Supplemental Appendix for: The End of Privilege: A Reexamination of the Net Foreign Asset Position of the United States

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1 Issues with the International Data

1.1 Measurement of Ownership of U.S. Resident Corporations and Cross Border Portfolio Equity

Several factors complicate the measurement of cross border holdings of claims on U.S. resident corporations.

First, as noted in Bertaut, Bressler, and Curcuru (2019), U.S. multinationals have increasingly chosen to incorporate in offshore tax havens in what are called "corporate inversions." As a result, a growing share of what are reported as cross-border equity holdings are, in fact, primarily claims on what are economically U.S. firms held by U.S. equity investors through their claims on the parent firm located in the offshore tax haven. See also Hanson et al. (2015) on how corporate inversions impact the U.S. economic accounts.

Second, again as noted in Bertaut, Bressler, and Curcuru (2019), cross border holdings of assets through mutual funds are classified as equity even if the mutual fund is a bond fund.

Third, Coppola et al. (2021), Beck et al. (2024) and Bertaut, Bressler, and Curcuru (2019) provide evidence on the impact of offshore financial centers on the measurement of cross border financial positions. They show that firms, especially those in developing economies, use subsidiaries located in low tax offshore financial centers to raise capital from investors in the U.S. and other developed economies. Coppola et al. (2021) show that this distorts the geographical of U.S. foreign asset holdings, but does not much impact U.S. *liabilities*, the size of which are a key input in our experiments. .

Fourth, as noted in Bertaut, Bressler, and Curcuru (2020), U.S. households hold portfolio equity in U.S. firms with international operations. In this regard, we underestimate the extend of U.S. residents' holdings of equity claims on corporations resident in the ROW.

Bertaut, Bressler, and Curcuru (2019) estimate that roughly \$2 trillion of the total \$12 trillion U.S. outward investment abroad in 2017, or 16 percent, was actually exposure to the U.S. It is unclear what the total adjustment of the estimated gross claims by foreigners on the U.S. would be if similar methods were applied to these data.

Zucman (2013) argues that official statistics substantially underestimate the net foreign asset positions of rich countries because miss most of the assets held by households in offshore tax havens. He argues that the true U.S. NFA position was 6 percentage points of GDP less negative than officially recorded over the 2001–2008 period (see the note to his Table VI).

1.2 Market Valuation of FDI Equity

Milesi-Ferretti (2021) raises concerns with the market valuation of ROW equity direct investment in U.S. resident corporations and the market valuation of U.S. residents' equity direct investment in corporations resident in the ROW reported in Table S.9. The market value of ROW equity direct investment in U.S. resident corporations is estimated using U.S. stock market indices and the market value of U.S. residents' equity direct investment in corporations resident in the ROW is estimated using foreign stock market indices. One might argue that it is more appropriate to use foreign stock market indices to value foreign equity direct investment equity in the United States and U.S. stock market indices to value U.S. direct investment equity in the rest of the world. In Figure 2, we show the evolution of U.S. net foreign assets with foreign direct investment into and out of the United States valued at current cost, as it was in the *Financial Accounts of the United States* until 2019. The net foreign asset position with FDI at current cost is computed using data from table 2.1 from the BEA international Investment Position. We first subtract from total assets and total liabilities the series for FDI equity assets and liabilities evaluated at market value (Lines 10 and 27, Inward and Outward equity direct investment, directional basis). We then add back to total assets and liabilities the series for FDI assets and liabilities at current cost (Lines 39 and 44, Equity Direct Investment at current cost, directional basis). This could be viewed as an intermediate case between the current method for valuing FDI and the alternative suggested above. The figure shows that valuating FDI at current cost has an impact on the measured evolution of the U.S. NFA position. In particular, negative valuations no longer apply to FDI, which accounts for about 50 percent of the gross equity positions. So, not surprisingly, the size of the decline of the U.S. NFA position is smaller (90 percent of corporate GVA instead of 120 percent). Nevertheless the main fact we highlight remains: since 2007, the U.S. NFA position has declined primarily because of negative valuation effects.

1.3 Sensitivity of baseline results to size of gross cross-border equity positions

In our baseline measurement exercise, we use data on the size of gross cross-border equity positions as reported in Table S9 of the Integrated Macroeconomic Accounts. The literature discussed above points to several reasons that these data may overstate the economically meaningful size of these gross cross-border positions. Here we conduct a sensitivity analysis of our baseline measurement and welfare results to an alternative smaller estimate of the size of these gross positions.

In our baseline analysis, we measured the parameter $(1 - \lambda_t)$ representing the share of ROW ownership of U.S. corporations using the ratio of the gross equity claims of the ROW on the U.S. as reported in Table S9 to our measure of U.S. corporations' enterprise value.

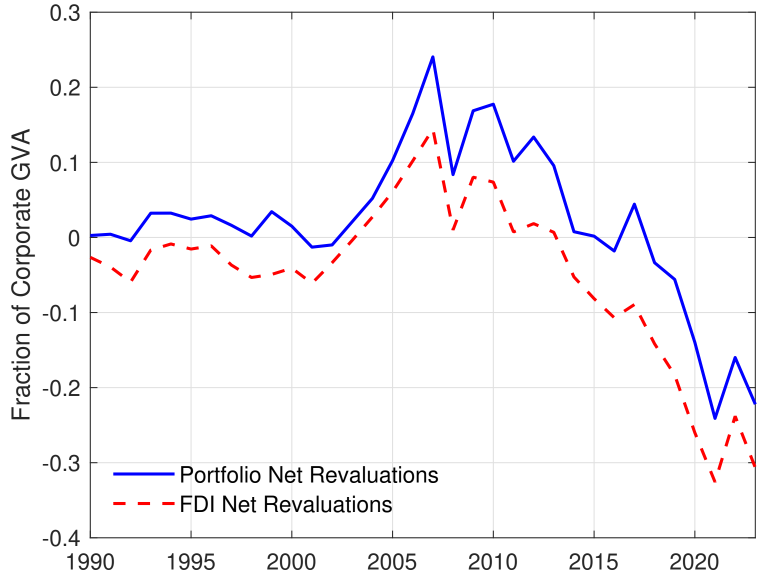

Figure 1: Cumulated Valuation Effects for Portfolio Equity and FDI Equity over GDP

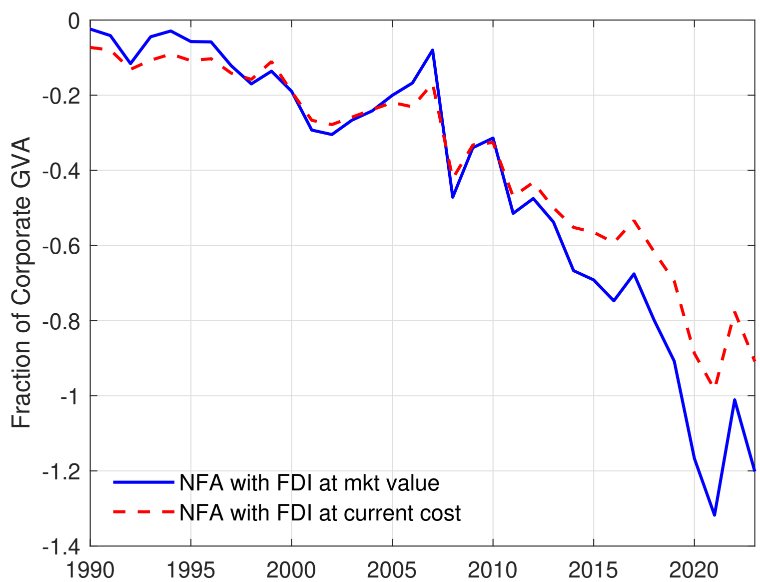

Figure 2: U.S. NFA over GVA: FDI Equity Valued At Market Value and At Current Cost

This procedure produces estimates of this share of U.S. corporate equity owned by the ROW that rise from 15% at the start of 1990 to over 40% at the end of our sample as shown in Figure A6 in the paper.

Bertaut, Bressler, and Curcuru (2019) estimate that from 2015 onward, around 20% of reported foreign equities held by U.S. investors actually reflects exposure to U.S. firms.[1] Thus, we now consider an alternative calibration in we impose a lower path for λ_t^*, setting $\tilde{\lambda}_t^* = 0.8\lambda_t^* \; \forall t$. We simultaneously adjust the path for λ_t so that the dynamics of the U.S. net foreign equity position are identical to the baseline calibration.

We first report the values of the parameters found in this alternative exercise in Figure 3. These parameters can be compared to our baseline parameters shown in Figure 5. We see that changing the size of cross border equity holdings does not impact our parameter estimates relative to our baseline measurement except for our measures of the size of gross cross border equity portfolios shown in the rightmost panel of the third row of Figures 3 and 5.

We next report on our experiment regarding the ex-post welfare impact on U.S. residents of the changes in parameter values of this time period with these alternative estimates of the extent of gross cross-border equity positions in Figure 4. In this figure, we see in the lower left panel that even with a reduced estimate of cross-border equity positions, the consumption of U.S. households is substantially reduced relative to the alternative with no cross-border equity holdings. These results can be compared to those in our baseline in Figure 14 in the paper.

1.4 The Income Puzzle

A long-standing puzzle in the international data is that while the U.S. net foreign asset position is large and negative, U.S. primary income from abroad as measured in the current account remains positive. There is a large literature on this topic. Curcuru, Thomas, and Warnock (2013) is an important paper in this literature that points out that a large portion of this discrepancy is due to a gap between the accounting income yields on U.S. direct investment assets and liabilities.

One hypothesis regarding the puzzlingly high accounting income on U.S. FDI equity in the ROW is that the valuation of U.S. direct investment equity assets recorded in the BEA's International Investment Position tables is too low, thus resulting in a high income yield as a matter of mismeasurement of the denominator of that ratio. This is often referred to as the "Dark Matter" hypothesis. See Hausmann and Sturzenegger (2007). See also Kozlow (2006)

[1] See the total_equity_data_table.csv file at https://www.federalreserve.gov/econres/notes/feds-notes/globalization-and-the-geography-of-capital-flows-20190906.html.

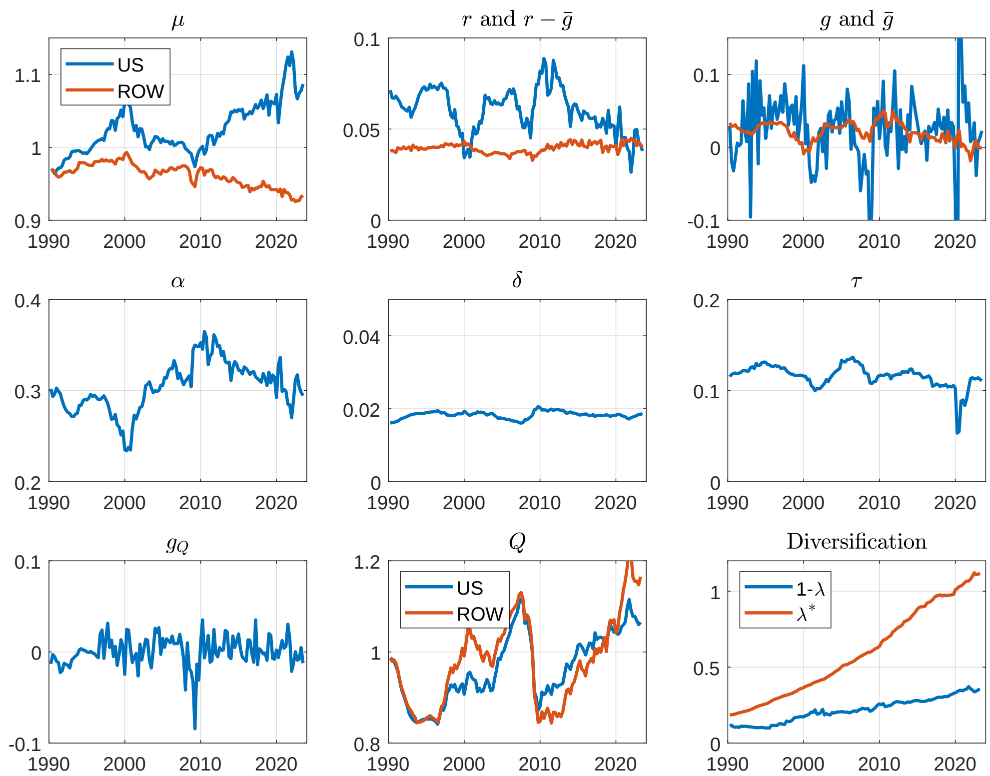

Figure 3: All Parameter Values with Reduced Cross-Border Equity Holdings

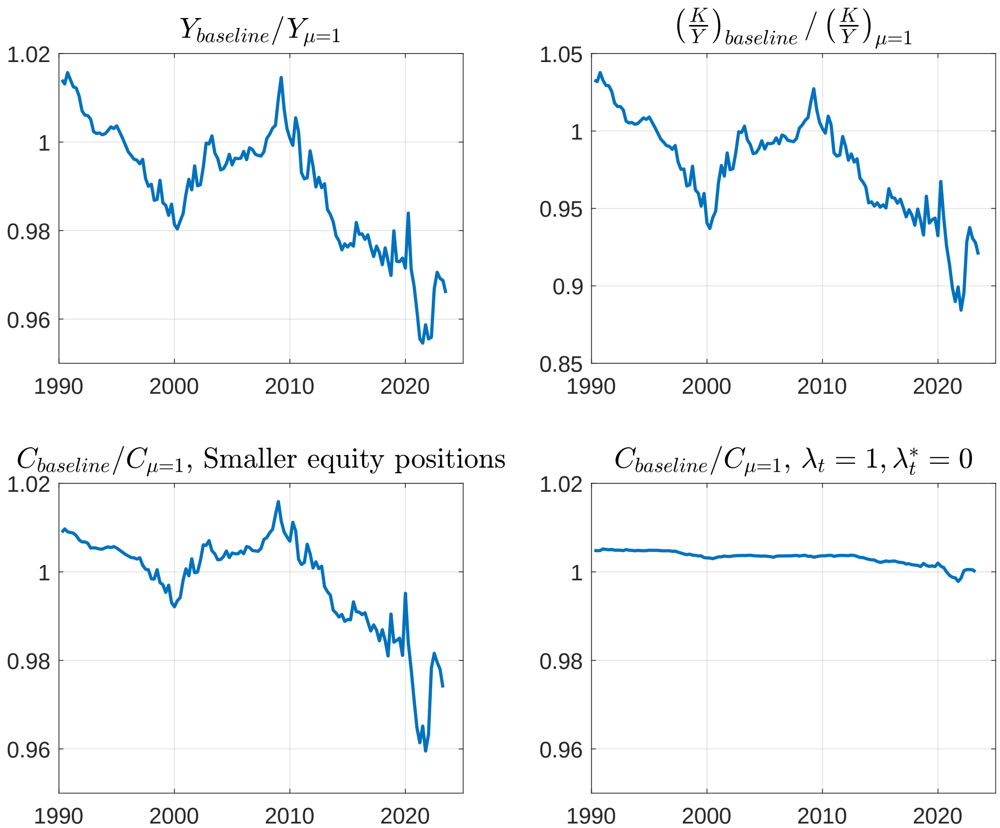

Figure 4: Effect of Output Wedges on Y, K/Y, and C. Effect on C Shown with Alternative Path for Diversification, and Zero Diversification Counterfactual.

and the following discussion from the BEA: https://www.bea.gov/help/faq/202.

Another hypothesis regarding this gap in income yields for Direct Investment Equity Assets and Liabilities is that for fiscal reasons, multinational firms tend to over-report income from foreign affiliates and under-report income generated in the United States. See, for example, Bosworth, Collins, and Chodorow-Reich (2007), Curcuru, Thomas, and Warnock (2013), Curcuru and Thomas (2015), Setser (2017), Setser (2019), Torslov, Wier, and Zucman (2022), Guvenen et al. (2022), and Garcia-Bernardo, Jansky, and Zucman (2022). According to this hypothesis, the numerator of the ratio that is the income yield is mismeasured. The upshot of some of these papers is that that these concerns affect the division of the current account between net exports and net foreign income but distort neither the measurement of the U.S. NFA position nor the current account.

One important point to note is that the accounting income yield on U.S. direct investment equity in the ROW is a ratio of corporate income as reported by the ROW subsidiaries of U.S. multinationals to the value of the corporation, not a measure of monetary dividends actually paid. The gap between accounting income on direct investment equity and the monetary dividends actually paid is accounted for as a capital flow titled "Reinvestment of Direct Investment Income". In our measurement, we use only the measure of monetary dividends paid, as discussed in appendix A in the paper. We do not use data on accounting income on direct investment equity in our measurement procedure.

2 Using the Model for Measurement: Full Detail

We now describe the details of our recursive calibration procedure. The data we use to calibrate our model is nominal. The model we laid out in the text is real. One could introduce fluctuations in the general price level in our model. And one could assume that non-equity assets in the model are nominal; non-equity assets and liabilities in the data are, in fact, mostly nominal. However, if model agents have perfect foresight over the path for the price level, price level fluctuations will have no impact on real allocations. In particular, nominal interest rates will move one-for-one with expected inflation, and the path for the equilibrium real interest rate will be invariant to the path for the price level.

2.1 Nominal Bonds

But there is one aspect where changes in the price level will affect our calibration, which has to do with how changes in the nominal values of gross foreign assets and liabilities due to inflation are divided between net asset purchases on the current account versus valuation changes in the balance of payments accounts. The measurement conventions used can affect the measured current account (see, for example, Box 1.1 in Obstfeld and Rogoff (1996). We will assume that all changes in the nominal values of equity assets and liabilities, including those reflecting changes in the general price level, are counted as valuation effects. In contrast, we assume that there are no valuation effects for bonds, so that all changes in the nominal bond position show up on the current account. This assumption is consistent with the absence of valuation effects for non-equity liabilities in the national accounts. We measure gross inflation as the growth in the GDP deflator, P_{t+1}:

$$\pi_{t+1} = \frac{P_{t+1}}{P_t}.$$

Consider a version of the model in which bonds are nominal, and in which the current account includes the change in the nominal bond position. Given perfect foresight regarding the price level, the gross interest rate on the nominal bond between t and $t+1$ is

$$1 + r_{t+1}^{*nom} = (1 + r_{t+1}^{*})\pi_{t+1}$$

so $r_{t+1}^{*nom} = (1 + r_{t+1}^{*})\pi_{t+1} - 1$. The current account expression in equation 23 in the paper now changes in that the term $\frac{1}{1+\rho}(r_t^* - \rho)B_t$ is replaced with $\frac{1}{1+\rho}(r_t^{*nom} - \rho)\frac{B_t^{nom}}{P_t}$. Note that for $\pi_{t+1} > 1$, this increases the measured current account (and the current account to gross value added ratio).

To understand why introducing nominal bonds changes the current account but does not change real consumption or the NFA position in real terms, consider the following simplified version of the model which abstracts from equity and human wealth.

In the "real" version of this simplified model, consumption is

$$C_t = \frac{\rho}{1+\rho}(1 + r_t^*)B_t,$$

the current account is

$$CA_t = r_t^* B_t - C_t = \frac{1}{1+\rho}(r_t^* - \rho)B_t,$$

and the end of period NFA position is

$$B_{t+1} = B_t + CA_t = B_t + \frac{1}{1+\rho}(r_t^* - \rho)B_t = \frac{1 + r_t^*}{1+\rho}B_t.$$

In the "nominal" version of the model, consumption is

$$C_t = \frac{\rho}{1+\rho}(1+r_t^{*nom})\frac{B_t^{nom}}{P_t},$$

Substituting in $\frac{B_t^{nom}}{P_{t-1}} = B_t$ and $r_t^{*nom} = (1+r_t^*)\pi_t - 1$ gives

$$C_t = \frac{\rho}{1+\rho}(1+r_t^*)\pi_t^D \frac{B_t P_{t-1}}{P_t} = \frac{\rho}{1+\rho}(1+r_t^*)B_t$$

which is identical to the expression in the "real" version of the model.

The current account is

$$\begin{aligned} CA_t &= r_t^{*nom}\frac{B_t^{nom}}{P_t} - C_t = r_t^{*nom}\frac{B_t^{nom}}{P_t} - \frac{\rho}{1+\rho}(1+r_t^{*nom})\frac{B_t^{nom}}{P_t} \\ &= \frac{1}{1+\rho}\left(r_t^{*nom} - \rho\right)\frac{B_t^{nom}}{P_t} \\ &= \frac{1}{1+\rho}\left((1+r_t^*) - \frac{(1+\rho)}{\pi_t}\right)B_t \end{aligned}$$

which differs from the expression in the "real" model.

The end of period NFA position is

$$\begin{aligned} \frac{B_{t+1}^{nom}}{P_t} &= \frac{B_t^{nom}}{P_t} + \frac{CA_t}{P_t} = \frac{P_{t-1}B_t}{P_t} + \frac{1}{1+\rho}\left(r_t^{*nom} - \rho\right)\frac{B_t^{nom}}{P_t} \\ &= \frac{P_{t-1}B_t}{P_t} + \frac{1}{1+\rho}\left((1+r_t^*)\pi_t - 1 - \rho\right)\frac{P_{t-1}B_t}{P_t} \\ &= \frac{1+r_t^*}{1+\rho}B_t \end{aligned}$$

which again is identical to the real version of the model.

2.2 Data Series used

In everything that follows, a superscript D denotes a data variable. From the data, we have series for (1) corporate taxes paid, (2) wages and salaries, (3) corporate investment, and (4) consumption of fixed capital, all as shares of corporate value added. Denote these

$$(1)\ \frac{Taxes_t^D}{GVA_t^D}$$

$$(2)\ \frac{WL_t^D}{GVA_t^D}$$

$$(3)\ \frac{X_t^D}{GVA_t^D}$$

$$(4)\ \frac{CFC_t^D}{GVA_t^D}$$

We define earnings relative to value added as

$$\frac{E_t^D}{GVA_t^D} = 1 - \frac{WL_t^D}{GVA_t^D} - \frac{Taxes_t^D}{GVA_t^D} - \frac{CFC_t^D}{GVA_t^D}.$$

We measure free cash flow from the corporate sector as

$$\frac{D_t^D}{GVA_t^D} = \frac{E_t^D}{GVA_t^D} + \frac{CFC_t^D}{GVA_t^D} - \frac{X_t^D}{GVA_t^D}$$

We also measure (5) growth in corporate value added, and (6) the replacement value of the capital stock, which is end of period, and whose model counter-part is $Q_t K_{t+1}$, and (7) U.S. corporate enterprise value. Denote these

$$(5)\ \frac{GVA_{t+1}^D}{GVA_t^D}$$

$$(6)\ \frac{K_t^D}{GVA_t^D}$$

$$(7)\ \frac{V_t^D}{GVA_t^D}$$

Note that from (3) and (4) we have net investment:

$$\frac{NetX_t^D}{GVA_t^D} = \frac{X_t^D}{GVA_t^D} - \frac{CFC_t^D}{GVA_t^D}$$

and from (3), (4) and (6) we can measure start of period capital (whose model counterpart is $Q_t K_t$) as

$$\frac{KS_t^D}{GVA_t^D} = \frac{K_t^D}{GVA_t^D} - \frac{X_t^D}{GVA_t^D} + \frac{CFC_t^D}{GVA_t^D} \tag{1}$$

We measure (8) the revaluation U.S. foreign equity assets in t in nominal dollar terms,

(9) the value of U.S.-owned foreign equity, and (10) the value of foreign-owned equity in the U.S.

$$(8)\ \frac{VAFA_t^D}{GVA_t^D}$$
$$(9)\ \frac{USFA_t^D}{GVA_t^D}$$
$$(10)\ \frac{USFL_t^D}{GVA_t}$$

Finally we have (11) the current account, and (12) a series for foreign corporate dividend income

$$(11)\ \frac{CA_t^D}{GVA_t^D}$$
$$(12)\ \frac{D_t^{*D}}{GVA_t^D}.$$

We use these 12 empirical time series to identify quarterly time series for 12 time-varying model parameters: $\tau_t, g_{t+1}, \delta_t, Q_t, \lambda_t^*, \lambda_t, \bar{g}_{t+1}, r_{t+1}^*, \alpha_{t+1}, \mu_{t+1}, \mu_{t+1}^*, Q_t^*$. To make the notation more compact, we henceforth use lower case letters to denote data ratios relative to value added; e.g., $x_t^D = X_t^D/GVA_t^D$.

2.3 Rate of Time Preference

We set ρ so that the sample average current dividend yield for U.S. corporations (current dividend over end of period enterprise value) is consistent with being on a balanced growth path. Suppose the economy is on a balanced growth path with a constant r^* and a constant growth rate g. For consumption to grow at rate g requires

$$1 = \frac{1}{1+\rho}\frac{1+r^*}{1+g}$$

so

$$\frac{1}{1+\rho} = \frac{1+g}{1+r^*}$$

The balanced growth path dividend yield D/V satisfies

$$1 = \frac{(1+g)}{(r^*-g)}\frac{D}{V},$$

which implies

$$r^* = (1+g)\frac{D}{V} + g$$

Substituting that expression into the discount factor expression gives

$$\frac{1}{1+\rho} = \frac{1+g}{(1+g)\frac{D}{V} + (1+g)} = \frac{1}{\frac{D}{V}+1}$$

so the discount rate consistent with consumption growth at rate g is

$$\rho = \frac{D}{V}$$

Thus, we set ρ equal to the average dividend yield over our sample period:

$$\rho = \mathbb{E}\left[\frac{d_t^D}{v_t^D}\right]$$

2.4 Time-Varying Parameters

We now describe how we recursively identify all 12 of our time-varying parameters.

1. τ_t : Our model assumes that taxes are proportional to value added. Thus, to ensure the model replicates the observed path for taxes paid we set

$$\tau_t = \frac{Taxes_t^D}{GVA_t^D}.$$

2. g_{t+1} : In our model, both Z_t and z_{Ht} impact the level of equilibrium output. At each date t, we specify z_{Ht} and z_{Ht}^* as parametric functions of other model parameters, where the functions have the property that in equilibrium $Y_t = Y_t^* = Z_t$. We describe those functions at the end of the calibration description. We can then identify g_{t+1} from

$$1 + g_{t+1} = \frac{Z_{t+1}}{Z_t} = \frac{GVA_{t+1}^D}{GVA_t^D}\frac{1}{\pi_{t+1}^D}$$

which ensures that model real value added tracks U.S. corporate real value added. We normalize $Z_0 = 1$.

3. δ_t : Model depreciation is proportional to the start of period capital stock. Thus,

$$\delta_t = \frac{cfc_t^D}{ks_t^D}$$

where start-of-period capital ks_t^D is given by equation 1.

4. Q_t : We can measure the growth rate for Q_t as follows. The perpetual inventory equation in units of capital is a model identity

$$K_{t+1} = (1-\delta_t)K_t + X_t$$

Thus

$$\begin{aligned} Q_t K_{t+1} &= Q_t K_t - \delta_t Q_t K_t + Q_t X_t \\ &= \frac{Q_t}{Q_{t-1}} Q_{t-1} K_t - \delta_t Q_t K_t + Q_t X_t \end{aligned}$$

which implies

$$\frac{Q_t}{Q_{t-1}} = \frac{Q_t K_{t+1} + \delta_t Q_t K_t - Q_t X_t}{Q_{t-1} K_t}$$

Recognizing that our data is nominal, we implement this as

$$\begin{aligned} \frac{Q_t}{Q_{t-1}} &= \frac{K_t^D - NetX_t^D}{\pi_t^D K_{t-1}^D} \\ &= \frac{(1+g_t)\left(k_t^D - netx_t^D\right)}{k_{t-1}^D} \end{aligned}$$

We normalize the initial $Q_0 = 1$.

5. λ_t^* : We measure the growth in the foreign enterprise value using equity asset revaluation data and the foreign equity position as follows.

   (a) Let V_t^{*D} denote the nominal data value of the foreign corporate sector at t. We have

$$VAFA_{t+1}^D = \lambda_t^*(V_{t+1}^{*D} - V_t^{*D})$$

   The value of U.S. owned foreign equity at the end of t is

$$USFA_t^D = \lambda_t^* V_t^{*D}$$

   Thus.we can identify the nominal growth rate of foreign enterprise value, V_{t+1}^{*D}/V_t^{*D}, by taking the ratio of valuation effects to the value of the stock at the end of the previous period:

$$\frac{GVA_{t+1}^D}{GVA_t^D}\frac{vafa_{t+1}^D}{usfa_t^D} = \frac{\lambda_t^*(V_{t+1}^{*D} - V_t^{*D})}{\lambda_t^* V_t^{*D}} = \frac{V_{t+1}^{*D}}{V_t^{*D}} - 1$$

(b) To pin down the *level* of foreign enterprise value we assume that the foreign Buffett ratio is initially equal to the U.S. value:

$$v_0^{*D} = v_0^D$$

(c) Given the assumption that foreign nominal value added grows at the value added in the U.S., the growth rate in the foreign Buffett ratio is then identified as

$$\frac{v_{t+1}^{*D}}{v_t^{*D}} = \frac{\frac{V_{t+1}^{*D}}{V_t^{*D}}}{(1+g_{t+1})\pi_{t+1}^D} = \frac{\frac{GVA_{t+1}^D}{GVA_t^D}\frac{vafa_{t+1}^D}{usfa_t^D}+1}{(1+g_{t+1})\pi_{t+1}^D} = \frac{vafa_{t+1}^D}{usfa_t^D} + \frac{1}{(1+g_{t+1})\pi_{t+1}^D}$$

which gives us the level of v_t^{*D} for each date t.

(d) Then we identify λ_t^* from

$$\lambda_t^* = \frac{usfa_t^D}{v_t^{*D}}$$

6. λ_t : We identify this from U.S. equity liabilities and U.S. enterprise value:

$$(1-\lambda_t) = \frac{usfl_t^D}{v_t^D}$$

7. $\bar{g}_{t+1}$: We identify $\bar{g}_{t+1}$ using (1) a valuation equation, and (2) the current account. The value of firms in the model is given by

$$V_t = \frac{\mathbb{E}_t\left[D_{t+1}\right]}{r_{t+1}^* - \bar{g}_{t+1}}$$

where expected dividends are given by expected earnings minus expected net investment:

$$\mathbb{E}_t\left[D_{t+1}\right] = \mathbb{E}_t\left[E_{t+1}\right] - \mathbb{E}_t\left[X_{t+1} - \delta_{t+1}Q_{t+1}K_{t+1}\right]$$

where

$$\mathbb{E}_t\left[E_{t+1}\right] = E_{t+1} + \delta_{t+1}Q_{t+1}K_{t+1}\left(1 - \frac{Q_t}{Q_{t+1}}\right) \qquad (2)$$

and

$$\mathbb{E}_t\left[X_{t+1} - \delta_{t+1}Q_{t+1}K_{t+1}\right] = \bar{g}_{t+1}Q_tK_{t+1}$$

Note that realized earnings differ from expected earnings because unexpected changes in the replacement cost of capital at $t+1$ affect realized consumption of fixed capital.

Thus,

$$\left(r^*_{t+1} - \bar{g}_{t+1}\right) V_t = E_{t+1} + \delta_{t+1} Q_{t+1} K_{t+1} \left(1 - \frac{Q_t}{Q_{t+1}}\right) - \bar{g}_{t+1} Q_t K_{t+1} \tag{3}$$

This equation has two unknowns: r^*_{t+1} and $\bar{g}_{t+1}$. Thus we need another equation to identify $\bar{g}_{t+1}$. In our baseline calibration, we use the model expression for the current account. Recall that the equilibrium model current account is very sensitive to $\bar{g}_{t+1}$: all else equal, a higher value for expected trend growth implies a higher value for human capital, $H_t = \frac{W_{t+!}L_{t+1}}{r^*_{t+1} - \bar{g}_{t+1}}$, translating to higher desired consumption, and a larger current account deficit. The current account expression, in the version of the model with nominal bonds explained above, is

$$CA_t = \frac{1}{1+\rho} \left[\left(\frac{D_t}{V_t} - \rho\right) \lambda_{t-1} V_t + \left(\frac{D^*_t}{V^*_t} - \rho\right) \lambda^*_{t-1} V^*_t + (r^{*nom}_t - \rho) \frac{B^{nom}_t}{P_t} + (W_t L_t - \rho H_t) \right] \tag{4}$$

where $H_t = \frac{W_{t+1}L_{t+1}}{r^*_{t+1} - \bar{g}_{t+1}}$ and $r^{*nom}_t = \left((1 + r^*_t)\pi^D_t - 1\right)$. Given equation 3, the denominator of the H_t term can be expressed as

$$r^*_{t+1} - \bar{g}_{t+1} = \frac{\mathbb{E}_t \left[E_{t+1}\right]}{V_t} - \bar{g}_{t+1} \frac{Q_t K_{t+1}}{V_t}$$

Substituting that into the current account expression, we can solve for $\bar{g}_{t+1}$ as

$$\begin{aligned} \bar{g}_{t+1} = {} & \frac{\mathbb{E}_t \left[E_{t+1}\right]}{Q_t K_{t+1}} - \frac{V_t}{Q_t K_{t+1}} \rho W_{t+1} L_{t+1} \\ & \times \left[\left(\frac{D_t}{V_t} - \rho\right) \lambda_{t-1} V_t + \left(\frac{D^*_t}{V^*_t} - \rho\right) \lambda^*_{t-1} V^*_t + \right. \\ & \left. \left[\left((1 + r^*_t)\pi^D_t - 1\right) - \rho\right] \frac{B^{nom}_t}{P_t} + W_t L_t - (1+\rho) CA_t \right]^{-1} \end{aligned}$$

The data analogue is (dividing date t nominal data variables by P_t and date $t+1$ variables by P_{t+1})

$$\begin{aligned} \bar{g}_{t+1} = {} & \frac{\mathbb{E}_t \left[E^D_{t+1}\right]}{\pi_{t+1} K^D_t} - \frac{V^D_t}{K^D_t} \rho \frac{WL^D_{t+1}}{P^D_{t+1}} \\ & \times \left[\left(\frac{D^D_t}{V^D_t} - \rho\right) \lambda_{t-1} \frac{V^D_t}{P^D_t} + \left(\frac{D^{*D}_t}{V^{*D}_t} - \rho\right) \lambda^*_{t-1} \frac{V^{*D}_t}{P^D_t} + \right. \\ & \left. \left[\left((1 + r^*_t)\pi^D_t - 1\right) - \rho\right] \frac{B^{nom}_t}{P^D_t} + \frac{WL^D_t}{P^D_t} - (1+\rho) \frac{CA^D_t}{P^D_t} \right]^{-1} \end{aligned}$$

Expressing data values relative to data value added gives

$$\bar{g}_{t+1} = (1+g_{t+1})\frac{\mathbb{E}_t\left[e^D_{t+1}\right]}{k^D_t} - (1+g_{t+1})\frac{v^D_t}{k^D_t}\rho w l^D_{t+1}$$

$$\times\left[\left(\frac{d^D_t}{v^D_t}-\rho\right)\lambda_{t-1}v^D_t + \left(\frac{d^{*D}_t}{v^{*D}_t}-\rho\right)\lambda^*_{t-1}v^{*D}_t + \left[\left((1+r^*_t)\pi^D_t - 1\right)-\rho\right]b^{nom}_t + wl^D_t - (1+\rho)ca^D_t\right]^{-1}$$

There are two variables on the right-hand side of this equation that are neither data objects nor parameters that we have recovered in previous steps. Those are r^*_t and b^{nom}_t (the non-equity position relative to value added carried into period t.) But we can recover these parameters sequentially through time: given r^*_t and b^{nom}_t, we can solve for $\bar{g}_{t+1}$ using the equation above, then for r^*_{t+1} (following step 8 below) and other date $t+1$ parameters, and finally for the equilibrium value for b^{nom}_{t+1}.

Alternatives

(a) We might have an external estimate for $\bar{g}_{t+1}$.

(b) We might have an external estimate for $\left(r^*_{t+1} - \bar{g}_{t+1}\right)$ – for example, $r^*_{t+1} - \bar{g}_{t+1} = average\left(\frac{D^D_{t+1}}{V^D_t}\right)$. We then immediately obtain $\bar{g}_{t+1}$ from equation 3

$$\begin{aligned}\bar{g}_{t+1} &= \frac{\mathbb{E}_t\left[E_{t+1}\right]}{Q_t K_{t+1}} - \left(r^*_{t+1}-\bar{g}_{t+1}\right)\frac{V_t}{Q_t K_{t+1}} \\ &= \frac{\mathbb{E}_t\left[E_{t+1}\right]}{Q_t K_{t+1}} - average\left(\frac{D^D_{t+1}}{V^D_t}\right)\frac{V_t}{Q_t K_{t+1}}\end{aligned}$$

In the data, that is identified as

$$\begin{aligned}\bar{g}_{t+1} &= \frac{\mathbb{E}_t\left[E^D_{t+1}\right]}{\pi^D_{t+1}K^D_t} - average\left(\frac{D^D_{t+1}}{V^D_t}\right)\frac{V^D_t}{K^D_t} \\ &= (1+g_{t+1})\frac{\mathbb{E}_t\left[e^D_{t+1}\right]}{k^D_t} - average\left(\frac{D^D_{t+1}}{V^D_t}\right)\frac{v^D_t}{k^D_t}\end{aligned}$$

(c) Suppose we want to identify $\bar{g}_{t+1}$ from an equation assuming perfect foresight about future dividends (note that this is NOT strictly consistent with our baseline expectations model – here we think of it as a separate auxiliary model which informs the parameter vector for $\{\bar{g}_{t+1}\}$.)

$$V_t = \frac{D_{t+1}}{r_{t+1} - \bar{g}_{t+1}}$$

Then we can replace $(r_{t+1} - \bar{g}_{t+1})$ in our model valuation equation (3) with $\frac{D_{t+1}}{V_t}$

$$
\begin{aligned}
(r_{t+1} - \bar{g}_{t+1}) V_t &= \mathbb{E}_t [E_{t+1}] - \bar{g}_{t+1} Q_t K_{t+1} \\
D_{t+1} &= \mathbb{E}_t [E_{t+1}] - \bar{g}_{t+1} Q_t K_{t+1}
\end{aligned}
$$

which we can operationalize empirically as

$$
\begin{aligned}
\bar{g}_{t+1} &= \frac{\mathbb{E}_t \left[E^D_{t+1}\right] - D^D_{t+1}}{\pi_{t+1} K^D_t} \\
&= (1 + g_{t+1}) \frac{\left(\mathbb{E}_t \left[e^D_{t+1}\right] - d^D_{t+1}\right)}{k^D_t}
\end{aligned}
$$

8. r^*_{t+1} : Given $\bar{g}_{t+1}$ we next identify r^*_{t+1}. The key valuation equation can rearranged as

$$
r^*_{t+1} = \frac{\mathbb{E}_t[E_{t+1}]}{V_t} + \bar{g}_{t+1} \left(\frac{V_t - Q_t K_{t+1}}{V_t} \right)
$$

But note that we are working with nominal data, and $\mathbb{E}_t[E_{t+1}]$ is dated one period later than the other variables. Thus we implement this as

$$
\begin{aligned}
r^*_{t+1} &= \frac{\mathbb{E}_t[E^D_{t+1}]}{\pi_{t+1} V^D_t} + \bar{g}_{t+1} \left(1 - \frac{K^D_t}{V^D_t}\right) \\
&= (1 + g_{t+1}) \frac{\mathbb{E}_t[e^D_{t+1}]}{v^D_t} + \bar{g}_{t+1} \left(1 - \frac{k^D_t}{v^D_t}\right)
\end{aligned}
$$

where expected earnings are given by eq. (2).

9. α_{t+1} : Given r_{t+1}, the expression for the labor share and the FOC for investment identify μ_{t+1} and α_{t+1}. The former can be expressed as

$$
\frac{W_{t+1} L_{t+1}}{Y_{t+1}} \frac{1}{(1 - \tau_{t+1})(1 - \alpha_{t+1})} = \frac{1}{\mu_{t+1}}
$$

The second is

$$
\begin{aligned}
Q_t (1 + r^*_{t+1}) &= \mathbb{E}_t[(1 - \tau_{t+1}) \frac{\alpha_{t+1}}{\mu_{t+1}} \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta_{t+1}) Q_{t+1}] \\
&= (1 - \tau_{t+1}) \frac{\alpha_{t+1}}{\mu_{t+1}} \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta_{t+1}) Q_t]
\end{aligned}
$$

which implies

$$
\frac{r^*_{t+1} + \delta_{t+1}}{\alpha_{t+1}(1 - \tau_{t+1})} \frac{Q_t K_{t+1}}{Y_{t+1}} = \frac{1}{\mu_{t+1}} \tag{5}
$$

Combining those two expressions gives

$$\alpha_{t+1} = \frac{\left(r^*_{t+1} + \delta_{t+1}\right) Q_t K_{t+1}}{W_{t+1} L_{t+1} + \left(r^*_{t+1} + \delta_{t+1}\right) Q_t K_{t+1}}$$

which we implement as

$$\begin{aligned}
\alpha_{t+1} &= \frac{\left(r^*_{t+1} + \delta_{t+1}\right) K^D_t / P^D_t}{W L^D_{t+1} / P^D_{t+1} + \left(r^*_{t+1} + \delta_{t+1}\right) K^D_t / P^D_t} \\
&= \frac{\left(r^*_{t+1} + \delta_{t+1}\right) k^D_t}{(1 + g_{t+1}) w l^D_{t+1} + \left(r^*_{t+1} + \delta_{t+1}\right) k^D_t}
\end{aligned}$$

10. μ_{t+1} : We can plug the solution for α_{t+1} into the labor's share expression for solve for μ_{t+1}.

$$\mu_{t+1} = \frac{(1 - \tau_{t+1})(1 - \alpha_{t+1})}{w l^D_{t+1}}$$

Given μ_{t+1} and $z_{H,t+1}$ from equation 6 we have $z_{L,t+1} = z_{H,t+1}/\mu_{t+1}$.

11. μ^*_{t+1} : We use the valuation formula to infer μ^*_{t+1}. Recall that we assume the rest of the world shares the U.S. tax rate and the U.S. growth rate. Recall that we have a series for V^{*D}_t / GVA^D_t. We know that

$$V^*_t = Q^*_t K^*_{t+1} + \frac{\Pi^*_{t+1}}{r^*_{t+1} - \bar{g}_{t+1}}$$

and

$$\begin{aligned}
Q^*_t K^*_{t+1} &= \frac{(1 - \tau_{t+1})\alpha_{t+1}}{(r^*_{t+1} + \delta_{t+1})\mu^*_{t+1}} Y_{t+1} \\
\Pi^*_{t+1} &= \frac{(1 - \tau_{t+1})(\mu^*_{t+1} - 1)}{\mu^*_{t+1}} Y_{t+1}
\end{aligned}$$

Thus

$$\mu^*_{t+1} = \frac{(1 - \tau_{t+1})(1 + g_{t+1}) \left(\frac{\alpha_{t+1}}{(r^*_{t+1} + \delta_{t+1})} - \frac{1}{\left(r^*_{t+1} - \bar{g}_{t+1}\right)} \right)}{v^{*D}_t - \frac{(1 - \tau_{t+1})(1 + g_{t+1})}{\left(r^*_{t+1} - \bar{g}_{t+1}\right)}}$$

(One might wonder why Q^*_t does not show up in the expression for $Q^*_t K^*_{t+1}$. The logic is that equilibrium K^*_{t+1} is proportional to Q^{*-1}_t; when Q^*_t is high, investment is low)

12. Q_t^* : We assume $Q_0^* = Q_0 = 1$. Foreign dividends at date t are given by

$$D_t^* = (1-\tau_t)Y_t^* - W_t^* L_t^* - Q_t^*(K_{t+1}^* - (1-\delta_t)K_t^*)$$

That can be rearranged to give

$$Q_t^* = \frac{D_t^* - (1-\tau_t)Y_t^* + W_t^* L_t^* + Q_t^* K_{t+1}^*}{(1-\delta_t)K_t^*}$$

At each date t (initially for $t = 0$) we can solve for K_{t+1}^* from the foreign FOC for investment (recall that agents expect $Q_{t+1}^* = Q_t^*$). In particular, the rest of world version of equation 5 gives

$$K_{t+1}^* = \frac{\alpha_{t+1}(1-\tau_{t+1})Z_{t+1}}{Q_t^*(r_{t+1}^* + \delta_{t+1})\mu_{t+1}^*}$$

Substituting that expression into the previous one, and dividing through by output (recall $Y_t = Y_t^*$) gives

$$Q_t^* = \frac{\frac{D_t^*}{Y_t} - (1-\tau_t) + \frac{W_t^* L_t^*}{Y_t^*} + \frac{\alpha_{t+1}(1-\tau_{t+1})}{(r_{t+1}^*+\delta_{t+1})\mu_{t+1}^*}(1+g_{t+1})}{(1-\delta_t)\frac{K_t^*}{Y_t^*}}$$

We have model expressions for $\frac{W_t^* L_t^*}{Y_t^*}$ and $\frac{K_t^*}{Y_t^*}$ and a data series for $\frac{D_t^{*D}}{GVA_t^D}$ which identify Q_t^* given Q_{t-1}^* :

$$Q_t^* = \frac{\frac{D_t^{*D}}{GVA_t^D} - (1-\tau_t) + \frac{(1-\tau_t)(1-\alpha_t)}{\mu_t^*} + \frac{\alpha_{t+1}(1-\tau_{t+1})}{(r_{t+1}^*+\delta_{t+1})\mu_{t+1}^*}(1+g_{t+1})}{(1-\delta_t)\frac{(1-\tau_t)\alpha_t}{(r_{t+1}^*+\delta_t)\mu_t^*}\frac{1}{Q_{t-1}^*}}.$$

Thus we can iteratively construct a sequence for Q_t^*.

2.5 Functions for $z_{H,t+1}$ and $z_{H,t+1}^*$

The functions for $z_{H,t+1}$ and $z_{H,t+1}^*$ are derived as follows.

1. (a) The optimality condition for investment, equation 12 in the paper, simplifies, given $E[Q_{t+1}] = Q_t$, to

$$r_{t+1}^* = \frac{R_{t+1}}{Q_t} - \delta_{t+1}$$

which pins down R_{t+1} given r_{t+1}^* (which is known at t).

(b) The first-order condition for capital (Equation 10 in the paper) in conjunction with the production function (Equation 7 in the paper) then pins down K_{t+1} as

$$K_{t+1} = Z_{t+1} \left(z_{H,t+1}\right)^{\frac{1}{1-\alpha_{t+1}}} \left(\frac{\left(r^*_{t+1} + \delta_{t+1}\right) \mu_{t+1} Q_t}{(1-\tau_{t+1})\alpha_{t+1}} \right)^{\frac{1}{\alpha_{t+1}-1}}$$

so output is given by

$$\begin{aligned} Y_{t+1} &= z_{H,t+1} K_{t+1}^{\alpha_{t+1}} Z_{t+1}^{1-\alpha_{t+1}} \\ &= Z_{t+1} \left(z_{H,t+1}\right)^{\frac{1}{1-\alpha_{t+1}}} \left(\frac{\left(r^*_{t+1} + \delta_{t+1}\right) \mu_{t+1} Q_t}{(1-\tau_{t+1})\alpha_{t+1}} \right)^{\frac{\alpha_{t+1}}{\alpha_{t+1}-1}} \end{aligned}$$

Note, from the expressions for capital and output, that Z_{t+1} and $z_{H,t+1}$ affect inputs and output symmetrically.

(c) It follows that $Y_{t+1} = Y^*_{t+1} = Z_{t+1}$ when

$$\begin{aligned} z_{H,t+1} &= \left(\frac{\left(r^*_{t+1} + \delta_{t+1}\right) \mu_{t+1} Q_t}{(1-\tau_{t+1})\alpha_{t+1}} \right)^{\alpha_{t+1}} \\ z^*_{H,t+1} &= \left(\frac{\left(r^*_{t+1} + \delta_{t+1}\right) \mu^*_{t+1} Q^*_t}{(1-\tau_{t+1})\alpha_{t+1}} \right)^{\alpha_{t+1}} \end{aligned} \tag{6}$$

2.6 All Parameter Values

We plot the full set of parameter values in our baseline analysis in Figure 5.

2.7 Model fit

Figure 6 illustrates the model's ability to replicate key macroeconomic time series for the U.S. corporate sector: value added, gross investment, labor earnings, and cash flow payable to firm owners (defined as in Equation 13 in the paper). By construction, this fit is exact. The model replicates the decline in the 2000s in labor's share of value added, $(1-\tau_t)(1-\alpha_t)/\mu_t$, via a mix of changes in the share of labor in costs determined by $1-\alpha_t$ and changes in the output wedge μ_t. The rise in free cash flow to firm owners is due in part to lower payments to labor, and in part to lower taxes; investment is a fairly stable share of value added.

Figure 7 illustrates the model's replication of key valuation metrics: the Buffett ratio, the replacement cost of capital, and the dividend and earnings yields. Again, by construction, this fit is exact.

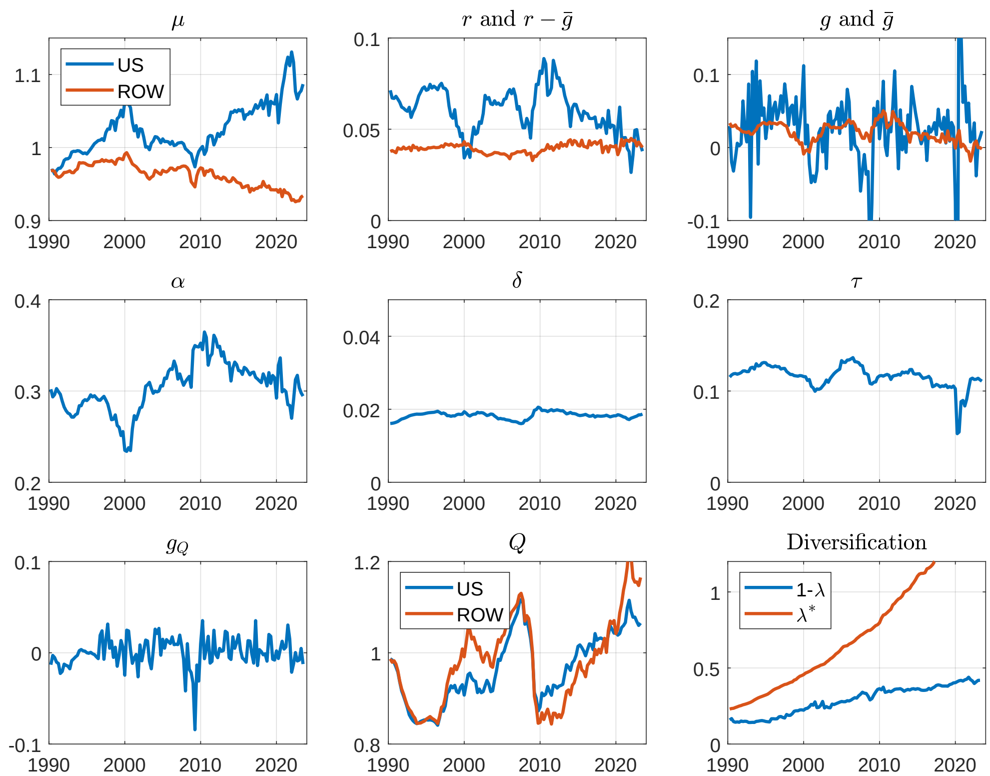

Figure 5: All Parameter Values

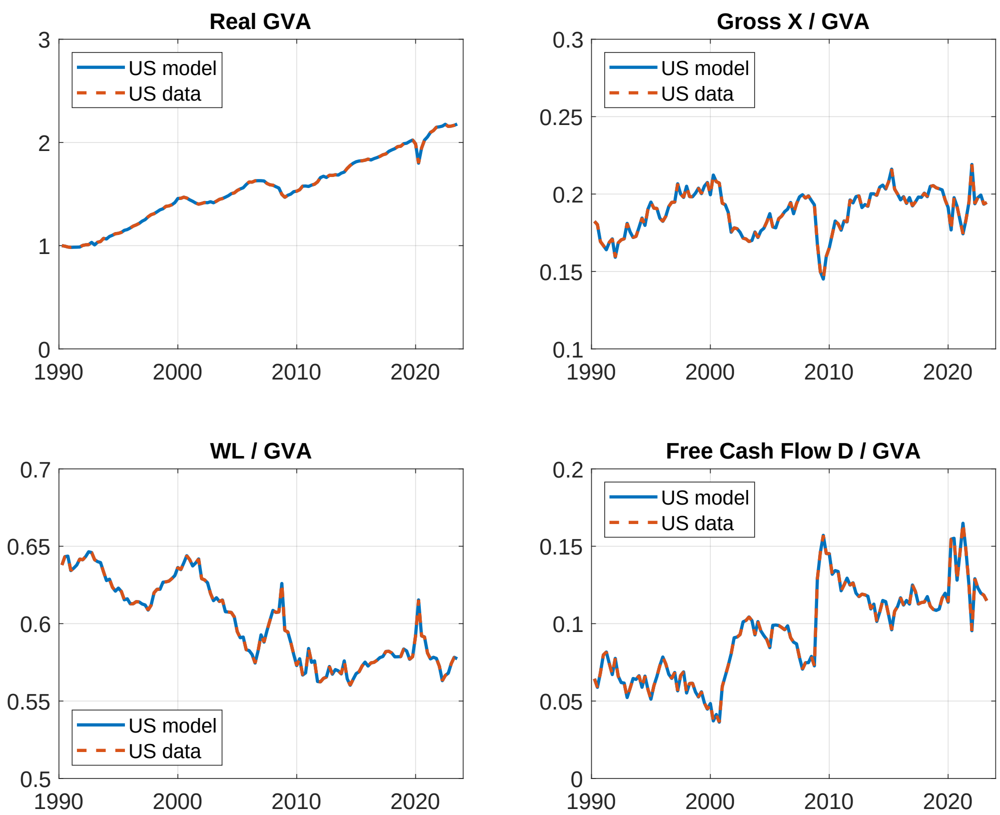

Figure 6: National Income Accounts for the Corporate Sector

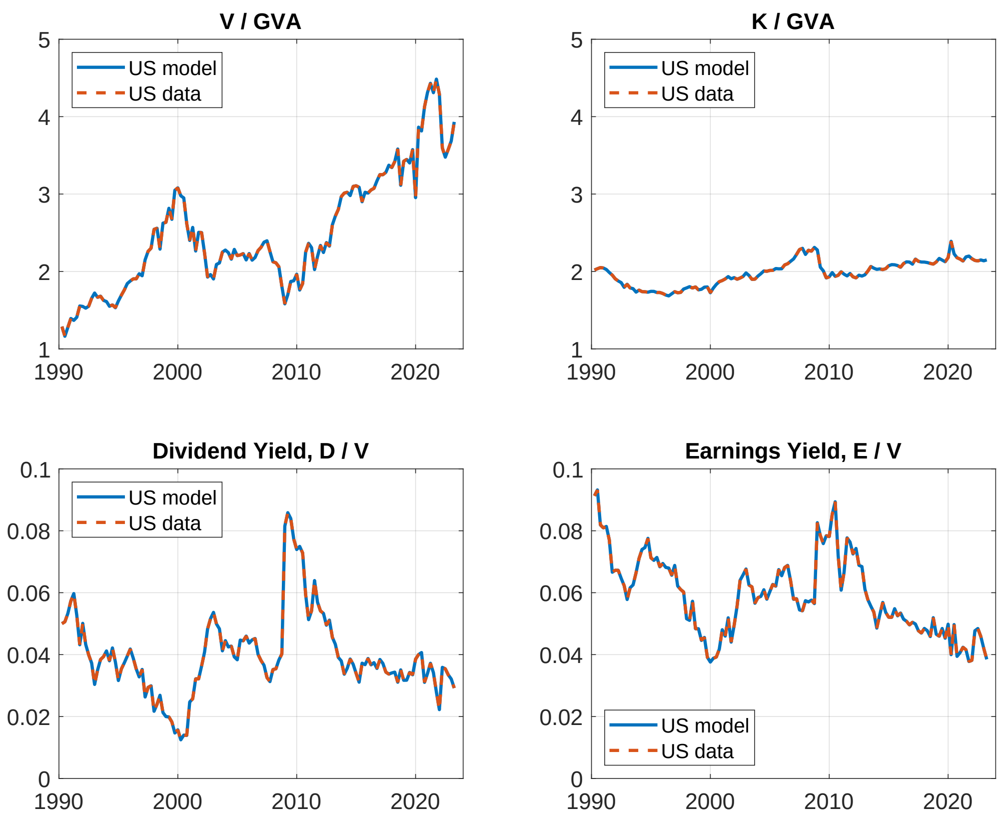

Figure 7: Key Asset Pricing Metrics

3 Comparison of our Measurement Procedure to that in Prior Papers

Our use of a simple macro finance model to measure factors driving the change in the division of income in the U.S. corporate sector into compensation for labor and physical capital and profits and the valuation of that sector has several antecedents in the literature. Here we describe how our work extends and refines this prior work.

Barkai (2020) and Karabarbounis and Neiman (2019) focus on measuring the division of income in the U.S. corporate sector into compensation for labor and physical capital and profits. These papers do not use data on the market valuation of the sector. Specifically, these papers start with estimates of the cost of capital r^*_{t+1} and then follow procedures analogous to those that we follow in steps 9 and 10 above to arrive at analogs of our estimates of the share of labor in costs $1 - \alpha_t$ and the share of corporate income left over to pay investors after deducting compensation of physical capital Π_t/Y_t. Karabarbounis and Neiman (2019) highlight that estimates of the "factorless income" share Π_t/Y_t derived using this procedure are very sensitive to the estimate of the cost of capital r^*_{t+1} used as an input into the measurement procedure. The principal measurement issue here is that it is difficult to arrive directly at an estimate of the appropriate cost of capital for the corporate sector r^*_{t+1} as it is difficult to measure the equilibrium gap between this cost of capital and the observed yields on government bonds due to considerations of risk and any liquidity or convenience yields on government bonds.

Our measurement procedure is more closely related to that in Farhi and Gourio (2018), Eggertsson, Robbins, and Wold (2021), and in the baseline case with no adjustment costs for investment studied in Crouzet and Eberly (2023).[2] Farhi and Gourio (2018) in particular argue that one need not build up an estimate of the cost of capital r^*_{t+1} from data on government bond yields and estimates of the equity premium and any convenience yield on those bonds. Instead, all three of these papers argue that one can proceed as we do by including measures of firm valuation V_t as well as the replacement value of the capital stock Q_tK_{t+1} in the analysis. These papers arrive at estimates of the cost of capital r^*_{t+1} using analogs of equation 26 in the paper and assumptions about expected growth from $t + 1$ on, $\bar{g}_{t+1}$.

We extend the measurement done in these papers in two respects.

First, we bring in the current account (equation 23 in the paper) as an additional data

[2] Greenwald, Lettau, and Ludvigson (forthcoming) conduct a related measurement exercise that develops a richer model of the dynamics that agents in the model expect but that does not use data on measures of the reproduction value of the stock of physical capital or investment. They conclude, as do these other papers, that a large portion of the increase in the market valuation of U.S. corporations is due to an increase in the share of value added paid to the owners of these firms.

series that is highly informative about $r^*_{t+1} - \bar{g}_{t+1}$. This additional data moment obviates the need for independent estimates of future trend growth.

Second, we conduct a sensitivity analysis of our measurement of r^*_{t+1} to alternative assumptions regarding the expected growth rate $\bar{g}_{t+1}$. Specifically, in Section 7 of the paper, we present measures of r^*_{t+1} using only U.S. corporate data and auxiliary alternative assumptions about either expected growth $\bar{g}_{t+1}$ or the valuation multiple for profits given by $1/(r^*_{t+1} - \bar{g}_{t+1})$. We consider four cases. In the first, expected growth $\bar{g}_{t+1}$ is set equal to the trend of growth rates of value added for the Corporate Sector from an HP filter of that time series. In the second, expected growth $\bar{g}_{t+1}$ is set equal to ten-year forecasts of GDP growth from the Survey of Professional Forecasters. In the third, we set the valuation multiple for profits equal to a constant $1/(r^*_{t+1} - \bar{g}_{t+1})$. In the fourth, we set the valuation multiple for profits $1/(r^*_{t+1} - \bar{g}_{t+1})$ equal to the realized value of dividends at $t+1$ over firm value at t (D_{t+1}/V_t). In this last case, we are assuming that agents' expectations for dividends realized at t are equal to the realized value of these dividends each period. In this way, we examine the sensitivity of the measurement procedure followed in Farhi and Gourio (2018), Eggertsson, Robbins, and Wold (2021), and in the baseline case with no adjustment costs for investment studied in Crouzet and Eberly (2023) to alternative assumptions about expected growth.

As shown in Section 7 in the paper, we find that the values of r^*_{t+1} obtained under these four alternative assumptions are remarkably similar outside of the period around the peak of the Tech boom in stocks in 2000. Accordingly, we find from this sensitivity exercise that the conclusion that profits or factorless income in the U.S. corporate sector have risen substantially over the past 10 years is robust to alternative assumptions about growth rates that agents expect going forward.

At the same time, as pointed out by Aguiar and Gopinath (2007), the implications of the model for the current account are highly sensitive to these four alternative assumptions for the expected growth rate $\bar{g}_{t+1}$ because the value of human wealth is highly sensitive to alternative assumptions for $r^*_{t+1} - \bar{g}_{t+1}$. Thus, in our baseline measurement in which we include the current account, we find a very stable value of $r^*_{t+1} - \bar{g}_{t+1}$.

In our measurement, we have abstracted from the role of unmeasured intangible capital in accounting for the increase in value of the U.S. corporate sector.[3] While we recognize that firms do make many investments that are not currently included in the measures that we use of the reproduction value of firm capital stocks and that firms likely generate substantial

[3] Hall (2001) argued that unmeasured intangible capital played an important role in accounting for the boom in the valuation of U.S. firms in the late 1990's. Eisfeldt and Papanikolaou (2014), Belo et al. (2022), Eisfeldt, Kim, and Papanikolaou (2022) and the papers cited therein argue that measured of intangible capital drawn from firms' accounting statements that is not included in the National Income and Product Accounts help account for the valuation of firms in the cross section.

quasi-rents from these past investments, we abstract from unmeasured capital for two reasons.

First, in the aggregate data on capital stocks not measured by the BEA cited in Corrado et al. (2022), there is no trend in the stock of such capital relative to value added over the past decade or more. Hence, incorporating these estimates of unmeasured capital would not serve to explain much of the rise in the market valuation of U.S. corporations over the past decade.[4]

Second, if one were to postulate that the observed increase in the valuation of U.S. corporations was accounted for by a large increase in investment in and accumulation of forms of capital that are not measured in the National Income and Product Accounts, then one would also have to postulate that U.S. corporations had simultaneously experienced a very large increase in productivity that allowed them to maintain measured value added growing along a smooth trend and large free cash flow as observed in the data. This would be required because, absent such an increase in productivity, and increase in investment in unmeasured capital would decrease measured output and measured free cash flow. Thus, while one could conduct a measurement exercise such as ours that matched observed flows, stocks, and market valuations of U.S. corporations and that attributed the large increase in the valuation and payouts from this sector to an increase in accumulated unmeasured capital rather than to profits (rents), such an exercise would require what seem like implausibly large increases in productivity to allow the U.S. corporate sector to maintain a steadily growing path of measured output while simultaneously dramatically increasing investment in forms of unmeasured capital. In the context of our model, these increases in productivity would be unexpected shocks from the perspective of model agents, and thus the model would still attribute a large portion of the increase in the valuation of U.S. corporations to unexpected capital gains to owners of firms rather than as an anticipated reward for previous investments.

4 The Impact of Shocks on the Current Account in the Model

In this appendix we derive an expression for the current account in terms of the underlying parameters of our model.

[4] This statement must be qualified in that we do not consider adjustment costs together with unmeasured forms of capital. Crouzet and Eberly (2023) argue that considering the interaction of these two model assumptions may have a significant impact on the conclusions drawn regarding the drivers of firm value in the aggregate.

We present the following equation for the current account

$$CA_t = \frac{1}{1+\rho}\left[\left(\frac{D_t}{V_t} - \rho\right)\lambda_{t-1}V_t + \left(\frac{D_t^*}{V_t^*} - \rho\right)\lambda_{t-1}^*V_t^* + (r_t^* - \rho)B_t + \left(\frac{W_tL_t}{H_t} - \rho\right)H_t\right] \quad (7)$$

with Human Wealth H_t given by

$$H_t \equiv \frac{W_{t+1}L_{t+1}}{r_{t+1}^* - \bar{g}_{t+1}}. \quad (8)$$

Note that in equation 7, the terms r_t^* and B_t are predetermined (set at $t-1$), so that we take them as given.

In taking the equation 7 to data, we combine the dividends from the intermediate goods firms and the firm that manages the capital stock into a single dividend D_t and computed enterprise value of these two types of firms into a single value V_t. To get intuition for how changes in model parameters impact the current account, it is more transparent to divide this dividend up into the component coming from intermediate goods firms, denoted by Π_t, and that coming from the firm that manages the capital stock, denoted by D_t^K in the paper, and to divide that enterprise value of US firms into the component due to intermediate goods firms, denoted by V_t^Π and the component coming from the end of period replacement cost of capital Q_tK_{t+1}. Thus, we study the following version of our equation for the current account

$$(1+\rho)CA_t = \left(\frac{\Pi_t}{V_t^\Pi} - \rho\right)\lambda_{t-1}V_t^\Pi + \left(\frac{D_t^K}{Q_tK_{t+1}} - \rho\right)\lambda_{t-1}Q_tK_{t+1} + \quad (9)$$

$$\left(\frac{D_t^*}{V_t^*} - \rho\right)\lambda_{t-1}^*V_t^* + (r_t^* - \rho)B_t + \left(\frac{W_tL_t}{H_t} - \rho\right)H_t$$

We make use of the following additional equations of the model.

Firm Valuation Equations:

$$V_t^\Pi = \frac{\Pi_{t+1}}{r_{t+1}^* - \bar{g}_{t+1}} \quad (10)$$

$$V_t^* = Q_t^*K_{t+1}^* + V_t^{\Pi *} \quad (11)$$

Definitions of Dividends and Investment

$$D_t^K = R_tK_t - X_t \quad (12)$$

$$X_t = Q_tK_{t+1} - (1-\delta_t)Q_tK_t \quad (13)$$

$$D_t^* = \Pi_t^* + R_t^*K_t^* - X_t^* \quad (14)$$

$$X_t^* = Q_t^*K_{t+1}^* - (1-\delta_t^*)Q_t^*K_t^* \quad (15)$$

Note that the terms $\delta_t, \delta_t^*, K_t, K_t^*$ are all determined at $t-1$ and that Q_t and Q_t^* are exogenous shocks realized in period t. Thus, the terms $Q_t(1-\delta_t)K_t$ and $Q_t^*(1-\delta_t^*)K_t^*$ for the replacement value of the capital stock remaining after depreciation are taken as given at time t.

Note as well that we define $1+g_{t+1} = Y_{t+1}/Y_t$ and $1+g_{t+1}^* = Y_{t+1}^*/Y_t^*$ and assume that these growth rates are known at t.

Factor Shares

$$\frac{\Pi_t}{Y_t} = \left(\frac{\mu_t - 1}{\mu_t}\right)(1-\tau_t), \tag{16}$$

$$\frac{W_t L_t}{Y_t} = \frac{(1-\alpha_t)}{\mu_t}(1-\tau_t), \tag{17}$$

$$\frac{R_t K_t}{Y_t} = \frac{\alpha_t}{\mu_t}(1-\tau_t), \tag{18}$$

and likewise for the factor shares in ROW.

Euler equations for Physical capital (with the assumptions that the parameters at $t+1$ other than Q_{t+1} and Q_{t+1}^* are known at t)

$$(1+r_{t+1}^*)Q_t K_{t+1} = R_{t+1}K_{t+1} + (1-\delta_{t+1})\mathbb{E}_t Q_{t+1}K_{t+1} \tag{19}$$

$$(1+r_{t+1}^*)Q_t^* K_{t+1}^* = R_{t+1}^* K_{t+1}^* + (1-\delta_{t+1}^*)\mathbb{E}_t Q_{t+1}^* K_{t+1}^* \tag{20}$$

Adding our assumption that $\mathbb{E}_t Q_{t+1} = Q_t$ and likewise for Q^*, we then have the following two equations for the replacement value of the capital stock

$$(r_{t+1}^* + \delta_{t+1})Q_t K_{t+1} = R_{t+1}K_{t+1} \tag{21}$$

$$(r_{t+1}^* + \delta_{t+1}^*)Q_t^* K_{t+1}^* = R_{t+1}^* K_{t+1}^* \tag{22}$$

4.1 Step 1: Current Account Relative to Output

The first step in terms of solving for the current account in terms of model parameters is to state the equations relative to output. If we divide all the equations above except the factor share equations by output and then use the factor share equations to get variables in terms of parameters, we have

$$(1+\rho)\frac{CA_t}{Y_t} = \left(\frac{\Pi_t}{V_t^{\Pi}} - \rho\right)\lambda_{t-1}\frac{V_t^{\Pi}}{Y_t} + \left(\frac{D_t^K}{Q_t K_{t+1}} - \rho\right)\lambda_{t-1}\frac{Q_t K_{t+1}}{Y_t} + \tag{23}$$
$$\left(\frac{D_t^*}{V_t^*} - \rho\right)\lambda_{t-1}^*\frac{V_t^*}{Y_t^*}\frac{Y_t^*}{Y_t} + (r_t^* - \rho)\frac{B_t}{Y_t} + \left(\frac{W_t L_t}{H_t} - \rho\right)\frac{H_t}{Y_t}$$

The ratio of Human Wealth to output is given by

$$\frac{H_t}{Y_t} \equiv \frac{(1-\alpha_{t+1})}{\mu_{t+1}}(1-\tau_{t+1})\frac{(1+g_{t+1})}{r_{t+1}^* - \bar{g}_{t+1}}. \tag{24}$$

and the income yield on human wealth is given by

$$\frac{W_t L_t}{H_t} = \frac{(1-\alpha_t)}{(1-\alpha_{t+1})}\frac{\mu_{t+1}}{\mu_t}\frac{(1-\tau_t)}{(1-\tau_{t+1})}\frac{(r_{t+1}^* - \bar{g}_{t+1})}{(1+g_{t+1})} \tag{25}$$

The ratio of the value of intermediate goods firms to output is given by

$$\frac{V_t^{\Pi}}{Y_t} = \frac{(\mu_{t+1}-1)}{\mu_{t+1}}(1-\tau_{t+1})\frac{(1+g_{t+1})}{(r_{t+1}^* - \bar{g}_{t+1})} \tag{26}$$

and the income yield on these firms is given by

$$\frac{\Pi_t}{V_t^{\Pi}} = \frac{(\mu_t - 1)}{\mu_t}\frac{\mu_{t+1}}{(\mu_{t+1}-1)}\frac{(1-\tau_t)}{(1-\tau_{t+1})}\frac{(r_{t+1}^* - \bar{g}_{t+1})}{(1+g_{t+1})} \tag{27}$$

The ratio of end of period capital to output is given (using equations 10, and 12 in the paper) by

$$\frac{Q_t K_{t+1}}{Y_t} = \frac{(1+g_{t+1})}{(r_{t+1}^* + \delta_{t+1})}\frac{\alpha_{t+1}}{\mu_{t+1}}(1-\tau_{t+1}) \tag{28}$$

The ratio of dividends from the firms that manage the capital stock to output is given by

$$\frac{D_t^K}{Y_t} = \frac{\alpha_t}{\mu_t}(1-\tau_t) - \frac{X_t}{Y_t}$$

The ratio of investment to output is given by

$$\frac{X_t}{Y_t} = \frac{(1+g_{t+1})}{(r_{t+1}^* + \delta_{t+1})}\frac{\alpha_{t+1}}{\mu_{t+1}}(1-\tau_{t+1}) - (1-\delta_t)\frac{(Q_t - Q_{t-1})K_t}{Y_t} - (1-\delta_t)\frac{Q_{t-1}K_t}{Y_t}$$

so

$$\frac{X_t}{Y_t} = \frac{(1+g_{t+1})}{(r_{t+1}^* + \delta_{t+1})}\frac{\alpha_{t+1}}{\mu_{t+1}}(1-\tau_{t+1}) - \frac{(1-\delta_t)}{(r_t^* + \delta_t)}\frac{\alpha_t}{\mu_t}(1-\tau_t) - (1-\delta_t)\frac{(Q_t - Q_{t-1})K_t}{Y_t} \tag{29}$$

These equations imply

$$\frac{D_t^K}{Y_t} = \frac{(1+r_t^*)}{(r_t^*+\delta_t)}\frac{\alpha_t}{\mu_t}(1-\tau_t) - \frac{(1+g_{t+1})}{(r_{t+1}^*+\delta_{t+1})}\frac{\alpha_{t+1}}{\mu_{t+1}}(1-\tau_{t+1}) + \tag{30}$$

$$(1-\delta_t)\frac{(Q_t - Q_{t-1})K_t}{Y_t}$$

Direct analogs of these equations hold for the ROW (D^* and V^*) as well.

4.2 Step 2: Solving for Balanced Growth Paths

In the second step of solving for the response of the model current account to changes in model parameters is to solve for a balanced growth path in the model. We assume that parameters are constant on a balanced growth path. Thus, we have $\tau_t = \tau_{t+1}$, $\mu_t = \mu_{t+1}$, $\alpha_t = \alpha_{t+1}$, $\delta_t = \delta_{t+1}$, $Q_t = Q_{t+1}$, $r_t^* = r_{t+1}^*$, and we assume that growth from t to $t+1$ is equal to long term growth so $g_{t+1} = \bar{g}_{t+1}$. We also assume that equity shares λ_t and λ_t^* are constant as well. We denote all of these variables with a bar over the top.

With these assumptions, we have the ratio of human wealth to output given by the labor share times its valuation multiple

$$\overline{\frac{H}{Y}} = \frac{(1-\bar{\alpha})}{\bar{\mu}}(1-\bar{\tau})\frac{(1+\bar{g})}{\bar{r}^*-\bar{g}}. \tag{31}$$

and the income yield on human wealth is given by

$$\overline{\frac{WL}{H}} = \frac{(\bar{r}^*-\bar{g})}{(1+\bar{g})} \tag{32}$$

For the intermediate goods firms, the ratio of their value to GDP is equal to the share of factorless income times its valuation multiple

$$\overline{\frac{V^\Pi}{Y}} = \frac{(\bar{\mu}-1)}{\bar{\mu}}(1-\bar{\tau})\frac{(1+\bar{g})}{\bar{r}^*-\bar{g}} \tag{33}$$

and the associated income yield is given by

$$\overline{\frac{\Pi}{V^\Pi}} = \frac{(\bar{r}^*-\bar{g})}{(1+\bar{g})} \tag{34}$$

The value of the firms managing the capital stock is given by the ratio of BGP capital at the end of the period to output

$$\frac{\overline{QK'}}{Y} = \frac{(1+\bar{g})}{(\bar{r}^* + \bar{\delta})} \frac{\bar{\alpha}}{\bar{\mu}}(1-\bar{\tau}) \tag{35}$$

The ratio of the dividends from these firms managing the capital stock to output is given by

$$\frac{\overline{D^K}}{Y} = \frac{(\bar{r}^* - \bar{g})}{\bar{r}^* + \bar{\delta}} \frac{\bar{\alpha}}{\bar{\mu}}(1-\bar{\tau})$$

Thus income yield on this capital is given by

$$\frac{\overline{D^K}}{\overline{QK'}} = \frac{(\bar{r}^* - \bar{g})}{(1+\bar{g})} \tag{36}$$

Note that on a BGP, the ROW has the same cost of capital and growth rate, so the income yield on the corporate sector in the ROW is given by

$$\frac{\overline{D^*}}{\overline{V^*}} = \frac{(\bar{r}^* - \bar{g})}{(1+\bar{g})} \tag{37}$$

These equations imply that on a balanced growth path, the ratio of the current account to output taking as given the outstanding stock of bonds maturing relative to output B_t/Y_t is given by

$$\frac{CA_t}{Y_t} = \frac{1}{1+\rho}\left[\left(\frac{(\bar{r}^* - \bar{g})}{(1+\bar{g})} - \rho\right)\left(\frac{\overline{H}}{Y} + \bar{\lambda}\frac{\overline{V^{\Pi}}}{Y} + \bar{\lambda}\frac{\overline{QK'}}{Y} + \bar{\lambda}^*\frac{\overline{V^*}}{Y}\right) + (\bar{r}^* - \rho)\frac{B_t}{Y_t}\right] \tag{38}$$

The change in the stock of bonds coming due is then equal to the current account, or

$$(1+\bar{g})\frac{B_{t+1}}{Y_{t+1}} - \frac{B_t}{Y_t} = \frac{CA_t}{Y_t} \tag{39}$$

4.3 Alternative BGP's

In the event that the income yield on human wealth and equity assets equals the rate of time preference, so

$$\frac{(\bar{r}^* - \bar{g})}{(1+\bar{g})} = \rho$$

then the term

$$\left(1 - \rho\frac{(1+\bar{g})}{(\bar{r}^* - \bar{g})}\right) = 0$$

and

$$\frac{(\bar{r}^* - \rho)}{(1+\rho)} = \bar{g}$$

so $CA_t/Y_t = \bar{g}B_t/Y_t$ and thus the ratio of net non-equity assets to output remains constant ($B'/Y' = B/Y$). In this case, the ratio of the current account to output remains constant over time at a level indexed by $B_t/Y_t = \overline{B/Y}$.

In the event that the income yield on human wealth and equity assets exceeds the rate of time preference, so

$$\frac{(\bar{r}^* - \bar{g})}{(1+\bar{g})} > \rho$$

then

$$\bar{r}^* - \rho > \bar{g}(1+\rho)$$

and, since the sum of human and equity assets must be non-negative, we have that

$$\frac{CA}{Y} > \bar{g}\frac{B}{Y}$$

and thus

$$\frac{B'}{Y'} > \frac{B}{Y}$$

That is, the US economy steadily acquires net non-equity claims on the ROW. If the income yield on human wealth and equity assets is smaller than the rate of time preference, then these inequalities are reversed and the US economy steadily depletes non-equity claims on the ROW. This process of trend accumulation or decumulation of net non-equity claims on the ROW would be very slow given the narrow range of fluctuations in the gap between the income yield on equity and the rate of time preference allowed in the model. A more complete model would specify a force to prevent the net non-equity position from growing without bound. One approach in the literature to address this issue is to include a quadratic cost to holding a large net non-equity position. Alternatively, if one models uncertainty explicitly, a full non-linear solution has a consumption rule out of wealth that varies with the level of wealth due to changes in the strength of the precautionary motive for saving as the level of wealth rises.

4.3.1 Magnitudes

To get a sense of magnitudes, consider the case in which $B/Y = 0$ and

$$\frac{(\bar{r}^* - \bar{g})}{(1+\bar{g})} = \rho$$

In this case, the current account balance on this BGP is zero and the terms

$$\frac{(1-\bar{\alpha})}{\bar{\mu}}(1-\bar{\tau}) + \lambda\frac{(\bar{\mu}-1)}{\bar{\mu}}(1-\bar{\tau}) + \lambda\frac{(\bar{r}^*-\bar{g})}{\bar{r}^*+\bar{\delta}}\frac{\bar{\alpha}}{\bar{\mu}}(1-\bar{\tau}) + \lambda^*\frac{D^*}{Y}$$

in total are equal to the ratio of consumption to output. If we consider changes in the term comparing the rate of time preference to the income yield on human wealth and equity assets

$$\left(1-\rho\frac{(1+\bar{g})}{(\bar{r}^*-\bar{g})}\right)$$

we see that the ratio of the current account to GDP is quite sensitive to such changes. For example, if the baseline value of ρ on an annual basis is 3.3% and the income yield on human wealth and equity assets drops to 3%, then the current account to output becomes

$$\frac{CA}{Y} = -0.1 * \frac{C}{Y} \approx -8\%$$

4.4 Responses of the Current Account to Shocks

In our model, at time t, a shock is the arrival of news that any of the parameters of the model dated $t+1$ have changed and will have that new value from $t+1$ on. These parameters include $\alpha_{t+1}, \delta_{t+1}, \mu_{t+1}, \tau_{t+1}, r^*_{t+1}$. We also consider permanent shocks to the expected growth rate $\bar{g}_{t+1}$ relevant for growth from $t+1$ on and transitory shocks to the growth rate from t to $t+1$ denoted by g_{t+1}. The only exception to this rule is that at time t, it is possible that the current value of the price Q_t is shocked relative to its prior value Q_{t-1} which is also the expectation of Q_t at time $t-1$. We do not shock the parameters λ and λ^*.

For shocks to $\alpha_{t+1}, \delta_{t+1}, \mu_{t+1}, \tau_{t+1}, r^*_{t+1}$ and $\bar{g}_{t+1}$, the response of the current account has two steps. At t, there is a transitory response of the current account as detailed below. From period $t+1$ on, the ratio of the current account to output is given by equations 38 with parameters held constant at their levels at $t+1$ and the ratio of net non-equity claims on the ROW evolves according to equation 39.

In the algebra below, we use ratios without dates (such as $\overline{H/Y}$) to denote the values of these ratios on the original BGP up to time t.

For the impact of such a shock on the current account in period t, we use equations 23, 24, 25, 26, 27, 28, and 30. We illustrate the application of these equations for several types of shocks next. In each case, we assume that the economy starts on a BGP with $B_t/Y_t = 0$ and with

$$\frac{(\bar{r}^*-\bar{g})}{(1+\bar{g})} = \rho,$$

4.4.1 A transitory growth shock

Consider the impact of a shock to the growth rate from t to $t+1$ denoted by g_{t+1}. We assume that this shock hits both countries (we mention where this matters below). Assume that the economy starts on a BGP as described above and that from $t+1$ on, the parameters continue on this original path. Thus, this corresponds to a one-time permanent shock to the level of productivity.

In this case, income levels $W_t L_t$, Π_t, and D_t^* are not impacted by the shock. The dividend from the firms managing the physical capital stock does fall because investment rises. Specifically, from equation 29, the ratio of investment to output rises by the shock to the growth rate times the capital output ratio

$$\frac{X_t}{Y_t} = \overline{\frac{X}{Y}} + \frac{(g_{t+1} - \bar{g})}{1 + \bar{g}} \overline{\frac{QK'}{Y}}$$

Thus, as an example, with a capital to output ratio of 2, a $50bp$ transitory shock to the growth rate leads to a jump in the ratio of investment to output of 1 percentage point. Plugging in this result for investment, we have that

$$\frac{D_t^K}{Y_t} = \overline{\frac{D^K}{Y}} - \frac{(g_{t+1} - \bar{g})}{1 + \bar{g}} \overline{\frac{QK'}{Y}}$$

Using equation 28 for the end of period t capital stock to output ratio, we have

$$\frac{Q_t K_{t+1}}{Y_t} = \frac{1 + g_{t+1}}{1 + \bar{g}} \overline{\frac{QK'}{Y}}$$

and the income yield on physical capital in period t becomes

$$\frac{D_t^K}{Q_t K_{t+1}} = \frac{1 + \bar{g}}{1 + g_{t+1}} \left(\rho - \frac{(g_{t+1} - \bar{g})}{1 + \bar{g}} \right)$$

Putting this together, the contribution of the impact of this shock on physical capital to the current account is given by

$$\frac{1}{1+\rho} \left(\frac{D_t^K}{Q_t K_{t+1}} - \rho \right) \lambda_{t-1} \frac{Q_t K_{t+1}}{Y_t} = \frac{1}{1+\rho} \left(\left(\rho - \frac{(g_{t+1} - \bar{g})}{1 + \bar{g}} \right) - \rho \frac{1 + g_{t+1}}{1 + \bar{g}} \right) \bar{\lambda} \overline{\frac{QK'}{Y}} =$$

$$- \left(\frac{g_{t+1} - \bar{g}}{1 + \bar{g}} \right) \bar{\lambda} \overline{\frac{QK'}{Y}}$$

That is, the increment to investment induced by this transitory shock to the growth rate is financed by a current account deficit in proportion to the US households' share of US equity $\bar{\lambda}$.

If we assume that the shock hits the ROW as well, then we have the equivalent term for the impact on the US current account

$$-\left(\frac{g_{t+1}-\bar{g}}{1+\bar{g}}\right)\bar{\lambda}^{*}\frac{\overline{Q^{*}K^{*\prime}}}{Y}$$

If the shock does not hit the ROW, then this term would be zero.

From equations 24, 26 the ratios of values of domestic assets H_t, V_t^{Π} relative to output at t all rise by the ratio

$$\frac{1+g_{t+1}}{1+\bar{g}}$$

relative to their BGP values. If the shock hits the ROW as well, then the same is true for $V_t^{\Pi*}/Y_t$ while Π_t^{*}/Y_t is unchanged.

As a result, the impact of changes in the terms involving human capital on the current account are given by

$$-\frac{\rho}{1+\rho}\left(\frac{g_{t+1}-\bar{g}}{1+\bar{g}}\right)\frac{\overline{H}}{Y}$$

and those involving US factorless income are given by

$$-\frac{\rho}{1+\rho}\left(\frac{g_{t+1}-\bar{g}}{1+\bar{g}}\right)\bar{\lambda}\frac{\overline{V^{\Pi}}}{Y}$$

while the terms involving factorless income in the ROW are given by

$$-\frac{\rho}{1+\rho}\left(\frac{g_{t+1}-\bar{g}}{1+\bar{g}}\right)\bar{\lambda}^{*}\frac{\overline{V^{\Pi*}}}{Y}$$

If the shock did not hit the ROW, this term would be zero.

Putting these results together, we have that if the transitory growth shock is common to both countries, then

$$\frac{CA_t}{Y_t}-\frac{\overline{CA}}{Y}=\frac{\rho}{1+\rho}\left(\frac{\bar{g}-g_{t+1}}{1+\bar{g}}\right)\left[\frac{\overline{H}}{Y}+\bar{\lambda}\frac{\overline{V^{\Pi}}}{Y}+\bar{\lambda}^{*}\frac{\overline{V^{\Pi*}}}{Y}\right]-\frac{(g_{t+1}-\bar{g})}{1+\bar{g}}\left[\bar{\lambda}\frac{\overline{QK'}}{Y}+\bar{\lambda}^{*}\frac{\overline{Q^{*}K^{*\prime}}}{Y}\right]$$

where the last term represents the negative impact on the current account of increased investment in physical capital in both the US and ROW. We see here that for reasonable values of the rate of time preference ρ, the dominant impact of this shock on the current account is through its impact on the ratio of investment to output (the second term) rather than through its impact on the ratio of human wealth and the value of factorless income to output.

Note that this shock feeds into the net bond position B_{t+1} and from $t+1$ on, the economy is on a BGP with the current account equal to $\bar{g}B_{t+1}/Y_{t+1}$.

4.4.2 A permanent growth shock

Now consider a shock to $\bar{g}_{t+1}$. Let $\bar{g}$ denote the long term growth rate expected on the initial BGP and $\bar{g}_{t+1}$ denote the new growth rate expected from period $t+1$ on. This shock is common to both countries. By definition, this shock only impact the growth in productivity from period $t+1$ on. Hence, it does not impact any flow variables at time t. Moreover, it does not impact the end of period t capital stock in either the US Q_tK_{t+1} or ROW $Q_t^*K_{t+1}^*$ and hence does not impact investment at time t. The shock does, however, impact the end of period t valuation of human wealth H_t and the end of period t valuations of US and ROW factorless income V_t^{Π} and $V_t^{\Pi *}$. In particular

$$\frac{H_t}{Y_t} = \frac{\bar{r}^* - \bar{g}}{\bar{r}^* - \bar{g}_{t+1}} \overline{\frac{H}{Y}}$$

$$\frac{V_t^{\Pi}}{Y_t} = \frac{\bar{r}^* - \bar{g}}{\bar{r}^* - \bar{g}_{t+1}} \overline{\frac{V^{\Pi}}{Y}}$$

$$\frac{V_t^{\Pi *}}{Y_t} = \frac{\bar{r}^* - \bar{g}}{\bar{r}^* - \bar{g}_{t+1}} \overline{\frac{V^{\Pi *}}{Y}}$$

Thus, the impact of this shock on the current account at time t is given by

$$\frac{CA_t}{Y_t} - \overline{\frac{CA}{Y}} = \frac{\rho}{1+\rho} \left(\frac{\bar{g} - g_{t+1}}{\bar{r} - \bar{g}_{t+1}} \right) \left[\overline{\frac{H}{Y}} + \bar{\lambda} \overline{\frac{V^{\Pi}}{Y}} + \bar{\lambda}^* \overline{\frac{V^{\Pi *}}{Y}} \right]$$

Note that this shock feeds into the net bond position B_{t+1} and that from period $t+1$, since $\bar{r}^* - \bar{g}_{t+1}$ differs from its level in the initial balanced growth path, the current account rise (or fall) relative to output depending on whether the new income yield $\frac{(\bar{r}^* - \bar{g}_{t+1})}{(1+\bar{g}_{t+1})}$ is larger or smaller than the rate of time preference ρ.

Note as well that the impact of this shock on the current account can be quite large as the ratio of human wealth to output is large and the term $\bar{r} - \bar{g}_{t+1}$ is on the order of ρ. Thus, the impact of such a shock on the ratio of the current account to output is of opposite sign and at least an order of magnitude larger than the magnitude of the shock itself $(g_{t+1} - \bar{g})$.

4.4.3 A shock to the discount rate r^*_{t+1}

Now consider a shock to the discount rate r^*_{t+1}. This shock arrives at t and impacts the discount rate between t and $t+1$ and all subsequent periods in the same way. Thus, this shock impacts the equilibrium capital to output ratio at the end of period t, and hence investment at t, in both the US and the ROW. It also impacts the valuation of future labor and factorless income as in the case of a permanent growth shock.

We have that this shock impacts the ratio of capital to output at the end of period t by

$$\frac{Q_t K_{t+1}}{Y_t} = \frac{\bar{r}^* + \bar{\delta}}{r^*_{t+1} + \bar{\delta}} \overline{\frac{QK'}{Y}}$$

Thus, the ratio of investment to output at t is given by

$$\frac{X_t}{Y_t} = \overline{\frac{X}{Y}} + \left(\frac{\bar{r}^* - r^*_{t+1}}{r^*_{t+1} + \bar{\delta}} \right) \overline{\frac{QK'}{Y}}$$

and the ratio of dividends from the firm managing the capital stock to output at t is given by

$$\frac{D^K_t}{Y_t} = \overline{\frac{D^K}{Y}} - \left(\frac{\bar{r}^* - r^*_{t+1}}{r^*_{t+1} + \bar{\delta}} \right) \overline{\frac{QK'}{Y}}$$

Thus, the impact on the current account at t due to the terms corresponding to physical capital are given by

$$\frac{1}{1+\rho} \left(\rho \frac{r^*_{t+1} + \bar{\delta}}{\bar{r}^* + \bar{\delta}} - \frac{\bar{r}^* - r^*_{t+1}}{\bar{r}^* + \bar{\delta}} - \rho \right) \bar{\lambda} \frac{\bar{r}^* + \bar{\delta}}{r^*_{t+1} + \bar{\delta}} \overline{\frac{QK'}{Y}} =$$

$$- \left(\frac{\bar{r}^* - r^*_{t+1}}{r^*_{t+1} + \bar{\delta}} \right) \overline{\frac{QK'}{Y}}$$

That is, these terms are equal to minus the increase in investment.

While current labor compensation and factorless income do not change, the end of period valuations of these income streams do change and are given by

$$\frac{H_t}{Y_t} = \frac{\bar{r}^* - \bar{g}}{r^*_{t+1} - \bar{g}} \overline{\frac{H}{Y}}$$

$$\frac{V^{\Pi}_t}{Y_t} = \frac{\bar{r}^* - \bar{g}}{r^*_{t+1} - \bar{g}} \overline{\frac{V^{\Pi}}{Y}}$$

$$\frac{V^{\Pi *}_t}{Y_t} = \frac{\bar{r}^* - \bar{g}}{r^*_{t+1} - \bar{g}} \overline{\frac{V^{\Pi *}}{Y}}$$

Thus, the impact on the current account from these terms is given by

$$\frac{\rho}{1+\rho}\left(\frac{r^*_{t+1}-\bar{r}^*}{r^*_{t+1}-\bar{g}}\right)\left[\frac{\overline{H}}{Y}+\bar{\lambda}\frac{\overline{V^{\Pi}}}{Y}+\bar{\lambda}^*\frac{\overline{V^{\Pi *}}}{Y}\right]$$

Putting these together, the overall impact on the ratio of the current account to output at time t is given by

$$\frac{CA_t}{Y_t}-\frac{\overline{CA}}{Y}=-\left(\frac{\bar{r}^*-r^*_{t+1}}{r^*_{t+1}+\bar{\delta}}\right)\frac{\overline{QK'}}{Y}+\frac{\rho}{1+\rho}\left(\frac{r^*_{t+1}-\bar{r}^*}{r^*_{t+1}-\bar{g}}\right)\left[\frac{\overline{H}}{Y}+\bar{\lambda}\frac{\overline{V^{\Pi}}}{Y}+\bar{\lambda}^*\frac{\overline{V^{\Pi *}}}{Y}\right]$$

The first term captures in the impact of the discount rate shock on investment at t while the second term captures the revaluation of future labor income and factorless income. Again, since human wealth is quite large and ρ and $r^*_{t+1}-\bar{g}$ are similar in magnitude, the impact of this shock on the current account through the second term is quite large.

Note also that this shock feeds into the net bond position B_{t+1} and that from period $t+1$, since $\bar{r}^*-\bar{g}_{t+1}$ differs from its level in the initial balanced growth path, the current account rise (or fall) relative to output depending on whether the new income yield $\frac{(\bar{r}^*-\bar{g}_{t+1})}{(1+\bar{g}_{t+1})}$ is larger or smaller than the rate of time preference ρ.

4.4.4 A shock to factorless income

We now consider a shock to the allocation of income due to an increase in μ_{t+1} in the U.S. News regarding this change in parameters arrives in period t. Households and firms perceive that $\mu_{t+k}=\mu_{t+1}$ for all periods $t+k$ for $k\geq 1$.

This shock has no impact in time t on wages W_tL_t or factorless income Π_t, output Y_t, or dividends received from the ROW, D^*_t. The shock does alter dividends paid by the US firm that manages the capital stock D^K_t. From equation 30, we have

$$\frac{D^K_t}{Y_t}=\frac{\overline{D^K}}{Y}+\left(1-\frac{\bar{\mu}}{\mu_{t+1}}\right)\frac{\overline{QK'}}{Y}$$

Moreover, the ratio of end of period capital to output in period t is given by

$$\frac{Q_tK_{t+1}}{Y_t}=\frac{\bar{\mu}}{\mu_{t+1}}\frac{\overline{QK'}}{Y}$$

Thus, the impact on the current account at t from the terms associated with investment in physical capital is given by

$$\frac{1}{1+\rho}\left(\rho\frac{\mu_{t+1}}{\bar{\mu}}+(\frac{\mu_{t+1}}{\bar{\mu}}-1)-\rho\right)\bar{\lambda}\frac{\bar{\mu}}{\mu_{t+1}}\frac{\overline{QK'}}{Y}=$$

$$\left(\frac{\mu_{t+1}-\bar{\mu}}{\mu_{t+1}}\right)\frac{\overline{QK'}}{Y}$$

That is, the impact of this shock on the current account at t coming through terms having to do with physical capital in the US is equal to the the drop in investment at t.

The change in μ_{t+1} also alters the value of human wealth and factorless income. We have

$$\frac{H_t}{Y_t}=\frac{\bar{\mu}}{\mu_{t+1}}\frac{\overline{H}}{Y}$$

and the income yield on this human wealth from equation 25 is given by

$$\frac{W_tL_t}{H_t}=\frac{\mu_{t+1}}{\bar{\mu}}\frac{\overline{WL}}{H}$$

so the impact on the current account at t from the terms associated with human wealth is given by

$$\frac{\rho}{1+\rho}\left(\frac{\mu_{t+1}-\bar{\mu}}{\mu_{t+1}}\right)\frac{\overline{H}}{Y}$$

Likewise, for factorless income, we have

$$\frac{V_t^{\Pi}}{Y_t}=\frac{\mu_{t+1}-1}{\mu_{t+1}}\frac{\bar{\mu}}{\bar{\mu}-1}\frac{\overline{V^{\Pi}}}{Y}.$$

This gives an income yield on the claim to factorless income of

$$\frac{\Pi_t}{V_t^{\Pi}}=\frac{\mu_{t+1}}{\mu_{t+1}-1}\frac{\bar{\mu}-1}{\bar{\mu}}\frac{\overline{\Pi}}{V^{\Pi}}$$

and the impact on the current account at t from the terms associated with factorless income given by

$$\frac{\rho}{1+\rho}\left(1-\frac{\mu_{t+1}-1}{\mu_{t+1}}\frac{\bar{\mu}}{\bar{\mu}-1}\right)\bar{\lambda}\frac{\overline{V^{\Pi}}}{Y}=\frac{\rho}{1+\rho}\left(\frac{\bar{\mu}-\mu_{t+1}}{\mu_{t+1}}\right)\frac{\bar{\lambda}}{\bar{\mu}-1}\frac{\overline{V^{\Pi}}}{Y}$$

From equations 31 and 33, we have

$$\frac{\overline{V^{\Pi}}}{Y}=\frac{\bar{\mu}-1}{1-\alpha}\frac{\overline{H}}{Y}$$

Thus, we can write this impact on the current account at t from the sum of the terms associated with factorless income and human wealth as

$$\frac{\rho}{1+\rho}\left(\frac{\bar{\mu}-\mu_{t+1}}{\mu_{t+1}}\right)\left(1-\frac{\bar{\lambda}}{1-\alpha}\right)\frac{H}{Y}$$

This gives the overall impact on the current account at t from this shock as

$$\frac{CA_t}{Y_t}-\frac{\overline{CA}}{Y}=\left(\frac{\mu_{t+1}-\bar{\mu}}{\mu_{t+1}}\right)\frac{\overline{QK'}}{Y}+\frac{\rho}{1+\rho}\left(\frac{\mu_{t+1}-\bar{\mu}}{\mu_{t+1}}\right)\left(1-\frac{\bar{\lambda}}{1-\alpha}\right)\frac{\overline{H}}{Y}$$

Note that this shock feeds into the net bond position B_{t+1} and from $t+1$ on, the economy is on a BGP with the current account equal to $\bar{g}B_{t+1}/Y_{t+1}$.

This formula for the response of the current account on impact (at t) to a shock to μ_{t+1} is the sum of the impact of this shock on investment in physical capital as captured in the first term and the impact of this shock on the combined value of the claims held by US Households on labor income and factorless income as captured in the second term. The first term positive when $\mu_{t+1} > \bar{\mu}$ because the capital to output ratio and thus investment falls. While this second term in theory can be large because the baseline value of human wealth relative to output is large, this effect is mitigated to the extent to which US residents hold claims to US firms (as indexed by $\bar{\lambda}$). If the share of equity US residents hold in US firms exceeds the share of labor in production costs, then this second term is negative. If this equity share is less than the share of labor in costs, then it is positive. These offsetting effects arise as this shock to μ_{t+1} reallocates income from labor compensation to factorless income.

5 Extended Model with Terms of Trade Effects

In our simple baseline model, all domestic intermediate varieties have the same price, and because domestic and foreign final output are the same good, the prices of domestic and foreign intermediates are identical. Thus, in that model, a rise in output wedges for U.S. firms does not change the price that consumers pay for U.S.-produced relative to foreign-produced goods.

We now briefly consider an extended version of the model, in which domestically produced intermediates produce a composite domestic good A, while foreign intermediates are combined to produce a composite foreign final good B. Goods A and B are traded and used symmetrically in each country as imperfectly substitutable inputs in the production of final consumption and investment goods. In this extended model, the equilibrium price of good

B relative to good A – the terms of trade – will depend on how much of good B is produced relative to good A. Thus, whether a rise in U.S. output wedges improves or worsens the terms of trade will depend on whether the rise in U.S. output wedges is associated with an expansion or a contraction in U.S. production.

A pure output wedge shock – one in which output wedges go up because follower firms become less productive and z_L falls – will be associated with a decline in U.S. output and an increase in the price of U.S.-produced goods relative to foreign ones. This terms of trade effect will ameliorate the negative welfare consequences of a pure output wedge shock for U.S. consumers. This is an optimal tariff argument: just like a tax on exports, a pure increase in domestic output wedges reduces the supply of U.S.-produced goods and increases their relative price. However, note that an increase in U.S. output wedges may be associated with either a decline or a rise in the production of U.S. goods, depending on whether the rise in output wedges reflects a decline in z_L (which reduces U.S. output) or a rise in z_H (which boosts U.S. output). In our baseline calibration of our baseline model, we constructed a combination of changes to z_L and z_H with the property that the rise in U.S. output wedges neither expands nor reduces U.S. output. We now show that if we were to follow the same strategy in the extended model in which goods A and B are imperfect substitutes, there would be no change in the equilibrium terms of trade. And in the absence of such a change, all the positive and normative implications of the increase in output wedges would be identical to those in the baseline model described in the main text.

In particular, consider an extension of the baseline model in which domestically produced varieties are combined to produce a composite domestic intermediate A and a composite foreign intermediate B, where the quantites of these composites are denoted by Y_A and Y_B. Thus,

$$\begin{aligned} Y_A &= \left[\int_0^1 Y_i^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)} \\ Y_B &= \left[\int_0^1 Y_i^{*(\theta-1)/\theta} di\right]^{\theta/(\theta-1)} \end{aligned}$$

These two composites are combined to produced the final consumption and investment goods using a CES aggregator function G. Let A and A^* and B and B^* denote the quantities of the two composite goods used in producing the final consumption and investment goods in the two countries. Thus,

$$\begin{aligned} C + K' - (1-\delta)K &= G(A, B) \\ C^* + K^{*\prime} - (1-\delta)K^* &= G^*(A^*, B^*). \end{aligned}$$

Assume the aggregators for producing final goods are identical in the two economies:

$$\begin{aligned} G(A,B) &= 2^{\frac{1}{1-\varepsilon}}\left[A^{(\varepsilon-1)/\varepsilon}+B^{(\varepsilon-1)/\varepsilon}\right]^{\frac{\varepsilon}{\varepsilon-1}} \\ G^*(A^*,B^*) &= 2^{\frac{1}{1-\varepsilon}}\left[A^{*(\varepsilon-1)/\varepsilon}+B^{*(\varepsilon-1)/\varepsilon}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \end{aligned}$$

Here, the parameter ε defines the elasticity of substitution between locally produced intermediates and foreign-produced ones.

Market clearing requires

$$\begin{aligned} Y_A &= A+A^* \\ Y_B &= B+B^*. \end{aligned}$$

Let P_A and P_B denote the prices of good A and B relative to the domestic final consumption good and similarly for P_A^* and P_B^*. Given that all intermediate varieties are symmetric, in equilibrium $Y_A = Y_i$, $Y_B = Y_i^*$, $P_A = P_i$ and $P_B^* = P_i^*$.

Note that because the aggregators for producing domestic and foreign consumption goods are identical, the relative price of foreign to domestic consumption (the real exchange rate) in this model is one, and thus $P_A = P_A^*$ and $P_B = P_B^*$.

The first order conditions for intermediate-good-producing firms in this economy are identical to those in the baseline model. But we cannot immediately equate the prices P_A and P_B to the price of the final consumption good, which is normalized to one. Rather, these prices are pinned down by two conditions. First, the first-order conditions for final-good-producing firms ties the relative price of B to A to the relative quantities produced:

$$\frac{Y_B}{Y_A}=\frac{B}{A}=\frac{B^*}{A^*}=\left(\frac{P_B}{P_A}\right)^{-\varepsilon}. \tag{40}$$

Second, final-good-producing firms are competitive, so that the price of producing one unit of final consumption must equal the price of one unit of consumption (which is normalized to one). If domestic firms are producing one unit of output, then the quantities A and B must satisfy

$$\begin{aligned} G(A,B) &= 1 = 2^{\frac{1}{1-\varepsilon}}\left[A^{(\varepsilon-1)/\varepsilon}+B^{(\varepsilon-1)/\varepsilon}\right]^{\frac{\varepsilon}{\varepsilon-1}} \\ 1 &= 2^{\frac{1}{1-\varepsilon}}A\left[1+\left(\frac{B}{A}\right)^{(\varepsilon-1)/\varepsilon}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \end{aligned}$$

which, given (40), implies

$$A = 2^{\frac{-1}{1-\varepsilon}} \left[1 + \left(\frac{P_B}{P_A} \right)^{1-\varepsilon} \right]^{\frac{-\varepsilon}{\varepsilon-1}}.$$

So the cost of producing one unit of the final consumption good is

$$\begin{aligned} & P_A 2^{\frac{-1}{1-\varepsilon}} \left[1 + \left(\frac{P_B}{P_A} \right)^{1-\varepsilon} \right]^{\frac{-\varepsilon}{\varepsilon-1}} + P_i^* 2^{\frac{-1}{1-\varepsilon}} \left[1 + \left(\frac{P_B}{P_A} \right)^{1-\varepsilon} \right]^{\frac{-\varepsilon}{\varepsilon-1}} \left(\frac{P_B}{P_A} \right)^{-\varepsilon} \\ = \; & 2^{\frac{-1}{1-\varepsilon}} \left(P_A^{1-\varepsilon} + P_B^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \end{aligned}$$

If this cost is to equal to the price of consumption, which is one, then

$$P_A^{1-\varepsilon} + P_B^{1-\varepsilon} = 2. \tag{41}$$

Proposition 1 *If*

$$\frac{z_H^*}{z_H} = \left(\frac{\mu^*}{\mu} \right)^{\alpha + \frac{1-\alpha}{1+\sigma}}, \tag{42}$$

then $P_A = P_B = 1$ and allocations are independent of ε and are identical to those in the one good model in the main text.

Proof:

Bertrand competition among intermediate-good-producing firms gives the same price expressions as in the one-good model, which we reproduce here:

$$\begin{aligned} P_A &= \frac{\mu}{z_H} \left(\frac{W}{Z(1-\alpha)} \right)^{1-\alpha} \left(\frac{R}{\alpha} \right)^{\alpha} \\ P_B &= \frac{\mu^*}{z_H^*} \left(\frac{W^*}{Z(1-\alpha)} \right)^{1-\alpha} \left(\frac{R^*}{\alpha} \right)^{\alpha}. \end{aligned}$$

Taking the ratio of these two prices (and recalling that $R = R^*$), we get

$$\frac{P_B}{P_A} = \frac{\mu^*}{\mu} \left(\frac{z_H^*}{z_H} \right)^{-1} \left(\frac{W^*}{W} \right)^{(1-\alpha)}. \tag{43}$$

From the two FOCs for labor supply, we have

$$\frac{L^*}{L} = \left(\frac{W^*}{W} \right)^{1/\sigma}.$$

Thus, the ratio of foreign to domestic output is

$$\frac{Y_B}{Y_A} = \frac{z_H^*}{z_H}\left(\frac{K^*}{K}\right)^{\alpha}\left(\frac{L^*}{L}\right)^{1-\alpha} = \frac{z_H^*}{z_H}\left(\frac{K^*}{K}\right)^{\alpha}\left(\frac{W^*}{W}\right)^{(1-\alpha)/\sigma}. \tag{44}$$

Multiplying together expressions (43) and (44), we get

$$\frac{P_B}{P_A}\times\frac{Y_B}{Y_A} = \frac{\mu^*}{\mu}\left(\frac{K^*}{K}\right)^{\alpha}\left(\frac{W^*}{W}\right)^{(1-\alpha)(1+\sigma)/\sigma}. \tag{45}$$

From equation 10 in the paper at home and abroad, with a common value of $R = R^*$, we have

$$\frac{K^*}{K} = \frac{\mu}{\mu^*}\frac{P_B}{P_A}\frac{Y_B}{Y_A} = \left(\frac{K^*}{K}\right)^{\alpha}\left(\frac{W^*}{W}\right)^{(1-\alpha)(1+\sigma)/\sigma}.$$

or

$$\frac{K^*}{K} = \left(\frac{W^*}{W}\right)^{(1+\sigma)/\sigma}$$

Substituting this into (45) gives

$$\frac{P_B}{P_A}\frac{Y_B}{Y_A} = \frac{\mu^*}{\mu}\left(\frac{W^*}{W}\right)^{(1+\sigma)/\sigma}$$

or, using equation (40) to substitute out Y_B/Y_A,

$$\left(\frac{P_B}{P_A}\right)^{1-\varepsilon} = \frac{\mu^*}{\mu}\left(\frac{W^*}{W}\right)^{(1+\sigma)/\sigma}. \tag{46}$$

Now we can combine eqs. (43) and (46) to solve for $\frac{W^*}{W}$ as a function of exogenous parameters:

$$\frac{W^*}{W} = \left(\left(\frac{z_H^*}{z_H}\right)^{-(1-\varepsilon)}\left(\frac{\mu^*}{\mu}\right)^{-\varepsilon}\right)^{\frac{1}{\frac{(1+\sigma)}{\sigma}-(1-\alpha)(1-\varepsilon)}} \tag{47}$$

Now recall equation (41),

$$P_A^{1-\varepsilon} + P_B^{1-\varepsilon} = 2,$$

which can be written as

$$P_A^{1-\varepsilon}\left(1+\left(\frac{P_B}{P_A}\right)^{1-\varepsilon}\right) = 2.$$

using eq: (46) again and then substituting in eq: (47) gives

$$
\begin{aligned}
P_A^{1-\varepsilon}\left(1+\frac{\mu^*}{\mu}\left(\frac{W^*}{W}\right)^{(1+\sigma)/\sigma}\right) &= 2 \\
P_A^{1-\varepsilon}\left(1+\frac{\mu^*}{\mu}\left(\left(\frac{z_H^*}{z_H}\right)^{-(1-\varepsilon)}\left(\frac{\mu^*}{\mu}\right)^{-\varepsilon}\right)^{\frac{\frac{1+\sigma}{\sigma}}{\frac{(1+\sigma)}{\sigma}-(1-\alpha)(1-\varepsilon)}}\right) &= 2.
\end{aligned}
$$

Now substitute in the expression for $\frac{z_H^*}{z_H}$ in the statement of the Proposition, equation (42), which gives

$$P_A^{1-\varepsilon}(2) = 2,$$

which implies $P_A = 1$. equation (41) then implies $P_B = 1$.

Given $P_B = P_A = 1$, it is immediate that the budget constraints for domestic and foreign households are identical to the baseline one-good model and thus that all equilibrium allocations are identical.

6 Extended Current Account Decomposition

The current account contribution from domestic equity in equation 23 in the paper can be expressed as

$$
\begin{aligned}
\frac{\lambda_{t-1}}{1+\rho}(D_t-\rho V_t) &= \frac{\lambda_{t-1}}{1+\rho}\left[D_t-\rho\left(\left(e_t+(1+r_t^*)\right)V_{t-1}-D_t\right)\right] \\
&= \lambda_{t-1}\left(\mathbb{E}_{t-1}[D_t]-(Q_tX_t-\mathbb{E}_{t-1}[Q_tX_t])-\frac{\rho}{1+\rho}e_tV_{t-1}-\frac{\rho}{1+\rho}(1+r_t^*)V_{t-1}\right) \\
&= \lambda_{t-1}\left(\frac{r_t^*-\rho}{1+\rho}V_{t-1}-\bar{g}_tV_{t-1}-\frac{\rho}{1+\rho}e_tV_{t-1}-(Q_tX_t-\mathbb{E}_{t-1}[Q_tX_t])\right)
\end{aligned}
$$

The first two terms here relate to predictable factors. If domestic equity pays the expected return r_t^*, desired net saving is given by $\frac{r_t^*-\rho}{1+\rho}V_{t-1}$. Fo this desired saving to boost foreign asset purchases, desired saving must exceed expected growth in the value of domestic assets, $\bar{g}_tV_{t-1}$. Thus higher (expected) returns or lower expected growth will both translate into a more positive current account.

The next two terms show the impact on the current account of news shocks at t. If domestic assets pay an unexpected positive excess return between $t-1$ and t ($e_t > 0$) then there is a wealth effect on desired consumption, which reduces desired saving by $-\frac{\rho}{1+\rho}e_tV_{t-1}$. In addition, if news at t leads to more investment than was expected at $t-1$, U.S. households will finance that difference by borrowing.

The contributions from all these effects are proportional to domestic ownership of domestic equity, λ_{t-1}. An analogous decomposition applies to the foreign equity term.

Thus, the model current account can be expressed as

$$\begin{aligned} CA_t &= \left(\frac{r_t^*-\rho}{1+\rho}-\bar{g}_t\right)\left(\lambda_{t-1}V_{t-1}+\lambda_{t-1}^*V_{t-1}^*\right) \\ &-\frac{\rho}{1+\rho}\left(\lambda_{t-1}e_tV_{t-1}+\lambda_{t-1}^*e_t^*V_{t-1}^*\right) \\ &-\lambda_{t-1}(Q_tX_t-\mathbb{E}_{t-1}[Q_tX_t])-\lambda_{t-1}^*(Q_t^*X_t^*-\mathbb{E}_{t-1}[Q_t^*X_t^*]) \\ &+\frac{r_t^*-\rho}{1+\rho}B_t+\frac{1}{1+\rho}\left(\frac{W_tL_t}{H_t}-\rho\right)H_t \end{aligned} \tag{48}$$

Figure 8 plots the novel terms in the current account decomposition according to 48. It shows that the low income yield on U.S. equity in the 1990s reflected unexpectedly strong U.S. investment (see also Figure 6), and widening current account deficits during this period reflect Americans borrowing from abroad to finance that investment. Conversely, unexpectedly weak U.S. investment explains some of the high income yield on U.S. equity around the Great Recession, and the associated narrowing of the U.S. current account.

We can similarly decompose valuation effects into a predictable component versus the impact of shocks. Note that the excess return to domestic equity between $t-1$ and t can be expressed as

$$\begin{aligned} e_t &= \frac{D_t+V_t}{V_{t-1}}-(1+r_t^*) \\ &= \frac{D_t+V_t}{V_{t-1}}-\frac{\mathbb{E}_{t-1}[D_t]+(1+\bar{g}_{t-1})V_{t-1}}{V_{t-1}} \end{aligned}$$

Thus, the equity liability revaluation term can be expressed as

$$\begin{aligned} -(1-\lambda_{t-1})(V_t-V_{t-1}) &= -(1-\lambda_{t-1})(\bar{g}_{t-1}V_{t-1}+e_tV_{t-1}-D_t+\mathbb{E}_{t-1}[D_t]) \\ &= -(1-\lambda_{t-1})(\bar{g}_{t-1}V_{t-1}+e_tV_{t-1}+Q_tX_t-\mathbb{E}_{t-1}[Q_tX_t]) \end{aligned}$$

A similar expression applies for the revaluation of U.S. foreign equity assets. In this expression $\bar{g}_{t-1}V_{t-1}$ captures the expected change in asset values due to trend growth, while the other terms reflect surprise components: a positive excess return on U.S. equity inflates U.S. liabilities, as does unexpected U.S. investment.[5]

[5] For example, if households learn at t that the cost of capital moving forward r_{t+1}^* will be lower, domestic investment and the value of domestic firms will increase. And this unexpected revaluation will occur even in an economy with no output wedges ($\mu=1$), and thus no excess returns ($e_t=0$).

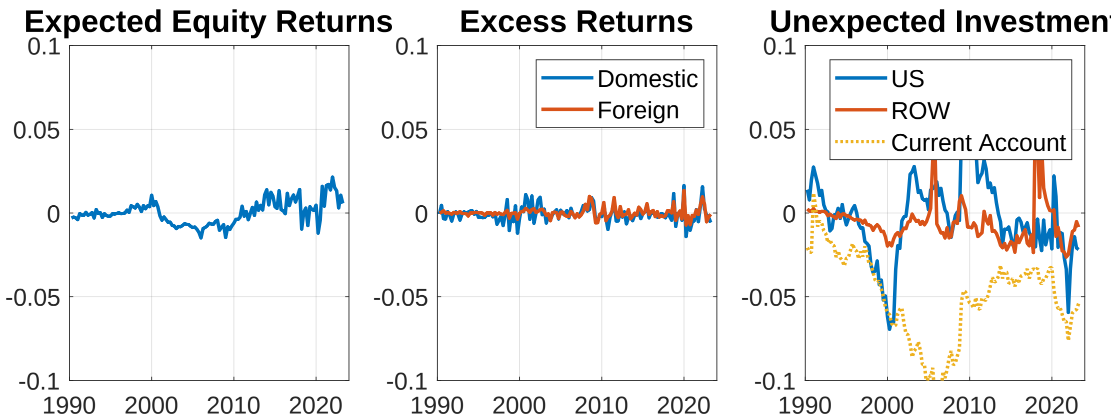

Figure 8: Alternative Current Account Decomposition. These panels plot the following components of equation (48): left panel plots the first line, middle panel plots the terms in the second line, right panel plots the terms in the third line.

Note that the expected equity return term plotted in Figure 8 is almost perfectly correlated with the return to human wealth term plotted in Figure 11 in the paper: both are approximately proportional to $r_t^* - \bar{g}_t - \rho$. However, human wealth, on average, is 6.8 times larger than the value of U.S. corporations, and thus fluctuations in $r_t^* - \bar{g}_t$ impact the current account primarily through that channel. Note also that the wealth effects associated with excess returns to equity also have only a modest impact on the current account.

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