Online Appendix for "Import Constraints"

Diego Comin Robert C. Johnson Callum Jones

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In this appendix, we provide details for the model that we use to compute impulse responses in Section II of the manuscript. We briefly describe the economic environment, sketch the foreign firm's pricing problem, and then present model equilibrium conditions. We skip the full derivation of the equilibrium conditions, as these straightforward.

1 Economic Environment

We consider a small open economy, with New Keynesian features. We assume there is a unit continuum of Home firms that compete under monopolistic competition. A given Home firm ($\omega \in (0,1)$) uses labor and intermediate inputs to produce, with production function $Y_{Ht}(\omega) = L_t(\omega)^{\alpha} N_t(\omega)^{1-\alpha}$, where $L_t(\omega)$ is labor and $N_t(\omega)$ is a CES composite input formed from domestic and foreign composite goods with elasticity $\eta > 0$. Domestic producers set prices in Dollars, subject to quadratic costs of price adjustment.

As described in the main text, there is a unit continuum of foreign firms; we discuss their pricing problem below. There are also intermediary firms that bundle varieties (with zero value added) into CES composite Home and Foreign goods, where the elasticity within varieties from a given source is $\kappa > 1$. These composites are sold to both firms and consumers in Home. The Home composite good is also exported to Foreign, where export demand is CES with elasticity $\sigma > 0$.

On the consumer side, Home agents are endowed with labor, which they supply to domestic firms, and they own shares domestic firms. Preferences over consumption and labor supply are given by: $U = \sum_{t=0}^{\infty} \beta^t \Theta_t \left[\frac{1}{1-\rho} C_t^{1-\rho} - \frac{\chi}{1+\psi} L^{1+\psi}\right]$, where Θ_t will serve to introduce discount rate (demand) shocks and other notation/parameters are standard. The consumption good C_t is a CES composite of domestic and foreign composite goods, with elasticity ϵ.

We assume financial markets are complete, so there is perfect risk sharing. Interest rates are determined by a monetary policy rule, in which the monetary authority targets inflation subject to interest rate smoothing.

2 Foreign Firm's Problem

The foreign firm chooses the price of its variety in home currency, $P_{Ft}(\varpi)$, subject to price adjustment frictions. The firm chooses a sequence for $\{P_{Ft}(\varpi)\}$ to solve the following problem:

$$\max_{\{P_{Ft}(\varpi)\}} \mathbf{E}_0 \sum_{t=0}^{\infty} S^*_{0,t} \left[\left(P_{Ft} - E_t MC^*_t - F_t\right) Y^*_t(\varpi) - \frac{\phi}{2}\left(\frac{P_{Ft}(\varpi)}{P_{Ft-1}(\varpi)} - 1\right)^2 P_{Ft} Y^*_t \right]$$
$$\text{s.t.} \quad Y^*_t(\varpi) = \left(\frac{P_{Ft}(\varpi)}{P_{Ft}}\right)^{-\kappa} Y^*_t$$

where $S^*_{0,t} = \beta^t \left(\frac{C^*_t}{C^*_0}\right)^{-\rho} \frac{1}{\mathbf{E}_t P^*_t}$ is the foreign stochastic discount factor, MC^*_t is foreign marginal costs in foreign currency, E_t is a the nominal exchange rate (units of home currency to buy one unit of foreign currency), P^*_t is the foreign price level (in foreign currency), and $P_{Ft} = \left(\int_0^1 P_{Ft}(\varpi)^{1-\kappa} d\varpi\right)^{1/(1-\kappa)}$ is the price of the foreign composite good. The first order condition for a representative firm can be log linearized to yield the import price Phillips Curve presented in the main text.

3 Equilibrium

The exogenous stochastic variables are $\{\Theta_t\}$, and the parameters are $\{\beta,\rho,\psi,\chi,\gamma,\epsilon,\eta,\kappa,\alpha,\xi,\phi,\sigma,\Xi,\bar{M},C^*,\rho_i,\omega\}$. With symmetric producers in each country and endogenous trade costs, an equilibrium is a collection of quantities $\{L_t,C_t,C_{Ht},C_{Ft},N_t,N_{Ht},N_{Ft},Y_t,X_t,Y^*_t\}$ and prices $\{W_t/P_t,MC_t/P_t,P_{Ht}/P_t,P_{Ft}/P_t,P_{Nt}/P_t,i_t,Q_t,\Pi_t,\Pi_{Ht},\Pi_{Ft}(s),F_t\}$ that satisfy the equilibrium conditions in Table 1.[1]

We solve the non-linear model using a piecewise log-linear approximation, as implemented by the OccBin toolkit in Dynare. To write the log-linear model, we introduce some additional notation for relative prices. Let $RP_{Ht} \equiv P_{Ht}/P_t$, $RW_t \equiv W_t/P_t$, $RMC_t \equiv MC_t/P_t$, $RP_{Ht} \equiv P_{Ht}/P_t$, $RP_{Ft} \equiv P_{Ft}/P_t$, and $RP_{Mt} \equiv P_{Mt}/P_t$. As in the text, lower cases denote logs, and the hat-notation indicates log deviations from steady state. Finally, define $\bar{F}_t = 1 + F_t/P_t$. The set of log-linear approximate equilibrium conditions common to all equilibria (both for slack and binding constraints) are included in Table 2. We then add different equilibrium conditions to this system depending on which regime prevails – whether constraints are slack or binding, and whether binding constraints are priced or unpriced. We outline these for the three cases that follow.

[1] For clarity, several terms that would ordinarily appear in these expressions are hidden by assumptions. For example, we assume foreign consumption (C^*) is constant, so it cancels in the foreign stochastic discount factor. It also disappears in the risk sharing condition, embedded in the parameter Ξ.

Table 1: Equilibrium Conditions

Euler Equation	$1 = \mathbf{E}_t \left[\beta \frac{\Theta_{t+1}}{\Theta_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{1+i_t}{\Pi_{t+1}} \right) \right]$
Labor Supply	$C_t^{-\rho} \left(\frac{W_t}{P_t} \right) = \chi L_t^{\psi}$
Consumer Allocation	$C_{Ht} = \gamma \left(\frac{P_{Ht}}{P_t} \right)^{-\epsilon} C_t$ $C_{Ft} = (1-\gamma) \left(\frac{P_{Ft}}{P_t} \right)^{-\epsilon} C_t$
Producer Allocation	$\frac{W_t}{P_t} L_t = \alpha \frac{MC_t}{P_t} Y_t$ $\frac{P_{Mt}}{P_t} N_t = (1-\alpha) \frac{MC_t}{P_t} Y_t$ $N_{Ht} = \xi \left(\frac{P_{Ht}/P_t}{P_{Nt}/P_t} \right)^{-\eta} M_t$ $N_{Ft} = (1-\xi) \left(\frac{P_{Ft}/P_t}{P_{Nt}/P_t} \right)^{-\eta} M_t$
Marginal Cost	$\frac{MC_t}{P_t} = \left(\frac{W_t}{P_t} \right)^{\alpha} \left(\frac{P_{Mt}}{P_t} \right)^{1-\alpha}$
Domestic Pricing	$0 = 1 - \kappa \left(1 - \frac{MC_t/P_t}{P_{Ht}/P_t} \right) - \phi \left(\Pi_{Ht} - 1 \right) \Pi_{Ht}$ $+ \mathbf{E}_t \left[\beta \frac{\Theta_{t+1}}{\Theta_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \frac{\phi}{\Pi_{t+1}} \left(\Pi_{Ht+1} - 1 \right) \Pi_{Ht+1}^2 \frac{Y_{t+1}}{Y_t} \right]$
Import Pricing	$0 = 1 - \kappa \left(1 - Q_t \frac{MC^*/P^*}{P_{Ft}/P_t} - \frac{F_t/P_t}{P_{Ft}/P_t} \right) - \phi \left(\Pi_{Ft} - 1 \right) \Pi_{Ft}$ $+ \mathbf{E}_t \left[\beta \left(\frac{Q_t}{Q_{t+1}} \right) \frac{\phi P_{F0}}{\Pi_{t+1}} \left(\Pi_{Ft+1} - 1 \right) \Pi_{Ft+1}^2 \frac{Y_{t+1}^*}{Y_t^*} \right]$
Composite Input	$N_t = \left(\xi^{1/\eta} N_{Ht}{}^{(\eta-1)/\eta} + (1-\xi)^{1/\eta} N_{Ft}{}^{(\eta-1)/\eta} \right)^{\eta/(\eta-1)}$
Composite Cons.	$C_t = \left(\gamma^{1/\epsilon} C_{Ht}{}^{(\epsilon-1)/\epsilon} + (1-\gamma)^{1/\epsilon} C_{Ft}{}^{(\epsilon-1)/\epsilon} \right)^{\epsilon/(\epsilon-1)}$
Market Clearing	$Y_t = C_{Ht} + N_{Ht} + X_t + \frac{\phi}{2} \left(\Pi_t - 1 \right)^2 Y_t$ $X_t = \left(\frac{P_{Ht}/P_t}{Q_t} \right)^{-\sigma} C^*$ $Y_t^* = C_{Ft} + N_{Ft} + \frac{\phi}{2} \left(\Pi_{Ft} - 1 \right)^2 Y_t^*$ $\Theta_t C_t^{-\rho} Q_t = \Xi$
Monetary Policy Rule	$1 + i_t = (1 + i_{t-1})^{\rho_i} \left(\Pi_t \right)^{(1-\rho_i)\omega}$
Import Constraint	$\min \left\{ F_t, \bar{M} - N_{Ft} - C_{Ft} \right\} = 0$
Auxiliary Definitions	$\Pi_{Ht} = \frac{P_{Ht}/P_t}{P_{Ht-1}/P_{t-1}} \Pi_t$ $\Pi_{Ft} = \frac{P_{Ft}/P_t}{P_{CFt-1}/P_{t-1}} \Pi_t$

Table 2: Log Linear Equilibrium Conditions Common to All Equilibria

Euler Equation	$0 = \mathbf{E}_t \hat{\Theta}_{t+1} - \hat{\Theta}_t - \rho\left(\mathbf{E}_t \hat{c}_{t+1} - \hat{c}_t\right) + i_t - \mathbf{E}_t \pi_{t+1}$
Labor Supply	$-\rho \hat{c}_t + \widehat{rw}_t = \psi \hat{l}_t$
Cons. of Home Good	$\hat{c}_{Ht} = -\epsilon \widehat{rp}_{Ht} + \hat{c}_t$
Producer Allocation	$\widehat{rw}_t + \hat{l}_t = \widehat{rmc}_t + \hat{y}_t$ $\widehat{rp}_{Nt} + \hat{n}_t = \widehat{rmc}_t + \hat{y}_t$ $\hat{n}_{Ht} = -\eta \widehat{rp}_{Ht} + \eta \widehat{rp}_{Mt} + \hat{m}_t$
Marginal Cost	$\widehat{rmc}_t = \alpha \widehat{rw}_t + (1-\alpha)\widehat{rp}_{Mt}$
Domestic Pricing	$\pi_{Ht} = \left(\frac{\kappa-1}{\phi}\right)\left(\widehat{rmc}_t - \widehat{rp}_{Ht}\right) + \beta \mathbf{E}_t \pi_{Ht+1}$
Import Pricing	$\pi_{Ft} = \left(\frac{\kappa-1}{\phi}\right)\left(\widehat{rmc}^*_t + \hat{q}_t - \widehat{rp}_{Ft}\right) + \frac{\kappa P_0}{\phi P_{F0}} \hat{\bar{f}}_t + \beta \mathbf{E}_t \pi_{t+1}$
Composite Input Price	$\hat{n}_t = \frac{P_{H0} N_{H0}}{P_{N0} N_0} \hat{n}_{Ht} + \frac{P_{F0} N_{F0}}{P_{N0} N_0} \hat{n}_{Ft}$
Composite Cons. Price	$\hat{c}_t = \frac{P_{H0} C_{H0}}{P_0 C_0} \hat{c}_{Ht} + \frac{P_{F0} C_{F0}}{P_0 C_0} \hat{c}_{Ft}$
Market Clearing	$\hat{y}_t = \frac{C_{H0}}{Y_0} \hat{c}_{Ht} + \frac{N_{H0}}{Y_0} \hat{n}_{Ht} + \frac{X_0}{Y_0} \hat{x}_t$ $\hat{x}_t = -\sigma \widehat{rp}_{Ht} + \sigma \hat{q}_t$ $\hat{\Theta}_t - \rho \hat{c}_t + \hat{q}_t = 0$
Monetary Policy Rule	$i_t = \varrho_i i_{t-1} + \omega(1-\varrho_i)\hat{\pi}_t$
Auxiliary Definitions	$\pi_{Ht} = \widehat{rp}_{Ht} - \widehat{rp}_{Ht-1} + \pi_t$ $\pi_{Ft} = \widehat{rp}_{Ft} - \widehat{rp}_{Ft-1} + \pi_t$

Case 1: Slack Import Constraint When the constraint is slack, then add the following equations to those in Table 2:

$$
\begin{gathered}
\hat{c}_{Ft} = -\epsilon \widehat{rp}_{Ft} + \hat{c}_t \\
\hat{n}_{Ft} = -\eta \widehat{rp}_{Ft} + \eta \widehat{rp}_{Nt} + \hat{n}_t \\
\hat{\bar{f}}_t = 0.
\end{gathered}
$$

Case 2: Binding Import Constraint with Endogenous Trade Cost When the constraint is priced, then add the following equations to those in Table 2:

$$
\begin{gathered}
\hat{c}_{Ft} = -\epsilon \widehat{rp}_{Ft} + \hat{c}_t \\
\hat{n}_{Ft} = -\eta \widehat{rp}_{Ft} + \eta \widehat{rp}_{Nt} + \hat{n}_t \\
\left(\frac{C_{F0}}{M_0}\right) \hat{c}_{Ft} + \left(\frac{N_{F0}}{M_0}\right) \hat{n}_{Ft} = \ln\left(\frac{\bar{M}}{M_0}\right).
\end{gathered}
$$

Case 3: Binding Import Constraint with Exogenous Trade Cost When the constraint is not priced, then add the following equations to those in Table 2:

$$
\begin{gathered}
\left(\frac{C_{F0}}{M_0}\right) \hat{c}_{Ft} + \left(\frac{N_{F0}}{M_0}\right) \hat{n}_{Ft} = \ln\left(\frac{\bar{M}}{M_0}\right) \\
\hat{c}_{Ft} - \hat{n}_{Ft} = -\epsilon \widehat{rp}_{Ft} + \hat{c}_t + \eta \widehat{rp}_{Ft} - \eta \widehat{rp}_{Nt} - \hat{n}_t \\
\hat{\bar{f}}_t = 0.
\end{gathered}
$$

The second equation here is the allocation rule for constrained imports, where we assume that the relative proportional increases for consumption and intermediate goods align with relative demand for them.

4 Quantitative Details

We set various structural parameters as follows: $\beta = 0.995$, $\rho = 2$, $\psi = 2$, $\epsilon = 2$, $\eta = 0.5$, $\kappa = 4$, $\alpha = 2/3$, $\sigma = 0.5$, $\phi = 35.468$, $\omega = 1.5$, $\rho_i = 0.5$, and $\chi = 1$. We assume that the demand shock follows an autoregressive process: $\hat{\Theta}_t = \rho_\Theta \hat{\Theta}_{t-1} + \varepsilon_t$, where $\rho_\Theta = 0.5$ and $\sigma_\varepsilon = .01$. Further, we assume that excess import capacity is equal to 2%, so $\bar{M}/M_0 = 1.02$. We set remaining prices and quantities to their steady-state values computed based on these parameters.