Online Appendix to "Public Liquidity and Financial Crises".

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A. Proofs and Properties

In this section, I will list all the proofs and properties of the model. I start with discussions of modeling assumptions and a summary list of notations.

A1. Discussions of Modeling Assumptions

Different Productivities

The assumption that banker-operated capital has higher productivity captures the downside of a weaker bank balance sheet. This assumption is common in the macro-finance literature, e.g., Brunnermeier and Sannikov (2014), Gertler and Kiyotaki (2015), Gertler, Kiyotaki and Prestipino (2020). In a richer model, the bank-held capital can be allowed to have a decreasing return to scale, which implies that the marginal return to the bank-held capital falls below the marginal return to the household-held capital at a certain point. This feature is also present in Kiyotaki and Moore (1997).

An alternative way of modeling is to assume that bankers have lower risk aversion, and therefore, risky capital is more valuable if bankers have higher wealth, as in Drechsler, Savov and Schnabl (2018). The higher value of capital will feed into investment and economic growth. In reality, both features matter: banks indeed provide a lending service that is not directly replaceable by households (Schwert, 2018), and, in general, financial institutions are less risk-averse than households, as is reflected in much higher leverage and risk-taking. For simplicity, the model captures the first feature. Introducing the second feature can potentially quantitatively improve the performance of the model, but it requires general forms of utility functions and is technically more challenging.

Financial frictions

The first financial friction is the equity issuance friction. The assumption of no equity issuance by banks in this paper is standard in the literature (Brunnermeier and Sannikov, 2014; Gertler and Kiyotaki, 2015). This friction can be microfounded as a limit case of an agency friction in which bankers can privately divert resources at the cost of depositors, as in He and Krishnamurthy (2011), Di Tella (2017, 2019). Moreover, the equity constraint makes the financial market incomplete. As a result, bank-owned capital and household-owned capital are

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two different types of segmented asset. An implicit assumption is that banks and households cannot write contracts on aggregate risks. Indeed, if we allow agents to write contracts on the aggregate state, the balance sheet channel is eliminated, as in Di Tella (2017).

The second financial friction is the bank-run incentive. In this model, the financial panic is due to the fear of bankruptcy that is driven by real shocks. Thus, the modeling approach is different from the sunspot equilibria in the literature (Diamond and Dybvig, 1983; Postlewaite and Vives, 1987; Peck and Shell, 2003), and provides a highly tractable alternative to model liquidity disruptions in the financial sector.

The third friction is the fire sale price discount, which is just a wealth transfer from sellers to buyers. I have assumed that the fire sale price is lower than the post fire sale equilibrium price, which can be microfounded by certain market frictions, such as search frictions (Lagos and Rocheteau, 2009) and slow-moving capital (Duffie, 2010). In the 2008 financial crisis, a temporary price discount was indeed quite prevalent during systemic fire sales, as documented by Stanton and Wallace (2011) and Merrill et al. (2013). The temporary discount is a salient feature of fire sales (Shleifer and Vishny, 2011).

Another interpretation for α^0 is a cost of obtaining emergency funding. Even if there is no actual fire sale, as long as banks have difficulty obtaining emergency financing without substantial costs, banks suffer losses and trigger a vicious cycle in capital price decline and bank equity drops.

Government

The government is modeled in a highly stylized way, with wealth taxation and simple rules of government spending. In this model, government bonds can be interpreted as the liabilities of a combined central bank and the government. Taxation provides government the flexibility to adjust the quantity of government bonds without distorting individual decisions. The model can be readily extended to allow for distortionary taxation such as capital income tax, wealth tax, and other types of distortions.

A2. Summary of Notations

I summarize all notations below.

1) Aggregate quantities:

- K_t: aggregate amount of capital in the economy.
- W_t^b: aggregate banker wealth.
- W_t^h: aggregate household wealth.
- Y_t: aggregate output.
- $A_t K_t$: total government holding of non-deposit bank debt.

- $\mathcal{T}_t$: total taxation.
- ψ_t: share of capital owned by bankers.
- $g_t K_t$: total government spending.

2) State variables
   - K_t: total amount of capital.
   - w_t: the fraction of banker wealth among total wealth.
   - $B_t K_t$: total amount of government bonds.

3) Aggregate shocks:
   - dZ_t: the Brownian shock that affects capital dynamics.
   - dN_t: the liquidity crisis shock.

4) Endogenous asset prices and returns:
   - q_t: the value for each unit of capital.
   - $d\bar{R}^K_{j,t}$: return of capital held by bankers.
   - $d\underline{R}^K_{j,t}$: return of capital held by households.
   - $dR^B_t = r^B_t dt$: the return on government bonds.
   - $dR^D_t = r^D_t dt$: the return on deposits.
   - $dR^{ND}_t = r^{ND}_t dt - \kappa^{ND}_{t-} dN_t$: the return on non-deposits, which has exposure to the liquidity crisis shock dN_t.
   - μ^R_t: the non-dividend component of capital return.

5) Drifts:
   - μ^q_t: the drift of capital price dynamics dq_t/q_t.

6) Volatilities:
   - σ^q_t: the volatility of capital price dynamics dq_t/q_t.

7) Jumps:
   - κ^q_t: the loss of capital value in a crisis at time t.
   - κ^{ND}_t: the loss of non-deposit value in a crisis at time t.
   - κ^{fs}_t: firesale benefits for households that purchase capital at a low price in a crisis at time t.

8) Individual choices.
   - $k_{j,t}$: unit of capital held by individual j.
   - $c^b_{j,t}$: banker consumption.

- $c^h_{j,t}$: household consumption.
- $\hat{c}^b_{j,t}$: banker consumption per unit wealth.
- $\hat{c}^h_{j,t}$: household consumption per unit of wealth.
- $x^K_{j,t}$: banker capital holding per unit of wealth.
- $y^K_{j,t}$: household capital holding per unit of wealth.
- $x^B_{j,t}$: banker government bond holding per unit of wealth.
- $y^B_{j,t}$: household government bond holding per unit of wealth.
- $x^D_{j,t}$: banker total deposit issuance per unit of wealth.
- $y^D_{j,t}$: household deposit holding per unit of wealth.
- $x^{ND}_{j,t}$: banker non-deposit funding per unit of wealth.
- $y^{ND}_{j,t}$: household investment in non-deposits per unit of wealth.

A3. Property of the Investment Function $i(q)$

The function $i(q)$ is solved from

$$\phi'(\mu^K) = q$$

Because $q \geq 0$, and the range of $\phi'(\mu^K)$ includes $\mathbb{R}^+$ from properties of $\phi(\cdot)$ in (2), we always have a solution from the above equation. Since $\phi'(\cdot)$ is a strictly increasing function, the solution is unique, and the unique growth rate of capital μ^K is an increasing function of capital price q.

Since capital price $q > 0$, we obtain $\phi'(\mu^K) > 0$. Therefore, the investment function $i(q) = \phi(\mu^K(q))$ increases in μ^K, which increases in q. The above implies that $i(q)$ is an increasing function of q.

A4. Proof of Lemma 1

By model assumption, when a crisis shock dN_t occurs, the capital holding among a θ fraction of banks is completely destroyed. The government takes over the banks, pays the insured depositors in full and bankers a small reservation value (ε fraction of pre-shock equity), and all other creditors obtain zero recovery in their debt.

Among non-deposits, a maximum fraction $\beta > 0$ is allowed to be withdrawn in a crisis while the rest $1-\beta$ stays with banks, which reflects the reality that banks issue longer-term debt that is not susceptible to short-term liquidity problems. In what follows, I prove that for non-deposits, households withdraw the maximum fraction β in equilibrium.

For an active non-deposit (i.e., immediately withdraw funding is allowed in a crisis), if the decision is to withdraw, the recovery is full value regardless, but it forgoes the interest payment in the dt interval by $r^{ND}dt$. If the decision is to stay with the bank, then with probability $1-\theta$, not only the principal amount is recovered, but also the $r^{ND}dt$ amount of interest payment is earned. However, with probability θ, the whole face value is lost.

Consequently, the difference of payoff between staying with the bank versus withdrawing the funding is $(1-\theta)r^{ND}dt-\theta$, which is negative regardless of r^{ND} and θ, because θ is of order 1 while $r^{ND}dt$ is of order dt. Thus, households choose to withdraw all of their active credit to banks (fraction β of all non-deposit lending to banks) and suffer losses for the rest $1-\beta$, resulting in the aggregate non-deposit return of

$$r^{ND}dt-\theta(1-\beta)dN_t$$

In normal times, asset markets are liquid by assumption. Therefore, whenever households withdraw funding from banks, banks can sell assets without the fire-sale discount, thus incurring zero loss. More importantly, because in normal times there is no capital destruction, bank funding market does not freeze. Whenever households run on banks, they can also immediately borrow from other households via non-deposits, which eliminates any sales of assets. In summary, banks suffer no losses if households withdraw funding in normal times, and all creditors are paid off in full. Therefore, households obtain no benefit by running on banks in normal times.

A5. HJB Equations

According to household's budget constraint, the scaling property of banker wealth remains, i.e. if we scale household's wealth by a factor of $\bar{\alpha}$, then by following the same strategy (consumption strategy is the consumption/wealth ratio), the new wealth at each time t will be just $\bar{w}^h_{j,t}=\bar{\alpha}w^h_{j,t}$. Regardless of the starting wealth, the portfolio choices and consumption/wealth ratio should be the same, because otherwise, we can use the scaling property to arrive at a contradiction. Denote $\bar{c}^h_{j,t}$ and $\bar{y}^D_{j,t}$ as the optimal consumption and portfolio choice in insured deposits, given an initial wealth of 1. By definition, the paths of $\bar{c}^h_{j,t}$ and $\bar{y}^D_{j,t}$ are not depending on $w_{j,h}$ but only on the aggregate state variables (w,B). Because of the scaling property, for any different initial wealth w^h_j, the optimal consumption at any time t is $\bar{c}^h_{j,t}w^h_j$ and the optimal total deposit holding is $\bar{y}^D_{j,t}w^h_j$.

Consequently, we obtain

(A1)
$$\begin{aligned}
V^h(w,B,w_j^h) &= E\left[\int_0^\infty e^{-\rho t}\log(w_j^h\bar{c}_{j,t}^h)dt|w_0=w,B_0=B,w_{j,0}^h=w_j^h\right]\\
&= E\left[\int_0^\infty e^{-\rho t}\left(\log(w_j^h)+\log(\bar{c}_{j,t}^h)\right)dt|w_0=w,B_0=B,w_{j,0}^h=w_j^h\right]\\
&= \frac{1}{\rho}\log(w_j^h)+v^h(w,B)
\end{aligned}$$

where $v^h(w,B)$ is a function that only depends on the aggregate state of the economy.

With the log-form of the value function, we use c_j^h to denote the current household consumption and y_j^D to denote the fraction of wealth currently invested in deposits. Then the HJB equation becomes

$$V^h(w,B,w_j^h) = \max\left\{\log(c_j^h)dt+(1-\rho dt)E\left[V(w,B,w_j^h)+\frac{1}{\rho}\cdot d\log(w_j^h)+dv^h(w,B)\right]\right\}$$

The greatest simplification comes from the property that individual state w_j^h is separable from the aggregate state (w,B), so the portfolio decisions are directly solvable. Rearranging terms, we obtain

$$\text{(A2)}\qquad \rho V^h(w,B,w_j^h) = \max\left\{\begin{array}{c}\rho\log(c_j^h)+\mu_j^h-\frac{1}{2}(\sigma_j^h)^2\\ +\lambda\left(\log(w_j^h+\Delta w_j^h)-\log(w_j^h)\right))+\cdots\end{array}\right\}$$

where we denote μ_j^h as the drift of dw_j^h/w_j^h, σ_j^h as the volatility of dw_j^h/w_j^h, and Δw_j^h as the jump of wealth during the crisis shock. At the end of (A2), we omit the terms related to aggregate states w and B because they do not involve individual household portfolio and consumption choices.

Therefore, we can plug in the dynamics of dw_j^h/w_j^h as in (13), and obtain the equivalent optimization problem as in (28). The verification step for the HJB equation is standard in the literature and thus omitted.

For banks, there is a transition to households at the rate of η. Therefore, the bank value function is

$$V^b(w,B,w_j^b) = E[\int_0^T e^{-\rho t}\log(w_j^b\bar{c}_{j,t}^b)dt+e^{-\rho T}V^h(w_T,B_T,w_{j,T}^h)|w_0=w,B_0=B,w_{j,0}^h=w_j^h]$$

where $\bar{c}_{j,t}^b$ is defined in the same way as $\bar{c}_{j,t}^h$, representing the optimal consumption when the initial wealth is one unit. The HJB equation is

$$V^b(w,B,w_j^b) = (1-\lambda dt)\cdot\Big(\log(\bar{c}_j^b)dt + \log(w_j^b)dt + (1-\rho dt)(V^b(w,B,w_j^b) + E[dV^b(w,B,w_j^b)])\Big) \\ + \lambda dt \cdot V^h(w,B,w_j^b)$$

Plugging in the functional form of V^h in (A1), we get

$$(\rho+\lambda)V^b(w,B,w_j^b) = (1+\frac{\lambda}{\rho})\log(w_j^b) + \cdots$$

where we omit other terms. We find that the following functional form of V^b is consistent with the HJB equation:

$$V^b(w,B,w_j^b) = \frac{1}{\rho}\log(w_j^b) + v^b(w,B)$$

With this value function, following similar derivations as in (A2), we obtain the equivalent optimization problem as in (30). Again, the verification step for the HJB equation is standard in the literature and thus omitted.

A6. First-Order Conditions

First, we solve for household benefit from fire sales κ^{fs} from the following equality:

$$\underbrace{(1-w)}_{\substack{\text{total household}\\ \text{wealth}}} \cdot \kappa^{\text{fs}} = \underbrace{w}_{\substack{\text{total banker}\\ \text{wealth}}} \cdot \underbrace{\frac{\Delta x}{(1-\alpha^0)q}}_{\substack{\text{fire sale quantity for each}\\ \text{unit of banker wealth}}} \cdot \underbrace{\alpha^0 q}_{\substack{\text{wealth transfer for}\\ \text{each unit sale}}}$$

$$\Rightarrow \kappa^{\text{fs}} = \frac{\alpha^0}{1-\alpha^0}\Delta x \frac{w}{1-w} \tag{A3}$$

Then we derive the first-order conditions according to the simplified problems as in (28) and (30).

For banks, the first-order condition for capital holding is

$$\mu^R + \frac{\bar{A}}{q} - r^{ND} = x^K(\sigma^q+\sigma^K)^2 + \lambda(1-\theta)\frac{\kappa^q + \frac{\alpha^0}{1-\alpha^0}\beta\cdot 1_{\Delta x>0}}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x} \tag{A4}$$

which suggests that the excess return earned by capital is due to both its volatility and compensation for its losses in a crisis, including both the decline of price κ^q, and the liquidation losses related to α^0.

The first-order condition for insure deposits supply x^D is

$$r^{ND} - r^D = c'_D(x^D) - \lambda(1-\theta)\frac{\frac{\alpha^0}{1-\alpha^0}\beta}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x}1_{\Delta x>0} \tag{A5}$$

which suggests that the deposit spread, $r^{ND} - r^D$, is driven mainly by two forces: 1) the marginal cost of producing deposits, $c'_D(x^D)$; and 2) the benefit of having extra deposits on reducing non-deposit funding. The former force makes deposit production more costly to banks and thus banks pay lower deposit rate. The latter increases banks' willingness to pay for deposits and thus increases the deposit rate. When the deposit production cost dominates, we obtain a positive deposit spread.

The first-order condition for holding government bonds x^B is

$$r^{ND} - r^B \geq \lambda(1-\theta)\frac{\frac{\alpha^0}{1-\alpha^0}(1-\beta)}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x}1_{\Delta x>0} \tag{A6}$$

where the equality holds if $x^B > 0$. We note from the right-hand side that government bonds is priced at lower yields because it provides a hedge to capital liquidation.

For households, the first-order condition for capital holding is

$$\mu^R + \frac{A}{q} - r^{ND} \leq y^K(\sigma^q + \sigma^K)^2 + \lambda\theta\frac{1-\kappa^{ND}}{1 - y^{ND}\kappa^{ND} - y^K + \kappa^{fs}} + \lambda(1-\theta)\frac{\kappa^q - \kappa^{ND}}{1 - y^{ND}\kappa^{ND} - y^K\kappa^q + \kappa^{fs}}$$

where the equality holds if $y^K > 0$. The above right-hand side composes three terms: 1) volatility risk; 2) compensation for the capital destruction (with probability θ in a crisis); 3) compensation for the price drop of capital (if capital is not directly destroyed). Since θ is quite small, quantitatively only terms 1) and 3) matter for the household pricing of capital.

The first-order condition over deposits holding y^D is

$$r^{ND} - r^D = v'_D(y^D) + \lambda\theta\frac{\kappa^{ND}}{1 - y^{ND}\kappa^{ND} - y^K + \kappa^{fs}} + \lambda(1-\theta)\frac{\kappa^{ND}}{1 - y^{ND}\kappa^{ND} - y^K\kappa^q + \kappa^{fs}} \tag{A7}$$

which implies that the deposit spread $r^{ND} - r^D$ should always be positive, for two reasons: 1) households have extra liquidity value when they hold deposits, reflected by $\beta_d\rho/y^D$; 2) non-deposits have a small amount of credit risk. Since $\kappa^{ND} = \theta(1-\beta)$ is a very small number, the main driving force is the household special demand for liquidity. Taking (A5) and (A7) together, we find that a larger household demand for deposits increases the deposit spread, which requires the banking sector to accommodate with higher x^D and thus larger marginal cost of deposit production, justifying the higher deposit spread in the first place.

A7. Proof of Proposition 1 and 2

The proofs of Proposition 1 and 2 are combined since they are both under the same assumptions and results are closely interlinked.

First, I show that x^K and capital value q are positively correlated, given asset price dynamics. Plugging the definition of ψ_t in (17), the wealth identity (23), and the consumption rule (31) into the resource constraint (26), we get

$$wx^K(\bar{A} - \underline{A}) = \rho q + \frac{q}{q + B_0}\left(i(q) + g - \underline{A}\right), \tag{A8}$$

where $i(q) = \phi(\mu^K(q))$ is the optimal investment as a function of q, which is an increasing function of q. We find that the right hand side of (A8) increases in q, which implies that in equilibrium, a higher bank capital holding x^K is associated with more valuable capital. Let this relationship be $q(x^K)$.

Denote

$$g(x^K, \Delta x) = x^K(\sigma^q + \sigma^K)^2 + \lambda(1-\theta)\frac{\kappa^q + \frac{\alpha^0}{1-\alpha^0}\beta}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x}1_{\Delta x > 0} - \left(\mu^R + \frac{\bar{A}}{q(x^K)} - r^{ND}\right) \tag{A9}$$

which clearly is an increasing function in x^K, but a decreasing function over Δx. Since the first-order condition in (A4) is equivalent to $g(x^K, \Delta x) = 0$, we find that Δx and x^K are negatively related. The idea is that given asset price dynamics, to justify banks holding capital, more vulnerability has to be compensated with higher bank leverage and thus more profits. Furthermore, we also find that the total crisis destruction, $x^K\kappa^q + \frac{\alpha^0}{1-\alpha^0}\Delta x$, as a whole according to (A9) decreases with x^K, and therefore increases with Δx.

Second, I show that financial fragility Δx is lower when there is a larger public liquidity supply B. I will prove by contradiction. Suppose that Δx becomes lower, which then implies lower x^K and larger $x^K\kappa^q + \frac{\alpha^0}{1-\alpha^0}\Delta x$. According to equation (A11), bank deposit production x^D expands. However, these changes are contradictory to the larger Δx, because by definition,

$$\Delta x = \beta(x^K - x^D - 1) - (1 - \beta)x^B \tag{A10}$$

When we simultaneously have higher x^B, lower x^K, and larger x^D, equation (A10) implies a lower Δx. Contradiction! As a result, a larger public liquidity supply B reduces financial fragility Δx, which implies a smaller total crisis destruction, $x^K\kappa^q + \frac{\alpha^0}{1-\alpha^0}\Delta x$. The first-order condition $g(x^K, \Delta x) = 0$ also implies that larger B increases bank lending x^K, which boosts the capital value q.

Third, the total productivity of the economy is

$$\left(\psi\bar{A} + (1 - \psi)\underline{A}\right) = \rho(q + B_0) + \phi(\mu^K) + g$$

which increases in q (note that μ^K is an increasing function of q). Therefore, the total productivity of the economy increases with public liquidity supply B. This finishes the proof for Proposition 2.

Fourth, we show that deposits are crowded out. Equating (A5) and (A7), we obtain

$$c'_D(x^D) - \beta_d \frac{\rho}{y^D} = \lambda(1-\theta)\frac{\frac{\alpha^0}{1-\alpha^0}\beta}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x}1_{\Delta x>0}\cdots \tag{A11}$$

where we have omitted the κ^{ND} terms since we take the stance of small direct capital destruction. Due to convexity, the function $c'_D(x^D)$ increases in x^D. Furthermore, the market clearing condition in (25) implies $y^D = \frac{w}{1-w}x^D$ and thus the left-hand side of (A11) increases in x^D. Therefore, we only need to prove that a larger B reduces the right-hand side of (A11). We have already proved that a larger B increases x^K. According to the first-order condition in (A4), which is equivalent to $g(x^K, \Delta x) = 0$, the right-hand side of (A11) is smaller with a larger x^K., which implies that larger B reduces the right-hand side of (A11). Consequently, a larger B decreases deposits x^D.

Fifth, since larger B simultaneously reduce Δx but increases x^K, according to (A10), we must have at least x^D increasing or x^B increasing. Since we have already proved that x^D decreases with B, equation (A10) implies that x^B increases with public liquidity B. This finishes the proof for Proposition 1.

Finally, since both x^B and x^K increases with public liquidity supply B, and bank leverage is x^K+x^B, we find that bank leverage increases with public liquidity supply B. This concludes the proof for Proposition 1.

A8. Proof of Proposition 3

Consider the hypothetical asset where only a portion $(1-\pi) \in (0,1)$ can be redeemed for liquidity in a crisis, while the value of this asset remains the same in a crisis. According to the bank pricing kernel, the spread between r^{ND} and the yield r^f of this asset is

$$r^{ND} - r^f = \lambda(1-\theta)\frac{\frac{\alpha^0}{1-\alpha^0}(1-\pi-\beta)}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x}1_{\Delta x>0}$$

Combining the above equation with (A6), we obtain

$$\ell \equiv r^f - r^B = \lambda(1-\theta)\frac{\frac{\alpha^0}{1-\alpha^0}\pi}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x}1_{\Delta x>0}$$

Given asset price dynamics, according to the proof for Proposition 1 and 2, we know that the term $x^K\kappa^q + \frac{\alpha^0}{1-\alpha^0}\Delta x$ decreases in B, and therefore, the liquidity premium ℓ is smaller when there is a larger public liquidity supply.

A9. Proof of Proposition 4

As a starting point, the mechanism of public liquidity supply affecting the economy starts from its impact on bank holding of public liquidity. The change of bank portfolio holding then affects the severity of a crisis, bank capital holding x^K, and the equilibrium value of capital q.

As the structure of the proposition, I will prove the proposition in three parts.

Case 1: banks can issue equity.

In this scenario, banks can share the risks with households through issuing equity, and households can also earn a higher return on productive capital by holding bank equity. In a benchmark setting where Modigliani–Miller holds, whether banks issue equity or debt does not matter. However, in the model, issuing equity is superior to issuing debt, because bank debt is associated with bank run risks. Furthermore, from the households perspective, investing in bank equity is superior to directly investing in productive capital, because bank-operated capital yields higher returns. Consequently, banks only issue equity, which allows perfect risk sharing between bankers and households.

Since there is no bank debt, bank runs and fire sales are all eliminated by the equity issuance. Consequently, the liquidity premium is zero. The Ricardian equivalence holds, and the amount of government bonds has no impact on the real economy.

Case 2: all deposits are sticky with $\beta = 0$.

When all deposits are sticky, we have the net funding withdrawal

$$\Delta x = (\beta x^{ND} - x^B)^+ = 0$$

As a result, bank holding of government bonds does not affect its losses in a crisis. In the first-order conditions of x^K and x^D, the supply of government bonds completely drops off.

Next, I discuss whether the bank run losses during a crisis are still related to the amount of public liquidity supply. Banker wealth jump during a crisis is

$$\kappa^b = (1-\theta)\left(x^K \kappa^q + \frac{\alpha^0}{1-\alpha^0}\Delta x\right) + \theta(1-\varepsilon)$$

Given $\beta = 0$, we have

$$\kappa^b = (1-\theta)x^K\kappa^q + \theta(1-\varepsilon)$$

and

$$\kappa^h = \theta y^K + (1-\theta)y^K\kappa^q + y^{ND}\kappa^{ND}$$

where the fires-sale benefit is zero since there are no actual fire sales when $\beta = 0$. As a result, the changes of banker and household wealth are not related to public liquidity holding, which implies that it does not affect crisis severity.

In summary,, liquidity supply has zero impact on the disruptions of a crisis shock when $\beta = 0$. Neither does it affect the capital price and banks' portfolio choices except for government bonds.

Case 3: no fire sale market pressure, or $\alpha^0 = 0$.

The proof for this case is similar to Case 2, since $\alpha^0 = 0$ leads to no fire-sale loss at all and no role of holding public liquidity for banks.

A10. Evolutions of State Variables

I will derive the dynamic evolutions of the aggregate state variables w and K. Define the evolution of state variable w as

$$dw_t \overset{\Delta}{=} \mu_t^w dt + \sigma_t^w dZ_t - \kappa_{t-}^w dN_t \tag{A12}$$

I will derive the explicit expressions for dw_t in two steps. First, with Ito's formula, the dynamics of W_t^b/W_t^h is

$$d(\frac{W_t^b}{W_t^h}) = \frac{W_{t-}^b}{W_{t-}^h}\left(\begin{array}{c}(\mu_t^b - \mu_t^h + (\sigma_t^h)^2 - \sigma_t^b\sigma_t^h - \eta(1+\frac{W_t^b}{W_t^h}))dt \\ +(\sigma_t^b - \sigma_t^h)dZ_t + \left(\frac{1-\kappa_{t-}^b}{1-\kappa_{t-}^h} - 1\right)dN_t\end{array}\right)$$

Second, from

$$w_t = \frac{W_t^b}{W_t^b + W_t^h} = 1 - \frac{1}{W_t^b/W_t^h + 1}$$

and Ito's lemma, the dynamics of wealth w_t is

$$\begin{aligned} dw_t = & \frac{\frac{W_{t-}^b}{W_{t-}^h}\left((\mu_t^b - \mu_t^h + (\sigma_t^h)^2 - \sigma_t^b\sigma_t^h - \eta(1+\frac{W_t^b}{W_t^h}))dt + (\sigma_t^b - \sigma_t^h)dZ_t\right)}{\left(W_t^b/W_t^h + 1\right)^2} \\ & - \frac{\left(\frac{W_t^b}{W_t^h}\left(\sigma_t^b - \sigma_t^h\right)\right)^2 dt}{\left(W_t^b/W_t^h + 1\right)^3} + \left(\frac{1}{1+\frac{W_{t-}^b}{W_{t-}^h}} - \frac{1}{1+\frac{W_{t-}^b}{W_{t-}^h}\frac{1-\kappa_{t-}^b}{1-\kappa_{t-}^h}}\right)dN_t \end{aligned}$$

Based on the connection between individual wealth dynamics and aggregate dynamics in (19) and (20), we can express dynamics of w_t as

(A13)

$$\begin{aligned} dw_t = & w_{t-}(1-w_{t-})\left(\mu_t^b - \mu_t^h + (\sigma_t^h)^2 - \sigma_t^b\sigma_t^h - w_t(\sigma_t^b - \sigma_t^h)^2 - \eta\frac{1}{1-w_t}\right)dt \\ & + w_{t-}(1-w_{t-})(\sigma_t^b - \sigma_t^h)dZ_t + w_{t-}(1-w_{t-})\left(\frac{\frac{1-\kappa_{t-}^b}{1-\kappa_{t-}^h} - 1}{1 + w_{t-}(\frac{1-\kappa_{t-}^b}{1-\kappa_{t-}^h} - 1)}\right)dN_t \end{aligned}$$

With wealth share dynamics expressed, we can write the equilibrium price fixed point equation as

$$\kappa^q = \frac{q(w,B) - q(w\frac{1-\kappa^b}{1-\kappa^h-w(\kappa^b-\kappa^h)}, B)}{q(w,B)} \tag{A14}$$

Next, we denote the aggregate capital process as

$$\frac{dK_t}{K_{t-}} = (\mu_t^K - \delta)dt + \sigma^K dZ_t - \theta dN_t \tag{A15}$$

B. Numerical Methods and Model Moments

To solve the model, I first list the equilibrium equation system. Then I present the numerical algorithm to solve the whole model. Finally, I present a summary of model moments and contrast them with data.

B1. System of Equations for Solving the Model

In equilibrium, we mainly have the following three types of equations: (1) Equilibrium conditions, such as capital market clearing. (2) Individual optimality, such as the optimality of public liquidity and bank debt holding. (3) Definitions, such as taking Ito's formula on price function $q(w, B)$ to get an expression of μ^q.

All of the portfolio choices, x^K, x^B, etc, are functions of the states (w, B), but for simplicity, we omit the explicit dependence in the expressions below.

1) Market clearing conditions for capital, government debt, and insured deposits:

(B1) $$wx^K + (1-w)y^K = \frac{q}{q+B_0}$$

(B2) $$wx^B + (1-w)y^B = \frac{B}{q+B_0}$$

(B3) $$wx^D = (1-w)y^D$$

By assumption, a fraction δ_B of government debt is held by banks,

(B4) $$wx^B = \delta_B \frac{B}{q+B_0}$$

The fraction of capital held by banks is

(B5) $$\psi = \frac{wx^K}{q/(q+B_0)}$$

2) Consumption good clearing:

(B6) $$\left(\psi\bar{A} + (1-\psi)\underline{A}\right) = \rho(q+B_0) + \phi(\mu^K) + g$$

where

(B7) $$q = \phi'(\mu^K)$$

3) First-order conditions over capital holdings,

$$\mu^R + \frac{\bar{A}}{q} - r^{ND} = x^K(\sigma^q + \sigma^K)^2 + \lambda(1-\theta)\frac{\kappa^q + \frac{\alpha^0}{1-\alpha^0}\beta \cdot 1_{\Delta x>0}}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x} \tag{B8}$$

$$\begin{aligned}\mu^R + \frac{\underline{A}}{q} - r^{ND} \leq\; & y^K(\sigma^q + \sigma^K)^2 + \lambda\theta\frac{1-\kappa^{ND}}{1 - y^{ND}\kappa^{ND} - y^K + \kappa^{fs}} \\ & + \lambda(1-\theta)\frac{\kappa^q - \kappa^{ND}}{1 - y^{ND}\kappa^{ND} - y^K\kappa^q + \kappa^{fs}}\end{aligned} \tag{B9}$$

4) First-order condition over government bond holding,

$$r^{ND} - r^B = \lambda(1-\theta)\frac{\frac{\alpha^0}{1-\alpha^0}(1-\beta)}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x}1_{\Delta x>0} \tag{B10}$$

where the equality comes from the assumption that banks always hold $\delta_B > 0$ fraction of government bonds. Households are assumed to passively hold government debt and therefore they do not have a pricing equation.

5) First-order conditions over insured deposits:

$$r^{ND} - r^D = c'_D(x^D) - \lambda(1-\theta)\frac{\frac{\alpha^0}{1-\alpha^0}\beta}{1 - x^K\kappa^q - \frac{\alpha^0}{1-\alpha^0}\Delta x}1_{\Delta x>0} \tag{B11}$$

(B12)

$$r^{ND} - r^D = v'_D(y^D) + \lambda\theta\frac{\kappa^{ND}}{1 - y^{ND}\kappa^{ND} - y^K + \kappa^{fs}} + \lambda(1-\theta)\frac{\kappa^{ND}}{1 - y^{ND}\kappa^{ND} - y^K\kappa^q + \kappa^{fs}}$$

6) Volatilities of capital growth σ^q, state variable w, household wealth growth σ^h, and banker wealth growth σ^b:

$$\begin{cases} \sigma^q = q_w w(1-w)(\sigma^b - \sigma^h) \\ \sigma^w = w(1-w)(\sigma^b - \sigma^h) \\ \sigma^h = y^K(\sigma^K + \sigma^q) \\ \sigma^b = x^K(\sigma^K + \sigma^q) \end{cases} \tag{B13}$$

7) Drifts of capital growth μ^q, state variable w, household wealth growth μ^h, and banker wealth growth μ^b:

$$\mu^q = q'_w\mu^w + \frac{1}{2}q''_{ww}\left(w(1-w)(\sigma^b - \sigma^h)\right)^2 + q'_B\mu^B \tag{B14}$$

$$\mu^w = w(1-w)\left(\mu^b - \mu^h + (\sigma_t^h)^2 - \sigma_t^b\sigma_t^h - w_t(\sigma_t^b - \sigma_t^h)^2\right) - w\eta \tag{B15}$$

$$\mu^h = y^K(\mu^R + \frac{\underline{A}}{q} - r^{ND}) + y^B(r^B - r^{ND}) + y^D(r^D - r^{ND}) + r^{ND} - \rho \tag{B16}$$

(B17)
$$\mu^b = x^K(\mu^R + \frac{\bar{A}}{q} - r^{ND}) + x^B(r^B - r^{ND}) - x^D(r^D + c_D(x^D) - r^{ND}) + r^{ND} - \rho$$

8) Jump of banker wealth is κ^b (drop in a crisis as a fraction of pre-crisis wealth),

$$\kappa^b = (1-\theta)\left(x^K\kappa^q + \frac{\alpha^0}{1-\alpha^0}\Delta x\right) + \theta(1-\varepsilon) \tag{B18}$$

and household wealth decline is κ^h

$$\kappa^h = \left(\theta y^K + (1-\theta)y^K\kappa^q\right) + y^{ND}\kappa^{ND} - \kappa^{fs} \tag{B19}$$

with

$$\kappa^{ND} = \theta(1-\beta) \tag{B20}$$

$$\kappa^{fs} = \frac{\alpha^0}{1-\alpha^0}\Delta x \cdot \frac{w}{1-w} \tag{B21}$$

Finally, the equilibrium fixed-point condition for capital value q is

$$\kappa^q = 1 - q(w\frac{1-\kappa^b}{1-\kappa^h - w(\kappa^b - \kappa^h)}, B + \kappa^B)/q(w,B) \tag{B22}$$

9) Balance sheet identities:

$$x^K + x^B = 1 + x^D + x^{ND} \tag{B23}$$

$$y^K + y^B + y^D + y^{ND} = 1 \tag{B24}$$

B2. Algorithm

Due to jumps in both state variables w_t and B_t, the equilibrium system cannot be translated into a standard partial differential equation. If the model has a single state variable w_t, then the system can be cast as a delayed ODE. A good reference is Huang (2018). Huang (2018) also features endogenous jumps in w_t and is also built on Brunnermeier and Sannikov (2014). Nevertheless, the problem in my paper involves delayed partial differential equations. I have designed a specific functional iteration algorithm to solve the model.

The structure of the algorithm is as follows.

1) Start with an initial function $q_{(0)}(w, B)$. The initial price function is constructed by interpolating solutions that take B as a constant.

2) For $n = 0, 1, \cdots$. With the given price function $q_{(n)}(\cdot)$, I can solve a new $x^K_{(n+1)}(\cdot)$ and a new jump function $\kappa^q_{(n+1)}(\cdot)$, where in the jump equation (B22), the q function is taken as the last round value $q_{(n)}(\cdot)$. The progressive property of the algorithm guarantees the existence of solutions in each step. Then we can calculate

$$\psi_{(n+1)} = \frac{w x^K_{(n+1)}}{w x^K_{(n+1)} + (1-w) y^K_{(n+1)}}$$

Finally, I solve the next-round capital value $q_{(n+1)}$ from the resource constraint:

$$\psi_{(n+1)} \bar{A} + (1 - \psi_{(n+1)}) \underline{A} = \rho q_{(n+1)} + \phi(\mu^K(q_{(n+1)})) + g$$

3) Update the new rounds of price function and the liquidity wealth function with a learning rate $\varpi \in (0, 1)$, so that the final updating is

$$q_{(n+1)} := \varpi q_{(n+1)} + (1 - \varpi) q_{(n)}$$

where the operator := means updating the value of a variable. Setting a learning rate not too close to 1 is very important to guarantee the stability of the whole algorithm.

4) Stop when the absolute error is smaller than a threshold ϵ:

$$\text{(B25)} \qquad \int \int \left| q_{(n+1)}(w, B) - q_{(n)}(w, B) \right| dw dB < \epsilon.$$

The core of the algorithm is step 2, where the capital value function is updated. Solving the fixed point problem in (B22) requires knowledge of the global property for $q(w, B)$, for which I use the last round price function. This approach is valid because eventually, the consecutive rounds of iteration have very similar price functions and thus converge to the solution.

Below we provide the details for steps 1 and 2 in the above algorithm.

Step 1: Initialization

The whole algorithm needs an initialization of the function $q(w, B)$. I will initialize the algorithm by solving a simple version of the model without any bank run, i.e. $\lambda = 0$. Then the first order conditions of x^K and y^K imply

$$\mu^R + \frac{\bar{A}}{q} - r^d = (\sigma^K + \sigma^q)^2 x^K$$

$$\mu^R + \frac{\underline{A}}{q} - r^d = (\sigma^K + \sigma^q)^2 y^K$$

we get

$$\sigma^K + \sigma^q = \sqrt{\frac{\bar{A} - \underline{A}}{q(x^K - y^K)}}$$

Then from the volatility equation system, we can solve $q_w(w)$ as

$$q_w = \frac{\sigma^q}{w(1-w)(x^K - y^K)(\sigma^K + \sigma^q)} \tag{B26}$$

The boundary condition is $q(0) = \underline{q}$, which is solved from

$$\rho \underline{q} + i(\underline{q}) + g = \underline{A}$$

Clearly, the above equation system for $q(w, B)$ does not depend on B, and thus we can simply solve an ODE for the function $q(w, B) = q(w)$.

Step 2: Update the Value function q

At round n, for each pair of (w, B), implement the following:

1) Start with last round price function $q_{(n)}(\cdot)$, solve for the current round x^K, y^K, and volatilities via equations (B1), (B6), (B4), and the definition of ψ in (17). Then the volatilities, σ^q, σ^h, and σ^b, can be derived from equation (B13).

2) Next, solve x^B via the market clearing conditions (B4). Solve x^K, x^D, and κ^q together for equations (B8), (B9), (B11), (B12), and (B22).

3) With the updated x^K, we can solve for ψ via (B5), where q uses the last-round value $q_{(n)}$.

4) With the updated ψ, we solve for the next-round capital value through the consumption good clearing in (B6),

$$\psi \bar{A} + (1 - \psi)\underline{A} = \rho q_{(n+1)} + \phi(\mu^K(q_{(n+1)})) + g$$

Thus, after the above steps, we obtain $q_{(n+1)}$.

After solving for the capital value function q, we can follow the same algorithm to obtain the portfolio choices x^K, x^D, x^B, and the jumps in the capital value κ^q. Next, we need to calculate other rates, including r^B, r^D, and r^{ND}.

1) First solve for the spreads, $\mu^R - r^{ND}$, $r^{ND} - r^B$,$r^{ND} - r^D$, via the first-order conditions in (B8), (B10), and (B11).

2) Solve the drifts μ^b, μ^h, and μ^w, through equations (B15), (B16), (B17). Then solve for μ^q via equation (B14). With μ^q, we can solve for μ^R via its definition in (29).

3) With μ^R solved, we then obtain the level of rates, r^B, r^D, r^{ND}.

B3. Calibration Moments

Calibrated model parameters are shown in Table B1, and estimated parameters together with moment values are shown in Table B2.

TABLE B1—CALIBRATED MODEL PARAMETERS

	Parameters	Choice	Data Moment
λ	Crises arrival rate	4%	Frequency of historical financial crises
α_0	Market illiquidity discount	21%	Loan haircut in the secondary market during the global financial crisis
β	Fraction of runnable funding	42 %	Runnable funding of banks from flow of funds, 1970 to 2016
δ	Depreciation rate	10%	Depreciation rate in the literature
ρ	Time discount rate	4%	Discount rate in the literature
χ	Investment adjustment cost	3	Adjustment cost in literature
ε	Shareholder bankruptcy leftover	10^{-3}	Small ε
θ	Crisis shock size	10^{-5}	Small θ

Table B2—Moments and Model Estimates

Panel A: Moments

Moment	Data	Model
Average of public liquidity/GDP	36%	36%
Minimum of public liquidity/GDP	18%	18%
Jump of public liquidity/GDP in crises	8%	8%
Average output change in a crisis	-8.0%	-7.7%
Average productivity	0.14	0.14
Average liquidity premium	0.36%	0.37%
Average deposit spread	1.8%	1.4%
Average insured deposits/total bank liability	40%	42%
Volatility of real GDP growth	2.1%	2.7%
Extra household growth due to liquidity value	0	0
Bank equity ratio	10%	12%

Panel B: Estimated Parameter Values

Parameter	Value	Parameter	Value
Banker productivity $\bar{A}$	0.15	Household productivity $\underline{A}$	0.12
Capital growth volatility σ^K	3%	Banker$\rightarrow$household transition η	0.2
Household deposits valuation β_D	0.075	Household deposits valuation $\underline{y}^D$	0.15
Persistence of liquidity supply, θ_B	0.018	Crisis-expansion of liquidity κ^B	0.015
Basic liquidity supply, $\bar{B}$	0.027	Government spending $\bar{g}$	0.046
Illiquidity parameter π	20%		

B4. Additional Quantitative Evaluations of the Model

In Table B3, I show additional untargeted moments and contrast them with the data. I simulate the model for 10000 years at monthly frequency and measure data moments on a sample that starts from 1920 (for most variables, this is the longest horizon of data I can obtain).

As shown in the panel A of Table B3, the model-implied volatilities are similar to the data counterparts for output growth, bank equity growth, consumption growth, investment growth, and the deposit spread. However, there are two major differences: First, the model-implied liquidity premium is not volatile enough compared to the data counterpart; Second, the model-implied risk-free rate is not volatile enough. These results are due to similar reasons explained in the previous exercise. We also note that although the underlying capital shock volatility is smaller than that of the previous exercise in Table 3, asset price volatilities are actually higher. The main reason is that the capital ratio data in the previous exercise is larger than the stationary state of the model, and thus the state vector is in the region with less volatile asset prices.

In panel B of Table B3, I report correlations among major quantities and prices. These correlations are of the same sign, and mostly of similar magnitudes. Nevertheless, the correlation between changes of deposit spread and output growth is much larger in the data than in the model. Deposit spread data for the long horizon are obtained from projections on federal funds rate, as in Krishnamurthy and Li (2023), and thus, highly correlated with output dynamics due to the endogenous reaction of monetary policy. However, there is no such link in the model and therefore the model-implied correlation is tiny.

TABLE B3—ADDITIONAL UNTARGETED MODEL MOMENTS

Moments	Model	Data
Panel A: volatilities (%)		
vol(liquidity premium)	0.18	0.30
vol(output growth)	2.64	5.01
vol(bank equity growth)	30.56	26.06
vol(consumption growth)	2.75	1.94
vol(investment growth)	4.28	6.80
vol(risk free rate)	1.13	3.28
vol(deposit spread)	1.32	1.12
Panel B: correlations		
corr(consumption growth, output growth)	0.88	0.85
corr(investment growth, output growth)	0.77	0.70
corr(bank equity growth, output growth)	0.14	0.31
corr(diff(liquidity premium), output growth)	-0.07	-0.08
corr(diff(risk free rate), output growth)	0.10	0.19
corr(diff(deposit spread), output growth)	0.01	0.19

Note: This table shows model moments from a model simulation of 10000 years at monthly frequency. All growth variables are calculated as yearly growth, and all differences are taken as yearly differences. Data are from 1920 to 2016, except for bank equity growth, which is the same as He and Krishnamurthy (2019) and starts from 1973. The risk free rate is the federal funds rate. The deposit spread is from Krishnamurthy and Li (2023).

C. Additional Empirics

In this section, I will present additional VAR analysis, regressions evidence about the liquidity premium, and discuss alternative measures of the liquidity premium.

C1. Robustness of the VAR Analysis

The VAR setup is

$$X_t = A_1 X_{t-1} + \cdots + A_p X_{t-p} + u_t \tag{C1}$$

$$\begin{aligned} X_t = [&\Delta\text{GDP}_t, \Delta\text{C\&I loans}_t, \Delta\text{investment}_t, (\text{public liquidity/GDP})_t, \\ &\text{inflation}_t, \text{federal funds rate}_t, \text{stock market return}_t, \\ &\text{bank leverage}_t, \text{excess bond premium}_t, \text{liquidity premium}_t] \end{aligned} \tag{C2}$$

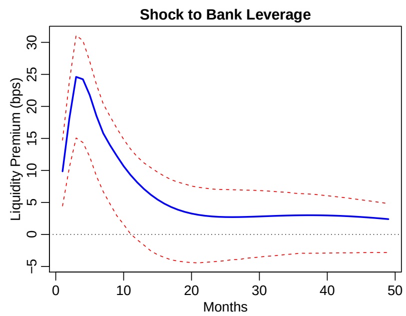
In the following, I will present more robustness checks on the main result – the liquidity premium significantly responds to a shock of bank leverage. As shown in Figure C1, the impulse response remains significant and positive after a variety of changes, including setting lag $p = 3$ and $p = 4$, excluding recession and crisis episodes (with $p = 2$), and putting bank leverage at the beginning of the variable list (with $p = 1$ to avoid duplication of Figure 1b).

Next, I show the impulse response results for other variables in Figure C2. Results are broadly sensible, with the liquidity premium positively reacting to a C&I loan growth shock, which proxies firms' demand for liquidity. Furthermore, the impulse response to a public liquidity/GDP shock is negative, consistent with the literature. The error bands are wider than other shocks, since in the data, we only have measures on quarterly government debt volume, while the analysis is done at a monthly frequency (government debt volume is interpolated within each quarter). A longer period analysis at a lower frequency generally reveals a stronger relationship, as in Krishnamurthy and Vissing-Jorgensen (2012). Finally, we find that initially, the liquidity premium positively responds to a credit spread shock, but then the response reverts and becomes slightly negative, and eventually converges towards zero.

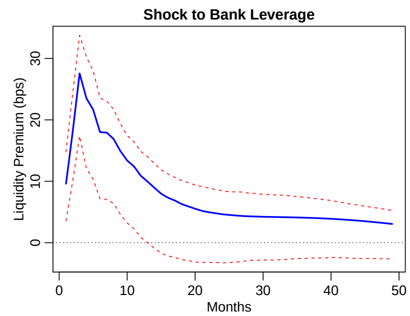
Since in the model, intermediaries charge a higher risk premium when leverage is high, we also expect that the excess bond premium positively responds to a positive leverage shock. Indeed, as shown in Figure C3, the initial response of excess bond premium to a bank leverage shock is significantly positive, although followed by a slight reversal that is not statistically different from zero.

C2. Narrative Restrictions

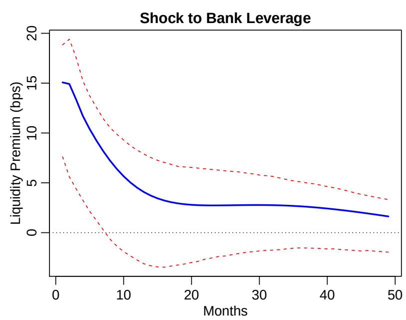
In this subsection, I follow the methodology of Antolín-Díaz and Rubio-Ramírez (2018) to implement the narrative restrictions on VAR analysis. The idea is that there are episodes of events when banking disruptions are the key and



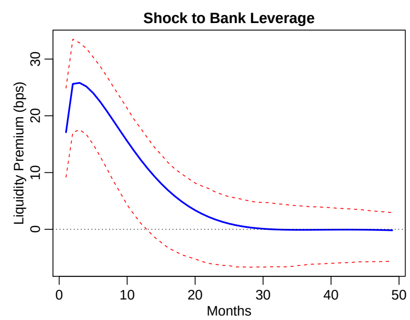
(a) lag $p = 3$



(b) lag $p = 4$



(c) Excluding Recession/Crises



(d) Ranking Bank Leverage at Front

Figure C1. Robustness on the Impulse Responses of Liquidity Premium to Bank Leverage.

Note: This figure illustrates various robustness checks on the impulse responses of the liquidity premium on bank leverage. Dashed red lines illustrate the 90% confidence interval.

dominate the response of the liquidity premium to other variables. By imposing such economic knowledge onto the VAR system, we are better able to identify how banking-sector shocks affect the pricing of liquidity and other dynamics of the economy. Since the key innovation of Antolín-Díaz and Rubio-Ramírez (2018) is to introduce narrative restrictions, not sign restrictions, I focus on the implementation of narrative restrictions.

The starting point of the analysis is a traditional VAR with rank restrictions, and then the algorithm imposes narrative restrictions with a Bayesian approach. I use the same baseline VAR as in Section I.A. Then, I pick the same events as in Section C.C3 and impose the restriction that during those events (since the VAR analysis is at monthly frequency, the restriction is on the event month), the absolute value of the liquidity premium response to the bank leverage shock dominates the absolute value of response to any other variable in the VAR. For identification purpose, I also include a sign restriction that the response of leverage to leverage shock is positive.

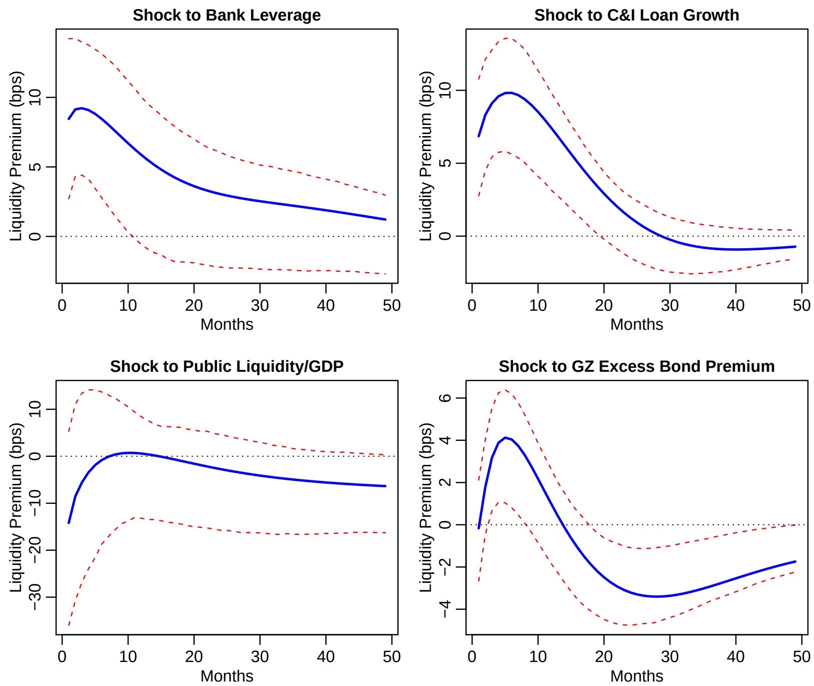


FIGURE C2. IMPULSES RESPONSES OF THE LIQUIDITY PREMIUM TO OTHER SHOCKS.

Note: This figure shows the impulse response of the liquidity premium to a one standard deviation shock to other variables, including C&I loan growth that proxies firm liquidity demand, public liquidity supply/GDP, and the shock to GZ excess bond premium. The dashed red lines illustrate the 90% confidence interval.

The resulting impulse responses are illustrated in Figure C4, with confidence interval of plus and minus one standard deviation. We find that the liquidity premium positively responds to bank leverage shocks and the effect persists for about a year. Consistent with the theory, the excess bond premium also positively responds to a positive bank leverage shock. Next, a positive leverage shock negatively affects bank lending, and therefore, reduces investment growth. Finally, the government will react to those events, leading to a higher public liquidity/GDP ratio afterward.

C3. Event Studies

Next, I study events that are closely linked to bank shocks and show the strong comovements between bank leverage and the liquidity premium. To get better identification of asset price changes around those events, I use daily data of both bank leverage (inverse of bank capital ratio from He, Kelly and Manela (2017))

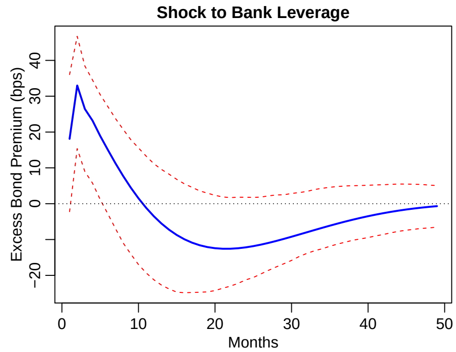


Figure C3. Impulse response of Excess Bond Premium to Bank Leverage.

Note: This figure shows the impulse response of excess bond premium as in Gilchrist and Zakrajšek (2012) to a one standard deviation shock of bank leverage.

and the liquidity premium from Nagel (2016).[17]

I use the following events:

- September 11, 2001: the terrorist attack that destroyed the World Trade Center and caused damage to the financial sector (McAndrews and Potter, 2002).
- September 15, 2008: the bankruptcy of Lehman Brothers during Global Financial Crisis.[18]
- May 7, 2009: release of bank stress testing results that reveal bank health during Global Financial Crisis.[19]
- November 1, 2009: the bankruptcy of CIT Group.[20]
- May 10, 2010: Eurozone leaders resolved in Brussels to take drastic action against the debt crisis.[21]
- July 26, 2012: the "whatever it takes" speech by ECB President Mario Draghi.

[17]The repo-based measure from Nagel (2016) has more sensitive high-frequency variation than the Refcorp-based measure in Longstaff (2004), so I use only the measure from Nagel (2016) for this high-frequency exercise.

[18]See Brunnermeier (2009) for more detailed analysis on Lehman bankruptcy and why it is such a major liquidity shock to the banking sector..

[19]Link to the Federal Reserve announcement: `https://www.federalreserve.gov/newsevents/pressreleases/bcreg20090507a.htm`

[20]Refer to Helwege and Zhang (2016) for further analysis on the significance of financial firms' bankruptcies.

[21]See a list of European debt crisis events in Table 2 of Stracca (2013). I only select a subset of those events that are most pronounced and mostly related to the banking sector.

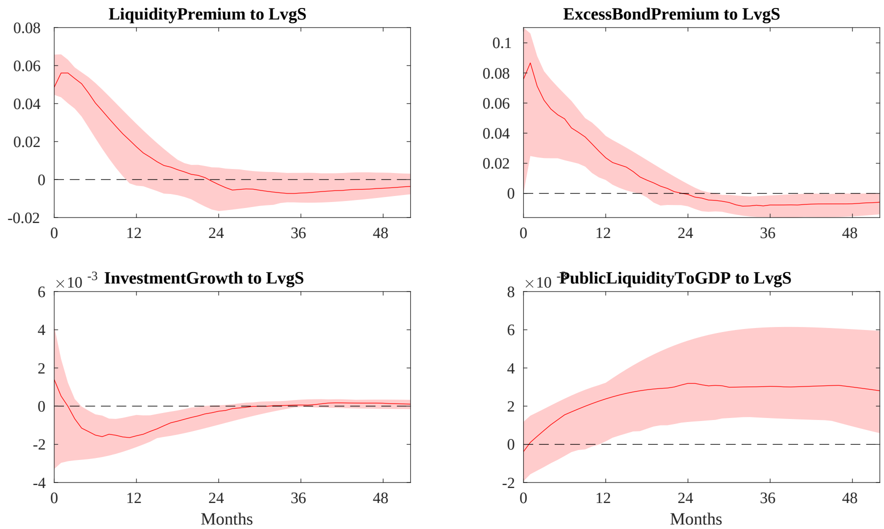


FIGURE C4. IMPULSE RESPONSES WITH NARRATIVE RESTRICTIONS ON THE VAR

Results are shown in Figure C5. I normalize both bank leverage and the liquidity premium for comparison. We find that in all of these events, bank leverage and the liquidity premium strongly comove. To the extent that these events represent shocks to banks, results provide supportive evidence on bank demand of liquidity driving the liquidity premium.

These events can also be integrated into the VAR analysis in Section I.A, using the narrative restriction approach in Antolín-Díaz and Rubio-Ramírez (2018). Since this approach is more involved, I illustrate the results in Appendix C.C2. With narrative restrictions, the liquidity premium response to bank leverage shocks is significant and positive, consistent with previous results.

C4. Regressions on the Liquidity Premium using a Long Sample

In the analysis of Table 2, we find that public liquidity has a small explanatory power on the liquidity premium. The reason is that the public liquidity supply is slow-moving, and there are not enough variations during that period. Once we extend the data sample to a longer horizon as in Krishnamurthy and Vissing-Jorgensen (2012), there is a significant negative relationship between public liquidity supply and the liquidity premium. As shown in Table C1 below, public liquidity supply can explain 12% of time-series variations in monthly liquidity premium, for the data sample from 1929 to 2016. Despite the difference in R^2, the coefficient on public liquidity/GDP is very similar to columns 1-3 of Table 2.

Table C1—Empirical Relationships between the Liquidity Premium and Public Liquidity Supply at a Longer Horizon

	Liquidity Premium
public liquidity/GDP	-0.51^{**}
	(0.20)
Constant	0.49^{***}
	(0.10)
Observations	1,056
R^2	0.12

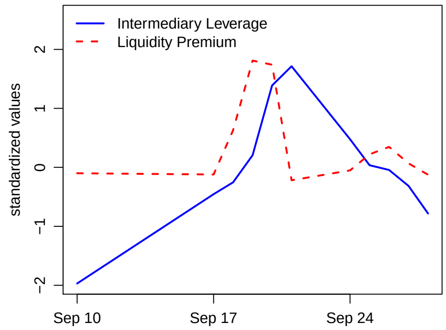
Note: Public liquidity is defined as the total government bonds held by the domestic private sector plus central bank reserves. Data are from 1929 to 2016 at monthly frequency.

C5. Alternative Measures of the Liquidity Premium

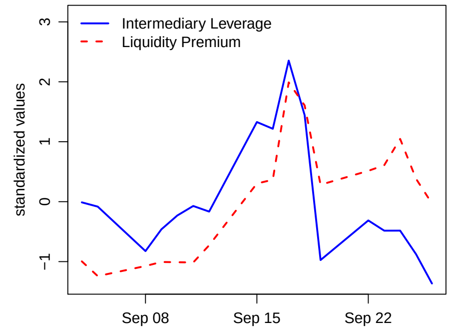
In the paper, I have used the principal component of GC Repo 3 month term loan spread with respect to treasury 3 months, as well as the Refcorp – Treasury spreads as the liquidity premium measure from 1991 to 2016. This is an ideal measure for the liquidity premium, because it has no credit risk in all components and is more robust by extracting the common variations. However, before 1991, we do not have such a measure, and the banker's acceptance might have a small credit risk component. Then it is necessary to check if the main results in the paper are robust to alternative measures to the liquidity premium before 1991. Since the 2008 counterfactual analyses are only based on the Repo spread data, none of the results in section V will be affected.

One alternative measure is the fed funds rate – treasury 3-month spread. It is not an ideal measure as well, because the uninsured interbank borrowing and lending also have credit risks. Furthermore, the fed funds rate is an overnight rate, which makes the measure subject to maturity mismatch. To fix this issue, I can also use another measure: the compounded 3-month fed funds rate – treasury 3-month spread. The compounded 3-month rate is an average of fed funds rate in the next 3 months. Absent from monetary policy shocks, this should be a good measure for the expectation of interest rate in the coming 3 months. However, since the fed funds rate has unexpected shocks, this measure might include too much noise and reduce the power of explanation.

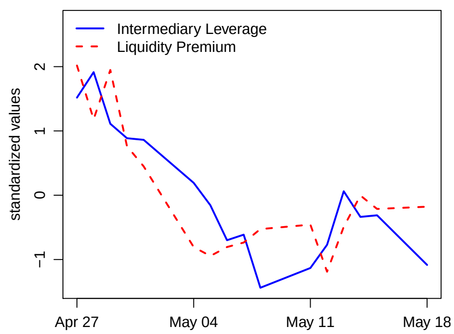
In Figure C6, I plot the three spreads based on three different measures, including the 3-month baker acceptance, fed funds rate, and compounded fed funds rate for 3 months. We find that the three measures are very close to each other from 1970 to 2000. Before 1970, the spreads based on the fed funds rate and the compounded fed funds rate are still similar, but the compounded fed funds rate–treasury spread has more fluctuations, indicating unexpected shocks. However, compared to the banker acceptance spread, the spreads based on the fed funds rate are much higher before 1950. Three reasons may lead to this phenomenon. First, since the banker's acceptance was widely used for international trade and accessible by a large group of investors, it is more liquid than the fed funds. Second, banker's acceptance was backed and directly purchased by the federal reserve for the majority of the period before 1977, making it safer than the interbank borrowing. Third, the payment from a banker's acceptance is double-backed by both the bank and the underlying firm. Consequently, the current measure of the liquidity premium, the banker's acceptance – treasury 3-month spread, is the best among these proposed alternatives.



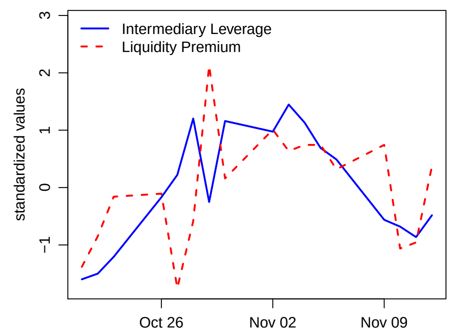
(a) 9-11 Terrorist Attack



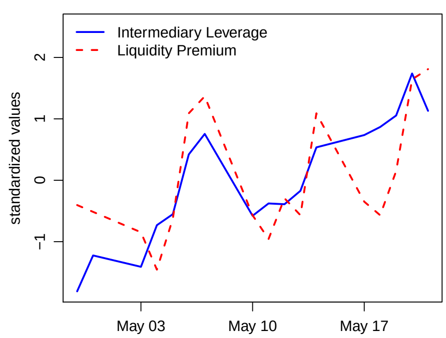
(b) Lehman Brothers bankruptcy



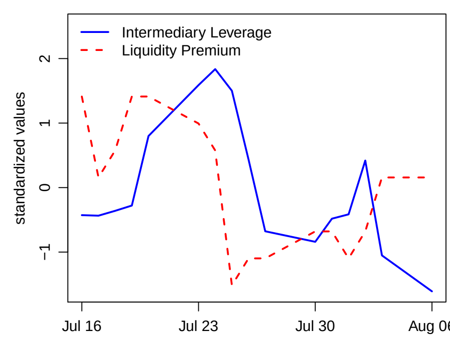
(c) Release of Bank Stress Testing Results



(d) Bankruptcy of CIT Group



(e) Eurozone Meeting on Debt Crisis



(f) "Whatever it takes" by Draghi

Figure C5. Event Studies.

Note: This figure plots the path of intermediary leverage and the liquidity premium around six events of banking-sector disruptions. Both variables are standardized as zero-mean and unit-volatility for comparison.

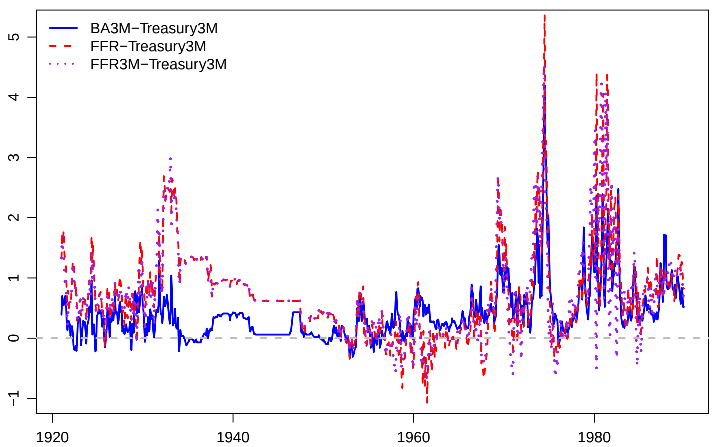


Figure C6. Alternative Measures of the Liquidity Premium.

Note: This figure illustrates the time series of several different measures of the liquidity premium. BA3M refers to the yield of the three-month banker acceptance. Treasury3M refers to the yield of three-month Treasurys. FFR is the federal funds rate, while FFR3M is the compounded 3-month federal funds rate.