Online Appendix: Self-fulfilling Prophecies in the Transition to Clean Technology

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OA1 Supply side equations in the decentralized equilibrium

Substituting the firm's demand curve (9) and research technology (5), we write the Hamiltonian for the firm's maximization problem as:

$$H_{ji} = \alpha P_j L_j^{1-\alpha} q_{ji} x_{ji}^{\alpha} - P_j q_{ji} x_{ji} - w s_{ji} + \lambda_{ji} \mu Q_j s_{ji}.$$

The first order condition (FOC) for s_{ji} is (11). The FOC for x_{ji} reads

$$\alpha P_j L_j^{1-\alpha} q_{ji} x_{ji}^{\alpha-1} \alpha = P_j q_{ji},$$

which after substitution of (9) gives (10) and

$$x_{ji} = \alpha^{\frac{2}{1-\alpha}} L_j. \tag{OA.1}$$

Substituting (10) and (OA.1) into the definition of profits gives (15). Substitution of (OA.1) into the production function for Y_j gives (13).

The FOC for q_{ji} gives (12). Since $\partial \pi_{ji} / \partial q_{ji}$ is the same across firms, λ_{ji} is also the same across firms. Omitting the subscript i in (12), dividing both sides by λ_j, and substituting (15), we find

$$\dot{\lambda}_j / \lambda_j \equiv \hat{\lambda}_j = r - (1-\alpha) \alpha P_j Y_j / (Q_j \lambda_j). \tag{OA.2}$$

Using (11), (8), and (13), we find (A.2) for the research active sector. From (11) we have $\hat{\lambda}_k = \hat{w} - \hat{Q}_k$, which combined with (A.2) gives (A.4).

Finally, for any sector j, (OA.2) can be rewritten as

$$\hat{\lambda}_j = r - \alpha\mu L_k \frac{P_j L_j/\lambda_j}{P_k L_k/\lambda_k}.$$

Using $P_c Q_c = P_d Q_d$ as derived from (8) and (13) and the definition of m_c, the above equation is equivalent to

$$\hat{\lambda}_j = r - \alpha\mu L_j \frac{Q_k \lambda_k}{Q_j \lambda_j} = r - \alpha\mu L_j \frac{m_k}{m_j}. \tag{OA.3}$$

OA2 Planner's solution

From (7), we write the carbon stock S as the sum of a non-decaying stock S_1 and a decaying stock S_2, where $\dot{S}_1 = \phi_L E$ and $\dot{S}_2 = \phi_D E - \delta S_2$. To allow for symmetric expressions (across clean and dirty sectors), we write emissions as $E = a_c Y_c + a_d Y_d$ but we maintain our assumption $a_c = 0$. The current value Hamiltonian of the planner's problem is given by

$$\begin{aligned}\mathscr{H}^{sp} = & \ln\left[\exp(-\gamma(S_1+S_2-\bar{S}))C\right] + \Omega_C\left[\left(\sum_{j\in\{c,d\}} C_j^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} - C\right] \\ & + \sum_{j\in\{c,d\}} \Omega_{Yj}\left[L_j^{1-\alpha}\int_0^1 q_{ji}x_{ji}^{\alpha}di - Y_j\right] + \sum_{j\in\{c,d\}} \zeta_j\left[Y_j - \int_0^1 q_{ji}x_{ij}di - C_j\right] \\ & + \sum_{j\in\{c,d\}} \int_0^1 \lambda_{ji}^s \mu s_{ji}\left(\int_0^1 q_{ji}di\right)di + \lambda_{S1}\phi_L \sum_{j\in\{c,d\}} a_j Y_j + \lambda_{S2}\left[\phi_D \sum_{j\in\{c,d\}} a_j Y_j - \delta S_2\right] \\ & + \zeta_L\left[1 - \sum_{j\in\{c,d\}} L_j - \sum_{j\in\{c,d\}}\int_0^1 s_{ji}di\right] + \sum_{j\in\{c,d\}}\int_0^1 \xi_{s_{ji}} s_{ji}di,\end{aligned} \tag{OA.4}$$

where C, C_j, Y_j, $\{x_{ji}\}_{i=0}^1$, L_j, $\{s_{ji}\}_{i=0}^1$ ($j \in \{c,d\}$) are the choice variables, $\{q_{ji}\}_{i=0}^1$ ($j \in \{c,d\}$) and S are the state variables, Ω_C and Ω_{Yj} are the shadow price associated with C and Y_j respectively, ζ_j and ζ_L are the shadow prices associated with the market clearing conditions, λ_{ji}^s, λ_{S1}, and λ_{S2} are the co-state variables, and finally,

$\xi_{s_{ji}}$ are the shadow prices associated with the non-negativity constraints.

The FOCs are given by

$$\frac{\partial \mathscr{H}^{sp}}{\partial C}: \quad C^{-1} = \Omega_C, \tag{OA.5}$$

$$\frac{\partial \mathscr{H}^{sp}}{\partial C_j}: \quad \zeta_j = \Omega_C \left(C_j/C\right)^{-1/\sigma}, \tag{OA.6}$$

$$\frac{\partial \mathscr{H}^{sp}}{\partial x_{ji}}: \quad \zeta_j q_{ji} = \Omega_{Yj} \alpha L_j^{1-\alpha} q_{ji} x_{ji}^{\alpha-1}, \tag{OA.7}$$

$$\frac{\partial \mathscr{H}^{sp}}{\partial Y_j}: \quad \Omega_{Yj} = \zeta_j + a_j \left[\phi_L \lambda_{S1} + \phi_D \lambda_{S2}\right], \tag{OA.8}$$

$$\frac{\partial \mathscr{H}^{sp}}{\partial L_j}: \quad \zeta_L = \Omega_{Yj}(1-\alpha) Y_j / L_j, \tag{OA.9}$$

$$\frac{\partial \mathscr{H}^{sp}}{\partial s_{ji}}: \quad \zeta_L = \lambda_{ji}^s \mu Q_j + \xi_{s_{ji}}, \tag{OA.10}$$

$$\frac{\partial \mathscr{H}^{sp}}{\partial q_{ji}}: \quad \dot{\lambda}_{ji}^s = \rho \lambda_{ji}^s - \mu \int_0^1 \lambda_{ji}^s s_{ji} di - \Omega_{Yj} L_j^{1-\alpha} x_{ji}^{\alpha} + \zeta_j x_{ji}, \tag{OA.11}$$

$$\frac{\partial \mathscr{H}^{sp}}{\partial S_1}: \quad \dot{\lambda}_{S1} = \gamma + \rho \lambda_{S1}, \tag{OA.12}$$

$$\frac{\partial \mathscr{H}^{sp}}{\partial S_2}: \quad \dot{\lambda}_{S2} = \gamma + (\rho + \delta)\lambda_{S2}. \tag{OA.13}$$

From (OA.7) we conclude that $x_{ji} = x_j$ for all i. From (OA.10) we conclude that all producers i in sector j that are active in R&D have the same shadow price λ_{ji}^s denoted λ_j^s. Hence $\int_0^1 \lambda_{ji}^s s_{ji} di = \lambda_j^s s_j$, where s_j is aggregate R&D labor as above. Using this and $x_{ji} = x_j$ in (OA.11), we conclude that we can drop all i subscripts:

$$x_{ji} = x_j,\ s_{ji} = s_j,\ \lambda_{ji}^s = \lambda_j^s.$$

OA2.1 Social cost of carbon

Solving (OA.12) and (OA.13) we find $\lambda_{S1} = -\gamma/\rho$ and $\lambda_{S2} = -\gamma/(\rho+\delta)$, respectively. Hence, the shadow values of the two carbon stocks are constant (because of the logarithmic exponential structure as in Golosov et al. (2014)) and negative (because excess carbon causes climate damage and reduces welfare). We use Φ to denote the social cost of carbon emissions (in utility terms):

$$\Phi \equiv \lambda_S \frac{\partial \dot{S}}{\partial E} = -\lambda_{S,1} \frac{\partial \dot{S}_1}{\partial E} - \lambda_{S,2} \frac{\partial \dot{S}_2}{\partial E} = \phi_L(-\lambda_{S1}) + \phi_D(-\lambda_{S2}) = \gamma(\phi_L/\rho + \phi_D/(\rho+\delta)).$$

We define the social cost of emissions in sector j, in terms of j-goods, as

$$\tau_j^s \equiv a_j \left[\phi_L(-\lambda_{S1}) + \phi_D(-\lambda_{S2})\right] / \zeta_j.$$

Thus, we find that in the optimum the emission costs equal:

$$\tau_d^s = a_d \Phi / \zeta_d; \quad \tau_c^s = a_c \Phi / \zeta_c = 0, \tag{OA.14}$$

where we refer to τ_j^s as the tax and introduce the zero tax in the clean sector τ_c^s to allow symmetry in our expressions below.

OA2.2 Optimal input mix and static allocation

Because $x_{ji} = x_j$, the production function and goods market equilibrium can be written as, respectively, $Y_j = L_j^{1-\alpha} Q_j x_j^{\alpha} = C_j + Q_j x_j$. Substituting (OA.7) and (OA.8), we find expressions (34) for the production function and we find the consumption-output ratio C_j/Y_j as a function of the tax:

$$Y_j = Q_j L_j [\alpha(1-\tau_j^s)]^{\alpha/(1-\alpha)} \tag{OA.15}$$

$$C_j = Y_j [1 - \alpha(1-\tau_j^s)]. \tag{OA.16}$$

Let $\chi_{C,j}$ denote the share of goods j in total value of consumption and χ_{Lj} denote the share of production labor hired in sector j, that is

$$\chi_{C,j} \equiv \frac{\zeta_j C_j}{\Omega_C C}, \qquad \chi_{L,j} \equiv \frac{L_j}{L}. \tag{OA.17}$$

$\chi_{C,j}$ is thus the direct counterpart of the expenditure share in the decentralized equilibrium and $\chi_{L,j}$ the production labor share. From (OA.9), (OA.5), (OA.16), and

(OA.17), we express the shadow price of labor as:

$$\zeta_L = \left(\frac{(1-\tau_j^s)(1-\alpha)}{1-\alpha(1-\tau_j^s)} \right) \frac{\chi_{Cj}}{L_j} \tag{OA.18}$$

We write the six equations (OA.9), (OA.7), (OA.6), (OA.8), (OA.15), and (OA.16) in relative terms (clean versus dirty) and solve for $\zeta_r, \Omega_{Yr}, C_r, Y_r, L_r, x_r$ in terms of τ_j^s and Q_r. Using these solutions with (20), the definition of θ_c, we find

$$\frac{\chi_{C,c}}{\chi_{C,d}} = \frac{\theta_c}{\theta_d}(1-\tau_d^s)^{-\frac{\sigma-1}{1-\alpha}}, \qquad \frac{\chi_{L,c}}{\chi_{L,d}} = \frac{\theta_c}{\theta_d}(1-\tau_d^s)^{-\frac{\sigma-\alpha}{1-\alpha}} \frac{1-(1-\tau_d^s)\alpha}{1-\alpha}. \tag{OA.19}$$

While in the decentralized equilibrium the relative technology fully captures the economic incentive for clean production and consumption, in the planner's solution these economic incentives must be augmented by the technology's contribution to carbon emission. Compared to the decentralized equilibrium, with the same level of relative technology θ_c, the planner will allocate more labor to the clean sector and consume a larger share of clean goods.

The allocation of labor for R&D is governed by (OA.10). For the research active sector, $\zeta_L = \lambda_k^s \mu Q_k$ must hold, while $\zeta_L > \lambda_{-k}^s \mu Q_{-k}$ holds for the research inactive sector. Using the definition of m_c^s, (35), we find that innovation is only active in the clean (dirty) sector if $m_c^s > 1/2$ (if $m_c^s < 1/2$). Thus, $m_c^s = 1/2$ separates the innovation regimes, just like in the decentralized equilibrium.

OA2.3 Static expression for the optimal tax

From (OA.5), (OA.6), (OA.17), and the definition of τ_d^s, we find $\tau_d^s = a_d\Phi/\zeta_d = a_d\Phi C_d/\chi_{C,d}$. Substituting $1/\chi_{C,d} = 1 + \chi_{C,c}/\chi_{C,d}$, (OA.16), (OA.15), and $L_d = \chi_{L,d}L$, we write:

$$\tau_d^s = a_d\Phi(\chi_{L,d}L)Q_d[\alpha(1-\tau_d^s)]^{\alpha/(1-\alpha)}(1+\chi_{C,c}/\chi_{C,d})[1-\alpha(1-\tau_d^s)].$$

Since, from (OA.19) we find $\chi_{C,c}/\chi_{C,d} = (\chi_{L,c}/\chi_{L,d})(1-\tau_d^s)(1-\alpha)/(1-\alpha(1-\tau_d^s))$, we can write:

$$\tau_d^s = a_d \Phi L Q_d [\alpha(1-\tau_d^s)]^{\alpha/(1-\alpha)} \left[(1-\alpha(1-\tau_d^s))\chi_{L,d} + (1-\tau_d^s)(1-\alpha)\chi_{L,c}\right]. \tag{OA.20}$$

This equation gives a relationship between the tax and other key variables. We are interested in the relationship with θ_c, L, and Q_d. We therefore substitute $\chi_{L,d} = 1-\chi_{L,c}$ at the RHS and divide both sides by τ_d^s. It can be easily seen that then the RHS declines with τ_d^s and with $\chi_{L,c}$, where the latter itself increases with τ_d^s and θ_c. Hence, there is a unique solution for τ_d^s as a function of θ_c, L, and Q_d with the following properties:

$$\tau_d^s = \tilde{\tau}(\theta_c, a_d \Phi L Q_d) \in [0,1), \quad \tilde{\tau}_1 < 0, \quad \tilde{\tau}_2 > 0. \tag{OA.21}$$

OA2.4 Dynamic allocation

Time differentiating (20) and (OA.17), we find

$$\dot{\theta}_c = (\sigma-1)\theta_c(1-\theta_c)\left(\hat{Q}_c - \hat{Q}_d\right), \tag{OA.22}$$

$$\dot{\chi}_{C,c} = (\sigma-1)\chi_{C,c}(1-\chi_{C,c})\left[\hat{Q}_c - \hat{Q}_d + \frac{1}{1-\alpha}\frac{\tau_d^s}{1-\tau_d^s}\hat{\tau}_d^s\right], \tag{OA.23}$$

$$\dot{\chi}_{L,c} = (\sigma-1)\chi_{L,c}(1-\chi_{L,c})\left[\hat{Q}_c - \hat{Q}_d + \left(\frac{1}{1-\alpha} + \frac{1}{(\sigma-1)(1-\alpha(1-\tau_d^s))}\right)\frac{\tau_d^s}{1-\tau_d^s}\hat{\tau}_d^s\right]. \tag{OA.24}$$

From (OA.10) and the definition of m_j^s, (35), we derive as the planner's counterpart of (26) that sector j is the research-active sector if its social market value exceeds cost: $m_j^s > \kappa_j \Leftrightarrow s_j > 0$. Since $j = k$ denotes the research-active sector, we must have $m_k^s \geq \kappa_k$ and $\zeta_L = \mu \lambda_k^s Q_k$. This implies:

$$k = \begin{cases} c, & \text{if } m_c^s > 1/2; \\ d, & \text{if } m_c^s < 1/2. \end{cases} \tag{OA.25}$$

$$(\hat{Q}_c, \hat{Q}_d) = \begin{cases} (\mu(1-L), 0), & \text{if } k = c; \\ (0, \mu(1-L)), & \text{if } k = d. \end{cases} \tag{OA.26}$$

Substituting (OA.7), (OA.9), and (OA.15) into (OA.11) gives $\hat{\lambda}_j^s = \rho - \mu s_j - \zeta_L L_j / \lambda_j^s Q_j$. For the research active sector ($j = k$) we have $s_k > 0$ and $\zeta_L = \lambda_k^s \mu Q_k$ from (OA.10). Together with the definition of m_j^s, (35), this gives:

$$\hat{\lambda}_j^s = \rho - \mu s_j - \frac{m_k^s}{m_j^s} \mu L_j. \tag{OA.27}$$

For $j = k$, (OA.27) implies $\hat{\lambda}_k^s = \rho - \mu(L_k + s_k)$. To derive the dynamics for m_j^s, note that $\hat{Q}_j = \mu s_j$ and $\hat{m}_k^s - \hat{m}_j^s = (\hat{\lambda}_k^s + \hat{Q}_k) - (\hat{\lambda}_j^s - \hat{Q}_j) = (m_k^s/m_j^s - L_k/L_j)\mu L_j$ where the second equality follows from (OA.27). Using $L_j = \chi_{L,j} L$ from (OA.17), we find

$$\dot{m}_c^s = \begin{cases} m_c^s(m_c^s - \chi_{L,c})\mu L, & \text{if } k = c; \\ 0, & \text{if } k = c, d; \\ (1 - m_c^s)(m_c^s - \chi_{L,c})\mu L, & \text{if } k = d. \end{cases} \tag{OA.28}$$

To derive the dynamics for L, we combine (OA.10) and $\hat{\lambda}_k^s = \rho - \mu(L_k + s_k)$ to arrive at $\hat{\lambda}_k^s + \hat{Q}_k = \hat{\zeta}_L = \rho - \mu L_k$, while time differentiating (OA.18) implies $\hat{\zeta}_L = \hat{\chi}_{C,c} - \hat{\chi}_{L,c} - \hat{L}$. Hence, we arrive at $\hat{L} = \mu L_k - \rho + \hat{\chi}_{C,c} - \hat{\chi}_{L,c}$. Substituting (OA.23) and (OA.24), we find:

$$\dot{L} = L \Big[\mu L_k - \rho + (\sigma - 1)(\chi_{L,c} - \chi_{C,c}) \left(\hat{Q}_c - \hat{Q}_d \right) + \left(\frac{(\sigma - 1)(\chi_{L,c} - \chi_{C,c})}{1 - \alpha} - \frac{1 - \chi_{L,c}}{1 - \alpha(1 - \tau_d^s)} \right) \frac{\tau_d^s}{1 - \tau_d^s} \hat{\tau}_d^s \Big]. \tag{OA.29}$$

Finally, we derive the dynamics of τ_d^s. From (OA.8) and (OA.9), we find $\zeta_d(1 - \tau_d^s) = \zeta_L L_d / ((1 - \alpha) Y_d)$ and after using the definition of τ_d^s to eliminate ζ_d using (OA.10) and (OA.15) to eliminate ζ_L and Y_d, respectively, we derive

$$(1 - \alpha) a_d \Phi = \tau_d^s [(1 - \tau_d^s)\alpha^\alpha]^{-1/(1-\alpha)} \lambda_k^s \mu Q_k / Q_d.$$

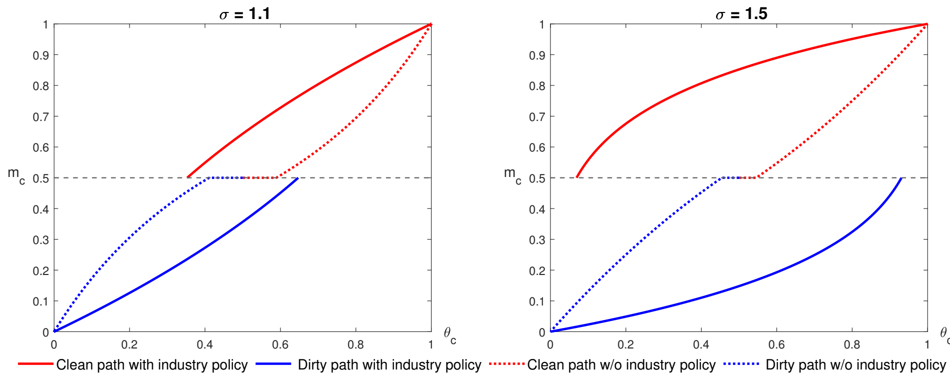

Figure 9: Size of overlap under industrial policy

Time differentiating this equation and substituting (OA.27) to eliminate $\hat{Q}_k$, we find the dynamics of the tax:

$$\dot{\tau}_d^s = -\frac{(1-\alpha)\tau_d^s(1-\tau_d^s)}{1-\alpha(1-\tau_d^s)}\left(\rho - \mu L_k - \hat{Q}_d\right). \tag{OA.30}$$

OA2.5 Characterising dynamics

Using (OA.19), (OA.21), and (OA.30) to eliminate χ_L, χ_C, τ_d^s, and $\dot{\tau}_d^s$, respectively, we find that (OA.22), (OA.26), (OA.28), and (OA.29) constitute four differential equations in four variables, namely θ_c, m_c, L, and Q_d.

The projection of the dynamics in the (θ_c, m_c^s) plane, used in Figure 8, can be characterised as follows. From (OA.25) we find the line $m_c^s = 1/2$ as the regime border which acts as the $\dot{\theta}_c = 0$ locus: above (below) the line there is clean (dirty) innovation only, and θ_c increases (decreases) over time if and only if $\sigma > 1$. Next, from (OA.28) we derive $m_c^s = \chi_{L,c}$ as the $\dot{m}_c^s = 0$ locus, with m_c increasing (decreasing) over time above (below) the locus. This locus is not a fixed line in the plane, since $\chi_{L,c}$ depends on not only θ_c but also τ_d^s, see (OA.19), which depends on the whole dynamics of the system, cf. (OA.21). Nevertheless, (OA.19) shows that $\chi_{L,c} > \theta_c$ and $\chi_{L,c} \to \theta_c$ at the corners $\theta_c \to 0$ and $\theta_c \to 1$, so that the $\dot{m}_c^s = 0$ locus cuts the 45 degree line in the corners and is above the 45 degree line for θ_c. From (OA.22) and (OA.28), we derive that the slope of the optimum path, $\dot{m}_c^s/\dot{\theta}_c$, in the corner $(1,1)$ equals 0. Hence the optimal path must approach the clean steady state from the south west.

OA2.6 Regulated market economy

With a tax τ_E on carbon emission (in real terms), the profit of final goods producers becomes

$$\pi_j = P_jY_j - wL_j - \int_0^1 P_{ji}x_{ji}di - \tau_E P a_j Y_j = (1-\tau_j)P_jY_j - wL_j - \int_0^1 P_{ji}x_{ji}di, \tag{OA.31}$$

where $\tau_j = a_j \tau_E P/P_j$ is the emission tax in terms of revenue (i.e. formulated as a value-added tax). Maximizing profits subject to the production function leads to a modified factor demand:

$$w = (1-\alpha)(1-\tau_j)P_j \frac{Y_j}{L_j}, \tag{OA.32}$$

$$P_{ji} = \alpha(1-\tau_j)P_jL_j^{1-\alpha}q_{ji}x_{ji}^{\alpha-1}. \tag{OA.33}$$

With this modified factor demand and the industry policy specified in the proposition, the Hamiltonian of the intermediate goods producers becomes

$$H_{ji} = (1+\tau_\alpha)\alpha(1-\tau_j)P_jL_j^{1-\alpha}q_{ji}x_{ji}^{\alpha} - P_jq_{ji}x_{ji} - ws_{ji} + \tau_{qj}q_{ji} + \lambda_{ji}\mu Q_j s_{ji}. \tag{OA.34}$$

where τ_α is the revenue subsidy, and τ_{qj} is the sector-specific technology subsidy. Accordingly, (10)-(12) change to

$$P_{ji} = \frac{1}{\alpha}(1+\tau_\alpha)^{-1}P_jq_{ji} = P_jq_{ji}, \tag{OA.35}$$

$$\mu Q_j \lambda_{ji} \leq w \perp s_{ji} \geq 0, \tag{OA.36}$$

$$\dot{\lambda}_{ji} = r\lambda_{ji} - \tau_{qj} - \frac{\partial \pi_{ji}}{\partial q_{ji}}. \tag{OA.37}$$

Combining the above results with (16) and (17), and using the symmetry result $x_{ji} = x_j, \lambda_{ji} = \lambda_j$, we can now summarize the regulated market economy by the following equations:

$$P_j/P = (C_j/C)^{-1/\sigma} \tag{OA.38}$$

$$1 = (1-\tau_j)\alpha L_j^{1-\alpha} x_j^{\alpha-1} \alpha(1+\tau_\alpha) \tag{OA.39}$$

$$w = (1-\tau_j)P_j(1-\alpha)Y_j/L_j \tag{OA.40}$$

$$w = \lambda_k \mu Q_k \tag{OA.41}$$

$$\hat{\lambda}_j - \hat{P} - \hat{C} = \rho - s_{qj}/\lambda_j - (1-\tau_j)P_j L_j^{1-\alpha} x_j^{\alpha}(1-\alpha)\alpha(1+\tau_\alpha) \tag{OA.42}$$

Combining the optimality conditions of the social planner's problem (OA.5)-(OA.11), using symmetry $x_{ji} = x_j, \lambda_{ji}^s = \lambda_j$ and the definition $\tau_j^s \equiv a_j \left[\phi_L(-\lambda_{S1}) + \phi_D(-\lambda_{S2})\right]/\zeta_j$, we find that the social planner's solution satisfies the following equations:

$$\zeta_j C = (C_j/C)^{-1/\sigma}, \tag{OA.43}$$

$$1 = (1-\tau_j)\alpha L_j^{1-\alpha} x_j^{\alpha-1}, \tag{OA.44}$$

$$\zeta_L = (1-\tau_j)\zeta_j(1-\alpha)Y_j/L_j, \tag{OA.45}$$

$$\zeta_L = \lambda_k^s \mu Q_k, \tag{OA.46}$$

$$\hat{\lambda}_j^s = \rho - \mu s_j - (1-\tau_j)\zeta_j L_j^{1-\alpha} x_j^{\alpha}(1-\alpha). \tag{OA.47}$$

Comparing (OA.43)-(OA.47) for the optimal economy to (OA.38)-(OA.42) for the regulated economy, we find that the latter replicates the former if the tax policies of proposition are imposed, $(1+\tau_\alpha)\alpha = 1$, $\tau_{qj} = ws_j/Q_j$, and $\tau_j = \tau_j^s$. Note that this implies $P_j/PC = \zeta_j$, $w/PC = \zeta_L$, $\lambda_j/PC = \lambda_j^s$, i.e. the real market prices in utility terms (market prices divided by P to make C the unit of account and then multiplied by marginal utility $1/C$ to make utility the unit of account) equal the corresponding shadow prices.

OA3 General condition for the overlap

This appendix, first, generalizes the production and innovation technology to allow for more general complementarities in innovation and, second, relaxes the patent length assumption to allow for variable patent length.

OA3.1 Generalizing the sources of complementarity

We generalize the model in three ways to allow for multiple sources of investment complementarities. First, we allow a direct effect of intermediate firms' innovation on productivity in their sector, by generalizing the final good production to be

$$Y_j = (Q_j^{\varepsilon} L_j)^{1-\alpha} \int_0^1 q_{ji} x_{ji}^{\alpha} di, \tag{OA.48}$$

where $\varepsilon \geq 0$ measures how labor-augmenting the direct innovation spillovers are ($\varepsilon = 0$ brings us back to the main text model).

Second, we allow for a more general input-output structure by assuming intermediate goods production requires both own sector goods and general goods. The unit cost (or equivalently, its monopoly price divided by markup) of an intermediate in sector j with quality q_{ji} is now

$$q_{ji} P_j^{\omega} P^{1-\omega} = \alpha P_{ji}, \tag{OA.49}$$

where $\omega \geq 0$ measures the share of own sector inputs in the production of specific inputs ($\omega = 1$ brings us back to the main text model; Acemoglu et al (2012) choose $\omega = 0$).

Third, we allow intersectoral knowledge spillovers in innovation such that

$$\dot{q}_{ji} = \mu s_{ji} Q_j^{\eta+\chi} Q_{-j}^{\chi} (Q_c + Q_d)^{1-\eta-2\chi}, \tag{OA.50}$$

where $Q_c + Q_d$ is the general knowledge stock, χ is the degree of cross-sectoral spillovers and η denotes how much more own-sector knowledge enhances research productivity than other-sector knowledge; we have maintained the linear homogeneity that was also assumed in the main text. The model presented in the main text can then be considered a special case where $\eta = 1, \chi = 0$.

Lemma OA1. *In a static equilibrium, intermediate goods profits are linear in firms own quality q_{ji}, i.e. $\pi_{ji} = \bar{\pi}_j q_{ji}$ with*

$$\bar{\pi}_r = (Q_r)^{\psi}, \psi \equiv (1+\varepsilon)(\sigma-1)\frac{1-\alpha}{1-\omega\alpha} - 1, \tag{OA.51}$$

where $\bar{\pi}_r \equiv \bar{\pi}_c/\bar{\pi}_d$ and $Q_r \equiv Q_c/Q_d$, while relative R&D costs are

$$\frac{ws_{di}/\dot{q}_{di}}{ws_{ci}/\dot{q}_{ci}} = \mu_r(Q_r)^{\eta}. \tag{OA.52}$$

Proof of Lemma OA1. We determine static equilibrium, i.e. the allocation of labor and profits, given the state variables q_{ji} and given the amount of labor in production L.[1]

From demand for intermediates ($P_j \partial Y_j/\partial x_{ji} = P_{ji}$) and supply (OA.49) we find $x_{ji} = \alpha^{2/(1-\alpha)} Q_j^{\varepsilon} L_j (P_j/P)^{(1-\omega)/(1-\alpha)}$. Plugging this into the production function we find $Y_j = \alpha^{2\alpha/(1-\alpha)} Q_j^{\varepsilon+1} L_j (P_j/P)^{(1-\omega)\alpha/(1-\alpha)}$. Hence, in relative terms:

$$x_r = Q_r^{\varepsilon} L_r (P_r)^{(1-\omega)/(1-\alpha)},$$

$$Y_r = Q_r^{\varepsilon+1} L_r (P_r)^{(1-\omega)\alpha/(1-\alpha)}.$$

Demand for labor implies $P_j \partial Y_j/\partial L_j = P_j(1-\alpha)Y_j/L_j = w$, or in relative terms

$$L_r = P_r Y_r.$$

Demand for Y-goods implies:

$$Y_r = (P_r)^{-\sigma}.$$

Hence we have four equations in P_r, L_r, Y_r, x_r which can be solved in terms of Q_r.

$$\begin{aligned}
P_r &= (Q_r)^{-(\varepsilon+1)(1-\alpha)/(1-\omega\alpha)} \\
L_r &= (Q_r)^{(\sigma-1)(\varepsilon+1)(1-\alpha)/(1-\omega\alpha)} \\
x_r &= (Q_r)^{\varepsilon+(\varepsilon+1)[(\sigma-1)(1-\alpha)-(1-\omega)]/(1-\omega\alpha)}
\end{aligned}$$

Now we turn to profits of intermediate firms. Since the markup is $1/\alpha$, profits are $\pi_{ji} = (1-\alpha)P_{ji}x_{ji}$. and the price P_{ji} from (OA.49), we find $\pi_{ji} = [(1-\alpha)\alpha^{-1} x_{ji} P_j^{\omega} P^{1-\omega}] q_{ji} \equiv \bar{\pi}_j q_{ji}$, where the latter step uses the result that x_{ji} in equilibrium is the same across firms. This shows that profits are linear in own quality

[1] Using this static allocation, below we turn to the dynamic equilibrium to determine the allocation of labor over production and innovation and the resulting dynamics of the state variable.

q_{ji}, which is stated in the lemma. Plugging in the solution for x_{ji} and taking relative variables, we find $\bar{\pi}_r = x_r(P_r)^{\omega}$ which together with above solutions gives (OA.51).

From (OA.50) we directly find (OA.52). □

Hence ψ reflects investment complementarities in production: if $\psi > 0$, an increase in relative knowledge stocks increases relative marginal profits (in the main text, $\omega = 1, \varepsilon = 0$ so that $\psi = \sigma - 2$). Complementarities arise from (i) demand externalities (σ) (ii) input-output mulitipliers (ω) and (iii) direct productivity spillovers (ε). Furthermore, η reflect investment complementarities in innovation: if $\eta > 0$ investment in sector j reduces the cost of subsequent investment more than in the other sector.

Lemma OA2. *SFPs in the unregulated market economy require* $\psi > \max\{0, -\eta\}$.

Proof of Lemma OA2. This proof turns to the dynamics of the model and exploits the static equilibrium solutions in terms of the state variables q_{ji} from the revious proof. The intermediate good producer's investment problem of choosing s_{ji} has the following Hamiltonian:

$$H_{ji} = \bar{\pi}_j q_{ji} - w s_{ji} + \lambda_{ji} \mu \bar{Q}_j s_{ji}.$$

where $\bar{Q}_j = Q_j^{\eta+\chi} Q_{-j}^{\chi} (Q_j + Q_{-j})^{1-\eta-2\chi}$ is the productivity of research labor. The firm takes variables without i subscript as given. Optimality conditions are:

$$\bar{Q}_j \mu \lambda_{ji} \leq w \perp s_{ji} \geq 0 \tag{OA.53}$$

$$\hat{\lambda}_{ji} = r - \bar{\pi}_j / \lambda_{ji} \tag{OA.54}$$

The two conditions show that all firms within a sector have the same shadow value of quality, $\lambda_{ji} = \lambda_j$. We define two variables, z_c and m_c:

$$z_c \equiv \frac{\bar{\pi}_c \bar{Q}_c}{\bar{\pi}_c \bar{Q}_c + \bar{\pi}_d \bar{Q}_d} = \frac{(Q_r)^{\psi+\eta}}{1 + (Q_r)^{\psi+\eta}}, \tag{OA.55}$$

$$m_c \equiv \frac{\lambda_c \bar{Q}_c}{\lambda_c \bar{Q}_c + \lambda_d \bar{Q}_d} = \frac{\lambda_r (Q_r)^{\eta}}{1 + \lambda_r (Q_r)^{\eta}}. \tag{OA.56}$$

Variable z_c captures current (green) market conditions. It is a predetermined state variable, i.e. a transformation of the relative technology state variable Q_r. The transformation ensures that z_c captures all channels through which the state variable affects the return to innovation: complementarities in production (ψ) and in innovation (η). Variable m_c captures future (green) market conditions. It is a forward-looking variable constructed such that its value directly pins down which innovation is active. Clean (dirty) innovation requires future green market conditions to be sufficiently good (poor) according to:[2]

$$m_c > (<) 1/2 \Leftrightarrow \hat{Q}_r > (<) 0.$$

From optimality condition (OA.54) we derive the relative growth rates $\hat{\lambda}_r = \frac{\bar{\pi}_d}{\lambda_d}\left(1 - \frac{\bar{\pi}_r}{\lambda_r}\right)$ which in terms of our new variables reads:[3]

$$\hat{\lambda}_r = \left(\frac{\bar{\pi}_d/\lambda_d}{(1-z_c)m_c}\right)(m_c - z_c).$$

To derive the dynamics of the model in terms of z_c and m_c, we time differentiate (OA.55) and (OA.56):

$$\dot{z}_c = z_c(1-z_c)(\psi+\eta)\hat{Q}_r \tag{OA.57}$$

$$\dot{m}_c = m_c(1-m_c)(\hat{\lambda}_r + \eta\hat{Q}_r), \tag{OA.58}$$

We now build the phase diagram in (z_c, m_c) plane. The regime border is the horizontal line $m_c = 1/2$. We first consider $\psi + \eta < 0$ and show that this rules out SFPs. If $m_c < 1/2$, innovation is brown, Q_r declines and z_c grows. Symmetric for $m_c > 1/2$. Hence the interior steady state with simultaneous research and $m_c = z_c = 1/2$ is stable, the corner steady states can never be reached, and no SFPs can arise.

We next show that an overlap requires $\psi > 0$. Assume $\psi + \eta > 0$. The slope of any time path is given by $\dot{m}_c/\dot{z}_c$. On the 45 degree line (with $m_c = z_c$ and hence

[2] From (OA.53) we derive the regime border condition $\lambda_r \bar{Q}_r \mu_r > (<) 1 \Leftrightarrow \hat{Q}_r > (<) 0$ which in terms of m_c gives the expression.

[3] Note $\bar{\pi}_r/\lambda_r = z_r/m_r = [z_c/(1-z_c)]/[m_c/(1-m_c)]$.

$\hat{\lambda}_r = 0$) this slope boils down to:

$$\left.\frac{\dot{m}_c}{\dot{z}_c}\right|_{m_c=z_c} = \frac{\eta}{\psi+\eta}. \tag{OA.59}$$

This means that, unless $\eta/(\psi+\eta) = 1 \Longleftrightarrow \psi = 0$, an equilibrium path can cross the 45 degree line only once. When tracing back the equilibrium path from a corner steady state (either the dirty staedy state $m_c = z_c = 0$ or the clean one $m_c = z_c = 1$), we start on the 45 degree line and never cross again; when the slope is smaller than 1, the path from the dirty (clean) steady state crosses the regime border to the right (left) of the 45 degree line, implying an overlap. Hence the condition for SFPs is $\eta/(\psi+\eta) < 1 \Leftrightarrow \psi > 0, \psi+\eta > 0$. □

Remark. This proof only uses the investment conditions and does not need consumer intertemporal utility maximization. This is becuase we only need to solve for relative variables. When we want to solve for all variables, in particular total - rather than relative - investment, as measured by $1-L$, we need the savings block of the model.

OA3.2 Variable patent length

While Acemoglu et al. (2012) assume one-period patents and our main text model assumes infinite patent length, in reality patents often last between 15 and 20 years. To model elementary aspects of patent protection issues, we assume all intermediate firms face a risk of losing their profits permanently because of patent infringement.[4] The infringement event occurs at Poisson rate ι, so that the arbitrage equation (12) now contains a risk premium:

$$\dot{\lambda}_{ji} = (r+\iota)\lambda_{ji} - \frac{\partial \pi_{ji}}{\partial q_{ji}}, \tag{OA.60}$$

[4] This modelling assumes that infringement is exogenous and uniform across firms; firms who "steal" the patent are immediately in the same position as robbed incumbent. A full modelling would require specifying who is successful in infringement, whether this costs effort etc. Moreover, (legal) patent length is not the same as (illegal) infringement. We leave these details for further research.

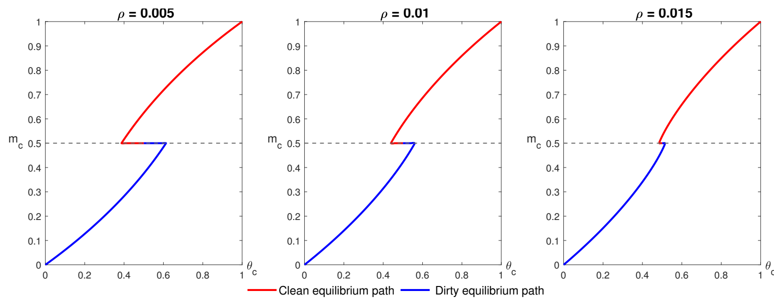

Figure 10: Equilibrium paths with different ρ values ($\sigma = 3$)

which implies that profits are discounted with the interest rate plus the infringement risk ι to calculate the value of investment λ_{ji}.

The patent infringement rate does not affect the analysis in Section 3. Intuitively, because patent infringement occurs with the same probability in all sectors, it does not affect the direction of investment.

However, the patent infringement rate affects the speed of overall investment as analyzed in Section 4. Following the procedure of Section A.2, we derive the counterparts of (A.2) and (A.4),

$$\hat{\lambda}_j = r + \iota - \alpha\mu L(m_k/m_j)\theta_c, \tag{OA.61}$$

$$r = \alpha\mu L\theta_k + \hat{w} - \hat{Q}_k - \iota. \tag{OA.62}$$

Continuing the same procedure as in Section A.2 to derive the reduced-form equilibrium dynamics, we find that in all equations in Lemma 1, ρ is replaced by $\rho + \iota$. Intuitively, a higher probability of loosing the patent right reduces investors' horizon as does an increase in the discount rate, so that the sum of discount rate and patent infringement rate governs the speed of innovation. The effect of a change in ρ and a change in ι are the same with respect to the equilibrium dynamics analyzed in Section 4. Hence, we conclude that a shorter average patent length (increase in ι) makes the overlap smaller.

Figure 10 shows the projection of the equilibrium paths for different time preference rates. As in Proposition 3, the larger the time preference rate, the smaller is the size of the overlap.

In the market economy, policy is needed to counteract the excessively short horizon of investors introduced by effect of finite patent length. A subsidy to R&D can do this job and is needed to decentralize the first-best.

OA4 Segmented labor market

Suppose labour market is segmented and the total supply of labour is $\bar{L}$ for workers and $\bar{s}$ for scientists, that is, $L_c + L_d \leq \bar{L}$ and $s_c + s_d \leq \bar{s}$, where $\bar{L}$ and $\bar{s}$ are now parameters. Wage in (8) now differs from wage in (11). Denote the former by w_L and the latter by w_s. From (11) and (24), we see that we can continue to use the variable m_j to determine the innovation regime.

Combining (12), (15), and (22), we find

$$\hat{\lambda}_j = r - (1-\alpha)\alpha \frac{\theta_j PY}{\lambda_j Q_j}. \tag{OA.63}$$

From (24), we find $\dot{m}_c = m_c(1-m_c)\left(\hat{\lambda}_c + \hat{Q}_c - \hat{\lambda}_d - \hat{Q}_d\right)$ or

$$\dot{m}_c = m_c(1-m_c)\mu(s_c - s_d) + \alpha\mu \frac{(1-\alpha)PY}{w_s} m_k(m_c - \theta_c), \tag{OA.64}$$

where k denotes the research active sector and we have used $\mu\lambda_c Q_c = w_s m_c/m_k$ based on (11). Finally, the clean market share evolves according to

$$\dot{\theta}_c = \theta_c(1-\theta_c)(\sigma-1)\mu(s_c - s_d). \tag{OA.65}$$

From (OA.64) and (OA.65), it is clear that the two variables θ_c and m_c are insufficient in describing the entire dynamics of the model due to the expression $(1-\alpha)PY/w_s$ in (OA.64). This expression can be rewritten as $w_L\bar{L}/w_s$. Compared to the integrated labour market, where relative wage in production and research is 1 and production labour L is endogenous, here $\bar{L}$ is a constant but the relative wage w_L/w_s is an endogenous variable. Assuming segmented labour market thus does not reduce the dimensionality of the model, because a third variable is needed in order to fully describe the dynamics of the model.

To more precisely compare the model with segmented labour market with the integrated labour market model in the main text, we define $L_e \equiv (1-\alpha)PY/w_s$. This new variable adopts a similar role to production labour L in the main text by determining the savings rate. With integrated labour market, the savings rate is given by $ws/(PC) \propto s/L = (1-L)/L$. Here, using (18), we find L_e to be inversely proportional to the savings rate:

$$L_e = \frac{\bar{s}}{1+\alpha}\left[\frac{w_s \bar{s}}{PC}\right]^{-1}.$$

Combining the above expression with (11), (OA.63) and (17), we find

$$\dot{L}_e = L_e\left[\alpha\mu L_e \theta_k - \mu\bar{s} - \rho\right]. \tag{OA.66}$$

Using L_e, (OA.64) can be written as

$$\dot{m}_c = \alpha\mu L_e m_k(m_c - \theta_c) + \mu(s_c - s_d)m_c(1-m_c). \tag{OA.67}$$

Together, (OA.65)-(OA.67) form a differential equation system in variables θ_c, L_e, and m_c that summarizes the dynamics of the model with segmented labor market. Comparing with Lemma 1, we see that the dynamics of both models are almost identical, the only difference being that where the constant $\bar{s}$ shows up in the equation for $\dot{L}_e$, in the integrated market model the variable $s_k = 1-L$ shows up.

We conclude that assuming segmented labour market does not reduce the dimensionality of the model. The reason is that, as long as the investment decision is dynamic, the expected present value of investment m_j will be affected by the savings rate. Even if the supply of scientists is fixed (measured as labor input), the savings rate (measured consumption equivalents) changes over time depending on the relative wage. If labour is mobile across the two sectors and the wage equalized, the savings rate depends on the allocation of scientists. Thus, to reduce the dimensionality, we either need to make investment decision static (e.g. with one-period patent) or assume a fixed savings rate.

OA5 Modeling innovation as creative destruction

In this appendix we model innovation as creative destruction and demonstrate that it generates the same qualitative result.

The R&D process is the following (see also Acemoglu et al., 2012). At the beginning of each period, each scientist decides whether to direct her research to clean or dirty technology. Each scientist is then randomly allocated to innovating at most one machine within their chosen sector with a success probability of μ.

The rest of the model is the same as in Section 2. Thus, equations (8)-(10), (13), (15)-(22) continue to hold. From (15), the average profit of a sector is given by

$$\pi_j = \int_0^1 \pi_{ji} di = (1-\alpha)\alpha P_j Y_j. \tag{OA.68}$$

Denote by V_j the value of the patent in sector j in case of successful innovation. The decision to target a particular sector is governed by the free entry condition:

$$\mu V_j \leq w \perp s_j \geq 0, \tag{OA.69}$$

which is the counterpart of (11). Note that for the research active sector k, $V_k = w/\mu$. From (8),

$$V_k = (1-\alpha)P_k Y_k/(\mu L_k) \tag{OA.70}$$

follows.

Equation (12) is replaced by the non arbitrage condition:

$$\dot{V}_j = rV_j - \pi_j + \mu s_j V_j. \tag{OA.71}$$

For the dynamic equilibrium, note that (31) and (32) continue to hold. Similar to (25), we can define

$$m_c \equiv \frac{V_c}{V_c + V_d}. \tag{OA.72}$$

From (OA.69), it is clear that $m_c \gtreqless 1/2$ separates the three innovation regimes.

Time differentiating m_c and using (OA.71), we find

$$\dot{m}_c = m_c(1-m_c)\left(\hat{V}_c - \hat{V}_d\right) = m_c(1-m_c)\left(-\frac{\pi_c}{V_c} + \frac{\pi_d}{V_d} + \hat{Q}_c - \hat{Q}_d\right). \quad \text{(OA.73)}$$

Using (OA.68), (OA.70), (21), and the definition of m_c, we can derive (30). Thus, Lemma 1 holds.

This analysis shows that creative destruction and inhouse R&D offer almost exactly the same equilibrium conditions, both statically and dynamically. The only difference is that quality improvement is evaluated at its marginal value of improving the patent in the case of inhouse R&D, as innovation can occur repeatedly, whereas in the case of creative destruction, quality improvement is valued at the total value of the patent. As Lemma 1 holds in both cases, this difference does not matter for the result.

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