Long-Term Securities and Banking Crises

Supplemental Appendix

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Appendix B. No-Run Equilibrium System

The no-run equilibrium is characterized by the following 36 equations in 36 variables $\{R_t^n,\ R_t,\ R_t^l,\ R_t^c,\ R_t^k,\ Q_t^l,\ Q_t^c,\ Q_t^k,\ B_{ht},\ B_{bt},\ B_t^c,\ B_{ht}^c,\ B_{bt}^c,\ S_t,\ S_{ht},\ S_{bt},\ N_t,\ \phi_t,\ W_t,\ Z_t,\ T_t,\ Y_t,\ C_t,\ K_t,\ L_t,\ I_t,\ p_t^*,\ \Pi_t,\ P_{wt},\ \Delta_t,\ \Gamma_t^a,\ \Gamma_t^b, \tau_t^l,\ M_t,\ \lambda_t,\ \nu_t\}$ for $t \geq 1$. The following variables $\{N_t,\ B_{bt},\ S_t, S_{bt},\ B_t^c,\ B_{bt}^c,\ Q_t^l,\ Q_t^c,\ Q_t^k,\ C_t,\ \Delta_t,\ R_t^n,\ A_t,\ v_t^m,\ v_t^\zeta,\ v_t^p,\ \tau_t^l\}$ are predetermined variables with their initial values at $t = 0$ being exogenously given. The shock innovations $\{\varepsilon_t^a, \varepsilon_t^p, \varepsilon_t^m, \varepsilon_t^\zeta\}$ are also exogenously given.

1) Household's first-order condition for deposit,

(B1)
$$\mathbb{E}_t \Lambda_{t,t+1} R_{t+1} = 1,$$

where

$$\Lambda_{t,t+1} \equiv \beta v_{t+1}^\zeta \frac{C_t - hC_{t-1}}{C_{t+1} - hC_t},$$

and

$$\ln v_t^\zeta = \rho_\zeta \ln v_{t-1}^\zeta + \varepsilon_t^\zeta.$$

2) Household's first-order condition for long-term government bonds,

(B2)
$$1 + \kappa \left(\frac{B_{ht}}{B_t} - \eta_B \right) = \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^l.$$

3) Household's first-order condition on firm equity,

(B3)
$$1 + \kappa \left(\frac{S_{ht}}{S_t} - \eta_S \right) = \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^k.$$

4) Household's first-order condition for long-term investment bonds,

(B4)
$$1 + \kappa \left(\frac{B_{ht}^c}{B_t^c} - \eta_C \right) = \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^c.$$

5) Labor supply,

(B5)
$$\frac{1}{C_t - hC_{t-1}} W_t = \chi L_t^\varphi.$$

6) Law of motion of bank net worth,

(B6)
$$N_t = \sigma\Bigg[\left(R_t^k - R_t\left(1+\frac{\psi}{2}\frac{Q_{t-1}^k S_{b,t-1}}{N_{t-1}}\right)\right)Q_{t-1}^k S_{b,t-1} + \left(R_t^l - R_t\left(1+\tau_{t-1}^l\right)\right)Q_{t-1}^l B_{b,t-1} + \left(R_t^c - R_t\right)Q_{t-1}^c B_{b,t-1}^c + R_t N_{t-1}\Bigg] + \omega N_{t-1}.$$

7) Bank's no-arbitrage condition 1,

$$\mathbb{E}_t\Omega_{t+1}\left(R_{t+1}^k - R_{t+1}\left(1+\psi\frac{Q_t^k S_{bt}}{N_t}\right)\right) = \lambda_t\theta, \tag{B7}$$

where
$$\Omega_{t,t+1} = \Lambda_{t,t+1}\left[(1-\sigma)+\sigma\phi_{t+1}\right].$$

8) Bank's no-arbitrage condition 2,

$$\mathbb{E}_t\Omega_{t+1}\left(R_{t+1}^l - R_{t+1}(1+\tau_t^l)\right) = \lambda_t\theta, \tag{B8}$$

9) Bank's no-arbitrage condition 3,

$$\mathbb{E}_t\Omega_{t+1}\left(R_{t+1}^c - R_{t+1}\right) = \lambda_t\theta, \tag{B9}$$

10) Marginal value of bank net worth,

$$\phi_t = \frac{\theta\mathbb{E}_t\Omega_{t+1}R_{t+1}\left(1+\frac{\psi}{2}\left(\frac{Q_t^k S_{bt}}{N_t}\right)^2\right)}{\theta - \mathbb{E}_t\Omega_{t+1}(R_{t+1}^l - R_{t+1}(1+\tau_t^l))}. \tag{B10}$$

11) Borrowing constraint for banks,

$$\theta Q_t^k S_{bt} + \theta Q_t^l B_{bt} + \theta Q_t^c B_{bt}^c \leq \phi_t N_t, \tag{B11}$$

with equality holds if $\mathbb{E}_t\Omega_{t,t+1}\left(R_{t+1}^l - R_{t+1}(1+\tau_t^l)\right) > 0$.

12) Capital producers optimization condition 1,

$$M_t = \nu_t F_{I,t}, \tag{B12}$$

where

$$F_{I,t} = 1 - \Omega_k \left(\frac{I_t}{K_t} - \delta \right).$$

13) Capital producers optimization condition 2,

$$Q_t^k - \nu_t = \mathbb{E}_t \Lambda_{t,t+1} \left[(1-\delta) Q_{t+1}^k - \nu_{t+1} \left(1 - \delta + F_{K,t+1} \right) \right], \tag{B13}$$

where

$$F_{K,t} = \frac{\Omega_k}{2} \left(\frac{I_t^2}{K_t^2} - \delta^2 \right).$$

14) Capital producers optimization condition 3,

$$M_t Q_t^c = \mathbb{E}_t \Lambda_{t,t+1} \left[1 + \rho_c M_{t+1} Q_{t+1}^c \right] \Pi_{t+1}^{-1}. \tag{B14}$$

15) Loan-in-advance constraint,

$$I_t \leq Q_t^c (B_t^c - \rho_c B_{t-1}^c \Pi_t^{-1}), \tag{B15}$$

with equality holds if $M_t > 1$.

16) Return on investment bonds,

$$R_t^c = \frac{1 + \rho_c Q_t^c}{Q_{t-1}^c} \Pi_t^{-1}. \tag{B16}$$

17) Law of motion of capital,

$$K_{t+1} = (1-\delta) K_t + \left[I_t - \frac{\Omega_k}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t \right]. \tag{B17}$$

18) Production function,

$$\Delta_t Y_t = A_t (K_t)^\alpha L_t^{1-\alpha}, \tag{B18}$$

where

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_t^a.$$

19) Labor demand,

$$W_t = P_{wt} (1-\alpha) \frac{Y_t}{L_t} \Delta_t. \tag{B19}$$

20) Profits of capital,

$$Z_t = P_{wt}\alpha\frac{Y_t}{K_t}\Delta_t. \tag{B20}$$

21) Real return on equity,

$$R_t^k = \frac{Z_t + (1-\delta)Q_t^k}{Q_{t-1}^k}. \tag{B21}$$

22) Optimal pricing of retailers,

$$p_t^* = \frac{\varepsilon}{\varepsilon - 1}\frac{\Gamma_t^a}{\Gamma_t^b}. \tag{B22}$$

23) Numerator of the pricing rule,

$$\Gamma_t^a = P_{wt}Y_t + \mathbb{E}_t\gamma\Lambda_{t,t+1}\Pi^{-\varepsilon}\Pi_{t+1}^{\varepsilon}\Gamma_{t+1}^a. \tag{B23}$$

24) Denominator of the pricing rule,

$$\Gamma_t^b = (1-\tau_t^s)Y_t + \mathbb{E}_t\gamma\Lambda_{t,t+1}\Pi^{1-\varepsilon}\Pi_{t+1}^{\varepsilon-1}\Gamma_{t+1}^b, \tag{B24}$$

where

$$\tau_t^s = 1 - \exp\left[-\frac{v_t^p}{\kappa_\pi} + \ln\left(\frac{\varepsilon}{\varepsilon-1}\right)\right],$$

$\kappa_\pi = (1-\beta\gamma)(1-\gamma)/\gamma$, and

$$v_t^p = \rho_p v_{t-1}^p + \varepsilon_t^p + \rho_{ma}\varepsilon_{t-1}^p.$$

25) Inflation and the pricing rule,

$$1 = \left[\gamma\left(\frac{\Pi}{\Pi_t}\right)^{1-\varepsilon} + (1-\gamma)p_t^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{B25}$$

26) Price dispersion,

$$\Delta_t = (1-\gamma)p_t^{*-\varepsilon} + \gamma\left(\frac{\Pi}{\Pi_t}\right)^{-\varepsilon}\Delta_{t-1}. \tag{B26}$$

27) Monetary policy,

$$R_t^n = \rho_r R_{t-1}^n + (1-\rho_r)\left(R^n + \phi_\pi \ln \frac{\Pi_t}{\Pi} + \phi_y \ln \frac{Y_t}{Y}\right) + v_t^m, \tag{B27}$$

where

$$v_t^m = \rho_m v_{t-1}^m + \varepsilon_t^m.$$

28) Real interest rate,

$$R_t = \frac{R_{t-1}^n}{\Pi_t}. \tag{B28}$$

29) Government budget constraint,

$$Q_{t-1}^l B_{t-1} R_t^l = (T_t - G_t) + Q_t^l B_t. \tag{B29}$$

where

$$G_t = G,\ t \geq 0;\quad B_t = B,\ t \geq 0.$$

30) Real return on long-term government bonds,

$$R_t^l = \frac{1 + \rho_l Q_t^l}{Q_{t-1}^l} \Pi_t^{-1}. \tag{B30}$$

31) Macroprudential policy tax on long-term government bonds,

$$\tau_t^l = \tau^l + \phi_l (R_t^n - R^n). \tag{B31}$$

32) Resource constraint,

$$Y_t = C_t + I_t + G_t. \tag{B32}$$

33) Issuance of equity claims,

$$K_t = S_{t-1}. \tag{B33}$$

34) Market clearing for firm equity,

$$S_t = S_{ht} + S_{bt}. \tag{B34}$$

35) Market clearing for long-term bonds,

$$B_t = B_{ht} + B_{bt}. \tag{B35}$$

36) Market clearing for investment bonds,

$$B_t^c = B_{ht}^c + B_{bt}^c. \tag{B36}$$

Appendix C. Steady State Equilibrium System

The steady-state values of Π, B, τ^l are exogenously given policy targets. The rest of the variables are determined by the following system of equations.

1) Household's first-order condition for deposit,

$$\beta R = 1. \tag{C1}$$

2) Household's first-order condition for long-term government bonds,

$$1 + \kappa \left(\frac{B_h}{B} - \eta_B \right) = \beta R^l. \tag{C2}$$

3) Household's first-order condition on firm equity,

$$1 + \kappa \left(\frac{S_h}{S} - \eta_S \right) = \beta R^k. \tag{C3}$$

4) Household's first-order condition for long-term investment bonds,

$$1 + \kappa \left(\frac{B_h^c}{B^c} - \eta_C \right) = \beta R^c. \tag{C4}$$

5) Labor supply,

$$\frac{1}{(1-h)C} W = \chi L^{\varphi}. \tag{C5}$$

6) Law of motion of bank net worth,

$$\begin{aligned} N = \sigma \Bigg[& \left(R^k - R \left(1 + \frac{\psi}{2} \frac{Q^k S_b}{N} \right) \right) Q^k S_b + \left(R^l - R \left(1 + \tau^l \right) \right) Q^l B_b \\ & + \left(R^c - R \right) Q^c B_b^c + RN \Bigg] + \omega N. \end{aligned} \tag{C6}$$

7) Bank's no-arbitrage condition 1,

$$\Omega \left(R^k - R \left(1 + \psi \frac{Q^k S_b}{N} \right) \right) = \lambda \theta, \tag{C7}$$

where

$$\Omega = \beta(1 - \sigma + \sigma\phi). \tag{C8}$$

8) Bank's no-arbitrage condition 2,

$$\Omega \left(R^l - R(1 + \tau^l) \right) = \lambda\theta. \tag{C9}$$

9) Bank's no-arbitrage condition 3,

$$\Omega \left(R^c - R \right) = \lambda\theta. \tag{C10}$$

10) Marginal value of bank net worth,

$$\phi(\theta - \Omega(R^l - R(1 + \tau^l))) = \theta\Omega R \left(1 + \frac{\psi}{2} \left(\frac{Q^k S_b}{N} \right)^2 \right). \tag{C11}$$

11) Borrowing constraint for banks,

$$\theta Q^k S_b + \theta Q^l B_b + \theta Q^c B_b^c = \phi N. \tag{C12}$$

12) Capital producers optimality condition 1,

$$M = \nu. \tag{C13}$$

13) Capital producers optimality condition 2,

$$Q^k = \nu. \tag{C14}$$

14) Capital producers optimality condition 3,

$$MQ^c = \beta \left[1 + \rho_c M Q^c \right] \Pi^{-1}. \tag{C15}$$

15) Loan-in-advance constraint,

$$I = Q^c(B^c - \rho_c B^c \Pi^{-1}). \tag{C16}$$

16) Return on investment bonds,

$$R^c = \frac{1 + \rho_c Q^c}{Q^c} \Pi^{-1}. \tag{C17}$$

17) Aggregate investment,

$$\delta K = I. \tag{C18}$$

18) Production function,

$$Y = A K^\alpha L^{1-\alpha}. \tag{C19}$$

19) Labor demand,

$$W = P_w (1-\alpha) \frac{Y}{L}. \tag{C20}$$

20) Profits of capital,

$$Z = P_w \alpha \frac{Y}{K}. \tag{C21}$$

21) Real return on equity,

$$R^k = \frac{Z + (1-\delta) Q^k}{Q^k}. \tag{C22}$$

22) Optimal pricing of retailers,

$$p^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\Gamma^a}{\Gamma^b}. \tag{C23}$$

23) Numerator of the pricing rule,

$$\Gamma^a = \frac{1}{1 - \beta\gamma} P_w Y. \tag{C24}$$

24) Denominator of the pricing rule,

$$\Gamma^b = \frac{1}{1 - \beta\gamma} (1 - \tau^s) Y. \tag{C25}$$

where

$$\tau^s = \frac{1}{1-\varepsilon}.$$

25) Inflation and the pricing rule,

$$p^* = 1 = P_w. \tag{C26}$$

26) Price dispersion,

$$\Delta = 1. \tag{C27}$$

27) Real interest rate,

$$R = \frac{R^n}{\Pi}. \tag{C28}$$

28) Government budget constraint,

$$Q^l B(R^l - 1) = (T - G). \tag{C29}$$

29) Real return on long-term government bonds,

$$R^l = \frac{1 + \rho_l Q^l}{Q^l} \Pi^{-1}. \tag{C30}$$

30) Resource constraint,

$$Y = C + I + G. \tag{C31}$$

31) Issuance of equity claims,

$$K = S. \tag{C32}$$

32) Market clearing for firm equity,

$$S = S_h + S_b. \tag{C33}$$

33) Market clearing for long-term bonds,

$$B = B_h + B_b. \tag{C34}$$

34) Market clearing for investment bonds,

$$B^c = B^c_h + B^c_b. \tag{C35}$$

Appendix D. Calibration

We need the following calibration targets: $\{R, R^k, R^l, L, B_h/B, Q^l B_b/Asset, Q^c B_b^c/Asset, Q^l B/Y, Lev\}$, where $Asset = Q^l B_b + Q^c B_b^c + Q^k S_b$ and Lev is bank leverage. The procedure for calculating the steady state and the calibrated parameters are as follows:

1) We calibrate the model without macroprudential policy: $\tau^l = 0$.
2) From (C1), we obtain $R = 1/\beta$. From (C26), we obtain $p^* = 1$. We normalize $P_w = 1$. Combining with (C23), (C24), and (C25), we obtain $\tau^s = 1/(1-\varepsilon)$. From (C27), we obtain $\Delta = 1$.
3) Combining $R = 1/\beta$ and the calibration targets of $R^k - R$ and $R^l - R$, we obtain R^k and R^l. Using (C10), we have $R^c = R^l$.
4) From (C17), we derive that $Q^c = 1/(R^c\Pi - \rho_c)$. From (C30), we obtain $Q^l = 1/(R^l\Pi - \rho_l)$. From (C14), $M = \frac{\beta}{(\Pi-\beta\rho_c)Q^c}$. We then have $Q^k = M$ from (C13). Using (C22), we obtain $Z = (R^k - 1 + \delta)Q^k$.
5) The value of Y/K then follows from (C21).

$$\frac{Y}{K} = \frac{Z}{\alpha}.$$

Combining (C19) and (C21), we obtain

$$\frac{K}{L} = \left(\frac{\alpha}{Z}\right)^{\frac{1}{1-\alpha}}.$$

It follows from (C19) and (C20) that

$$W = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}.$$

6) Given the calibration target $L = 0.33$ and the value of K/L, one can obtain the value of K and Y. From (C32), $S = K$.
7) From (C18), we have $I = \delta K$. From (C16), we have

$$B^c = \frac{\delta K \Pi}{Q^c(\Pi - \rho_c)}.$$

8) Using the calibration targets $Q^l B/Y$, we can obtain the value of $Q^l B$ and hence the value of B. Using the calibration target B_h/B, we get B_h and B_b. Using the calibration G/Y, we get the value of G. We then obtain T from (C29).

9) Denote the total asset value of the banking sector by $Asset = Q^l B_b + Q^c B_b^c + Q^k S_b$. Using the calibration target $Q^l B_b/Asset$, we have $Asset = Q^l B_b/(Q^l B_b/Asset)$. Then using the calibration target $Q^c B_b^c/Asset$, we have $Q^c B_b^c = (Q^c B_b^c/Asset) \cdot Asset$ and $Q^k S_b = (1 - Q^l B_b/Asset - Q^c B_b^c/Asset) \cdot Asset$. We then obtain the value of B_b^v and S_b. From the market clearing conditions, we have B_h^v and S_h.

10) Using the bank's leverage as a calibration target, we have $N = Asset/Lev$. Using (C7), we obtain:

$$\psi = \frac{R^k - R^l}{R\frac{Q^k S_b}{N}}.$$

Using (C6), we can obtain the value ω:

$$\omega = 1 - \sigma N^{-1}\left[\left(R^k - R\left(1 + \frac{\psi}{2}\frac{Q^k S_b}{N}\right)\right) Q^k S_b + \left(R^l - R\right) Q^l B_b + (R^c - R)\, Q^c B_b^c + RN\right].$$

Using (C12), we can derive

$$\frac{\phi}{\theta} = Lev.$$

Substituting it into (C11) yields the value of θ:

$$\theta = \frac{\beta(1-\sigma)\left[\frac{\phi}{\theta}\left(R^l - R\right) + R\left(1 + \frac{\psi}{2}\left(\frac{Q^k S_b}{N}\right)^2\right)\right]}{\frac{\phi}{\theta} - \frac{\phi}{\theta}\beta\sigma\left[\frac{\phi}{\theta}\left(R^l - R\right) + R\left(1 + \frac{\psi}{2}\left(\frac{Q^k S_b}{N}\right)^2\right)\right]}.$$

The value of ϕ is hence determined.

11) Using the first-order conditions (C2), (C3) and (C4), one can get

$$\eta_B = \frac{B_h}{B} + \frac{1 - \beta R^l}{\kappa},$$
$$\eta_C = \frac{B_h^c}{B^c} + \frac{1 - \beta R^c}{\kappa},$$
$$\eta_S = \frac{S_h}{S} + \frac{1 - \beta R^k}{\kappa},$$

where we calibrate κ to match the responsiveness of long-term bond yield to monetary policy shocks.

12) From (C31), we can obtain the value of C from

$$C = Y - I - G.$$

We can obtain the value of χ from (C5):

$$\chi = \frac{1}{1-h}\frac{W}{C}L^{-\varphi}.$$

Appendix E. Amplification Effects for Other Shocks

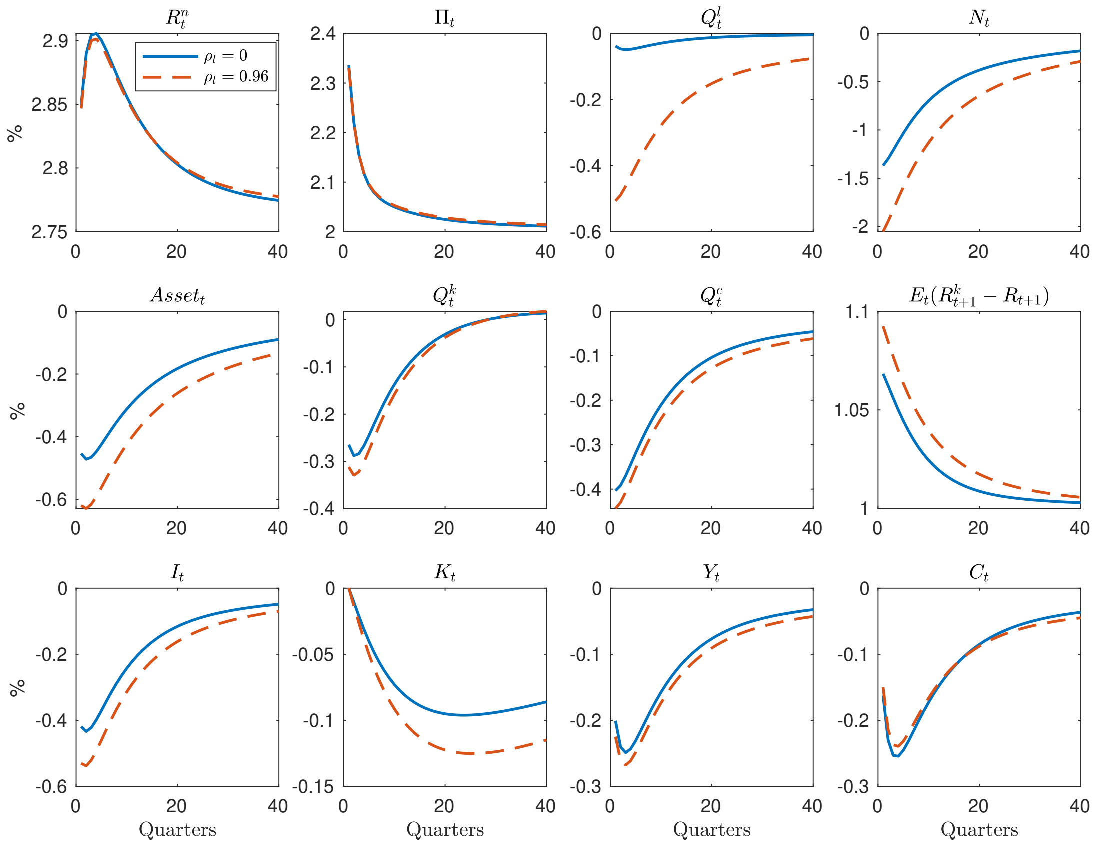

Figure E1. Technology shocks for $\rho_l = 0$ and $\rho_l = 0.96$.

Note: This figure plots the impulse responses of selected variables to a one-standard deviation negative technology shock for $\rho_l = 0$ and $\rho_l = 0.96$. The total bank asset is defined by $Asset_t = Q_t^k S_{bt} + Q_t^l B_{bt} + Q_t^c B_{bt}^c$. Nominal interest rate R_t^n, inflation Π_t, and excess return on capital $\mathbb{E}_t(R_{t+1}^k - R_{t+1})$ are in annualized percentage points. The rest variables are in percentage deviation from steady states.

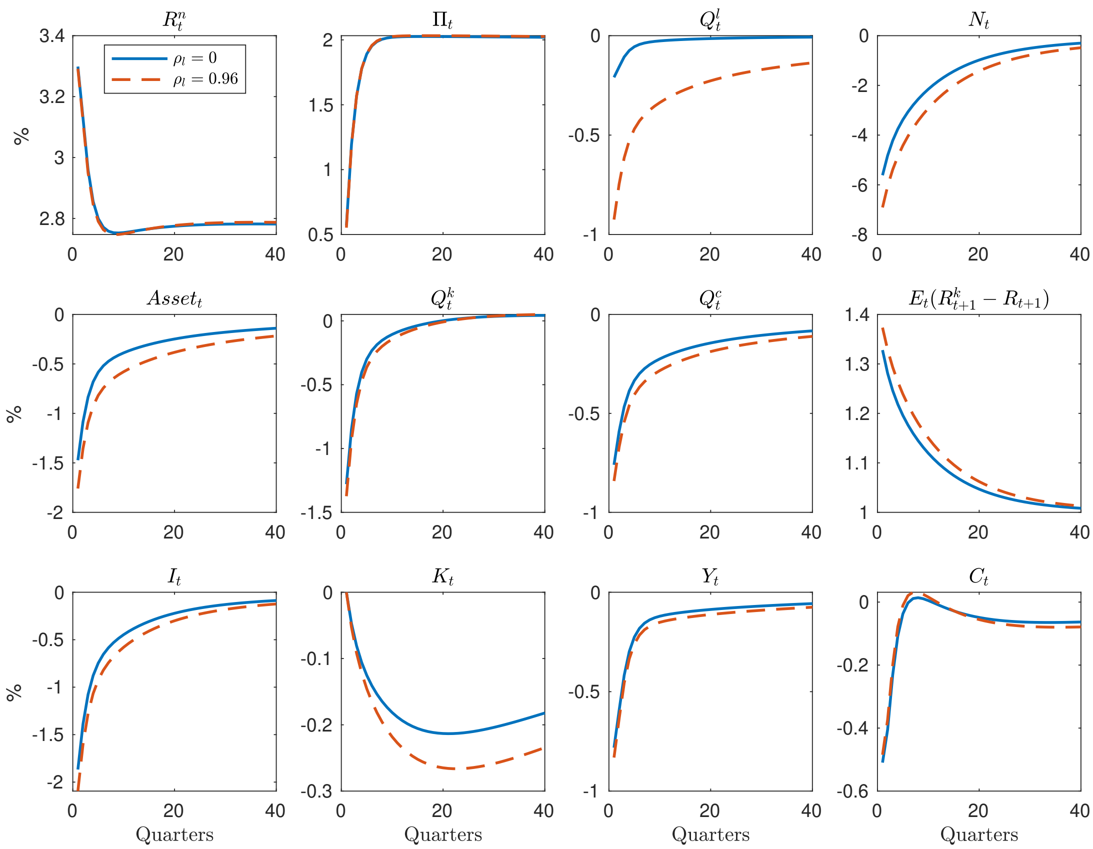

Figure E2. Monetary policy shocks for $\rho_l = 0$ and $\rho_l = 0.96$

Note: This figure plots the impulse responses of selected variables to a 25-bp positive monetary policy shock for $\rho_l = 0$ and $\rho_l = 0.96$. The total bank asset is defined by $Asset_t = Q_t^k S_{bt} + Q_t^l B_{bt} + Q_t^c B_{bt}^c$. Nominal interest rate R_t^n, inflation Π_t, and excess return on capital $\mathbb{E}_t(R_{t+1}^k - R_{t+1})$ are in annualized percentage points. The rest of variables are in percentage deviation from steady states.

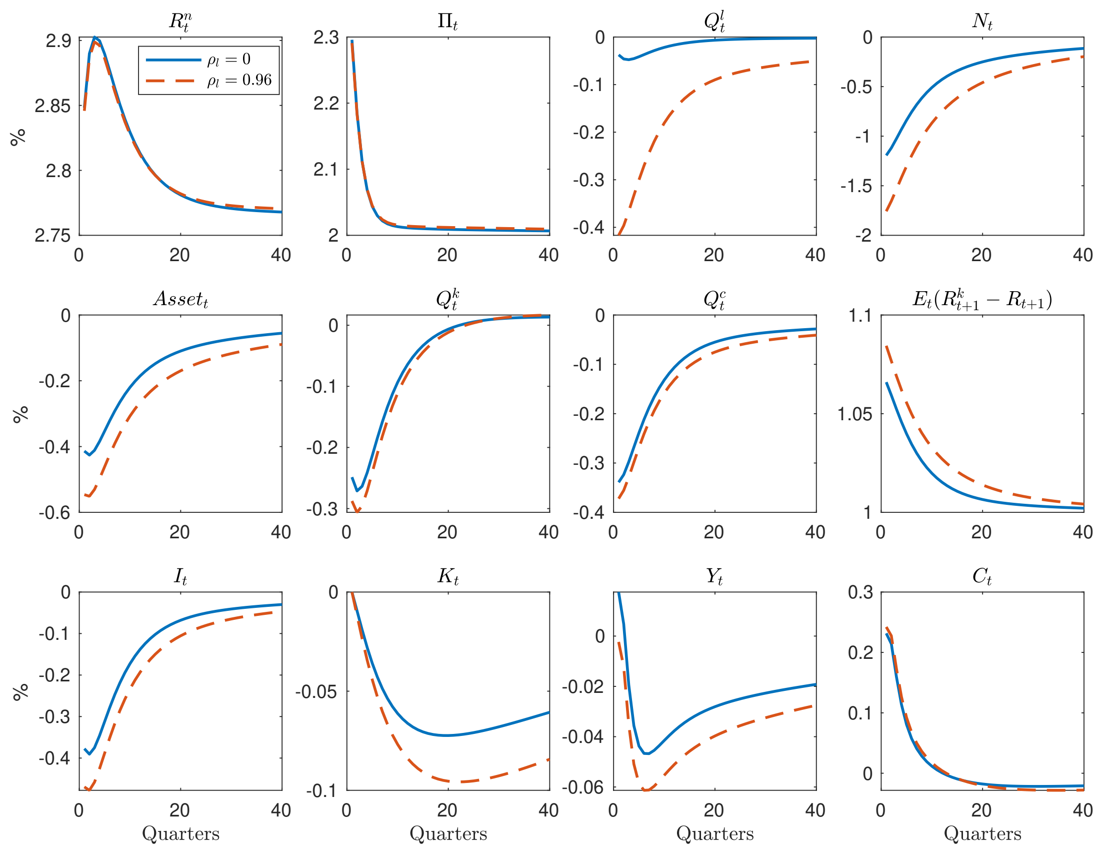

Figure E3. Preference shocks for $\rho_l = 0$ and $\rho_l = 0.96$

Note: This figure plots the impulse responses of selected variables to a one-standard deviation negative preference shock for $\rho_l = 0$ and $\rho_l = 0.96$. The total bank asset is defined by $Asset_t = Q_t^k S_{bt} + Q_t^l B_{bt} + Q_t^c B_{bt}^c$. Nominal interest rate R_t^n, inflation Π_t, and excess return on capital $\mathbb{E}_t(R_{t+1}^k - R_{t+1})$ are in annualized percentage points. The rest variables are in percentage deviation from steady states.

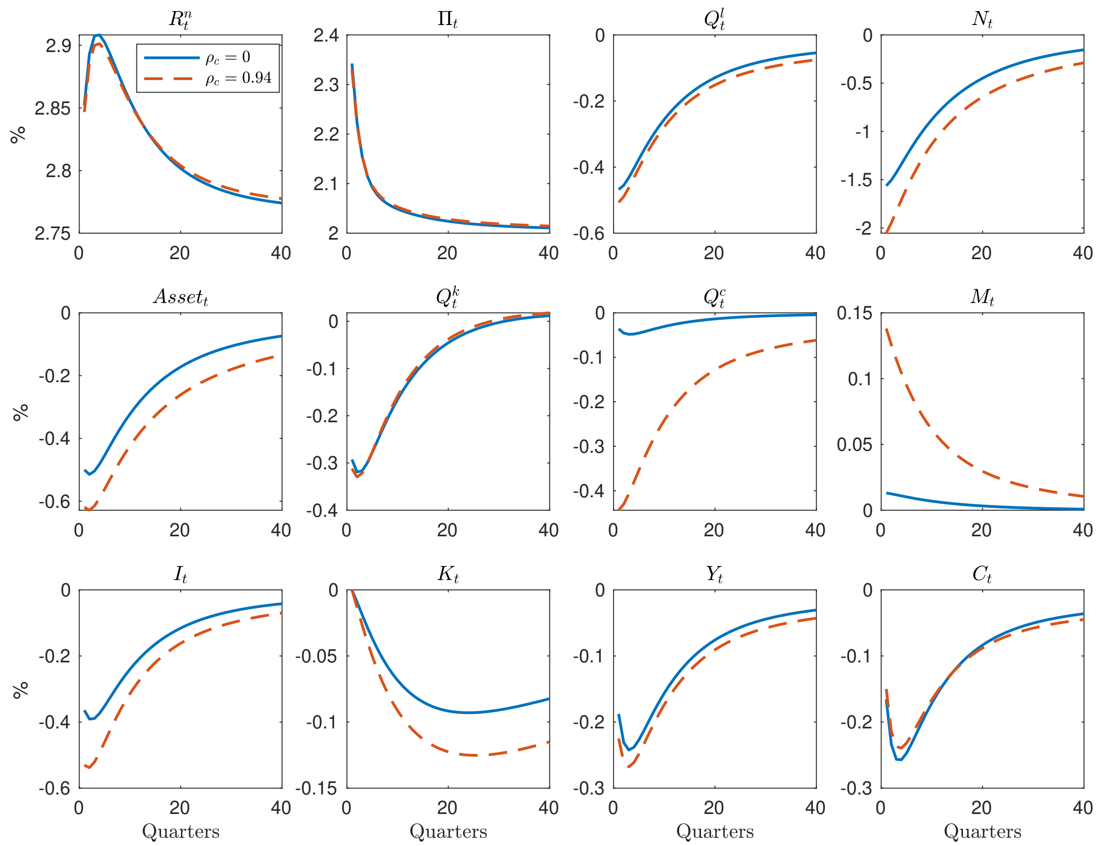

FIGURE E4. TECHNOLOGY SHOCKS FOR $\rho_c = 0$ AND $\rho_c = 0.94$

Note: This figure plots the impulse responses of selected variables to a one-standard deviation negative technology shock for $\rho_c = 0$ and $\rho_c = 0.96$. The total bank asset is defined by $Asset_t = Q_t^k S_{bt} + Q_t^l B_{bt} + Q_t^c B_{bt}^c$. Nominal interest rate R_t^n and inflation Π_t are in annualized percentage points. The rest variables are in percentage deviation from steady states.

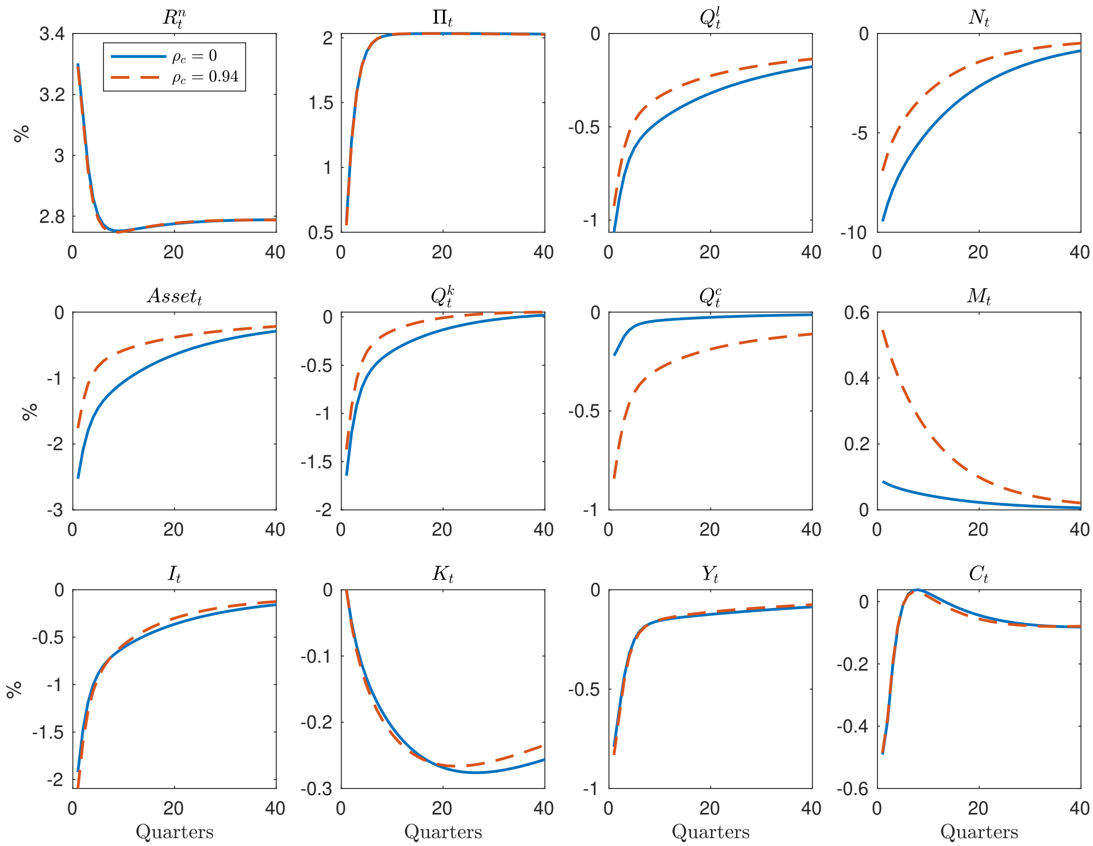

FIGURE E5. MONETARY POLICY SHOCKS FOR $\rho_c = 0$ AND $\rho_c = 0.94$

Note: This figure plots the impulse responses of selected variables to a one-standard deviation negative monetary policy shock for $\rho_c = 0$ and $\rho_c = 0.96$. The total bank asset is defined by $Asset_t = Q_t^k S_{bt} + Q_t^l B_{bt} + Q_t^c B_{bt}^c$. Nominal interest rate R_t^n and inflation Π_t are in annualized percentage points. The rest variables are in percentage deviation from steady states.

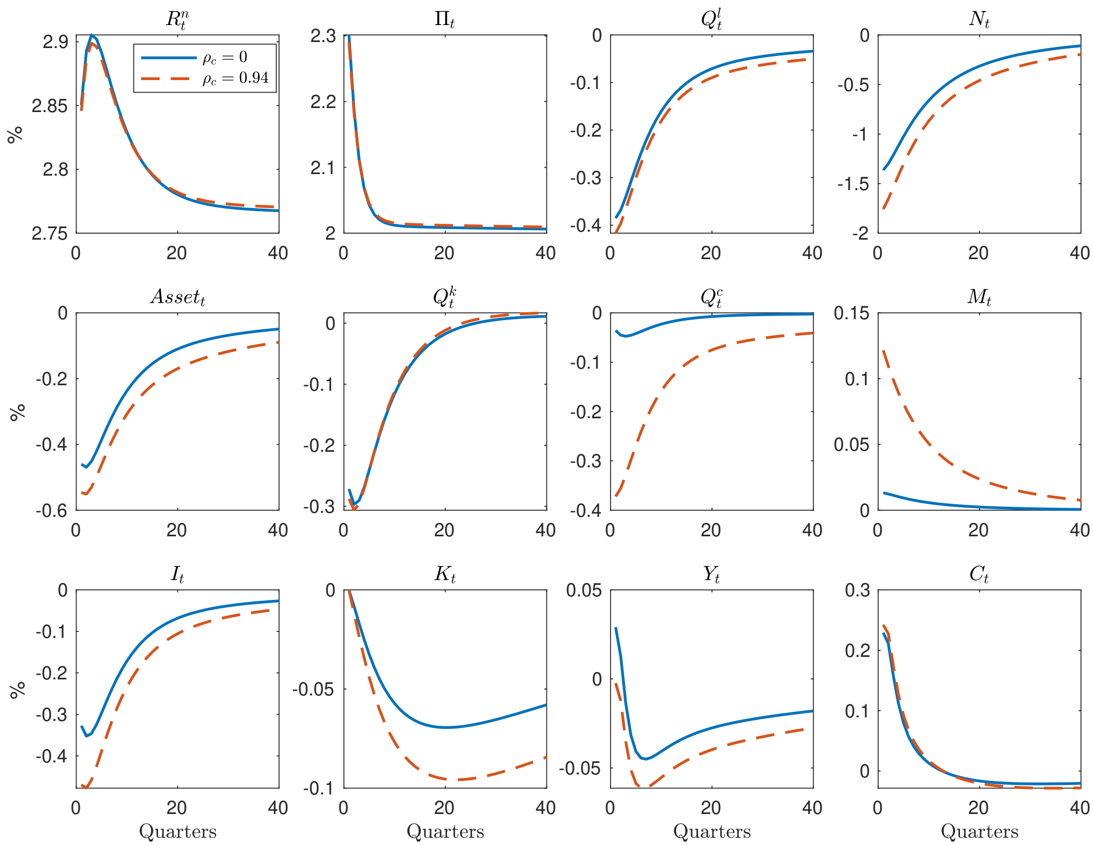

Figure E6. Preference shocks for $\rho_c = 0$ and $\rho_c = 0.94$

Note: This figure plots the impulse responses of selected variables to a one-standard deviation positive preference shock for $\rho_c = 0$ and $\rho_c = 0.96$. The total bank asset is defined by $Asset_t = Q_t^k S_{bt} + Q_t^l B_{bt} + Q_t^c B_{bt}^c$. Nominal interest rate R_t^n and inflation Π_t are in annualized percentage points. The rest variables are in percentage deviation from steady states.

Appendix F. Algorithm to Compute Bank-Run Equilibrium

This section describes the numerical method to compute the transition dynamics of the economy subject to bank runs. We study the following experiment. Suppose that the economy is at the no-run equilibrium steady state at $t = 1$. A one-time positive cost-push shock innovation $\varepsilon_1^p = 0.01$ hits the economy at $t = 1$, followed by two monetary policy innovations $\varepsilon_1^m = \varepsilon_2^m = 0.0025$. Thus v_t^p and v_t^m follow deterministic processes that converge to the steady state in period $T + 1$, where T is a large enough number. We shut down all other shocks throughout the experiment.

Suppose that a bank run may occur in any period and once it occurs it will not happen again in the future. We suppose that it occurs in period J and check $x_J < 1$ is satisfied. After the cost-push shock and interest rate hikes, the economy moves along the transition path until a bank run occurs in period J. After the bank run, the economy starts a new transition path from period $J + 1$ until it converges back to the no-run equilibrium steady state in period $T + 1$.

We compute the equilibrium path in the following steps:

1) Compute the transition path of the economy after the cost-push and monetary shocks when no bank run ever happens, denoted as $\{X_t\}_{t=1}^T$ where X_t stands for the vector of all endogenous variables.

2) Compute the transition path of the economy recovering from a bank run. Suppose that a bank run occurs in period J, $1 \leq J \leq T$. New banks restart in period $J + 1$. We denote the transition path after a period-J bank run as $\{{}_JX_t^*\}_{t=J+1}^T$.

3) Using the equilibrium conditions in period J, we solve for the endogenous variables X_J^*.

4) Finally, the transition path when a bank run takes place in period J is the combination of three pieces:
$$\left\{ \{X_t\}_{t=1}^{J-1}, \quad X_J^*, \quad \{{}_JX_t^*\}_{t=J+1}^T \right\}.$$

We use the usual nonlinear deterministic simulation in Dynare to compute the transition path without a bank run $\{X_t\}_{t=1}^T$ using the equilibrium conditions in Supplemental Appendix B. We next describe the method for computing the transition path after the bank run $\{{}_JX_t^*\}_{t=J+1}^T$ and the variables during the bank run X_J^*.

F1. Transition Path after Bank Run

The transition path after the bank run $\{{}_JX_t^*\}_{t=J+1}^T$ solves the deterministic version of the equilibrium conditions in Supplemental Appendix B with one modification: the bank starts with net worth $N_{J+1} = N^{new}$. We need to determine the proper boundary conditions for $t = J$.

In addition to exogenous shocks, the model has 13 endogenous predetermined state variables. We need to determine their values at time J, $\{Q^l_J$, Q^k_J, Q^c_J, $S_{b,J}$, $B_{b,J}$, $B^c_{b,J}$, N_J, B_J, B^c_J, S_J, C_J, R^n_J, $\Delta_J\}$. First, we have $S_{b,J} = B_{b,J} = B^c_{b,J} = N_J = 0$ since banks do not operate during the period of the bank run. We also have $B_J = B$ by our assumed fiscal policy. We are left with the vector of eight values, $\mathcal{S} = \{Q^k_J, Q^l_J, Q^c_J, S_J, B^c_J, C_J, R^n_J, \Delta_J\}$, to be determined. We use the following iterative procedure to compute $\mathcal{S}$ together with other variables in X^*_J.

Step 1. Take the vector of the steady-state values $\mathcal{S}^0 \equiv \{Q^k, Q^l, Q^c, S, B^c, C, R^n, \Delta\}$, as an initial guess for $\mathcal{S}$.

Step 2. Given the values for the endogenous state variables in the i-th iteration $\mathcal{S}^i$, $i \geq 0$, we compute the transition path after the bank run: $\{{}_J X^{*i}_t\}^{J+T}_{t=J+1}$. Using the equilibrium conditions at $t = J$, we can solve for the vector of endogenous variables X^{*i}_J. In the next subsection we describe the detailed procedure. We then update the values for the state variables in the $(i+1)$-th iteration $\mathcal{S}^{i+1}$ using the solution in X^{*i}_J.

Step 3. We repeat the above two steps until convergence according to the criterion $|\mathcal{S}^i - \mathcal{S}^{i+1}| < 10^{-6}$. After convergence, we obtain X^*_J from the last iteration.

F2. Computing the Updated Values

In this subsection, we describe the procedure to compute X^{*i}_J, the implied endogenous variables in period J, given the state vector for the current iteration $\mathcal{S}^i$. We then describe how we update the state vector $\mathcal{S}^{i+1}$. As described above, we can solve for the transition path after the bank run in period J given state $\mathcal{S}$, $\{{}_J X^*_t\}^{J+T}_{t=J+1}$. We can also compute the transition path before the bank run $\{X_t\}^{J-1}_{t=1}$. We also have the transition path for all exogenous variables as they follow deterministic processes in our perfect foresight setup. We now use the following equilibrium conditions at $t = J$ to compute X^{*i}_J. For simplicity, we suppress the number of iteration superscript i and the bank-run superscript *. We use the superscript $^+$ to denote the implied value of the state variables.

1) First of all, $\{Q^k_J, Q^l_J, Q^c_J, S_J, B^c_J, C_J, R^n_J, \Delta_J\} = \mathcal{S}^i$.

2) Compute C^+_J using
$$C^+_J = \frac{\Lambda_{J,J+1} C_{J+1} + \beta v^{\zeta}_{J+1} h C_{J-1}}{\beta v^{\zeta}_{J+1} + \Lambda_{J,J+1} h},$$
where $\Lambda_{J,J+1} = 1/R_{J+1}$.

3) Compute Q_J^{k+}, Q_J^{l+} and Q_J^{c+} using

$$Q_J^{k+} = \Lambda_{J,J+1}(Z_{J+1} + (1-\delta)Q_{J+1}^k)\left[1 + \kappa\left(\frac{S_{hJ}}{S_J} - \eta_S\right)\right]^{-1},$$
$$Q_J^{l+} = \Lambda_{J,J+1}(1 + \rho_l Q_{J+1}^l)\Pi_{J+1}^{-1}\left[1 + \kappa\left(\frac{B_{hJ}}{B_J} - \eta_B\right)\right]^{-1},$$
$$Q_J^{c+} = \Lambda_{J,J+1}(1 + \rho_c Q_{J+1}^c)\Pi_{J+1}^{-1}\left[1 + \kappa\left(\frac{B_{hJ}^c}{B_J^c} - \eta_C\right)\right]^{-1},$$

where $S_{hJ} = S_J$, $B_{hJ} = B_J$, $B_{hJ}^c = B_J^c$, and $B_J = B$.

4) Using the capital producer's optimization conditions, we have

$$M_J = \frac{1}{Q_J^c}\Lambda_{J,J+1}\left(1 + \rho_c M_{J+1} Q_{J+1}^c\right)\Pi_{J+1}^{-1},$$
$$\nu_J = -\Lambda_{J,J+1}\left[(1-\delta)Q_{J+1}^k - \nu_{J+1}\left(1 - \delta + \frac{\Omega_k}{2}\left(\frac{I_{J+1}^2}{K_{J+1}^2} - \delta^2\right)\right)\right] + Q_J^k,$$
$$I_J = \left(\frac{\nu_J - M_J}{\Omega_k \nu_J} + \delta\right)K_J,$$

where $K_J = S_{J-1}$. We then compute S_J^+ using

$$S_J^+ = (1-\delta)K_J + \left[I_J - \frac{\Omega_k}{2}\left(\frac{I_J}{K_J} - \delta\right)^2 K_J\right].$$

5) Using the resources constraint, we have:

$$Y_J = C_J + I_J + G_J.$$

Then we can compute the rest variables accordingly:

$$\begin{aligned}
L_J &= \left(\frac{\Delta_J Y_J}{A_J K_J^\alpha}\right)^{\frac{1}{1-\alpha}}, \\
W_J &= \chi L_J^\varphi (C_J - hC_{J-1}), \\
P_{wJ} &= \frac{W_J L_J}{(1-\alpha) Y_J \Delta_J}, \\
\Gamma_J^a &= P_{wJ} Y_J + \gamma \Lambda_{J,J+1} \Pi^{-\varepsilon} \Pi_{J+1}^\varepsilon \Gamma_{J+1}^a, \\
\Gamma_J^b &= (1-\tau_J^s) Y_J + \gamma \Lambda_{J,J+1} \Pi^{1-\varepsilon} \Pi_{J+1}^{\varepsilon-1} \Gamma_{J+1}^b, \\
p_J^* &= \frac{\varepsilon}{\varepsilon - 1} \frac{\Gamma_J^a}{\Gamma_J^b}, \\
\Pi_J &= \left[\frac{1-(1-\gamma) p_J^{*1-\varepsilon}}{\gamma}\right]^{\frac{1}{\varepsilon-1}} \Pi.
\end{aligned}$$

6) We compute B_J^{c+}, R_J^{n+} and Δ_J^+ using

$$\begin{aligned}
B_J^{c+} &= \frac{I_J}{Q_J^c} + \rho_c B_{J-1}^c \Pi_J^{-1}, \\
R_J^{n+} &= \rho_r R_{J-1}^n + (1-\rho_r)\left(R^n + \phi_\pi \ln \frac{\Pi_J}{\Pi} + \phi_y \ln \frac{Y_J}{Y}\right) + v_J^m, \\
\Delta_J^+ &= (1-\gamma) p_J^{*\,-\varepsilon} + \gamma \left(\frac{\Pi}{\Pi_J}\right)^{-\varepsilon} \Delta_{J-1}.
\end{aligned}$$

7) We can determine the remaining variables in X_J^* :

$$\begin{aligned}
R_J &= R_{J-1}^n / \Pi_J, \\
Z_J &= P_{wJ} \alpha Y_J \Delta_J / K_J, \\
R_J^k &= \frac{Z_J + (1-\delta) Q_J^k}{Q_{J-1}^k}, \\
R_J^l &= \frac{1 + \rho_l Q_J^l}{Q_{J-1}^l} \frac{1}{\Pi_J}, \\
R_J^c &= \frac{1 + \rho_c Q_J^c}{Q_{J-1}^c} \frac{1}{\Pi_J}, \\
T_J &= Q_{J-1}^l B_{J-1} R_J^l - Q_J^l B_J + G_J.
\end{aligned}$$

8) Since all banks fail in period J, we have $N_J = S_{bJ} = B_{bJ} = B_{bJ}^c = 0$. The bank's

marginal value of net worth ϕ_J, the Lagrange multiplier of the borrowing constraint λ_J, and the macroprudential policy tax rates τ^l_J are meaningless and are set to zero.

In sum, we have computed all endogenous variables during the bank run X^*_J. We then update the state vector as the weighted average of the implied state variables and the state vector of the previous iteration:

$$\mathcal{S}^{i+1} = (1-a)\{Q^{k+}_J, Q^{l+}_J, Q^{c+}_J, S^+_J, B^{c+}_J, C^+_J, R^{n+}_J, \Delta^+_J\} + a \cdot \mathcal{S}^i,$$

where $a \in [0, 1)$.

Appendix G. Interest Rate Smoothing

In this section, we study the role of interest rate smoothing in the interest rate rule. Figure G1 plots impulse responses of the economy to a one-standard deviation positive cost-push shock for $\rho_r = 0.85$ and $\rho_r = 0$. We compare our benchmark model for $\rho_r = 0.85$ with the one for $\rho_r = 0$. We find that a monetary policy rule without interest rate smoothing leads to a much larger increase in the interest rate in response to the cost-push shock. The higher interest rate leads to lower long-term asset prices and lower bank net worth, resulting in lower investment and output.

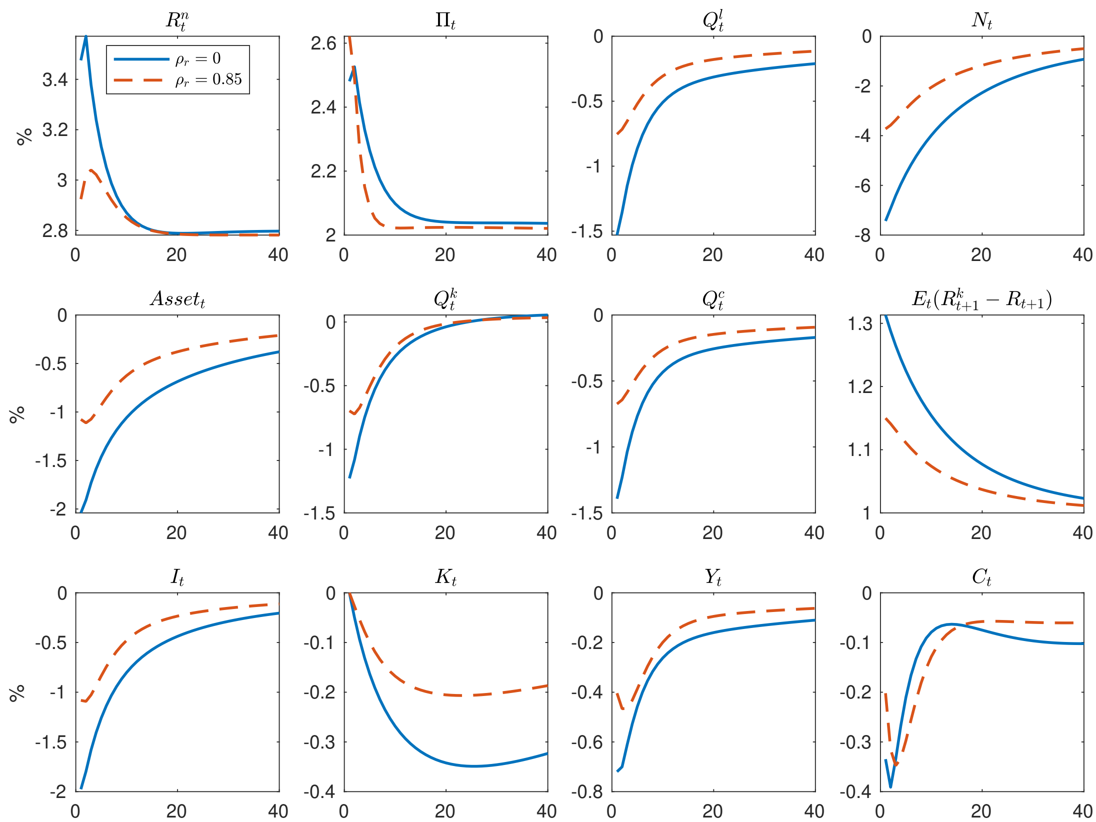

Figure G1. Effects of monetary policy smoothing

Note: This figure plots the impulse responses of selected variables to a one-standard deviation positive cost-push shock for $\rho_r = 0$ and $\rho_r = 0.85$. The total bank asset is defined by $Asset_t = Q_t^k S_{bt} + Q_t^l B_{bt} + Q_t^c B_{bt}^c$. Nominal interest rate R_t^n, inflation Π_t, and excess return on capital $\mathbb{E}_t(R_{t+1}^k - R_{t+1})$ are in annualized percentage points. The rest of variables are in percentage deviation from steady states.

Appendix H. Welfare Measure

To evaluate macroprudential policies, we need a measure of welfare. We calculate the expected welfare conditional on the set of adverse shocks. Specifically, the economy is at the no-run steady state in period 0. Then a 1% cost-push shock hits the economy in period 1 followed by 2 consecutive 25-bp monetary policy shocks in period 1 and 2. After the shocks, asset prices fall and bank net worth shrinks, and thus the recovery rate x_t may fall below one so that a bank run can happen.

Let U_t^e be the expected period-t utility taking into account the possibility of a bank run. Then the expected utility in period 1 is

$$U_1^e = (1-p_1)U_1 + p_1 U_1^*(1),$$

where

$$U_t = \ln(C_t - hC_{t-1}) - \frac{\chi}{1+\varphi} L_t^{1+\varphi}$$

is the household's period-t utility when bank run never happens and $U_t^*(J)$ denotes the period-t utility when a bank run happens in period $J \leq t$. For simplicity, we assume that if a bank run happens in any period J, it will never happen again in the future. Then, the expected period-2 utility is

$$U_2^e = (1-p_1)(1-p_2)U_2 + (1-p_1)p_2 U_2^*(2) + p_1 U_2^*(1).$$

Accordingly, the expected period-t utility taking into account the possibility of bank run is thus

$$U_t^e = \prod_{j=1}^{t}(1-p_j)U_t + \prod_{j=1}^{t-1}(1-p_j)p_t U_t^*(t) + \prod_{j=1}^{t-2}(1-p_j)p_{t-1}U_t^*(t-1) + ... + p_1 U_t^*(1).$$

We can then calculate the expected household's welfare as the sum of discounted expected period utility U_t^e:

$$welfare = \sum_{t=0}^{\infty} \beta^t U_t^e.$$

As is standard in the literature, we measure welfare gains in terms of consumption equivalent when comparing different macroprudential policies. Let $welfare$ and $welfare^{policy}$ denote the equilibrium expected lifetime utilities before and after a macroprudential policy conditional on the realized shocks. Let Ξ denote the consumption gain from the policy, i.e., the percentage increase in the whole consumption path after the policy. Then we can show that

$$\Xi = \exp[(welfare^{policy} - welfare)(1-\beta)] - 1.$$

We calculate the probability of a bank run happening in any period over an infinite horizon as

$$\sum_{t=1}^{\infty} \prod_{i=1}^{t-1} (1 - p_i) p_t,$$

where $p_0 = 0$.

Appendix I. Additional Analysis of Macroprudential Policies

I1. Financing Subsidies

In the main text, we assume that subsidies are financed by lump-sum taxes on households. In this appendix, we study the effects of several different ways of financing subsidies. We first prove that a steady state does not exist when the permanent subsidy rate on investment bonds ($\tau^c < 0$) is sufficiently large and when the subsidy is financed by the lump-sum tax on households. We shut down the tax on the long-term government bonds by setting $\tau_t^l = 0$ for all t. The bank's optimality conditions and the equilibrium system are similar to those for the case with τ_t^l presented in Appendix A in the paper and Supplemental Appendices B and C. We will not present them explicitly here.

Intuitively, when the subsidy rate $-\tau^c > 0$ increases in the steady state, a bank raises its demand for investment bonds so that their price rises and return R^c declines. When R^c declines to R, it follows from (C15) that $M = 1$ so that the loan-in-advance constraint for the capital producer is not binding. In the following derivations, we focus on sufficiently large subsidy rates such that the loan-in-advance constraint is not binding in the steady state.

We substitute the bank's first-order-conditions with τ^c, analogous to (C7), (C9), and (C10), into the law of motion of bank net worth with τ^c analogous to (C6), to derive

$$1 = \sigma \left[\frac{\lambda\theta}{\Omega N} \left(Q^k S_b + Q^l B_b + Q^c B_b^c \right) + \frac{\psi}{2} R \left(\frac{Q^k S_b}{N} \right)^2 + R \right] + \omega.$$

We then use conditions similar to (C9), (C11), and (C12) to derive that

$$1 = \sigma \frac{\phi}{\beta(1 - \sigma + \sigma\phi)} + \omega, \tag{I1}$$

where we have substituted (C8). Equation (I1) suggests that the marginal value of bank net worth ϕ does not depend on τ^c.

We can derive the bank's first-order-condition for investment bonds in the steady state:

$$\Omega \left(R^c - (1 + \tau^c) R \right) = \lambda\theta. \tag{I2}$$

When the loan-in-advance constraint does not bind (i.e., $R^c = R = 1/\beta$), we use (I2) to derive

$$\lambda = \frac{\Omega R}{\theta} (-\tau^c) = \frac{1 - \sigma + \sigma\phi}{\theta} (-\tau^c) \geq 0, \tag{I3}$$

where we have used (C8) to derive the second equality. It follows that an increase in the subsidy rate $-\tau^c$ raises the Lagrange multiplier λ. Intuitively, when the subsidy rate is larger, the bank is more eager to hold investment bonds and increase its balance sheet. As a result, its borrowing constraint is tighter.

Using equations (C11) and (C9) without τ^l, we derive that

$$1+\frac{\psi}{2}\left(\frac{Q^k S_b}{N}\right)^2=(1-\lambda)\frac{\phi}{1-\sigma+\sigma\phi}. \tag{I4}$$

An increase in the subsidy rate $-\tau^c$ raises λ proportionally and hence reduces the right-hand side of (I4). When the subsidy rate $-\tau^c$ is large enough, the right-hand side becomes negative. However, the left-hand side of (I4) cannot be smaller than one as the $Q^k S_b/N \geq 0$. Therefore, a steady state does not exist. The intuition is that the bank is earning a profit from the subsidy, and if the subsidy is large enough, the profit will be too large for the whole banking sector to maintain a stable size.

Using (I1) and (I3), we can calculate the limit negative tax (subsidy) rate below which the model has no steady state:

$$\tau^c_{\min}=\frac{\theta[\sigma-\beta(1-\omega)][1-\beta(1-\omega)]}{\beta(1-\omega)(1-\sigma)},$$

which is around -0.15% under our calibration.

Now we suppose that the subsidy on the bank holdings of investment bonds is financed by other sources of taxes. We consider several cases.

Taxing Incumbent Bank Net Worth. — Following Aoki, Benigno and Kiyotaki (2016), we tax the incumbent bank net worth to finance the subsidy. Denote by τ^n_t the tax rate on the bank's net worth (if $\tau^n_t<0$, it is rebate). Then the bank balance sheet becomes

$$Q^k_t s_t+\frac{\psi}{2}\frac{\left(Q^k_t s_t\right)^2}{n_t}+Q^l_t b_t+Q^c_t b^c_t(1-\tau^c_t)=(1-\tau^n_t)n_t+d_t,$$

where the tax rates satisfy

$$\tau^c_t Q^c_t B^c_{bt}+\tau^n_t N_t=0.$$

The bank net worth satisfies the law of motion:

$$N_t=\sigma\Bigg[\left(R^k_t-R_t\left(1+\frac{\psi}{2}\frac{Q^k_{t-1}S_{b,t-1}}{N_{t-1}}\right)\right)Q^k_{t-1}S_{b,t-1}+\left(R^l_t-R_t\right)Q^l_{t-1}B_{b,t-1}$$
$$+\left(R^c_t-(1+\tau^c_{t-1})R_t\right)Q^c_{t-1}B^c_{b,t-1}+(1-\tau^n_{t-1})R_t N_{t-1}\Bigg]+\omega N_{t-1}.$$

We first consider the impact of the permanent component τ^c on the steady state. For

$\tau^c > 0$, the government subsidizes the bank net worth by taxing bank holdings of investment banks. When $\tau^c > 0$ is sufficiently large, the subsidy on the bank net worth is large enough such that the bank borrowing constraint does not bind in the steady state. We verify this result numerically and find the critical value of τ^c is around $\tau^c_{\lim} = 0.3\%$. We will focus on the more interesting case with $\tau^c < \tau^c_{\lim}$ such that the bank borrowing constraint binds in the steady state and also on the transition path locally around the steady state.

Figure I1 shows the effects of τ^c_t. We find that there are welfare gains when $\tau^c > 0$ and $\phi_c > 0$ and the welfare gains increase with $\tau^c > 0$ and $\phi_c > 0$. This suggests that it is beneficial to tax bank holdings of investment bonds and subsidize the bank's net worth both in the steady state and on the transition path in response to interest rate hikes. Intuitively, subsidizing bank holdings of investment bonds financed by taxing bank net worth will relax capital producers' loan-in-advance constraints but reduce bank lending by tightening the bank borrowing constraints, while taxing bank holdings of investment bonds to subsidize bank net worth has the opposite effect. It turns out that the latter policy improves welfare. We also find that an increase in $\tau^c > 0$ raises the bank run probability even though it raises bank net worth. The reason is that the bank shifts its holdings of investment bonds to long-term government bonds. The latter is prone to the bank run risk. By contrast, an increase in $\phi_c > 0$ can mitigate this impact and reduce the bank run probability. However, there is no optimal policy rule on the transition path no matter whether the bank run possibility is taken into account. This is because the benefit from increased bank lending dominates the cost of potential bank runs.

Taxing New Banker Net Worth. — Suppose that the government taxes the net worth of the new bankers in a lump-sum manner to finance the subsidy on bank holdings of long-term investment bonds. Then the net worth of the banking sector follows

$$N_t = \sigma \Bigg[\left(R^k_t - R_t \left(1 + \frac{\psi}{2} \frac{Q^k_{t-1} S_{b,t-1}}{N_{t-1}} \right) \right) Q^k_{t-1} S_{b,t-1} + \left(R^l_t - R_t \right) Q^l_{t-1} B_{b,t-1}$$
$$+ \left(R^c_t - (1 + \tau^c_{t-1}) R_t \right) Q^c_{t-1} B^c_{b,t-1} + R_t N_{t-1} \Bigg] + \omega N_{t-1} - X_t,$$

where X_t is the lump-sum tax on the startup funds of the new banker, which satisfies

$$\tau^c_t Q^c_t B^c_{bt} + X_t = 0.$$

For the permanent subsidy, we find an optimal subsidy rate at $\tau^c = -0.03\%$ that maximizes the expected welfare when a potential bank run is taken into account. Intuitively, the main cost of this policy is to reduce bank lending, but this cost is smaller than the distortionary tax on incumbent bank net worth. Thus, the benefit and cost of the macroprudential policy can be balanced at an interior solution $\tau^c = -0.03\%$.

For the cyclical component of the policy rule, we find that the expected welfare is de-

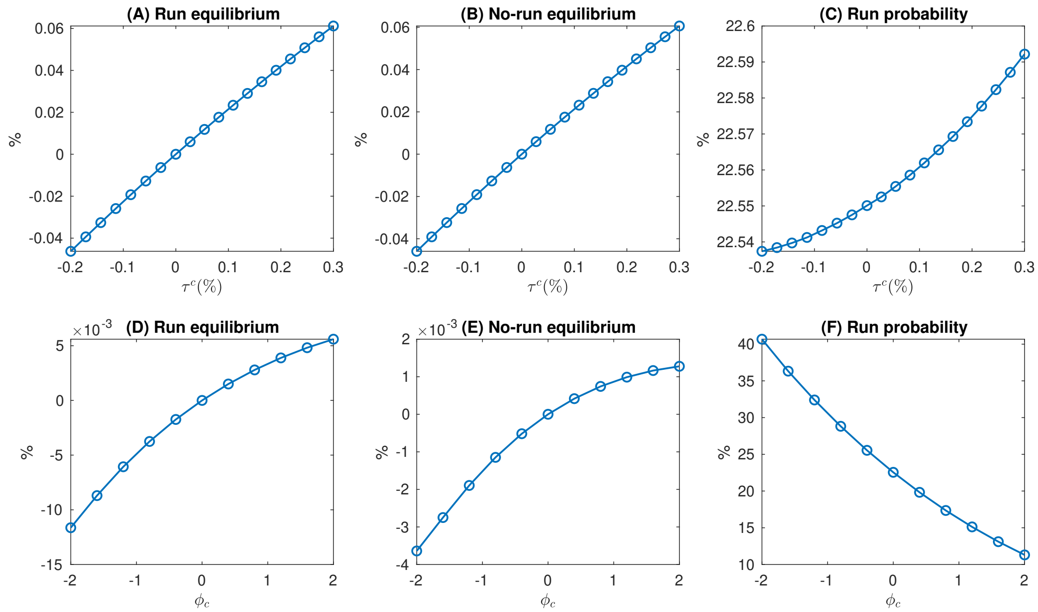

FIGURE I1. SUBSIDIZING INVESTMENT BONDS FINANCED BY TAXING INCUMBENT BANK NET WORTH

Note: The top three panels are for the case of permanent tax $\tau_t^c = \tau^c$ for all t. The bottom three panels are for the case of cyclical tax $\tau_t^c = \phi_c(R_t^n - R^n)$. The first column shows the welfare gain when the possibility of bank run is considered. The second column shows the welfare gain assuming bank run never happens. The third column shows the probability that a bank run happens over the entire transition path. Welfare numbers are presented in percentage of consumption equivalent relative to the case of no macroprudential policy.

creasing in the responsiveness parameter ϕ_c. In particular, the government should raise the subsidy rate ($\phi_c < 0$) in response to the cost-push shock and interest rate hikes. Intuitively, subsidizing investment bonds with $\phi_c < 0$ can stabilize their prices and alleviate the decline in capital and output in a downturn. Thus, it improves welfare in the no-run equilibrium as illustrated in Panel E of Figure I2. On the other hand, since the subsidy is financed by taxing the new banker, the net worth of the entire banking sector is lower when the economy is recovering from the recession. A lower bank net worth makes the bank more vulnerable to bank runs so that bank run probability increases as ϕ_c decreases. Panel E of Figure I2 shows that the benefit of the subsidy dominates the cost. There is no interior solution for ϕ_c.

TAXING LABOR OR CAPITAL INCOMES. — We have also analyzed the cases in which the government taxes labor income only or taxes both labor and capital incomes to finance the subsidy on bank holdings of long-term investment bonds. We have not found an interior constrained optimal policy rule. Again the reason is that the benefit and cost cannot be balanced in the same magnitude. See Figures I3 and I4 for the results. We will omit a detailed discussion on the intuition as it is similar to that discussed previously.

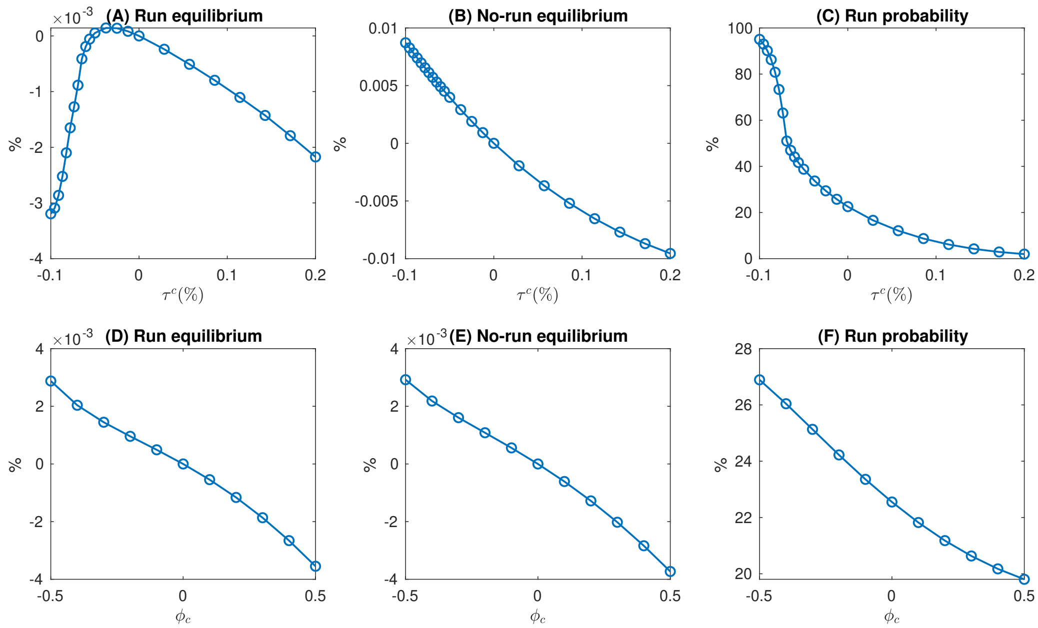

FIGURE I2. SUBSIDIZING INVESTMENT BONDS FINANCED BY TAXING NEW BANKERS

Note: The top three panels are for the case of permanent tax $\tau_t^c = \tau^c$ for all t. The bottom three panels are for the case of cyclical tax $\tau_t^c = \phi_c(R_t^n - R^n)$. The first column shows the welfare gain when the possibility of bank run is considered. The second column shows the welfare gain assuming bank run never happens. The third column shows the probability that a bank run happens over the entire transition path. Welfare numbers are presented in percentage of consumption equivalent relative to the case of no macroprudential policy.

I2. Macroprudential Policies when Investment Bonds are Short-Term

In this appendix, we study the effects of macroprudential policies when the investment bonds are short-term. Figure I5 and I6 compare the welfare gains of macroprudential policies when $\rho_c = 0.94$ and $\rho_c = 0$.

As we have analyzed in Section IV in the paper, the permanent tax on long-term government bonds has the tradeoff between lowering the bank run probability and improving the welfare in the no-run equilibrium, resulting in an optimal tax rate around 1% when $\rho_c = 0.94$. When the nominal loans are short-term ($\rho_c = 0$), the loan prices are less sensitive to adverse shocks (see Figure 3 in the paper) and the bank run probability is much reduced. In our simulation, the bank run is not possible at all even without any macroprudential policy (see Figure 6 in the paper). Thus, increasing the permanent tax rate has only costs without any benefit of reducing the bank run probability. A permanent subsidy $\tau^l < 0$, on the other hand, is welfare improving because it increases the welfare in the no-run equilibrium without increasing the bank run probability much. We find that the optimal rate of permanent subsidy is around 2.5% ($\tau^l = -2.5\%$). The tradeoff now encourages the bank to hold more long-term government bonds under the subsidy since

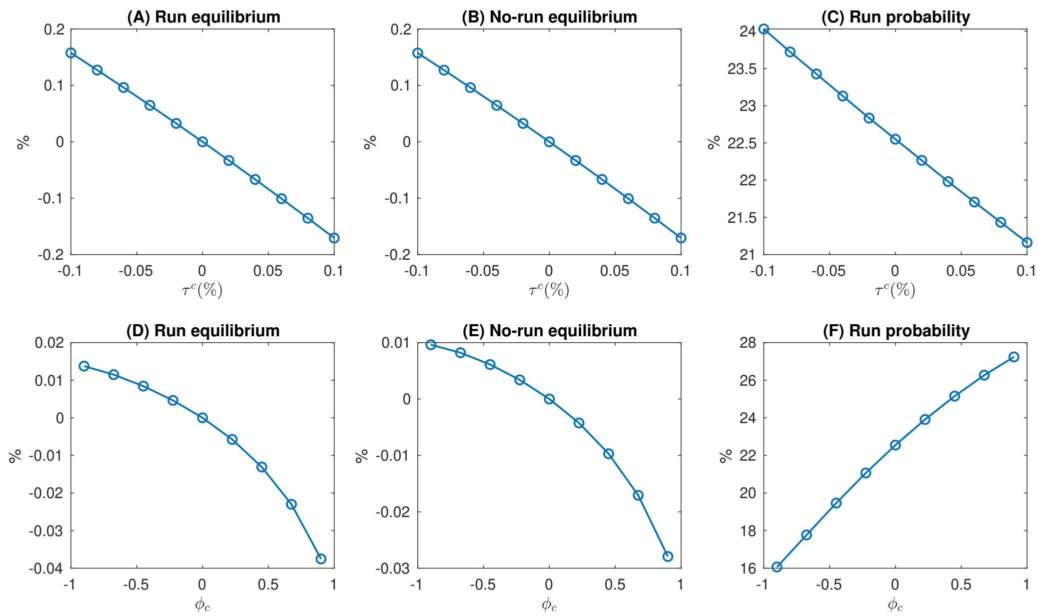

FIGURE I3. WELFARE EFFECTS OF SUBSIDIZING INVESTMENT BONDS FINANCED BY TAXING LABOR INCOME

Note: The top three panels are for the case of permanent tax $\tau_t^c = \tau^c$ for all t. The bottom three panels are for the case of cyclical tax $\tau_t^c = \phi_c(R_t^n - R^n)$. The first column shows the welfare gain when the possibility of bank run is considered. The second column shows the welfare gain assuming bank run never happens. The third column shows the probability that a bank run happens over the entire transition path. Welfare numbers are presented in percentage of consumption equivalent relative to the case of no macroprudential policy.

the bank is holding too few long-term assets when the nominal loans are short-term.

Regarding the cyclical policy on government bonds $\tau_t^l = \phi_l(R_t^n - R^n)$, we find that subsidizing government bonds when interest rates are high ($\phi_l < 0$) improves the welfare in the no-run equilibrium and also reduces the bank run probability, thus improving the expected welfare unambiguously. When the nominal loans are short-term, the cyclical subsidy ($\phi_l < 0$) loses the benefit of reducing the bank run probability but still improves welfare in the no-run equilibrium. The expected welfare still decreases monotonically in the response coefficient ϕ_l, though the magnitude is smaller.

For macroprudential policies targeted on nominal loans, we find that subsidizing long-term loans improves welfare in Section IV in the paper. When the nominal loans are short-term ($\rho_c = 0$), the loan prices are less sensitive to adverse shocks and a bank run cannot occur in our simulations. Thus, subsidizing short-term nominal loans improves the welfare in the no-run equilibrium without any cost, and hence improves the expected welfare unambiguously. The magnitude of the welfare improvement of the subsidy is very small because the debt overhang problem for credit-constrained capital producers is minimal for short-term nominal loans (see Figure I6).

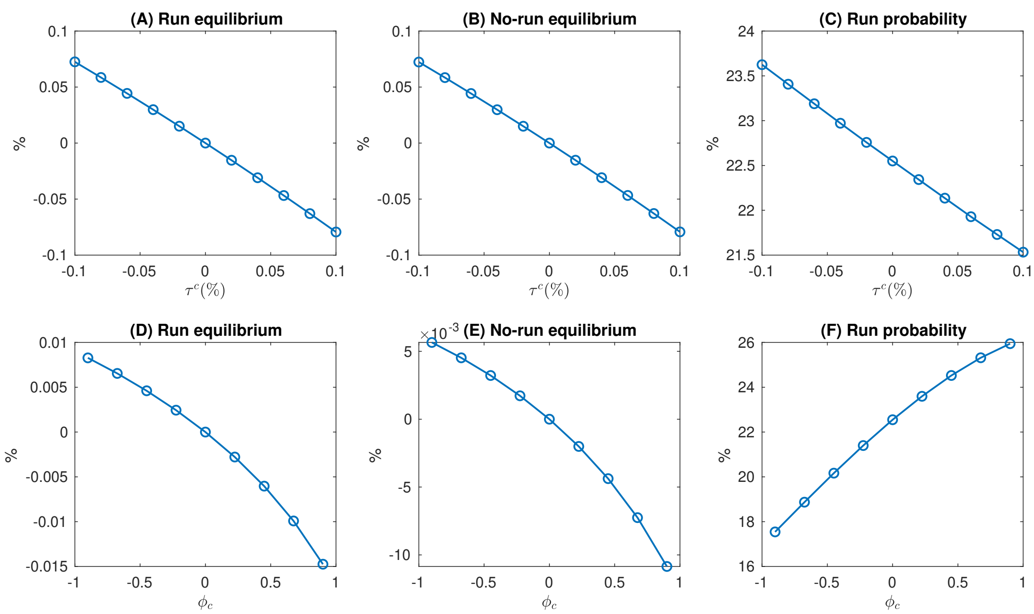

Figure I4. Welfare effects of subsidizing investment bonds financed by taxing labor and capital income

Note: The top three panels are for the case of permanent tax $\tau_t^c = \tau^c$ for all t. The bottom three panels are for the case of cyclical tax $\tau_t^c = \phi_c(R_t^n - R^n)$. The first column shows the welfare gain when the possibility of bank run is considered. The second column shows the welfare gain assuming bank run never happens. The third column shows the probability that a bank run happens over the entire transition path. Welfare numbers are presented in percentage of consumption equivalent relative to the case of no macroprudential policy.

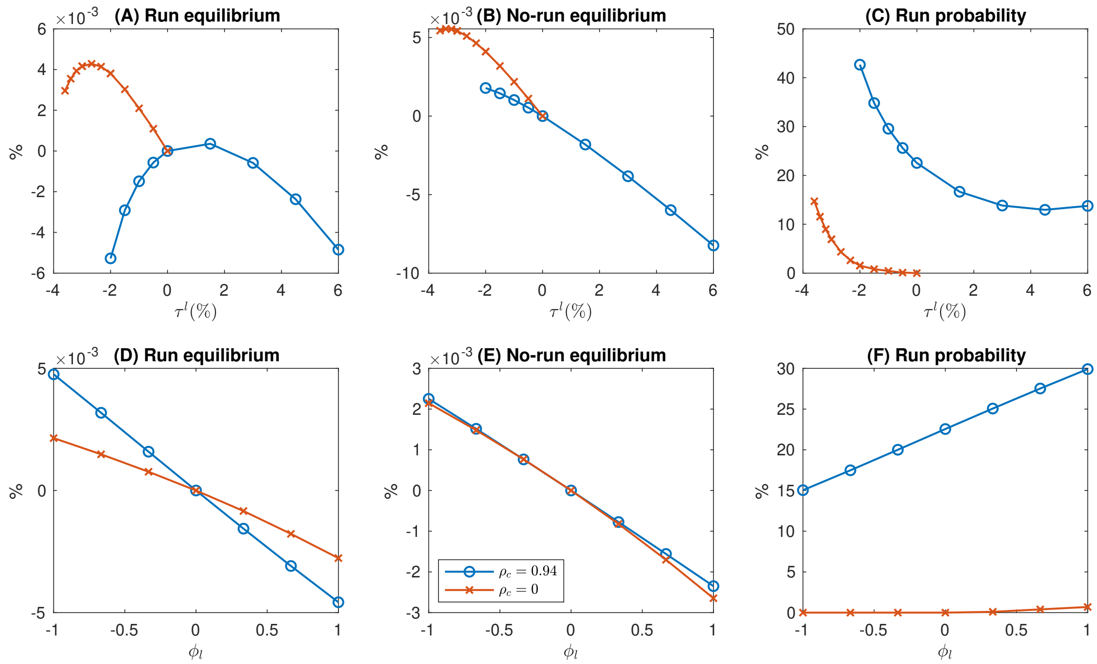

FIGURE I5. WELFARE GAINS OF TAXES/SUBSIDIES ON GOVERNMENT BONDS UNDER DIFFERENT ρ_c

Note: The top three panels are for the case of permanent tax $\tau_t^l = \tau^l$ for all t. The bottom three panels are for the case of cyclical tax $\tau_t^l = \phi_l(R_t^n - R^n)$. The first column shows the welfare gain when the possibility of bank run is considered. The second column shows the welfare gain assuming a bank run never happens. The third column shows the probability that a bank run happens over the infinite horizon. Welfare gains are measured in percentage of consumption equivalent relative to the case of no macroprudential policy.

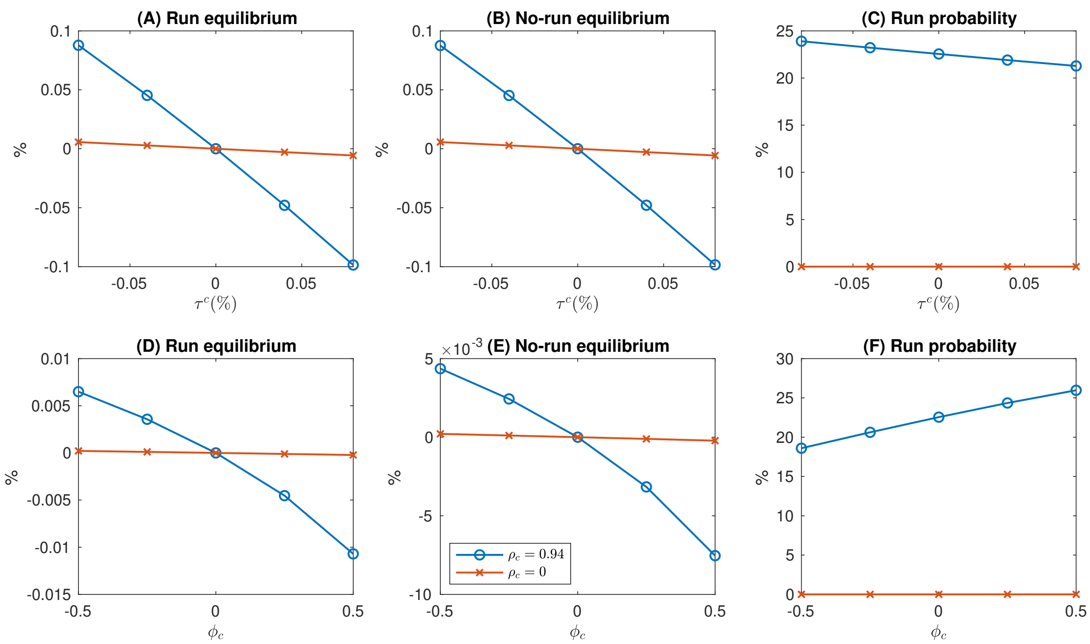

FIGURE I6. WELFARE GAINS OF TAXES/SUBSIDIES ON INVESTMENT BONDS UNDER DIFFERENT ρ_c

Note: The top three panels are for the case of permanent tax $\tau_t^c = \tau^c$ for all t. The bottom three panels are for the case of cyclical tax $\tau_t^c = \phi_c(R_t^n - R^n)$. The first column shows the welfare gain when the possibility of bank run is considered. The second column shows the welfare gain assuming a bank run never happens. The third column shows the probability that a bank run happens over the infinite horizon. Welfare gains are measured in percentage of consumption equivalent relative to the case of no macroprudential policy.

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