Supplemental Appendix: Changing Income Risk across the US Skill Distribution: Evidence from a Generalized Kalman Filter*

J. Carter Braxton   Kyle Herkenhoff   Jonathan Rothbaum   Lawrence Schmidt

June 5, 2025

*Braxton: University of Wisconsin. Herkenhoff: University of Minnesota, FRB Minneapolis, IZA, and NBER. Rothbaum: US Census Bureau. Schmidt: MIT. The Census Bureau has reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product (Data Management System (DMS) number: P-7503840, Disclosure Review Board (DRB) approval number: CBDRB-FY23-SEHSD003-071, CBDRB-FY24-SEHSD003-009 and CBDRB-FY24-0469.)

A Filtering under augmented sequential exogeneity

In this appendix we layout the statistical assumptions for the employment process that our approach requires. We introduce the concept of augmented sequential exogeneity to govern our employment process and show that under this assumption we obtain the same estimates for persistent earnings at the individual level via the the Kalman filter and Kalman smoother as under the assumption of strict exogeneity. Further, we show that we obtain the same parameter estimates for our income process under augmented sequential exogeneity and strict exogeneity.

A.1 Income process and definitions

We start by defining our income process and the definitions of exogeneity that we will be using. In this section, we adopt two notational conventions for brevity. First, we omit individual subscripts. Second, we use the letter f to denote a conditional density function and allow its arguments to specify to which distribution we referring–i.e., we use $f(y_t|x_t)$ for the conditional pdf of $y_t|x_t$ rather than $f_{y_t|x_t}(y_t, x_t)$. These functions may depend on an unknown set of parameters $\{\theta, \lambda\}$; when these arguments are omitted, it is implied that we are evaluating at parameters of the true DGP. Our income process is given by a linear state space model **(LS)**,

$$
\begin{aligned}
y_t &= H\left(l_t\right)\left(z_t + \omega_t\right), && \omega_t \sim N\left(0, R\left(x_t\right)\right) \quad \text{[Meaurement]}\\
z_t &= F z_{t-1} + B\left(l_t; x_t\right) + \nu_t, && \nu_t \sim N\left(0, Q\left(l_t; x_t\right)\right) \quad \text{[State]}\\
z_0 &\sim N\left(0, Q_0(x_0)\right).
\end{aligned}
$$

We assume that ω_t and ν_t are independently drawn, conditional on (l_t, x_t). For this appendix, we assume that $H(l_t)$ is a scalar that is equal to one when an individual is employed (i.e., $H(l_t) = 1$ if $l_t = 1$), and is equal to zero otherwise. When an individual is unemployed (i.e., $l_t = 0$), we assume their earning observation is set to missing. It is also straightforward to allow the initial mean of z_0 to depend on x_0, though one needs to attend to the possibility that this introduces additional identification issues.

Let θ denote the parameters which appear in the equations of the state space model ([Measurement] and [State]), $\theta = \{R\left(\cdot\right), Q\left(\cdot\right), Q_0\left(\cdot\right), B\left(\cdot\right), F\}$. Let superscript t denote the cumulative history up to time t (e.g., $y^t = \{y_t, \ldots, y_0\}$). Given our maintained assumption **(LS)**,

$$
\begin{aligned}
f\left(y_t|y^{t-1}, z^t, l^t, x^t; \{\theta, \lambda\}\right) &= f\left(y_t|z_t, l_t, x_t; \theta\right)\\
f\left(z_t|l_t, x_t, y^{t-1}, z^{t-1}, l^{t-1}, x^{t-1}; \{\theta, \lambda\}\right) &= f\left(z_t|z_{t-1}, l_t, x_t; \theta\right)
\end{aligned}
$$

because $y_t \perp (y^{t-1}, z^{t-1}, l^{t-1}, x^{t-1})|(z_t, l_t, x_t)$ and $z_t \perp (y^{t-1}, z^{t-2}, l^{t-1}, x^{t-1})|(z_{t-1}, l_t, x_t)$.

In order to fully specify the dynamics of the DGP, we need to specify a law of motion for employment status l_t and other conditioning variables x_t. We assume that the conditional density $f(l_t, x_t|y^{t-1}, z^{t-1}, l^{t-1}, x^{t-1}; \{\theta, \lambda\})$ potentially depends on both θ and an additional vector of parameters λ. We can thus write the joint likelihood as:

$$\begin{aligned}
f\left(y^T, z^T, l^T, x^T; \{\theta, \lambda\}\right) &= f\left(y_T\,|z_T, l_T, x_T; \theta\right) f\left(z_T|\, z_{T-1}, l_T, x_T; \theta\right) \\
&\times\; f\left(l_T, x_T\,|\, y^{T-1}, z^{T-1}, l^{T-1}, x^{T-1}; \{\theta, \lambda\}\right) f\left(y^{T-1}, z^{T-1}, l^{T-1}, x^{T-1}; \{\theta, \lambda\}\right) \\
f\left(y^T, z^T, l^T, x^T; \{\theta, \lambda\}\right) &= \Pi_{t=1}^{T}\Big[f\left(y_t\,|\, z_t, l_t, x_t; \theta\right) f\left(z_t\,|\, z_{t-1}, l_t, x_t; \theta\right) \\
&\times f\left(l_t, x_t\,|\, y^{t-1}, z^{t-1}, l^{t-1}, x^{t-1}; \{\theta, \lambda\}\right)\Big] f\left(z_0|x_0; \theta\right) f\left(x_0; \{\theta, \lambda\}\right).
\end{aligned}$$

We next discuss potential restrictions one could place on the dynamics of (l_t, x_t), where our first two definitions follow Gourieroux and Monfort (1995) (Ch 1.5.2):

1. **Strict exogeneity (S):** Given **LS**, (l_t, x_t) satisfy

$$f\left(l_t, x_t|y^{t-1}, z^{t-1}, l^{t-1}, x^{t-1}; \{\theta, \lambda\}\right) = f\left(l_t, x_t|l^{t-1}, x^{t-1}; \lambda\right)$$

   There are two key components of the **S** assumption. First, the law of motion for (l_t, x_t) only depends on a disjoint subset of the parameter space λ, so it conveys no incremental information about θ. Second, the law of motion for (l_t, x_t) does not depend on $\{y_1, \ldots, y_T\}$ or $\{z_1, \ldots, z_T\}$ conditional on its own lags. In other words, $(l_t, x_t) \perp (y^{t-1}, z^{t-1})|(l^{t-1}, x^{t-1})$.

2. **Sequential Exogeneity (SQ):** Given **LS**, (l_t, x_t) satisfy

$$f\left(l_t, x_t|y^{t-1}, z^{t-1}, l^{t-1}, x^{t-1}; \{\theta, \lambda\}\right) = f\left(l_t, x_t|y^{t-1}, z^{t-1}, l^{t-1}, x^{t-1}; \lambda\right)$$

   The **SQ** assumption allows for dependence of l_t and x_t on the history of observable y^{t-1}, l^{t-1}, x^{t-1}, and unobservable z^{t-1}, while maintaining the assumption that (l_t, x_t) only depends on a disjoint subset of the parameter space λ.

3. **Augmented Sequential Exogeneity (ASQ):** Given **LS**, (l_t, x_t) satisfy

$$f\left(l_t, x_t|y^{t-1}, z^{t-1}, l^{t-1}, x^{t-1}; \{\theta, \lambda\}\right) = f\left(l_t, x_t|y^{t-1}, l^{t-1}, x^{t-1}; \lambda\right)$$

   The **ASQ** assumption is weaker than **S** but stronger than **SQ**, in that it allows (l_t, x_t) to depend on observed earnings y^{t-1} in addition to the history of l^{t-1} and x^{t-1}. However, its key restriction is that $(l_t, x_t) \perp z^{t-1}|(y^{t-1}, l^{t-1}, x^{t-1})$, so employment (and other conditioning variables x_t) don't depend on the latent states conditional on $(y^{t-1}, l^{t-1}, x^{t-1})$. Again, we assume that the conditional distribution of (l_t, x_t) only depends on a disjoint subset of the parameter space λ.

In order to preserve the tractability of the standard Kalman filter, we assume that the DGP satisfies the augmented sequential exogeneity assumption **ASQ** defined above. We demonstrate next that one obtains identical likelihoods, posteriors, and EM recursions under assumptions **S** and **ASQ**.

The second economic restriction imposed by all three assumptions discussed above is that the law of motion for (l_t, x_t) doesn't depend directly on parameters of the income process θ. In other words, there are no cross-equation restrictions which convey information about θ through the dynamics of the conditioning variables. When this is the case, one can factor the likelihood into a part which depends on θ and another which depends on λ, then analyze each piece separately. Our Kalman filter estimator estimates θ while treating λ as a vector of nuisance parameters. In the presence of cross-equation restrictions, there would be potential efficiency gains from jointly estimating the law of motion for the conditioning variables and parameters of the state space model.

A.2 Kalman filter under ASQ

In this section, we derive the Kalman filter under the assumption of **ASQ**. We show that the output of the Kalman filter under the **ASQ** assumption is identical to its output if we assume strict exogeneity **S**. For notational convenience, we omit covariates (x_t) in the proofs; however, all results and proofs are identical with augmented sequentially exogenous covariates. This derivation is based on Hamilton (1994b).

- **Timing:** l_t is observed, then shocks (ω_t and ν_t) are drawn and y_t is observed.

- **Notation:** We define the following expectations

$$\begin{aligned} \widehat{z}_{t|t-1} &= E\left[z_t|l^t, y^{t-1}\right] \\ \widehat{z}_{t|t} &= E\left[z_t|l^t, y^t\right] \\ \widehat{y}_{t|t-1} &= E\left[y_t|l^t, y^{t-1}\right]. \end{aligned}$$

Note that we treat l^t as a component of the date $t-1$ information set since it is drawn before date t shocks (ω_t and ν_t) are realized.

Kalman filter algorithm:

1. $t = 1$: To initialize the filter, we assume $y_0 = \emptyset$, $l_0 = \emptyset$, $z_0 \sim N(0, Q_0)$, and $l_1 = 1$. We begin the recursion with $\widehat{z}_{1|0}$ which is our forecast of z_1 based only on the information contained in l_1 (which by assumption equals 1 for all in our sample):

$$\widehat{z}_{1|0} = E[z_1|l_1] = E[z_1] = B(1)$$
$$m_{1|0} = E\left[(z_1 - E(z_1|l_1))^2 |l_1\right] = F^2 Q_0 + Q(1).$$

   We forecast y_1 as $\widehat{y}_{1|0} = \widehat{z}_{1|0}$. We can then iterate forward.

2. $t = 2$. We observe y_1, which allows us to update our inference of persistent income, $\widehat{z}_{1|1} = E[z_1|l^1, y^1]$ (*we derive the formulas for these expectations in the next step*). We then observe l_2, which allows us to forecast persistent income as $\widehat{z}_{2|1} = E[z_2|l^2, y^1]$. We then forecast income y_2 as $\widehat{y}_{2|1} = E[y_2|l^2, y^1]$.

3. Generic step t. We observe y_t, which allows us to update our inference of persistent income $\widehat{z}_{t|t}$. We do so by deriving the joint distribution $f(y_t, z_t|l^t, y^{t-1})$ and computing the conditional normal distribution of $z_t|y_t, l^t, y^{t-1}$. We re-derive the joint distribution $f(y_t, z_t|l^t, y^{t-1})$ under the assumption of **ASQ**.

   (a) We first derive the mean of $z_t|l^t, y^{t-1}$,

$$\begin{aligned} E\left[z_t|l^t, y^{t-1}\right] &= E\left[Fz_{t-1} + B(l_t) + \nu_t|l^t, y^{t-1}\right] \\ &= E\left[Fz_{t-1}|l_t, l^{t-1}, y^{t-1}\right] + B(l_t) \\ [\textbf{by ASQ}] &= E\left[Fz_{t-1}|l^{t-1}, y^{t-1}\right] + B(l_t) \\ \widehat{z}_{t|t-1} &= F\widehat{z}_{t-1|t-1} + B(l_t) \end{aligned}$$

   and its variance

$$E\left[\left(z_t - \widehat{z}_{t|t-1}\right)\left(z_t - \widehat{z}_{t|t-1}\right)|l^t, y^{t-1}\right] = m_{t|t-1} = F^2 m_{t-1|t-1} + Q(l_t)$$

(b) Next we derive the mean of $y_t|l^t, y^{t-1}$

$$E\left[y_t|l^t, y^{t-1}\right] = H\left(l_t\right)\widehat{z}_{t|t-1} + 0$$

and its variance

$$E\left[\left(y_t - \widehat{y}_{t|t-1}\right)\left(y_t - \widehat{y}_{t|t-1}\right)|l^t, y^{t-1}\right] = H\left(l_t\right)\left(m_{t|t-1} + R\right)H\left(l_t\right).$$

(a) Lastly, we derive the covariance of $z_t|l^t, y^{t-1}$ and $y_t|l^t, y^{t-1}$,

$$E\left[\left(z_t - \widehat{z}_{t|t-1}\right)\left(y_t - \widehat{y}_{t|t-1}\right)|l^t, y^{t-1}\right] = m_{t|t-1}H\left(l_t\right).$$

Putting this together, we have the joint distribution

$$\left[\begin{array}{c} y_t \\ z_t \end{array}\middle| l^t, y^{t-1}\right] \sim N\left(\left[\begin{array}{c} H\left(l_t\right)\widehat{z}_{t|t-1} \\ \widehat{z}_{t|t-1} \end{array}\right], \left[\begin{array}{cc} H\left(l_t\right)\left(m_{t|t-1} + R\right)H\left(l_t\right) & m_{t|t-1}H\left(l_t\right) \\ m_{t|t-1}H\left(l_t\right) & m_{t|t-1} \end{array}\right]\right) \tag{1}$$

which yields the updating formula:[1]

$$\widehat{z}_{t|t} = \widehat{z}_{t|t-1} + \frac{m_{t|t-1}H\left(l_t\right)}{m_{t|t-1}H\left(l_t\right) + R}\left[y_t - H\left(l_t\right)\widehat{z}_{t|t-1}\right] \tag{2}$$

We then observe l_{t+1}, which allows us to forecast persistent income as $\widehat{z}_{t+1|t} = F\widehat{z}_{t|t} + B\left(l_{t+1}\right)$. We then forecast income y_{t+1} as $\widehat{y}_{t+1|t} = H\left(l_{t+1}\right)\widehat{z}_{t+1|t}$. Finally to complete the recursion, the updating formula for $m_{t|t}$, can be written as,

$$m_{t|t} = m_{t|t-1} - \frac{\left(m_{t|t-1}H\left(l_t\right)\right)^2}{m_{t|t-1}H\left(l_t\right) + R}$$

4. This completes the Kalman filter recursion.

Discussion Notice that assumption **ASQ** is already implied by strict exogeneity **S**. Hence, the Kalman filter recursions are identical under the stronger (**S**) and weaker assumptions (**ASQ**).

[1] Note that since $H\left(l_t\right)$ is an indicator, we replace $H\left(l_t\right)\left(m_{t|t-1} + R\right)H\left(l_t\right)$ with $m_{t|t-1}H\left(l_t\right) + R$ in the updating formula to avoid division by zero. This is without loss. We use the same convention in the updating formula for $m_{t|t}$.

By contrast, filtering under **SQ** is *not* identical to that of **S**, since under **SQ** observations of l_t would provide extra information about z_{t-1} beyond $y^{t-1}$and l^{t-1}. For example, if the relationship between l_t and z_{t-1} was known to be linear, one would want to first use l_t as an additional observation equation in an intermediate Kalman filtering step in order to refine one's estimate of the predictive distribution of $z_{t-1}|l^t, y^{t-1}$. Assumption **ASQ** rules out this sort of dependence.

Partial likelihood function under ASQ. As is discussed in section II.A of the main text, we estimate θ by maximizing the sum across agents of the partial log-likelihood function

$$\sum_{t=1}^{T} \ln f(y_t|y^{t-1}, l^t; \theta),$$

where each term in the sum will be equal to zero when $l_t = 0$ and the expression in equation (8), which depends on $\widehat{z}_{t|t-1}$ and $m_{t|t-1}$ from the forward filter, otherwise. Under assumption (**ASQ**), maximizing the partial likelihood is equivalent to maximizing the log likelihood,

$$f\left(y^T, l^T; \{\theta, \lambda\}\right) = \left[\Pi_{t=1}^{T} f\left(y_t \,|\, y^{t-1}, l^t; \theta\right)\right] \left[\Pi_{t=1}^{T} f\left(l_t \,|\, y^{t-1}, l^{t-1}; \lambda\right)\right], \tag{3}$$

since the second term only depend on the nuisance parameters λ unrelated to θ (Greene (2003), Chapter 14.5). As discussed above, this separation allows one to separately analyze the employment (and, when applicable, x_t) process and the income process.

A.3 Kalman smoother under ASQ

In this section, we derive the Kalman smoother under the **ASQ** assumption. We show that the output of the Kalman smoother is identical under **ASQ** and **S.** This derivation is based on Särkkä and Svensson (2023).

Under **ASQ**, our state space model can be compactly rewritten as:

$$\begin{aligned} y_t &\sim f\left(y_t|z_t, l_t; \theta\right) \\ z_t &\sim f\left(z_t|z_{t-1}, l_t; \theta\right) \\ z_0 &\sim N\left(0, Q_0\right). \end{aligned}$$

We want to re-derive the smoothed distribution under our augmented sequential exogeneity assumption **ASQ**, where the smoothed distribution is defined to be:

$$z_t|y^T, l^T \sim f\left(z_t|y^T, l^T; \theta\right).$$

Property M1: Note that the Markov assumption of our model implies[2]

$$f\left(z_t|z_{t+1}, y^T, l^T; \theta\right) = f\left(z_t|z_{t+1}, y^t, l^{t+1}; \theta\right)$$

and

$$f\left(z_t|l_t, y^{t-1}, l^{t-1}, z^{t-1}; \theta\right) = f\left(z_t|l_t, z_{t-1}; \theta\right)$$

Property M2: Using the definition of conditional densities, $f\left(z_t|z_{t+1}, y^t, l^{t+1}; \theta\right)$ can be written as,

$$f\left(z_t|z_{t+1}, y^t, l^{t+1}; \theta\right) = \frac{f\left(z_t, z_{t+1}|y^t, l^{t+1}; \theta\right)}{f\left(z_{t+1}|y^t, l^{t+1}; \theta\right)} = \frac{f\left(z_{t+1}|z_t, y^t, l^{t+1}; \theta\right) f\left(z_t|y^t, l^{t+1}; \theta\right)}{f\left(z_{t+1}|y^t, l^{t+1}; \theta\right)}.$$

Smoother algorithm:

1. $t = T$. We begin with $z_T|y^T, l^T \sim f\left(z_T|y^T, l^T; \theta\right)$.
2. Step t. Assume that the smoothing distribution of the prior step is available $f\left(z_{t+1}|y^T, l^T; \theta\right)$, then we can use properties **M1** and **M2** to derive a recursion:

$$\begin{aligned}
f\left(z_t, z_{t+1}|y^T, l^T; \theta\right) &= f\left(z_t|z_{t+1}, y^T, l^T; \theta\right) f\left(z_{t+1}|y^T, l^T; \theta\right) \\
[\textbf{by M1}] &= f\left(z_t|z_{t+1}, y^t, l^{t+1}; \theta\right) f\left(z_{t+1}|y^T, l^T; \theta\right) \\
[\textbf{by M2}] &= \left(\frac{f\left(z_{t+1}|z_t, y^t, l^{t+1}; \theta\right) f\left(z_t|y^t, l^{t+1}; \theta\right)}{f\left(z_{t+1}|y^t, l^{t+1}; \theta\right)}\right) f\left(z_{t+1}|y^T, l^T; \theta\right) \\
[\textbf{by ASQ}] &= \left(\frac{f\left(z_{t+1}|z_t, y^t, l^{t+1}; \theta\right) f\left(z_t|y^t, l^t; \theta\right)}{f\left(z_{t+1}|y^t, l^{t+1}; \theta\right)}\right) f\left(z_{t+1}|y^T, l^T; \theta\right).
\end{aligned}$$

[2] z_t can solved in terms of gaussian shocks from our system of equations if $\{z_{t+1}, y^t, l^{t+1}; \theta\}$ are known. This can be seen by rewriting our system of equations as:

$$\begin{aligned}
y_t &= H(l_t)\left(z_t + R^{1/2}\omega_t^*\right), & \omega_t^* &\sim N(0,1) \quad \text{[Meaurement]} \\
z_{t+1} &= Fz_t + B(l_{t+1}) + Q(l_{t+1})^{1/2} \nu_{t+1}^*, & \nu^*{}_{t+1} &\sim N(0,1) \quad \text{[State]} \\
z_0 &\sim N(0, Q_0).
\end{aligned}$$

We then integrate over z_{t+1} to obtain our smoothed estimates

$$f\left(z_t|y^T,l^T;\theta\right)=\int f\left(z_t,z_{t+1}|y^T,l^T;\theta\right)dz_{t+1}=f\left(z_t|y^t,l^t;\theta\right)\int\frac{f\left(z_{t+1}|z_t,y^t,l^{t+1};\theta\right)}{f\left(z_{t+1}|y^t,l^{t+1};\theta\right)}f\left(z_{t+1}|y^T,l^T;\theta\right)dz_{t+1}.$$

3. Iterating backwards, this completes the Kalman smoother derivation under **ASQ.**

Discussion. The Kalman smoothing procedure under **ASQ** is identical to that of **S**. The reason for this is that $f\left(z_T|y^T,l^T;\theta\right)$ is identical, and the integrand associated with the smoothing step in (2) involves densities computed from the Kalman filter, which are also identical (as shown in Section A.2).

Putting the results of Section A.2 and A.3 together, we have shown that the output from the Kalman filter and Kalman smoother is identical under strict exogeneity **(S)** and augmented sequential exogeneity (**ASQ)**. So far we have assumed the parameters of the income process are known. We next write out the likelihood that we will maximize to obtain the parameters of our income process. In the following section, we show that will will obtain the same parameters to maximize the likelihood under strict exogeneity **(S)** and augmented sequential exogeneity (**ASQ)**. Further, we will show that we can focus on the income process parameters (θ) because of the **ASQ** assumption.

A.4 Full information likelihood and E-Step under ASQ

Our preferred approach to maximizing the likelihood is to use the Expectations Maximization (EM) algorithm. The EM algorithm requires using the full information log likelihood as well as estimates from the Kalman smoother. In this section, we show that the E-step of our EM algorithm is identical under **ASQ** and **S.**

First, we define the full information log likelihood (given initial conditions):

$$\begin{aligned}
f\left(\{y_t,z_t,l_t\}_{t=1}^T;\{\theta,\lambda\}\right)&=f\left(y_T,z_T,l_T|y^{T-1},l^{T-1},z^{T-1};\{\theta,\lambda\}\right)f\left(y^{T-1},l^{T-1},z^{T-1};\{\theta,\lambda\}\right)\\
&=f\left(y_T|z_T,l_T,y^{T-1},l^{T-1},z^{T-1};\{\theta,\lambda\}\right)f\left(z_T|l_T,y^{T-1},l^{T-1},z^{T-1};\{\theta,\lambda\}\right)f\left(l_T|y^{T-1},l^{T-1},z^{T-1};\{\theta,\lambda\}\right)f\left(y^{T-1},l^{T-1},z^{T-1};\{\theta,\lambda\}\right)\\
[\textbf{by LS and ASQ}]&=f\left(y_T|z_T,l_T,y^{T-1},l^{T-1},z^{T-1};\theta\right)f\left(z_T|l_T,y^{T-1},l^{T-1},z^{T-1};\theta\right)f\left(l_T|y^{T-1},l^{T-1};\lambda\right)f\left(y^{T-1},l^{T-1},z^{T-1};\{\theta,\lambda\}\right)\\
[\textbf{by LS and M1}]&=f\left(y_T|z_T,l_T;\theta\right)f\left(z_T|l_T,z_{T-1};\theta\right)f\left(l_T|y^{T-1},l^{T-1};\lambda\right)f\left(y^{T-1},l^{T-1},z^{T-1};\{\theta,\lambda\}\right)\\
[\text{Iterating back}]&=\Pi_{t=1}^T f\left(y_t|z_t,l_t;\theta\right)f\left(z_t|l_t,z_{t-1};\theta\right)f\left(l_t|y^{t-1},l^{t-1};\lambda\right)f\left(z_0;\theta\right)
\end{aligned}$$

Partial log likelihood. We then take logs and define the partial log likelihood:

$$\ln\Pi_{t=1}^T f\left(y_t|z_t,l_t;\theta\right)f\left(z_t|l_t,z_{t-1};\theta\right)f\left(z_0;\theta\right).$$

We denote each generic element of the partial log likelihood:

$$L\left(y_t, z_t | l_t, z_{t-1}; \theta\right) = \ln f\left(y_t | z_t, l_t; \theta\right) f\left(z_t | l_t, z_{t-1}; \theta\right).$$

It is efficient to maximize the partial log likelihood since our only parameters of interest are in θ (Greene (2003), Chapter 14.5).

E-step under ASQ: Based on our Kalman smoother under **ASQ**, we obtain the joint distribution $f\left(z_t, z_{t-1} | y^T, l^T; \theta\right)$. We then integrate each element of the partial log likelihood with respect to $f\left(z_t, z_{t-1} | y^T, l^T; \theta\right)$, yielding

$$\int L\left(y_t, z_t | l_t, z_{t-1}; \theta\right) df\left(z_t, z_{t-1} | y^T, l^T; \theta\right) = \int \ln f\left(y_t | z_t, l_t; \theta\right) df\left(z_t, z_{t-1} | y^T, l^T; \theta\right) + \int \ln f\left(z_t | l_t, z_{t-1}; \theta\right) df\left(z_t, z_{t-1} | y^T, l^T; \theta\right).$$

which we then sum across people and time and optimize, as in Dempster et al. (1977).

Discussion. The E-step under **ASQ** is identical to that of **S** since the smoothed posteriors are identical.

B Additional details: Estimation

In this appendix, we provide additional details on our estimation procedure. In Appendix B.1 we discuss the identification of our income process. In Appendix B.2 we present the "Kalman smoother." In Appendix B.3 we discuss the role of normally distributed shocks in our estimation procedure. In Appendix B.4, we discuss how we compute standard errors.

B.1 Identification

In this appendix, we discuss the identification of model parameters under various employment processes. We first formally demonstrate identification under strict exogeneity and the assumption of 1st order Markov employment (Appendix B.1.1). We provide a comparable proof under strict exogeneity of the employment process alone (Appendix B.1.2). We then provide a theoretical argument that identification under strict exogeneity will imply identification under augmented sequential exogeneity under modest conditions on the employment process (Appendix B.1.3). Next, we conduct two Monte Carlo exercises. First, we illustrate that the model works well in simulations which satisfy augmented sequential exogeneity of the employment process given a relationship between lagged earnings and unemployment disciplined by the data (Appendix B.1.4). Finally, we conduct similar simulations to demonstrate the ability of our filtering-EM procedure to recover the correct parameters when employment depends on lagged latent states and so augmented sequential exogeneity is not satisfied (Appendix B.1.5). Lastly, we provide an alternate set of results based on the moments which pin down the model parameters when employment is assumed to follow a first order Markov process (Appendix B.1.6). Despite declining earnings risk at 1-year and 2-year horizons, we infer rising persistent earnings risk in this scenario.

For this appendix, we adopt a notational convention for brevity with regards to employment. In this appendix, let $l_{i,t} = 1$ when an individual is employed (i.e., when they have strictly positive earnings ($Y_{i,t} > 0$)) and let $l_{i,t} = 0$ when they are unemployed (i.e., have zero/missing earnings).

B.1.1 Identification with 1st order Markov employment

Suppose $l_{i,t}$ follows a first order Markov process, a special case of strict exogeneity (assumption **S**). This is the case in our model in Section I if we were to make the fairly common assumption that the separation rate is the same across jobs (i.e., $\delta(Y_{-1}) = \bar{\delta}$). First, we discuss how the persistence parameter F can be identified. Observe that $cov(y_{i,t}, y_{i,t-1}|l_{i,t} = l_{i,t-1} = 1) = Fvar(z_{i,t-1} \,|\, l_{i,t-1} = 1)$ when $l_{i,t} = 1$ and $l_{i,t-1} = 1$ and that $cov(y_{i,t+1}, y_{i,t-1}|l_{i,t+1} = l_{i,t-1} = 1) = F^2 var(z_{i,t-1} \,|\, l_{i,t-1} = 1)$ when $l_{i,t+1} = 1$ and $l_{i,t-1} = 1$.[3] We can recover F as a ratio of covariances of nonmissing earnings:

$$F = \frac{cov(y_{i,t+1}, y_{i,t-1}|l_{i,t+1} = l_{i,t-1} = 1)}{cov(y_{i,t}, y_{i,t-1}|l_{i,t} = l_{i,t-1} = 1)} = \frac{F^2 var(z_{i,t-1} \,|\, l_{i,t-1} = 1)}{Fvar(z_{i,t-1} \,|\, l_{i,t-1} = 1)}. \tag{4}$$

With F identified, we can define $\tilde{\Delta} y_{i,t} \equiv y_{i,t} - Fy_{i,t-1}$ to be the "quasi-difference" in earnings for an individual i in year t (where $\tilde{\Delta} y_{i,t}$ is non-missing when earnings is observed in both

[3] The second expression follows from the law of total covariance:

$$\begin{aligned} cov(y_{i,t+1}, y_{i,t-1}|l_{i,t+1} = l_{i,t-1} = 1) &= F^2 cov(z_{i,t-1}, z_{i,t-1} \,|\, l_{i,t+1} = l_{i,t-1} = 1) + E[cov(v_{i,t+1} + Fv_{i,t}, z_{i,t-1} \,|\, l_{i,t}) \,|\, l_{i,t+1} = l_{i,t-1} = 1] \\ &+ cov(B_E + FB(l_{i,t}), E[z_{i,t-1}|l_{i,t}] \,|\, l_{i,t+1} = l_{i,t-1} = 1) = F^2 cov(z_{i,t-1}, z_{i,t-1}|l_{i,t-1} = 1), \end{aligned}$$

where the last equality holds because l_{it} follows a first order Markov process, so $l_{i,t+1}$, $l_{i,t}$, and shocks from $t+1$ and t convey no information about $z_{i,t-1}$.

periods). Using the income process specified in equations (4) and (5), we then have that

$$\text{mean}(\tilde{\Delta} y_{i,t} | l_{i,t} = l_{i,t-1} = 1) = B_E \tag{5}$$
$$\text{var}(\tilde{\Delta} y_{i,t} | l_{i,t} = l_{i,t-1} = 1) = Q_E + (1 + F^2)R. \tag{6}$$

Next, consider the second "quasi-difference", $\tilde{\Delta}^2 y_{i,t+1} \equiv y_{i,t+1} - F^2 y_{i,t-1}$, which is also defined only for those with observed income in periods $t+1$ and $t-1$. For those individuals employed in t, the conditional variance of $\tilde{\Delta}^2 y_{i,t+1}$ is

$$\text{var}(\tilde{\Delta}^2 y_{i,t+1} | l_{i,t+1} = l_{i,t} = l_{i,t-1} = 1) = (1 + F^2) Q_E + (1 + F^4) R, \tag{7}$$

For the individuals unemployed in period t, the conditional mean and variance satisfy

$$\text{mean}(\tilde{\Delta}^2 y_{i,t+1} | l_{i,t} = 0,\ l_{i,t+1} = l_{i,t-1} = 1) = F B_U + B_E, \tag{8}$$
$$\text{var}(\tilde{\Delta}^2 y_{i,t+1} | l_{i,t} = 0,\ l_{i,t+1} = l_{i,t-1} = 1) = F^2 Q_U + Q_E + (1 + F^4) R. \tag{9}$$

Finally, we have initial income observations for all individuals (recall $l_{i,1} = 1$ for all individuals):

$$y_{i,1} = z_{i,1} + \omega_{i,1} = F z_{i,0} + B_E + \nu_{i,1} + \omega_{i,1}. \tag{10}$$

Taking the variance of equation (10), we have:

$$\text{var}(y_{i,1}) = F^2 Q_0 + Q_E + R. \tag{11}$$

The income process specified in Section II contains 7 parameters $(Q_E, Q_U, Q_0, R, B_E, B_U, F)$. The structure of the income process and these parameters then make predictions about the mean and variance of (quasi) earnings changes at different horizons and for different employment statuses, which are summarized by the following 7 equations: (4), (5), (6), (7), (8), (9), and (11).

B.1.2 Identification with strict exogeneity

Above, we required that $l_{i,t}$ followed a 1st order Markov process, which is a stronger assumption than our definition of strict exogeneity. Here, we discuss how the identification argument changes for a general $l_{i,t}$ process which satisfies strict exogeneity. Relative to the above section, only one adjustment is required to the procedure. Specifically, we need to recover F by computing a ratio of covariances for a subset of individuals who are consecutively employed for three periods. Observe that $cov(y_{i,t}, y_{i,t-1}|l_{i,t+1} = l_{i,t} = l_{i,t-1} = 1) = Fvar(z_{i,t-1}|l_{i,t+1} = l_{i,t} = l_{i,t-1} = 1)$ and that $cov(y_{i,t+1}, y_{i,t-1}|l_{i,t+1} = l_{i,t} = l_{i,t-1} = 1) = F^2var(z_{i,t-1}|l_{i,t+1} = l_{i,t} = l_{i,t-1} = 1)$. Thus, we can recover F via,

$$F = \frac{cov(y_{i,t+1}, y_{i,t-1}|l_{i,t+1} = l_{i,t} = l_{i,t-1} = 1)}{cov(y_{i,t}, y_{i,t-1}|l_{i,t+1} = l_{i,t} = l_{i,t-1} = 1)} = \frac{F^2var(z_{i,t-1}|l_{i,t+1} = l_{i,t} = l_{i,t-1} = 1)}{Fvar(z_{i,t-1}|l_{i,t+1} = l_{i,t} = l_{i,t-1} = 1)}. \tag{12}$$

All remaining moment restrictions for means and variance of quasi-differences fully condition on the relevant sequences of $l_{i,t}$ and therefore are still valid under arbitrary strict exogeneity. However, we do require that the relevant sequences of $l_{i,t}$ discussed above occur with positive probability in the data, so that we can still compute the relevant moments needed to identify the parameters. A sufficient condition is that for all $t > 1$, $P(l_{i,t} = 1|l_i^{t-1}) \in (0, 1)$.

These arguments are straightforward to extend to a specification in which variances vary depending on both age and calendar time. Since age is deterministic conditional on its initial level, these specifications satisfy strict exogeneity. See for instance, Karahan and Ozkan (2013) who use similar moment restrictions to those which appear above to prove that an income process with shocks that vary by time and age can be identified. While Karahan and Ozkan (2013) abstract from unemployment risk, additional parameters capturing means and variances of shocks conditional on unemployment can be estimated using similar long differences of earnings around a year with zero earnings for people of the same age observed in the same years.

B.1.3 Identification of econometric framework under augmented sequential exogeneity.

In this section we show identification of our econometric framework under augmented sequential exogeneity (**ASQ**). For brevity, we omit individual subscripts in this subsection. In Section A.2 above, we established that the partial log likelihood function $\ln f(y_t|y^{t-1}, l^{t-1}; \theta)$ is identical under strict exogeneity (**S**) and augmented sequential exogeneity (**ASQ**). However, the joint distribution of the conditioning variables (y^{t-1}, l^{t-1}) will not be the same. To see this, taking the limit as the number of individuals in the sample gets large, the cross sectional average of the partial log likelihoods across people will converge to

$$\mathcal{L}(\theta) \equiv \sum_{t=1}^{T} \int \log f\left(y_t \,|\, y^{t-1}, l^t; \theta\right) f\left(y^t, l^t; \{\theta_0, \lambda_0\}\right) dy^t dl^t = \sum_{t=1}^{T} E\left[\log f\left(y_t \,|\, y^{t-1}, l^t; \theta\right)\right].$$

Let θ_0 be the true vector of income process parameters and denote by $f^*\left(y^t, l^t; \{\theta_0, \lambda_0\}\right)$ the joint distribution of l^t and y^t which would have obtained in a hypothetical world in which 1) l^t

has the same marginal distribution and 2) (l^t, y^t) satisfies assumption (**S**), which we define as:

$$f^*\left(y^t, l^t; \{\theta_0, \lambda_0\}\right) = \left[\Pi_{t=1}^t f\left(y_t \,|\, y^{t-1}, l^t; \theta_0\right)\right]\left[\Pi_{t=1}^t f\left(l_t \,|\, l^{t-1}; \{\theta_0, \lambda_0\}\right)\right], \tag{13}$$

where $f\left(l_t \,|\, l^{t-1}; \{\theta_0, \lambda_0\}\right)$ captures the marginal distribution of l_t given l^{t-1} (i.e., after integrating out y^{t-1}). We can therefore rewrite $\mathcal{L}(\theta)$ as

$$\begin{aligned}
\mathcal{L}(\theta) &= \sum_{t=1}^{T} \int \log f\left(y_t \,|\, y^{t-1}, l^t; \theta\right) \frac{f\left(y^t, l^t; \{\theta_0, \lambda_0\}\right)}{f^*\left(y^t, l^t; \{\theta_0, \lambda_0\}\right)} f^*\left(y^t, l^t; \{\theta_0, \lambda_0\}\right) dy^t dl^t \\
&= \sum_{t=1}^{T} \int \log f\left(y_t \,|\, y^{t-1}, l^t; \theta\right) \frac{\Pi_{k=1}^t f\left(l_k \,|\, y^{k-1}, l^{k-1}; \lambda_0\right)}{\Pi_{j=1}^t f\left(l_j \,|\, l^{j-1}; \{\theta_0, \lambda_0\}\right)} f^*\left(y^t, l^t; \{\theta_0, \lambda_0\}\right) dy^t dl^t \quad (14) \\
&\equiv \sum_{t=1}^{T} E^*\left[\frac{\Pi_{k=1}^t f\left(l_k \,|\, y^{k-1}, l^{k-1}; \lambda_0\right)}{\Pi_{j=1}^t f\left(l_j \,|\, l^{j-1}; \{\theta_0, \lambda_0\}\right)} \log f\left(y_t \,|\, y^{t-1}, l^t; \theta\right)\right],
\end{aligned}$$

where $E^*(\cdot)$ denotes the expectation which would obtain under $f^*\left(y^t, l^t; \{\theta_0, \lambda_0\}\right)$.[4] Equation (14) illustrates that the objective function being maximized under ASQ is a weighted average of the one which would have obtained under strict exogeneity and the same distribution of l^t. We can also write

$$\mathcal{L}(\theta) - \mathcal{L}(\theta_0) = \sum_{t=1}^{T} E^*\left[\frac{\Pi_{k=1}^t f\left(l_k \,|\, y^{k-1}, l^{k-1}; \lambda_0\right)}{\Pi_{j=1}^t f\left(l_j \,|\, l^{j-1}; \{\theta_0, \lambda_0\}\right)} \log \frac{f\left(y_t \,|\, y^{t-1}, l^t; \theta\right)}{f\left(y_t \,|\, y^{t-1}, l^t; \theta_0\right)}\right].$$

Each term in the sum is a weighted average of the Kullback-Leibler divergence (under E^*) between the distribution of y_t given the parameters θ relative to its true conditional distribution, which is minimized and equal to zero at the true θ_0. Imposing mild restrictions on the employment process, essentially that the two distributions have the same support, is sufficient to guarantee that identification under (**S**) will imply identification under (**ASQ**). For example, a sufficient condition would be $f\left(l_k \,|\, y^{k-1}, l^{k-1}; \lambda_0\right) \in (0,1)$ for all y^{k-1}, l^{k-1}. Our estimates of the probability of experiencing a zero earnings draw given lagged income in Figure A19 are consistent with this sufficient condition holding.

What is the implication of this result? If the likelihood function has a unique maximizer under (**S**), the maximizer of the likelihood under (**ASQ**) will also be unique provided the support restrictions discussed above are satisfied. This implies one can use GMM-type moment restrictions to establish identification, as is common practice in the income process literature. As we note in the main text, these moment conditions have the potential to be biased under ASQ,

[4] Note the second equality comes from using equations 3 and 13.

which can create problems for the standard GMM approach. However, the result above implies that the model will be identified under the weaker ASQ assumption after imposing plausible restrictions on the employment process.

In the next two sections, we also conduct simulation exercises to demonstrate that the model works well, even in small samples, under an employment process satisfying ASQ and also under a version with unemployment probabilities depending on latent z.

B.1.4 Monte carlo evidence under augmented sequential exogeneity

In this appendix, we numerically establish identification under augmented sequential exogeneity. We follow Altonji et al. (2013) and provide Monte Carlo experiments in Table A1 below to establish local identification. Given a hypothesized set of parameters, we simulate data from the income process and then verify maximum likelihood estimation based on equations (2) and (3) yields parameters close to the hypothesized values.[5]

For this MC exercise, we simulate data for 2,500 individuals using the parameters of our baseline income process presented in Table 2. We simulate data for 2,500 individuals as this is a typical sample size in panel data sets such as the PSID. We vary the number of observations for each individual between 35 and 5 to examine the degree to which our method can recover the parameters of the income process in panels with shorter and longer time dimensions.

For this MC exercise, we must specify a law of motion for employment. We use a law of motion that is consistent with our augmented sequential exogeneity (ASQ) assumption. In particular, we define the probability of becoming unemployed to be a function of prior earnings. We use the coefficient estimates from Section F.1, which are presented in Table A12 and in Figure A19 we show the implied probability of entering unemployment by prior earnings.[6] In this employment process, we also set the probability an unemployed worker becomes employed (p) to be 43.9%.

Table A1 presents the average parameter values across the estimations as well as the t-statistic for the difference between the estimated parameter and the true parameter. The method is able to very accurately recover the parameters of the income process. Even when there are very few observations for each individual, the method can recover the parameters within the 95% confidence interval. In Appendix D.6 we report additional results from this MC exercise to

[5] As Altonji et al. (2013) write, "...Consequently, we use Monte Carlo experiments extensively to establish local identification and analyze the adequacy of our auxiliary model given the sample size and demographic structure of the available data and to check for bias. For a hypothesized vector of parameter values, we simulate data and then verify that the parameter values that maximize the likelihood function of the auxiliary model are close to the hypothesized values."

[6] Note since we are considering the full set of years, we set $\alpha_E = 0$.

show that our estimation routine is able to accurately recover estimates of persistent earnings at the individual level.

Table A1: Monte Carlo Exercise: Recovering Parameters

	True Value	Estimated Parameters						
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
T		35	30	25	20	15	10	5
Q_E	0.0891	0.0879	0.0878	0.0878	0.0873	0.0868	0.0860	0.0779
		(-1.3594)	(-1.3645)	(-0.9824)	(-1.1995)	(-1.6992)	(-1.4223)	(-2.0062)
Q_U	0.3169	0.3164	0.3154	0.3146	0.3165	0.3147	0.3142	0.2894
		(-0.0408)	(-0.1396)	(-0.1676)	(-0.0314)	(-0.1221)	(-0.1309)	(-0.9424)
R	0.0336	0.0345	0.0346	0.0346	0.0349	0.0353	0.0358	0.0405
		(1.6325)	(1.4199)	(1.0388)	(1.3371)	(1.7794)	(1.6228)	(2.1952)
B_E	0.0017	0.0018	0.0016	0.0018	0.0019	0.0020	0.0010	0.0014
		(0.1089)	(-0.0630)	(0.0932)	(0.1318)	(0.1570)	(-0.3152)	(-0.0616)
B_U	-0.2015	-0.2009	-0.2000	-0.2004	-0.1998	-0.2028	-0.2013	-0.1929
		(0.0795)	(0.1913)	(0.1206)	(0.1645)	(-0.1103)	(0.0129)	(0.3942)
Q_0	0.6594	0.6545	0.6523	0.6581	0.6570	0.6587	0.6595	0.6577
		(-0.1601)	(-0.2023)	(-0.0387)	(-0.0796)	(-0.0250)	(0.0043)	(-0.0463)
F	0.9240	0.9247	0.9247	0.9244	0.9248	0.9248	0.9262	0.9333
		(0.5000)	(0.4351)	(0.2322)	(0.5058)	(0.3789)	(0.6723)	(1.5238)

Notes: Table presents the average parameter values recovered by the estimation procedure in the simulated data for the Monte Carlo exercise with an ASQ employment process. T denotes the number of periods simulated, and in each simulation 2,500 individuals are simulated. We repeat the simulations 50 times. In parentheses we report the t-statistics that the average value of the recovered parameter is statistically different from the true value.

B.1.5 Numerical performance of algorithm when ASQ fails.

In this appendix, we relax our assumption on augmented sequential exogeneity for the employment process and examine how well our procedure performs in recovering the parameters of the income process. In this MC exercise, we make the probability that an individual transitions into unemployment in a period t a function of their lagged persistent earnings ($z_{i,t-1}$) rather than their lagged (residual) earnings ($y_{i,t-1}$) as in Appendix B.1.4. This form of the employment process satisfies sequential exogeneity but not our stronger assumption of augmented sequential exogeneity. We use the same parameter estimates for the employment process as well as the income process as in the MC exercise above. Table A2 presents the results of estimating the income process parameters under the sequentially exogenous income process. The table shows that we continue to be able to recover the parameters of the income process very well under the assumption of sequential exogeneity.[7]

[7] In results that are available upon request, we also find that we are able to recover the individual level estimates of persistent earnings under the assumption of sequential exogeneity.

Table A2: Monte Carlo Exercise: Recovering Parameters Under Sequential Exogeneity

	True Value	Estimated Parameters						
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
T		35	30	25	20	15	10	5
Q_E	0.0891	0.0879	0.0878	0.0879	0.0873	0.0868	0.0861	0.0779
		(-1.2445)	(-1.3205)	(-0.9552)	(-1.1637)	(-1.7017)	(-1.3391)	(-1.9723)
Q_U	0.3169	0.3184	0.3170	0.3182	0.3186	0.3170	0.3172	0.2914
		(0.1273)	(0.0135)	(0.0899)	(0.1111)	(0.0064)	(0.0158)	(-0.8282)
R	0.0336	0.0344	0.0346	0.0346	0.0349	0.0353	0.0358	0.0405
		(1.5516)	(1.4219)	(1.0204)	(1.3045)	(1.7906)	(1.5641)	(2.1951)
B_E	0.0017	0.0027	0.0025	0.0028	0.0029	0.0029	0.0019	0.0025
		(0.8781)	(0.6894)	(0.9396)	(0.7102)	(0.5918)	(0.0905)	(0.1695)
B_U	-0.2015	-0.2114	-0.2096	-0.2104	-0.2089	-0.2097	-0.2064	-0.1953
		(-1.2718)	(-0.9885)	(-0.9163)	(-0.7145)	(-0.7396)	(-0.3818)	(0.2843)
Q_0	0.6594	0.6568	0.6544	0.6596	0.6588	0.6607	0.6611	0.6602
		(-0.0861)	(-0.1430)	(0.0057)	(-0.0196)	(0.0493)	(0.0524)	(0.0220)
F	0.9240	0.9234	0.9235	0.9231	0.9236	0.9235	0.9249	0.9317
		(-0.4836)	(-0.3378)	(-0.5215)	(-0.2700)	(-0.2259)	(0.2801)	(1.2106)

Notes: Table presents the average parameter values recovered by the estimation procedure in the simulated data for the Monte Carlo exercise with a sequentially exogenous (SQ) employment process. T denotes the number of periods simulated, and in each simulation 2,500 individuals are simulated. We repeat the simulations 50 times. In parentheses we report the t-statistics that the average value of the recovered parameter is statistically different from the true value.

B.1.6 GMM results

In this section, we show results from using the moments from the first order Markov employment case (Section B.1.1) to inform the parameters of our income process. In particular, we measure the simple moments in equations (4), (5), (6), (7), (8), (9), and (11) and then compute the implied persistent and temporary income risk paths over time.

Panel (a) of Figure A1 plots the variance of the quasi-difference of changes in log earnings over a 1-year horizon (black, solid line) and a 2-year horizon (red, dashed line) among individuals who are employed in the middle year.[8] The figure shows that the variance of earnings changes over a 1-year horizon has declined between 1985 and 2015, while the variance of earnings changes over a two-year horizon have declined more modestly.

In panel (b) of Figure A1, we plot the path of persistent income risk (black, solid line) and temporary earnings risk (red, dashed line) implied by the identification argument in Appendix B.1. The figure shows that persistent income risk rises between 1985 and 2015, while temporary earnings risk decreases. The figure also shows that we obtain similar cyclical patterns using these "simple moments" as in our full estimation from Section IV. In particular, the implied path of persistent earnings risk experiences a decline after the Great Recession, while temporary earnings risk spikes around the 2001 and 2008-09 recessions.

We next present additional results about how trends in the risk faced by unemployed workers evolve over time using the simple moments highlighted above. In Panel (a) of Figure A2, we plot the variance of the (quasi) earnings change over two-years for individuals who are unemployed in the middle year. The figure shows that the variance of these earnings changes has increases substantially over the sample period. Accordingly, panel (c) of Figure A2 shows that the implied variance of shocks to persistent earnings among the unemployed has increased over the sample period. In panel (b) of Figure A2, we plot the mean of the (quasi) difference in earnings over two years around an unemployment spell. The figure shows that the decline in earnings around these unemployment spells has gotten larger over our sample period. This acceleration in earnings declines around unemployment spells generates a larger mean decline in persistent earnings during unemployment, which we show in panel (d) of Figure A2.

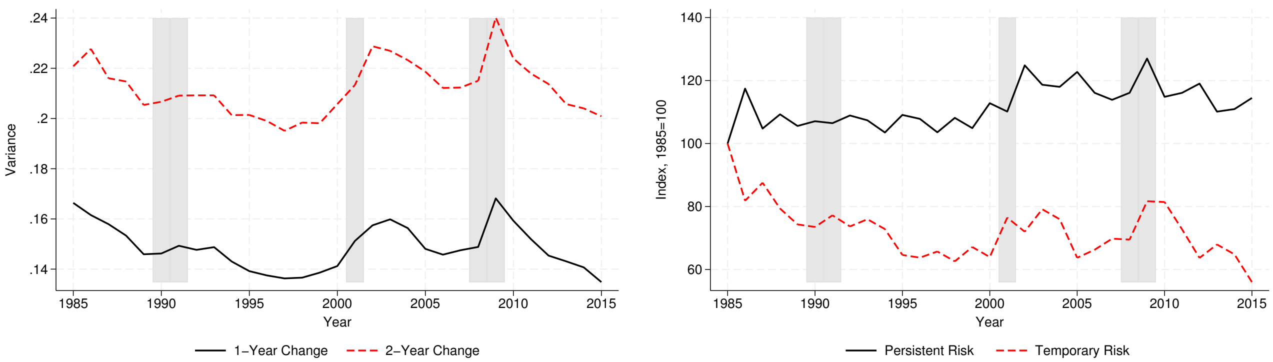
B.2 Kalman smoother

As discussed in Section II.A after running the Kalman filter, we run the Kalman smoother to update our estimates of the unobserved state variable (i.e., persistent earnings and its lag.) In

[8] For the quasi-differences, we use the measure of persistence (F) as implied by equation (4), which measures F using the ratio of covariances of earnings over different horizons. This produces an estimate of $F = 0.924$. Given F, we then treat each period as if the model is in steady state.

Figure A1: Identifying changes in risk over time among employed

(a) Variance of Earnings Changes At Different Horizons (b) Implied Path of Temporary and Persistent Risk



Note: Panel (a) plots of the variance of the quasi-difference in log earnings over a one year horizon (black, solid line) and a two year horizon (red, dashed line), where the individual was employed in the middle year. Panel (b) plots the implied path of persistent income risk (black, solid line) and temporary income risk (red, dashed line) using the moments from Panel (a) and the identification argument in Appendix B.1. Gray bars denote NBER recession dates.

Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

this appendix, we present the algorithm for running the Kalman smoother.

The steps for the smoothed Kalman filter are:

1. Run the Kalman filter as presented in Section II.A storing the sequences $\{M_{i,t|t-1}\}_{t=1}^{T}$ and $\{M_{i,t|t}\}_{t=1}^{T}$ as well as $\{\hat{\zeta}_{i,t|t-1}\}_{t=1}^{T}$ and $\{\hat{\zeta}_{i,t|t}\}_{t=1}^{T}$.

2. Store the element $\hat{\zeta}_{i,T|T}$ from $\{\hat{\zeta}_{i,t|t}\}_{t=1}^{T}$.

3. Calculate the sequence of smoothed estimations $\{\hat{\zeta}_{i,t|T}\}_{t=1}^{T-1}$ in reverse order by iterating on:

$$\hat{\zeta}_{i,t|T} = \hat{\zeta}_{i,t|t} + J_{i,t}(\hat{\zeta}_{i,t+1|T} - \hat{\zeta}_{i,t+1|t})$$

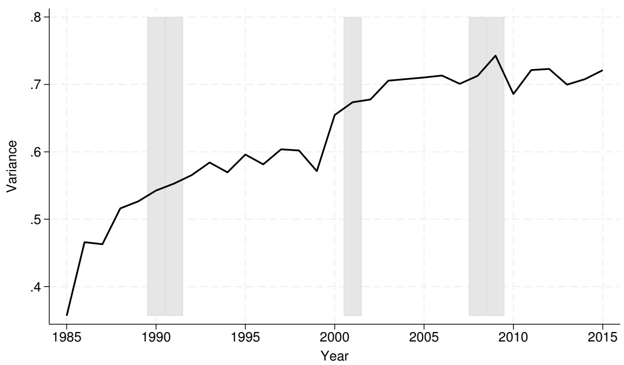
for $t = T-1, T-2, ..., 1$, where $J_{i,t} = M_{i,t|t}\hat{F}'M_{i,t+1|t}^{-1}$.

4. Update the sequence of MSE by iterating on:

$$M_{i,t|T} = M_{i,t|t} + J_{i,t}(M_{i,t+1|T} - M_{i,t+1|t})J_{i,t}'.$$

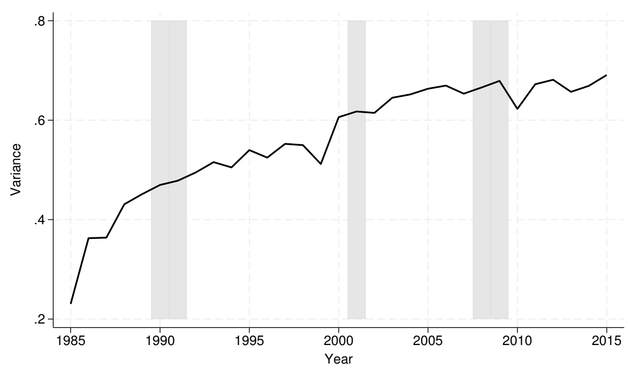
Figure A2: Identifying changes in risk over time among unemployed

(a) Variance Earnings Changes

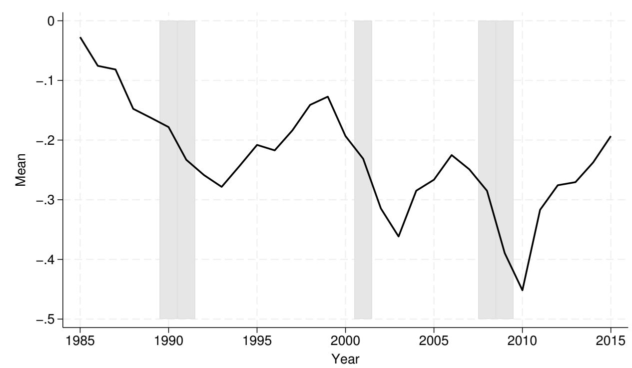


(b) Mean Earnings Changes

(c) Implied Variance of Persistent Shock



(d) Implied Mean of Persistent Shocks



Note: Panel (a) plots the variance of the quasi-difference of log earnings over a two year horizon where the individual was unemployed in the middle year. Panel (b) plots the mean of the quasi-difference of log earnings over a two year horizon where the individual was unemployed in the middle year. Panel (c) plots the implied path of the variance of shocks to persistent earnings among the unemployed, while panel (d) plots the implied path of the mean of shocks to persistent earnings among the unemployed using the identification argument in Appendix B.1. Gray bars denote NBER recession dates.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

B.3 Role of normally distributed shocks

In this appendix, we discuss the role of distributional assumptions for our analysis: (1) we discuss an interpretation of the filter as a linear projection under non-normality, (2) we show that our filtering procedure is robust under non-normal distributions through Monte Carlo simulations, and (3) we show that our benchmark income process exhibits non-normal distributions of persistent and temporary innovations.

Linear projection interpretation. In deriving the likelihood function used to estimate parameters and compute posteriors, we assume that the shocks to temporary and persistent earnings (for both the employed and unemployed) are normally distributed; however, each step of our estimation procedure has a natural interpretation even when shocks are not normally distributed.

The key step of the Kalman filter updates estimates of the unobserved state variable (persistent earnings) using an updating formula which takes the form of a linear projection (detailed in Chapter 4.5 and Chapter 13.2 of Hamilton (1994a)). This allows us to go from equation (1) describing the joint distribution of latent $z_{i,t}$ and observed $y_{i,t}$ given l_i^t, y_i^{t-1} to the conditional distribution of $z_{i,t}$ given l_i^t, y_i^t in equation (2). This updating equation is exact when initial conditions and shocks are normally distributed (conditional on l_i^t, y_i^{t-1}), yielding normally distributed posteriors and conditional mean which can be written as an affine function of y_i^t, where the slope and intercept which depend on l_i^t. Likewise, the posterior variance is a known function of l_i^t captured by the Kalman filter.

When initial conditions and shocks are not normally distributed, the filter which minimizes mean-squared error—the true conditional expectation of $z_{i,t}|y_i^t, l_i^t$—may not necessarily take the linear form in (2). Thus, the output from the Kalman filter $\widehat{z}_{i,t|t}$ may differ from the true conditional expectation of $z_{i,t}$ given y_i^t, l_i^t. However, as is discussed further in Chapter 5 of Anderson and Moore (1979), the update associated with the Kalman filter in a model with parameters that are known functions of time has an interpretation as a *linear minimum variance estimator*.[9] Under strict exogeneity, the model for the evolution of y_i^T conditional on l_i^T falls precisely into this class of estimators. Accordingly, under assumption strict exogeneity the Kalman filter has an interpretation as an optimal linear projection *conditional on the path of employment status* l_i^T–i.e., the projection coefficients are allowed to vary depending on the individual's employment history. Under the weaker ASQ assumption, the recursion from the Kalman filter may no longer

[9]Hamilton (1994b) writes the following about linear state space models, "Thus, while the Kalman filter forecasts need no longer be optimal for systems that are not normal, no other forecast based on a linear function of $[z_t]$ will have a smaller mean squared error... These results parallel the Gauss-Markov theorem for ordinary least squares regression."

be the minimum variance linear projection, though it nonetheless yields unbiased estimates of the mean and variance of the latent state.

In the case in which the state space model is linear, the Kalman filter has a quasi-maximum likelihood interpretation, and maximizing the Gaussian likelihood function yields consistent estimates of state space parameters under nongaussianity (see Hamilton, 1994a, p. 389 for additional discussion). While we conjecture that the extension to strictly exogeneous l_i^T follows almost immediately, we do not establish such a result formally here. When we make the weaker ASQ assumption, we assume that the model is conditionally normal.

Monte Carlo evidence. In Appendix D.6, we test the performance of our filter in non-Gaussian settings with Monte Carlo simulation exercises. We simulate non-normal innovations to equations (4) and (5) for $= 2500$ individuals and $T = 30$ years (comparable to samples from the PSID). We use Guvenen et al. (2021)'s estimated mixture distribution of innovations to both persistent and temporary earnings (parameters presented in their Table 4, column (3)). We then apply our filtering methods to the simulated data. We regress the true persistent earnings (z_{it}) on the recovered estimate of persistent earnings ($\hat{z}_{it}$) and we report the mean and standard deviation of that coefficient in order to assess the algorithm's performance. Our method produces an extremely good fit of the true latent states, with a bias of less than 0.1%. We vary time horizons and find similar results.

Non-normal shocks in estimated process. In Appendix F.5, we show that our estimated income process yields higher order moments that are consistent with Guvenen et al. (2021). In our estimates and Guvenen et al. (2021)'s, the standard deviation and skewness of earnings are negatively correlated with an individual's lagged ranking in the earnings distribution. Moreover, kurtosis is positively correlated with an individual's lagged ranking in the earnings distribution. Thus our simple income process yields non-degenerate higher-order moments that mirror the data, and so we contribute a tractable income process that allows researchers to incorporate rich earnings dynamics into theoretical frameworks.

Non-normality of recovered shocks. Recent work has emphasized that log income changes are non-Gaussian, and exhibit negative skewness as well as excess kurtosis (e.g., Guvenen et al. (2021)). While the shocks to temporary and persistent earnings in our income process are drawn from normal distributions, our income process produces skewness and kurtosis in log earnings changes by incorporating unemployment spells as well as making the shocks functions of other observables. By conditioning on these observables, we naturally estimate mixture distributions; therefore, integrating out these observables yields non-Gaussian shock distributions even if shocks were Gaussian conditional on $l_{i,t}$ and $x_{i,t}$. In an earlier version of this paper

(Braxton et al. (2021), Figure 1), we showed that our estimates of persistent and temporary earnings shocks exhibit negative skewness as well as excess kurtosis relative to a normal distribution. These deviations became especially stark when we incorporate observables such as job switching into the estimation.

B.4 Computing standard errors

In this appendix, we discuss how we compute standard errors of our model parameters. We obtain standard errors on our parameter estimates using a block bootstrap procedure. In the bootstrap procedure, we draw a 5% random sample and run the filtering algorithm on this sub-sample of the data. To obtain greater variation, we also randomly draw weights from an exponential distribution and rescale the survey weights by multiplying the survey weights by these draws (e.g., Barbe and Bertail (2012)). We repeat this exercise 100 times. We obtain standard errors by taking the standard deviation of the parameter estimates across the 100 replications and multiply by the square root of the sampling probability (5%).

C Model appendix

In this appendix, we provide a micro-foundation of the piece-rate wage assumption in Section I under the assumption of strict exogeneity ($\delta(Y) = \bar{\delta}$). Piece-rate earnings can be micro-founded using generalized Nash bargaining (e.g., Kaplan and Menzio (2016)) and is a common assumption in the literature. Let $J(z, \omega)$ be the value of a firm matched with a worker with human capital vector (z, ω). Let α denote worker bargaining power. Kaplan and Menzio (2016) assume that the outside options of the workers and firm are to continue the match and renegotiate next period. The generalized Nash bargaining objective is

$$\max_{Y} \Big\{ Y + \beta(1-\bar{\delta})EW(z',\omega') + \beta\bar{\delta}EU(z',\omega') - \underbrace{be^{z+\omega} - \beta(1-\bar{\delta})EW(z',\omega') - \beta\bar{\delta}EU(z',\omega')}_{\text{Keep match, collect UI}} \Big\}^{\alpha} \ldots$$

$$\ldots \times \Big\{ \underbrace{e^{z+\omega} - Y + \beta(1-\bar{\delta})EJ(z',\omega')}_{=J(z,\omega)} - \underbrace{0 - \beta(1-\bar{\delta})EJ(z',\omega')}_{\text{Keep match, dont produce}} \Big\}^{1-\alpha}$$

By solving the above maximization problem, we arrive at an expression for the wage that yields $Y = (\alpha + b(1-\alpha))e^{z+\omega}$ and thus $\gamma = (\alpha + b(1-\alpha)$ is the piece-rate. We then have that log income y (residualized to remove the piece-rate), can be written as,

$$y \equiv \ln Y - \ln \gamma = z + \omega.$$

D EM algorithm

In this appendix, we outline the EM algorithm we use to estimate the parameters of the income process presented in Section II. In Appendix D.1, we give an overview of the EM algorithm. In Appendix D.2, we present the full-information log likelihood. In Appendix D.3, we derive the expressions for updating the mean (drift) parameters (B) and the persistence parameter F. In Appendix D.4, we drive the expression for updating the variance parameters (Q and R). In Appendix D.5, we write out the full EM algorithm. In Appendix D.6 we present a series of additional Monte-Carlo exercises to validate that our estimation procedure is able to accurately recover the path of persistent earnings at the individual level.

D.1 Overview of EM algorithm

The EM algorithm is an iterative algorithm to update the parameters that govern the income process. To start the algorithm we make an initial guess of the parameters of the income process, and using these parameters create an estimate of the state vector using the Kalman filter presented in Section II.A. The next step in the EM algorithm is to use the estimates of the state vector along with the data to update the estimates of the parameters. The parameters are updated using a series of equations that we derive below. The algorithm then repeats by using the new parameters to update the estimate of the state vector, and then using the estimated state vector and data to update the parameters. This process continues until the log likelihood has been maximized. In the subsections below we derive the equations that will allow for closed form updating of the parameters. Finally, we provide a detailed description of the EM algorithm.

D.2 Log likelihood

The EM algorithm uses a set of closed form updating equations to uncover parameters which allow the log likelihood function to be maximized. To derive these formulas we start with the full-information conditional log likelihood, which is the likelihood function *if* the state-variables are observed. For an individual i, the full information log likelihood appears as:[10]

[10] Note the full information log likelihood is used to derive the equations which update the parameters of the income process via the EM algorithm. The likelihood that is maximized as part of the estimation is given by (8).

$$
\begin{aligned}
LL_i(\{y_{i,t}\}_{t=0}^{T}, \{z_{i,t}\}_{t=0}^{T} \mid \{l_{i,t}\}_{t=1}^{T}, \theta_0) = &-\frac{T+1}{2}\log(2\pi) \\
&-\frac{1}{2}\log(Q_0) - \frac{1}{2}\frac{(z_{i0})^2}{Q_0} \\
&-\frac{1}{2}\sum_{t=1}^{T}\log(Q(l_{i,t})) - \frac{1}{2}\sum_{t=1}^{T}\frac{(z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2}{Q(l_{i,t})} \\
&-\frac{1}{2}\sum_{t=1}^{T}\log(R) - \frac{1}{2}\sum_{t=1}^{T}\frac{(l_{E,i,t})(y_{i,t} - z_{i,t})^2}{R}
\end{aligned}
$$

D.3 Updating means

In this appendix, we derive the expressions that are used to update the mean parameters of the income process (e.g. the persistence of persistent earnings, and drifts of persistent earnings when employed/unemployed). Before deriving the formulas we present a series of useful expressions that will ease the derivations of the updating equations. Additionally, note the following notation. Define $E_T\left[z_{it}|\{y_{it}, l_{it}\}\right] = \hat{z}_{it|T}$, that is the expected value of individual i's persistent earnings in period t (given the data) is denoted by $\hat{z}_{it|T}$, which corresponds to the output of the smoothed Kalman filter. Define $\Sigma_{i0|T}(1,1)$ to be the estimated the variance of initial persistent earnings. Define $\Sigma_{it|T}(1,2)$ to be the estimated covariance between $\hat{z}_{it|T}$ and $\hat{z}_{it-1|T}$.[11]

For simplicity and ease of notation, we will first discuss in detail how to update parameters for the simple income process outlined in section II. Then, we discuss how things extend to the more general case in which F and B are both assumed to be linear in a set of unknown parameters in Section D.3.3.

D.3.1 Useful expressions

In this section, we derive a series of useful expressions that will aid in the derivation of the updating equations in the following subsections.

[11]Note this covariance term is the $(1,2)$ element of the matrix $M_{i,t|T}$.

First, we show that $E_T\left[z_{i0}^2|\{y_{it},l_{it}\}\right] = \Sigma_{i0|T}(1,1) + \hat{z}_{i0|T}^2$,

$$\begin{aligned}
E_T\left[z_{i0}^2|\{y_{it},l_{it}\}\right] &= E_T\left[\left(z_{i0}-\hat{z}_{i0|T}+\hat{z}_{i0|T}\right)^2|\{y_{it},l_{it}\}\right] \\
&= E_T\left[\left(z_{i0}-\hat{z}_{i0|T}\right)^2+\hat{z}_{i0|T}^2+2\left(z_{i0}-\hat{z}_{i0|T}\right)\hat{z}_{i0|T}|\{y_{it},l_{it}\}\right] \\
&= E_T\left[\left(z_{i0}-\hat{z}_{i0|T}\right)^2+\hat{z}_{i0|T}^2|\{y_{it},l_{it}\}\right] \\
&= \Sigma_{i0|T}(1,1)+\hat{z}_{i0|T}^2 \qquad (15)
\end{aligned}$$

where in the third equality we used the fact that $E_T\left[z_{i0}-\hat{z}_{i0|T}|\{y_{it},l_{it}\}\right] = 0$.

Next, we show that $E_T\left[z_{it}z_{i,t-1}|\{y_{it},l_{it}\}\right] = \Sigma_{it|T}(1,2) + \hat{z}_{it|T}\hat{z}_{it-1|T}$,

$$\begin{aligned}
E_T\left[z_{it}z_{i,t-1}|\{y_{it},l_{it}\}\right] &= E_T\left[\left(z_{it}-\hat{z}_{it|T}+\hat{z}_{it|T}\right)\left(z_{i,t-1}-\hat{z}_{it-1|T}+\hat{z}_{it-1|T}\right)|\{y_{it},l_{it}\}\right] \\
&= E_T\left[\left(z_{it}-\hat{z}_{it|T}\right)\left(z_{i,t-1}-\hat{z}_{it-1|T}\right)+\hat{z}_{it|T}\hat{z}_{it-1|T}|\{y_{it},l_{it}\}\right] \\
&= \Sigma_{it|T}(1,2)+\hat{z}_{it|T}\hat{z}_{it-1|T} \qquad (16)
\end{aligned}$$

where in the second equality we have used the fact that $E_T\left[\left(z_{it}-\hat{z}_{it|T}\right)|\{y_{it},l_{it}\}\right] = 0$ and $E_T\left[\left(z_{it-1}-\hat{z}_{it-1|T}\right)|\{y_{it},l_{it}\}\right] = 0$.

D.3.2 Updating F, B_E, B_U

In this section, we derive the expression we will use to update the parameters $\{F, B_E, B_U\}$. The relevant part of the log likelihood for updating the parameters $\{F, B_E, B_U\}$ is given by:

$$\frac{1}{Q(l_{i,t})}\sum_{t=1}^{T}\left(z_{i,t}-Fz_{i,t-1}-B(l_{i,t})\right)^2$$

The expected value can be written as:

$$\frac{1}{Q(l_{i,t})}E_T\left[\left(z_{i,t}-Fz_{i,t-1}-B(l_{i,t})\right)^2|\{y_{it},l_{it}\}\right]$$

Completing the square we obtain the following expression:

$$\frac{1}{Q(l_{i,t})}E_T\left[z_{i,t}^2 - z_{i,t}Fz_{i,t-1} - z_{i,t}B(l_{i,t}) \right.$$
$$+F^2z_{i,t-1}^2 - Fz_{i,t-1}z_{i,t} + Fz_{i,t-1}B(l_{i,t})$$
$$\left. +B(l_{i,t})^2 - B(l_{i,t})z_{i,t} + FB(l_{i,t})z_{i,t-1}|\{y_{it}, l_{it}\}\right]$$

Combining terms we have:

$$\frac{1}{Q(l_{i,t})}E_T\left[z_{i,t}^2 - 2Fz_{i,t}z_{i,t-1} - 2z_{i,t}B(l_{i,t}) \right. \tag{17}$$
$$+F^2z_{i,t-1}^2 + 2Fz_{i,t-1}B(l_{i,t})$$
$$\left. +B(l_{i,t})^2|\{y_{it}, l_{it}\}\right]$$

We will next use expressions from Section D.3.1 to simplify equation (17). First using equation (15) (adjusted for period t, and period $t+1$), we have:

$$\frac{1}{Q(l_{i,t})}\left(\Sigma_{it|T}(1,1) + \hat{z}_{it|T}^2 + F^2\left[\Sigma_{it-1|T}(1,1) + \hat{z}_{it-1|T}^2\right]\right)$$
$$+\frac{1}{Q(l_{i,t})}E_T\left[-2Fz_{i,t}z_{i,t-1} - 2z_{i,t}B(l_{i,t}) + 2Fz_{i,t-1}B(l_{i,t}) \right.$$
$$\left. +B(l_{i,t})^2|\{y_{it}, l_{it}\}\right]$$

Next using equation (16), we have:

$$\frac{1}{Q(l_{i,t})}\left(\Sigma_{it|T}(1,1) + \hat{z}_{it|T}^2 + F^2\left[\Sigma_{it-1|T}(1,1) + \hat{z}_{it-1|T}^2\right]\right)$$
$$+\frac{1}{Q(l_{i,t})}\left(-2F\left[\Sigma_{it|T}(1,2) + \hat{z}_{it|T}\hat{z}_{it-1|T}\right]\right)$$
$$+\frac{1}{Q(l_{i,t})}E_T\left[-2z_{i,t}B(l_{i,t}) + 2Fz_{i,t-1}B(l_{i,t}) + B(l_{i,t})^2|\{y_{it}, l_{it}\}\right]$$

Then taking the expectation over the remaining terms we have:

$$\frac{1}{Q(l_{i,t})}\left(\Sigma_{it|T}(1,1)+\hat{z}^2_{it|T}+F^2\left[\Sigma_{it-1|T}(1,1)+\hat{z}^2_{it-1|T}\right]\right) \qquad (18)$$
$$\frac{1}{Q(l_{i,t})}\left(-2F\left[\Sigma_{it|T}(1,2)+\hat{z}_{it|T}\hat{z}_{it-1|T}\right]\right)$$
$$+\frac{1}{Q(l_{i,t})}\left[\left(-2\hat{z}_{it|T}B(l_{i,t})+2F\hat{z}_{it-1|T}B(l_{i,t})+B(l_{i,t})^2\right)\right]$$

We want to optimize equation (18) with respect to F, B_E and B_U. For ease of exposition, we drop the terms in equation (18) that do not include F, B_E and B_U, which returns:

$$\frac{1}{Q(l_{i,t})}\left(F^2\left[\Sigma_{it-1|T}(1,1)+\hat{z}^2_{it-1|T}\right]\right) \qquad (19)$$
$$\frac{1}{Q(l_{i,t})}\left(-2F\left[\Sigma_{it|T}(1,2)+\hat{z}_{it|T}\hat{z}_{it-1|T}\right]\right)$$
$$+\frac{1}{Q(l_{i,t})}\left[\left(-2\hat{z}_{it|T}B(l_{i,t})+2F\hat{z}_{it-1|T}B(l_{i,t})+B(l_{i,t})^2\right)\right]$$

The expression in (19) gives the expected contribution to the likelihood for individual i in period t. We want to maximize the likelihood across all individuals and time periods. To perform this optimization it will be convenient to define the following vectors and matrices. Define:

$$X_C \equiv \begin{bmatrix} \hat{z}_{1,0|T} & l_{E,1,1} & l_{U,1,1} \\ \hat{z}_{1,1|T} & l_{E,1,2} & l_{U,1,2} \\ \vdots & \vdots & \vdots \\ \hat{z}_{1,T-1|T} & l_{E,1,T} & l_{U,1,T} \\ \hat{z}_{2,0|T} & l_{E,2,1} & l_{U,2,1} \\ \vdots & \vdots & \vdots \\ \hat{z}_{N,T-1|T} & l_{E,N,T} & l_{U,N,T} \end{bmatrix}_{NT\times 3} \qquad C \equiv \begin{bmatrix} F \\ B_E \\ B_U \end{bmatrix}_{3\times 1} \qquad Y_C \equiv \begin{bmatrix} \hat{z}_{1,1|T} \\ \vdots \\ \hat{z}_{1,T|T} \\ \vdots \\ \hat{z}_{N,T|T} \end{bmatrix}_{NT\times 1} \qquad (20)$$

We can rewrite terms in matrix notation as follows:

$$
\begin{aligned}
C^{'}X_C^{'}X_C C &= \sum_{i=1}^{N}\sum_{t=1}^{T}\left(F\hat{z}_{it-1|T} + B_E l_{Eit} + B_U l_{Uit}\right)^2 \\
&= \sum_{i=1}^{N}\sum_{t=1}^{T}\left(F^2\hat{z}^2_{it-1|T} + 2FB_E\hat{z}_{it-1|T}l_{Eit} + 2FB_U\hat{z}_{it-1|T}l_{Uit} + (B_E l_{Eit})^2 + (B_U l_{Uit})^2\right) \\
&= \sum_{i=1}^{N}\sum_{t=1}^{T}\left(F^2\hat{z}^2_{it-1|T} + 2F\hat{z}_{it-1|T}B(l_{i,t}) + B(l_{i,t})^2\right) \qquad \text{using } B(l_{it}) \text{ def. from above.}
\end{aligned}
$$

We also have,

$$
\begin{aligned}
Y_C^{'}X_C C &= F\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{z}_{it|T}\hat{z}_{it-1|T} + B_E\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{z}_{it|T}l_{Eit} + B_U\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{z}_{it|T}l_{Uit} \\
&= F\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{z}_{it|T}\hat{z}_{it-1|T} + \sum_{i=1}^{N}\sum_{t=1}^{T}\hat{z}_{it|T}B(l_{i,t}) \qquad \text{using } B(l_{it}) \text{ def. from above.}
\end{aligned}
$$

To complete writing the sum of the log likelihood across individuals it will be convenient to define the following vectors:

$$
\vec{\sigma}_{t-1}(1,1) \equiv \begin{bmatrix} \Sigma_{1,0|T}(1,1) \\ \Sigma_{1,1|T}(1,1) \\ \vdots \\ \\ \Sigma_{N,T-1|T}(1,1) \end{bmatrix}_{NT\times 1} \qquad \vec{\sigma}_{t}(1,2) \equiv \begin{bmatrix} \Sigma_{1,2|T}(1,1) \\ \Sigma_{1,2|T}(1,2) \\ \vdots \\ \\ \Sigma_{N,T|T}(1,2) \end{bmatrix}_{NT\times 1} \qquad e_1^3 \equiv \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad e^{NT} \equiv \underbrace{\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}}_{(NT\times 1)}
$$

We can make further progress with matrix notation by noting,

$$
\sum_{i=1}^{N}\sum_{t=1}^{T}\left(F^2\Sigma_{it-1|T}(1,1)\right) = C^{'}e_1^3 e_1^{3'}C\vec{\sigma}^{'}_{t-1}(1,1)e^{NT} \tag{21}
$$

and

$$
\sum_{i=1}^{N}\sum_{t=1}^{T}\left(F\Sigma_{it|T}(1,2)\right) = e^{NT^{'}}\vec{\sigma}_t(1,2)e_1^{3'}C. \tag{22}
$$

Using (19) and the definitions above we have the following:

$$\sum_{i=1}^{N}\sum_{t=1}^{T} E_T\left[(z_{i,t}-Fz_{i,t-1}-B(l_{i,t}))^2\,|\{y_{it},x_{it},l_{it}\}\right] = C^{'}X_C^{'}X_CC - 2Y_C^{'}X_CC$$
$$- 2e^{NT^{'}}\vec{\sigma}_t(1,2)e_1^{3'}C + C^{'}e_1^3e_1^{3'}C\vec{\sigma}_{t-1}^{'}(1,1)e^{NT}$$

Finally, define:

$$Q^{-1} \equiv \begin{bmatrix} \frac{1}{Q_1(l_{1,1})} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \frac{1}{Q_2(l_{1,2})} & 0 & \vdots & \vdots & \vdots & \vdots \\ \vdots & 0 & \ddots & 0 & \vdots & \vdots & \vdots \\ \vdots & \vdots & 0 & \frac{1}{Q_T(l_{1,T})} & 0 & \vdots & \vdots \\ \vdots & \vdots & \vdots & 0 & \frac{1}{Q_1(l_{2,1})} & 0 & \vdots \\ \vdots & \vdots & \vdots & \vdots & 0 & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & \frac{1}{Q_T(l_{N,T})} \end{bmatrix}$$

Using (19) and the definitions above we have the following:

$$\sum_{i=1}^{N}\sum_{t=1}^{T} \frac{1}{Q(l_{i,t})} E_T\left[(z_{i,t}-Fz_{i,t-1}-B(l_{i,t}))^2\,|\{y_{it},l_{it}\}\right] = C^{'}X_C^{'}Q^{-1}X_CC - 2Y_C^{'}Q^{-1}X_CC$$
$$- 2e^{NT^{'}}Q^{-1}\vec{\sigma}_t(1,2)e_1^{3'}C$$
$$+ C^{'}e_1^3e_1^{3'}C\vec{\sigma}_{t-1}^{'}(1,1)Q^{-1}e^{NT}$$

Taking the FOC with respect to C returns:

$$0 = 2C^{'}X_C^{'}Q^{-1}X_C - 2Y_C^{'}Q^{-1}X_C - 2e^{NT^{'}}Q^{-1}\vec{\sigma}_t(1,2)e_1^{3'} + 2C^{'}e_1^3e_1^{3'}\vec{\sigma}_{t-1}^{'}(1,1)Q^{-1}e^{NT} \tag{23}$$

Rearranging equation (23) returns:

$$C^{'}\left[X_C^{'}Q^{-1}X_C + e_1^3e_1^{3'}\vec{\sigma}_{t-1}^{'}(1,1)Q^{-1}e^{NT}\right] = Y_C^{'}Q^{-1}X_C + e^{NT^{'}}Q^{-1}\vec{\sigma}_t(1,2)e_1^{3'} \tag{24}$$

Taking the transpose of both sides of equation (24) returns:

$$\left[X_C^{'}Q^{-1}X_C + e_1^3e_1^{3'}\vec{\sigma}_{t-1}^{'}(1,1)Q^{-1}e^{NT}\right]C = X_C^{'}Q^{-1}Y_C + e_1^3\vec{\sigma}_t^{'}(1,2)Q^{-1}e^{NT} \tag{25}$$

where we have exploited the fact that the matrices on the LHS of equation (24) are symmetric.

Equation (25) gives us a closed form equation for updating the parameters $\{F, B_E, B_U\}$.

D.3.3 Extension to the general case

Above, we assumed that the income process was quite simple. In particular, one could write $F_{it} = [0, 0, 1]C = e_1^{3'}C = F$ and $B_{it} = [0, l_{E,i,t}, l_{U,i,t}]C$. It turns out that it is straightforward to extend to a much more flexible setting in which

$$F(l_{it}; x_{it}) \equiv f_F(l_{i,t}; x_{i,t}) = g_F(l_{i,t}; x_{i,t})'\Lambda_{B,F} \tag{26}$$

$$B(l_{i,t}; x_{i,t}) \equiv f_B(l_{i,t}; x_{i,t}) = g_B(l_{i,t}; x_{i,t})'\Lambda_{B,F}, \tag{27}$$

where $g_F(l_{i,t}; x_{i,t})$ and $g_B(l_{i,t}; x_{i,t})$ are *known* functions of l_{it} and a strictly exogenous set of covariates x_{it}. $\Lambda_{B,F}$ is the set of unknown parameters which captures information which is relevant for the conditional mean in the state equation, which involves both the AR(1) coefficient on lagged persistent income as well as the drift in the state equation. Here, we allow for considerably more flexibility, but simply require that both $F(l_{it}; x_{it})$ and $B(l_{i,t}; x_{i,t})$ are linear in these parameters. In the vast majority of applications, one would tend to expect that things are partitioned so that the j^{th} element of $g_F(l_{i,t}; x_{i,t})$ is always zero if the j^{th} element of $g_B(l_{i,t}; x_{i,t})$ is nonzero with positive probability, but we don't need to require this per se.[12]

Let us define X_F as the design matrix constructed by concatenating the $[g_F(l_{i,t}; x_{i,t})]'$ vectors vertically, and X_B be the analogous object constructed by concatenating the $[g_B(l_{i,t}; x_{i,t})]'$ vectors vertically. Then, let us redefine

$$X_C \equiv \text{diag}(\vec{z}_{t-1})X_F + X_B,$$

where $\vec{z}_{t-1}$ is the first column of the definition of X_C in equation ((20))–i.e., the vector of lagged posterior means. In the special case in which F is constant, $g_F(l_{i,t}; x_{i,t})$ has a 1 in its first element and a zero otherwise, and $g_B(l_{i,t}; x_{i,t})$ has a zero in its first element, we can use this extended X_C matrix in place of the one defined above, and the updating formulas defined in ((25)) apply without modification.

If F_{it} is not constant, we also need to make a minor modification to the additional terms which appear in the likelihood function which involve filtering uncertainty about current and

[12] For example, in our base case above, F was assumed to be the first element of C and the remaining two parameters captured the unknown parameters which governed $B(l_{it})$.

lagged z_{it}. In the more general case, the expressions in equations ((21)-(22)) simplify to

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\left(F_{it}^2\Sigma_{it-1|T}(1,1)\right)=\sum_{i=1}^{N}\sum_{t=1}^{T}\left(C'g_F(l_{i,t};x_{i,t})\Sigma_{it-1|T}(1,1)[g_F(l_{i,t};x_{i,t})]'C\right)=C'X_F'\text{diag}(\vec{\sigma}_{t-1}(1,1))X_FC$$

and

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\left(F_{it}\Sigma_{it|T}(1,2)\right)=\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\Sigma_{it|T}(1,2)[g_F(l_{i,t};x_{i,t})]'C\right)=\vec{\sigma}_t(1,2)'X_FC.$$

If we use the alternative formulation which allows for F to vary as a linear function of $g_F(l_{i,t};x_{i,t})$, we obtain the closely related expression to equation ((25)):

$$\left[X_C'Q^{-1}X_C+X_F'Q^{-1}\text{diag}(\vec{\sigma}_{t-1}(1,1))X_F\right]C=X_C'Q^{-1}Y_C+X_F'Q^{-1}\vec{\sigma}_t(1,2),\tag{28}$$

which still resembles a GLS regression equation. Clearly, this will not work for completely arbitrary X_F and X_B; we will need to be able to impose restrictions which ensure that the matrix $\left[X_C'Q^{-1}X_C+X_F'Q^{-1}\text{diag}(\vec{\sigma}_{t-1}(1,1))X_F\right]$ is invertible.

D.4 Updating variances

In this appendix, we derive the expressions that will be used to update the variance parameters. As above, we will economize on notation by restricting attention to the notation of the model in Section II. However, the extension to the general case is immediate. Notice that, below, we already assume that log variances are linear in unknown sets of parameters. As such, allowing for a more flexible linear-in-parameters structure simply requires reinterpreting l_{it} as a broader set of observables than just employment/unemployment dummies.[13]

D.4.1 Shocks to persistent earnings when employed and unemployed (Q_E and Q_U)

In this section we discuss how we update the variance of persistent earnings for the employed and unemployed. We can write the variance of persistent earnings as:

$$Q(l_{it}) \quad = \quad \exp(l_{it}'\Lambda_Q)$$

where $\Lambda_Q=[\Lambda_{Q,E},\Lambda_{Q,U}]$. The relevant part of the negative log likelihood which depends on Λ_Q is:

[13] Also, in expressions below, we would need to replace F with F_{it} and $B(l_{it})$ with B_{it} where appropriate.

$$\Theta(\Lambda_Q; \beta, \omega) \equiv \sum_{i=1}^{N}\sum_{t=1}^{T} \log(Q(l_{it})) + \sum_{i=1}^{N}\sum_{t=1}^{T} \frac{(z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2}{Q(l_{it})} \tag{29}$$

To arrive at an updating formula for the variance of persistent earnings, we will take the conditional expectation using the posterior distribution of the latent states given all of the missing data, and then take FOC with respect to Λ_Q. Taking the conditional expectation using the posterior distribution of latent states (given all of the missing data) returns:

$$\Theta(\Lambda_Q; \beta, F) \equiv \sum_{i=1}^{N}\sum_{t=1}^{T} \log(Q(l_{it})) + \sum_{i=1}^{N}\sum_{t=1}^{T} \frac{E_T\left[(z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2 | y_{it}, l_{it}\right]}{Q(l_{it})} \tag{30}$$

Observe that this function is convex in Λ_Q. Therefore, if we take a second order approximation of the objective, we obtain the following:

$$\Theta(\Lambda_Q; \beta, F) - \Theta(\Lambda_{Q,0}; \beta, \omega) \equiv (\Lambda_Q - \Lambda_{Q,0})' \nabla\Theta + \tfrac{1}{2}(\Lambda_Q - \Lambda_{Q,0})' \, \nabla^2\Theta(\Lambda_Q - \Lambda_{Q,0}),$$

where the Jacobian matrix is defined as

$$\nabla\Theta(\Lambda_{Q,0}; \beta, F) \equiv \sum_{i=1}^{N}\sum_{t=1}^{T} \left[1 - \frac{E\left[(z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2 | \{y_{it}, l_{it}\}\right]}{Q(l_{it})}\right] l_{i,t}$$

and the Hessian matrix $\nabla^2\Theta$ is defined as

$$\nabla^2\Theta(\Lambda_{Q,0}; \beta, F) \equiv \sum_{i=1}^{N}\sum_{t=1}^{T} \left[\frac{E\left[(z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2 | \{y_{it}, l_{it}\}\right]}{Q(l_{it})}\right] l_{i,t} l_{i,t}'$$

Taking first order conditions, we obtain the familiar expressions for Newton's method:

$$\nabla^2\Theta\Lambda_Q = \nabla^2\Theta\Lambda_{Q,0} - \nabla\Theta \qquad \Longleftrightarrow \qquad \Lambda_Q = \Lambda_{Q,0} - \left[\nabla^2\Theta\right]^{-1} \nabla\Theta, \tag{31}$$

which gives us a simple way of updating Λ_Q.

Implementation Note that we can write the conditional expectation term as:

$$\begin{aligned} E\left[(z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2 | \{y_{it}, l_{it}\}\right] &= E\left[z_{i,t} - Fz_{i,t-1} - B(l_{i,t}) | \{y_{it}, l_{it}\}\right]^2 \\ &\quad + var(z_{i,t} - Fz_{i,t-1} - B(l_{i,t}) | \{y_{it}, l_{it}\}) \end{aligned} \tag{32}$$

Let $A_{it} = z_{it} - Fz_{i,t-1}$, then we can write the conditional variance expression as follows:

$$\begin{aligned} var(z_{i,t} - Fz_{i,t-1} - B(l_{i,t})|\{y_{it}, l_{it}\}) &= var(A_{it} - B(l_{i,t})|\{y_{it}, l_{it}\}) \\ &= var(A_{it}|\{y_{it}, l_{it}\}) + var(B(l_{i,t})|\{y_{it}, l_{it}\}) \\ &\quad - 2cov(A_{it}, B(l_{i,t})|\{y_{it}, l_{it}\}) \\ &= var(A_{it}|\{y_{it}, l_{it}\}) \end{aligned}$$

where in the final equality we are using the fact that we are conditioning on l_{it}. Then using the definition of A_{it}, we have:

$$\begin{aligned} var(z_{it} - Fz_{it-1} - B(l_{i,t})|\{y_{it}, l_{it}\}) = var(z_{it}|\{y_{it}, l_{it}\}) + F^2 var(z_{i,t-1}|\{y_{it}, l_{it}\}) \\ - 2Fcov(z_{it}, z_{i,t-1}|\{y_{it}, l_{it}\}) \end{aligned} \tag{33}$$

Combining equations (32) and (33), we have the following expression for the conditional expectations terms.

$$\begin{aligned} E\left[(z_{it} - Fz_{it-1} - B(l_{i,t}))^2|\{y_{it}, l_{it}\}\right] &= E\left[(z_{it} - Fz_{it-1} - B(l_{i,t}))|\{y_{it}, l_{it}\}\right]^2 \\ &\quad + var(z_{it}|\{y_{it}, l_{it}\}) + F^2 var(z_{i,t-1}|\{y_{it}, l_{it}\}) \\ &\quad - 2Fcov(z_{it}, z_{i,t-1}|\{y_{it}, l_{it}\}) \end{aligned} \tag{34}$$

Closed form expression for Q. Using equation 34 when $Q(\cdot)$ depends only on labor market status, we can obtain a closed form expression for the variance of persistent shocks to the employed and unemployed as a function of the individual level estimates of persistent earnings ($z_{i,t|T}$), its lag ($z_{i,t-1|T}$), and the variance-covariance matrix ($M_{i,t|T}$).

$$Q_E = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} l_{E,i,t}\left[\left(\hat{z}_{i,t|T} - F\hat{z}_{i,t-1|T} - B_E\right)^2 + [1, -F]M_{i,t|T}[1, -F]'\right]}{\sum_{i=1}^{N}\sum_{t=1}^{T} l_{E,i,t}} \tag{35}$$

$$Q_U = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} l_{U,i,t}\left[\left(\hat{z}_{i,t|T} - F\hat{z}_{i,t-1|T} - B_U\right)^2 + [1, -F]M_{i,t|T}[1, -F]'\right]}{\sum_{i=1}^{N}\sum_{t=1}^{T} l_{U,i,t}}, \tag{36}$$

D.4.2 Updating variance of temporary earnings (R)

In this section we discuss how we update the variance of temporary earnings. We can write the variance of persistent earnings as:

$$R = \exp(\Lambda_R).$$

The relevant part of the negative log likelihood which depends on Λ_R is:

$$\Theta(\Lambda_R) \equiv \sum_{i=1}^{N}\sum_{t=1}^{T}\log(R) + \sum_{i=1}^{N}\sum_{t=1}^{T}\frac{H(l_{i,t})(y_{i,t}-z_{i,t})^2}{R} \tag{37}$$

Taking the conditional expectation using the posterior distribution of latent states (given all of the missing data) returns:

$$\Theta(\Lambda_R) \equiv \sum_{i=1}^{N}\sum_{t=1}^{T}\log(R) + \sum_{i=1}^{N}\sum_{t=1}^{T}H(l_{i,t})\frac{E\left[(y_{i,t}-z_{it})^2|\{y_{it},l_{it}\}\right]}{R} \tag{38}$$

Similar to above, observe that this function is convex in Λ_R. Therefore, if we take a second order approximation of the objective, we obtain the following:

$$\Theta(\Lambda_R) - \Theta(\Lambda_{R,0}) \equiv (\Lambda_R - \Lambda_{R,0})'\nabla\Theta + \tfrac{1}{2}(\Lambda_R - \Lambda_{R,0})'\,\nabla^2\Theta(\Lambda_R - \Lambda_{R,0}),$$

where the Jacobian matrix is defined as

$$\nabla\Theta(\Lambda_{R,0}) \equiv \sum_{i=1}^{N}\sum_{t=1}^{T}\left[1 - \frac{E\left[(y_{i,t}-z_{it})^2|\{y_{it},l_{it}\}\right]}{R}\right] l_{E,i,t}$$

and the Hessian matrix $\nabla^2\Theta$ is defined as

$$\nabla^2\Theta(\Lambda_{R,0}) \equiv \sum_{i=1}^{N}\sum_{t=1}^{T}\left[\frac{E\left[(y_{i,t}-z_{it})^2|\{y_{it},l_{it}\}\right]}{R}\right] l_{E,i,t}l'_{E,i,t}$$

Taking first order conditions, we obtain the familiar expressions for Newton's method:

$$\nabla^2\Theta\Lambda_R = \nabla^2\Theta\Lambda_{R,0} - \nabla\Theta \qquad \Longleftrightarrow \qquad \Lambda_R = \Lambda_{R,0} - \left[\nabla^2\Theta\right]^{-1}\nabla\Theta, \tag{39}$$

Implementation Note that we can write the conditional expectations term as:

$$E\left[(y_{i,t}-z_{it})^2|\{y_{it},l_{it}\}\right] = E\left[y_{i,t}-z_{it}|\{y_{it},l_{it}\}\right]^2 + var(y_{i,t}-z_{it}|\{y_{it},l_{it}\})$$

Since we condition on $y_{i,t}$, the conditional variance term can be written as:

$$var(y_{i,t} - z_{it}|\{y_{it}, l_{it}\}) = var(z_{it}|\{y_{it}, l_{it}\})$$

Then we have that:

$$E\left[(y_{i,t} - z_{it})^2|\{y_{it}, l_{it}\}\right] = E\left[y_{i,t} - z_{it}|\{y_{it}, l_{it}\}\right]^2 + var(z_{it}) \tag{40}$$

Closed form expression for R. Using equation 40 when R depends only on labor market status, we can obtain a closed form expression for the variance of temporary shocks as a function of the individual level estimates of persistent earnings ($z_{i,t|T}$) and the variance-covariance matrix ($M_{i,t|T}$).

$$R = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} l_{E,i,t}\left[\left(y_{i,t} - \hat{z}_{i,t|T}\right)^2 + [1,0]M_{i,t|T}[1,0]'\right]}{\sum_{i=1}^{N}\sum_{t=1}^{T} l_{E,i,t}}, \tag{41}$$

D.4.3 Updating variance of initial persistent earnings draw (Q_0)

In this section we discuss how we update the variance of the initial draw of persistent earnings. We can write the variance of initial persistent earnings as:

$$Q_0 = exp(l^0_{E,i,t}\Lambda_{u_{z0}})$$

where $l^0_{E,i,t}$ is a dummy variable that is equal to 1 if individual i is employed E for the first time in the sample in period t.

The relevant part of the negative log likelihood which depends on $\Lambda_{u_{z0}}$ is:

$$\Theta(\Lambda_{u_{z0}}) \equiv \sum_{i=1}^{N} \log(Q_0) + \sum_{i=1}^{N} \frac{(z_{i0})^2}{Q_0}$$

Taking the conditional expectation using the posterior distribution of latent states (given all of the missing data) returns:

$$\Theta(\Lambda_{u_{z0}}) \equiv \sum_{i=1}^{N} \log(Q_0) + \sum_{i=1}^{N} \frac{E\left[(z_{i0})^2|\{y_{it}, l_{it}\}\right]}{Q_0}$$

Similar to above, observe that this function is convex in $\Lambda_{u_{z0}}$. Therefore, if we take a second order approximation of the objective, we obtain the following:

$$\Theta(\Lambda_{u_{z0}}) - \Theta(\Lambda_{u_{z0},0}) \equiv (\Lambda_{u_{z0}} - \Lambda_{u_{z0},0})'\nabla\Theta + \tfrac{1}{2}(\Lambda_{u_{z0}} - \Lambda_{u_{z0},0})'\,\nabla^2\Theta(\Lambda_{u_{z0}} - \Lambda_{u_{z0},0}),$$

where the Jacobian matrix is defined as

$$\nabla\Theta(\Lambda_{u_{z0},0}) \equiv \sum_{i=1}^{N}\left[1 - \frac{E\left[(z_{i0})^2|\{y_{it}, l_{it}\}\right]}{Q_0}\right] l^0_{E,i,t}$$

and the Hessian matrix $\nabla^2\Theta$ is defined as

$$\nabla^2\Theta(\Lambda_{u_{z0},0}) \equiv \sum_{i=1}^{N}\left[\frac{E\left[(z_{i0})^2|\{y_{it}, l_{it}\}\right]}{Q_0}\right] l^0_{E,i,t} l^{0'}_{E,i,t}$$

Taking first order conditions, we obtain the familiar expressions for Newton's method:

$$\nabla^2\Theta\Lambda_{u_{z0}} = \nabla^2\Theta\Lambda_{u_{z0},0} - \nabla\Theta \qquad \Longleftrightarrow \qquad \Lambda_{u_{z0}} = \Lambda_{u_{z0},0} - \left[\nabla^2\Theta\right]^{-1}\nabla\Theta, \tag{42}$$

Implementation Note that we can write the conditional expectations term as:

$$E\left[(z_{i0})^2|\{y_{it}, l_{it}\}\right] = E\left[z_{i0}|\{y_{it}, l_{it}\}\right]^2 + var\left[z_{i0}|\{y_{it}, l_{it}\}\right]$$

D.4.4 Extension of variance to the general case

Similar to Section D.3.3, the variances can depend on a linear-in-variables function of strictly exogeneous covariates, $\{x_{i,t}\}_{t=0}^{T}$. We assume that the variances are exponential-affine transformations of x_{it} as before, e.g.:

$$R(x_{i,t}) = \exp(x'_{i,t}\Lambda_R^x) \text{ if } l_{E,i,t} = 1, \text{ missing otherwise}$$

D.5 Algorithm

In this section, we present the EM algorithm we use to recover the estimate of persistent earnings as well as the parameters which govern the income process.

1. Guess an initial set of parameters $\theta_0 = [F, Q_E, Q_U, B_E, B_U, R, Q_0]'$.

2. Using the parameter guess θ_0 use the Kalman filter for the state-space system in equations (5) and (4) to obtain an estimate of $\{\{z_{i,t}\}_{i=1}^{N}\}_{t=0}^{T}$, and estimate the log likelihood.

3. Using estimated persistent earnings $\{\{z_{i,t}\}_{t=0}^{T}\}_{i=1}^{N}$ and data $\{\{y_{i,t}\}_{t=0}^{T}, \{l_{i,t}\}_{t=0}^{T}, \{x_{i,t}\}_{t=0}^{T}\}_{i=1}^{N}$, update the parameter vector as follows:
   (a) Update F, B_U, B_E using equation (25).
   (b) Update the shocks to persistent earnings by iterating on equation (31).
   (c) Update the shocks to temporary earnings by iterating on equation (39).
   (d) Update the initial draw of persistent earnings by solving (42).
4. Repeat steps (2) and (3) until the log likelihood is maximized.

D.6 Additional: Monte Carlo exercises

In this appendix we provide results from a series of additional Monte Carlo (MC) exercises.

Individual level estimates of persistent earnings We first examine how well our method can recover the path of persistent earnings at the individual level. We follow the same procedures for the MC exercise as in Appendix B.1. After simulating the data, we perform our estimation on the simulated data and recover the estimated parameters of the income process and the path of persistent earnings for each simulated individual. Let $\hat{z}_{i,t}$ denote the estimate of persistent earnings for individual i in period t, and let $z_{i,t}$ denote the true value of persistent earnings. To summarize the accuracy of the method, we estimate the following OLS regression:

$$z_{i,t} = \alpha + \beta^{MC}\hat{z}_{i,t} + \epsilon_{i,t} \tag{43}$$

If $\beta^{MC} \approx 1$ then we have evidence that our method is accurately recovering persistent earnings. For each time panel length T we repeat the process outlined above 50 times to examine the variability in our estimates.

Table A3 summarizes the results for the individual level estimates of persistent earnings. The first column of Table A3 presents the results from simulating data for $T = 35$ years. Across the 50 simulations the average β^{MC} from estimating equation 43 is 1.0004. This coefficient indicates that, on average, our estimates of persistent earnings differ from the true value by 0.04 percent. The fact that this coefficient is so close to 1 indicates that our method can accurately recover the path of persistent earnings at the individual level. The remaining columns of Table A3 show the results for estimations with different time series lengths. We continue to find that even as the panel dimension gets very small (5 observations) we are still able to recover persistent earnings with a high degree of accuracy. For example with 5 observations, on average, our estimate of persistent earnings differs from the true value by approximately than 0.7 percent.

Table A3: Monte Carlo Exercise: Recovering Persistent Earnings

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
T	35	30	25	20	15	10	5
Avg. β^{MC}	1.0005	1.0013	1.0010	1.0009	1.0012	1.0018	1.0070
	(0.2510)	(0.6725)	(0.4574)	(0.3540)	(0.4117)	(0.4426)	(0.9785)

Notes: Table presents the average coefficient β^{MC} from estimating equation 43 for the Monte Carlo exercise with an ASQ employment process. T denotes the number of periods simulated, and in each simulation 2500 individuals are simulated. We repeat the simulations 50 times. In parentheses we report the t-statistics that the average β^{MC} is statistically different from 1.

What if shocks are non-normal? In the above MC exercise, we are simulating data using the parameter estimates from the baseline model. In this simulation the shocks to temporary and persistent earnings are drawn from normal distributions. Recently, the literature has found that labor income changes have substantial deviations from a normal distribution (e.g., Guvenen et al. (2014), Guvenen et al. (2021)). Further, the literature finds that using a mixture of normal distributions can help recover the non-Gaussian features of the data. In this appendix, we examine how well our estimation procedure uncovers the path of persistent earnings when the labor income process has non-Gaussian shocks.

For this exercise, we need to simulate data that exhibits deviations from a Gaussian distribution. We simulate data from a distribution with non-Gaussian features by using the estimates from Guvenen et al. (2021). In particular, we use the parameter estimates from their estimation where there is a mixture distribution for both persistent and temporary earnings shocks (parameters presented in their Table 4, column (3)). In this estimation, the shock to persistent and temporary earnings are mixture distributions, where one distribution is drawn with a low probability and has a very negative mean and wide variance, while the other distribution has a near zero mean and a very tight variance. The latter distributions represents how most individuals have very small income fluctuations, while the first set of distributions induces severe negative shocks for a segment of the population. Using this labor income process, we repeat the MC exercise.

Table A4 presents the results of the MC exercise where the shocks to labor income are non-normal. Similar to the case of normally distributed shocks, our estimation routine is able to very accurately recover the path of persistent earnings at the individual level. For all time spans considered (35 observations per person to 5 observations) the average value of β^{MC} is not statistically different from one, indicating the method can successfully estimate persistent earnings when shocks are non-normal.

Table A4: Monte Carlo Exercise: Recovering Persistent Earnings with Non-normal Shocks

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
T	35	30	25	20	15	10	5
Avg. β^{MC}	0.9998	0.9999	0.9996	0.9993	1.0003	1.0005	1.0004
	(-0.0822)	(-0.0484)	(-0.1073)	(-0.1467)	(0.0401)	(0.0509)	(0.0136)

Notes: Table presents the average coefficient β^{MC} from estimating equation 43 where the data are simulated from an income process with non-normal shocks to persistent and temporary earnings. T denotes the number of periods simulated, and in each simulation 2500 indiviudals are simulated. We repeat the simulations 50 times. In parentheses we report the t-statistics that the average β^{MC} is statistically different from 1.

E Additional details: Empirics

In this appendix, we present additional details about our empirical exercise. In Appendix E.1, we discuss the representativeness of our sample. In Appendix E.2, we examine how observable shocks (e.g., layoffs, job switching) align with our estimates of temporary and persistent shocks at the individual level. In Appendix E.3, we present additional results from the estimation of Model 2. In Appendix E.4, we presents results for the risk in persistent earnings faced by individuals who report being unemployed in the CPS. In Appendix E.5, we present results for how risk has evolved across the age distribution over time. In appendix E.6, we present results where we show how shocks to temporary and persistent earnings have evolved over time for job switchers and job stayers. In Appendix E.7, we present additional results about how risk differs by gender. In Appendix E.8, we show that our results are robust to different values of the minimum earnings adjustment. In Appendix E.9, we show that our results are robust to altering the start and end dates of our sample. In Appendix E.10, we present results for testing Hypothesis 2 which explores geographic variation in persistent risk. In Appendix E.11, we present results for testing Hypothesis 3 which examines if routine occupations have seen larger increases in persistent risk. In Appendix E.12, we presents results for testing Hypothesis 4 where we allow income risk to evolve differently by education and gender. In Appendix E.13, we show that our results for the high skill hypothesis are robust to exploiting industry variation. In Appendix E.14, we present more details on ONET job zones. In Appendix E.15, we show that our results supporting the high skill worker hypothesis are robust to focusing on workers who are younger when we observe their occupation in the CPS. Finally, in Appendix E.16 we discuss how we estimate the change in computer requirements over the 1985-2015 time period.

E.1 Representativeness

In this appendix, we assess the representativeness of our linked SSA-CPS sample. We show that moments from our sample align with estimates from the full SSA database.

In particular, we show that the standard deviation of earnings closely mirror those reported in Guvenen, Kaplan, Song, and Weidner (2022), hereafter referred to as GKSW, whose sample is the universe of SSA earnings records. Importantly, GKSW impose very similar earnings criteria for sample inclusion: (1) ages 25 to 55 during the panel period (1957-2013), (2) earnings are larger than a year specific minimum earnings criterion, denoted $\underline{Y}_t$ in at least 15 years between the ages of 25 and 55, and (3) had a total lifetime earnings of at least $31 \times \underline{Y}_t$. GKSW set their minimum earnings criterion to the equivalent of working part-time at the real federal minimum wage for 1 quarter of the year. When the earnings history is truncated condition (2) is updated to 50% of years since 25, and condition (3) is updated to number of years since age 25 times $Y_{min,t}$.

We compare estimates of the standard deviation of log earnings by gender from our linked SSA-CPS sample to the moments reported in GKSW.[14] Panels (a) and (b) of Figure A3 present the standard deviation of log earnings among men and women, respectively. The black solid line presents the standard deviation of log earnings among individuals in our baseline sample, while the red line presents the standard deviation from GKSW. The trend in the standard deviation of log earnings from our sample closely tracks the estimates from GKSW.[15] In results that are available upon request, we show that we obtain similar trends in both median earnings and the standard deviation of log earnings by selected age in our samples as well as in the GKSW sample.

The results of this section show that we obtain similar estimates in our linked sample of SSA-CPS earnings as in the full SSA database. We view these results as providing evidence that our linked SSA-CPS sample is representative of the full sample of SSA individuals.

E.2 What drives persistent and temporary income shocks?

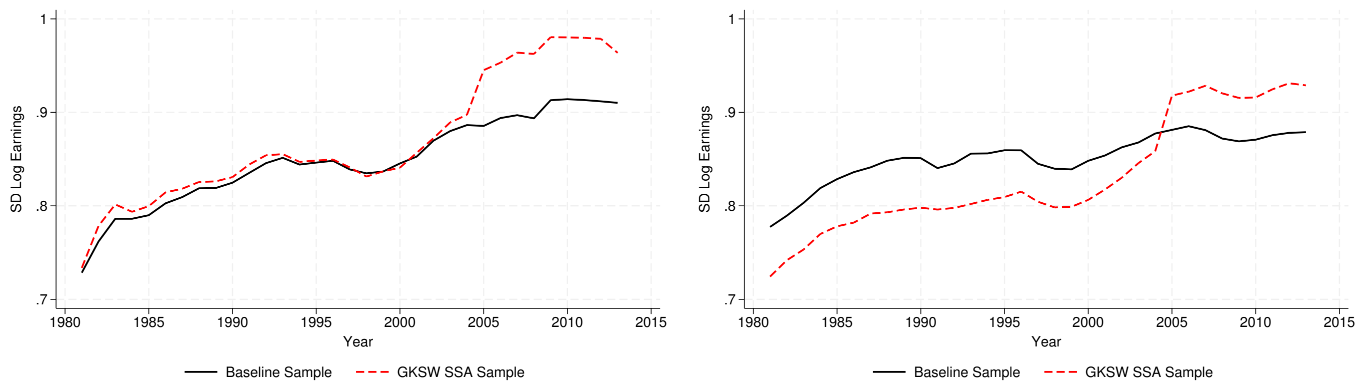
In this appendix, we examine how our filtered estimates of temporary and persistent earnings shocks align with job switches and job loss in our SSA-CPS database. For each labor market event, we report the joint density of persistent and temporary shocks. We illustrate these joint

[14] Note for this comparison, we plot the standard deviation of $\log(Y_{i,t}^{raw})$ and we only include an individual in a given year if their earnings are over the minimum earnings criterion from Guvenen et al. (2022) (i.e., $Y_{i,t} > \underline{Y}_t$).

[15] The differences at the end of the sample have to do with a minor difference in sample construction, where we require at least 5 years of data to be included in our sample. GKSW do not have a corresponding sample condition.

Figure A3: Representativeness of SSA-CPS linked sample vs. Guvenen et al. (2022)

(a) Std. dev. log earnings, men (b) Std. dev. log earnings, women



Note: Panels (a) and (b) report standard deviations of log earnings in levels for men and women, respectively. The GKSW data are taken from the accompanying supporting documents to Guvenen et al. (2022).

densities as heatmaps whose colors correspond to the mass of individuals with a given combination of persistent and temporary shocks.

As we are showing results from the distribution of shocks to temporary and persistent earnings from our filtering method, we must take into account *filtering uncertainty*. The Kalman smoother returns an estimate of persistent earnings for individual i in period t and the lag of persistent earnings, i.e., $\hat{\zeta}_{i,t|T} = \begin{bmatrix} \hat{z}_{i,t|T} & \hat{z}_{i,t-1|T} \end{bmatrix}'$. The Kalman smoother also produces an estimate about the uncertainty of this estimate, which is given by the MSE matrix $M_{i,t|T}$. To obtain an estimate of persistent earnings for individual i in period t taking into account filtering uncertainty, denoted $\hat{z}_{i,t}$, we draw normal noise, denoted $\xi_{i,t}$, from a bi-variate normal distribution with mean zero and variance-covariance matrix $M_{i,t|T}$. Let $\xi_{1,i,t}$ and $\xi_{2,i,t}$ denote the first and second elements of $\xi_{i,t}$, respectively. For each individual i in period t, we recover persistent earnings $\hat{z}_{i,t} = \hat{z}_{i,t|T} + \xi_{1,i,t}$, the persistent earnings innovation $\Delta\hat{z}_{i,t} = \hat{z}_{i,t|T} + \xi_{1,i,t} - (\hat{z}_{i,t-1|T} + \xi_{2,i,t})$ and the temporary earnings innovation $\hat{\omega}_{i,t} = y_{i,t} - \hat{z}_{i,t}$.[16]

[16]Note that in every time period t, filtering uncertainty is not fully resolved for the current and lagged innovation. Filtering uncertainty is not iid, and any comparison of current and lagged values of persistent earnings requires a correlated draw of filtering uncertainty for the current and lagged persistent earnings. The bivariate filtering uncertainty distribution is over the current and lagged innovation. The primary benefit of including lagged persistent earnings in the state vector is that this bivariate distribution of filtering uncertainty is estimated by the Kalman filter.

E.2.1 Job switching

We first plot heatmaps of the shocks to persistent and temporary earnings for individuals who remain at the same primary employer (EIN) across two consecutive years (Panel (a) of Figure A4) and for individuals who switch their primary employer (Panel (b) of Figure A4).[17] Panel (a) shows that among job stayers, the majority of individuals have small shocks to temporary and persistent earnings. These shocks likely reflect changes in hours and weeks worked, as well as promotions, and raises, etc. Conversely, Panel (b) shows that among job switchers, the mass of individuals spreads out of the middle of the distribution towards more extreme persistent and temporary shocks (either positive or negative). To facilitate comparison, Panel (c) of Figure A4 subtracts the joint density in Panel (a) from Panel (b). The resulting difference in densities more clearly illustrates that job switching is associated with larger shocks (both positive and negative) to persistent and temporary earnings. Among non-switchers, roughly 6% have the most extreme earnings outcomes (lowest or highest persistent and temporary shocks). Among switchers, 17 percent have the most extreme earnings outcomes, representing approximately a threefold increase.

We further split job switchers by the type of job switch. Using data from the Longitudinal Business Database (LBD), we measure average earnings per employer.[18] We separate job switchers into those who move to an employer with average earnings per worker that are 25% lower (higher) than their previous employer. Panel (d) (Panel (e)) of Figure A4 shows that when an individual moves to a lower (higher) paying employer, they become more likely to experience a large negative (positive) shock to persistent earnings. To facilitate comparison, Panel (f) of Figure A4 subtracts the joint density in Panel (d) from Panel (e). Panel (f) demonstrates that moving to a higher-paying firm is associated with positive shocks, especially to persistent earnings.

The results of this section demonstrate that the estimates of temporary and persistent shocks align with observable labor market events. In particular, the estimates align with job ladder models of the labor market in which job switching is associated with shocks to temporary and persistent earnings that are larger than those associated with remaining at the same employer. Further, the direction of the job switch (i.e., moving to a higher or lower paying employer) aligns with the notion of climbing up and falling down the job ladder.

[17] An individual's primary employer in a given year is the defined as the EIN where the individual had the largest share of earnings in that year.

[18] See Jarmin and Miranda (2002) for details on the construction of the LBD.

E.2.2 Layoffs

In this section, we examine temporary and persistent earnings shocks around layoff. While many papers have studied the average response of earnings to layoffs, we examine the heterogeneous behavior of temporary and persistent earnings following layoff. We document substantial heterogeneity in earnings following layoff and how it correlates with observable features of the layoff.

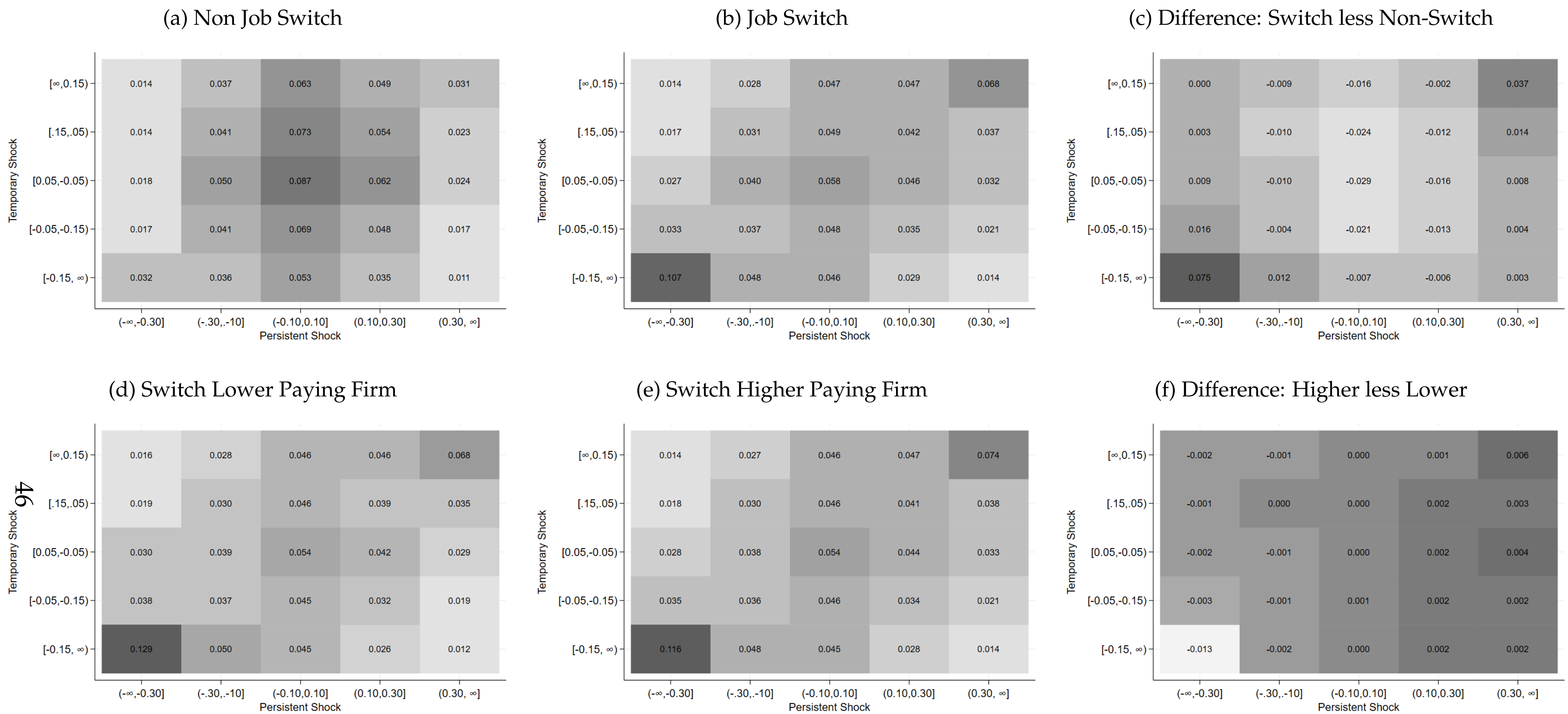
We identify layoffs using an individual's self-reported CPS responses. In particular, we define an individual to have been laid off in year t if they report having positive weeks on layoff in year t and zero weeks on layoff in year $t-1$. We impose the requirement that an individual have zero weeks on layoff in year $t-1$ so that we are able to accurately measure the inflow of individuals into unemployment. In Panel (a) of Figure A5, we plot the heatmap of persistent and temporary earnings shocks in year t for individuals we identify as laid off in the CPS. The figure shows that there is a large mass of individuals in the bottom left hand corner of the heatmap, which indicates that a sizeable mass of laid off individuals have large negative persistent and temporary shocks. Interestingly, there is also a large mass of individuals with small shocks, and there are even some individuals with positive shocks.

We investigate the heterogeneity in shocks around layoffs by distinguishing *recalls* from *non-recalls*. We define an individual to be recalled if their primary employer in the year before layoff is also their primary employer in the year after layoff.[19] We define an individual to be non-recalled if they have different primary employers in the years before and after layoff. Panel (b) of Figure A5 plots the heatmap of persistent and temporary shocks among recalled individuals, while Panel (c) of Figure A5 plots the heatmap for non-recalled individuals, and Panel (d) illustrates the difference. Comparing Panels (b) and (c) shows that relative to individuals who are not recalled after layoff, individuals who are recalled exhibit much smaller negative shocks to temporary and persistent earnings, and they are more likely to have a positive shock.

Taking stock. The results of this section showed that our estimates of temporary and persistent earnings align with observable shocks that individuals face in the labor market. As the filter is unaware of the shocks individuals face, we view these results as a validation of the method.

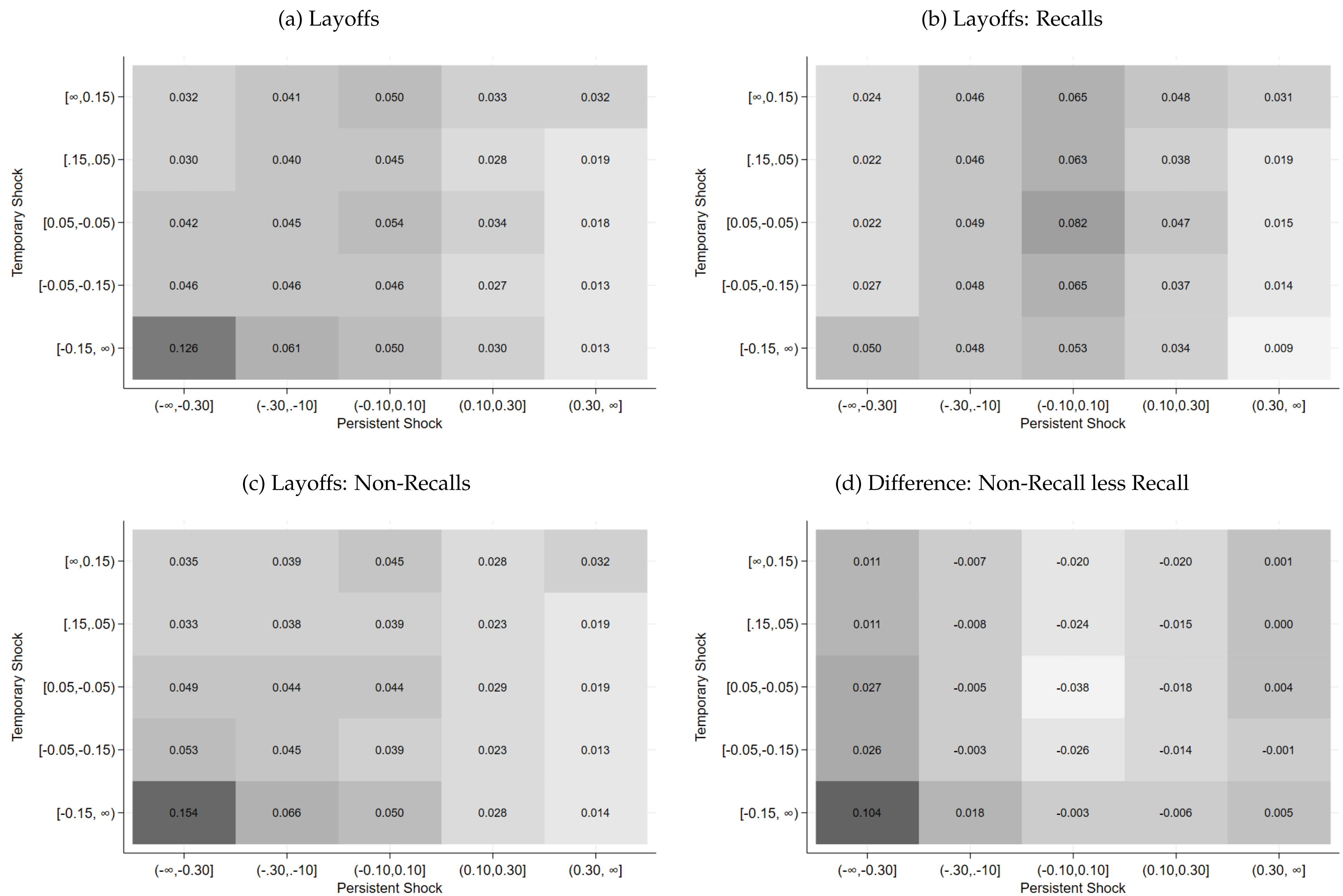
[19] As in section E.2.1, we define an individual's primary employer in a given year as the EIN where they have the largest labor earnings.

Figure A4: Shocks to temporary and persistent earnings around job switching



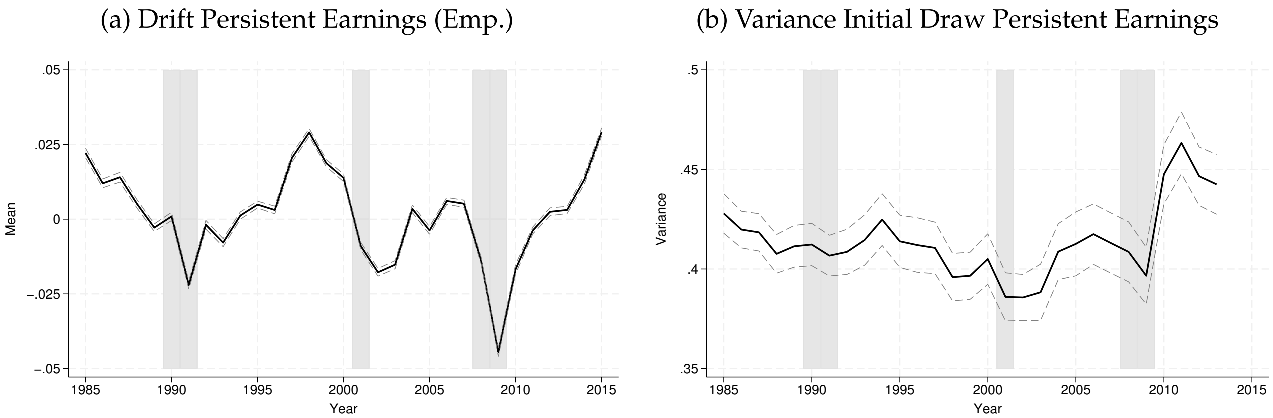
Note: Figure plots a heatmap of temporary and persistent shocks by observable labor market event. Higher (lower) paying firms are identified by moving to an employer with average earnings that are 25% above (below) an individuals current employer.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

Figure A5: Shocks to temporary and persistent earnings around layoff



Note: Figure plots a heatmap of temporary and persistent shocks around layoff. Layoffs are identified using the CPS. Individuals are defined as "recalled" if their primary employer in the year after layoff is the same as the year before layoff.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

Figure A6: Additional time series from Model 2



Note: Figure presents time series estimates from estimating Model 2. Panel (a) presents the mean (drift) of shocks to persistent earnings among the employed. Panel (b) presents the variance of the initial draw of persistent earnings. Dashed gray lines denote a 95% confidence interval. Gray bars denote NBER recessions.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

E.3 Additional results: Estimating model 2

In this appendix, we present additional results from the estimation of Model 2. We first present additional time series results, and then present the estimates of how risk evolves over the life-cycle.

Additional time series results. In Figure A6, we present additional time series produced in the estimation of Model 2. Panel (a) of Figure A6 presents the mean (drift) of shocks to persistent earnings while employed by year. The figure shows that the mean of shocks to persistent earnings is highly cyclical, with the average shock to persistent earnings declining in recessions and increasing in expansions. This cyclical behavior aligns with work by Guvenen et al. (2014), who show that the distribution of shocks to earnings shifts to the left during recessions relative to expansions. In panel (b) of Figure A6, we plot the variance of the initial draw of persistent earnings. The figure shows that there has been a relatively stable level of the initial draw of persistent earnings since the mid-1980s.

Shocks by age. In Figure A7, we present the age profiles of the means and variances of shocks to temporary and persistent earnings over the life-cycle as estimated by Model 2.[20] Panel (a) of Figure A7 shows that the variance of shocks to persistent earnings among the employed is "U-

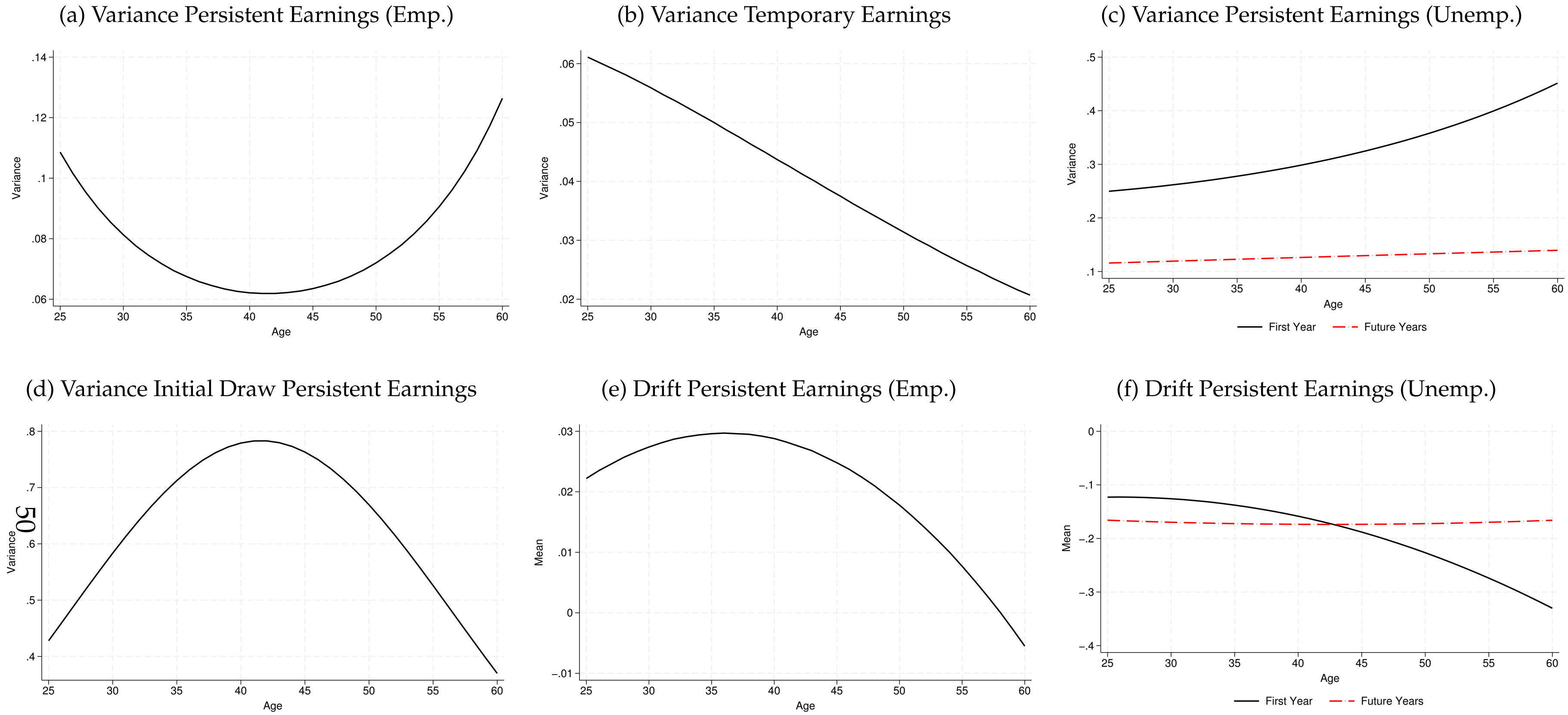
[20] Note the age profiles are plotted for the year 1985.

shaped" over the life-cycle.[21] The shape of this profile is in part influenced by how individuals transition across jobs over the life-cycle. We show in Appendix E.6 that the variance of shocks to persistent earnings is larger for job switchers relative to job stayers. Given that the likelihood of job switching decreases over the life-cycle, this contributes to the decline in persistent earnings risk over the first half of the life-cycle. We also show in Appendix E.6 that the increase in persistent risk among individuals over the age of 45 occurs among both job switchers and job stayers.

Panel (b) of Figure A7 presents the profile of temporary earnings risk over the life-cycle. The figure shows that temporary earnings risk decreases over the life-cycle. As for persistent risk, this shape is largely influenced by the declining nature of job switching over the life-cycle, as job switchers undergo much larger temporary shocks relative to job stayers (see Appendix E.6). Panel (c) shows that the variance of shocks to persistent earnings is increasing over the life-cycle in both the first year of unemployment (black, solid line) and in all future years of unemployment (red, dashed line). Panel (d) of Figure A7 shows that the variance of the initial draw of persistent earnings is increasing in the age in which an individuals shows up in our sample. Panel (e) of Figure A7 shows that the mean shock to persistent earnings is decreasing over the life-cycle. Finally, panel (f) of Figure A7 shows that the mean shocks to persistent earnings during the first year of unemployment is decreasing with age, i.e., older individuals see a larger decline from entering unemployment relative to younger individuals.

[21] Karahan and Ozkan (2013) also find that shocks to persistent earnings are U-shaped over the life-cycle using the PSID.

Figure A7: Persistent and temporary earnings risk by age



Note: Figure presents parameter estimates of the shocks to earnings over the life-cycle from estimating Model 2. Panel (a) plots the variance of persistent earnings among the employed. Panel (b) plots the variance of temporary shocks. Panel (c) plots the variance of persistent shocks to the unemployed in the first year of unemployment (black, solid line) and in all future years of unemployment (red, dashed line). Panel (d) plots the variance of the initial draw of persistent earnings. Panel (e) plots the mean (drift) of shocks to persistent earnings among the employed. Panel (f) plots the mean of persistent shocks to the unemployed in the first year of unemployment (black, solid line) and in all future years of unemployment (red, dashed line).
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

E.4 Additional results: CPS unemployed

In this appendix, we further discuss persistent earnings risk among the unemployed. In Section IV, we showed that the variance of persistent earnings shocks to the unemployed has increased over time, and that there are larger declines in persistent earnings during spells of unemployment. One potential concern is that our time series patterns may be attributable to individuals with no observable labor market shocks (i.e., no 'true' labor market risk), and thus the time trends reflect misspecification and/or life events that are unrelated to labor market risk. In this appendix, we use our ability to link with the CPS-ASEC to examine persistent earnings shocks among individuals who self-report having an unemployment spell, i.e., individuals who are facing labor market risk. We first show that persistent earnings risk and persistent earnings scarring are increasing in the length of an individual's unemployment spells. We then validate our time series result of rising persistent earnings risk among the unemployed by showing that those who self-reported job loss in the CPS-ASEC exhibit rising persistent earnings risk and greater scarring effects of being unemployed.

In what follows, we define an individual to be *CPS unemployed* in year t if they report being on layoff for at least one week in year t or report not working in year t because they cannot find a job.[22] We additionally require that an individual have positive earnings in year $t-1$ to be considered CPS unemployed in year t. With this definition, we examine the shocks to persistent earnings of individuals that we classify as CPS unemployed. For this analysis, we use our individual level of estimates of persistent earnings shocks from the Kalman filter. We first examine how the shocks to persistent earnings vary with the length of an individual's unemployment spell and then examine trends in persistent shocks for the CPS unemployed.

Persistent risk and unemployment durations. In Table A5, we first examine how the shocks to persistent earnings evolve among the CPS unemployed by the length of their unemployment duration. For individuals with an unemployment spell of between 1 and 13 weeks, there is an average decline in persistent earnings of approximatley 5%, and the variance of shocks to persistent earnings is 0.120 log points. As the length of an unemployment spell increases we see larger scarring effects of persistent earnings as well as greater persistent risk. For unemployment spells that span 40 to 52 weeks, we see a decline in persistent earnings of over 37 percent on average and the variance of persistent shocks is almost 0.30 log points.

Trends in risk among CPS unemployed. We next examine trends in risk among the CPS unemployed. In Figure A8, we compare the time series of persistent earnings risk among our

[22]Note that the individuals that we classify as CPS unemployed need not have zero earnings as in Section IV.

Table A5: Persistent risk among CPS unemployed by unemployment duration

Unemployment Definition	**Mean**	**Variance**
CPS Unemployed, 1-13 weeks	-0.047	0.120
CPS Unemployed, 14-26 weeks	-0.126	0.146
CPS Unemployed, 27-39 weeks	-0.226	0.175
CPS Unemployed, 40-52 weeks	-0.376	0.298

Note: Table presents summary statistics for the shocks to persistent earnings among the CPS unemployed by the duration of their unemployment spell. See Appendix E.4 for definitions of CPS Unemployed.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

full sample of unemployed individuals (who are in their first year of unemployment) to the individuals we classify as CPS unemployed. Panel (a) of Figure A8 compares the time series of the variance of shocks to persistent earnings among individuals in the full sample whom we classify as unemployed (black, solid line) to individuals whom we classify as CPS unemployed (red, dashed line).[23] The figure shows that the variance of persistent earnings shocks has been increasing over the sample period and exhibits a similar trend to the full sample of unemployed individuals (correlation = 0.75). Hence, under the alternative definition of CPS unemployment, we see a similar trend increase in the standard deviation of shocks to persistent earnings among the unemployed.

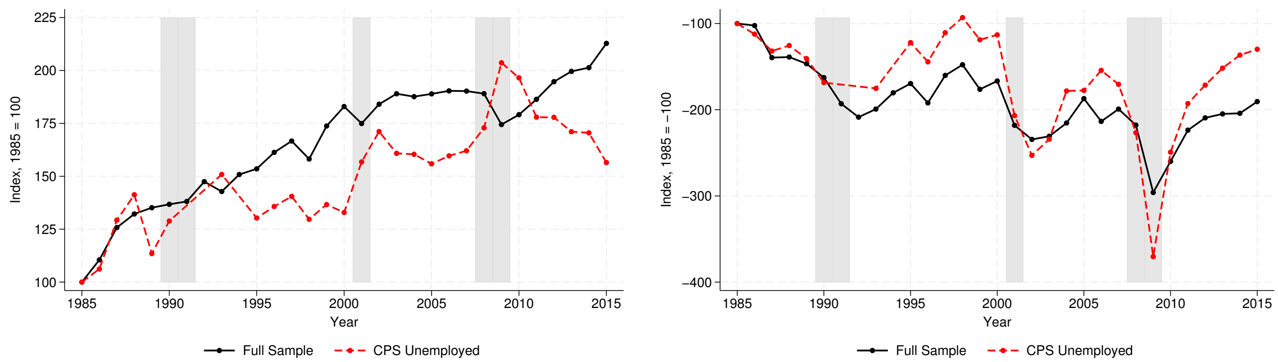
Next, we examine the mean change in persistent earnings among the full sample of unemployed individuals and among individuals who report being unemployed in the CPS. Panel (b) of Figure A8 compares the average shock to persistent earnings among individuals in the full sample whom we classify as unemployed (black, solid line) to individuals whom we classify as CPS unemployed (red, dashed line). The figure shows that those who self-report CPS unemployment have experienced a similar acceleration over time of persistent earnings losses during unemployment (correlation = 0.85).

In summary, we show that individuals with observable labor market risk in the CPS (i.e., those who self-report positive weeks on layoff) are precisely those who have rising persistent earnings risk implied by our filter. We view this as a demonstration of our filter's ability to capture economically meaningful labor market risk not just cross-sectionally but also over time.

[23] An individual is defined as full sample unemployed in year t if they have zero earnings in year t, and positive earnings in year $t-1$.

Figure A8: Persistent earnings risk of CPS unemployed over time

(a) Variance of Persistent Earnings Shocks (b) Mean of Persistent Earnings Shocks



Note: Figure compares the time series of the variance and mean of (filtered) shocks to persistent earnings among unemployed individuals in the full estimation sample (black, solid line) and individuals whom we classify as CPS unemployed. See Appendix E.4 for definitions of CPS Unemployed as well as Full Sample unemployed.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

E.5 Additional results: Shocks by age over time

In this appendix, we examine how income risk has evolved over time across the age distribution. To do so, we estimate a fourth model of income risk, which we specify below:

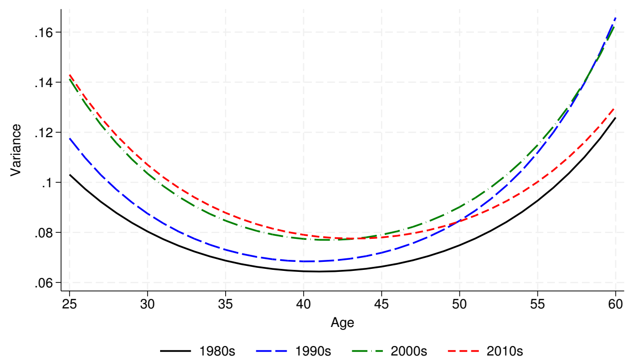
- **Model 4.** The income process presented in Section II, extended so that unemployment spells are split into the first year of unemployment and all future years of unemployment. Additionally, we will allow the variance parameters (Q_E, Q_U, Q_0 and R) and mean parameters (B_E and B_U) to vary by age via an age quadratic, where the age quadratics are specific to a decade (e.g., 1981-1989, 1990-1999, etc.).

Figure A9 presents the results of estimating Model 4. Panel (a) of Figure A9 plots the variance of persistent shocks to the employed by age as well as by decade. Comparing the 2010s (red, dashed line) to the 1980s (black, solid line), we find that persistent earnings risk among the employed has increased the most among young workers. In particular, persistent earnings risk among the young increases by nearly 40 percent. Alternatively for older workers in our sample there has been a small increase in persistent earnings risk, e.g., a 8 percent increase among 55-year olds.

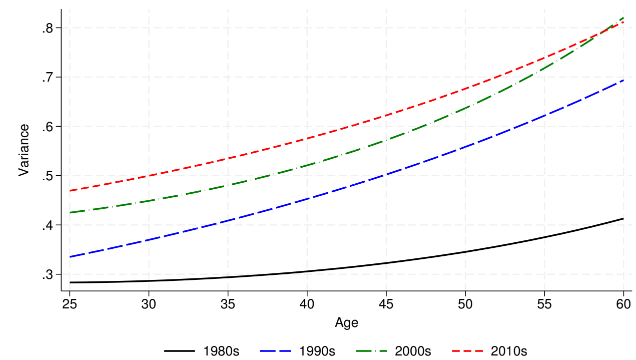
Panel (b) of Figure A9 plots the variance of shocks to persistent earnings among the unemployed in their first period of unemployment by age and decade. The figure shows that the increase in persistent income risk among the unemployed over time has occurred for workers

Figure A9: Persistent and temporary risk over time by age

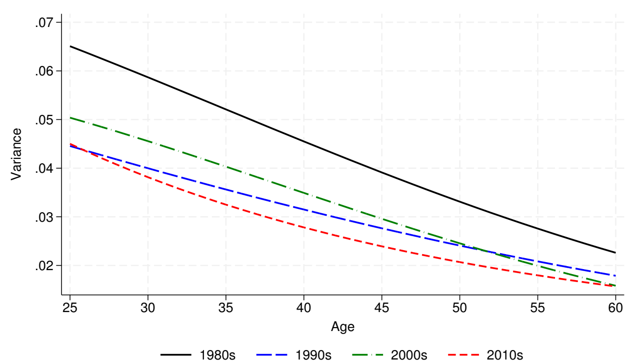
(a) Variance Persistent Shocks Employed



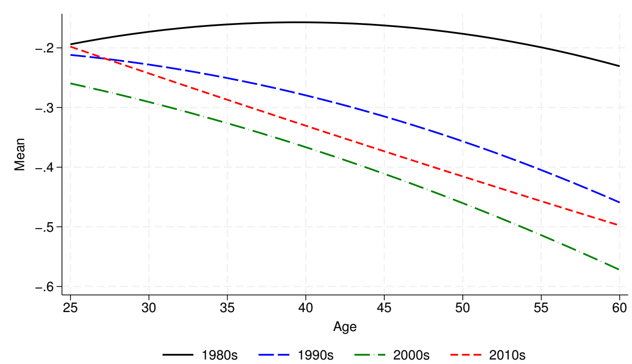
(b) Variance Persistent Shocks Unemp., First Period



(c) Variance Temporary Shocks



(d) Mean Persistent Shocks Unemp., First Period

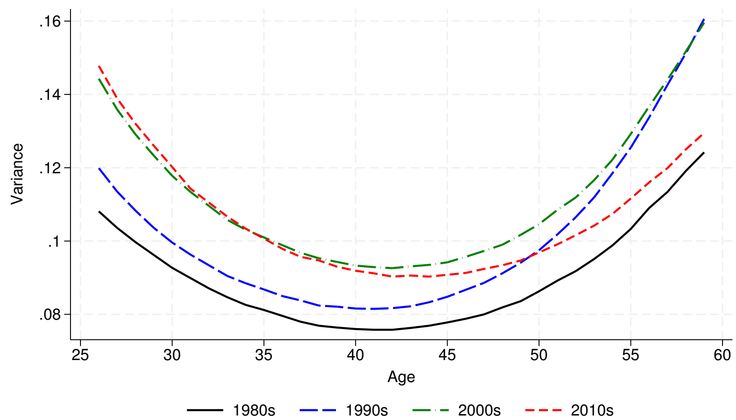


Note: Figure presents the results of estimating Model 4. Panel (a) plots the variance of shocks to persistent earnings among the employed. Panel (b) plots the variance of shocks to persistent earnings among the unemployed, in their first period of unemployment. Panel (c) plots the variance of shocks to temporary earnings. Panel (d) plots the mean of shocks to persistent earnings among the unemployed, in their first period of unemployment. The black, solid line corresponds to 1980s, the blue, long dashed line corresponds to the 1990s, the green, dashed-dotted line corresponds to the 2000s, and the red, dashed line corresponds to the 2010s.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

of all ages but is most pronounced among older workers. In panel (c) of Figure A9, we show the evolution of temporary risk over time by age. The figure shows that the decline in temporary earnings risk between the 1980s and 2010s has occurred for workers of all ages, but is most pronounced among the young. Finally, panel (d) of Figure A9 presents the drift in persistent earnings among the unemployed in their first period of unemployment. The figure shows that the decline in persistent earnings during unemployment spells has become larger for workers of all ages since the 1980s, but that the decline has accelerated the most among older workers.

Using the parameter estimates from Model 4 and the shares of workers across employ-

Figure A10: Combined persistent risk over time by age



Note: Figure presents combined persistent risk over time by age and decade. The black, solid line corresponds to 1980s, the blue, long dashed line corresponds to the 1990s, the green, dashed-dotted line corresponds to the 2000s, and the red, dashed line corresponds to the 2010s.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

ment/unemployment by age and decade we can compute how combined persistent earnings risk has evolved over time. Figure A10 plots combined persistent risk by age over the life-cycle for each decade from the 1980s to the 2010s. Between the 2010s and the 1980s, combined persistent income risk has increased for workers of all ages, but the increase has been most pronounced among younger workers.

E.6 Additional results: Role of job switching and staying

In this appendix, we examine how income risk has changed over time among job stayers and job switchers across the age distribution. We define an individual to be a *job stayer* in a year t, if they have the same primary employer (EIN) in year t and year $t-1$, while a job switcher in a year t has a different primary employer (EIN) in year t and $t-1$. To examine how income risk has evolved over the age distribution among switcher and stayers, we estimate a version of Model 4, where we split the employed in job switchers and stayers.[24]

We first present estimates for how shocks to persistent earnings have evolved over time among switchers and stayers. Panel (a) of Figure A11 presents the variance of shocks to per-

[24]We examine job switchers and stayers using Model 4, which allows for means and variance of persistent and temporary shocks to evolve by age and decade, given that the likelihood job switching is strongly predicted by age. We find that the likelihood of job switching over the life-cycle declines from nearly 35% (in a year) for younger workers to just over 10% for older workers.

sistent earnings among job stayers by age for the 1980s (black, solid line) and the 2010s (red, dashed line). The figure shows that persistent earnings risk among stayers has increased between the 2010s and 1980s for workers under the age of 50. Panel (b) of Figure A11 presents the variance of shocks to persistent earnings among job switchers by age for the 1980s (black, solid line) and the 2010s (red, dashed line). First, observe that the variance of shocks to job switchers is substantially larger than for job stayers. Second, the figure shows that persistent risk among job switchers has increased from the 1980s to the 2010s for younger and older workers.[25]

We next examine how shocks to temporary earnings have evolved over time among switchers and stayers. Panel (c) of Figure A11 presents the variance of temporary earnings shocks among job stayers by age in the 1980s (black, solid line) and the 2010s (red, dashed line). The figure shows that there has been a decline in temporary earnings risk among job stayers across the age distribution, with the decline being slightly larger for younger workers. Finally, panel (d) of Figure A11 presents the variance of temporary earnings shocks among job switchers by age in the 1980s (black, solid line) and the 2010s (red, dashed line). As for persistent shocks, job switchers face substantially more dispersion in temporary shocks relative to job stayers. Across time, we find that the variance of shocks to temporary earnings among job stayers has declined between the 1980s and 2010s for workers who are less than 50-years old.

E.7 Additional results: Gender

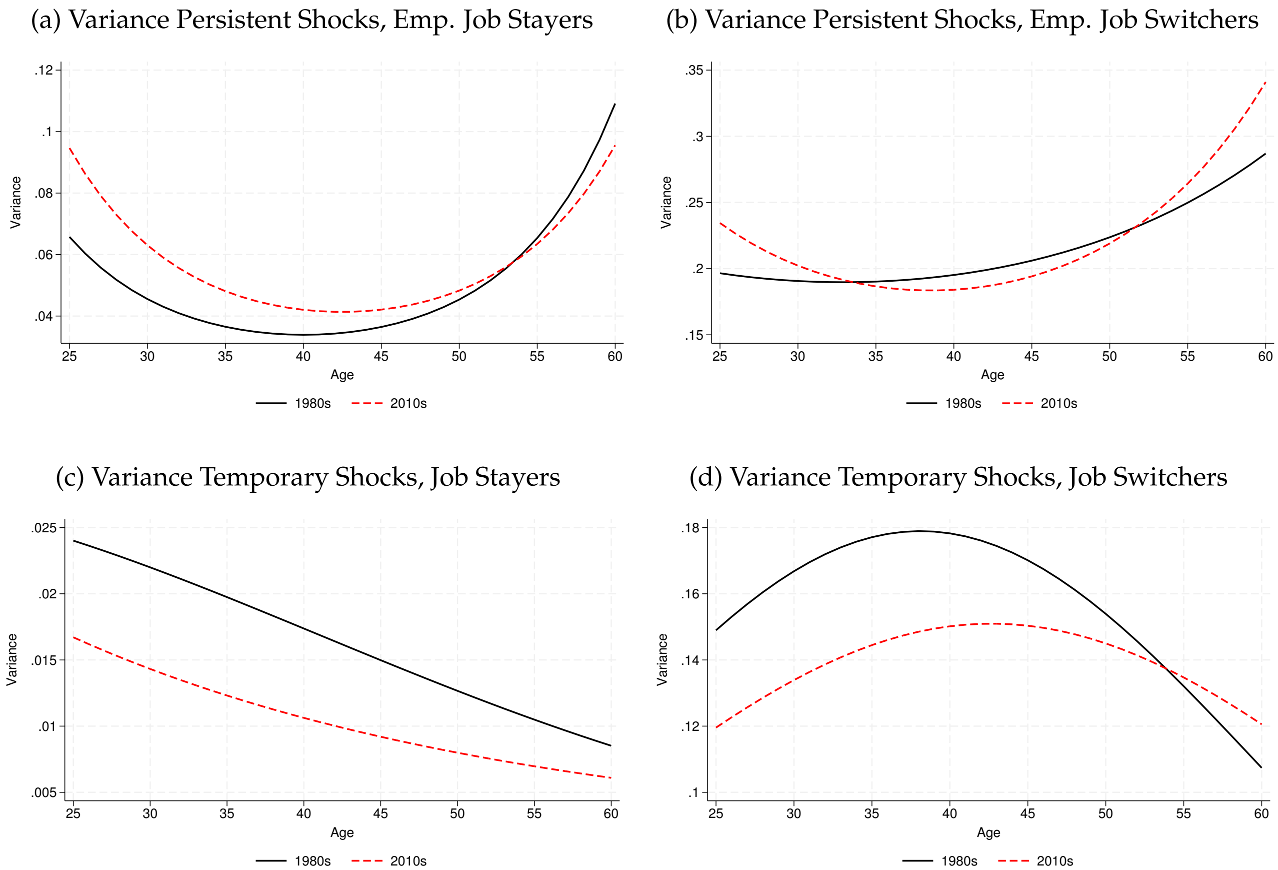
In this appendix, we examine how income risk has evolved for men and women separately. Figure A12 presents the results of estimating Model 2 separately for men and women. The figure shows that we see similar increases in persistent earnings risk for both men (black, solid line) and women (red, dashed line) while employed (panel (a)) and while unemployed (panel (b)). Similarly, there is a decline in temporary earnings risk between 1985 and 2015 for both men and women (panel (c)).

E.8 Additional results: Minimum earrings adjustment

In this appendix, we examine the robustness of our results to alternative values of the minimum earnings adjustment (Y^{min}). We find that we obtain similar time trends if we decrease the value of the minimum earnings adjustment to ($1473), which corresponds to the value of minimum earnings considered in Guvenen et al. (2014) and which we refer to as the *low adjustment*. Additionally, we find similar result if we increase the value of the minimum earnings adjustment

[25] Changes in persistent earnings risk among the unemployed evolve in the same manner as presented in Appendix E.5. These results are available upon request.

Figure A11: Persistent and temporary risk over time for job switchers and stayers



(a) Variance Persistent Shocks, Emp. Job Stayers

(b) Variance Persistent Shocks, Emp. Job Switchers

(c) Variance Temporary Shocks, Job Stayers

(d) Variance Temporary Shocks, Job Switchers

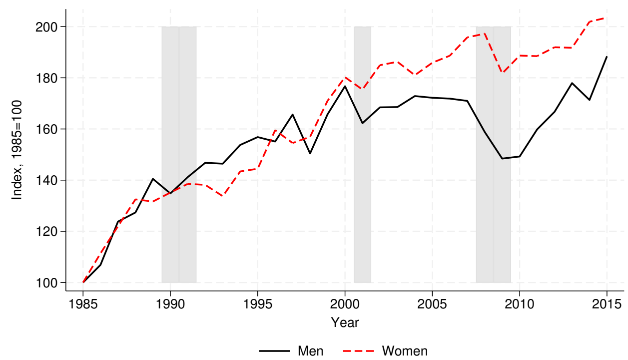
Note: Figure presents the results of estimating Model 4, where the employed are split into job switchers and stayers. Panel (a) plots the variance of shocks to persistent earnings employed, job stayers. Panel (b) plots the variance of shocks to persistent earnings among employed, job switchers. Panel (c) plots the variance of shocks to temporary earnings among job stayers. Panel (d) plots the variance of shocks to temporary earnings among job switchers. The black, solid line corresponds to 1980s, while the red, dashed line corresponds to 2010s.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

to $5893, which corresponds to the minimum value of earnings considered in Juhn et al. (1993) as well as Autor et al. (2008) and which we refer to as the *high adjustment*. Figure A13 presents the parameters of our income process for different levels of the minimum earnings adjustment, and Figure A14 presents combined persistent income risk across different values of the minimum earnings adjustment. Putting these results together, we find that we obtain similar trends of rising persistent risk and declining temporary risk across values of the minimum earnings adjustment.

Figure A12: Persistent and temporary risk over time by gender

(a) Variance Persistent Shocks Employed

(b) Variance Persistent Shocks Unemp., First Period



(c) Variance Temporary Shocks

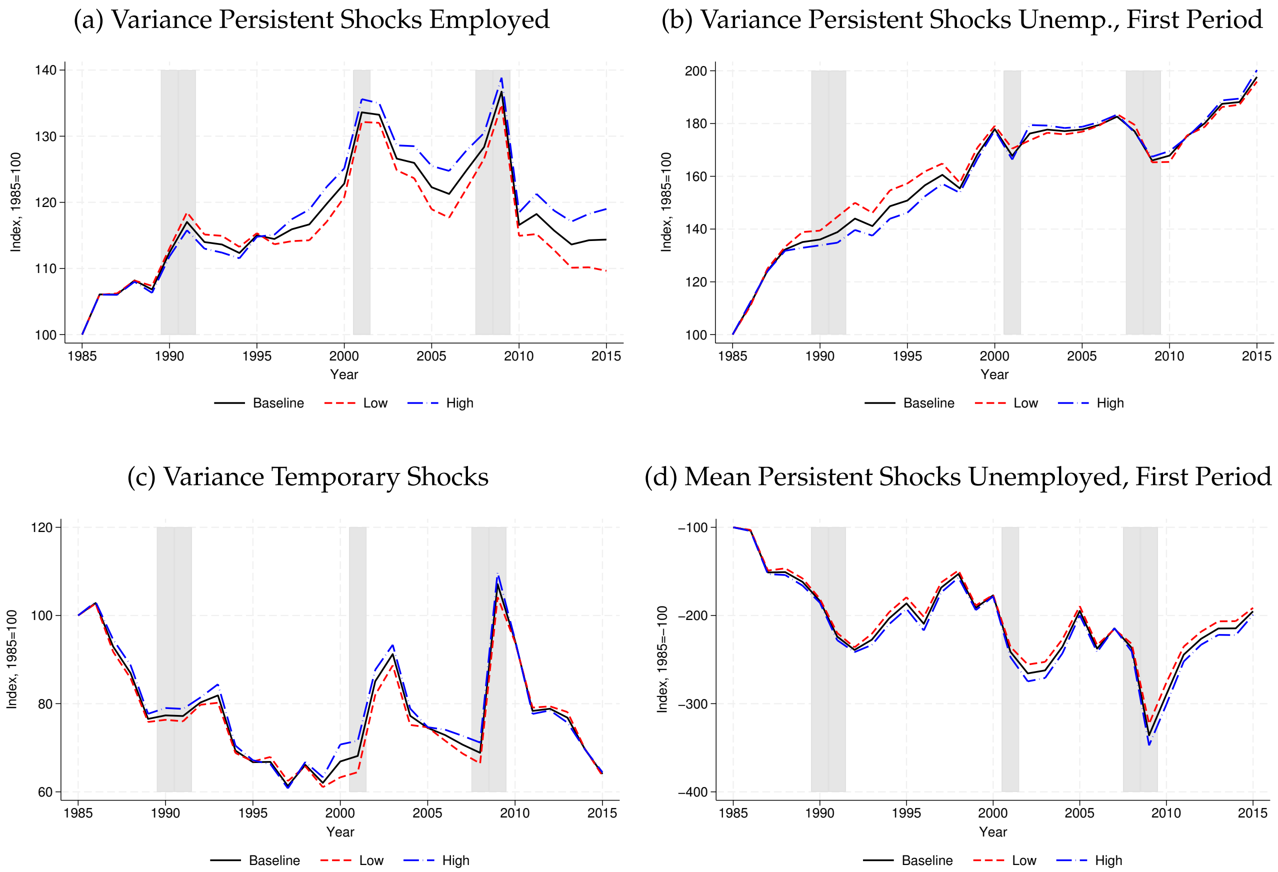
Note: The figure presents the results of estimating Model 3, where the parameters vary by gender. Panel (a) plots the variance of shocks to persistent earnings among the employed. Panel (b) plots the variance of shocks to persistent earnings among the unemployed, in their first period of unemployment. Panel (c) plots the variance of shocks to temporary earnings. The black, solid line corresponds to men, while the red, dashed line corresponds to women.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

E.9 Additional results: Sample start and end dates

In this appendix, we show that our results are robust to changing the start/end dates of our sample. In Figure A15, we present our baseline estimates of the parameters of the income process as well as estimates from the 1985-2019 time period (red, dashed line), which we refer to as "Start 1985", and from the 1981-2015 time period (blue, long dash-dotted line), which we refer to as "End 2015."[26] The figure shows that we obtain virtually identical results when we

[26]Note that because of the way we bin together the first four years and last four years in creating time fixed effects the Start 1985 series is presented from 1989-2015, and similarly the End 2015 series is presented from 1985-

Figure A13: Persistent and temporary risk over time by minimum earnings adjustment

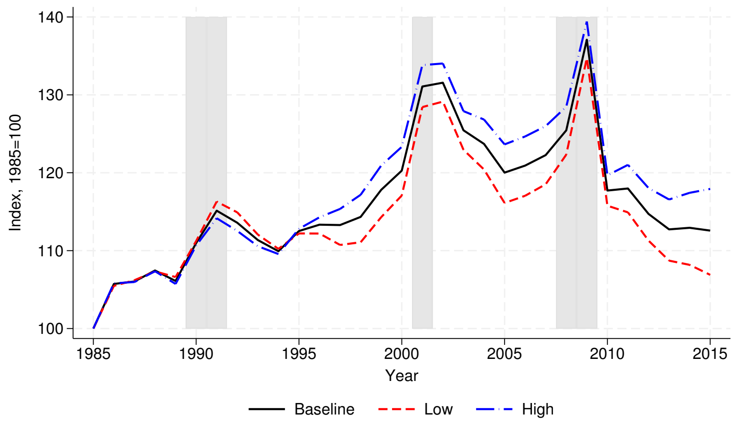


Note: The figure presents the results of estimating Model 2 across different values of the minimum earnings adjustment. Panel (a) plots the variance of shocks to persistent earnings among the employed. Panel (b) plots the variance of shocks to persistent earnings among the unemployed, in their first period of unemployment. Panel (c) plots the variance of shocks to temporary earnings. Panel (d) plots the mean of shocks to persistent earnings among the unemployed, in their first period of unemployment. The black, solid line corresponds to the baseline value of the minimum earnings adjustment ($3,350). The red, dashed line corresponds to low adjustment ($1,473), and the blue, long dash-dotted line corresponds to the high adjustment ($5,893). All dollars amounts are in 2005 PCE dollars.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

change the start/end year of the our sample.

2011.

Figure A14: Combined persistent risk over time by minimum earnings adjustment



Note: Figure presents combined persistent risk over time for the baseline minimum earnings adjustment (black, solid line), the low minimum earnings adjustment (red, dashed line), and the high minimum earnings adjustment (blue, long dashed-dotted line). See notes to Figure A13 for dollar amounts associated with each minimum earnings adjustment.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

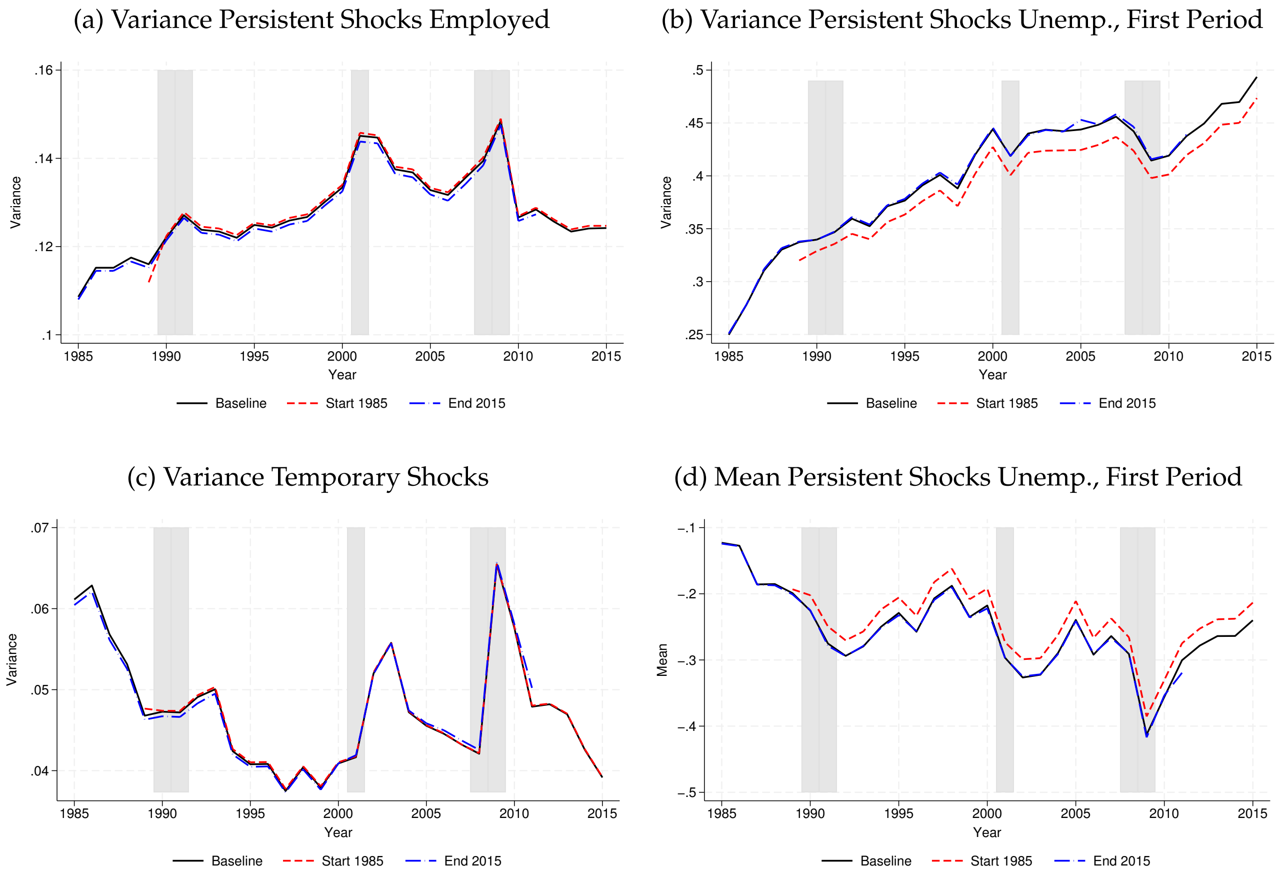
E.10 Additional results: Geographic variation

In this appendix, we test the second hypothesis that rising persistent earnings risk is related to the declines in manufacturing employment and union coverage. To do so, we start by estimating Model 3 by state. Given the large number of parameters, we use 5-year windows for the time fixed effects. We then relate changes in the parameters of the income process over time to changes in union coverage and manufacturing employment in a given state. Let X_s be the change in union membership (manufacturing employment) in state s between 1985-1989 and 2010-2015. The share of employed workers that are members of a union is measured in the CPS, while manufacturing employment is based upon Fort and Klimek (2016) industry classifications in the DER data. Let $\Delta Y_s = Y_{s,(2010-2015)} - Y_{s,(1985-1989)}$ denote the change in parameter Y (e.g. the variance of shocks to persistent earnings among employed etc.) for state s between $2010-2015$ and $1985-1989$. The specification we use is of the form,

$$\Delta Y_s = \alpha + \eta X_s + \epsilon_s \tag{44}$$

The parameter of interest is η which reports the correlation between the change in union coverage (manufacturing employment) in a state and measures of earnings risk in that state. If $\eta < 0$, then we have evidence that in states with larger declines in union coverage (manufacturing em-

Figure A15: Persistent and temporary risk over time by sample start/end date



Note: The figure presents the results of estimating Model 2 for different starting and ending dates. Panel (a) plots the variance of shocks to persistent earnings among the employed. Panel (b) plots the variance of shocks to persistent earnings among the unemployed, in their first period of unemployment. Panel (c) plots the variance of shocks to temporary earnings. Panel (d) plots the mean of shocks to persistent earnings among the unemployed, in their first period of unemployment. The black, solid line corresponds to our baseline estimate which uses a sample from 1981-2019. The red, dashed line uses a sample from 1985-2019, while the blue, long dashed-dotted line uses a sample from 1981-2015.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

ployment) there have been larger increases in earnings risk.

Tables A6 and A7 present the results of estimating equation (44) for changes in union coverage and manufacturing employment, respectively. The tables show that changes in union coverage and manufacturing employment are largely uncorrelated with changes in earnings risk. For the declines in manufacturing employment we find some evidence that decreasing manufacturing employment is associated with greater persistent risk while employed, but we do not find evidence that it is associated with increasing persistent risk (or scarring) in unem-

Table A6: Change in union coverage and changes in earnings risk

	(1) ΔQ	(2) ΔQ_E	(3) ΔQ_U	(4) ΔB_U
Change Union Coverage	0.000388	0.00106	-0.0186*	0.0108
	(0.00231)	(0.00216)	(0.0110)	(0.00928)
Round N (states)	100	100	100	100
R-squared	0.001	0.007	0.050	0.031

*Note: Table presents parameter results of estimating equation (44), where the independent variable is the change in union coverage in a state between 1985-1989 and 2010-2015 as measured in the CPS. Changes in union coverage are normalized to be mean zero and have unit standard deviation. Changes in income risk are measured between 1985-1989 and 2010-2015. Regressions are weighted by number individuals in each state, in each column. Robust standard errors in parentheses. ***$p < 0.01$, **$p < 0.05$,*$p < 0.1$.*

Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

Table A7: Change in manufacturing employment and changes in earnings risk

	(1) ΔQ	(2) ΔQ_E	(3) ΔQ_U	(4) ΔB_U
Change in Manufacturing Emp.	-0.00422*	-0.00446**	0.00669	0.0122
	(0.00225)	(0.00212)	(0.0140)	(0.00836)
Round N (states)	100	100	100	100
R-squared	0.083	0.099	0.005	0.032

*Note: Table presents parameter results of estimating equation (44), where the independent variable is the change in manufacturing employment in a state between 1985-1989 and 2010-2015. Changes in manufacturing employment are normalized to be mean zero and have unit standard deviation. Changes in income risk are measured between 1985-1989 and 2010-2015. Regressions are weighted by number individuals in each state, in each column. Robust standard errors in parentheses. ***$p < 0.01$, **$p < 0.05$,*$p < 0.1$.*

Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

ployment. We view these results as providing evidence against the second hypothesis that the increase in persistent income risk is related to the declines in manufacturing employment and union coverage.

E.11 Additional results: Routine occupations

In this appendix, we examine the third hypothesis that the increase in persistent income risk is occurring in routine occupations. To do so, we split occupations by their routine task content

Table A8: Routine skills and changes in earnings risk

	(1) ΔQ	(2) ΔQ_E	(3) ΔQ_U	(4) ΔB_U
Routine Skills	0.00123	0.00131	-0.0356**	0.0170*
	(0.00197)	(0.00199)	(0.0142)	(0.00922)
Round N (Occupations)	300	300	300	300
R-squared	0.002	0.003	0.027	0.015

Note: Table presents parameter results of estimating equation (13), where the independent variable is the degree of routine skills in an occupation as measured by Acemoglu and Autor (2011). Routine skills are normalized to be mean zero and unit standard deviation. Changes in income risk are measured between 1985-1989 and 2010-2015. Regressions are weighted by number individuals in each occupation, in each column. Robust standard errors in parentheses. ***$p < 0.01$, **$p < 0.05$,*$p < 0.1$.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

as measured in Acemoglu and Autor (2011). Acemoglu and Autor (2011) provide a measure of routine manual as well as routine cognitive task content of an occupation. We combine their estimates into a single measure of the routine task content of an occupation by averaging the two measures. As in Acemoglu and Autor (2011) we normalize the index to be mean zero and have unit variance. Table A8 presents the results of estimating equation (13) where the independent variable is the routine task content of an occupation. The results presented in Table A8 shows that the degree of routine task content is not correlated with changes in combined persistent risk (column (1)), or changes in persistent risk among the employed (column (2)). Column (3) shows that occupations with more routine task content have seen declines in the persistent risk from entering unemployment, which goes in the opposite direction of the aggregate trends presented in Section IV. Similarly, column (4) shows that occupations with more routine task content have seen smaller declines in persistent earnings from entering unemployment, which also goes in the opposite direction of the aggregate trends presented in Section IV. We view these results as providing strong evidence against the third hypothesis that the increase in persistent income risk is related to the declines in routine employment.

E.12 Additional results: Education and gender

In this appendix, we examine how income risk has evolved for men and women separately by education group in order to test hypothesis (4) that rising persistent risk is among low-skill men.[27] For this exercise, we estimate Model 3 in which the parameters of the income process differ by an individual's recorded education level in the CPS as well as by gender. As in Section V.A, we consider five educational groups: (1) less than high school, (2) high school graduate, (3) some college, (4) college graduate, and (5) more than a college degree.[28] Figure A16 presents the results of this estimation, where for ease of presentation we only present the time series for less than a high school degree and more than a college degree for each gender.[29] Panel (a) presents the time series of combined persistent risk by education and gender. The figure shows that for both men and women, more highly educated individuals has seen a larger increase in combined persistent income risk. Among men, we see a decline in combined persistent earnings risk for those without a high school degree. Conversely for men with more than a college degree, combined persistent income risk has increased by almost 30 percent by the end of the sample. Panel (b) presents the evolution of the mean shock to persistent earnings during the first year of unemployment for men and women by education level. The figure shows that the "scarring effect" on persistent earnings of unemployment has accelerated more for both highly educated men and women relative to their less educated counterparts. From this appendix, we do not find evidence that the increase in persistent risk is being driven by low-skill men. Additionally, we find that the evidence for the high skill hypothesis holds for both men and women.

E.13 Additional results: Industry

In this appendix, we show that our results for the high skill hypothesis in Section V.C are robust to using detailed industry instead of detailed occupation. We repeat the analysis of Section V.C for industries, by splitting the sample by an individuals 4-digit industry in their first CPS year. We are able to obtain an individual's industry by using industry classification from the LBD at an individual's EIN, which we are able to observe as part of the DER. At the industry level, we are able to measure the log of mean earnings.[30]

Table A9 shows the results of estimating equation 13 using industry level variation where the independent variable is log of mean earnings in an industry between 1985-1989. The table shows that industries with higher initial earnings have seen larger increases in combined persistent earnings, persistent earnings risk among the employed and unemployed, as well as a

[27] Work by Binder and Bound (2019) document the declining employment prospects of men in the U.S. over the past 50 years.

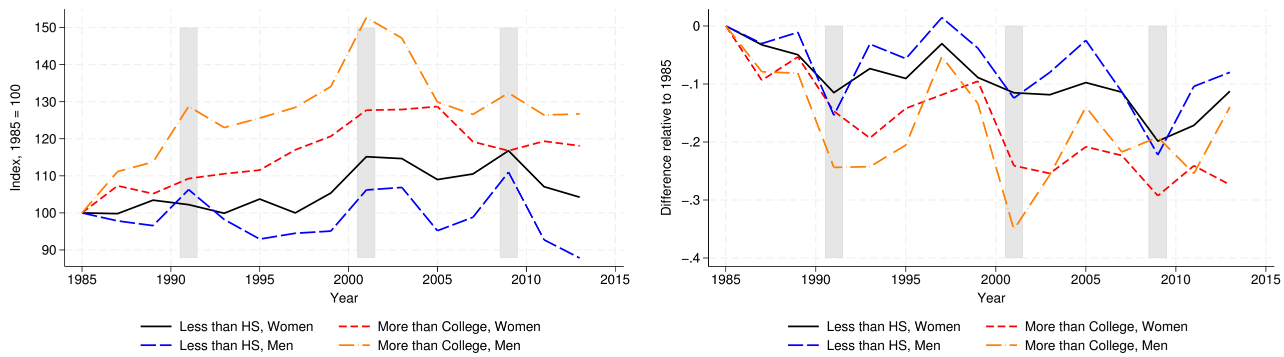
[28] As in Section V.A, we only use an individual's reported education if they are 30 or older in their CPS observation and use 2-year binned fixed effects in the estimation given the larger number of groups.

[29] The time series for all education groups and genders are available upon request.

[30] Note that the degree of non-routine cognitive task content is measured in the ONET database, which is based upon occupations. We are unaware of any attempts to create an ONET style database by industry. Similarly, the measure of cognitive skill content from Braxton and Taska (2023) is measured at the occupation level.

Figure A16: Changes in persistent risk over time by gender and education

(a) Combined Persistent Risk (b) Drift First Period of Unemployment



Note: The figure presents the results of estimating Model 3, where parameters vary by education and gender. Panel (a) presents combined persistent earnings risk over time by education and gender. Panel (b) presents the drift to persistent earnings during the first period of unemployment by gender and edcuation. The black, solid line corresponds to women with less than a high school degree. The red, dashed line corresponds to women with more than a college degree. The blue, long dashed line corresponds to men with less than a college degree. The orange, long dashed-dotted line corresponds to men with more than a college degree. Gray bars denote NBER recession dates.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

Table A9: Log mean earnings by industry and changes in earnings risk

	(1) ΔQ	(2) ΔQ_E	(3) ΔQ_U	(4) ΔB_U
Log Mean Earnings	0.00339**	0.00288*	0.0717***	-0.0244***
	(0.00149)	(0.00152)	(0.0116)	(0.00819)
Round N (Industry)	300	300	300	300
R-squared	0.022	0.019	0.030	0.040

Note: Table presents parameter results of estimating equation (13), where the independent variable is the log of mean earnings in an industry in the years 1985-1989. Log mean earnings are normalized to be mean zero and unit standard deviation. Changes in income risk are measured between 1985-1989 and 2010-2015. Regressions are weighted by number individuals in each industry, in each column. Robust standard errors in parentheses. ***$p < 0.01$, **$p < 0.05$,*$p < 0.1$.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

larger decline in persistent earnings from entering unemployment.

E.14 Additional details: ONET job zones

In this appendix, we provide additional details about ONET job zones. We first present the description of each ONET job zone that is included in the ONET package. We then present summary statistics about the mean level of earnings and education by ONET job zone as reported in the 2005 ACS.

E.14.1 Description and examples of each ONET Job Zone

Job Zone 1: Little or No Preparation Needed

- **Experience.** Little or no previous work-related skill, knowledge, or experience is needed for these occupations. For example, a person can become a waiter or waitress even if he/she has never worked before.

- **Education.** Some of these occupations may require a high school diploma or GED certificate.

- **Job Training.** Employees in these occupations need anywhere from a few days to a few months of training. Usually, an experienced worker could show you how to do the job.

- **Job Zone Examples.** These occupations involve following instructions and helping others. Examples include taxi drivers, amusement and recreation attendants, counter and rental clerks, construction laborers, continuous mining machine operators, and waiters/waitresses.

Job Zone 2: Some Preparation Needed

- **Related Experience.** Some previous work-related skill, knowledge, or experience is usually needed. For example, a teller would benefit from experience working directly with the public.

- **Education.** These occupations usually require a high school diploma.

- **Job Training.** Employees in these occupations need anywhere from a few months to one year of working with experienced employees. A recognized apprenticeship program may be associated with these occupations.

- **Job Zone Examples.** These occupations often involve using your knowledge and skills to help others. Examples include sheet metal workers, forest fire fighters, customer service representatives, physical therapist aides, salespersons (retail), and tellers.

Job Zone 3: Medium Preparation Needed

- **Related Experience.** Previous work-related skill, knowledge, or experience is required for these occupations. For example, an electrician must have completed three or four years of apprenticeship or several years of vocational training, and often must have passed a licensing exam, in order to perform the job.
- **Education.** Most occupations in this zone require training in vocational schools, related on-the-job experience, or an associate's degree.
- **Job Training.** Employees in these occupations usually need one or two years of training involving both on-the-job experience and informal training with experienced workers. A recognized apprenticeship program may be associated with these occupations.
- **Job Zone Examples.** These occupations usually involve using communication and organizational skills to coordinate, supervise, manage, or train others to accomplish goals. Examples include food service managers, electricians, agricultural technicians, legal secretaries, interviewers, and insurance sales agents.

Job Zone 4: Considerable Preparation Needed

- **Related Experience.** A considerable amount of work-related skill, knowledge, or experience is needed for these occupations. For example, an accountant must complete four years of college and work for several years in accounting to be considered qualified.
- **Education.** Most of these occupations require a four-year bachelor's degree, but some do not.
- **Job Training.** Employees in these occupations usually need several years of work-related experience, on-the-job training, and/or vocational training.
- **Job Zone Examples.** Many of these occupations involve coordinating, supervising, managing, or training others. Examples include accountants, sales managers, database administrators, teachers, chemists, environmental engineers, criminal investigators, and special agents.

Job Zone 5: Extensive Preparation Needed

- **Related Experience.** Extensive skill, knowledge, and experience are needed for these occupations. Many require more than five years of experience. For example, surgeons must complete four years of college and an additional five to seven years of specialized medical training to be able to do their job.

- **Education.** Most of these occupations require graduate school. For example, they may require a master's degree, and some require a Ph.D., M.D., or J.D. (law degree).

- **Job Training.** Employees may need some on-the-job training, but most of these occupations assume that the person will already have the required skills, knowledge, work-related experience, and/or training.

- **Job Zone Examples.** These occupations often involve coordinating, training, supervising, or managing the activities of others to accomplish goals. Very advanced communication and organizational skills are required. Examples include librarians, lawyers, aerospace engineers, wildlife biologists, school psychologists, surgeons, treasurers, and controllers.

E.14.2 Summary statistics by ONET Job Zone

We next present summary statistics by ONET job zone from the 2005 ACS. Panel (a) of Figure A17 shows mean annual earnings by ONET job zone. The figure shows that the average level of earnings is steeply increasing in job zone. Panel (b) shows that mean years of education in the 2005 ACS by ONET job zone. The figure shows that the average level of education is steeply increasing by ONET job zone.

E.15 Additional results: Occupations and younger workers

In this appendix, we examine the high skill worker hypothesis using workers who are younger when we observe their occupation in the CPS.

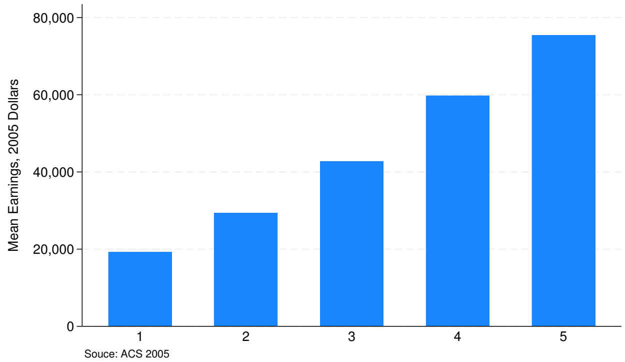
E.15.1 ONET job zones.

In this section, we repeat the ONET job zone specification from Section V.B but only classify an individual into a job zone if they are between the ages of 25 and 40 when their occupation is first recorded in the CPS-ASEC.[31] Figure A18 presents the results. Panel (a) of Figure A18 shows combined persistent earnings risk for each ONET job zone over time, and shows that

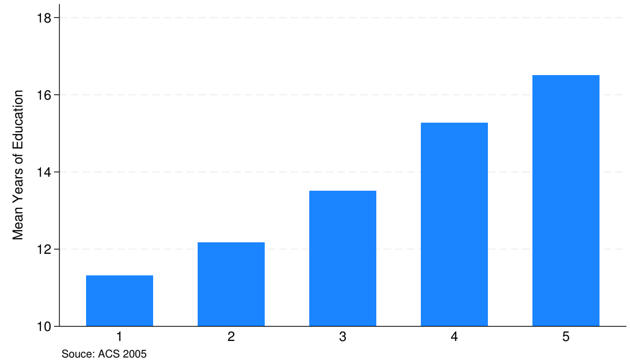
[31] In results that are available upon request we obtain similar results using workers between the ages of 30 and 45.

Figure A17: Summary statistics by ONET Job Zone

(a) Average earnings



(b) Average years of education



Note: Figure presents average annual earnings (panel (a) and average years of education (panel (b)) by ONET job zone.

combined persistent income risk has increased the most for job zones (4) and (5), which require considerable and extensive preparation, respectively. In panel (b) of Figure A18 we show the evolution of the mean shock to persistent earnings during the first period of unemployment.[32] The figure shows that the scarring effect of job loss has accelerated in a fairly common manner across job zones.

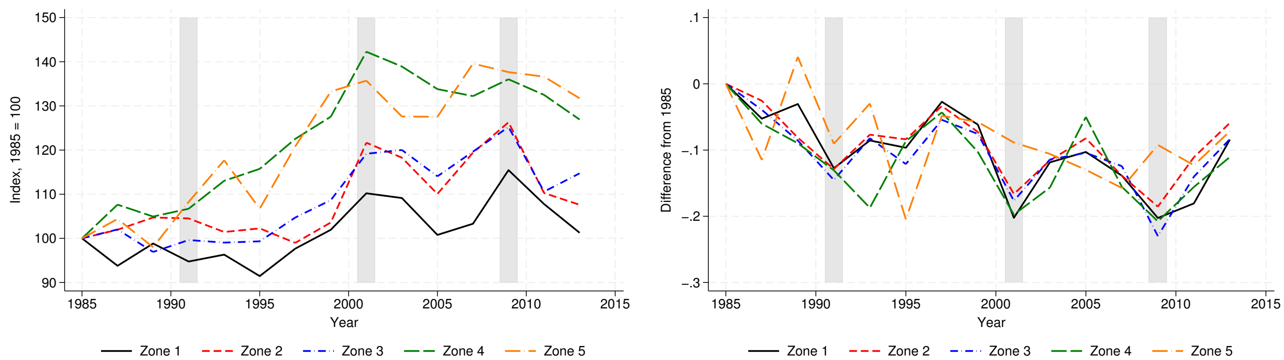
E.15.2 Detailed occupation results.

In this section, we repeat the analysis of Section V.C but only classify an individual into an occupation if they are between the ages of 25 and 40 when their occupation is first recorded in the CPS-ASEC. Table A10 presents the results. The table shows that high-skill occupations as proxied by: (A) non-routine cognitive skills, (B) cognitive skills, and (c) the log of mean earnings are associated with higher combined persistent risk (column (1)), persistent risk among the employed (column (2)), and persistent risk among the unemployed (column (3)). When the outcome of interest is the change in the drift to persistent earnings from entering unemployment (column (4)) we find a negative coefficient in each specification, as in Section V.C, however the coefficients are not statistically significant.

[32]Note in panel (b) we present the level difference since 1985 as the coefficient for job zone (5) is small and positive, which makes relative comparisons as we have done previously difficult to interpret.

Figure A18: Changes in persistent risk over time by ONET job zone, younger workers

(a) Combined Persistent Risk (b) Drift First Period of Unemployment



Note: The figure presents the results of estimating Model 3, where parameters vary by ONET Job zone and we only classify an individuals into a job zone if they are between the ages of 25 and 40 in their first CPS-ASEC observation. Panel (a) presents combined persistent earnings risk over time by ONET job zone. Panel (b) presents the drift to persistent earnings during the first period of unemployment by ONET job zone. The black, solid line corresponds to individuals in job zone 1. The red, dashed line corresponds to individuals in job zone 2. The blue, dash-dotted line corresponds to individuals in job zone 3. The green, long dashed line corresponds to individuals in job zone 4. The orange, long dashed-dotted line corresponds to individuals in job zone 5. Gray bars denote NBER recession dates.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

E.16 Estimating changes in computer requirements 1985-2015

In this appendix, we discuss how we estimate the change in computer requirements over the 1985-2015 time period. A challenge in examining the spread of computers (and/or software) as a proxy for technological change is finding consistent measures which span a long time horizon. Our data from Burning Glass covers 2007 to 2017, while the CPS Computer Supplement data, which asks individuals about direct computer use at work (as used by Krueger (1993)) covers 1984 to 2003 at an irregular frequency.[33] In this appendix we create a time series of computer requirements that spans our sample period (1985 to 2015) by combining the Burning Glass (BG) and CPS data series. Let $C_{o,BG,2007}$ denote the share of vacancies listing a computer or software requirement for occupation o in the Burning Glass (BG) database in the year 2007, and let $C_{o,CPS,2003}$ denote the share of workers in an occupation o who report using a computer as part of their job in the CPS in the year 2003. To get a mapping between the BG and CPS

[33]Over this period, the CPS computer supplement was performed in the years 1984, 1989, 1993, 1997, 2001, and 2003. The CPS computer supplement has been performed after 2003 but it no longer asks the question about direct computer usage at work.

Table A10: Changes in earnings risk and measures of high skill, younger workers

	(1) ΔQ	(2) ΔQ_E	(3) ΔQ_U	(4) ΔB_U
(A) Non-routine cognitive	0.00682***	0.00701***	0.0821***	-0.0144
	(0.00222)	(0.00214)	(0.0146)	(0.0104)
(B) Cognitive	0.00680***	0.00729***	0.0671***	-0.0176
	(0.00151)	(0.00152)	(0.0174)	(0.0111)
(C) Log mean earnings	0.00558***	0.00586***	0.0550***	-0.00159
	(0.00194)	(0.00201)	(0.0144)	(0.00891)

*Note: Table presents the coefficient η from estimating equation (13) for different measures of the skill content of an occupation and measures of income risk. Note in these regressions we only classify an individuals into an occupation if they are between the ages of 25 and 40 in their first CPS-ASEC observation. In panel (A) the measure of skill content is the non-routine cognitive index from Acemoglu and Autor (2011). Panel (b) uses the cognitive skill index from Braxton and Taska (2023), and panel (C) uses the log of mean earnings. All measures of skill content are normalized to be mean zero and unit standard deviation. Changes in income risk are measured between 1985-1989 and 2010-2015. Regressions are weighted by number individuals in each occupation, in each column. Robust standard errors in parentheses. ***$p < 0.01$, **$p < 0.05$,*$p < 0.1$*

Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

Table A11: Computer requirements in Burning Glass and CPS

	(1) Computer Req.BG ($C_{o,BG,2007}$)
Computer Usage CPS ($C_{o,CPS,2003}$)	0.206***
	(0.0194)
Constant	0.0122
	(0.00826)
Round N (Occupations)	300
R-squared	0.256

*Note: Table presents parameter results of estimating equation (A11). Robust standard errors in parentheses. ***$p < 0.01$, **$p < 0.05$,*$p < 0.1$.*

computer measures, we run the following cross-sectional regression:

$$C_{o,BG,2007} = \alpha + \beta C_{o,CPS,2003} + \epsilon_o \quad (45)$$

Table A11 reports the results of estimating equation 45. Using the recovered parameters $\hat{\alpha}$ and $\hat{\beta}$, and the historical CPS data on computer usage, we project an estimate of the BG measure of computer usage for earlier time periods. For example, we recover a 1984 estimate of computer requirements in the BG data, denoted $C_{o,BG,1984}$, via $C_{o,BG,1984} = \hat{\alpha} + \hat{\beta} C_{CPS,o,1984}$. We then linearly interpolate the projected time series and compute an estimated level of computer and software requirements for the 1985 to 1989 time period.[34] We then measure the change in computer requirements over our sample period as: $\Delta C_o = C_{o,BG,10-15} - C_{o,BG,85-89}$.

F Welfare effects of changing earnings risk

In this appendix, we use a stationary, finite-lifecycle, Bewley-Huggett-Aiyagari model to examine the welfare effects of changes in earnings volatility between the 1980s and 2010s. We treat the 1980s and 2000s as two different steady states. We assume there are $T \geq 2$ overlapping generations of agents, and let $t \in \{1, ..., T\}$ denote the age of an agent. Agents exit the model exogenously at age T, and there is no retirement.

Agents are heterogeneous along several dimensions. Let $e \in \{E, U\}$ denote the employment status of an agent, where $e = E$ denotes employed and $e = U$ denotes unemployed. Let $b \in \mathbb{R}$ denote the net asset position of an agent. When $b > 0$, the agent is saving, and when $b < 0$, the agent is borrowing. The agent's asset choice is constrained by a borrowing limit $\underline{b}$. Agents save and borrow at the risk-free rate, denoted r_f. Let $z \in \mathbb{R}$ denote an agent's persistent earnings. Let $\epsilon \in \mathbb{R}$ denote an agent's temporary shock to earnings.

Let $w_t(z, \epsilon, e)$ be a function that maps an individual's (i) age, (ii) persistent earnings, (iii) temporary earnings, and (iv) employment status into a wage. We define the wage $w_t(z, \epsilon, e)$ such that

$$w_t(z, \epsilon, e) = \begin{cases} \exp(\kappa_t + z + \epsilon) & \text{if } e = E \\ \gamma \exp(\kappa_t + z) & \text{if } e = U, \end{cases}$$

where κ_t is a deterministic age profile of log earnings. $\gamma \in [0, 1]$ can be thought of as a replacement rate of persistent income for the unemployed.[35] Wages are subject to labor income taxation. Let $\widetilde{w}_t(z, \epsilon, e)$ denote the after-tax income for an age t agent with persistent earnings z, temporary shock ϵ and employment status e. We model taxes following Heathcote et al. (2017), where after-tax income is given by

$$\widetilde{w}_t(z, \epsilon, e) = \lambda w_t(z, \epsilon, e)^{1-\alpha}.$$

[34] As noted above, estimates from the CPS are available in the years 1984, 1989, 1993, 1997, 2001 and 2003.

[35] Alternatively, one can model home production as proportional to z, consistent with our empirical interpretation of z as capturing "income risk" among the unemployed. See Section I for such a model.

The parameter $\alpha > 0$ governs the degree of tax progressivity.

At the start of each period, agents observe their employment status, as well as the shocks to persistent and temporary earnings. Define $y = z + \epsilon$ as residual earnings for an employed individual, and let $\delta(y, e) \in [0, 1]$ denote the probability that an agent becomes unemployed. The probability that an agent becomes unemployed depends upon their (residual) earnings and employment status from the prior period. In Section F.2, we discuss how we estimate the function $\delta(y, e)$ using prior earnings and employment status. Finally, when an agent enters into the labor market, they start as an employed agent and they draw their persistent earnings from a normal distribution with mean zero and variance Q_0.

Value functions. We next define the value function for agents in the model. We write the value function for agents after the shocks to employment status as well as those to temporary and persistent earnings have been realized. Let $V_t(b, z, \epsilon, e)$ denote the value of being an age t agent with employment status e, persistent earnings z and temporary earnings ϵ.[36] The agent makes a consumption savings decision in the current period, taking into account the set of potential income shocks next period. The value function for an age t agent is given by,

$$\begin{aligned} V_t(b, z, \epsilon, e) &= \max_{c, b' \geq \underline{b}} u(c) + \beta \mathbb{E}_{z', \epsilon', e'} \left[V_{t+1}(b', z', \epsilon', e') \right] \quad \forall t \leq T \\ V_{T+1}(b, z, \epsilon, e) &= 0, \end{aligned}$$

subject to the budget constraint,

$$c + b' \leq b(1 + r_f) + \widetilde{w}_t(z, \epsilon, e);$$

the law of motion for employment status,

$$e' = \begin{cases} E & \text{w. prob } 1 - \delta(y, e) \\ U & \text{w. prob } \delta(y, e); \end{cases}$$

and the law of motion for persistent earnings,

$$z' = \begin{cases} Fz + \nu_{E,t+1} & \text{if } e' = E \ \& \ e = E \\ Fz + \nu_{U,t+1} & \text{if } e' = U \ \& \ e = E \\ Fz + \nu_{N,t+1} & \text{if } e' = U \ \& \ e = U, \end{cases}$$

[36] Note for the unemployed the value of temporary earnings ϵ is irrelevant.

where $\nu_{e,t+1} \sim N(B_{e,t+1}, Q_{e,t+1})$.[37] Note that the mean and variance to the shock depends on the agent's employment status and age. Finally, the law of motion for temporary earnings is given by,

$$\epsilon' = \begin{cases} \epsilon_{t+1} & \text{if } e' = E \\ 0 & \text{if } e' = U, \end{cases}$$

where $\epsilon_{t+1} \sim N(0, R_{t+1})$. The variance of the shock to temporary earnings depends on the agent's age $t+1$.

F.1 Employment status law of motion

Before discussing the calibration of the quantitative model, we next discuss how we estimate a law of motion for employment status. We show empirically that the likelihood of becoming unemployed is a function of prior earnings. Let $U_{i,t}$ be an indicator for an individual i being in their first period of unemployment in period t. We model the likelihood that an individual enters into unemployment using the following functional form:

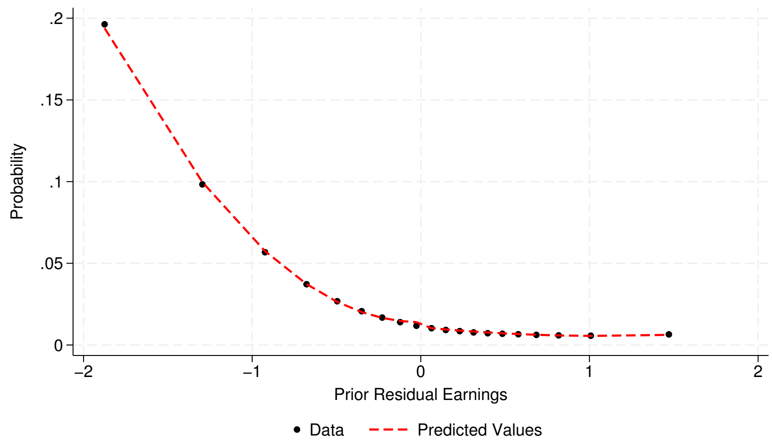
$$U_{i,t} = \mathbb{I}\{y_{i,t-1} \geq 0\} \left[\alpha_0^+ + \alpha_1^+ y_{i,t-1} + \alpha_2^+ y_{i,t-1}^2\right] + \mathbb{I}\{y_{i,t-1} < 0\} \left[\alpha_0^- + \alpha_1^- y_{i,t-1} + \alpha_2^- y_{i,t-1}^2\right] \quad (46)$$

The functional form in equation (46) allows for the probability of entering unemployment to be a quadratic function of prior (residual) earnings ($y_{i,t-1}$) estimated separately for positive (residual) earnings or negative prior (residual) earnings.

To gauge the plausibility of the functional form, Figure A19 compares the observed share of individuals entering unemployment (from employment) by ventile of prior earnings (black, solid line) to the predicted value based on equation (46) (red, dashed line). The fit is excellent. Finally, we must also define the probability of remaining unemployed. We use a constant probability of remaining unemployed. This employment process is consistent with the modeling assumptions made in Section I.

[37] As in Section E.5, we allow the shocks an individual draws while unemployed to depend on whether they are in their first year of unemployment ($e' = U$ & $e = E$) or after their first year of unemployment ($e' = U$ & $e = U$).

Figure A19: Probability of entering unemployment



Note: This figure shows the predicted probability of entering unemployment as estimated from equation (46) plotted against the observed probability of entering unemployment.
Source: 1973, 1979, 1981-1991, 1994, and 1996-2020 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1981 to 2019.

F.2 Calibration

We next discuss the calibration of the model.[38] Some parameters are assigned using estimates from the literature, while others are calibrated to be consistent with the U.S. labor market in the 1980s.

Demographics and preferences. To align with the sample in Section III.A, agents enter the model at age 25 ($t = 1$) , and work until age 60 ($T = 36$). When agents enter the model, they begin with zero assets and are employed.

Agents receive utility from consumption, with preferences given by,

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

We set the risk aversion parameter to a standard value, $\sigma = 2$. Agents discount the future at rate $\beta = 0.968$. The parameter β is calibrated to match the average ratio of net worth to income. As in Kaplan and Violante (2010), we target a value of 2.5.

Income process. Agents receive wages that are a function of their age, persistent earnings, and temporary earnings. The fixed age component of earnings is estimated as part of the residual-

[38]We solve the model using VFI on discrete grids. In Appendix F.4, we discuss how we discretize the income process.

ization process in Section III.B. Panel (a) of Figure A20 plots the deterministic path of earnings that is used in the model.

When agents are unemployed they receive (pre-tax and transfers) a fraction $\gamma \in [0,1]$ of their persistent earnings. The parameter γ can be thought as the replacement rate of unemployment insurance (UI). We set $\gamma = 0.4$, as in Shimer (2005). We next discuss the estimation of the stochastic process that governs how earnings evolve in the model.

Shocks to income. We model shocks to labor income as a function of an individual's age as in Appendix E.5. In our baseline estimation, we use the shocks to labor income that correspond to the 1980s. In the welfare experiment in Appendix F.3, we sequentially adjust the parameters of the income process to their 2010 values. Panels (d)-(h) of Figure A20 present the age profiles of shocks to temporary and persistent earnings that are used in the baseline estimation of the quantitative model.

Probability of unemployment. We model the likelihood that an employed individual becomes unemployed as a non-linear function of their prior (residual) earnings as in Section F.1. In particular, we use the parameter estimates from estimating equation (46) to discipline the following equation for $\delta(y,E)$,

$$\delta(y,E) = \alpha_E + \mathbb{I}\{y \geq 0\}\left[\alpha^{+}_{0,E} + \alpha^{+}_{1,E}y + \alpha^{+}_{2,E}y^2\right] + \mathbb{I}\{y < 0\}\left[\alpha^{-}_{0,E} + \alpha^{-}_{1,E}y + \alpha^{-}_{2,E}y^2\right] \tag{47}$$

To be consistent with the labor market in the 1980s, we include a constant term α_E in equation (47). We calibrate the constant (α_E) to match the probability that an individual transitioned from employment to unemployment in the 1980s in our sample, which we find to be 3.5%. Panel (c) of Figure A20 presents the profile for the probability of entering into unemployment as a function of prior (residual) earnings. We set the likelihood that an unemployed individual remains unemployed to be a constant ($\delta(y,U)) = \alpha_U$) equal to 56.1%.

Taxes. We model taxes as in Heathcote et al. (2017). Following Heathcote et al. (2017), we set the tax progressivity (α) parameters to be equal to 0.181. In addition to financing the UI system, we model the government as having exogenous expenditures G that are equal to share $g \in [0,1]$ of before-tax labor income. Using NIPA data on personal income and government consumption expenditure and investment, we set $g = 0.260$. We set the level parameter (λ) so that government revenue from taxes is equated to government spending on transfers and the exogenous government spending. Panel (b) of Figure A20 presents the implied tax function in the model economy. Agents with pre-tax incomes below approximately \$10K receive transfers from the government, while individuals with pre-tax incomes greater than \$10K pay labor income taxes.

Table A12: Model parameters

Variable	Value	Description
β	0.968	Discount factor
r_f	4%	Risk-free interest rate
σ	2	Coefficient of relative risk-aversion
α	0.181	Progressivity of tax function
γ	0.4	Replacement Rate UI
g	0.260	Ratio of government expenditure to pre-tax income
α_E	0.0135	Constant in unemployment probability
$\alpha_{0,E}^{+}$	0.0109	Constant in unemployment probability, positive prior earnings
$\alpha_{1,E}^{+}$	-0.0099	Linear term in unemployment probability, positive prior earnings
$\alpha_{2,E}^{+}$	0.0046	Square term in unemployment probability, positive prior earnings
$\alpha_{0,E}^{-}$	0.0140	Constant in unemployment probability, negative prior earnings
$\alpha_{1,E}^{-}$	0.0004	Linear term in unemployment probability, negative prior earnings
$\alpha_{2,E}^{-}$	0.0514	Square term in unemployment probability, negative prior earnings
α_U	0.561	Probability unemployed remain unemployed

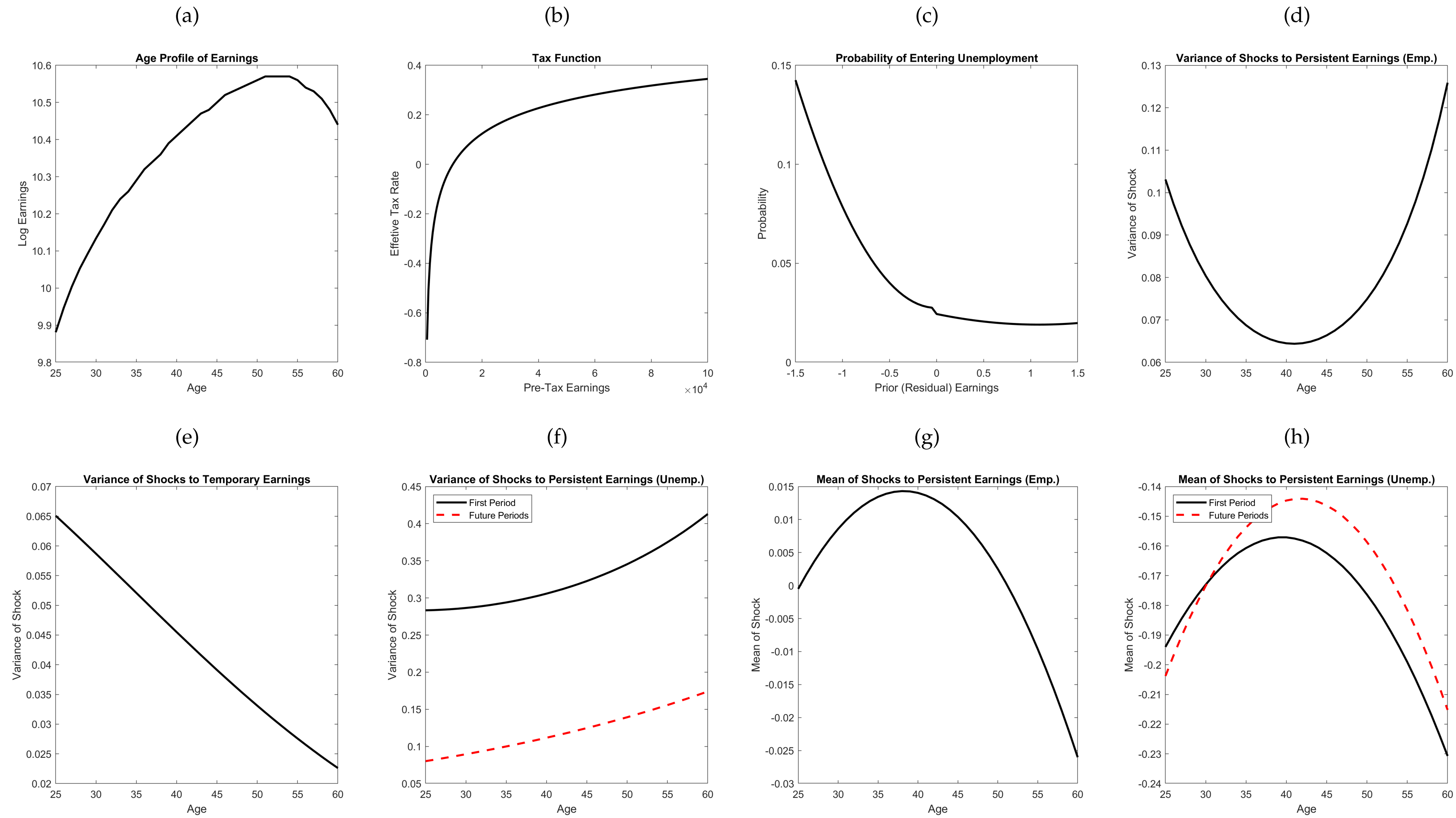
Note: Table presents model parameters for the baseline estimation of the quantitative model.

Asset markets. Agents are able to save and borrow at the risk-free rate of 4%. We set the borrowing limit $\underline{b}$ to the natural borrowing limit.[39] This yields an upper bound to the extent to which agents can use borrowing to smooth shocks to income.

Table A12 and Figure A20 present the parameters that govern the baseline model economy. In the next section, we conduct the welfare experiment of adjusting labor income risk as documented in Section IV.

[39] In our model, implementing the natural borrowing limit is equivalent to requiring that individuals exogenously exit the model with zero debt.

Figure A20: Model parameters



Note: Figure plots parameters used in the baseline estimation of the quantitative model.

F.3 Welfare implications of changing earnings risk

In this section, we examine the welfare implications of rising persistent income risk. We measure the welfare effects of changing income risk using lifetime consumption equivalents behind the veil-of-ignorance (i.e., before the initial draws of persistent earnings are drawn). In Section IV, we documented three facts about the changing nature of persistent income risk: (1) persistent earnings risk among the employed increased, (2) persistent earnings risk among the unemployed increased, and (3) the scarring effect (decline in persistent earnings) from entering unemployment accelerated. We also showed that the rate of entry into unemployment declined and that temporary earnings risk declined. We model these changes in our quantitative model by using the estimates for the 2010s from the income process in Appendix E.5 for Q_E, Q_U, B_U and R.[40] To match the change in the rate of entry into unemployment, we adjust α_E in the unemployment transition probability (equation 47) to match the EU rate from 2010-2015, which we measure to be 2.3%. Column (2) of Table A13 presents the results of modeling these changes. We find that the increase in persistent earnings risk has reduced welfare by 6% of lifetime consumption as individuals increase their precautionary savings in response to the increase in risk.

Two factors mitigated welfare losses from rising persistent earnings risk since the 1980s: (1) declining temporary risk, and (2) declining rates of entry into unemployment. We sequentially remove these mitigating factors and examine the implications of rising persistent risk. Column (3) of Table A13 shows the effects of rising persistent risk without the decline in temporary risk. We find that keeping temporary earnings risk at its 1980s value had a very modest role in mitigating the effects of rising persistent risk. If temporary earnings risk has stayed at its 1980s levels, then the welfare losses from the rise in persistent earnings risk would have been almost 6.3% of lifetime consumption.

The second mitigating factor is slower entry into unemployment in the 2010s. Column (4) of Table A13 shows the effects of keeping the entry rate into unemployment at its 1980s level. We find that if the entry rate into unemployment had remained at its 1980s level, the rise in persistent earnings risk would have caused over a 9% welfare loss. Hence, the decline in entry into unemployment played a substantial role in mitigating the impact of rising persistent earnings risk within employment and unemployment spells. Finally in column (5), we show that increasing persistent earnings risk among the employed results in over a 6% welfare decline (while keeping all other parameters fixed at their 1980s values), highlighting that it is the major contributor to the welfare losses associated with rising persistent risk.

[40] Given that we work with the log of earnings, changes in risk can change mean earnings due to Jensen's inequality. To highlight the role of risk, we adjust the age earnings profile κ so that mean earnings are held constant in the welfare experiment.

Table A13: Welfare experiment: changes in earnings risk

	(1)	(2)	(3)	(4)	(5)
	Baseline	$Q_E, Q_N, B_N, R, \alpha_E$	$Q_E, Q_N, B_N\ \alpha_E$	Q_E, Q_N, B_N, R	Q_E
Welfare chg. from baseline	-	-6.00%	-6.27%	-9.16%	-6.21%
R	0.04	0.03	0.04	0.03	0.04
Q_E	0.08	0.10	0.10	0.10	0.10
Q_N	0.33	0.63	0.63	0.63	0.33
B_N	-0.18	-0.36	-0.36	-0.36	-0.18
Q	0.10	0.11	0.11	0.13	0.11
EU Rate	0.035	0.022	0.023	0.038	0.036

Note: Table presents the results of the welfare experiment in Section F.3. Welfare is measured as a percent of lifetime consumption.

F.4 Computational Details

In this appendix, we discuss how we solve the lifecycle Bewley model. We solve the model using value function iteration on grids. Below we discuss the process for discretizing income shocks.

F.4.1 Discretization process (persistent earnings)

In this section, we outline our process for discretizing shocks to persistent earnings. For ease of presentation, we abstract from allowing the drift and variance of shocks to vary by age. For an income process that allows the mean and variance of shocks to vary by age, simply repeat these steps for each age.

At the start of the period an agent draws whether or not they will be employed for the period. Recall that the process for persistent earnings is given by

$$z^{'} = \rho z + B_e + \nu_e$$

where $e \in \{E, U\}$ denotes employment status, B_e denotes the drift of persistent earnings while in employment status e, and ν_e is the shock to persistent earnings while in employment status e. We assume that the drifts to persistent earnings and the variance of the shocks to persistent earnings differ by employment status. That is $\nu_U \sim N(0, Q_U)$, and $\nu_E \sim N(0, Q_E)$. Finally, the parameter ρ governs the degree of persistence in the process.

Define a transition matrix for agents classified as employed, denoted π^E, and a transition matrix for agents classified as unemployed, denoted π^U. The elements of π^e_{jk} define the probabilities that an agent with employment status e, moves from persistent earnings state j **today** to persistent earnings state k **tomorrow**.

Assume for now that we have specified a grid of values for z with N grid points, which are given by $[z_1, z_2, ..., z_N]$. Let the points be evenly spaced, with distance between grid points denoted by d.[41] The transition probability of going from state j **today** to state k **tomorrow** for an individual with employment status e is given by

$$\begin{aligned}\pi^e_{jk} &= P(\widetilde{z}_t = z_k | \widetilde{z}_{t-1} = z_j, e) \\ &= P(z_k - \frac{d}{2} < \rho z_j + B_e + \nu_e < z_k + \frac{d}{2}) \\ &= P(z_k - \frac{d}{2} - \rho z_j - B_e < \nu_e < z_k + \frac{d}{2} - \rho z_j - B_e) \end{aligned} \tag{48}$$

For an interior point on the grid, the probability in equation (48) is given by

$$\pi^e_{jk} = F\left(\frac{z_k + \frac{d}{2} - \rho z_j - B_e}{\sigma_{\nu,e}}\right) - F\left(\frac{z_k - \frac{d}{2} - \rho z_j - B_e}{Q_e^{1/2}}\right)$$

where $F(\cdot)$ is the standard normal distribution. For the end points of the grid, define the probabilities using:

$$\begin{aligned}\pi^e_{j1} &= F\left(\frac{z_1 + \frac{d}{2} - \rho z_j - B_e}{Q_e^{1/2}}\right) \\ \pi^e_{jN} &= F\left(\frac{z_N - \frac{d}{2} - \rho z_j - B_e}{Q_e^{1/2}}\right)\end{aligned}$$

F.4.2 Discretization Process (Temporary Earnings)

To discretize the process for temporary earnings, we use Tauchen's method with the persistence of the shock set to zero.

F.5 Additional quantitative results: Higher order moments

In this section, we briefly discuss estimates of the higher order moments from the income process used in our quantitative model.

Guvenen et al. (2021), hereafter referred to as GKOS, showed that labor income changes exhibit substantial deviations from a normal distribution and that the scope of these deviations

[41] In practice, we define the endpoints of the grid using $z_N = m\left(\frac{Q_U}{1-\rho}\right)^{\frac{1}{2}}$, setting $m = 3$, and $z_1 = -z_N$.

varies by an individual's prior earnings, which they refer to as recent earnings. In estimating the higher order moments from the simulated data of our income process, we closely follow the setup of GKOS. First, for each simulated individual, we measure their recent earnings, which is the sum of their earnings over the prior 5 years. We then remove the age specific component. To align with GKOS, in measuring recent earnings in a year $t-1$ we require that the individual have earnings above the minimum cutoff in year $t-1$ as well as in at least two of the years between $t-2$ and $t-5$. We then measure moments of the distribution of changes in earnings between t and $t+1$ by decile of recent earnings. To align with the estimated income process in GKOS, we use arc percent changes in earnings.

Before discussing the trends in higher order moments for our income process, we briefly review the findings of GKOS. They find that the standard deviation of 1 year earnings changes decreases in prior earnings up to the 80^{th} percentile of recent earnings distribution and increases slightly with recent earnings at the top of the distribution. GKOS find that labor income shocks are negatively skewed. GKOS show that as recent earnings increase, the shocks become more negatively skewed up to the 80th percentile of recent earnings and then there is a moderate reversal at the top of the distribution. Finally, GKOS show that labor income shocks have excess kurtosis and that kurtosis increases up to the 80^{th} percentile of recent earnings and then declines at the top of the distribution.

In Figure A21 below, we present estimates of the 2^{nd}, 3^{rd}, and 4^{th} moments of the 1-year arc change in labor earnings and find that the higher order moments of our income process qualitatively align with the pattern documented by GKOS. The far left panel of the figure shows the standard deviation of 1-year arc earnings changes by decile of recent earnings. In our simulated data, the standard deviation of earnings changes decreases with recent earnings. The middle panel presents the skewness of earnings changes in our simulated data. Our income process generates a U-shaped pattern of negative skewness, with the greatest negative skewness at the median of the distribution. Finally, the right panel plots the kurtosis of the simulated earnings changes and shows that in the simulated data kurtosis increases with recent earnings.

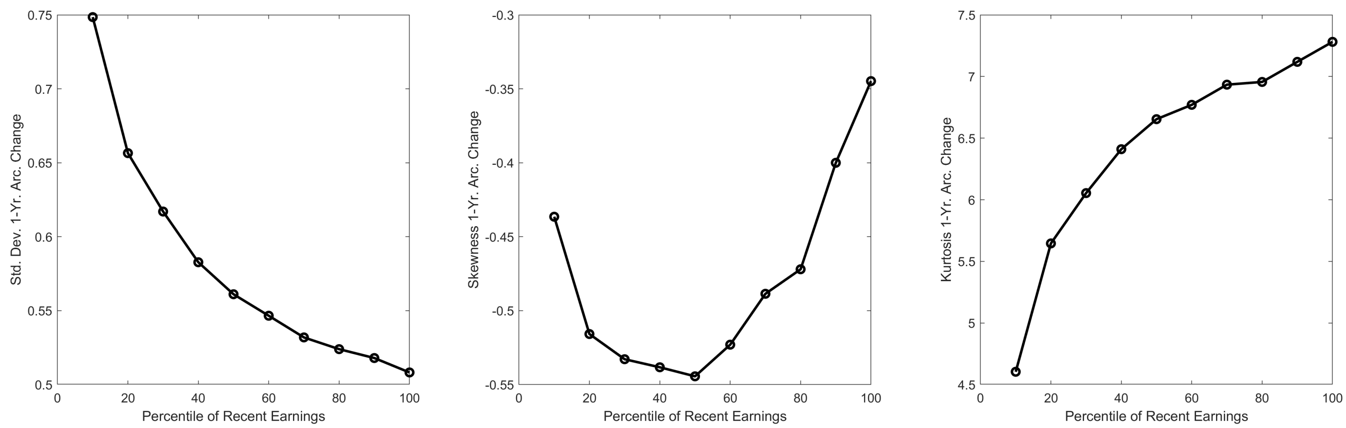
References

Acemoglu, D. and D. Autor (2011). Skills, tasks and technologies: Implications for employment and earnings. In *Handbook of labor economics*, Volume 4, pp. 1043–1171. Elsevier.

Altonji, J. G., A. A. Smith Jr, and I. Vidangos (2013). Modeling earnings dynamics. *Econometrica 81*(4), 1395–1454.

Figure A21: Higher order moments for baseline income process

(a) Std. dev. by recent earnings (b) Skewness by recent earnings (c) Kurtosis by recent earnings



Note: Panel (a) plots the standard deviation of log residualized arc income changes from the model simulation based on the income process from Appendix F. Panels (b) and (c) report the skewness and kurtosis of model simulated log residualized arc income changes, respectively.

Anderson, B. D. O. and J. B. Moore (1979). *Optimal Filtering*. Prentice-Hall.

Autor, D. H., L. F. Katz, and M. S. Kearney (2008). Trends in us wage inequality: Revising the revisionists. *The Review of economics and statistics 90*(2), 300–323.

Barbe, P. and P. Bertail (2012). *The weighted bootstrap*, Volume 98. Springer Science & Business Media.

Binder, A. J. and J. Bound (2019). The declining labor market prospects of less-educated men. *Journal of Economic Perspectives 33*(2), 163–90.

Braxton, J. C., K. F. Herkenhoff, J. L. Rothbaum, and L. Schmidt (2021). Changing income risk across the us skill distribution: Evidence from a generalized kalman filter. Technical report, National Bureau of Economic Research.

Braxton, J. C. and B. Taska (2023). Technological change and the consequences of job loss. *American Economic Review 113*(2), 279–316.

Dempster, A. P., N. M. Laird, and D. B. Rubin (1977). Maximum likelihood from incomplete data via the em algorithm. *Journal of the Royal Statistical Society: Series B (Methodological) 39*(1), 1–22.

Fort, T. C. and S. D. Klimek (2016). The effects of industry classification changes on us employment composition.

Gourieroux, C. and A. Monfort (1995). *Statistics and econometric models*, Volume 1. Cambridge University Press.

Greene, W. H. (2003). Econometric analysis. *Prentice Hall google schola 3*, 116–125.

Guvenen, F., G. Kaplan, J. Song, and J. Weidner (2022). Lifetime earnings in the united states over six decades. *American Economic Journal: Applied Economics*.

Guvenen, F., F. Karahan, S. Ozkan, and J. Song (2021). What do data on millions of us workers reveal about lifecycle earnings dynamics? *Econometrica 89*(5), 2303–2339.

Guvenen, F., S. Ozkan, and J. Song (2014). The nature of countercyclical income risk. *Journal of Political Economy 122*(3), 621–660.

Hamilton, J. (1994a). *Time series analysis*. Princeton University Press, Princeton, NJ.

Hamilton, J. D. (1994b). State-space models. *Handbook of econometrics 4*, 3039–3080.

Heathcote, J., K. Storesletten, and G. L. Violante (2017). Optimal tax progressivity: An analytical framework. *The Quarterly Journal of Economics 132*(4), 1693–1754.

Jarmin, R. and J. Miranda (2002). The longitudinal business database. Technical report, US Census Bureau, Center for Economic Studies.

Juhn, C., K. M. Murphy, and B. Pierce (1993). Wage inequality and the rise in returns to skill. *Journal of political Economy 101*(3), 410–442.

Kaplan, G. and G. Menzio (2016). Shopping externalities and self-fulfilling unemployment fluctuations. *Journal of Political Economy 124*(3), 771–825.

Kaplan, G. and G. L. Violante (2010). How much consumption insurance beyond self-insurance? *American Economic Journal: Macroeconomics 2*(4), 53–87.

Karahan, F. and S. Ozkan (2013). On the persistence of income shocks over the life cycle: Evidence, theory, and implications. *Review of Economic Dynamics 16*(3), 452–476.

Krueger, A. B. (1993). How computers have changed the wage structure: evidence from microdata, 1984–1989. *The Quarterly Journal of Economics 108*(1), 33–60.

Särkkä, S. and L. Svensson (2023). *Bayesian filtering and smoothing*, Volume 17. Cambridge university press.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American economic review*, 25–49.