

A Simple Explanation of Countercyclical Uncertainty

Online Appendix*

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ABSTRACT

This appendix provides the derivations for the Nash bargaining equation and employment uncertainty, our data sources, a description of the solution and estimation methods, and a proof of the Hosios condition. It also shows our results are robust to including home production and variable search intensity. Finally, it shows whether capital adjustment costs create time-varying uncertainty and the implications of our results for VAR estimates under recursive identification.

Keywords: Endogenous Uncertainty; Uncertainty Shocks; Variance Decomposition; Nonlinear

JEL Classifications: C13; D81; E32; E37; J64

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A NASH BARGAINING

To derive the wage rate under Nash bargaining, define the total surplus of a new match as $\Lambda_t = \lambda_{n,t} + J_{N,t}^H - J_{U,t}^H$. $J_{N,t}^H$ and $J_{U,t}^H$ satisfy the employment and unemployment envelope conditions,

$$\begin{aligned} J_{N,t}^H &= w_t + E_t[x_{t+1}((1 - \bar{s}(1 - \chi f_{t+1}))J_{N,t+1}^H + \bar{s}(1 - \chi f_{t+1})J_{U,t+1}^H)], \\ J_{U,t}^H &= b + E_t[x_{t+1}(f_{t+1}J_{N,t+1}^H + (1 - f_{t+1})J_{U,t+1}^H)], \end{aligned}$$

which are derived from the household's optimization problem given the following laws of motion:

$$\begin{aligned} n_{t+1} &= (1 - \bar{s}(1 - \chi f_{t+1}))n_t + f_{t+1}u_t, \\ u_{t+1} &= \bar{s}(1 - \chi f_{t+1})n_t + (1 - f_{t+1})u_t. \end{aligned}$$

The equilibrium wage rate maximizes $(J_{N,t}^H - J_{U,t}^H)\eta\lambda_{n,t}^{1-\eta}$, where $\eta \in [0, 1]$ is the household's bargaining weight. Optimality implies $J_{N,t}^H - J_{U,t}^H = \eta\Lambda_t$ and $\lambda_{n,t} = (1 - \eta)\Lambda_t$. After combining the optimality conditions with $J_{N,t}^H$, $J_{U,t}^H$, and (12), and defining tightness as $\theta_t = v_t/u_t^s$, we obtain

$$w_t = \eta((1 - \alpha)y_t/n_t + \kappa(1 - \chi\bar{s})E_t[x_{t+1}\theta_{t+1}]) + (1 - \eta)b.$$

The household's wage rate in period t is a weighted average of the firm's value of a new match and the worker's outside option b . The firm's value of a new worker includes the additional output produced plus the discounted expected value of the worker net of separations that occur in period $t + 1$.

B EMPLOYMENT UNCERTAINTY DERIVATION

Under the assumptions in Section 4, the model collapses to

$$u_t^s = 1 - (1 - \chi\bar{s})n_{t-1}, \tag{B.1}$$

$$m_t = \xi(u_t^s)^\phi v_t^{1-\phi}, \tag{B.2}$$

$$n_t = (1 - \bar{s})n_{t-1} + m_t, \tag{B.3}$$

$$q_t = m_t/v_t, \tag{B.4}$$

$$\lambda_{n,t} = a_t - b + \beta(1 - \bar{s})E_t[\lambda_{n,t+1}], \tag{B.5}$$

$$\lambda_{n,t} = \kappa/q_t, \tag{B.6}$$

$$a_t = (1 - \rho_a)\bar{a} + \rho_a a_{t-1} + \sigma_{a,t-1}\varepsilon_{a,t}, \tag{B.7}$$

$$\sigma_{a,t} = (1 - \rho_{sv})\bar{\sigma}_a + \rho_{sv}\sigma_{a,t-1} + \sigma_{sv}\varepsilon_{sv,t}. \tag{B.8}$$

Note that the TFP processes are written in levels rather than logs to permit a closed-form solution.

Guess a solution of the form $\lambda_{n,t} = \delta_0 + \delta_1(a_t - \bar{a})$. Equating coefficients in (B.5) implies

$$\delta_0 = \frac{\bar{a} - b}{1 - \beta(1 - \bar{s})} > 0, \quad \delta_1 = \frac{1}{1 - \beta(1 - \bar{s})\rho_a} > 0.$$

Therefore, match value uncertainty, $\sqrt{V_t[\lambda_{n,t+1}]} = \delta_1\sigma_{a,t}$. From (B.2), (B.4), and (B.6), we obtain

$$q_t = \xi(u_t^s/v_t)^\phi = \kappa/\lambda_{n,t} \quad \rightarrow \quad v_t = u_t^s(\xi\lambda_{n,t}/\kappa)^{1/\phi}.$$

Therefore, the job finding rate is given by

$$f_t = m_t/u_t^s = q_t v_t/u_t^s = (\xi\lambda_{n,t}/\kappa)^{1/\phi}(\kappa/\lambda_{n,t}) = \xi^{1/\phi}(\lambda_{n,t}/\kappa)^{(1-\phi)/\phi}.$$

If we further assume $\phi = 0.5$, then uncertainty surrounding the job finding rate is given by

$$\sqrt{V_t[f_{n,t+1}]} = (\xi^2/\kappa)\delta_1\sigma_{a,t}.$$

From (B.3), employment uncertainty is then given by

$$\sqrt{V_t[\hat{n}_{t+1}]} = \frac{1}{n_t}u_{t+1}^s\sqrt{V_t[f_{t+1}]} = \frac{1}{n_t}u_{t+1}^s(\delta_1\xi^2/\kappa)\sigma_{a,t},$$

where the number of unemployed searching is predetermined according to (B.1).

C DATA SOURCES AND TRANSFORMATIONS

We use the following time-series from 1963-2019 provided by Haver Analytics (2021):

1. **Civilian Noninstitutional Population: 16 Years & Over**
Not Seasonally Adjusted, Quarterly, Thousands (LN16N@USECON)
2. **Gross Domestic Product: Implicit Price Deflator**
Seasonally Adjusted, Quarterly, 2012=100 (DGDP@USNA)
3. **Gross Domestic Product**
Seasonally Adjusted, Quarterly, Billions of Dollars (GDP@USECON)
4. **Personal Consumption Expenditures: Nondurable Goods**
Seasonally Adjusted, Quarterly, Billions of Dollars (CN@USECON)
5. **Personal Consumption Expenditures: Services**
Seasonally Adjusted, Quarterly, Billions of Dollars (CS@USECON)
6. **Personal Consumption Expenditures: Durable Goods**
Seasonally Adjusted, Quarterly, Billions of Dollars (CD@USECON)

7. **Private Fixed Investment**
Seasonally Adjusted, Quarterly, Billions of Dollars (F@USECON)
8. **Output Per Person**, Non-farm Business Sector, All Persons,
Seasonally Adjusted, Quarterly, 2012=100 (LXNFS@USNA)
9. **Labor Share**, Non-farm Business Sector, All Persons,
Seasonally Adjusted, Quarterly, Percent (LXNFBL@USNA)
10. **Unemployed, 16 Years & Over**
Seasonally Adjusted, Monthly, Thousands (LTU@USECON)
11. **Civilian Labor Force: 16 yr & Over**
Seasonally Adjusted, Monthly, Thousands (LF@USECON)
12. **Civilians Unemployed for Less Than 5 Weeks**
Seasonally Adjusted, Monthly, Thousands (LU0@USECON)
13. **Job Openings**, Job Openings and Labor Turnover Survey,
Seasonally Adjusted, Monthly, Thousands (LJJTLA@USECON)
14. **SPF Forecast Dispersion: Real GDP Growth**,
Quarterly, 1-Quarter Ahead Growth Rate (ASAQ1GC@SURVEYS)
15. **CBOE Market Volatility Index: VIX**, Monthly, Index (SPVIX@USECON)
16. **Net Stock: Private Fixed Assets**, Annual, Billions of Dollars (EPT@CAPSTOCK)
17. **Net Stock: Durable Goods**, Annual, Billions of Dollars (EDT@CAPSTOCK)
18. **Depreciation: Private Fixed Assets**, Annual, Billions of Dollars (KPT@CAPSTOCK)
19. **Depreciation: Durable Goods**, Annual, Billions of Dollars (KDT@CAPSTOCK)

We also used the following data from other sources:

1. **Help Wanted Advertising Index (HWI)**, based on Barnichon (2010). The series is available at <https://sites.google.com/site/regisbarnichon/> (Barnichon, 2020).
2. **Real Uncertainty**, 3-month horizon, based on Ludvigson et al. (2021). The series is available at <https://www.sydneyludvigson.com/> (Ludvigson and Ng, 2022). The monthly series is averaged to a quarterly frequency.
3. **FRED-MD**, Monthly Databases for Macroeconomic Research. The data is available at <https://research.stlouisfed.org/econ/mccracken/fred-databases/> (McCracken, 2021). Under Monthly Data, we use the June 2021 vintage.
4. **Fama-French**, Data Library. The data is available at https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html (Fama and French, 2021).

5. **WRDS**, Stock Market Indexes and Fama-Bliss Discount Bonds. Data is available at <https://wrds-www.wharton.upenn.edu> (Wharton Research Data Services, 2021).

We applied the following transformations to the above data sources:

1. **Per Capita Real Output:** $Y_t = GDP_t / (DGDP_t \times LN16N_t)$.
2. **Per Capita Real Consumption:** $C_t = (CN_t + CS_t) / (DGDP_t \times LN16N_t)$.
3. **Per Capita Real Investment:** $I_t = (F_t + CD_t) / (DGDP_t \times LN16N_t)$.
4. **Unemployment Rate:** $U_t = 100(LTU_t / LF_t)$.
5. **Vacancy Rate:** HWI from 1963M1-2000M12 and $LJTTLA / LF$ from 2001M1-2019M12.
6. **Short-term Unemployed (U^s):** The redesign of the Current Population Survey (CPS) in 1994 reduced u_t^s . To correct for this bias, we follow Elsby et al. (2009) and scale u_t^s by the time average of the ratio of u_t^s / u_t for the first and fifth rotations groups to u_t^s / u_t across all rotation groups. Using IPUMS-CPS data (Flood et al., 2022), we extract EMPSTAT (“Employment Status”), DURUNEMP (“Continuous weeks unemployed”) and MISH (“Month in sample, household level”). Unemployed persons have EMPSTAT equal to 20, 21, or 22. Short-term unemployed are persons who are unemployed and have DURUNEMP equal to 4 or less. Incoming rotation groups have MISH equal to 1 or 5. Using the final weights, WTFINL, we calculate unemployment rates conditional on the appropriate values of MISH and DURUNEMP. We then apply the X-12 seasonal adjustment function in STATA to the time series for the ratio. Finally, we take an average of the seasonally adjusted series from 1994-2019. This process yields an average of 1.1725, so U^s equals $LU0$ prior to 1994 and $1.1725 \times LU0$ after 1994.
7. **Job-Finding Rate:** Following Shimer (2005), $f_t = 1 - (LTU_{t+1} - U_{t+1}^s) / LTU_t$.
8. **Job Separation Rate:** Following Shimer (2012), $s_t = 1 - \exp(-\tilde{s}_t)$, where \tilde{s}_t satisfies

$$LTU_{t+1} = \frac{(1 - \exp(-\tilde{f}_t - \tilde{s}_t))\tilde{s}_t LF_t}{\tilde{f}_t + \tilde{s}_t} + \exp(-\tilde{f}_t - \tilde{s}_t)LTU_t, \quad \tilde{f}_t = -\log(1 - f_t).$$

9. **Real Wage:** Following Hagedorn and Manovskii (2008), $w_t = LXNFB L_t \times LXNFS_t$.
10. **Wage Elasticity:** Slope coefficient from regressing w_t on an intercept and $LXNFS_t$.
11. **Depreciation Rate:** $\delta = (1 + \frac{1}{T/12} \sum_{t=1}^{T/12} (KPT_t + KDT_t) / (EPT_{t-1} + EDT_{t-1}))^{1/12} - 1$.
12. **Capital Share of Income:** $\alpha = 1 - \frac{1}{T/3} \sum_{t=1}^{T/3} LXNFB L_t$.
13. **Inflation Rate:** $\bar{\pi} = \frac{1}{T/3} \sum_{t=1}^{T/3} (1 + \log(DGDP_t / DGDP_{t-1}))^{1/3}$.

All monthly time series are averaged to a quarterly frequency. The data is detrended using a Hamilton filter with an 8 quarter window. All empirical targets are computed using quarterly data.

D SOLUTION METHOD

The equilibrium system of the model is summarized by $E[g(\mathbf{x}_{t+1}, \mathbf{x}_t, \varepsilon_{t+1}) | \mathbf{z}_t, \vartheta] = 0$, where g is a vector-valued function, \mathbf{x}_t is a vector of variables, ε_t is a vector of shocks, \mathbf{z}_t is a vector of states, and ϑ is a vector of parameters. There are many ways to discretize the TFP level shock and volatility process. We use the Markov chain in Rouwenhorst (1995), which Kopecky and Suen (2010) show outperforms other methods for approximating autoregressive processes. For our estimated model, the bounds on a_t and k_{t-1} are set to $\pm 8\%$ of their deterministic steady states, while n_{t-1} ranges from 0.88 to 0.98. These bounds ensure that simulations contain at least 99% of the ergodic distribution. We specify 9 states for $\sigma_{a,t}$, 9 states for $\varepsilon_{a,t+1}$, and discretize a_t , k_{t-1} , and n_{t-1} into 13, 9, and 9 evenly-spaced points, respectively.¹ The product of the points in each dimension, D , is the total nodes in the state space ($D = 9,477$). The realization of \mathbf{z}_t on node d is denoted $\mathbf{z}_t(d)$. The Rouwenhorst method provides integration nodes, $[\varepsilon_{a,t+1}(m), \sigma_{a,t+1}(m)]$, with weights, $\phi(m)$, for $m \in \{1, \dots, M\}$. The realizations of $\sigma_{a,t+1}$ are the same as $\sigma_{a,t}$ because it is a Markov chain.

Since vacancies $v_t \geq 0$, we introduce an auxiliary variable, μ_t , such that $v_t = \max\{0, \mu_t\}^2$ and $\lambda_t = \max\{0, -\mu_t\}^2$, where λ_t is the Lagrange multiplier on the non-negativity constraint. If $\mu_t \geq 0$, then $v_t = \mu_t^2$ and $\lambda_t = 0$. When $\mu_t < 0$, the constraint is binding, $v_t = 0$, and $\lambda_t = \mu_t^2$. Therefore, the constraint on v_t is transformed into a pair of equalities (Garcia and Zangwill, 1981).

The vector of policy functions and the realization on node d are denoted by \mathbf{pf}_t and $\mathbf{pf}_t(d)$, where $\mathbf{pf}_t \equiv [\mu_{v,t}(\mathbf{z}_t), c_t(\mathbf{z}_t)]$. The following steps outline our policy function iteration algorithm:

1. Use Sims's (2002) `gensys` algorithm to solve the log-linear model. Then map the solution for the policy functions to the discretized state space. This provides an initial conjecture.
2. On iteration $j \in \{1, 2, \dots\}$ and each node $d \in \{1, \dots, D\}$, use Chris Sims's `csolve` to find $\mathbf{pf}_t(d)$ to satisfy $E[g(\cdot) | \mathbf{z}_t(d), \vartheta] \approx 0$. Guess $\mathbf{pf}_t(d) = \mathbf{pf}_{j-1}(d)$. Then apply the following:
 - (a) Solve for all variables dated at time t , given $\mathbf{pf}_t(d)$ and $\mathbf{z}_t(d)$.
 - (b) Linearly interpolate the policy functions, \mathbf{pf}_{j-1} , at the updated state variables, $\mathbf{z}_{t+1}(m)$, to obtain $\mathbf{pf}_{t+1}(m)$ on every integration node, $m \in \{1, \dots, M\}$.
 - (c) Given $\{\mathbf{pf}_{t+1}(m)\}_{m=1}^M$, solve for the other elements of $\mathbf{s}_{t+1}(m)$ and compute

$$E[g(\mathbf{x}_{t+1}, \mathbf{x}_t(d), \varepsilon_{t+1}) | \mathbf{z}_t(d), \vartheta] \approx \sum_{m=1}^M \phi(m) g(\mathbf{x}_{t+1}(m), \mathbf{x}_t(d), \varepsilon_{t+1}(m)).$$

When `csolve` converges, set $\mathbf{pf}_j(d) = \mathbf{pf}_t(d)$.

3. Repeat step 2 until $\text{maxdist}_j < 10^{-6}$, where $\text{maxdist}_j \equiv \max\{|\mathbf{pf}_j - \mathbf{pf}_{j-1}|\}$. When that criterion is satisfied, the algorithm has converged to an approximate nonlinear solution.

¹We also tried using a grid that was more than 10 times denser, but it had very little effect on our quantitative results.

E ESTIMATION METHOD

The estimation procedure has two stages. The first stage estimates moments in the data using a 2-step Generalized Method of Moments (GMM) estimator with a Newey and West (1987) weighting matrix with 5 lags. The second stage is a Simulated Method of Moments (SMM) procedure that searches for a parameter vector that minimizes the distance between the GMM estimates in the data and short-sample predictions of the model, weighted by the diagonal of the GMM estimate of the variance-covariance matrix. The second stage is repeated for many different draws of shocks to obtain the standard errors on the parameter estimates. The following steps outline the algorithm:

1. Use GMM to estimate the targets, $\hat{\Psi}_T^D$, and the diagonal of the covariance matrix, $\hat{\Sigma}_T^D$.
2. Use SMM to estimate the nonlinear DMP model. Given a random seed, h , draw a $B + T$ period sequence for each shock in the model, where $B = 1,000$ is a burn-in period and $T = 687$ is the length of the monthly time series. Denote the shock matrix by $\mathcal{E}^s = [\varepsilon_a^s, \varepsilon_{sv}^s]_{t=1}^{B+T}$. For shock sequence $s \in \{1, \dots, N_s\}$, run the following steps:

- (a) Evaluate the loss function for $i \in \{1, \dots, N_m\}$ random draws in the parameter space.
 - i. Draw a vector of parameters $\hat{\mathcal{P}}_i$ from a multivariate normal distribution centered at a specified mean parameter vector, $\bar{\mathcal{P}}$, with diagonal covariance matrix, Σ_0 .
 - ii. Solve the model using the algorithm in [Appendix D](#) given $\hat{\mathcal{P}}_i$. Return to step i if the linear solution does not exist or the nonlinear algorithm does not converge.
 - iii. Given $\mathcal{E}^s(r)$, simulate the model R times for $B + T$ periods. We draw initial states from the ergodic distribution by burning off the first B periods and aggregate to a quarterly frequency. For each repetition r , calculate the moments $\Psi_T^M(\hat{\mathcal{P}}_i, \mathcal{E}^s(r))$.
 - iv. Calculate the mean moments $\bar{\Psi}_{R,T}^M(\hat{\mathcal{P}}_i, \mathcal{E}^s) = \frac{1}{R} \sum_{r=1}^R \Psi_T^M(\hat{\mathcal{P}}_i, \mathcal{E}^s(r))$ and the fit

$$J_i = [\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\hat{\mathcal{P}}_i, \mathcal{E}^s)]' [\hat{\Sigma}_T^D (1 + 1/R)]^{-1} [\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\hat{\mathcal{P}}_i, \mathcal{E}^s)].$$

- (b) Find a guess, $\hat{\mathcal{P}}_0$, for the N_p estimated parameters and the covariance matrix, Σ_0 :
 - i. Find the parameter draw $\hat{\mathcal{P}}_0$ that corresponds to $\min\{J_i\}_{i=1}^{N_m}$.
 - ii. Find all J_i below the median, stack the corresponding draws in a $N_m/2 \times N_p$ matrix, $\hat{\Theta}$, and define the (i, j) element as $\tilde{\Theta}_{i,j} = \hat{\Theta}_{i,j} - \sum_{i=1}^{N_m/2} \hat{\Theta}_{i,j} / (N_m/2)$.
 - iii. Calculate $\Sigma_0 = \tilde{\Theta}' \tilde{\Theta} / (N_m/2)$.
- (c) Minimize J with simulated annealing. For $i \in \{0, \dots, N_d\}$, repeat the following steps:

i. Draw a candidate vector of parameters, $\hat{\mathcal{P}}_i^{cand}$, where

$$\hat{\mathcal{P}}_i^{cand} \sim \begin{cases} \hat{\mathcal{P}}_0 & \text{for } i = 0, \\ \text{N}(\hat{\mathcal{P}}_{i-1}, c_0 \Sigma_0) & \text{for } i > 0. \end{cases}$$

We set c_0 to target an average acceptance rate of 50% across seeds.

ii. Repeat steps 2a, ii-iv.

iii. Accept or reject the candidate draw according to

$$(\hat{\mathcal{P}}_i, J_i) = \begin{cases} (\hat{\mathcal{P}}_i^{cand}, J_i^{cand}) & \text{if } i = 0, \\ (\hat{\mathcal{P}}_i^{cand}, J_i^{cand}) & \text{if } \min(1, \exp(J_{i-1} - J_i^{cand})/c_1) > \hat{u}, \\ (\hat{\mathcal{P}}_{i-1}, J_{i-1}) & \text{otherwise,} \end{cases}$$

where c_1 is the temperature and \hat{u} is a draw from a uniform distribution.

(d) Find $\hat{\mathcal{P}}_0^{up}$ and Σ_0^{up} following step 2b.

(e) Repeat steps 2c-d N_{SMM} times, initializing at $\hat{\mathcal{P}}_0 = \hat{\mathcal{P}}_0^{up}$ and $\Sigma_0 = \Sigma_0^{up}$. Gradually decrease the temperature. Across all N_{SMM} stages, find the lowest J value, denoted J^{guess} , and the corresponding draw, \mathcal{P}^{guess} .

(f) Minimize the same loss function with MATLAB's `fminsearch` starting at \mathcal{P}^{guess} . The minimum is $\hat{\mathcal{P}}^{\min}$ with a loss function value of J^{\min} . Repeat, each time updating the guess, until $J^{guess} - J^{\min} < 0.001$. The parameter estimates correspond to J^{\min} .

Given $\{\hat{\mathcal{P}}^s\}_{s=1}^{N_s}$, we report the mean, $\bar{\mathcal{P}} = \sum_{s=1}^{N_s} \hat{\mathcal{P}}^s / N_s$ and standard errors of the estimates.

The reported moments are then based on the mean parameter estimates, $\bar{\Psi}_T^M = \bar{\Psi}_{R,T}^M(\bar{\mathcal{P}}, \mathcal{E})$.

We set $N_s = 200$, $R = 1,000$, $N_{SMM} = 3$, $N_m = 1,000$, $N_d = 500$, and $N_p = 10$. For each simulated annealing stage, c_0 is 0.1, 0.7, and 1.0, and c_1 is 1.0, 0.5, and 0.25, respectively. The algorithm was programmed in Fortran and executed with Open MPI on the BigTex supercomputer.

F HOME PRODUCTION

This section shows our results are robust to alternative sources of unemployment volatility. To demonstrate this, we extend our baseline model to include home production following Petrosky-Nadeau et al. (2018). The representative household derives utility from the consumption of both the final market good $c_{m,t}$ and home production $c_{h,t}$. It has log utility over composite consumption $c_t = (\omega c_{m,t}^e + (1 - \omega) c_{h,t}^e)^{1/e}$, where $\omega \in (0, 1)$ is the preference weight on the market good and $e \leq 1$ governs the elasticity of substitution $1/(1 - e)$. The home production technology is $c_{h,t} = a_h u_t$, where $a_h > 0$ is productivity. The rest of the model is identical to the baseline model.

Household optimization yields the pricing kernel $x_{t+1} = \beta(c_{m,t}/c_{m,t+1})^{1-e}(c_t/c_{t+1})^e$. The flow value of unemployment becomes $z_t = a_h((1-\omega)/\omega)(c_{m,t}/c_{h,t})^{1-e} + b$, so the Nash wage satisfies

$$w_t = \eta((1-\alpha)y_t/n_t + \kappa(1-\chi\bar{s})E_t[x_{t+1}\theta_{t+1}]) + (1-\eta)z_t.$$

The remaining equilibrium conditions are unchanged from the baseline model shown in Section 3.

We set $b = 0.4$ to reflect the value of unemployment benefits (Shimer, 2005). The remaining baseline parameters are set to their estimated values shown in Table 2a. We set a_h to the steady-state marginal product of labor in final good production. We then calibrate $\omega = 0.7$ and $e = 0.9$ to target the standard deviations of unemployment and final good output in the baseline DMP model.

| Moment | Baseline | Home Production |
|--------------------------------|----------|-----------------|
| $SD(\mathcal{U})$ | 6.06 | 5.84 |
| $SD(\tilde{y})$ | 3.65 | 3.64 |
| $SD(\tilde{u})$ | 21.14 | 21.29 |
| $Corr(\mathcal{U}, \tilde{y})$ | -0.62 | -0.59 |

Table F.1: Key Moments

| Contribution | Output | Uncertainty |
|-------------------|--------|-------------|
| Level Total | 100.00 | 36.72 |
| Volatility Total | 0.19 | 63.50 |
| Level Direct | 99.81 | 36.50 |
| Volatility Direct | 0.00 | 63.28 |

Table F.2: Home production variance decomposition

Results Table F.1 compares the moments from the home production model to the baseline model and Table F.2 shows the variance decomposition. The model continues to generate a strong negative correlation between output and uncertainty and similar business cycle moments. Level shocks continue to explain almost all of the output variance and about 40% of the uncertainty variance. Crucially, these results stem from a much lower value of b that only reflects unemployment benefits, as in Shimer (2005). This shows our mechanism is robust to other sources of labor market volatility.

G EFFECT OF CAPITAL ADJUSTMENT COSTS

Given the importance of the employment law of motion, this section studies the effect of adjustment costs in the capital law of motion using textbook Real Business Cycle and New Keynesian models.

G.1 REAL BUSINESS CYCLE MODEL A representative household chooses $\{c_t, n_t, k_t\}_{t=0}^{\infty}$ to maximize expected lifetime utility, $E_0 \sum_{t=0}^{\infty} \beta^t [\ln c_t - \vartheta \frac{n_t^{1+\gamma}}{1+\gamma}]$, where ϑ determines steady-state labor hours and $1/\gamma$ is the Frisch elasticity of labor supply. The household's choices are constrained by $c_t + i_t = w_t n_t + r_t^k k_{t-1}$ and the law of motion for capital in (9). Optimality implies (10) and

$$w_t = \vartheta n_t^\gamma c_t. \tag{G.1}$$

The representative firm produces output with the following technology,

$$y_t = a_t k_{t-1}^\alpha n_t^{1-\alpha}, \quad (\text{G.2})$$

where α is the capital share of income. The firm chooses $\{n_t, k_{t-1}\}$ to maximize current profits, $y_t - w_t n_t - r_t^k k_{t-1}$, subject to the production function. The two optimality conditions are given by

$$w_t = (1 - \alpha)y_t/n_t, \quad (\text{G.3})$$

$$r_t^k = \alpha y_t/k_{t-1}. \quad (\text{G.4})$$

The aggregate resource constraint is given by

$$c_t + i_t = y_t. \quad (\text{G.5})$$

A competitive equilibrium consists of infinite sequences of quantities $\{y_t, k_t, c_t, n_t, i_t\}_{t=0}^\infty$, prices $\{w_t, r_t^k\}_{t=0}^\infty$, and exogenous variables $\{a_t, \sigma_{a,t}\}_{t=0}^\infty$ that satisfy (1), (2), (9), (10), and (G.1)-(G.5), given the state of the economy $\{k_{-1}, a_{-1}, \sigma_{a,-1}\}$ and the sequences of TFP shocks $\{\varepsilon_{a,t}, \varepsilon_{\sigma_{a,t}}\}_{t=1}^\infty$.

G.2 NEW KEYNESIAN MODEL The production sector now consists of a continuum of monopolistically competitive intermediate firms and a representative final good firm. Intermediate firm $f \in [0, 1]$ produces a differentiated good, $y_{f,t} = a_t k_{f,t-1}^\alpha n_{f,t}^{1-\alpha}$, where $n_{f,t}$ and $k_{f,t-1}$ are the labor and capital inputs used by firm f . The final good firm purchases output from each intermediate firm to produce the final good, $y_t \equiv [\int_0^1 y_{f,t}^{(\theta-1)/\theta} df]^\theta / (\theta-1)$, where $\theta > 1$ is the elasticity of substitution.

Profit maximization by the final good firm determines the demand for intermediate good f , $y_{f,t} = (p_{f,t}/p_t)^{-\theta} y_t$, where $p_t = [\int_0^1 p_{f,t}^{1-\theta} df]^{1/(1-\theta)}$ is the price level. Following Rotemberg (1982), intermediate firms pay a price adjustment cost, $\Lambda_{f,t}^p \equiv \varphi(p_{f,t}/(\bar{\pi} p_{f,t-1}) - 1)^2 y_t / 2$, where $\varphi > 0$ scales the cost and $\bar{\pi}$ is the steady-state inflation rate. Given this cost, the value of firm f satisfies

$$V_{f,t} = \max_{n_{f,t}, k_{f,t-1}, p_{f,t}} p_{f,t} y_{f,t} / p_t - w_t n_{f,t} - r_t^k k_{f,t-1} - \Lambda_{f,t}^p + E_t[x_{t+1} V_{f,t+1}],$$

subject to $y_{f,t} = a_t k_{f,t-1}^\alpha n_{f,t}^{1-\alpha}$ and $y_{f,t} = (p_{f,t}/p_t)^{-\theta} y_t$. In a symmetric equilibrium where $p_{f,t} = p_t$, optimality implies the input demand schedules and New Keynesian Phillips curve are given by

$$w_t = (1 - \alpha) m c_t y_t / n_t, \quad (\text{G.6})$$

$$r_t^k = \alpha m c_t y_t / k_{t-1}, \quad (\text{G.7})$$

$$\varphi(\pi_t/\bar{\pi} - 1)(\pi_t/\bar{\pi}) = 1 - \theta + \theta m c_t + \varphi E_t[x_{t+1}(\pi_{t+1}/\bar{\pi} - 1)(\pi_{t+1}/\bar{\pi}) y_{t+1}/y_t], \quad (\text{G.8})$$

where $\pi_t = p_t/p_{t-1}$ is the gross inflation rate. If $\varphi = 0$, then the real marginal cost of producing a unit of output, $m c_t$, equals $(\theta-1)/\theta$, which is the inverse of the markup of price over marginal cost.

In addition to capital, the household has access to a one-period nominal bond, so the budget constraint is $c_t + i_t + b_t = w_t n_t + r_t^k k_{t-1} + r_{t-1} b_{t-1} / \pi_t + d_t$, where r_t is the gross nominal interest rate and d_t is dividends from ownership of firms. The optimality conditions imply (10), (G.1), and

$$1 = E_t[x_{t+1}(r_t/\pi_{t+1})]. \quad (\text{G.9})$$

The bond is in zero net supply and the central bank sets the nominal interest rate according to

$$r_t = \bar{r}(\pi_t/\bar{\pi})^{\phi_\pi}, \quad (\text{G.10})$$

where \bar{r} is the nominal interest rate target and ϕ_π governs the strength of the response to inflation.

A competitive equilibrium consists of infinite sequences of quantities $\{y_t, k_t, c_t, n_t, i_t, mc_t\}_{t=0}^\infty$, prices $\{w_t, r_t^k, r_t, \pi_t\}_{t=0}^\infty$, and exogenous variables $\{a_t, \sigma_{a,t}\}_{t=0}^\infty$ that satisfy (1), (2), (9), (10), (G.1), (G.2), and (G.5)-(G.10), given the state $\{k_{-1}, a_{-1}, \sigma_{a,-1}\}$ and sequences of shocks $\{\varepsilon_{a,t}, \varepsilon_{\sigma_{a,t}}\}_{t=1}^\infty$.

| Parameter | RBC | NK | Calibration Target |
|---|--------|--------|--------------------------------|
| Frisch Elasticity ($1/\gamma$) | 0.5 | 0.5 | Chetty et al. (2012) |
| Capital Adjustment Cost (ν) | 20 | 20 | Investment Standard Deviation |
| Elasticity of Substitution (θ) | — | 11 | 10% Price Markup |
| Monetary Response to Inflation (ϕ_π) | — | 1.5 | Leduc and Liu (2016) |
| Price Adjustment Costs (φ) | — | 1,296 | Leduc and Liu (2016) |
| Steady-State Hours (\bar{n}) | 0.33 | 0.33 | Standard Value |
| Steady-State Inflation Rate ($\bar{\pi}$) | — | 1.0028 | Average Inflation Rate |
| Level Shock Persistence (ρ_a) | 0.95 | 0.95 | Output Autocorrelation |
| Volatility Shock Persistence (ρ_{sv}) | 0.87 | 0.87 | Leduc and Liu (2016) |
| Level Shock SD ($\bar{\sigma}_a$) | 0.0079 | 0.0078 | Output Standard Deviation |
| Volatility Shock SD (σ_{sv}) | 0.0309 | 0.0307 | Uncertainty Standard Deviation |

Table G.1: Real Business Cycle and New Keynesian model calibrations.

G.3 CALIBRATION Both models are calibrated at a monthly frequency. Table G.1 summarizes the parameter values. We set the level shock persistence (ρ_a) and standard deviation ($\bar{\sigma}_a$) to match the autocorrelation and volatility of detrended output. We calibrate the volatility shock standard deviation (σ_{sv}) to match the volatility of real uncertainty and the capital adjustment cost parameter (ν) to target the volatility of investment. Other parameters are calibrated in line with the literature.

G.4 RESULTS Table G.2 shows that neither model generates countercyclical uncertainty, as the correlation between output and uncertainty is slightly positive in both models. This holds even though both models match the volatility of uncertainty and generate realistic investment dynamics. Thus, capital adjustment costs are too weak to generate endogenous fluctuations in uncertainty. Instead, Table G.3 shows uncertainty dynamics are mostly driven by the exogenous volatility shocks.

| Moment | Data | Model | |
|--------------------------------|-------|-------|------|
| | | RBC | NK |
| $SD(\mathcal{U})$ | 5.93 | 5.93 | 5.93 |
| $SD(\tilde{y})$ | 3.15 | 3.15 | 3.15 |
| $SD(\tilde{i})$ | 8.68 | 7.36 | 7.80 |
| $Corr(\mathcal{U}, \tilde{y})$ | -0.60 | 0.01 | 0.11 |

Table G.2: Key Moments

| Contribution | Output | | Uncertainty | |
|-------------------|--------|--------|-------------|-------|
| | RBC | NK | RBC | NK |
| Level Total | 100.00 | 100.00 | 0.03 | 1.43 |
| Volatility Total | 0.38 | 0.37 | 99.97 | 98.58 |
| Level Direct | 99.62 | 99.63 | 0.03 | 1.42 |
| Volatility Direct | 0.00 | 0.00 | 99.97 | 98.57 |

Table G.3: Variance decompositions

H HOSIOS CONDITION

The equilibrium of a search and matching model is generally inefficient due to two externalities in the matching process (Hosios, 1990). First, when a firm posts a new vacancy, it imposes a positive externality on unemployed workers who face a higher job finding rate. Second, the vacancy posting imposes a negative externality on other firms who face lower job filling rates and a higher marginal cost of vacancy creation today and in the future. The Hosios (1990) condition restores efficiency.

Social Planner Problem The social planner maximizes

$$W_t = \ln c_t + \beta E_t W_{t+1}$$

subject to

$$\begin{aligned} c_t &\leq a_t k_{t-1}^\alpha n_t^{1-\alpha} - \kappa v_t - i_t + b u_t - \tau_t, \\ n_t &\leq (1-s)n_{t-1} + \xi(1-n_{t-1} + \chi \bar{s} n_{t-1})^\phi v_t^{1-\phi}, \\ k_t &\leq (1-\delta)k_{t-1} + \left(a_1 + \frac{a_2}{1-1/\nu} \left(\frac{i_t}{k_{t-1}} \right)^{1-1/\nu} \right) k_{t-1}, \\ u_t &= 1 - n_t, \quad v_t \geq 0. \end{aligned}$$

The optimality conditions for job creation and investment are given by

$$\frac{\kappa - \lambda_{v,t}}{(1-\phi)q_t} = (1-\alpha)\frac{y_t}{n_t} - b + E_t \left[x_{t+1} \left(\frac{\kappa - \lambda_{v,t+1}}{(1-\phi)q_{t+1}} \right) (1-s - \phi(1-\chi\bar{s})f_{t+1}) \right], \quad (\text{H.1})$$

$$\frac{1}{a_2} \left(\frac{i_t}{k_{t-1}} \right)^{1/\nu} = E_t \left[x_{t+1} \left(\alpha \frac{y_{t+1}}{k_t} + \frac{1}{a_2} \left(\frac{i_{t+1}}{k_t} \right)^{1/\nu} (1-\delta + a_1) + \frac{1}{\nu-1} \frac{i_{t+1}}{k_t} \right) \right], \quad (\text{H.2})$$

where $\lambda_{v,t}$ is the multiplier on the non-negativity constraint for v_t , and $x_{t+1} \equiv \beta(c_t/c_{t+1})$.

Competitive Equilibrium In competitive equilibrium, the job creation condition is given by

$$\frac{\kappa - \lambda_{v,t}}{q_t} = (1-\alpha)y_t/n_t - w_t + (1-\bar{s})E_t \left[x_{t+1} \frac{\kappa - \lambda_{v,t+1}}{q_{t+1}} \right], \quad (\text{H.3})$$

while (H.2) is unchanged. The wage rule resulting from Nash bargaining is given by

$$w_t = \eta((1 - \alpha)y_t/n_t + \kappa(1 - \chi\bar{s})E_t[x_{t+1}\theta_{t+1}]) + (1 - \eta)b. \quad (\text{H.4})$$

Note that labor market tightness is $\theta_t = v_t/u_{t-1} = f_t/q_t$. Combine (H.3) and (H.4) to obtain

$$\frac{\kappa - \lambda_{v,t}}{q_t} = (1 - \eta) \left((1 - \alpha) \frac{y_t}{n_t} - b \right) + E_t \left[x_{t+1} \left(\frac{\kappa - \lambda_{v,t+1}}{q_{t+1}} \right) \left(1 - \bar{s} - \eta\kappa(1 - \chi\bar{s}) \frac{f_{t+1}}{\kappa - \lambda_{v,t+1}} \right) \right] \quad (\text{H.5})$$

Hosios Condition If $v_{t+1} > 0$, then $\lambda_{v,t+1} = 0$. If $v_{t+1} = 0$, then $m_{t+1} = f_{t+1} = 0$. In either case, (H.5) is equivalent to (H.1) when the Nash bargaining weight equals the matching elasticity, $\eta = \phi$.

I VAR ESTIMATES UNDER RECURSIVE IDENTIFICATION

Recursive identification schemes are often used to identify the effect of uncertainty shocks on real activity. To show the implications of our theoretical results, consider the following bivariate VAR,

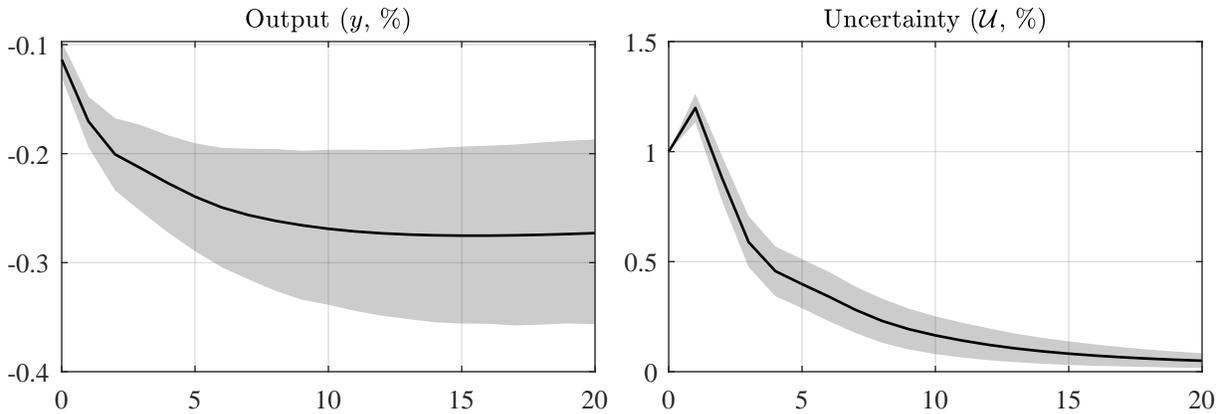
$$Y_t = \sum_{l=1}^L A_l Y_{t-l} + v_t, \quad (\text{I.1})$$

where Y_t is a vector of output and uncertainty, v_t is a vector of reduced-form shocks, and $\{A_l\}_{l=1}^L$ are parameter matrices. Suppose uncertainty is ordered first in Y_t and a Cholesky decomposition is used to identify the structural shocks. Under these assumptions, uncertainty shocks can affect uncertainty and output on impact, while output shocks can only affect output contemporaneously.

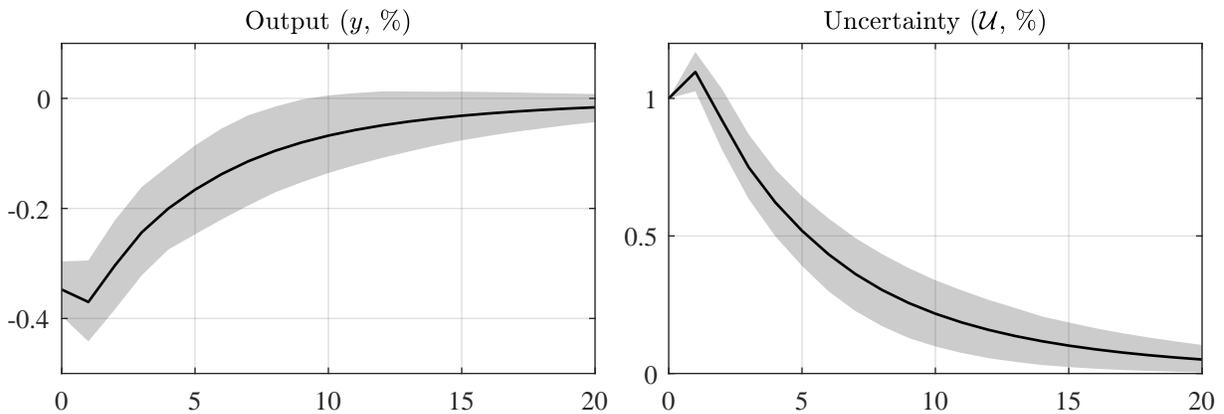
To test whether this recursive identification scheme can identify the true structural output responses, we estimate the VAR model on quarterly actual and simulated data from our estimated DMP model and compare the identified responses. The number of lags $L = 3$ is based on the Akaike Information Criterion. We simulate our model 1,000 times to produce artificial data series with 228 quarterly (684 monthly) observations, the same number used for our structural estimation.

Figure I.1 reports the responses to an uncertainty shock using actual and simulated data at a quarterly frequency under recursive identification. The responses with actual data are similar to those in the literature: a positive uncertainty shock raises uncertainty and lowers output on impact. We obtain similar responses using our simulated data VAR even though the model violates the identification assumption. Only the level shock affects output on impact in the simulated data, so the identified “structural” shock must be correlated with the level shock from the DMP model. We confirm this intuition by estimating the VAR model on simulated *monthly* data (i.e., the frequency of the model) and correlating the identified structural shocks with the true structural shocks. While the identified uncertainty shock has a correlation of 0.84 with the true structural uncertainty shock, it also has a correlation of -0.47 with the true structural level shock, confirming it is contaminated.

Alternatively, when uncertainty is ordered last in the VAR, the identified output shock has a correlation of 0.99 with the true level shock and 0 with the true uncertainty shock. The identified



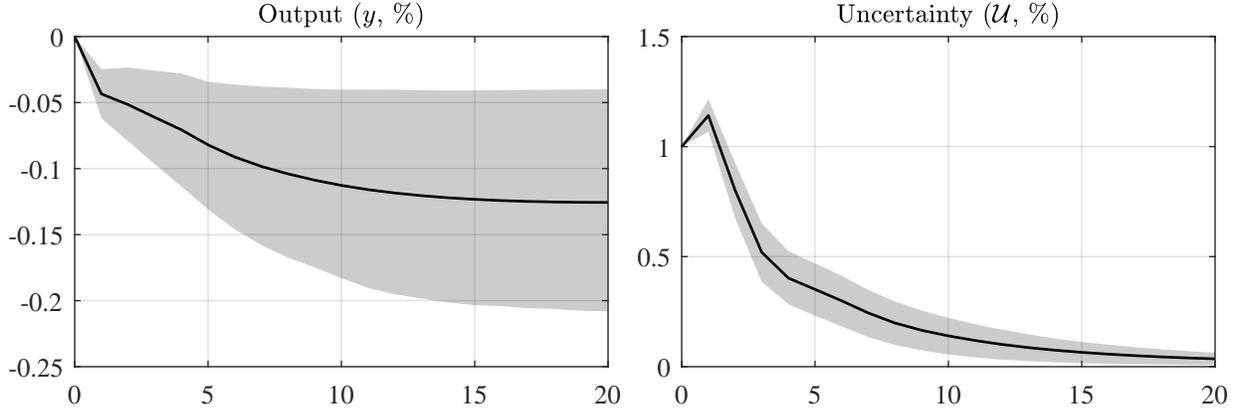
(a) Responses based on actual data. Shaded regions are 68% confidence intervals.



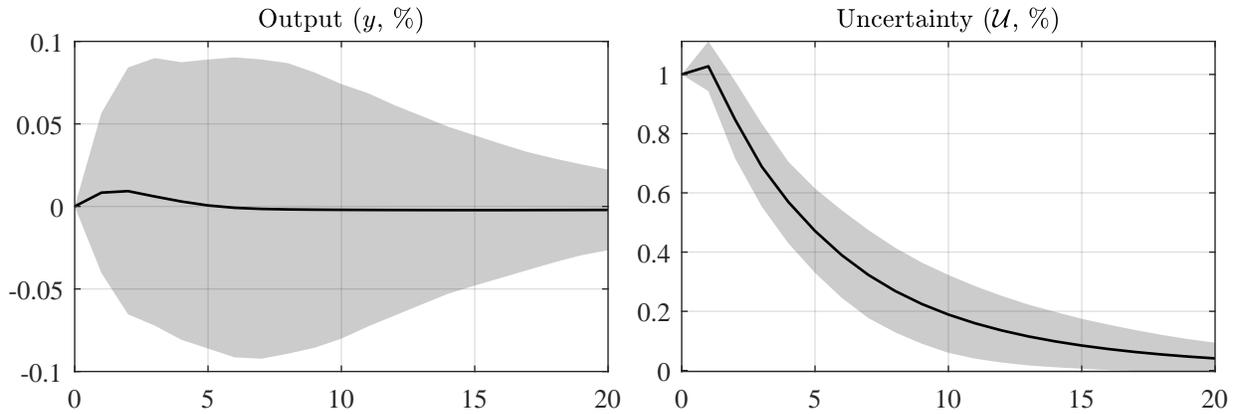
(b) Responses based on simulated data. Shaded regions are [16%, 84%] credible sets.

Figure I.1: Bivariate VAR responses to an uncertainty shock when uncertainty is ordered first.

uncertainty shock has a correlation of 0.96 with the true uncertainty shock and 0 with the level shock. These correlations show the recursive identification scheme properly identifies the structural shocks under this ordering because uncertainty is almost entirely endogenous in the DMP model. [Figure I.2](#) shows the impulse responses to an uncertainty shock using actual and simulated data in this case. Consistent with the impulse responses from the DMP model, there is almost no real effect of uncertainty using the simulated data VAR. Using actual data, the real effects are smaller than when uncertainty is ordered first, but there is still a statistically significant decline in output. One potential explanation is that uncertainty in the data has meaningful endogenous *and* exogenous components. In this setting, neither recursive ordering will properly identify the structural shocks.



(a) Responses based on actual data. Shaded regions are 68% confidence intervals.



(b) Responses based on simulated data. Shaded regions are [16%, 84%] credible sets.

Figure I.2: Bivariate VAR responses to an uncertainty shock when uncertainty is ordered last.

J VARIABLE SEARCH INTENSITY MODEL

This section shows our results are robust to adding search effort to the baseline DMP model following Leduc and Liu (2020). Define z_t as average search intensity, so new matches are given by

$$\mathcal{M}_t = \xi(z_t u_t^s)^\phi v_t^{1-\phi}.$$

The household chooses consumption, investment, capital, and search intensity to solve

$$J_t^H = \max_{c_t, i_t, k_t, z_t} \ln c_t + \beta E_t[J_{t+1}^H]$$

subject to (6), (9), and

$$c_t + i_t = w_t n_t + r_t^k k_{t-1} + b(1 - n_t) - h(z_t) u_t^s - \tau_t,$$

where $h(z_t)$ is the resource cost of search effort, which is increasing and concave. For a worker

with search effort z_{it} , the job finding rate is $f(z_{it}) = z_{it}m_t/(z_{it}u_t^s)$, so the marginal effect of raising search intensity is $\partial f(z_{it})/\partial z_{it} = f_t/z_t$. The optimality and envelope conditions produce (10) and

$$h'(z_t) = \frac{f_t}{z_t} \left(w_t - b + E_t \left[x_{t+1}(1 - f_{t+1} - \bar{s}(1 - \chi f_{t+1})) \tilde{J}_{n,t+1} \right] \right), \quad (\text{J.1})$$

$$\tilde{J}_{n,t} = w_t - b + E_t[x_{t+1}(1 - f_{t+1} - \bar{s}(1 - \chi f_{t+1})) \tilde{J}_{n,t+1}] + h(z_t) \frac{1 - \chi \bar{s}}{1 - f_t - \bar{s}(1 - \chi f_t)}, \quad (\text{J.2})$$

where $\tilde{J}_{n,t}^H \equiv J_{n,t}^H/\mu_t$ is the employment surplus and μ_t is the multiplier on the budget constraint.

The firm's problem is unchanged, but the Nash bargaining problem maximizes $(\tilde{J}_{n,t})^\eta (\lambda_{n,t})^{1-\eta}$, which implies $\tilde{J}_{n,t} = \eta \lambda_{n,t}/(1 - \eta)$. After combining with (12) and (J.1), the wage rate is given by

$$w_t = \eta((1 - \alpha)y_t/n_t + \kappa(1 - \chi \bar{s})E_t[x_{t+1}\theta_{t+1}]) + (1 - \eta) \left(b - \frac{(1 - \chi \bar{s})h(z_t)}{1 - f_t - \bar{s}(1 - \chi f_t)} \right), \quad (\text{J.3})$$

which replaces (15). Finally, the aggregate resource constraint becomes $c_t + i_t + \kappa v_t + h(z_t)u_t^s = y_t$.

| Moment | Baseline | Search Effort |
|--------------------------------|----------|---------------|
| $SD(\mathcal{U})$ | 6.06 | 5.94 |
| $SD(\tilde{y})$ | 3.65 | 3.77 |
| $SD(\tilde{u})$ | 21.14 | 24.14 |
| $Corr(\mathcal{U}, \tilde{y})$ | -0.62 | -0.59 |

Table J.1: Key Moments

| Contribution | Output | Uncertainty |
|-------------------|--------|-------------|
| Level Total | 100.00 | 39.23 |
| Volatility Total | 0.20 | 61.01 |
| Level Direct | 99.80 | 38.99 |
| Volatility Direct | 0.00 | 60.77 |

Table J.2: Search effort variance decomposition

We assume $h(z_t) = \zeta_0 + \zeta_1 z_t + \zeta_2 z_t^2/2$, where ζ_0 and ζ_1 are set so steady-state search intensity, $\bar{z} = 1$ and there are no search costs in steady state ($h(\bar{z}) = 0$). We set ζ_2 to match the standard deviation of detrended search intensity (2.15), where the time series is constructed by following the approach in Leduc and Liu (2020). This implies $\zeta_2 = 4$, given the parameter estimates in Table 2.

Table J.1 compares moments from this model to the baseline model and Table J.2 shows the variance decomposition. The model continues to generate a strong negative correlation between output and uncertainty and similar business cycle moments. Level shocks still explain almost all of the output variance and about 40% of the uncertainty variance. Therefore, the countercyclical fluctuations in uncertainty remain endogenous and volatility shocks continue to have small real effects.

K IMPULSE RESPONSES: EXTENDED DMP MODELS

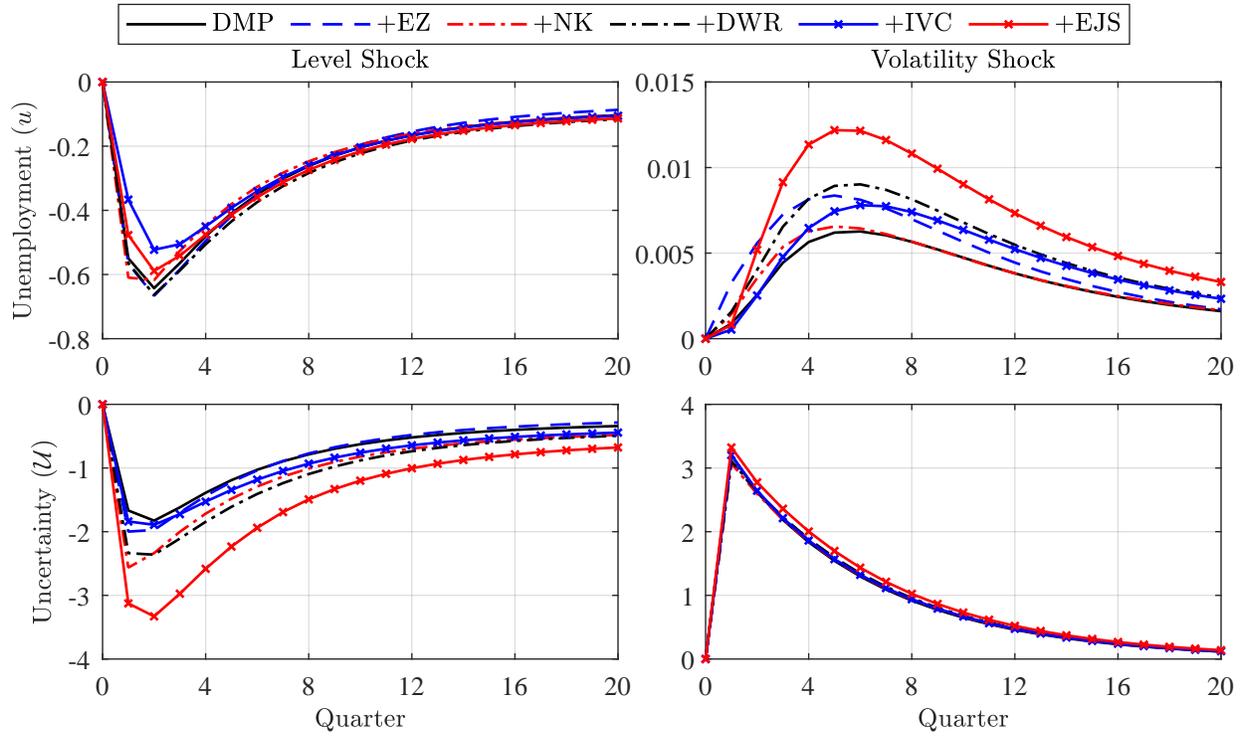


Figure K.1: Generalized impulse responses to 2 standard deviation shocks in the extended DMP models.

L ENDOGENOUS JOB SEPARATIONS MODEL

The following summarizes the equilibrium system for the model with endogenous job separations:

$$\begin{aligned}
 v_t &= \max\{0, \mu_t\}^2 \\
 \lambda_{v,t} &= \max\{0, -\mu_t\}^2 \\
 n_t &= (1 - F(\underline{z}_t))((1 - \bar{s})n_{t-1} + m_t) \\
 u_t &= 1 - n_t \\
 u_t^s &= u_{t-1} + \chi\bar{s}n_{t-1} \\
 \theta_t &= v_t/u_t^s \\
 \mathcal{M}_t &= \xi(u_t^s)^\phi v_t^{1-\phi} \\
 m_t &= \min\{\mathcal{M}_t, u_t^s, v_t\} \\
 q_t &= m_t/v_t \\
 w_t(z_t) &= \eta(w_{f,t}z_t + \kappa(1 - \chi\bar{s})E_t[x_{t+1}\theta_{t+1}]) + (1 - \eta)b \\
 \lambda_{n,t} &= \int_{\underline{z}_t}^\infty (w_{f,t}z_t - w_t(z_t))dF(z_t) + (1 - F(\underline{z}_t))(1 - \bar{s})E_t[x_{t+1}\lambda_{n,t+1}] \\
 w_{f,t}\underline{z}_t - w_t(\underline{z}_t) + (1 - \bar{s})E_t[x_{t+1}\lambda_{n,t+1}] &= 0 \\
 q_t\lambda_{n,t} &= \kappa - \lambda_{v,t} \\
 \ell_t &= ((1 - \bar{s})n_{t-1} + m_t) \int_{\underline{z}_t}^\infty z_t dF(z_t) \\
 y_t &= a_t k_{t-1}^\alpha \ell_t^{1-\alpha} \\
 c_t + i_t + \kappa v_t &= y_t \\
 \frac{1}{a_2} \left(\frac{i_t}{k_{t-1}}\right)^{1/\psi} &= E_t \left[x_{t+1} \left(r_{t+1}^k + \frac{1}{a_2} \left(\frac{i_{t+1}}{k_t}\right)^{1/\psi} (1 - \delta + a_1) + \frac{1}{\psi-1} \frac{i_{t+1}}{k_t} \right) \right] \\
 k_t &= (1 - \delta)k_{t-1} + \left(a_1 + \frac{a_2}{1-1/\psi} \left(\frac{i_t}{k_{t-1}}\right)^{1-1/\psi} \right) k_{t-1} \\
 r_t^k &= \alpha y_t / k_{t-1} \\
 w_{f,t} &= (1 - \alpha)y_t / \ell_t \\
 \ln a_t &= (1 - \rho_a) \ln \bar{a} + \rho_a \ln a_{t-1} + \sigma_{a,t} \varepsilon_{a,t} \\
 \ln \sigma_{a,t} &= (1 - \rho_{sv}) \ln \bar{\sigma}_a + \rho_{sv} \ln \sigma_{a,t-1} + \sigma_{sv} \varepsilon_{sv,t}
 \end{aligned}$$

When $\underline{z} \rightarrow 0$, $F(\underline{z}_t) = 0$ and $\int_{\underline{z}_t}^\infty z_t dF(z_t) = 1$, so the model collapses to our baseline DMP model.

REFERENCES

- BARNICHON, R. (2010): “Building a Composite Help-Wanted Index,” *Economics Letters*, 109, 175–178, <https://doi.org/10.1016/j.econlet.2010.08.029>.
- (2020): “Composite help-wanted index [dataset],” Accessed May 18, 2020.
- CHETTY, R., A. GUREN, D. MANOLI, AND A. WEBER (2012): “Does Indivisible Labor Explain the Difference between Micro and Macro Elasticities? A Meta-Analysis of Extensive Margin Elasticities,” in *NBER Macroeconomics Annual 2012, Volume 27*, ed. by D. Acemoglu, J. Parker, and M. Woodford, MIT Press, Cambridge, 1–56.
- ELSBY, M. W. L., G. SOLON, AND R. MICHAELS (2009): “The Ins and Outs of Cyclical Unemployment,” *American Economic Journal: Macroeconomics*, 1, 84–110, <https://doi.org/10.1257/mac.1.1.84>.
- FAMA, E. F. AND K. R. FRENCH (2021): “Data Library [dataset],” Accessed June 1, 2021.
- FLOOD, S., M. KING, R. RODGERS, S. RUGGLES, AND J. R. WARREN (2022): “Integrated Public Use Microdata Series, Current Population Survey: Version 10.0 [dataset],” Minneapolis, MN: IPUMS.
- GARCIA, C. B. AND W. I. ZANGWILL (1981): *Pathways to Solutions, Fixed Points and Equilibria*, Prentice-Hall series in computational mathematics, Prentice-Hall.
- HAGEDORN, M. AND I. MANOVSKII (2008): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited,” *American Economic Review*, 98, 1692–1706, <https://doi.org/10.1257/aer.98.4.1692>.
- HAYER ANALYTICS (2021): “Databases [dataset],” Accessed June 1, 2021.
- HOSIOS, A. J. (1990): “On The Efficiency of Matching and Related Models of Search and Unemployment,” *Review of Economic Studies*, 57, 279–298, <https://doi.org/10.2307/2297382>.
- KOPECKY, K. AND R. SUEN (2010): “Finite State Markov-Chain Approximations to Highly Persistent Processes,” *Review of Economic Dynamics*, 13, 701–714, <https://doi.org/10.1016/j.red.2010.02.002>.
- LEDUC, S. AND Z. LIU (2016): “Uncertainty Shocks are Aggregate Demand Shocks,” *Journal of Monetary Economics*, 82, 20–35, <https://doi.org/10.1016/j.jmoneco.2016.07.002>.
- (2020): “The Weak Job Recovery in a Macro Model of Search and Recruiting Intensity,” *American Economic Journal: Macroeconomics*, 12, 310–343, <https://doi.org/10.1257/mac.20170176>.
- LUDVIGSON, S. C., S. MA, AND S. NG (2021): “Uncertainty and Business Cycles: Exogenous Impulse or Endogenous Response?” *American Economic Journal: Macroeconomics*, 13, 369–410, <https://www.doi.org/10.1257/mac.20190171>.
- LUDVIGSON, S. C. AND S. NG (2022): “Macro, real, financial uncertainty indexes 1960:07-2021:12 [dataset],” Accessed Mar 11, 2022.
- MCCRACKEN, M. W. (2021): “Monthly and Quarterly Databases for Macroeconomic Research [dataset],” Accessed June 1, 2021.
- NEWKEY, W. K. AND K. D. WEST (1987): “A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix,” *Econometrica*, 55, 703–708, <https://www.doi.org/10.2307/1913610>.
- PETROSKY-NADEAU, N., L. ZHANG, AND L.-A. KUEHN (2018): “Endogenous Disasters,” *American Economic Review*, 108, 2212–2245, <https://doi.org/10.1257/aer.20130025>.
- ROTEMBERG, J. J. (1982): “Sticky Prices in the United States,” *Journal of Political Economy*, 90, 1187–1211, <https://doi.org/10.1086/261117>.

- ROUWENHORST, K. G. (1995): “Asset Pricing Implications of Equilibrium Business Cycle Models,” in *Frontiers of Business Cycle Research*, ed. by T. F. Cooley, Princeton, NJ: Princeton University Press, 294–330.
- SHIMER, R. (2005): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, 95, 25–49, <https://doi.org/10.1257/0002828053828572>.
- (2012): “Reassessing the Ins and Outs of Unemployment,” *Review of Economic Dynamics*, 15, 127–148, <https://doi.org/10.1016/j.red.2012.02.001>.
- SIMS, C. A. (2002): “Solving Linear Rational Expectations Models,” *Computational Economics*, 20, 1–20, <https://doi.org/10.1023/A:1020517101123>.
- WHARTON RESEARCH DATA SERVICES (2021): “Stock Market Indexes and Fama-Bliss Discount Bonds [dataset],” Accessed June 1, 2021.