ONLINE APPENDIX

Central Banks as Dollar Lenders of Last Resort: Implications for Regulation and Reserve Holdings

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APPENDIX B: PROOFS

(FOR ONLINE PUBLICATION ONLY)

B1. Derivation of equations (32), (33) and (34)

Take the case of the local central bank, which takes the dollar spread S as given, allowing banking and currency crises to be correlated. The local planner's objective function is given by equation (23) in the text, dropping the term corresponding to household utility from dollar assets, $\left(f(D_\$) - D_\$ f'(D_\$)\right)$:

$$W_L = B_\$(Q_\$ - \beta) + B_h(Q_h - \beta) - \beta\left\{\frac{\left(1-p(q+h)\right)\gamma B_\$^2}{2I} + S_K R_\$ + \Omega(\tau)\right\},$$

where the deadweight cost of taxation is:

$$\Omega(\tau) = \frac{\psi}{2}((q+h)(pB_h + (1+z)pB_\$ - zR_\$)^2$$

$$+ (q-h)(pB_h + (1-z)pB_\$ + zR_\$)^2).$$

We are interested in the case where the planner chooses the level of dollar reserves ($R_\$$) and capital requirements (B_h). In this case, $B_\$ = I\big((1-qp)S +$

$hpz)/\left((1-p(q+h))\gamma\right)$ is set by the unregulated bank. Take the first-order condition of W_L with respect to B_h and we recover:

$$(Q_h-\beta)-\beta\frac{d\Omega(\tau)}{dB_h}=0,$$

where $dB_\$/dB_h=0$ and $dR_\$/dB_h=0$. Plugging in for $d\Omega(\tau)/dB_h$ and solving for B_h^{**} :

$$(Q_h-\beta)-\beta\psi p[(q+h)(pB_h+(1+z)pB_\$-zR_\$)$$

$$+(q-h)(pB_h+(1-z)pB_\$+zR_\$)]=0,$$

$$(Q_h-\beta)-\beta\psi p[2qpB_h+2(qp+zhp)B_\$-2hzR_\$]=0,$$

$$B_h^{**}=\frac{(Q_h-\beta)}{2\beta\psi qp^2}-\left(1+\frac{zh}{q}\right)B_\$+\frac{zh}{pq}R_\$.$$

Next, we take the first-order condition of W_L with respect to $R_\$$. Note that in the case of the local planner, they do not internalize the effect of $R_\$$ on the dollar spreads S and S_K.

$$-\beta\frac{d(S_KR_\$)}{dR_\$}-\beta\frac{d\Omega(\tau)}{dR_\$}=0$$

Using that $S_KR_\$=((Q_\$/\beta)-1)R_\$$, this is equal to:

(B1.1) $$-(Q_\$-\beta)-\beta\frac{\Omega(\tau)}{dR_\$}=0.$$

Plug in for $d\Omega(\tau)/dR_\$$ and re-express the first term using the spread S_K:

$$-S_K - z\psi[-(q+h)(pB_h + (1+z)pB_\$ - zR_\$)$$

$$+ (q-h)(pB_h + (1-z)pB_\$ + zR_\$)] = 0,$$

$$[-2hpB_h - 2p(qz+h)B_\$ + 2qzR_\$] = -\frac{S_K}{\psi z}$$

$$R_\$^{**} = \frac{hp}{qz}[B_h + B_\$] + pB_\$ - \frac{S_K}{2\psi qz^2}.$$

We can rewrite $R_\$^{**}$ as

$$R_\$^{**} = \frac{2\beta\psi zhp(B_\$+B_h)+2\beta\psi qz^2pB_\$-\beta S_K}{2\beta\psi qz^2}.$$

Now, we can write the first order conditions for the small open economy as:

$$B_\$^{*} = \frac{I((1-qp)S+hpz)}{(1-p(q+h))\gamma} \equiv a_1 S + a_2,$$

$$B_h^{**} = \frac{(Q_h-\beta)}{2\beta\psi qp^2} - \left(1+\frac{zh}{q}\right)B_\$ + \frac{zh}{pq}R_\$,$$

$$R_\$^{**} = \frac{hp}{qz}[B_h + B_\$] + pB_\$ - \frac{S_K}{2\psi qz^2}.$$

Note that $B_\$$ is a linear function of S where $a_1 \equiv I(1-qp)/(\gamma(1-p(q+h)))$ and $a_2 \equiv hpzI/(\gamma(1-p(q+h)))$. Using the expression for B_h^{**}, we can write the term $(h/(qz))(B_h^{**} + B_\$^{**}) + B_\$^{**}$, which appears in the simplified version of $R_\$^{**}$, as:

$$\frac{h}{qz}(B_h^{**} + B_\$^{**}) + B_\$^{**} = \frac{h(Q_h-\beta)}{2\beta\psi q^2p^2z} - \frac{h^2}{q^2}B_\$ + \frac{h^2}{pq^2}R_\$ + B_\$.$$

Plug this and $S_K = (1 + (\theta_d/\beta))S + (\theta_d/\beta)$ into the expression for $R_\$^{**}$,

$$R_\$^{**} = p\left(\frac{hp}{qz}\left[B_h^{**} + B_\$^{**}\right] + B_\$^{**}\right) - \frac{S_K}{2\psi q z^2},$$

$$R_\$^{**} = p\left(\frac{h(Q_h-\beta)}{2\beta\psi q^2 p^2 z} - \frac{h^2}{q^2}B_\$ + \frac{h^2}{pq^2}R_\$ + B_\$\right) - \frac{S\,(\beta+\theta_d)}{2\beta\psi q z^2} - \frac{\theta_d}{2\beta\psi q z^2},$$

$$\left(1 - \frac{h^2}{q^2}\right)R_\$^{**} = p\left(\frac{h(Q_h-\beta)}{2\beta\psi q^2 p^2 z} + \left(1 - \frac{h^2}{q^2}\right)B_\$\right) - \frac{S\,(\beta+\theta_d)}{2\beta\psi q z^2} - \frac{\theta_d}{2\beta\psi q z^2},$$

$$R_\$^{**} = pB_\$ + \frac{h(Q_h-\beta)}{2\beta\psi p z(q^2-h^2)} - \frac{\theta_d q}{2\beta\psi z^2(q^2-h^2)} - \frac{Sq(\beta+\theta_d)}{2\beta\psi z^2(q^2-h^2)}.$$

Plug in for $B_\$^{**}$ to solve explicitly for the optimal level of dollar reserves as a function of the dollar spread, S:

$$R_\$^{**} = p\frac{I\big((1-qp)S+hpz\big)}{(1-p(q+h))\gamma} + \frac{h(Q_h-\beta)}{2\beta\psi p z(q^2-h^2)} - \frac{\theta_d q}{2\beta\psi z^2(q^2-h^2)} - \frac{Sq(\beta+\theta_d)}{2\beta\psi z^2(q^2-h^2)},$$

$$R_\$^{**} \equiv b_1 S + b_2,$$

where:

$$b_1 = \frac{I(1-qp)p}{\gamma\big(1-p(q+h)\big)} - \frac{q(\beta+\theta_d)}{2\beta\psi z^2(q^2-h^2)},\; b_2 = \frac{hp^2 zI}{\gamma\big(1-p(q+h)\big)} + \frac{h(Q_h-\beta)}{2\beta\psi p z(q^2-h^2)} - \frac{\theta_d q}{2\beta\psi z^2(q^2-h^2)}.$$

We want to solve for the equilibrium dollar spread. Note that:

$$B_\$^{**} - R_\$^{**} = a_1 S + a_2 - (\,b_1 S + b_2),$$

$$B_\$^{**} - R_\$^{**} = (a_1 - b_1)S + (a_2 - b_2).$$

To solve for the equilibrium spread in the local planner case, we use the equilibrium spread condition given by equation (28). Since we assume a unit mass of identical local planners, we plug in for the local planner's optimal decision (found above) and solve for the equilibrium spread. We have from equation (28):

$$S = \frac{\theta_{\$1} - \theta_{\$2}\left(B_\$ + X_\$ - R_\$\right)}{\beta + \theta_d} = \frac{\theta_{\$1} - \theta_{\$2}\left(X_\$ + (a_1 - b_1)S + (a_2 - b_2)\right)}{\beta + \theta_d}.$$

Hence, we can pin down the explicit equilibrium solution as follows:

$$S = \frac{\theta_{\$1} - \theta_{\$2}\left(X_\$ + a_2 - b_2\right)}{\beta + \theta_d + \theta_{\$2}(a_1 - b_1)},$$

$$B_\$^{**} = \frac{I\left((1 - qp)S + hpz\right)}{\left(1 - p(q + h)\right)\gamma},$$

$$R_\$^{**} = pB_\$ + \frac{h(Q_h - \beta)}{2\beta\psi pz(q^2 - h^2)} - \frac{Sq}{2\psi z^2(q^2 - h^2)},$$

$$B_h^{**} = \frac{(Q_h - \beta)}{2\beta\psi qp^2} - \left(1 + \frac{zh}{q}\right)B_\$ + \frac{zh}{pq}R_\$.$$

B2. Derivation of equation (42)

In this section, we solve for the system of equations that implicitly define the equilibrium solution for the global planner problem when the planner chooses the amount of dollar reserves, $R_\$$, and capital requirements, B_h, allowing for correlated banking and currency crises. This is the global planner equivalent of Appendix B.1. The explicit solution to this system of equations in terms of primitive parameters is derived in Appendix B.6. In this case, $B_\$$ is chosen by the banking sector and given by:

$$B_\$^* = \frac{I((1-qp)S+hpz)}{(1-p(q+h))\gamma}.$$

Note that the equilibrium dollar spread will solve:

$$S = \frac{\theta_{\$1}-\theta_{\$2}(X_\$+B_\$-R_\$)}{\beta+\theta_d},$$

where $B_\$$ is that given above and $R_\$$ will come from the optimization problem of the global planner. The welfare function for the global planner is given by equation (35) in the text:

$$W_G = D_\$(Q_\$ - \beta) + B_h(Q_h - \beta) + \left(f(D_\$) - D_\$\, f'(D_\$)\right)$$

$$- \beta\left(\frac{(1-p(q+h))\gamma B_\$^2}{2I} + \Omega(\tau)\right).$$

Consider first the first-order condition with respect to B_h. We can see from the welfare function above that the first-order condition for B_h will take the same form as that for the local planner in Appendix B.1. Hence, we have:

$$B_h^{***} = \frac{(Q_h-\beta)}{2\beta\psi qp^2} - \left(1 + \frac{zh}{q}\right)B_\$ + \frac{zh}{pq}R_\$.$$

Next, we need to determine the equilibrium dollar reserve policy for the global planner. In the global planner case, we must now take into account that the global planner internalizes the impact $R_\$$ has on the dollar spread, S. The global planner's first-order condition with respect to $R_\$$ is given by:

$$\frac{dW_G}{dR_\$} = \frac{d}{dR_\$}\left((B_\$ + X_\$ - R_\$)(Q_\$ - \beta)\right) - \beta \frac{d}{dR_\$}\left(\frac{\left(1-p(q+h)\right)\gamma B_\$^2}{2I}\right)$$

$$+ \frac{d}{dR_\$}\left(f(D_\$) - D_\$ f'(D_\$)\right) - \beta \frac{d}{dR_\$}\Omega(\tau) = 0.$$

Note that $B_\$$ is a linear function of S where $a_1 \equiv I(1-qp)/(\gamma(1-p(q+h)))$ and $a_2 \equiv hpzI/(\gamma(1-p(q+h)))$. Moving forward, using *that* $S = (Q_\$/Q_h) - 1$ and $Q_\$ = \beta + \theta_d + \theta_{\$1} - \theta_{\$2} D_\$$ we have:

$$B_\$ = \frac{I\left((1-qp)\left(\frac{\left(\beta+\theta_d+\theta_{\$1}-\theta_{\$2}\left(B_\$+X_\$-R_\$\right)\right)}{Q_h}-1\right)+hpz\right)}{\left(1-p(q+h)\right)\gamma},$$

which leads to:

$$\frac{dB_\$}{dR_\$} = \frac{I(1-qp)\theta_{\$2}}{\left(\left(1-p(q+h)\right)\gamma Q_h + I(1-qp)\theta_{\$2}\right)} \equiv \phi,$$

$$\frac{dD_\$}{dR_\$} = \frac{dB_\$}{dR_\$} - 1 = \phi - 1,$$

$$\frac{dQ_\$}{dR_\$} = \theta_{\$2}(1 - \phi).$$

Using these expressions and equation (28) in the text for $f(D_\$)$, we have that the derivatives of each term in W_G with respect to $R_\$$ are below (and given by equations (37)-(40) in the text):

$$\frac{d}{dR_\$}\left((B_\$ + X_\$ - R_\$)(Q_\$ - \beta)\right) = (\phi - 1)(Q_\$ - \beta)$$

$$+ (B_\$ + X_\$ - R_\$)\left(\theta_{\$2}(1 - \phi)\right),$$

$$\frac{d}{dR_\$}\left(\frac{\left(1-p(q+h)\right)\gamma B_\$^2}{2I}\right)=\frac{\phi\left(1-p(q+h)\right)\gamma B_\$}{I},$$

$$\frac{d}{dR_\$}\left(f(D_\$)-\ D_\$ f'(D_\$)\ \right)=-(1-\phi)\theta_{\$2}(B_\$+X_\$-R_\$),$$

$$\frac{d}{dR_\$}\Omega(\tau)=\left(2\psi\phi qp^2-2\psi zhp(1-p\phi)\right)(B_h+B_\$)$$

$$+\left(2\psi\phi zph-2\psi qz^2(1-p\phi)\right)(pB_\$-R_\$),$$

and where:

$$\phi\equiv\frac{dB_\$}{dR_\$}\ =\frac{\left(\frac{\theta_{\$2}I(1-qp)}{\gamma}\right)}{\left(\left(1-p(q+h)\right)(\beta+\theta_d)+\frac{\theta_{\$2}I(1-qp)}{\gamma}\right)}.$$

Summing up these terms, we have

(B2.1) $$\frac{dW_G}{dR_\$}=-(1-\phi)(Q_\$-\beta)-\frac{\beta\phi\left(1-p(q+h)\right)\gamma B_\$}{I}-\beta\ \left(\frac{\partial\Omega}{\partial R_\$}+\phi\frac{\partial\Omega}{\partial B_\$}\right)=0.$$

Arranging the terms, we can write this as

$$\frac{dW_G}{dR_\$}=\underbrace{-(Q_\$-\beta)-\beta\frac{\partial\Omega}{\partial R_\$}}_{\text{Local Planner's FOC}}+\underbrace{\phi\left((Q_\$-\beta)\ -\frac{\beta\left(1-p(q+h)\right)\gamma B_\$}{I}-\beta\frac{\partial\Omega}{\partial B_\$}\right)}_{\text{Wedge Between Global and Local Planner}},$$

where, from equation (B1.1) in Appendix B.1., we can see that the first to terms are the same expression as the local planner's first order condition with respect to $R_\$$.

The equations that implicitly express the equilibrium solution to the global planner problem are

$$S^{***} = \frac{\theta_{\$1} - \theta_{\$2}(X_\$ + B_\$^* - R_\$^{***})}{\beta + \theta_d},$$

$$B_\$^* = \frac{I((1-qp)S^{***} + hpz)}{(1-p(q+h))\gamma},$$

$$B_h^{***} = \frac{(Q_h - \beta)}{2\beta\psi q p^2} - \left(1 + \frac{zh}{q}\right) B_\$^* + \frac{zh}{pq} R_\$^{***},$$

$$-(1-\phi)(Q_\$^{***} - \beta) - \frac{\beta\phi(1-p(q+h))\gamma B_\$^*}{I} - \beta\Big(\frac{\partial\Omega}{\partial R_\$}|_{R_\$^{***}, B_\$^*, B_h^{***}}$$

$$+ \phi \frac{\partial\Omega}{\partial B_\$}|_{R_\$^{***}, B_\$^*, B_h^{***}}\Big) = 0,$$

where $|_{R_\$^{***}, B_\$^*, B_h^{***}}$ denotes that the term is to be evaluated at the equilibrium values, $R_\$^{***}$, $B_\$^*$, and B_h^{***}.

B3. Proof of Proposition 1

Proposition 1 follows directly from equation (42).

B4. Proof of Proposition 3

The global planner's first-order condition with respect to $R_\$$ is again given by:

$$\frac{dW_G}{dR_\$} = \frac{d}{dR_\$}\big((B_\$ + X_\$ - R_\$)(Q_\$ - \beta)\big) - \beta \frac{d}{dR_\$}\left(\frac{(1-p(q+h))\gamma B_\$^2}{2I}\right)$$

$$+ \frac{d}{dR_\$}(f(D_\$) - D_\$ f'(D_\$)) - \beta \frac{d}{dR_\$}\Omega(\tau) = 0.$$

Note that:

$$\frac{dD_\$}{dR_\$} = -1, \qquad \frac{dQ_\$}{dR_\$} = \theta_{\$2}.$$

We have that the derivatives of each term in W_G with respect to $R_\$$ are:

$$\frac{d}{dR_\$}\big((B_\$ + X_\$ - R_\$)(Q_\$ - \beta)\big) = -(Q_\$ - \beta) + (B_\$ + X_\$ - R_\$)\theta_{\$2},$$

$$\frac{d}{dR_\$}\left(\frac{\big(1-p(q+h)\big)\gamma B_\$^2}{2I}\right) = 0,$$

$$\frac{d}{dR_\$}\left(f(D_\$) - D_\$ f'(D_\$)\right) = -\theta_{\$2}(B_\$ + X_\$ - R_\$),$$

$$\frac{d}{dR_\$}\Omega(\tau) = -2\psi zhp(B_h + B_\$) - 2\psi qz^2(pB_\$ - R_\$),$$

in light of:

$$\Omega(\tau) = \frac{\psi}{2}((q+h)(pB_h + (1+z)pB_\$ - zR_\$)^2$$

$$+(q-h)(pB_h + (1-z)pB_\$ + zR_\$)^2).$$

Arranging the terms, we have

$$-(Q_\$ - \beta) - 2\psi\beta z\big(-hp(B_h + B_\$) - qz(pB_\$ - R_\$)\big) = 0.$$

Or,

(B4.1) $$R_\$ = pB_\$ + \frac{hp}{qz}(B_h + B_\$) - \frac{(Q_\$ - \beta)}{2\psi\beta qz^2}.$$

Turning to the first-order condition with respect to $B_\$$, we have

$$\frac{d}{dB_\$}\left((B_\$ + X_\$ - R_\$)(Q_\$ - \beta)\right) = (Q_\$ - \beta) - (B_\$ + X_\$ - R_\$)\theta_{\$2},$$

$$\frac{d}{dB_\$}\left(\frac{(1-p(q+h))\gamma B_\$^2}{2I}\right) = \frac{(1-p(q+h))\gamma B_\$}{I},$$

$$\frac{d}{dB_\$}(f(D_\$) - D_\$ f'(D_\$)) = \theta_{\$2}(B_\$ + X_\$ - R_\$),$$

$$\frac{d}{dB_\$}\Omega(\tau) = \psi(q+h)p(1+z)(pB_h + (1+z)pB_\$ - zR_\$)$$

$$+\psi(q-h)p(1-z)(pB_h + (1-z)pB_\$ + zR_\$)$$

$$= 2\psi(q+hz)p^2(B_h + B_\$) + 2\psi(qz+h)pz(pB_\$ - R_\$)$$

$$= 2\psi(q+hz)p^2 B_h + 2\psi p^2(q(1+z^2) + 2hz)B_\$ - 2\psi(qz+h)pzR_\$.$$

Arranging the terms, we have

(B4.2) $$(Q_\$ - \beta) - 2\psi\beta(q+hz)p^2 B_h - 2\psi\beta p^2(q(1+z^2) + 2hz)B_\$$$

$$+2\psi\beta(qz+h)pzR_\$ = 0.$$

The first order condition with respect to B_h is given by

$$(Q_h - \beta) - \beta\frac{d\Omega(\tau)}{dB_h} = 0,$$

which can be rearranged into:

(B4.3) $$B_h = \frac{(Q_h - \beta)}{2\beta\psi q p^2} - \left(1 + \frac{zh}{q}\right) B_\$ + \frac{zh}{pq} R_\$.$$

Finally, we have

(B4.4) $$Q_\$ = \beta + \theta_d + \theta_{\$1} - \theta_{\$2}(B_\$ + X_\$ - R_\$).$$

Equations (B4.1), (B4.2), (B4.3) and (B4.4) constitute a linear system of equations. Turning to the local planner's objective function, we have:

$$W_L = B_\$(Q_\$ - \beta) + B_h(Q_h - \beta) - \frac{\beta\big(1 - p(q+h)\big)\gamma}{2I} B_\$^2 - (Q_\$ - \beta)R_\$ - \beta\Omega(\tau).$$

The FOCs for the local planner are:

$$\frac{dW_G}{dB_h} = (Q_h - \beta) - \frac{\beta d\Omega(\tau)}{dB_h} = 0,$$

(B4.5) $$\frac{dW_G}{dB_\$} = (Q_\$ - \beta) - \frac{\beta\big(1 - p(q+h)\big)\gamma}{I} B_\$ - \frac{\beta d\Omega(\tau)}{dB_\$} = 0,$$

$$\frac{dW_G}{dR_\$} = -(Q_\$ - \beta) - \frac{\beta d\Omega(\tau)}{dR_\$} = 0.$$

Comparing these FOCs, we can see that they are the same as those of the global planner, where the derivatives of the deadweight cost of taxation are the same across the two planner cases. Thus, in the full regulation case, the local and global planner problems will yield the same solutions.

B5. Proof of Proposition 2

From equation (B4.5) just above, we have $dW_G/dB_\$ = (Q_\$ - \beta) - (\beta(1 - p(q+h))\gamma/I)B_\$ - \beta(d\Omega(\tau)/dB_\$)$.

Proposition 1 states that if, starting from the local planner's optimum, it is the case that $\left(Q_\$^{**} - \beta\right) - \beta\left(1 - p(q+h)\right)\gamma B_\$^{*}/I - \beta(\partial\Omega/\partial B_\$) < 0$, then $R_\$^{***} < R_\$^{**}$. The condition that is required for $R_\$^{***} < R_\$^{**}$ thus implies that, starting from the local planner's optimum, $dW_G/dB_\$ < 0$. This in turn means that starting from this point, if the planner could choose $B_\$$ directly, they would choose to reduce it. This is precisely our definition of mismatch being excessive.

B.6. Derivation of equations (43), (44) and (45)

Unpacking the terms, we can write equation (B2.1) as:

$$\phi(Q_\$ - \beta) - \frac{\beta\phi\left(1-p(q+h)\right)\gamma B_\$}{I} - (Q_\$ - \beta)$$

$$-\beta\left(2\psi\phi q p^2 - 2\psi z h p(1 - p\phi)\right)(B_h + B_\$)$$

$$-\beta\left(2\psi\phi z p h - 2\psi q z^2(1 - p\phi)\right)(pB_\$ - R_\$) = 0.$$

Isolate the $R_\$$ terms to get:

$$[-2\beta\psi\phi z p h + 2\beta\psi q z^2(1 - p\phi)]R_\$ =$$

$$\phi(Q_\$ - \beta) - (Q_\$ - \beta) + (2\beta\psi z h p(1 - p\phi) - 2\beta\psi\phi q p^2)(B_h + B_\$)$$

$$-\beta\left(2\psi\phi z p^2 h - 2\psi q z^2(1 - p\phi)p + \frac{\phi\left(1-p(q+h)\right)\gamma}{I}\right)B_\$.$$

Plug in that $B_h^{***} = ((Q_h - \beta)/(2\beta\psi q p^2)) - (1 + zh/q)B_\$ + (zh/(pq))R_\$$:

$$[-2\beta\psi\phi zph + 2\beta\psi qz^2(1 - p\phi)]R_\$ = \phi(Q_\$ - \beta) - (Q_\$ - \beta)$$

$$+(2\beta\psi zhp(1 - p\phi) - 2\beta\psi\phi qp^2)\left(\frac{(Q_h-\beta)}{2\beta\psi qp^2} - \frac{zh}{q}B_\$ + \frac{zh}{pq}R_\$\right)$$

$$-\beta\left(2\psi\phi zp^2h - 2\psi qz^2(1 - p\phi)p + \frac{\phi(1-p(q+h))\gamma}{I}\right)B_\$.$$

Isolate the $R_\$$ terms and combine the $B_\$$ terms to get:

$$\left[-\frac{2\beta\psi z^2h^2(1-p\phi)}{q} + 2\beta\psi\phi pzh - 2\beta\psi\phi zph + 2\beta\psi qz^2(1 - p\phi)\right]R_\$ =$$

$$\phi(Q_\$ - \beta) - (Q_\$ - \beta) + (2\beta\psi zhp(1 - p\phi) - 2\beta\psi\phi qp^2)\left(\frac{(Q_h-\beta)}{2\beta\psi qp^2}\right)$$

$$-\left(\left(\frac{zh}{q}\right)(2\beta\psi zhp(1 - p\phi) - 2\beta\psi\phi qp^2) + 2\beta\psi\phi zp^2h\right.$$

$$\left.-2\beta\psi qz^2(1 - p\phi)p + \frac{\beta\phi(1-p(q+h))\gamma}{I}\right)B_\$.$$

Rearranging and combining terms results in:

(B6.1) $$\left[-\frac{2\beta\psi z^2h^2(1-p\phi)}{q} + 2\beta\psi qz^2(1 - p\phi)\right]R_\$ =$$

$$\left(2\beta\psi qz^2(1 - p\phi)p - \frac{2\beta\psi z^2h^2p(1-p\phi)}{q} - \frac{\beta\phi(1-p(q+h))\gamma}{I}\right)B_\$$$

$$+\phi(Q_\$ - \beta) - \phi(Q_h - \beta) - (Q_\$ - \beta) + \left(\frac{zh(1-p\phi)(Q_h-\beta)}{qp}\right).$$

Substituting $B_\$ = I((1-qp)S + hpz)/((1-p(q+h))\gamma)$ and $Q_\$ = Q_h(S+1)$ into the equation,

$$\left(2\beta\psi qz^2 - 2\beta p\psi\phi qz^2 - \frac{2\beta\psi z^2h^2(1-p\phi)}{q}\right)R_\$ =$$

$$\left(2\beta\psi qz^2p(1-p\phi) - \frac{2\beta\psi z^2h^2p(1-p\phi)}{q} - \frac{\beta\phi(1-p(q+h))\gamma}{I}\right)\frac{I((1-qp)S+hpz)}{(1-p(q+h))\gamma}$$

$$-(Q_\$ - \beta) + \phi Q_h S + \left(\frac{zh(1-p\phi)(Q_h-\beta)}{qp}\right).$$

Plug in that $(Q_\$ - \beta) = Q_h(S+1) - \beta$ to get:

$$\left(\frac{2\beta\psi z^2(1-p\phi)(q^2-h^2)}{q}\right)R_\$ = \left(\frac{2\beta\psi z^2p(1-p\phi)(q^2-h^2)}{q} - \frac{\beta\phi(1-p(q+h))\gamma}{I}\right)\frac{I((1-qp)S+hpz)}{(1-p(q+h))\gamma}$$

$$+(\beta - Q_h) + (\phi - 1)Q_h S + \left(\frac{zh(1-p\phi)(Q_h-\beta)}{qp}\right).$$

We can explicitly solve for $R_\$$ and express it as a linear function of S:

$$R_\$ = b_3 S + b_4,$$

where:

$$b_3 \equiv \frac{q(\phi-1)Q_h + \left(2\beta\psi z^2p(1-p\phi)(q^2-h^2) - \frac{\beta q\phi(1-p(q+h))\gamma}{I}\right)\frac{I(1-qp)}{(1-p(q+h))\gamma}}{2\beta\psi z^2(1-p\phi)(q^2-h^2)},$$

$$b_4 \equiv \frac{q(-Q_h+\beta) + \frac{zh(1-p\phi)(Q_h-\beta)}{p} + \left(2\beta\psi z^2p(1-p\phi)(q^2-h^2) - \frac{\beta q\phi(1-p(q+h))\gamma}{I}\right)\frac{hpzI}{(1-p(q+h))\gamma}}{2\beta\psi z^2(1-p\phi)(q^2-h^2)}.$$

Finally, recall that:

$$S = \frac{\theta_{\$1} - \theta_{\$2}\left(X_{\$} + a_1 S + a_2 - b_3 S - b_4\right)}{\beta + \theta_d}.$$

The explicit solution in the global planner case is therefore described by the following equations:

$$S = \frac{\theta_{\$1} - \theta_{\$2}\left(X_{\$} + a_2 - b_4\right)}{\beta + \theta_d + \theta_{\$2}(a_1 - b_3)},$$

$$B_{\$}^{*} = \frac{I((1 - qp)S + hpz)}{\left(1 - p(q + h)\right)\gamma},$$

$$R_{\$}^{***} = b_3 S + b_4,$$

$$B_h^{***} = \frac{(Q_h - \beta)}{2\beta\psi q p^2} - \left(1 + \frac{zh}{q}\right) B_{\$}^{**} + \frac{zh}{pq} R_{\$}^{***}.$$

APPENDIX C: DETAILS OF NUMERICAL EXERCISE

(FOR ONLINE PUBLICATION ONLY)

Preliminary comments about our data sources:

- We use all known sources of dollar reserve shares to construct the sample for our analysis. Consistent with the papers referenced in Goldberg and Hannaoui (2024), they are: IMF (2020), Chinn, Ito and Macauley (2022), Arslanalp, Eichengreen and Simpson-Green (2022), and S.A.F.E. (State Administration of Foreign Exchange 2018-2022) (for China).
- This yields a sample of 71 countries for which we have an unbalanced time series of dollar reserve shares between 2013 and 2020.[1] When estimating the regressions, we drop 12 euro area countries (see text), 3 countries that are outliers (Hong Kong SAR, Mauritius and Seychelles; (see Figure 1) and 3 countries for which the financial openness Chinn-Ito index is unavailable in any year (Brunei, Serbia, Taiwan POC). The resulting regression sample has 53 countries of which 12 are advanced, 30 are emerging, and 11 are developing economies.
- For the attribution calculations, we divided the data into two samples: those for which we have dollar reserve shares ("*dollar shares known*" sample) and those for which we don't ("*dollar shares unknown"* sample).
- For the *dollar shares known* sample: we start with the regression sample, then drop the 11 developing economies, but include the 12 euro area countries which were dropped from the regression,[2] as well as the 6 countries dropped either because they were outliers (Hong Kong SAR, Mauritius and Seychelles) or

[1] These 71 countries do not include 3 nations (Kazakhstan, Pakistan, and Sri Lanka) for which reported dollar reserve shares are negative in some years. We exclude these countries for all years due to unreliability of the data.

[2] They are Belgium, Estonia, France, Germany, Italy, Latvia, Luxembourg, Netherlands, Finland, Portugal, Slovenia and Spain. Croatia, which is now in the euro area, was not in the euro area prior to 2023, and is included in our EM sample.

because they were missing financial openness data (Brunei, Serbia, and Taiwan POC). This results in a sample of 60 countries.

- For the *dollar shares unknown* sample, we begin with all the advanced and emerging economies (excluding those in the *dollar shares known* sample) for which the BIS reports cross-border dollar liabilities. We then drop the Marshall Islands which, with 2020 cross-border dollar liabilities reported at more than 10000% of GDP is a significant outlier, and Turkmenistan, which does not disclose its international reserves. This results in a sample of 69 countries.

Attribution calculations:

- Appendix Table A3 reports all the inputs used in calculating the attribution of dollar reserve holdings to our proposed mechanism. The 60 advanced economies and emerging markets in the *dollar shares known* sample are listed first, and the 69 advanced and emerging markets in the *dollar shares unknown* sample are listed below them.
- Our calculations are done using the latest data available, which is dictated by the dollar reserves share variable for the *dollar shares known* sample. That year is 2020 for all countries except Nigeria (2015), India (2017), and China, Turkey, and Taiwan POC (all 2018). Data for the other variables in Appendix Table A3 (cross-border dollar liabilities in percent of GDP, nominal GDP, and international reserves in percent of GDP) are drawn from the same year that the latest dollar reserve shares are available. For the *dollar shares unknown* sample, data for all variables and all countries are from 2020, with the exception of Tonga (2014), Tuvalu (2015) and Palau (2018).
- We first describe the calculations for the *dollar shares known* sample.
- The first step is calculating the predicted value of dollar reserves in % of GDP by multiplying the country's cross-border dollar liabilities in percent of GDP

(reported in Column 3) with its estimated coefficient from Table 2.[3] These coefficient estimates are: 3.428 for advanced economies (Table 2, Column 6) and 1.737 for emerging markets (Table 2, Column 7). The predicted values are reported in Column 4.

- Next, we compare the predicted values to actual dollar reserve holdings in % of GDP. The actual dollar reserves holdings are the product of dollar reserve shares (Column 5) and total international reserves in % of GDP (Column 6). We then select the minimum of the predicted dollar reserves in % of GDP and the actual dollar reserves in % of GDP, reporting that minimum in Column 7.
- To calculate the predicted dollar reserves in levels, we use the product of the minimum (Column 7) and nominal GDP (Column 8), reporting the predicted dollar reserves in levels (USD) in Column 9. The sum of the numbers in Column 9 for the first 60 countries (Australia through Uruguay) is $1.10 trillion.
- We next describe the calculations for the *dollar shares unknown* sample: this pertains to the 61st through 129th countries in Table A3 (Albania through Venezuela).
- The predicted value of dollar reserves in % of GDP is calculated exactly as for the *dollar shares known* sample: as the product of cross-border dollar liabilities (Column 3) with either 3.428 (for advanced economies) or 1.737 (for emerging markets).
- As we do not have dollar shares for this subsample, we next compare the predicted values (reported in Column 4) with actual *total* international reserves in % of GDP (Column 6) and report the minimum of the two values in Column 7.

[3] Note that we refer to "predicted values" although strictly speaking, as a product of the covariate and its estimated coefficient only (without the addition of the estimated intercept), it is perhaps more accurately the "marginal predicted value".

- We then calculate the predicted dollar reserves in levels for the *dollar shares unknown* sample as the product of the minimum (Column 7) and nominal GDP (Column 8), reporting that value in Column 9. The sum of the numbers in Column 9 for the 61st through 129th country (Albania through Venezuela) is $509 billion.

Appendix D: Data on Fed Swap Lines and

Indirect Regulation of Non-Financial Firm Mismatch

(For Online Publication Only)

Fed liquidity swap lines

- We assemble a database of countries and years in which a Fed liquidity swap line was provided to their central banks using information from the Credit and Liquidity Programs of the Federal Reserve System (Board of Governors of the Federal Reserve System 2010, 2013, 2020, 2024).

- In our unbalanced regression sample (which runs from 2013-20), this yields 32 country-years in which there is a Fed bilateral swap: 30 in advanced economies, 2 in emerging markets:

Country	Years the Fed swap is in place
Advanced economies	
Australia	2020
Canada	2013-20
Denmark	2020
Korea	2020
New Zealand	2020
Norway	2020
Sweden	2020
Switzerland	2013-20
United Kingdom	2013-20
Emerging markets	
Brazil	2020
Mexico	2020

Indirect FX regulation of non-financials

- We assemble a database of measures of indirect regulation of non-financials (via regulations of the activities of banks/financial institutions) using information on macroprudential measures tracked by the IMF Monetary and Capital Markets department.
- We know the year these measures become effective, and we also know that all these measures are currently effective. *However, our database may not include measures that were put into place and also removed at some point within our sample period.*
- The measures are grouped into: (1) limits on lending and borrowing denominated in FX; (2) FX denominated loans; and (3) Other broad-based measures to increase resilience or address risks from broad-based credit booms.
- We start with a database of 159 measures.
  - We remove measures that did not read to us as FX regulation of non-financials (nearly all removed are from category (3)). This leaves us with 68 measures.
  - We remove all measures that became effective after our sample period—that is, measures put in place after 2020. This leaves us with 60 measures.
  - We remove all countries that are not in our sample (largely those which did not have either dollar reserves data or NFC dollar liabilities data in the BIS, or both). This leaves us with 29 measures taken by 15 countries.
  - *The full database of measures, as well as each of the reductions above, will be provided in the replication package for the paper.*
- This produces 92 country-years of indirect FX measures: 65 EM, 22 LIC, 5 AE (of which all are from Iceland).
- We create a dummy variable, "Indirect FX regulation", equal to "1" for each country-year where indirect regulation is in place and 0 otherwise. We re-estimate Table 2, columns (5)-(8) adding this dummy variable.
- We find that:

- The estimated coefficient on our key variable, NFC dollar liabilities, is nearly unchanged in magnitude / significance from Table 2 results in the paper.
- The "Indirect regulation" dummy is insignificant in the pooled and EM regressions, and significant and positive for developing economies. The coefficient is not identified for advanced economies, as it is conflated with an Iceland dummy.

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