Appendix to "The Empirical Implications of the Interest-Rate Lower Bound"*

Christopher Gust
Federal Reserve Board

Edward Herbst
Federal Reserve Board

David López-Salido
Federal Reserve Board

Matthew E. Smith[†]
Hutchin Hill Capital

January 2017

Abstract

This appendix provides additional details and results for the paper, "The Empirical Implications of the Interest-Rate Lower Bound."

*The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or any other person associated with the Federal Reserve System.

[†]This paper represents the personal work product and views of Matthew E. Smith and have neither been reviewed nor endorsed by Hutchin Hill Capital, LP, its affiliates, any funds advised by them, or any of their officers, directors, employees, agents, partners, principals, owners, or shareholders (collectively, "Hutchin Hill Parties"). No representation or warranty is made by Matthew E. Smith or any Hutchin Hill Party about the completeness or accuracy of any information provided herein, and such parties expressly disclaim all liability for any of the information contained in these materials. These materials have been prepared for educational use only and may not be relied upon for any other purpose (including without limitation any commercial purpose). These materials do not constitute an offer, or the solicitation of an offer, to buy or sell any security or financial instrument or investment advisory services. No Hutchin Hill Party shall be liable or have any obligation with respect to these slides or any reliance thereon.

1 Introduction

This appendix describes the data used to estimate the model discussed in "The Empirical Implications of the Interest Rate Lower Bound." It also shows the prior distributions for the model's parameters, the parameter estimates from the linearized version of the model, and the parameter estimates from the nonlinear model with lower measurement error. It displays the path of the output gap derived from estimating the nonlinear model and characterizes the estimated degree of nominal rigidities in terms of the frequency of price adjustment.

2 Data

Output growth is measured by quarter-to-quarter changes in the log of real GDP (chained 2005 dollars, seasonally adjusted, converted to per capita terms using the civilian non-institutional population ages 16 and over). Non-durable consumption is measured by personal consumption expenditures, and investment corresponds to fixed private investment in the National Income and Product Accounts. The inflation rate is measured as the quarter-to-quarter change in the log of the GDP deflator, seasonally adjusted. The short-term nominal interest rate is measured by quarterly averages of daily readings on the three-month U.S. Treasury bill rate, converted from an annualized yield on a discount basis to a quarterly yield to maturity. The three-month T-bill rate tracks the federal funds rate closely over our sample period, and at the end of the sample, after the FOMC established a target range from 0 to 25 basis points for the federal funds rate, the quarterly average federal funds rate and three-month T-bill rate were within a few basis point of each other.

3 Prior Distribution of the Parameters

Table 1 shows the values for the fixed parameters in the estimation. Table 2 displays the prior distribution for the estimated parameters.

Table 1: Fixed Parameters

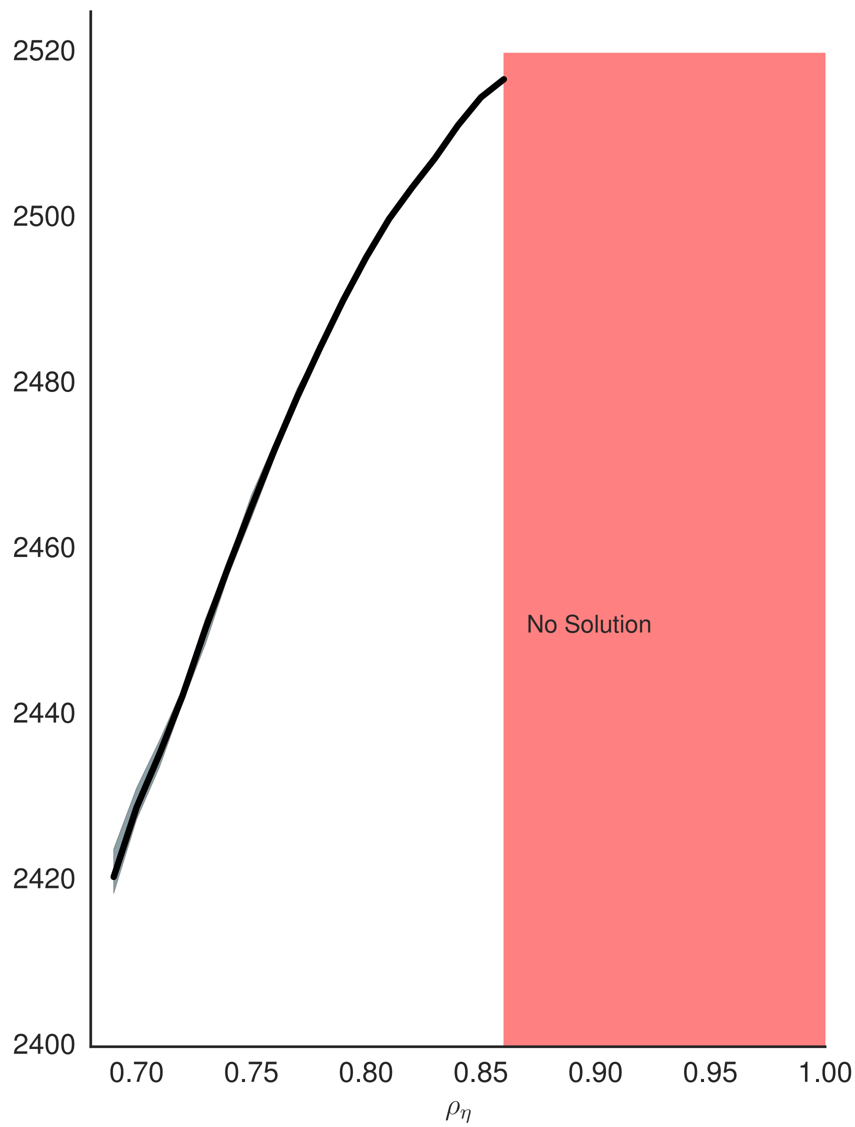
Parameter	Value	Description
δ	0.025	Depreciation of capital stock.
g	1.25	Steady state government spending ($G/Y = 0.2$)
$1/(\varepsilon_p - 1)$	0.20	Steady state net price markup.
$1/(\varepsilon_w - 1)$	0.20	Steady state net wage markup.
ψ_L	1	Disutility of Labor.
ρ_η	0.85	Persistence of the Liquidity Shock.

Table 2: Prior Distribution

Parameter	Dist.	Para(1)	Para(2)	Parameter	Dist.	Para(1)	Para(2)
			Steady State				
$100(\beta^{-1}-1)$	Gamma	0.25	0.10	$100(\bar{\Pi}-1)$	Normal	0.62	0.10
$100\ln(G_z)$	Normal	0.50	0.03	α	Normal	0.30	0.05
			Monetary Policy Rule				
ρ_R	Beta	0.60	0.20	γ_Π	Normal	1.70	0.30
γ_g	Normal	0.40	0.30	γ_x	Normal	0.40	0.30
			Endogenous Propagation				
γ	Beta	0.60	0.10	σ_L	Gamma	2.00	0.75
σ_a	Gamma	5.00	1.00	φ_I	Gamma	4.00	1.00
φ_p	Normal	100.00	25.00	$1-a$	Beta	0.50	0.15
φ_w	Normal	3000.00	5000.00	$1-a_w$	Beta	0.50	0.15
			Exogenous Processes				
ρ_g	Beta	0.60	0.20	ρ_μ	Beta	0.60	0.20
$100\sigma_g$	Inv. Gamma	0.33	2.00	$100\sigma_\mu$	Inv. Gamma	0.33	2.00
$100\sigma_Z$	Inv. Gamma	0.33	2.00	$100\sigma_R$	Inv. Gamma	0.33	2.00
$100\sigma_\eta$	Inv. Gamma	0.33	2.00				

Notes: Para (1) and Para (2) correspond to the mean and standard deviation of the Beta, Gamma, and Normal distributions and to the upper and lower bounds of the support for the Uniform distribution. For the Inverse (Inv.) Gamma distribution, Para (1) and Para (2) refer to s and ν, where $p(\sigma|\nu,s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$.

Figure 1: Likelihood Function of ρ_η



Notes. Figure shows the mean estimates (black line) of the likelihood function from the particle filter, as well as 90% bands (grey region), as a function of ρ_η, with all other parameters held fixed at their posterior mean level. The red shaded region shows the area for which the model does not solve.

4 Parameter Estimates from the Linearized Version of the Model

Table 3: Posterior Distribution of Parameter Estimates from the Linearized Version of the Model

Parameter	Mean	[05, 95]	Parameter	Mean	[05, 95]
		Steady State			
$100(\beta^{-1}-1)$	0.16	[0.07, 0.26]	$100(\bar{\Pi}-1)$	0.64	[0.56, 0.72]
$100\ln(G_z)$	0.50	[0.46, 0.55]	α	0.20	[0.17, 0.23]
		Policy Rule			
ρ_R	0.77	[0.69, 0.83]	γ_π	1.78	[1.34, 2.21]
γ_g	0.57	[0.27, 0.87]	γ_x	0.07	[0.01, 0.13]
		Endogenous Propagation			
γ	0.76	[0.68, 0.83]	σ_L	2.05	[1.03, 3.36]
σ_a	5.27	[3.79, 7.04]	φ_I	3.51	[2.08, 5.22]
φ_p	102.25	[64.01, 142.54]	$1-a$	0.64	[0.44, 0.82]
φ_w	5102.38	[2025.78, 9502.62]	$1-a_w$	0.55	[0.33, 0.76]
		Exogenous Processes			
ρ_g	0.70	[0.37, 0.94]	$100\sigma_g$	0.15	[0.12, 0.20]
ρ_μ	0.73	[0.61, 0.84]	$100\sigma_\mu$	9.12	[4.84, 14.80]
$100\sigma_\eta$	0.50	[0.31, 0.73]	$100\sigma_Z$	0.50	[0.31, 0.76]
$100\sigma_R$	0.17	[0.13, 0.21]			

Notes. The table reports the mean, fifth, and ninety-fifth percentiles of the posterior distribution under the linearized model. The model was estimated using a Sequential Monte Carlo algorithm tailored towards linearized DSGE models; see Herbst and Schorfheide (2015).

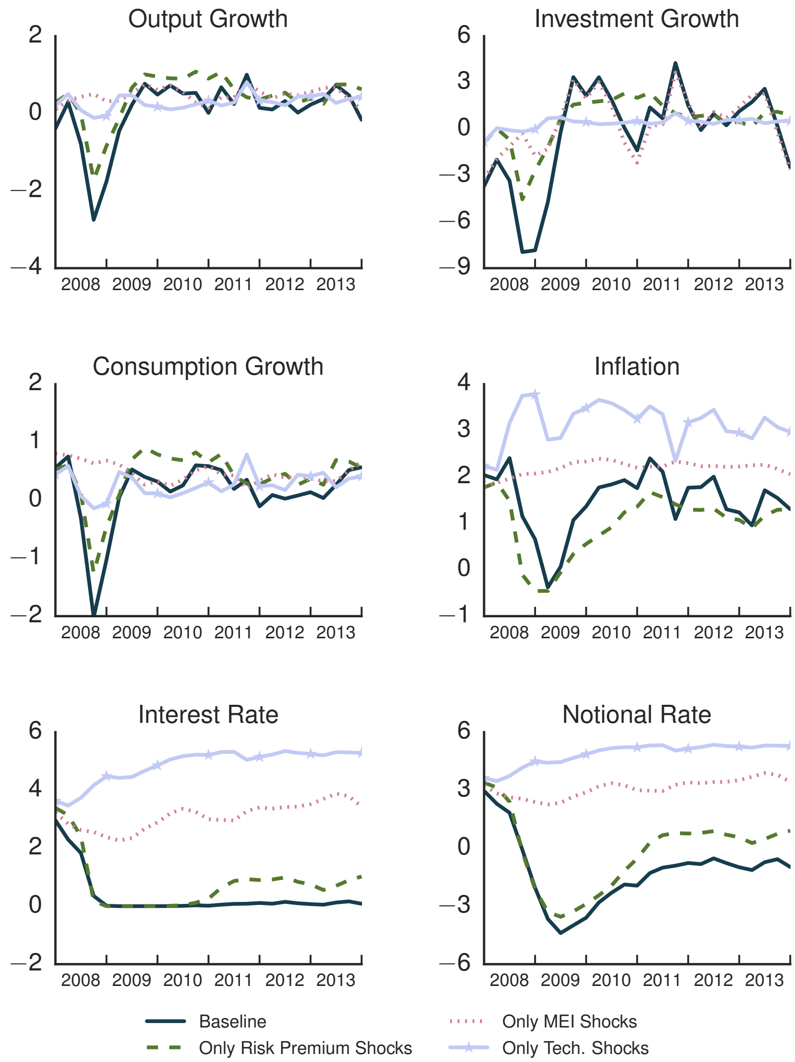
5 Estimates Using Low Measurement Error

Table 4: Posterior Distribution of Parameter Estimates Using Low Measurement Error ($m_e = 0.1$)

Parameter	Mean	[05, 95]	Parameter	Mean	[05, 95]
		Steady State			
$100(\beta^{-1}-1)$	0.14	[0.07, 0.23]	$100(\bar{\Pi}-1)$	0.62	[0.55, 0.70]
$100\log(G_z)$	0.50	[0.44, 0.54]	α	0.18	[0.16, 0.21]
		Policy Rule			
ρ_R	0.78	[0.71, 0.82]	γ_Π	1.60	[1.21, 2.01]
γ_g	0.67	[0.40, 0.95]	γ_x	0.24	[0.11, 0.46]
		Endogenous Propagation			
γ	0.67	[0.61, 0.72]	σ_L	2.00	[0.95, 3.30]
σ_a	5.64	[4.04, 7.55]	φ_I	3.95	[2.74, 5.42]
φ_p	77.41	[38.31, 116.64]	$1-a$	0.15	[0.07, 0.26]
φ_w	1287.68	[275.41, 3100.21]	$1-a_w$	0.34	[0.16, 0.53]
		Exogenous Processes			
ρ_G	0.71	[0.39, 0.97]	$100\sigma_G$	0.17	[0.13, 0.23]
ρ_{μ_I}	0.72	[0.54, 0.86]	$100\sigma_{\mu_I}$	3.81	[2.68, 5.43]
$100\sigma_\eta$	0.42	[0.33, 0.52]	$100\sigma_Z$	0.94	[0.73, 1.20]
$100\sigma_R$	0.16	[0.13, 0.19]			

Notes. The table reports the mean, fifth, and ninety-fifth percentiles of the posterior distribution for the model with low measurement error using an MCMC chain of length 75,000 after a burn in period of 5,000 draws.

Figure 2: The Contribution of the Estimated Shocks to the Great Recession
Low Measurement Error ($m_e = 0.1$)



Notes. Figure shows counterfactual trajectories of output growth (upper left panel), investment growth (upper right panel), consumption growth (middle left panel), inflation (middle right panel), the interest rate (bottom left panel), and the notional rate (bottom right panel). The trajectories are computed using smoothed estimates of the states in 2007:Q4 as initial conditions and the smoothed shock estimates from 2008:Q1 to 2014:Q1 for only liquidity shocks (dashed lines), only MEI shocks (dotted line), only technology shocks (line with triangles), and all of the structural shocks (solid line).

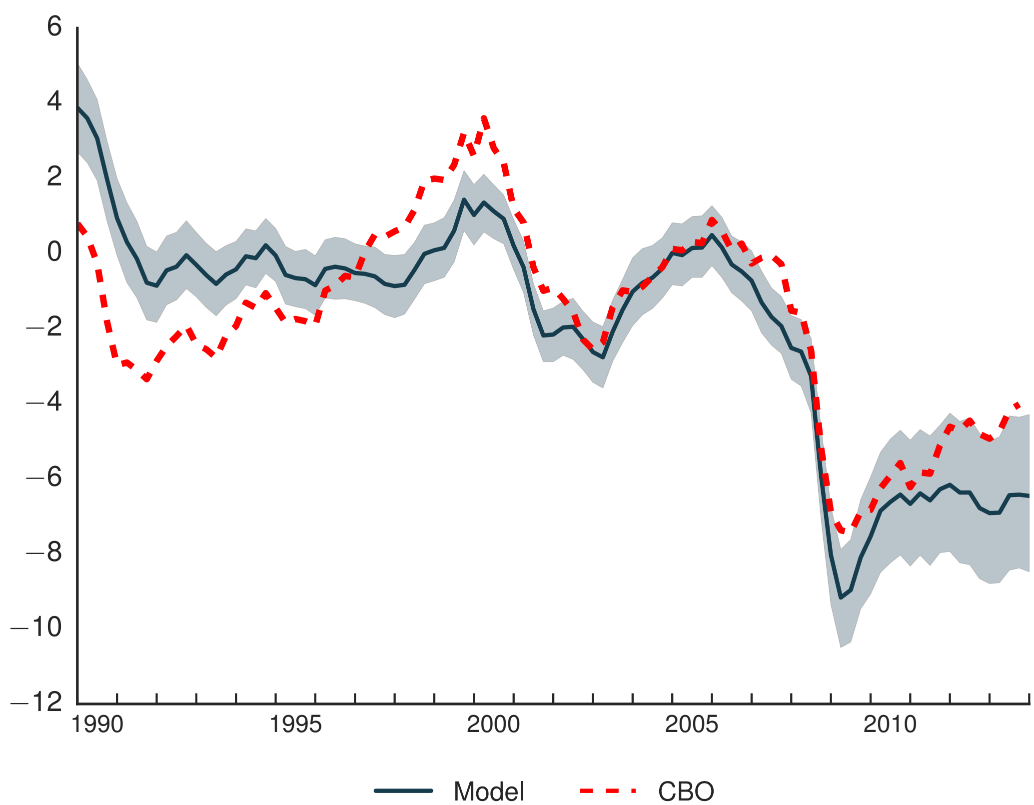
6 The Paths of the Output Gap and Additional Shocks

Figure 3 plots the posterior median and 68 percent pointwise credible sets for model-implied output gap,

$$x_t^g = \alpha \log u_t + (1-\alpha) \log \left(\frac{N_t}{N} \right),$$

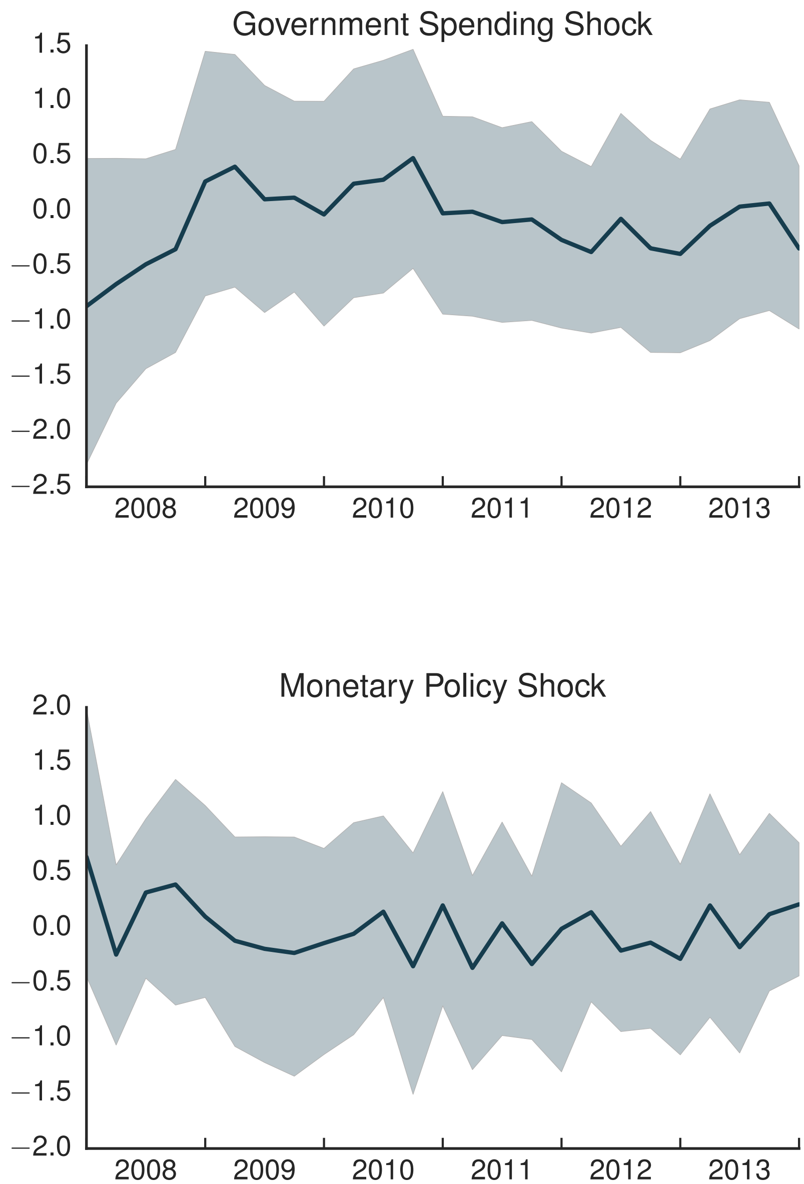
along with the Congressional Budget Office's (CBO) estimate of the output gap from February 2014. This vintage of the CBO output gap is chosen to be consistent with the end of our sample. The model's estimated output gap shares the same general features as the one estimated by the CBO, though the CBO output gap often lies outside of the 68 percent bands associated with the model estimate. In particular, the CBO's output gap closes more quickly in the aftermath of the Great Recession.

Figure 3: The Path of the Estimated Output Gap



Notes. Figure shows the time series of the mean (solid line) and 68% bands (shaded region) of the smoothed distributions of the model's output gap. The red dashed line shows the February 2014 vintage of the CBO output gap, constructed using real potential GDP from the CBOs February 2014 report, The Budget and Economic Outlook: 2014 to 2024, and real GDP from Table 1.1.6 of the BEAs February 2014 release of National Accounts (NIPA) data.

Figure 4: The Path of Two Estimated Shocks During the Great Recession



Notes. Figure shows the time series of the mean (solid line) and 68% bands (shaded region) of the smoothed distributions of $\ln(g_t)$ (top panel) and $\epsilon_{R,t}$ (bottom panel). Both are normalized by their unconditional standard deviations.

7 Rotemberg and Calvo Estimates of Nominal Rigidities

Under Calvo, the slope of the price inflation equation is given by:

$$\kappa_p^{Calvo} = \frac{(1-\xi_p\beta)(1-\xi_p)}{\xi_p(1+\iota_p\beta)},$$

where ξ_p corresponds to the probability of changing prices and ι_p represents the degree of price indexation. Evaluating this expression at $\xi_p = 0.787$, $\iota_p = 0.131$, and $\beta = 0.9986$, the posterior medians estimated by Justiniano, Primiceri, and Tambalotti (2011) yields $\kappa_p^{Calvo} = 0.0512$. Under Rotemberg contracts, the slope coefficient is:

$$\frac{\varepsilon_p - 1}{(1+\beta(1-a))\varphi_p}$$

Setting $\beta = 0.9986$, $(1-a) = \iota_p = 0.131$, and $\varepsilon_p - 1 = \frac{1}{0.180}$ as in Justiniano, Primiceri, and Tambalotti (2011) implies a parameter of price adjustment cost of:

$$\varphi_p = \frac{\varepsilon_p - 1}{(1+\beta(1-a))\kappa_p^{Calvo}} \approx 93.5.$$

In comparison, our posterior mean estimate of this parameter is 100, and 93.5 is well within the 90 percent credible band.

Regarding the slope of the wage inflation equation, under Calvo, the expression is given by:

$$\kappa_w^{Calvo} = \frac{(1-\xi_w\beta)(1-\xi_w)}{\xi_w(1+\beta)(1+\nu(1+\frac{1}{\lambda_w}))}.$$

If this expression is evaluated at the posterior medians estimated by Justiniano, Primiceri, and Tambalotti (2011) in which $\xi_w = 0.777$, $\lambda_w = 0.144$ (which corresponds to $\varepsilon_w - 1 = \frac{1}{\lambda_w}$, $\varepsilon_w \simeq 7.9$), and $\nu = 4.492$, then $\kappa_w^{Calvo} \approx 0.0009$, which implies wage inflation responds very little to labor market slack.

The equivalent slope coefficient using Rotemberg contracts is given by:

$$\frac{\varepsilon_w - 1}{\varphi_w} mc \frac{(1-\alpha)}{\frac{c}{y}},$$

where the steady state marginal costs is given by: $mc = \frac{\varepsilon_p - 1}{\varepsilon_p}$. To relate this to Calvo contracts, we use:

$$\varphi_w = \frac{\varepsilon_w - 1}{\kappa_w^{Calvo}} \frac{(1-\alpha)mc}{\frac{c}{y}}.$$

The estimates of Justiniano, Primiceri, and Tambalotti (2011) imply that $\frac{(1-\alpha)}{\frac{c}{y}} \simeq 1.5$ and $mc \simeq 0.84$ implying extremely high nominal wage adjustment costs as $\frac{\varphi_w}{1000} \approx 9936$. This estimate of wage adjustment costs is outside the upper end of the 90 percent credible set and is substantially higher than our posterior mean estimate.

References

Herbst, E. and F. Schorfheide (2015): *Bayesian Estimation of DSGE Models*, Princeton: Princeton University Press.

Justiniano, A., G. Primiceri, and A. Tambalotti (2011): "Investment Shocks and the Relative Price of Investment," *Review of Economic Dynamics*, 14, 101–121.