

# Online Appendix for: Discounts as a Barrier to Entry

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## Abstract

There are seven sections in this online Appendix (all references are in the paper). The first section considers rebate contracts with unconditional transfer in the rent-shifting setup of Aghion and Bolton (1987). The section also considers the case of unconditional transfers subject to an exogenous limit. Section 2 provides details of the no-contract payoffs in the naked-exclusion setting of Rasmusen et al. (1990) and Segal and Whinston (2000). Section 3 considers rebate contracts with unconditional transfers in the naked-exclusion setup. It also explores how unconditional transfers can be used to extract rents rather than to exclude efficient entrants. Section 4 extends the naked-exclusion analysis of rebates to a world of unknown entry costs, both with and without unconditional transfers. Section 5 considers rebates with unconditional transfers in the downstream-competition setup of Simpson and Wickelgren (2007) and Asker and Bar-Isaac (2014). As in the previous section, it also covers the use of unconditional transfers to extract rents. Section 6 analyze the downstream pricing game between a retailer carrying only  $E$ 's products, and a vertically integrated firm selling  $I$ 's product, when discrimination between different segments of demand is not possible. Finally, Section 7 presents a model of a single buyer with a downward-sloping demand.

## 1 Unconditional transfers in rent shifting

We first show how unconditional transfers restore the full anticompetitive potential of rebate contracts in the rent-shifting setup of Aghion and Bolton (1987), and then, look at how that potential gets diminished as we impose an exogenous limit on those transfers.

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## 1.1 Unconditional transfers in rebate contracts

A fundamental issue raised by Proposition 1 in the paper is what explains that  $I$  can write an anticompetitive exclusive but not an anticompetitive rebate. Both contracts are signed on date 1 before  $E$  shows up, so the difference lies in when exclusivity is committed. The exclusive does so ex-ante by committing  $B$  on date 1 to pay a penalty in case the exclusivity is breached. The rebate, by contrast, does so ex-post by rewarding  $B$  only after the exclusivity is observed. Another way to put it is that under an exclusive,  $B$  does not benefit at all from the downward pressure that penalty  $D$  exerts on  $E$ 's offer because the penalty goes directly to  $I$ , whereas under a rebate,  $B$  is the one that directly benefits from  $E$ 's low offer (i.e., lower than  $c_I$ ), and nothing in the contract commits  $B$  to share all or part of this benefit with  $I$ . This, however, suggests a way for  $I$  to amend the rebate contract  $(r, R)$  and restore its anticompetitive potential:

**Lemma 1.** *I can implement the anticompetitive outcome of Aghion-Bolton (see expression (3) in the paper) with the rebate contract  $(r, R, Z)$ , where  $Z$  is an unconditional transfer from  $B$  to  $I$  that is agreed at the time the rebate contract is signed on date 1. I has flexibility to write this anticompetitive rebate from the one with the lowest unconditional transfer ( $r = v$ ,  $R = \lambda(v - \tilde{c}_E)$ ,  $Z = \lambda(v - \tilde{c}_E) - \pi_B^{NC}$ ), to the one with the highest ( $r = \tilde{c}_E$ ,  $R = 0$ ,  $Z = v - \tilde{c}_E - \pi_B^{NC}$ ).*

**Proof.** Since  $Z$  is unconditional,  $I$ 's problem is to maximize ( $x \equiv r - R/\lambda$ )

$$\mathbb{E}\pi_I(r, R, Z) = Z + (1 - \lambda)(r - c_I) + \lambda(x - c_I)[1 - G(x)] \quad (1)$$

subject to  $r \leq v$  and  $v - r + R - Z \geq \pi_B^{NC}$ . Since this latter participation constraint will be binding, replacing  $R = \lambda(r - x)$  in it and substituting  $Z = v - (1 - \lambda)r - \lambda x - \pi_B^{NC}$  in (1) leads to

$$\mathbb{E}\pi_I(r, R, Z) = v - c_I - \pi_B^{NC} + \lambda(c_I - x)G(x) = (1 - \lambda)(v - c_I) + \lambda(c_I - x)G(x)$$

and solving for  $x$  yields (3) in the paper. The rest of the proof follows from the fact that  $Z$  and  $r \leq v$  are freely chosen from any combination that satisfies  $Z + (1 - \lambda)r = v - \lambda\tilde{c}_E - \pi_B^{NC}$ . ■

By committing  $B$  to make an unconditional transfer at the time the contract is signed,  $I$  can now offer an exclusionary rebate with large rewards ex-post, i.e.,  $r - R/\lambda < c_I$ , without sacrificing profits, i.e.,  $\mathbb{E}\pi_I(r, R, Z) > \pi_I^{NC}$ .<sup>1</sup> By trading ex-post rewards for ex-ante transfers,  $I$  can now extract rents from those entrants that decide to enter ( $c_E \leq \tilde{c}_E$ ) while leaving  $B$  with

<sup>1</sup>In a previous version of the paper, we show this is the only way to amend the  $(r, R)$  contract to obtain the Aghion-Bolton outcome.

enough to sign the contract. As long as  $I$  and  $B$  have some uncertainty about  $c_E$ , extracting rents from  $E$  will not be perfect and some exclusion will necessarily occur.<sup>2</sup>

Whether the transfers required in Lemma 1 are feasible to implement in practice is not obvious, which may explain why we rarely see them, if at all. Leaving aside cases that do not require these transfers (see Section V in the paper), good reasons may restrict their use, such as limited liability and asymmetric information. Since  $I$  has some flexibility to design these rebates, as Lemma 1 indicates, we now show how the optimal contract  $(r, R, Z)$  varies as we (exogenously) restrict the value of  $Z$  (see also Ide and Montero 2015).

## 1.2 Restricted unconditional transfers

In this section we look at the design of rebate contracts but under the assumption that unconditional transfers are subject to some exogenous limit.  $I$ 's problem can be written as (recall that  $F = 0$ )

$$\max_{r, x, Z} \mathbb{E}\pi_I = (1 - \lambda)(v - c_I) + \lambda(x - c_I)[1 - G(x)] + Z$$

subject to  $r \leq v$ ,  $Z \leq \bar{Z}(\theta)$  and

$$v - (1 - \lambda)r - \lambda x - Z \geq \pi_B^{NC}$$

where  $\theta \in [0, +\infty)$  is some parameter that captures how problematic is for  $I$  to demand an unconditional transfer from  $B$  on date 1. For instance,  $\theta$  could represent some limited liability faced by retail buyers, or the presence of asymmetric information (Ide and Montero 2015), to name a few. For generality we allow  $\bar{Z}(\theta)$  to take any value, even negative, which would make it a slotting allowance. We assume that the larger the  $\theta$  the more difficult is for  $I$  to demand an unconditional transfer so that  $\lim_{\theta \downarrow 0} \bar{Z}(\theta) = +\infty$ ,  $\lim_{\theta \uparrow \infty} \bar{Z}(\theta) = -\infty$ , and  $Z'(\theta) < 0$ .

Let us first define

$$\begin{aligned} \bar{\theta} & : \quad \bar{Z}(\bar{\theta}) = \lambda(v - \tilde{c}_E) - \pi_B^{NC} \\ \bar{\bar{\theta}} & : \quad \bar{Z}(\bar{\bar{\theta}}) = \lambda(v - c_E^o) - \pi_B^{NC} \end{aligned}$$

where  $\tilde{c}_E = c_I - G(\tilde{c}_E)/g(\tilde{c}_E) < c_I$ , and  $c_E^o = c_I + [1 - G(c_E^o)]/g(c_E^o) \in (c_I, v)$ .

It can be shown that the optimal rebate contract  $(r^*, x^*, Z^*)$  will depend on  $\theta$  as follows:

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<sup>2</sup>This partial-exclusion result connects nicely with a recent paper by Calzolari and Denicolo (2015). In their model, partial exclusion arises because it helps the incumbent to leave less information rents with privately informed buyers. Here, it arises because it helps the incumbent to extract more entry rents.

1. If  $\theta \leq \bar{\theta}$  (i.e.  $\lambda(v - \tilde{c}_E) - \pi_B^{NC} \leq \bar{Z}(\theta)$ ), then  $\{r^* \leq v, x^* = \tilde{c}_E, Z^* \leq \bar{Z}(\theta)\}$  such that

$$(1 - \lambda)r^* + Z^* = v - \lambda\tilde{c}_E - \pi_B^{NC}$$

2. If  $\bar{\theta} < \theta < \bar{\bar{\theta}}$  (i.e.  $\lambda(v - c_E^o) - \pi_B^{NC} < \bar{Z}(\theta) < \lambda(v - \tilde{c}_E) - \pi_B^{NC}$ ), then

$$\left\{ r^* = v, x^* = v - \left( \frac{\bar{Z}(\theta) + \pi_B^{NC}}{\lambda} \right), Z^* = \bar{Z}(\theta) \right\}$$

Notice moreover that there exists a unique  $\hat{\theta} \in (\bar{\theta}, \bar{\bar{\theta}})$  given by  $\bar{Z}(\hat{\theta}) = \lambda(v - c_I) - \pi_B^{NC} > 0$  such that for all  $\theta \geq \hat{\theta}$  the contract is not anticompetitive ( $x^* \geq c_I$ ).

3. If  $\bar{\bar{\theta}} \geq \theta$  (i.e.  $\bar{Z}(\theta) \leq \lambda(v - c_E^o) - \pi_B^{NC}$ ), then  $\{r^* = v, x^* = c_E^o, Z^* = \bar{Z}(\theta)\}$ .

To demonstrate that these are indeed the optimal rebate contracts, write first the Lagrangian of  $I$ 's problem

$$\begin{aligned} \mathcal{L} = (1 - \lambda)(v - c_I) + \lambda(x - c_I)[1 - G(x)] + Z + \mu_1(v - (1 - \lambda)r - \lambda x - Z - \pi_B^{NC}) \\ + \mu_2(\bar{Z}(\theta) - Z) + \mu_3(v - r) \end{aligned}$$

to obtain the first order conditions

$$\frac{\partial \mathcal{L}}{\partial r} = (1 - \lambda)(1 - \mu_1) - \mu_3 = 0 \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial x} = \lambda[1 - G(x) - g(x)(x - c_I) - \mu_1] = 0 \quad (3)$$

$$\frac{\partial \mathcal{L}}{\partial Z} = 1 - \mu_1 - \mu_2 = 0 \quad (4)$$

$$\frac{\partial \mathcal{L}}{\partial \mu_1} = v - (1 - \lambda)r - \lambda x - Z - \pi_B^{NC} \geq 0 \quad \text{and} \quad \mu_1 \frac{\partial \mathcal{L}}{\partial \mu_1} = 0 \quad (5)$$

$$\frac{\partial \mathcal{L}}{\partial \mu_2} = \bar{Z}(\theta) - Z \geq 0 \quad \text{and} \quad \mu_2 \frac{\partial \mathcal{L}}{\partial \mu_2} = 0 \quad (6)$$

$$\frac{\partial \mathcal{L}}{\partial \mu_3} = v - r \geq 0 \quad \text{and} \quad \mu_3 \frac{\partial \mathcal{L}}{\partial \mu_3} = 0 \quad (7)$$

with  $\mu_1, \mu_2, \mu_3 \geq 0$ . Depending on the value of  $\theta$ , we can now characterize three candidates for the optimum.

1. Candidate 1:  $\mu_1^* > 0$ , and  $\mu_2^* = \mu_3^* = 0$

Either from (2) or (3) we get  $\mu_1^* = 1 > 0$ . Moreover from (3):

$$G(x) + g(x)(x - c_I) = 0 \implies x^* = \tilde{c}_E$$

From (6) and (7) we get  $r^* \leq v$  and  $Z^* \leq \bar{Z}(\theta)$ . And from (5) we have

$$(1 - \lambda)r^* + Z^* = v - \lambda\tilde{c}_E - \pi_B^{NC}$$

Finally, since  $r^* \leq v$ , this latter implies that

$$Z^* = v - \lambda\tilde{c}_E - \pi_B^{NC} - (1 - \lambda)r^* \geq \lambda(v - \tilde{c}_E) - \pi_B^{NC}$$

and since  $Z^* \leq \bar{Z}(\theta)$ , this requires

$$\bar{Z}(\theta) \geq \lambda(v - \tilde{c}_E) - \pi_B^{NC}$$

2. Candidate 2:  $\mu_1^*, \mu_2^*, \mu_3^* > 0$

(5), (6) and (7) imply that  $Z^* = \bar{Z}(\theta)$ ,  $r^* = v$ , and

$$x^* = v - \left( \frac{\bar{Z}(\theta) + \pi_B^{NC}}{\lambda} \right) \quad (8)$$

Moreover (2), (3) and (4) give  $\mu_2^* = 1 - \mu_1^*$ ,  $\mu_3^* = (1 - \lambda)(1 - \mu_1^*)$  and

$$\mu_1^* = 1 - G(x^*) - g(x^*)(x^* - c_I)$$

Finally, for this to be indeed the optimum we require that  $\mu_1^* \in (0, 1)$ , but this is the case whenever  $\tilde{c}_E < x^* < c_E^\circ$ . Combining the latter with (8) we get that

$$\lambda(v - c_E^\circ) - \pi_B^{NC} < \bar{Z}(\theta) < \lambda(v - \tilde{c}_E) - \pi_B^{NC}$$

3. Candidate 3:  $\mu_1^* = 0$ , and  $\mu_2^*, \mu_3^* > 0$

From (2) and (4) we get  $\mu_3^* = (1 - \lambda) > 0$ , and  $\mu_2^* = 1 > 0$ . Moreover from (3)

$$1 - G(x) - g(x)(x - c_I) = 0 \implies x^* = c_E^\circ$$

and from (6) and (7) we get  $r^* = v$  and  $Z^* = \bar{Z}(\theta)$ . Finally from (5) we get

$$v - (1 - \lambda)r - \lambda x - Z - \pi_B^{NC} \geq 0 \implies \bar{Z}(\theta) \leq \lambda(v - c_E^\circ) - \pi_B^{NC}$$

Depending on the value of  $\bar{Z}(\theta)$ , one of these three candidates will characterize the optimum, as stated in the lemma. The first candidate applies when the transfer restriction is not binding ( $\theta \leq \bar{\theta}$ ), so  $I$  can achieve the Aghion-Bolton outcome. In the region where  $\theta$  is above  $\bar{\theta}$  the transfer restriction becomes active and  $I$  must distort the effective price upwards, leading to a less anticompetitive outcome. And when  $\theta$  reaches  $\hat{\theta} \in (\bar{\theta}, \bar{\bar{\theta}})$  the rebate contract is not longer

anticompetitive. Notice though, that at  $\hat{\theta}$  the scheme still entails a positive, but substantially small, unconditional transfer. A further tightening of the transfer restriction, for example to  $\bar{Z}(\theta) = 0$ , forces  $I$  to raise the effective price even further, which is when we arrive to our result that  $(r, R)$  contracts in the rent-shifting model of Aghion-Bolton cannot be anticompetitive (see Proposition 1 in the paper).

## 2 No-contract payoffs in the naked-exclusion setup

In the absence of contracts,  $I$  and  $E$  compete in the spot market as in the rent-shifting setup. In addition, it can be established that if  $I$  does not contract with  $B1$  and  $B2$  on date 1, then  $E$  is indifferent between making offers to either of the two buyers or going directly to the spot on date 4. In either case, the no-contract payoffs are:  $\pi_I^{NC} = (1 - \lambda)(v - c_I)$ ,  $\pi_E^{NC} = \lambda(c_I - c_E) - F$ , and  $\pi_{Bi}^{NC} = \lambda(v - c_I)/2$  for  $i = 1, 2$ .

To formally show that  $E$  cannot do better, notice that the maximum available surplus for a “coalition” of  $E$  and the two buyers ( $EBB$ ) is  $V_{EBB} = \lambda(v - c_E) - F$ . The question now is how that surplus gets divided between  $E$  and the buyers. In particular, can a “sub-coalition” of  $E$  and one of the buyers appropriate all of it by exploiting the other buyer? Suppose on date 2  $E$  makes simultaneous public price offers  $w_{E1}$  and  $w_{E2}$  to both buyers  $B1$  and  $B2$ , respectively (sequential offers do not change the result). There are three relevant cases to consider. The first is when both offers are slightly below  $c_I$ . It is clearly an equilibrium for both buyers to accept because  $E$  will enter in any case, which secures each buyer at least  $\lambda(v - c_I)/2$ . The second is when both offers are above  $c_I$ , in which case the equilibrium is for each of the two buyers to reject, regardless of what the other does, because they know they can always get  $\lambda(v - c_I)/2$ . The most interesting case is when  $E$  offers the pair  $w_{E1} < c_I < w_{E2}$  such that  $\pi_E = \lambda(w_{E1} - c_E)/2 + \lambda(w_{E2} - c_E)/2 - F > \pi_E^{NC} = \lambda(c_I - c_E) - F$ , which would be  $E$ ’s payoff in case both buyers accept, and

$$\pi_E = \lambda(w_{E1} - c_E)/2 + \lambda(c_I - c_E)/2 - F < 0,$$

which would be  $E$ ’s payoff in case only  $B1$  accepts but he enters anyway. If  $E$  could commit not to renegotiate with  $B1$ , then  $B2$  would, in equilibrium, accept any offer  $w_{E2} \leq v - \epsilon$  to make sure  $E$  enters. But the problem is that  $B2$  anticipates that if she rejects,  $E$  and  $B1$  will renegotiate the terms of  $B1$ ’s offer to make sure  $E$  enters, which in turn makes  $B2$ ’s rejection a best response. Anticipating buyers’ equilibrium responses,  $E$  gains nothing from making offers

on date 2.<sup>3</sup>

### 3 Unconditional transfers in naked exclusion

We now consider rebate contracts with unconditional transfers in the naked-exclusion setup of Rasmusen et al (1990) and Segal and Whinston (2000b). We first study how unconditional transfers can be used to restore the full anticompetitive potential of rebates (i.e., can replicate the work of exclusives) and then how they can also be used to extract rents.

#### 3.1 Using unconditional transfers for exclusion

Unlike in Rasmusen et al (1990) and in Segal and Whinston (2000b), the reason  $I$  fails to implement a divide-and-conquer strategy with rebates is because the unlucky buyer ( $B2$ ) can no longer be used to finance the rebates going to the lucky buyer ( $B1$ ). Given that rebates do not lock up buyers ex-ante,  $E$  has always the opportunity on date 2 to make counter offers to fight off exclusion. Moreover, because  $B2$  anticipates that she will be fully exploited by  $I$ , she is ready to give up almost her entire surplus to  $E$ , forcing  $I$  to offer even larger rewards to lock up  $B1$ , so large that they are not profitable. In other words, if  $I$  tries to exploit  $B2$  in the first place  $E$  will do likewise, so the same (ex-post) rents coming from  $B2$  that  $I$  is using to pay for the rebate to  $B1$  are also used by  $E$  to induce  $B1$  to switch. As with Lemma 1, however, there is a way for  $I$  to go around this funding problem and deter  $E$ 's entry:

#### **Proposition 2.**

**Lemma 3.** *In the naked-exclusion setup of multiple buyers and scale economies,  $I$  can profitably deter  $E$ 's entry with a pair of discriminatory rebate contracts  $(r_i, R_i, Z_i)$  for  $i = 1, 2$ , where  $Z_i$  is an unconditional transfer from  $B_i$  to  $I$  that is agreed at the time the rebate contract is signed on date 1.*

**Proof.** It is easy to see that  $I$  can replicate the exclusionary outcome of Proposition 2 in the paper with no offer to  $B2$  and a rebate offer  $(r_1 \leq v, R_1, Z_1)$  to  $B1$ , the lucky buyer, that satisfies both

$$(v - r_1 + R_1)/2 - Z_1 \geq \pi_{B1}^{NC} \tag{9}$$

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<sup>3</sup>Note that if in fact  $E$  could commit not to renegotiate with  $B1$ , then buyers outside options would be smaller, on average, which would make exclusion a bit cheaper for  $I$ .

and “the no-switching” condition (7) in the paper for  $w_{E1} = 2F/\lambda + 2c_E - v$ . To foreclose  $E$ ’s entry at minimum cost,  $\pi_{B1}^{NC}$ ,  $I$  will set (9) to equality. ■

Again, an unconditional transfer from the lucky buyer is what allows the incumbent to offer very competitive rewards ex-post, making it impossible for  $E$  to persuade that buyer to switch. But if such large unconditional transfers are feasible to implement, one can go further and ask if  $I$  can do better than just deterring  $E$ ’s entry.

### 3.2 Using unconditional transfers for rent extraction

We now establish the form of the most profitable rebate contracts  $I$  can offer when unconditional transfers have no restrictions. Consistent with the exclusives in Innes and Sexton (1994) and Spector (2011, section 4) that include breach penalties as vehicles to transfer rents,  $I$  can write a pair of efficient discriminatory  $(r_i, R_i, Z_i)$  contracts and pocket the entire social surplus, except for  $\pi_{B1}^{NC} + \epsilon$ . By locking up just one retailer,  $B1$ ,  $I$  can fully exploit  $B2$  while simultaneously extracting all of  $E$ ’s efficiency rents. This non-exclusionary result, however, is clearly not robust. For instance, rent extraction would be incomplete when  $c_E$  is unknown, in which case, again, there will be exclusion of some efficient entrants even when  $(r_i, R_i, Z_i)$  contracts are at hand. We leave that case for the next section.

When  $I$  is free to charge unconditional transfers to buyers on date 1, he will optimally let  $E$  to come in with rebate offers  $(r_1, x_1, Z_1)$  and  $(r_2, x_2, Z_2)$  that share the following characteristics: (i)  $Z_1 = ((1 - \lambda)(v - r_1) + \lambda(c_I - x_1))/2 - \epsilon$ , (ii)  $Z_2 = (v - (1 - \lambda)r_2 - \lambda x_2)/2$ , (iii)  $\lambda(x_1 - c_E)/2 + \lambda(x_2 - c_E)/2 - F = 0$ , (iv)  $r_i \leq v$ , (v)  $x_i \geq c_E$ , and (vi)  $x_1 < 2c_E - c_I + 2F/\lambda$ .

Note first that independently of whether  $Bj$  accepts or not, and irrespective of  $E$ ’s entry, if  $Bi$  accepts  $I$ ’s offer she gets  $v - r_i + R_i = v - (1 - \lambda)r_i - \lambda x_i$  ex-post, and therefore,  $v - (1 - \lambda)r_i - \lambda x_i - Z_i$  ex-ante. If, however,  $Bi$  rejects  $I$ ’s offer, her payoff does depend on whether  $Bj$  accepts or not. If both buyers reject, we are in the no-contract benchmark where each gets  $\pi_{Bi}^{NC} = \lambda(v - c_I)/2$ . But if  $Bi$  rejects and  $Bj$  accepts, her payoff will ultimately depend on the offers that  $E$  will present to each of them on date 2. Since  $x_i \geq c_E$  for both  $i$  and  $j$ ,  $E$  will find it optimal to slightly undercut the terms of the rebate contract of the “captive” buyer ( $Bj$  in this case) with a price offer slightly below  $x_i$ . As for  $E$ ’s offer to the free buyer, notice that  $Bj$ ’s outside option depends on  $x_i$ . In fact, if  $\lambda(x_i - c_E)/2 + \lambda(c_I - c_E)/2 - F \geq 0$ ,  $Bj$ ’s outside option is  $\lambda(v - c_I)/2$  because if she rejects  $E$ ’s offer,  $E$  will still find it optimal to enter. But if  $\lambda(x_i - c_E)/2 + \lambda(c_I - c_E)/2 - F < 0$ ,  $Bj$ ’s outside option is zero, because if she



rejects,  $E$  will not enter.

These outside options can be summarized in the following payoff matrix:

$B1 \setminus B2$	Accept	Reject
Accept	$\lambda(v - c_I)/2 + \epsilon, 0$	$\lambda(v - c_I)/2 + \epsilon, 0$
Reject	$\lambda(v - c_I)/2, 0$	$\lambda(v - c_I)/2, \lambda(v - c_I)/2$

Notice that the reason  $B2$  gets zero if she rejects and  $B1$  accepts (upper-right corner) is because (vi) leads to  $\lambda(x_1 - c_E)/2 + \lambda(c_I - c_E)/2 - F < 0$ . Similarly, the reason  $B1$  gets  $\lambda(v - c_I)/2$  if she rejects but  $B2$  does not (lower-left corner) is because

$$\lambda(x_2 - c_E) + \lambda(c_I - c_E)/2 - F = \lambda(c_I - c_E)/2 - \lambda(x_1 - c_E)/2 > 0$$

where the equality follows from (iii) and the inequality from  $x_1 < c_I$ , which follows directly from the efficient-entry condition and (vi). Hence, in equilibrium both buyers accept the contract (if  $B2$  rejects makes no difference),  $E$  enters and slightly undercuts both terms. Equilibrium payoffs, as of date 1, are  $\pi_I = v - c_I + \lambda(c_I - c_E) - F - \lambda(v - c_I)/2 - \epsilon$ ,  $\pi_{B1} = \lambda(v - c_I)/2 + \epsilon$ ,  $\pi_{B2} = 0$  and  $\pi_E = 0$ . Since  $I$  must leave  $B1$  with at least  $\pi_{B1}^{NC}$ , he cannot improve upon these rebate contracts.

Finally, notice that characteristics (i)-(vi) accommodate to different rebate contracts; for example, the pair  $(r_1 = v, x_1 = c_E, Z_1 = \lambda(c_I - c_E)/2 - \epsilon)$  and  $(r_2 = v, x_2 = c_E + 2F/\lambda, Z_2 = \lambda(v - c_E)/2 - F)$ . Note that  $Z_2 < 0$ , an slotting allowance, but it is easy to construct another example with  $Z_2 > 0$  by simply lowering the list price  $r_2$  accordingly. In any case, with any of these contracts  $I$  pockets all the social surplus but  $\pi_{B1}^{NC} + \epsilon$ , which is what he needs to give up in order to induce  $B1$  to sign.

## 4 Unknown entry costs in naked exclusion

In this section we look at the work of rebate contracts in a naked-exclusion setup, both with and without unconditional transfers, when  $c_E$  is unknown on date 1; very much as in a rent-shifting setting.

### 4.1 Rebates without unconditional transfers

To simplify matters assume that  $c_E$  is distributed according to the cumulative function  $G(\cdot)$  over  $[v - 2F/\lambda, c_I - F/\lambda]$ , so that  $\lambda(v - c_E)/2 - F \leq 0$  and  $0 \leq \lambda(c_I - c_E) - F$  for any

realization of  $c_E$ . That is, irrespective of  $c_E$ : (i)  $E$  needs to serve both buyers to find it profitable to enter the market, and (ii) entry is always efficient. This interval is well defined because  $v - 2F/\lambda < c_I - F/\lambda$ , or  $\lambda(v - c_I) < F$ .

Consider the scenario where  $I$  offers just one rebate contract, say  $(r_1, x_1)$  to  $B1$ .  $E$  will find it profitable to enter whenever

$$\lambda(x_1 - c_E)/2 + \lambda(v - c_E)/2 - F \geq 0 \implies \chi \equiv \frac{v + x_1}{2} - \frac{F}{\lambda} \geq c_E$$

as  $B2$  gets charged  $v$  for each unit. Therefore, the probability of entry is  $G(\chi)$ . Anticipating this,  $I$ 's expected profit is

$$\begin{aligned} \mathbb{E}\pi_I &= \left(\frac{1}{2}\right) G(\chi)(1 - \lambda)(r_1 + v - 2c_I) + \left(\frac{1}{2}\right) [1 - G(\chi)](r_1(1 - \lambda) + \lambda x_1 + v - 2c_I) \\ &= (1 - \lambda) \left(\frac{v + r_1}{2} - c_I\right) + \lambda[1 - G(\chi)] \left(\chi - \left[c_I - \frac{F}{\lambda}\right]\right) \end{aligned}$$

But  $I$ 's no-contract payoff is  $\pi_I^{NC} = (1 - \lambda)(v - c_I)$ , so

$$\mathbb{E}\pi_I - \pi_I^{NC} = -(1 - \lambda) \left(v - \frac{v + r_1}{2}\right) + \lambda[1 - G(\chi)] \left(\chi - \left[c_I - \frac{F}{\lambda}\right]\right)$$

Since  $r_1 \leq v$ , a necessary condition for a rebate contract to be profitable ( $\mathbb{E}\pi_I - \pi_I^{NC} \geq 0$ ) is that  $\chi > c_I - F/\lambda$ , that is, that entry must be inefficient.

Finally it is easy to see that  $I$  cannot improve upon offering a second contract. If  $I$  goes on and offers  $B2$  the contract  $(r_2, x_2)$ , in addition to the contract  $(r_1, x_1)$  to  $B1$ , then he will end up extracting  $(1 - \lambda)r_2$  and  $r_2 - R_2$  from  $B2$  in the event of entry and no-entry, respectively, which is less than what he is currently extracting,  $(1 - \lambda)v$  and  $v$ , respectively. Hence the new entry cutoff  $\chi' \equiv (x_1 + x_2)/2 - F/\lambda$  will have to be set even higher for  $I$  to be able to profit from the contract.

## 4.2 Rebates with unconditional transfers

Assume again that  $c_E$  distributes according to  $G(\cdot)$  over  $[v - 2F/\lambda, c_I - F/\lambda]$ . Suppose  $I$  offers only one contract, say  $(r_1, x_1, Z_1)$  to  $B1$ . The main difference with the previous section is that now  $I$  can use the unconditional transfer  $Z_1$  to extract rents from  $B1$ . Since buyers can communicate and coordinate their actions,  $I$  must leave  $B1$  with no less than her no-contract payoff, that is,  $(v - (1 - \lambda)r_1 - \lambda x_1)/2 - Z_1 \geq \pi_{B1}^{NC} = \lambda(v - c_I)/2$ . Since this constraint is binding, we can rewrite  $I$ 's expected payoff as

$$\begin{aligned} \mathbb{E}\pi_I &= (2(v - c_I) - \lambda G(\chi)[v + x_1 - 2c_I])/2 - \lambda(v - c_I)/2 \\ &= (v - c_I) - \lambda G(\chi) \left(\chi - \left[c_I - \frac{F}{\lambda}\right]\right) - \lambda(v - c_I)/2 \end{aligned} \tag{10}$$

Differentiating (10) with respect to  $\chi$  yields

$$\frac{d\mathbb{E}\pi_I}{d\chi} = G'(\chi) \left( \chi - \left[ c_I - \frac{F}{\lambda} \right] \right) + G(\chi) = 0 \quad (11)$$

which implies that  $\chi^* < c_I - F/\lambda$ . This optimal rebate contract blocks moderately efficient entrants, i.e.,  $c_E \in [\chi^*, c_I - F/\lambda]$ .

This result follows the Aghion-Bolton logic for a single buyer. The only difference is that now the  $IB1$  coalition acts as a monopoly over the  $EB2$  coalition, since  $EB2$  needs  $B1$  to enter. Because buyers cannot sign binding commitments ex-ante,  $I$  can get  $B1$  to sign for not less than her outside option  $\pi_{B1}^{NC}$ . Notice again that  $I$  cannot do better by offering a second contract since in any case must leave one of the buyers, say  $Bi$ , with no less than  $\pi_{Bi}^{NC}$ .

## 5 Unconditional transfers in downstream competition

We now considers rebate contracts with unconditional transfers in the downstream-competition setup of Simpson and Wickelgren (2007) and Asker and Bar-Isaac (2014). We first study how unconditional transfers can be use to restore the full anticompetitive potential of rebates and then how they can also be used to extract rents.

### 5.1 Using unconditional transfers for exclusion

Following the results in the rent-shifting and naked-exclusion settings (Lemmas 1 and 2 above), the obvious question is whether exclusion can again be restored by introducing unconditional transfers. In rent-shifting and naked-exclusion, unconditional transfers were less a problem for retailers because they could perfectly anticipate the monopoly rents available to pay for these transfer ex-post. Here it is not that simple, because Bertrand competition tends to dissipate those rents, so a retailer anticipating zero profits ex-post will not be willing to commit to any payment up-front. As the next proposition establishes, however,  $I$  can get around this problem with the proper contract design.

**Lemma 4.** *In the downstream-competition setup of Bertrand competitors,  $I$  can profitably deter  $E$ 's entry with a pair of discriminatory rebate contracts  $(r_i, R_i, Z_i)$  for  $i = 1, 2$ .*

**Proof.** This proof follows directly from the discussion at the end of the proof of Proposition 5 in the paper. Suppose on date 1 that  $I$  approaches  $B1$  with a rebate offer  $(r_1, R_1, Z_1)$  with an effective price  $r_1 - R_1$  slightly below  $E$ 's break-even price  $c_E + F/\lambda$ . At the same time, he

offers  $B2$  a uniform price in the form of a rebate offer  $(r_2, R_2, Z_2)$  with  $R_2 = 0$  ( $Z_1$  and  $Z_2$  are defined shortly). It is clear that if  $B1$  accepts the rebate and conforms to the exclusivity,  $E$  will not enter because he cannot use  $B2$  to compete with  $B1$ 's marginal cost of  $r_1 - R_1$  in the contestable portion of the retail market (notice, as discussed in the proof of Proposition 7, that the same applies if  $I$  offers nothing to  $B2$  on date 1). On the other hand, since retailers' outside options are zero, by setting  $r_1 = r_2 = v$ ,  $Z_1 = R_1 - \epsilon$ , and  $Z_2 = -\epsilon$ ,  $I$  can induce both buyers to sign for as little as  $\epsilon \rightarrow 0$ .  $B2$  is willing to sign because her outside option is zero. And having observed  $B2$  doing so (or at least the offer she received),  $B1$  is also willing to sign and commit to the unconditional transfer  $Z_1$  because she understands that  $B2$ 's contract commits  $I$  to sell units to  $B2$  at price  $r_2 = v$ , nothing less, which is what allows  $B1$  to pocket  $R_1 = Z_1 + \epsilon$  ex-post.  $B1$  also understands that any later effort by  $I$  to renegotiate  $B2$ 's contract to expropriate part or all of these ex-post rents is fruitless because she can always sell  $I$ 's units for as low as  $r_1 - R_1$ . Finally, it is easy to compute that with these rebates,  $I$  obtains an ex-ante payoff of  $v - c_I - 2\epsilon > (1 - \lambda)(v - c_I) = \pi_I^{NC}$ , which is exactly what he obtains with the exclusives of Proposition 4. ■

Notice that, due to the intense downstream competition,  $I$  just needs to make one of the retailers ( $B1$ ) aggressive enough in the contestable segment for  $E$  not to enter. But since this benefits  $B1$  greatly, as she can now charge  $v$  for all units,  $I$  uses the unconditional transfer  $Z_1$  to extract this benefit while maintaining the large rebate  $R_1$  that prevents  $E$ 's entry in the first place. In a way, this is similar to what happens in naked exclusion —where exclusion did not require  $I$  to lock up both retailers— but for a very different reason. In naked exclusion it was due to scale economies, while here it is due to the intense retail competition. Also, as in rent shifting and naked exclusion, the introduction of unconditional transfers is what allows  $I$  to offer large rewards ex-post; but unlike there,  $I$  needs here to purposefully soften retail competition to make sure retailers can recover those unconditional payments later. The way to do this is by offering one retailer ( $B2$ ) a rebate contract under such unfavorable conditions that she cannot compete; a contract that she is nevertheless willing to take for virtually nothing, precisely because of the intense retail competition.

## 5.2 Using unconditional transfers for rent extraction

In this section we look at  $I$ 's problem when unconditional transfers are not used to foreclose  $E$ 's entry but to extract efficiency rents from him. Suppose  $B1$  is the retailer approached

by  $E$  to enter the market. Consider the following pair of rebate contracts that  $I$  will offer both buyers on date 1, respectively  $(r_1, R_1, Z_1)$  and  $(r_2, R_2, Z_2, q_2 \leq \bar{q})$ , where: (i)  $r_1 = v$ , (ii)  $r_1 - R_1 = c_E + F/\lambda + 2\epsilon$ , (iii)  $Z_1 = \lambda R_1 - \epsilon$ , (iv)  $r_2 = v$ , (v)  $r_2 - R_2 < r_1 - R_1$ , (vi)  $Z_2 = (1 - \lambda)R_2 - \epsilon$ , and (vii)  $q_2 \leq 1 - \lambda$ , i.e.,  $B2$  can at most buy  $1 - \lambda$  from  $I$ .

To see why these pair of contracts allow  $I$  to extract  $E$ 's entry rents, notice first that the worst entry scenario for  $E$  is to approach  $B1$ , which according to (v) is the buyer with the higher effective price  $r - R$ , with a price offer that slightly undercuts (ii), that is

$$w_{E1} = c_E + F/\lambda + \epsilon$$

This leaves  $E$  with virtually nothing as  $\epsilon \rightarrow 0$ .  $B1$  accepts this offer, since it is  $\epsilon$  more attractive than  $I$ 's current offer (ii), and becomes the only one selling to the contestable portion. This latter is possible because the quantity limit (vii) prevents  $B2$  from selling to the contestable portion given that she is already selling  $1 - \lambda$  units to the non-contestable portion (note  $B1$  cannot compete in the non-contestable portion because (iv) and the fact that she gave up the rebate  $R_1$  once she accepted  $E$ 's offer). Thus,  $B1$  pockets an ex-post rent equal to  $\lambda(v - w_{E1})$ , so her profit as of date 1, i.e., her profit net of the unconditional transfer (iii), is  $\epsilon(1 + \lambda)$ . On the other hand,  $B2$  sticks to the exclusivity selling at  $v$  to the non-contestable portion, obtaining a profit net of the up-front (vi) equal to  $\epsilon$ . Hence,  $I$  gets the full social surplus using the list prices in (i) and (iv) and letting  $\epsilon \rightarrow 0$ .

Notice that this works for  $I$  as long as he can impose the quantity limit (vii) in  $B2$ 's contract, which seems odd; if anything, rebate contracts typically include deeper discounts as buyers sell more, as opposed to the radical price increase suggested by (vii). The problem is that in the absence of that limit there is no way for  $I$  to extract rents from  $E$  because  $B1$  and  $B2$  will always compete for the contestable units, preventing the retail price from going all the way to  $v$ .

## 6 Downstream pricing: retailer vs. vertically integrated firm

In this section we prove the claim made in the Appendix of the paper, that when price discrimination between contestable and non-contestable consumers in the downstream market is not possible, equilibrium profits of a vertically integrated firm selling  $I$ 's products (denoted  $B_I$ ) are always equal to  $(1 - \lambda)(v - c_I)$ , when facing a retailer carrying only  $E$ 's products (denoted  $B_E$ ) that has marginal cost/wholesale price  $w_E \leq c_I$ .

First, it is clear that no pure strategy equilibrium exists. Indeed, let  $p_E$  and  $p_I$  be the prices set by  $B_E$  and  $B_I$  respectively. If  $p_E < p_I$ , then  $B_E$  has incentives to rise its price; if  $w_E < p_I < p_E$ ,  $B_E$  has the incentive to undercut  $I$ 's price; if  $p_I \leq p_E = w_E$ ,  $B_I$  prefers to sell  $1 - \lambda$  units a price  $v$ , given that  $w_E \leq c_I < \hat{c} \equiv c_I + (1 - \lambda)(v - c_I)$ . And if  $p_I = p_E = v$  then  $B_I$  has incentives to undercut  $B_E$ 's price. Hence all possible candidates for a pure-strategy equilibrium are discarded. By a similar argument, it is also easy to see that both firms must be randomizing in equilibrium: if  $B_I$  plays  $p_I$  with probability 1 (w.p.1), then  $B_E$  has incentives to play  $p_E = p_I - \epsilon$  w.p.1, but if so, then  $B_I$  undercuts  $E$  if  $p_I > \hat{c}$ ; or raises its price to  $v$  if  $p_I < \hat{c}$ .

Suppose then that  $B_I$  and  $B_E$  randomize according to the pdf (cdf)  $h_I(\cdot)$  and  $h_E(\cdot)$  ( $H_I(\cdot)$  and  $H_E(\cdot)$ ) respectively, denote  $\sigma_i$  the support of  $H_i(\cdot)$ , and let  $r_i \equiv \inf \sigma_i$  and  $R_i \equiv \sup \sigma_i$  ( $R_i \leq v$ , for  $i = I, E$  obviously). It is clear that in any equilibria, equilibrium profits must satisfy  $\pi_{B_I}^* \geq (1 - \lambda)(v - c_I)$  and  $\pi_{B_E}^* > 0$ . The first inequality comes from the fact that  $B_I$  can always charge  $p_I = v$  and sell at least  $1 - \lambda$  units. The second, from the fact that  $B_E$  can always charge  $w_E < p_E < \hat{c}$  and prevent being undercut by  $B_I$ , earning therefore  $\lambda(p_E - w_E) > 0$ .

We now claim that  $\pi_{B_I}^* = (1 - \lambda)(v - c_I)$ . Suppose not. Then it must be that  $\pi_{B_I}^* > (1 - \lambda)(v - c_I)$ . This in turn means that:

$$\pi_{B_I}^* \equiv \pi_I(p_I) = (1 - \lambda)(p_I - c_I) + [1 - H_E(p_I)]\lambda(p_I - c_I) > (1 - \lambda)(v - c_I), \text{ for all } p_I \in \sigma_I \quad (12)$$

But since  $(1 - \lambda)(p_I - c_I) \leq (1 - \lambda)(v - c_I)$ , this is only possible if  $1 - H_E(p_I) > 0$  for all  $p_I \in \sigma_I$ . This latter is only possible if  $R_I < R_E$  or if  $R_I = R_E$  and  $h_E(\cdot)$  has an atom over  $R_E$ .

The first case immediately leads to a contradiction, since it would imply that  $B_E$  earns zero profit when playing  $R_E$ , which contradicts  $\pi_{B_E}^* > 0$ . The same is true if  $R_I = R_E = R$ ,  $h_E(\cdot)$  has an atom over  $R$ , but  $h_I(\cdot)$  is atomless at  $R$ . Finally, it neither can be that  $R_I = R_E = R$  and both  $h_E(\cdot)$  and  $h_I(\cdot)$  have an atom at  $R$ , since either  $B_I$  or  $B_E$  (depending on the tie-rule) will not play  $R$ , as playing  $R - \epsilon$  gives (almost) the same profit, but avoid the tie altogether, strictly increasing profit. Hence  $1 - H_E(p_I) > 0$  for all  $p_I \in \sigma_I$  leads to a contradiction, which implies that  $\pi_{B_I}^* = (1 - \lambda)(v - c_I)$ .

## 7 Single buyer with downward sloping demand

In this section we show that our non-exclusionary results (Propositions 1, 3, and 5 in the paper) do not depend on the full-surplus extraction over the non-contestable segment that

follows from the unit-demand model. Let then  $Q(p)$  be  $B$ 's downward sloping demand and  $P(q)$  the corresponding inverse demand. Suppose also that  $E$  is sufficiently small that can supply at most  $\lambda < Q(c_I)$  units. The timing of the game is as before (see section II of the paper).

Note first that to just block the entry of a rival of cost  $x$ , the discount contract must leave  $B$  with at least  $\int_0^\lambda (P(q) - x) dq$ , which is the most  $E$  can transfer  $B$  by pricing at cost his  $\lambda$  units. A simple discount contract that implements this outcome is a two-part tariff  $(x, T)$ , where  $x$  is the unit price and  $T = \int_\lambda^{Q(x)} (P(q) - x) dq$  is the (conditional) fixed fee equal to area ABD in Figure 1 (market-share and quantity-discount contracts work equally well; see, for example, Kolay et al. 2004). Thus, if  $I$  offers the two-part-tariff contract  $(x, T)$  with  $x < c_I$ , his payoff will be

$$\int_\lambda^{Q(x)} (P(q) - c_I) dq$$

when  $c_E < x$ , since he will only be selling  $Q(x) - \lambda$  units, and

$$\int_\lambda^{Q(x)} (P(q) - c_I) dq + \lambda(x - c_I)$$

when  $c_E \geq x$ , since he will now be selling  $Q(x)$  units.

Putting things together,  $I$ 's expected payoff reduces to

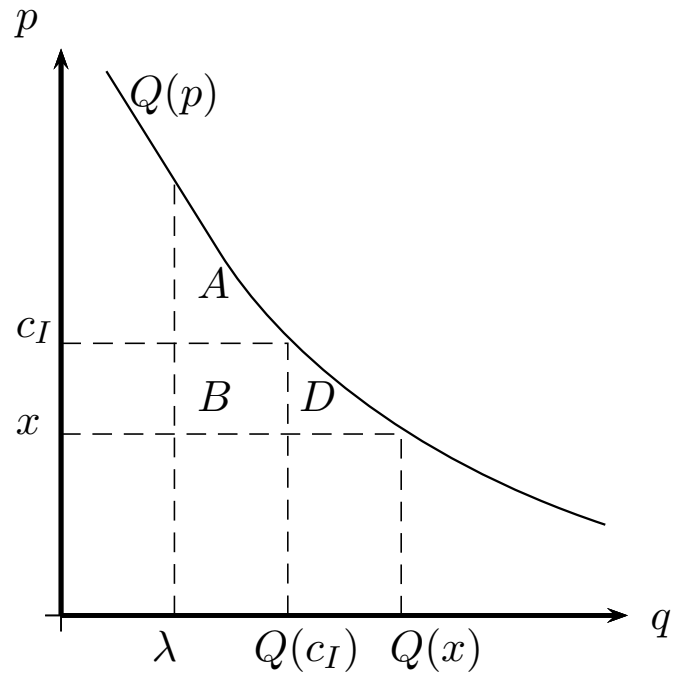
$$\int_\lambda^{Q(c_I)} (P(q) - c_I) dq + \int_{Q(c_I)}^{Q(x)} (P(q) - c_I) dq + \lambda(x - c_I)[1 - G(x)] \quad (13)$$

which differs from expression (6) in the proof of Proposition 1 in the paper only in the extra term in the middle, which captures the profit loss from selling additional units at an effective price  $x$  below  $c_I$ . This loss adds to the loss of selling  $\lambda$  units below cost when  $x < c_I$  and  $E$  does not enter, which happens with probability  $1 - G(x)$ . As in (6), the first term in (13) captures the residual monopoly profit, that is,  $I$ 's largest possible payoff when  $B$  has already  $\lambda$  units in her pocket.

Even without computing the best possible discount contract that  $I$  could offer  $B$  —which would require to know agents' outside options—, it is clear from (13) that  $I$  will never offer an anticompetitive one, that is, one with an effective price  $x < c_I$ . Now, to see whether  $I$  will still find it profitable to offer a two-part tariff  $(x, T)$  with  $x = c_I$ , we would need to know his outside option. If, in the absence of a contract the competition in the spot market is in nonlinear prices, then there is no reason for  $I$  to offer a discount contract because he can always secure the residual monopoly profit in the spot anyway. If, however, competition is in linear prices, it

is then profitable for  $I$  to offer the discount contract because he would otherwise obtain strictly less than  $\int_{\lambda}^{Q(c_I)} (P(q) - c_I) dq$ .<sup>4</sup>

Figure 1: Downward Sloping Demand




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<sup>4</sup>In the unit-demand case it was never profitable to offer the contract  $(r, R)$  because  $I$  always secured  $(1 - \lambda)(v - c_I)$  in the spot regardless of whether competition was in linear or non-linear prices. The mixed strategy equilibrium for the linear case can be found in an earlier version of the paper.



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