low type seller receives a payoff v_L and the high type receives a payoff $Q_H^1 = (1-\delta)\, c_H + \delta v_H$, where we denote by Q_θ^λ the continuation value of type θ seller when correlation is λ. This equilibrium exists since the low type seller does not want to deviate when $\pi^{off} = 0$:

$$v_L > Q_L^1 = (1-\delta)\, c_L + \delta v_L.$$

We also know from Theorem 1 that with imperfect correlation the high trade equilibrium exists for λ large. We now show that this equilibrium is the limit of the high trade equilibria in Theorem 1 as $\lambda \to 1$. Recall that in the high trade equilibrium, given $\lambda < 1$, the trading probability must equal x_λ such that $\pi_i(b; x_\lambda, x_\lambda) = \bar{\pi}$. But this implies that $\lim_{\lambda\to 1} x_\lambda = 1$, and thus the trading probabilities converge to the pure strategy of trading immediately. For convergence of welfares, note that the low type has a payoff of v_L for any $\lambda < 1$, while the high type has a payoff:

$$Q_H^{\lambda} = (1-\delta)\, c_H + \delta \left[\rho_H\,(b)\, c_H + (1-\rho_H\,(b))\, V\,(\pi_i(g; x_\lambda, x_\lambda))\right].$$

Since $\lim_{\lambda\to 1} x_\lambda = 1$ implies that $\lim_{\lambda\to 1} \pi_i(g; x_\lambda, x_\lambda) = 1$ and $\lim_{\lambda\to 1} \rho_H\,(b) = 0$, we have that:

$$\lim_{\lambda\to 1} Q_H^{\lambda} = (1-\delta)\, c_H + \delta v_H = Q_H^1.$$

This establishes the result. ■

2 Intermediate Post-trade Transparency

Here, we extend the model and analysis in Section III.A to intermediate levels of post-trade transparency. In particular, we assume that if a transaction for asset j occurs at $t = 1$, then buyers on platform i observe it only with some intermediate probability $\xi \in [0, 1]$. As a result, the event of observing a transaction for asset j in $t = 1$ is bad news, $z_i = b$, whereas the event of not observing a transaction is good news, $z_i = g$.

We can extend our analysis from Sections 2 and 3 by modifying the distribution of news in news in equation (5). In particular, with intermediate transparency, the probability of bad news, conditional on seller i being of type $\theta_i = \theta$ is

$$\rho_\theta^i\,(b) = \xi \sigma_j P\,(\theta_j = L | \theta_i = \theta)\,. \tag{1}$$

Notice that the effect of ξ on buyer's updating (see equation (6)) and seller i's continuation value (see equation (8)) is the same as the effect of σ_j since both enter (as a product $\xi\sigma_j$) only through the distribution of news. It is then straightforward to show that Lemmas 1-4 and Propositions 1-3 extend to the setting with $\xi \in [0,1]$, i.e., all equilibria are symmetric, satisfy equation (9), and are either low, medium, or high trade as labeled in Definition 2.

Our first result extends Theorem 1 to this setting and shows that the level of post-trade transparency needs to be sufficiently high in order for multiple equilibria to arise.

Proposition 2.1 *Suppose that $\delta > \bar{\delta}$ and $\lambda > \overline{\lambda}_\delta$ as in Theorem 1. Then the following hold.*

(i) There exists a $\overline{\xi} < 1$ such that the low, medium, and high trade equilibria as defined in Definition 2 coexist if $\xi > \overline{\xi}$.

(ii) If ξ is sufficiently small, then the equilibrium is unique, low trade, and converges to the one in Proposition 1 as ξ goes to zero.

Proof. The proof of (i) proceeds along the same steps as the proof of Theorem 1, using the continuity of posteriors and news distribution in ξ. For (ii), suppose that there is an equilibrium that features $\pi_i(g) > \bar{\pi}$. In such an equilibrium, the low type's continuation value must satisfy

$$Q_L^i \geq (1-\delta)c_L + \delta(\lambda\xi\sigma v_L + (1-\lambda\xi\sigma)V(\pi_i(g)),$$

where the inequality follows from the supposition that $\pi_i(g) > \bar{\pi}$ and (2). The RHS converges to $(1-\delta)c_L + \delta V(\pi_{\sigma_i}) \geq (1-\delta)c_L + \delta c_H > v_L$ as ξ goes to zero. Thus, for ξ small enough, $Q_L^i > v_L$, which violates Lemma 4 since, by Proposition 3, $\sigma_i > 0$. As a result, any equilibrium must feature $\pi_i(g) = \bar{\pi}$. Clearly, there is a unique $\sigma_i = \sigma_j = \sigma$ that achieves this equality. The convergence of the equilibrium to the one in Proposition 1 then follows from the convergence of posterior $\pi_i(g)$ to π_σ as ξ goes to zero. ■

Our final result extends Proposition 6 regarding the effects of post-trade transparency on welfare and trading volume to the setting with intermediate ξ.

Proposition 2.2 *The following statements hold.*

(i) Introducing a positive level of post-trade transparency to an otherwise opaque market (i.e., increasing ξ from 0 to any strictly positive level) leads to weakly higher welfare.

(ii) If the three equilibria coexist: $Q_H^{high} > Q_H^{med} > Q_H^{low}$. Moreover, Q_H^{high} increases with ξ while σ^{high} is independent of ξ, both Q_H^{med} and σ^{med} decrease with ξ, and $Q_H^{low} = c_H$ for all ξ while σ^{low} decreases with ξ.

Proof. If $\xi = 0$, then the equilibrium is unique as in Proposition 1 and the high type's payoff is c_H. If $\xi > 0$, the high type's payoff is weakly greater than c_H, and strictly so if $\pi_i(g) > \bar{\pi}$, i.e., if either medium or high trade equilibrium exists and is played. The argument for welfare ranking across the three equilibria (whenever these coexist) is the same as in the proof of Proposition 5.

In order to prove the comparative statics with respect to ξ, several properties are useful to note:

(a) Q_L^i is strictly decreasing in σ at σ^{med} when the three equilibria coexist (see argument in proof of Theorem 1);

(b) Q_L^i is strictly decreasing in $\xi\sigma_j$ when $\pi_i(b) < \bar{\pi} < \pi_i(g)$ by an argument analogous to that for the effect of σ_j on Q_L^i in Lemma A.1(i);

(c) Q_H^i is strictly increasing in $\xi\sigma_j$ when $\pi_i(b) \leq \bar{\pi} \leq \pi_i(g)$ also by an argument analogous to that for the effect of σ_j on Q_H^i in Lemma A.1(ii).

In the low trade equilibrium, $\pi_i(g) = \overline{\pi}$ and thus $Q_H^{low} = c_H$ independently of ξ. As $\pi_i(g)$ increases in ξ and in σ, it follows that σ^{low} decreases in ξ.

In the high trade equilibrium, $\pi_i(b) = \bar{\pi}$ and $\pi_i(b)$ is independent of ξ, and it thus follows that σ^{high} is independent of ξ. Since σ^{high} remains unchanged as ξ increases, by property (c) it follows that Q_H^{high} increases in ξ.

In the medium trade equilibrium, holding σ_i and σ_j fixed, an increase in ξ to $\xi' > \xi$ would imply, via property (b), that $Q_L^i < v_L$. Now, leaving σ_i fixed, suppose one were to decrease σ_j to σ_j' such that $\xi'\sigma_j' = \xi\sigma_j$. From (b), we would have that $Q_L^i = v_L$ but $Q_L^j < v_L$, since Q_L^j is decreasing in σ_j. Next suppose we decreased σ_i such that $\sigma_i' = \sigma_j'$; this would imply that $Q_L^i = Q_L^j < v_L$. From (a), in order to have $Q_L^i = Q_L^j = v_L$, it must be that the new equilibrium trading probability decreases below σ_j'. Hence, this implies that σ^{med} and $\xi\sigma^{med}$ decrease in ξ, which via (c) implies that Q_H^{med} decreases in ξ. ■

References

Asriyan, Vladimir, William Fuchs, and Brett Green. 2016. "Information Spillovers in Asset Markets with Correlated Values." *American Economic Review (forthcoming).*