Supplemental Appendix to "Entrepreneurial Investment Dynamics and the Wealth Distribution", by Eugene Tan

A Data Construction

In this section, I briefly discuss the types of firms that are included in the KFS survey. I then discuss how the measures for capital stock, revenue, value added, average revenue product of capital, investment rates, and net leverage ratios are constructed. In addition, I describe the residualization method used to avoid confounding cross-industry heterogeneity in my results.

A.I Survey Inclusion

As discussed in the main text, the universe of firms considered for survey inclusion in the KFS was all newly registered firms in 2004 from the Dun and Bradstreet database. However, given that the focus of the KFS is on new entrepreneurs, this universe is too broad, capturing a wide range of firms from newly registered subsidiaries to established firms spun off from family inheritances. Therefore, for actual inclusion into the survey, a firm must then satisfy at least one of the following conditions:

1. The business was started as independent business, or by the purchase of an existing business, or by the purchase of a franchise in the 2004 calendar year.
2. The business was *not* started as a branch or a subsidiary owned by an existing business, that was inherited, or that was created as a not-for-profit organization in the 2004 calendar year.
3. The business had a valid business legal status (sole proprietorship, limited liability company, subchapter S corporation, C-corporation, general partnership, or limited partnership) in 2004.
4. The business reported at least one of the following activities:
    (a) Acquired employer identification number during the 2004 calendar year

(b) Organized as sole proprietorships, reporting that 2004 was the first year they used Schedule C or Schedule C-EZ to report business income on a personal income tax return

(c) Reported that 2004 was the first year they made state unemployment insurance payments

(d) Reported that 2004 was the first year they made federal insurance contribution act payments

All firms that satisfy at least one of these conditions then make up the sample population of the KFS.

A.II Variable Construction

1 Capital Stock

In order to construct the average revenue product of capital, I first need to construct the capital stock of the firm. The KFS provides the researcher the balance sheet of the firm, and it provides a breakdown of the type of capital asset that the firm owns. The full range of asset types are product inventories, land and buildings and structures, vehicles, equipment or machinery, other properties, cash, and “others”.

However, as in most standard models, I consider only a single generic capital asset of interest. As such, in order render the results comparable, I construct a representative single asset, real capital stock, $K_{i,t}$, using the nominal value of capital assets as follows:

$$K_{i,t} = \sum_{s} \frac{K_{i,s,t}}{P_{s,t}},$$

where $P_{s,t}$ is the relative price of each capital type s and survey year t. Subscript i indexes the firm. The relative prices are taken from the BEA. For the aggregated capital stock, I consider the firm’s holdings of equipment or machinery, vehicles, land and buildings and structures, product inventories, and other properties. The value of product inventories are deflated using the GDP deflator.

2 Investment rates

As discussed in the main text, I approximate investment rates using the first difference in log capital, namely $\log K_{i,t} - \log K_{i,t-1}$. Note that t here indicates the survey year, so $K_{i,t}$ would correspond to the capital stock at the end of year t. Therefore, the appropriate measure of investment in year t is the difference between the capital stock in year t and the preceding year. This timing notation is adopted because the KFS only surveys the firm at the start of the following year for information regarding the current year. So, for instance, K_{2005} refers to the end-of-period capital stock for 2005, and K_{2004} refers to the beginning-of-period capital stock for 2005..

Investment spikes The definition of investment spikes follow Cooper and Haltiwanger (2006), who define a spike as gross investment in excess of 20%. For my context, this is equivalent to $\log K_{i,t} - \log K_{i,t-1} > \log(1 - \delta_e + 0.2)$, where δ_e is the depreciation rate of the capital stock and is evaluated at 0.11 per the measurement approach documented in Section 1.

Disinvestment spikes Disinvestment spikes are defined analogously as in Cooper and Haltiwanger (2006), who define a spike as gross disinvestment in excess of -20%. For my context, this is equivalent to $\log K_{i,t} - \log K_{i,t-1} < \log(1 - \delta_e - 0.2)$.

3 Revenue and Value Added

Construction of the average revenue product of capital also necessitates the construction of a measure of the firm's real value added. To do so, I first deflate nominal revenue using the GDP deflator to obtain real revenue. Next, as the KFS does not provide information on the firm's material expenses, I follow the literature by assuming that a fixed share of revenue is spent on material inputs. In other words, let $Y^R_{i,j,t}$ denote real revenue for firm i, in industry j, at time t. Then value added $Y_{i,j,t}$ is simply computed as

$$Y_{i,j,t} = (1 - \beta^j_m) \times Y^R_{i,j,t},$$

where β^m_j is the share of revenue spent on material expenses, which I allow to vary across industry j. β^m_j is estimated directly from national accounting data (the NIPA

KLEMS tables).[20] For the rest of this paper, I will use the term "revenue" and "value added" interchangeably, unless explicitly stated otherwise.

4 Average Revenue Product of Capital

Having constructed both capital and revenue, the (log) average revenue product of capital is simply

$$\log ARPK \equiv \log\left(\frac{Y_{it}}{K_{i,t-1}}\right).$$

For the main body of the paper, the moments I report are constructed using a pooled measure of log ARPK. In practice, moments constructed using pooled log ARPK might not be a good measure since there is likely to be large heterogeneity in capital share and returns to scale across industries. However, due to the small sample size of the KFS relative to the number of industries, there is simply insufficient statistical power to draw any useful inference if one restricted analysis only to the 6 digit NAICS industry level.[21] Instead, I address this issue through two methods.

Residualized ARPK. All the benchmark empirical results utilize a residualized measure of log ARPK rather than the raw measure. Here, the pooled log ARPK variables are residualized by regressing the raw log ARPK on two digit NAICS industry level fixed effects and time dummies. The residuals of this regression then form my measure of log ARPK for analysis. This avoids the issue where permanent differences across industries (such as heterogeneity in capital share and returns to scale) would distort the distribution, and thus introduce spurious correlations and moments. Moreover, it also removes some of the common aggregate shocks that might distort the distribution of log ARPK over time. The results presented in the following sub-sections are constructed using this residualized measure.[22]

[20] One might justifiably be concerned that this measurement of value added is very noisy, and introduces extra measurement error. As a robustness check, I repeat the empirical exercises using the "raw" revenue measure. The qualitative results obtained using the value added measure is replicated when I use the raw revenue measure.

[21] There were 3140 firms in 2004 when the survey started; by 2011, there were only 1630 firms left. Moreover, there are typically about 1500 firms per year on average reporting positive revenue and capital. In contrast, there are 659 six digit level NAICS code industries.

[22] As a robustness check, I also construct extended residualized measures of log ARPK by using more regressors that could potentially systematically distort the distribution of log ARPK (for instance, the legal form of the firm, or the gender of the primary owner of the firm). I find that

Industry level moments. A big concern about my findings relate to the fact that there is substantial permanent heterogeneity across industries that the residualization process is unable to fully purged. To address these concerns, I also directly investigate the relevant moments at the two digit industry level. Due to the smaller sample size, only one industry showed statistically significant results; however, most industries showed economically significant results. Moreover, the qualitative findings at the aggregate level also holds broadly across industries. The results at the industry level are reported in this appendix (see Figure C7).

5 Net Liquid Asset

The real net liquid asset of the firm is computed as the difference between the total cash holdings ($Cash_{i,t}$) and the business debt of the firm ($BusDebt_{i,t}$), deflated by the GDP deflator.

6 Net Leverage Ratios

Net leverage ratio at the firm level is defined as the ratio of the net debt ($D_{i,t}$) of the firm to the total collateral value ($Coll_{i,t}$) of the firm. Here, net debt is simply defined as $D_{i,t} = \max\{0, BusDebt_{i,t} - Cash_{i,t}\}$; that is, for firms who have net positive liquid asset holdings, the debt level is simply 0. For collateral, I assume that the physical capital stock of the firm serves as collateral. Therefore, net leverage is simply defined as $\frac{D_{i,t}}{K_{i,t}} = \frac{\max\{0, BusDebt_{i,t} - Cash_{i,t}\}}{K_{i,t}}$. This is the benchmark definition used in the main text.[23]

However, given that the bulk of the KFS firms operate as private firms (i.e., sole proprietorships, S corporations, or partnerships), the distinction between personal and business debt can be relatively unclear. This is especially relevant given that individuals (and their business partners) might be taking out substantial personal debt in order to finance their business. Therefore, one could be justifiably concerned that I am artificially lowering the firm's leverage by simply restricting $D_{i,t}$ to business debt.

my benchmark results are robust to alternative measures. Results computed using these extended measures are available upon request.

[23] All results, including the alternative definitions of leverage here, were also computed using a winsorized measure of the leverage ratios, at the 1st and 99th percentiles. Moments computed using the un-winsorized measures, as well as alternative measures that only include firms that are in debt (i.e., $D_{i,t} > 0$) are available upon request.

To allay these concerns, I also constructed alternative measures of leverage. Specifically, I define net debt as two additional alternative definitions:

$$D_{i,t} = \begin{cases} \max\{0, BusDebt_{i,t} - Cash_{i,t}\} & \text{(Benchmark definition)} \\ \max\{0, PerDebt_{i,t} + BusDebt_{i,t} - Cash_{i,t}\} & \text{(Alternate definition 1)} \\ \max\{0, OODebt_{i,t} + PerDebt_{i,t} + BusDebt_{i,t} - Cash_{i,t}\} & \text{(Alternate definition 2)} \end{cases} \tag{53}$$

where $PerDebt_{i,t}$ is the stock of personal debt of the primary owner-operator at time t, and $OODebt_{i,t}$ is the stock of personal debt of all the other owners of the firm at time t. Likewise, I define total collateral using three alternative definitions:

$$Coll_{i,t} = \begin{cases} K_{i,t} & \text{(Benchmark definition)} \\ \frac{1}{\kappa} PerDebt_{i,t} + K_{i,t} & \text{(Alternate definition 1)} \\ \frac{1}{\kappa}(OODebt_{i,t} + PerDebt_{i,t}) + K_{i,t} & \text{(Alternate definition 2)} \end{cases} \tag{54}$$

where $Coll_{i,t}$ is total collateral value. κ is a parameter used to back out the implied collateral required to sustain the observed level of personal debt. I use $\kappa = 0.3$, which corresponds to the typical collateral constraint used in the literature. Therefore, this imputation strategy implies that the owner(s) fully exhaust their borrowing capacity, and could be considered the most stringent definition (i.e., the largest leverage possible). I also consider allowing for $\kappa = \infty$, in other words, no collateral is needed for personal debt.[24] The results are report in Table A1.

[24] Note that while this is highly unlikely, I report these figures in the interest of transparency and robustness.

Table A1: Net leverage.

This table reports net average leverage, computed using benchmark and alternative definitions.

Average leverage at year	Benchmark Defn	Alt Defn 1 ($\kappa = 0.3$)	Alt Defn 2 ($\kappa = 0.3$)	Alt Defn 1 ($\kappa = \infty$)	Alt Defn 2 ($\kappa = \infty$)
1	0.093	0.098	0.100	0.459	0.479
2	0.042	0.053	0.054	0.210	0.222
3	0.038	0.051	0.052	0.222	0.232
4	0.031	0.040	0.040	0.169	0.173
5	0.032	0.040	0.041	0.162	0.174
6	0.031	0.039	0.039	0.168	0.172
7	0.030	0.033	0.033	0.114	0.116
8	0.014	0.019	0.020	0.070	0.071

A.III Additional Data Sources

My analysis draws on additional data beyond the KFS, which I summarize here. These sources are well-known in the current literature, and so I present only a short summary.

1 Survey of Consumer Finances

The Survey of Consumer Finances (SCF, Board of Governors of the Federal Reserve System, 1989-2022) is a triennial cross-sectional survey of families in the United States. The survey is conducted by the Federal Reserve Board of the United States. It includes information on families' balance sheets, pensions, income, and demographic characteristics. I use the surveys between the years 1989 and 2021 for my analysis.

2 Panel Survey of Income Dynamics

The Panel Survey of Income Dynamics (PSID) is a nationally representative survey that was conducted annually in the United States from 1968 to 1997 and every 2 years thereafter. The survey is conducted by the University of Michigan Institute for Social Research.

I use samples from the 2003 wave to construct the entrant wealth distribution for comparison with the KFS. "Entrepreneurs" here are defined as all self-employed business owners, following the definition in Cagetti and De Nardi (2006). Entrants are individuals were were not entrepreneurs in the previous survey wave (i.e., 2001), and are entrepreneurs in the current survey wave (i.e., 2003).

3 Business Dynamics Statistics

The Business Dynamics Statistics (BDS, U.S. Census Bureau, 2000-2008) is product from the U.S. Census Bureau. It draws from Longitudinal Business Database to track aggregate measures of establishment openings and closings, firm startups and shutdowns, and job creation and destruction. The database is still publicly available as of Jan 2026.

4 Quarterly Census of Employment and Wages

The Quarterly Census of Employment and Wages (QCEW, U.S. Bureau of Labor Statistics, 2002-2015) is product from the U.S. Bureau of Labor Statistics. It is a database that tabulates on the number of establishments, monthly employment and quarterly wages for workers covered by State unemployment insurance (UI) laws and Federal workers covered by the Unemployment Compensation for Federal Employees (UCFE) program. The database is still publicly available as of Jan 2026.

5 Survey of U.S. Businesses

The Survey of U.S. Businesses (SUSB, U.S. Census Bureau, 2002-2015) is a product from the U.S. Census Bureau. It is an annual series that provides subnational economic data by establishment industry and firm size. The database is still publicly available as of Jan 2026.

6 National Accounting Data

National accounts data is obtain from the Buerau of Economic Analysis (BEA, U.S. Bureau of Economic Analysis, 2025). The database is still publicly available as of Jan 2026.

B Predictions of Standard Frictionless Investment Model for ARPK Persistence

In this section, I derive a result showing that in a standard frictionless investment model with only time-to-build, log ARPK will (a) not exhibit any persistence (or conditional persistence) and (b), under typical assumptions on the distribution of productivity, log ARPK will not be left-skewed. Crucially, I show that this holds true if we allow for asymmetric persistence in the underlying productivity process. I also briefly discuss how heterogeneity in the cost of capital, factor shares, and returns to scale, affects our inference using the transition matrix approach as discussed in the main text.

I begin by stating below, in recursive notation, a generic firm i's investment problem:

$$\begin{aligned}\Pi\left(k,z\right) &= \max_{k'} D + \frac{1}{1+r_i}\mathbb{E}\left[\Pi\left(k',z'\right)|z\right] \\ &s.t. \\ D &= zk^{\alpha_i} + \left(1-\delta_i\right)k - k' \\ \log z' &= \rho(z)\log z + \epsilon' \\ k' &> 0,\end{aligned}$$

where $'$ denotes variables for the next period; ϵ' is any iid random variable; and $\rho(z)$ captures persistence in TFP z, is allowed to vary across state z, but restricted to $0 \leq \rho < 1$. This assumption on ρ allows me to capture potential asymmetry in TFP persistence. I also assume that the choice of capital is not measurable with respect to next-period innovations. Therefore, investment has a "time-to-build" element, since the payoff to investment today (k') is only realized tomorrow.[25] D here is per-period dividend flow, and the firm's problem is to maximize lifetime dividend flow.[26] Finally, I also denote the parameters controlling the capital share / returns to scale (α), the depreciation rate δ, and the interest rate r with a subscript i to denote that these parameters are unique to firm i (i.e., they are so-called firm fixed effects).

Claim 1. *Consider an investment model as summarized above. Then (log) ARPK can be expressed as*

$$\log ARPK = \vartheta_i + \epsilon, \tag{55}$$

[25]Trivially, if current choice of capital is measurable with next-period innovations, a frictionless model would generate a degenerate distribution for log ARPK.

[26]Implicit here is that the firm is risk-neutral. Note that all derivations hold if we include labor as an input or allow for endogenous entry and exit, but I ignore it in the interest of algebraic clarity. I also choose this simplified model to illustrate my point, as these models allow me to analytically characterize the skewness and persistence of ARPK. In contrast, a full-scale model like that in this paper does not easily admit an analytical expression.

where ϑ_i is a function of the parameters r_i, δ_i, and α_i, but does not include $\rho(z)$.

Proof. To begin, notice that the first order condition for capital yields (suppressing the argument z into ρ, and the various i's unless where necessary, for clarity)

$$\begin{aligned}
\mathbb{E}\left[z'\left(K'\right)^{\alpha-1}|z\right] &= \frac{r+\delta}{\alpha} \\
\implies \log K' &= \frac{1}{\alpha-1}\left(\log\left(\frac{r+\delta}{\alpha}\right) - \rho\log z - \log\mathbb{E}\left[\exp\left(\epsilon'\right)|z\right]\right) \\
\Leftrightarrow \log K &= \frac{1}{\alpha-1}\left(\log\left(\frac{r+\delta}{\alpha}\right) - \rho\log z_{-1} - \log\mathbb{E}\left[\exp\left(\epsilon\right)|z_{-1}\right]\right),
\end{aligned}$$

where for the second last line, since $z' = z^{\rho}\exp\left(\epsilon'\right)$, I used the relationship $\log\mathbb{E}\left[z'|z\right] = \log\left(z^{\rho}\mathbb{E}\left[\exp\left(\epsilon'\right)|z\right]\right) = \rho\log z + \log\mathbb{E}\left[\exp\left(\epsilon'\right)|z\right]$. The last line is simply a change of time notation, with $_{-1}$ denoting "last period" variables.

Recall that $\log ARPK = \log\left(\frac{Y}{K}\right) = \log z + (\alpha-1)\log K$, combining the two equations, we get

$$\log ARPK = \log z - \rho\log z_{-1} + \log\left(\frac{r+\delta}{\alpha}\right) - \log\mathbb{E}\left[\exp\left(\epsilon\right)|z_{-1}\right].$$

Defining $\vartheta \equiv \log\left(\frac{r+\delta}{\alpha}\right) - \log\mathbb{E}\left[\exp\left(\epsilon\right)|z_{-1}\right]$, and recalling that $\log z = \rho\log z_{-1} + \epsilon$, we can simplify the above relation to

$$\begin{aligned}
\log ARPK &= \epsilon + \log\left(\frac{r+\delta}{\alpha}\right) + \underbrace{\rho\log z_{-1} - \rho\log z_{-1}}_{\text{cancels out}} - \log\mathbb{E}\left[\exp\left(\epsilon\right)|z_{-1}\right] \\
&= \vartheta_i + \epsilon
\end{aligned}$$

□

With Claim 1 in hand, I now discuss the implications for log ARPK persistence for two cases, when parameters are homogeneous, and when they are heterogeneous.

B.I Homogeneous parameters

Proposition 1. *log ARPK has no persistence regardless of the autocorrelation structure of the underlying TFP shocks, as long as firms have homogeneous parameters.*

Proof. The result is a trivial outcome of the relationship derived in Claim 1, and is a result of the fact that ϑ depends on r, δ, and α, but does not include $\rho(z)$. When parameters are homogeneous across firms, ϑ is constant across firms. In that case, the distribution of log ARPK is simply a mean-shifted distribution of the underlying innovations ϵ. Trivially then, log ARPK exhibits no persistence. □

Corollary 1. *The skewness of log ARPK follows the skewness of the underlying innovations to TFP, as long as firms have homogeneous parameters.*

Proof. This corollary follows directly from the result in Proposition 1. □

Comment In most standard firm dynamics models, innovations are typically assumed to be distributed Gaussian or Pareto—the implication then is that log ARPK has zero or positive skewness. Therefore, under common assumptions used in the literature, log ARPK will not be left-skewed.

B.II Heterogeneous parameters

In this case, the exact structure of the persistence of log ARPK across the distribution is now fully dependent on the distribution of ϑ and ϵ. Instead of deriving a formal proposition, I briefly argue here that under standard assumptions with log-normal innovations, it is unlikely that heterogeneous parameters are the driver of this result.

To be precise, in the case where ϵ is log-normal (and hence drawn from a symmetric distribution), we can consider three generic cases:

1. ϑ has a symmetric distribution. In this case, log ARPK will exhibit symmetric persistence.

2. ϑ has a right-skewed distribution. In this case, log ARPK will exhibit higher right-tail persistence.

3. ϑ has a left-skewed distribution. In this case, log ARPK will exhibit higher left-tail persistence.

Notice that only case 3 can give rise to the observation in the KFS data. However, a left-skewed distribution implies that there is a fraction of firms that receive substantially lower effective cost of capital relative to the majority of firms, or that they have substantially larger capital shares / returns to scale than the rest of the distribution. The latter is unlikely given that the average product of labor does not feature asymmetric persistence.[27] The former is also unlikely. As I showed earlier, the bulk of low-ARPK firms are low-revenue firms that persist in a low-revenue state. As such, it is unlikely that these firms are receiving unusually low capital financing. Taken together, this implies that it is unlikely that case 3 is the driver of my empirical findings.

C Additional Empirical Results

In this section, I report the additional figures and tables referenced in the main text, as well as the additional robustness checks with regards to my empirical findings.

C.I Additional Data Figures and Tables

1 Diversification of Entrepreneurs Along the Wealth Distribution

In Figure C4, I report the fraction of an entrepreneur's wealth held as business wealth across the wealth distribution. The takeaway here is that the level of asset diversification is decreasing in wealth. As we move up the wealth distribution, an increasing fraction of an individual's wealth comes from business wealth.

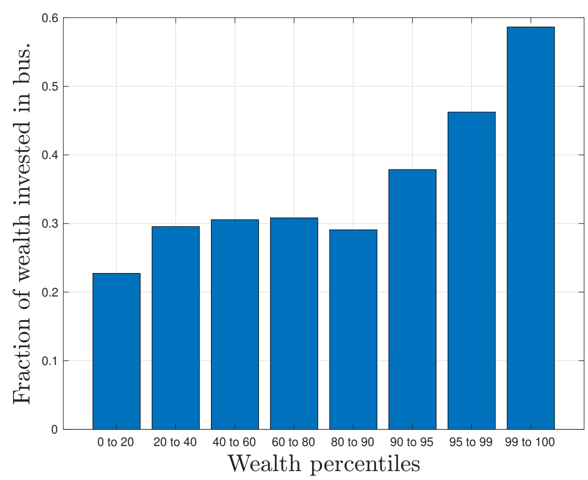


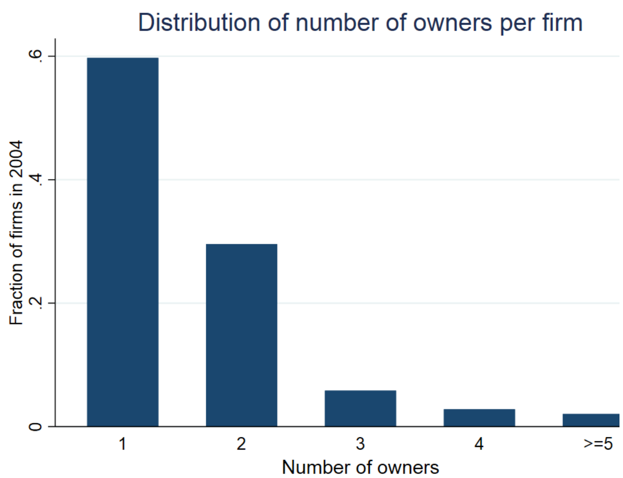
Figure C4: Entrepreneurial wealth is concentrated in own business wealth

The figure plots the fraction of the entrepreneur's portfolio that comes from business wealth, within each wealth percentile range. Values are reported for the median within each percentile range.
Source. SCF.

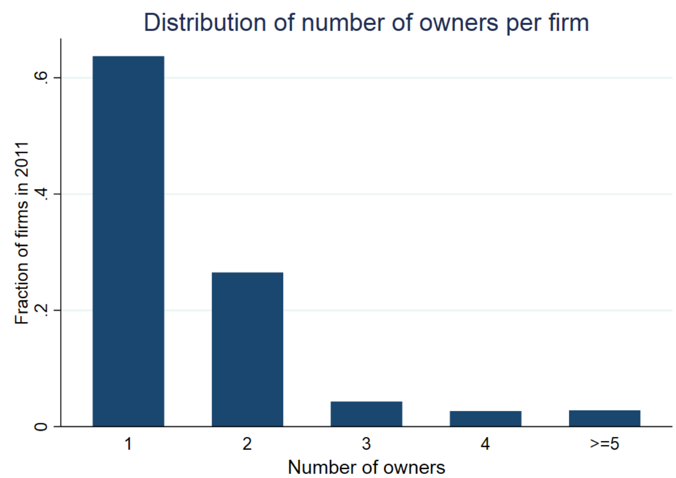
[27]Regardless, as discussed earlier, my benchmark results are estimated using residualized measures to avoid this from confounding my results.

2 Number of Owners At the Start and End of the Sample

In Figure C5, I report the number of owners per firm at the start of the sample (i.e., beginning of operations in year 2004), and at the end of the sample in 2011. As we can see, the number of owners per firm is relatively stable over time, with the vast majority of firms owned by one or two owners. This adds further argument that entrepreneurs largely bear the full risk of each business they own.



(a) 2004



(b) 2011

Figure C5: Distribution of the number of owners.

This figure plots a bar chart of the number of owners per firm.
Source. KFS.

C.II Robustness checks

1 Skewness: Across Time and Definitions

The skewness moments that I report in the main text takes the entire distribution as a pooled sample. I show here that this left-skewness is in fact a robust feature of the data. To that end, I estimated the skewness period-by-period over the entire sample, again for all three measures of skewness. These results are reported in Figure C6. From the graph, we see that regardless of the choice of skewness measure and time period, the distribution of log ARPK is left-skewed.

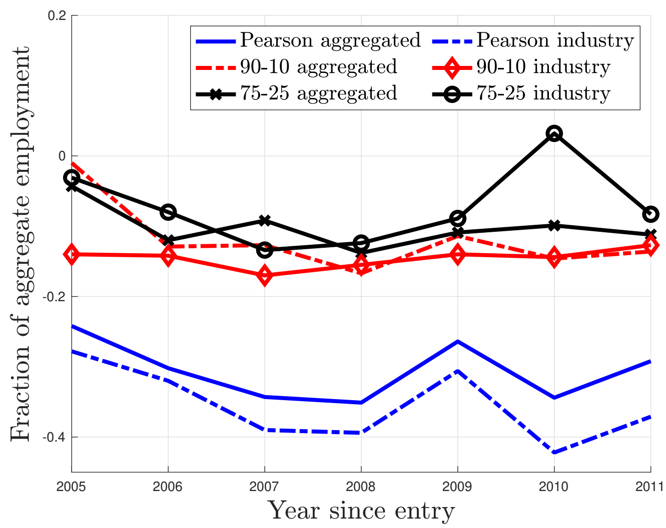


Figure C6: Time series of various measures of skewness.

This figure plots the skewness of ARPK measured on a year-by-year basis using two different levels of aggregation. *Source. KFS.*

2 Asymmetric Persistence: Using Alternative Definitions

The estimation of the persistence of relative rankings was estimated using the full sample, and the quantiles shift over time as I estimated the quantiles period-by-period.[28] This might lead to a few concerns, specifically:

1. Quantile construction. The results could be an artifact of the shifting quantiles.

2. Sample selection. The results could be biased by very small firms. Very small firms could be operated by individuals with no desire to maximize profits, and these firms are also most likely to be in the left tail. As a result, the higher left tail persistence that I report in the main text could be driven by these firms.

I address these issues in Table C2 by reporting a series of robustness checks. I address the first point by simply fixing the quantiles to the ones computed in period 1 (year 2005), and estimate the transition probabilities. To address the second point, I conduct the original estimation using an increasingly stricter cut off for firms. In the first specification, I consider stricter cut-offs defined in terms of assets. I first conduct the estimation only for firms reporting more than $5000 in assets, and then for firms with more than $20,000 in assets, and finally $50,000 in assets. In a second

[28] The quantiles themselves are still estimated at the industry level, not across the pooled sample.

specification, I conduct the cut-off using revenue. I start with a threshold of \$10,000, then \$50,000, and finally \$75,000. Finally, I consider only firms that survived for all seven years. In Table C2, I only report the persistence in the left tail (i.e., probability of staying in rank 1) and the right tail (i.e., probability of staying in rank 5) since these are the key moments of interest. I find that the asymmetry (i.e., higher left-tail persistence) is a robust feature of the data.

Table C2: Persistence of relative rankings.

Model	Fixed quantiles	Assets ≥			Revenue ≥			Survived all years
		\$5k	\$20k	\$50k	\$10k	\$50k	\$75k	
$Pr(1 \to 1)$	0.61	0.57	0.57	0.58	0.54	0.55	0.53	0.56
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)
$Pr(5 \to 5)$	0.50	0.51	0.52	0.49	0.49	0.50	0.51	0.47
	(0.02)	(0.02)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)	(0.03)

This table reports persistence of relative rankings for firms in the first and fifth quintiles under different cut off assumptions. For fixed quintiles, the quintiles are fixed to the cutoffs from year 2005. For the other versions, the quintile is constructed using the full sample as in the main text, but the estimation is done using only firms that pass the various selection cut-off. The standard errors are in parenthesis. All reported numbers are rounded to two decimal places for clarity of presentation.
Source. KFS.

3 Asymmetric Persistence: Using two- and three-year transition matrices

Table C3: Two-year transitions

		Quintile at $t+2$				
		1	2	3	4	5
Quintile at t	1	0.55	0.25	0.086	0.062	0.055
		(0.019)	(0.016)	(0.011)	(0.010)	(0.008)
	2	0.22	0.37	0.24	0.12	0.059
		(0.015)	(0.018)	(0.016)	(0.012)	(0.008)
	3	0.11	0.20	0.33	0.26	0.10
		(0.011)	(0.015)	(0.018)	(0.017)	(0.01)
	4	0.063	0.12	0.23	0.34	0.25
		(0.009)	(0.011)	(0.015)	(0.017)	(0.015)
	5	0.078	0.078	0.13	0.26	0.45
		(0.010)	(0.009)	(0.012)	(0.016)	(0.018)

This table reports the two-year transition probabilities for ARPK. All reported numbers are rounded to two significant figures or three decimal places for clarity of presentation, so rows might not sum to one.
Source. KFS.

Table C4: Three-year transitions

		Quintile at $t+3$				
		1	2	3	4	5
Quintile at t	1	0.52 (0.022)	0.24 (0.019)	0.11 (0.014)	0.077 (0.012)	0.050 (0.010)
	2	0.24 (0.018)	0.33 (0.020)	0.24 (0.018)	0.13 (0.014)	0.059 (0.008)
	3	0.12 (0.013)	0.20 (0.017)	0.32 (0.020)	0.23 (0.018)	0.13 (0.014)
	4	0.085 (0.011)	0.12 (0.013)	0.22 (0.017)	0.33 (0.020)	0.25 (0.018)
	5	0.062 (0.010)	0.10 (0.012)	0.15 (0.015)	0.26 (0.019)	0.43 (0.021)

This table reports the three-year transition probabilities for ARPK. All reported numbers are rounded to two significant figures or three decimal places for clarity of presentation, so rows might not sum to one.
Source. KFS.

4 Asymmetric Persistence: Industry Level

Here, I report the results for the persistence in relative rankings measured at the industry level. I only report the probability of staying in the same quintile for the first and last quintile; the results of the full estimation of the transition matrices are available on request. The blue diamonds correspond to the probability of staying in the first quintile, given that the firm was in the first quintile. The red diamonds correspond to the probability of staying in the last quintile, given that the firm was in the last quintile. Each pair corresponds to a single two digit NAICS industry code. As one can see, the asymmetric persistence does not just apply to the pooled sample. The vast majority of industries also feature economically significant asymmetry.

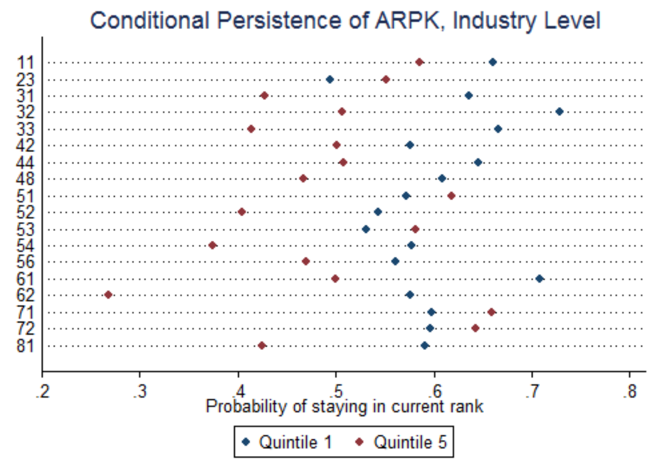


Figure C7: Conditional persistence at the industry level.

This figure plots the probability of staying in the same quintile for every 2-digit NAICS industry. Blue corresponds to the first quintile, red to the last quintile.
Source. KFS.

5 Asymmetric Persistence: Conditional Autocorrelation

A potential issue with estimating transition matrices, and using them to understand mobility at the tails of the distribution, is that transition matrices tend to overstate immobility at the tails (relative to other parts of the distribution). This happens because individuals can only move in one direction when they are at either ends of the distribution (i.e., the floor-ceiling effect).

To ensure that my results are not an artifact of this effect, I conduct a second analysis where I estimate the autocorrelation of log ARPK conditional on the firm's position in the distribution. Specifically, I estimate a regression of the form

$$\log ARPK_{i,t} = \alpha + \sum_{q=1}^{5} \rho_q \log ARPK_{i,t-1} + \varepsilon_{it}$$

where α is the intercept term, and ρ_q is a coefficient that depends on the log ARPK quantile q that the firm is in currently.

Table C5 summarizes my findings, for both the baseline estimates and the additional sensitivity analyses using different cut-offs for assets and revenue as in section 2. As the table shows, ρ_1 (autocorrelation when the firm is in quantile 1) is always much larger than ρ_5. A Wald test also rejects the null that the two estimates are equivalent, as such supporting the evidence that there is greater persistence in log ARPK at the bottom quintile relative to the top quintile.

Table C5: Conditional autocorrelation.

Model	Assets ≥						Revenue ≥				
	\$5k	\$10k	\$20k	\$50k	\$75	\$100	\$10k	\$20k	\$50k	\$75k	\$100k
ρ_1	0.68	0.71	0.73	0.74	0.75	0.77	0.71	0.74	0.78	0.77	0.76
	(0.04)	(0.03)	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)	(0.06)	(0.07)
ρ_5	0.40	0.37	0.34	0.27	0.25	0.27	0.42	0.42	0.43	0.43	0.43
	(0.04)	(0.04)	(0.05)	(0.06)	(0.06)	(0.06)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)

This table reports conditional autocorrelation for firms in the first and fifth quintiles under different cut off assumptions. The standard errors are in parenthesis. All reported numbers are rounded to two decimal places for clarity of presentation.
Source. KFS.

6 Distribution of capital growth rates

The figure below plots a kernel density of the distribution of capital growth rates of firms who are in the first quintile of ARPK, conditioned on their transitions. As we can see, firms that stay in the first quintile are firms that are, on average, doing zero disinvestment.

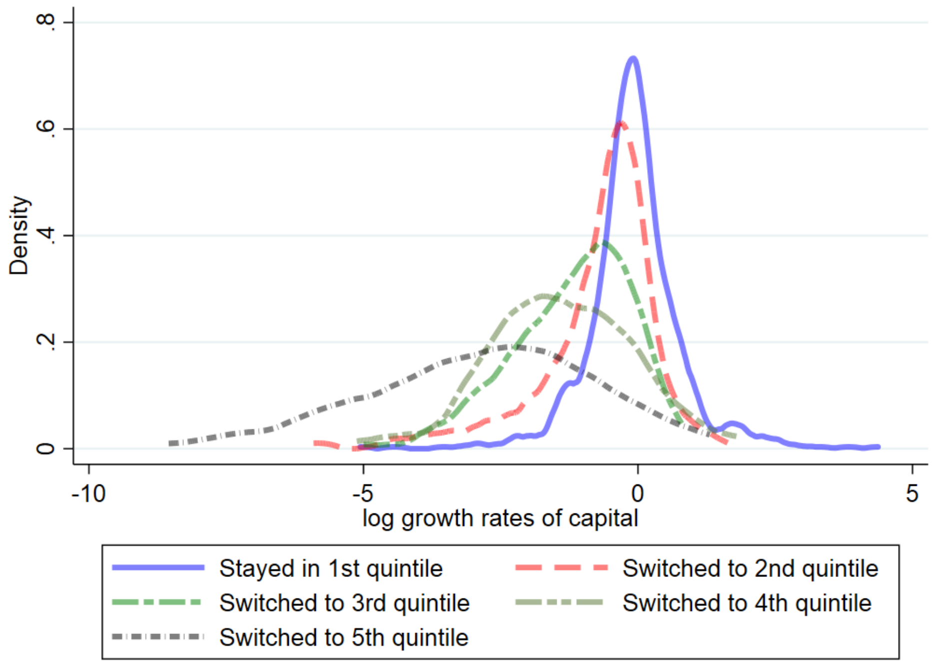


Figure C8: Distribution of capital growth rates.

Source. KFS.

7 Investment response to shocks, by ARPK quintile

The table below reports the regression results of estimating Equation 1 in Section I. As discussed in the main text. β_q (ξ_q) is the semi-elasticity of capital growth rates responses to negative (positive) value added "shocks", conditioned on the firm's current ARPK quintile. The first column reports results using the first-difference in log value added as a proxy for value added shocks, and the second column uses the residuals of an AR(1) regression of log value added.

We see that $\beta_1 < \xi_5$, indicating that (dis)investment responses are weaker for firms in the first quintile relative to the fifth. Noticeably, the semi-elasticities for the rest of the distribution are much smaller in magnitude, or simply statistically insignificant. The results are consistent with an investment model in which firms follow an sS-type investment policy (i.e., firms only react to "extreme" realizations of shocks). Taken as a whole, this suggests that firm investment might indeed face sharp frictions in downsizing in the form of capital irreversibilities.

Table C6: Investment responses to productivity shocks.

	(1)	(2)
β_1	0.077 (0.035)	0.070 (0.037)
β_2	0.003 (0.070)	-0.006 (0.074)
β_3	-0.020 (0.067)	-0.022 (0.072)
β_4	-0.123 (0.072)	-0.119 (0.079)
β_5	-0.155 (0.122)	-0.153 (0.131)
ξ_1	0.029 (0.063)	0.058 (0.072)
ξ_2	0.119 (0.052)	0.135 (0.057)
ξ_3	0.103 (0.046)	0.108 (0.050)
ξ_4	0.105 (0.048)	0.110 (0.052)
ξ_5	0.248 (0.039)	0.258 (0.041)
Observations	8028	8028
R^2	0.161	0.161

This table reports investment responses to value added shocks across the ARPK distribution. Regressions include quantile, industry and year fixed effects, which are suppressed in the interest of space. Standard errors are in parentheses. All values are rounded to three decimal places.
Source. KFS.

8 Asymmetric persistence is robust to controls for financial frictions

As discussed in the main text, I first estimate a regression of the form

$$\log ARPK_{i,t} = \alpha + \beta \log debt\_issued_{i,t-1} + \gamma_t + \eta_j + \epsilon_{i,t} \tag{56}$$

where γ_t and η_j are year and industry fixed effects, and $debt\_issued_{i,t-1}$ is the debt the firm issued in the previous period. Table C7 reports the results, where we see that the coefficient is negative and statistically significant, implying that firms which issued more debt in the last period have lower ARPK on average in the current period.

Table C7: Regression results.

	(1)
β	-0.029
	(0.004)
Observations	8582
R^2	0.09

This table reports the result of the regression described by equation 56. Standard errors are in parentheses.
Source. KFS.

Having estimated the regression model, I computed the residuals $\epsilon_{i,t}$, and estimate a transition matrix in quintiles. Table C8 reports the results. As we can see, the transition matrix is largely similar to the baseline transition matrix, and the asymmetric persistence still features prominently.

Table C8: Transition probabilities of adjusted log ARPK.

		Quintile tomorrow				
		1	2	3	4	5
Quintile today	1	0.60 (0.017)	0.23 (0.015)	0.091 (0.010)	0.036 (0.007)	0.046 (0.008)
	2	0.23 (0.015)	0.38 (0.017)	0.23 (0.015)	0.10 (0.010)	0.061 (0.008)
	3	0.072 (0.009)	0.24 (0.015)	0.37 (0.017)	0.22 (0.014)	0.10 (0.010)
	4	0.056 (0.008)	0.10 (0.010)	0.21 (0.014)	0.37 (0.017)	0.26 (0.015)
	5	0.064 (0.009)	0.066 (0.008)	0.13 (0.012)	0.27 (0.016)	0.47 (0.018)

This table reports transition probabilities of the residuals of the regression given by equation 56. Standard errors are in parentheses. Rows may not sum to 1, as entries have been rounded to two significant figures.
Source. KFS.

D Definition of Recursive Stationary Equilibrium

The state space of the model comprises liquid asset holdings $b \in \mathbb{B}$, capital holdings $k \in \mathbb{K}$, occupational choice $h \in \mathbb{H}$, entrepreneurial productivity $z \in \mathbb{Z}$, and labor productivity $\theta \in \Theta$. Denote by $\mathbb{S} = \mathbb{B} \times \mathbb{K} \times \mathbb{H} \times \mathbb{Z} \times \Theta$ the complete state space, and $\mathbf{s} \in \mathbb{S}$ the state vector representing each individual agent. I now proceed to define the equilibrium of this model.

Definition 1. *A stationary equilibrium of the model is defined by*

1. *The interest rate r and wage rate w which are time-invariant*
2. *Value functions: $V_e, V_{ee}, V_{ew}, V_w, V_{ww}, V_{we}, \Pi$*
3. *Policy functions: $k'(\mathbf{s})$, $b'(\mathbf{s})$, $h'(\mathbf{s})$*
4. *Optimal static profit function of the entrepreneur $\pi^*(\mathbf{s})$*
5. *Adjustment cost function $\mathcal{C}(\mathbf{s}', \mathbf{s})$*

6. *Labor demand* $l(\mathbf{s})$ *from entrepreneurs and labor supply* θ *from workers*

7. *Factor demand* K^c *and* L^c *from the corporate sector*

8. *Invariant distribution of households* $\Lambda(\mathbf{s})$

such that

1. *Taking* r *and* w *as given, the households' decision rules and value functions, as in equations 10, 11, 17, 23, 24, and 29, solve the individual problems.*

2. *Taking* r *and* w *as given, the representative corporate firm's decision rules and value function, as given in equation 34, solve the firm's problem.*

3. *Factor markets clear, where*

   (a) *Liquid asset holdings:* $\int b' d\Lambda = K^c$

   (b) *Labor:* $\int \theta 1_{\{h=W\}} d\Lambda = \int l 1_{\{h=E\}} d\Lambda + L^c$

4. *The aggregate resource constraint is satisfied, where*

$$\int c + k' 1_{\{h'=E\}} + b' + \mathcal{C}(k', k) 1_{\{h'=E\}} d\Lambda = \int \pi^* 1_{\{h=E\}} + \theta w 1_{\{h=W\}} + (1+r)b + (1-\delta_k) k 1_{\{h=E\}} d\Lambda$$

5. *The decision rules of the households, along with the exogenous Markov and i.i.d. processes, generate the time-invariant Markov transition kernel* Γ*, which, given any initial distribution of households* Λ_0*, generates the time-invariant distribution* Λ*; that is,*

$$\Lambda = \Gamma(\Lambda)$$

E Additional Calibration Materials

E.I Fixed and Externally Estimated Parameters

Table E9 reports the parameter values and their source. Panel A reports parameters that are fixed according to the literature. Panel B reports parameters that are externally estimated using the KFS.

Table E9: Parameter values.

Parameter	Description	Value	Source
		Panel A: Fixed Parameters	
γ	Risk aversion	2	Standard.
ρ_θ	Persistence of labor productivity	0.90	Floden and Linde (2001), Storesletten, Telmer and Yaron (2004)
σ_θ	Conditional variance of labor productivity	0.20	Floden and Linde (2001), Storesletten, Telmer and Yaron (2004)
α	Capital share of corporate sector	0.33	Cagetti and De Nardi (2006)
$\underline{b}$	Unsecured borrowing limit	0	Cagetti and De Nardi (2006)
δ	Depreciation rate of corporate sector capital	0.10	Standard
		Panel B: Externally Estimated Parameters	
δ_e	Depreciation rate	0.11	From data
α_e	Capital intensity	0.418	From data
ν	Returns to scale	0.786	From data

This table reports parameter values that are either fixed to values in the prior literature, or externally estimated. *Source. Author's calculations.*

E.II Identification of α_e and ν

In the main text, I argued that there is no bias in my estimate for Θ_k (the equation is reproduced here as equation 57). I will now provide evidence for this.

$$\log y = \Theta_0 + \Theta_k \log k + \Theta_z \log z. \tag{57}$$

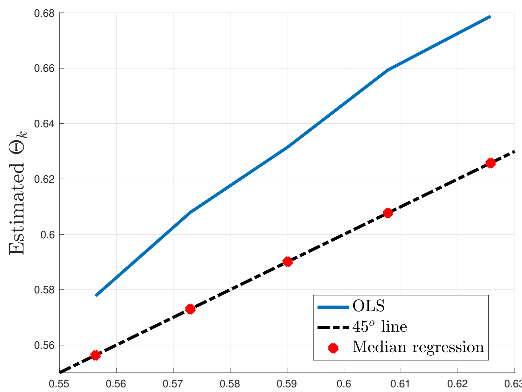
For the sake of the general interest reader, I will first explain the issue at hand. A well-known issue with estimating equation 57 is that $cov(\log z, \log k)$ is generally larger than 0: Large firms tend to have higher productivity. Since I don't observe z, estimating this equation via ordinary least-squares (OLS) therefore creates an endogeneity issue that leads to an upward bias on the estimate for Θ_k (i.e., it violates the weak exogeneity condition). This effect is indeed present in my model. In Panel A of Figure E9, I plot the estimated Θ_k against the actual Θ_k for a series of comparative static exercises in my model. That is, I solve multiple versions of my model for different values of Θ_k, and then estimate equation 57 to obtain an estimate for Θ_k. The black dashed line is the reference line (i.e., when the estimated Θ_k = true Θ_k) The blue solid line plots the results when equation 57 is estimated via OLS, showing an upward bias.

For further illustration, the blue solid line in Panel B of Figure E9 plots the average productivity of entrepreneurs conditional on a given capital stock in my baseline calibrated model. For the purposes of computing this figure, I bin capital into deciles. The blue line plots the conditional average productivity, showing that productivity is largely increasing in capital. In other words, $cov(\log z, \log k) > 0$, which gives rise to the aforementioned upward bias.

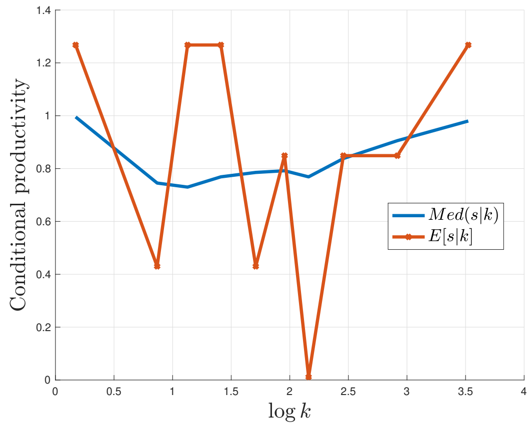
Turning back to Panel A, I now plot the estimated Θ_k against the true Θ_k when equation 57 is estimated via median regression (red crosses). The median regression does not generate any bias–the estimated Θ_k is always exactly equal to the true Θ_k. Note that the weak exogeneity condition for median regressions is $\mathbb{E}\left[\log k \left(0.5 - \mathcal{I}(\log z < 0)\right)\right] = 0$, where $\mathcal{I}$ is an indicator variable that evaluates to one when the argument is true. The red line with crosses in Panel B plots the median productivity conditional on capital, and it shows that this condition is likely satisfied in my model: There is no discernible pattern in the conditional median productivity. As a result, estimating equation 57 via median regression does not generate any bias.

A theoretical reason for why the usual bias is not present in my model is the strong presence of financial frictions and capital resale frictions. Financial frictions imply that there is a disproportionally large number of small firms with high productivity, while capital resale frictions mean that there is a disproportionally large number of large firms with low productivity. In combination, this breaks the usual problem whereby capital is strongly and positively correlated with productivity. Notice that even for the case of the conditional mean productivity (Panel B), the relationship is weakly U-shaped, although the increasing arm of the U-shape dominates (each point in the plot represents 10% of the population, so the majority of the population sees a positive correlation between productivity and capital). This gives rise to the aforementioned upward bias in the case of the OLS estimator.

Figure E9: Comparison of estimates for Θ_k using OLS and median regression



(a) Estimated Θ_k vs true Θ_k



(b) Mean and median z conditional on capital decile in the baseline model

Panel A plots the estimated Θ_k using OLS (blue line) and median regression (red crosses) for various true values of Θ_k in the model. The comparative static exercise is done holding fixed other parameters at their baseline calibrated values. The black dashed line is the reference line when the estimated and true values coincide. Panel B plots the mean (blue lines) and median (red lines with crosses) productivity conditional on a given decile of capital stock against the average capital stock in the same decile.
Source: Author's calculation.

E.III Derivation of Identification Argument

Consider the following statistical process below (the notation follows from the main text).

$$x_t = x_t^* + \sigma_u u_t \tag{58}$$

$$x_t^* = \rho_{x^*} x_{t-1}^* + \sigma_\epsilon \epsilon_t, \tag{59}$$

where in particular, only x_t is observed, but x_t^* is not. I also assume that u and ϵ are i.i.d processes drawn from a standard normal distribution. The parameters are ρ_{x^*}, σ_u, and σ_ϵ. The goal of the econometrician is to estimate ρ_{x^*}, σ_u, and σ_ϵ. Note that this means the variance of x^* is $\sigma_{x^*}^2 = \frac{1}{1-\rho_{x^*}^2}\sigma_\epsilon^2$.

It is clear that simply using the variance of x as an estimate for $\sigma_{x^*}^2$ leads to an upward biased for σ_ϵ, since $\sigma_x^2 = \sigma_{x^*}^2 + \sigma_u^2$. So even an unbiased estimate for ρ_{x^*} will not give us an unbiased estimate for σ_ϵ^2.

For ρ_{x^*}, using the standard OLS formula, we can express the estimate for ρ_{x^*} as

$$
\begin{aligned}
\hat{\rho_{x^*}} &= \frac{cov\left(x_{t-1}, \sigma_u u_t + \rho_{x^*} x^*_{t-1} + \sigma_\epsilon \epsilon_t\right)}{var\left(x_{t-1}\right)} \\
&= \frac{cov\left(x^*_{t-1} + \sigma_u u_{t-1}, \sigma_u u_t + \rho_{x^*} x^*_{t-1} + \sigma_\epsilon \epsilon_t\right)}{var\left(x^*_{t-1} + \sigma_u u_{t-1}\right)} \\
&= \rho_{x^*} \frac{var\left(y^*\right)}{var\left(y^*\right) + \sigma_u^2} \qquad (60)
\end{aligned}
$$

where the last line tell us that the estimate is biased downwards due to the presence of σ_u^2.

So in summary, a simple AR(1) model over-states volatility and under-states persistence.

The two moments I choose, however, can overcome this problem.

Let's start with estimating ρ_{x^*}. Notice that the two-year autocorrelation (regression of current y on twice-lagged y, which I denote here by $\hat{\rho_{x^*}}_{two}$) gives us

$$
\begin{aligned}
\hat{\rho_{x^*}}_{two} &= \frac{cov\left(x_{t-2}, \sigma_u u_t + \rho_{x^*} x^*_{t-1} + \sigma_\epsilon \epsilon_t\right)}{var\left(x_{t-1}\right)} \\
&= \frac{cov\left(y^*_{t-2} + \sigma_u u_{t-2}, \sigma_u u_t + \rho_{x^*}^2 y^*_{t-2} + \rho_{x^*} \sigma_\epsilon \epsilon_{t-1} + \sigma_\epsilon \epsilon_t\right)}{var\left(y^*_{t-2} + \sigma_u u_{t-2}\right)} \\
&= \rho_{x^*}^2 \frac{var\left(y^*\right)}{var\left(y^*\right) + \sigma_u^2} \qquad (61)
\end{aligned}
$$

So dividing line 61 by line 60, we obtain ρ_{x^*}, the true auto-correlation of output. This division is the moment denoted as $\Delta\rho_{x^*}(y)$. The reason this is possible is because, as long as the transitory shocks are drawn from a time-invariant distribution, the bias induced into the OLS estimator is constant at all horizons.

Now let's turn to estimating σ_ϵ. For the first difference in growth rate of output, we can write

$$
\begin{aligned}
x_t - x_{t-1} &= x_t^* - x_{t-1}^* + (u_t - u_{t-1}) \\
&= \rho_{x^*} x_{t-1}^* + \epsilon_t - x_{t-1}^* + (u_t - u_{t-1}) \\
&= (\rho_{x^*} - 1)\, x_{t-1}^* + \epsilon_t + (u_t - u_{t-1}) \qquad (62) \\
\implies var\,(x_t - x_{t-1}) &= (\rho_{x^*} - 1)^2\, var(x^*) + var\,(\epsilon) + 2var\,(u) \\
&= (\rho_{x^*} - 1)^2 \frac{1}{1-\rho_{x^*}^2} \sigma_\epsilon^2 + \sigma_\epsilon^2 + 2\sigma_u^2 \qquad (63)
\end{aligned}
$$

For the second difference in growth rate of output, we can write

$$
\begin{aligned}
x_t - x_{t-2} &= x_t^* - x_{t-2}^* + (u_t - u_{t-2}) \\
&= \rho_{x^*}^2 x_{t-2}^* + \epsilon_t + \rho_{x^*}\epsilon_{t-1} - x_{t-2}^* + (u_t - u_{t-2}) \\
&= \left(\rho_{x^*}^2 - 1\right) x_{t-2}^* + \epsilon_t + \rho_{x^*}\epsilon_{t-1} + (u_t - u_{t-2}) \qquad (64) \\
\implies var\,(x_t - x_{t-2}) &= \left(\rho_{x^*}^2 - 1\right)^2 var(x^*) + var\,(\epsilon) + \rho_{x^*} var\,(\epsilon) + 2var\,(u) \\
&= \left(\rho_{x^*}^2 - 1\right)^2 \frac{1}{1-\rho_{x^*}^2} \sigma_\epsilon^2 + (1 + \rho_{x^*})\, \sigma_\epsilon^2 + 2\sigma_u^2 \qquad (65)
\end{aligned}
$$

So the difference between the two variances is given by

$$
var\,(x_t - x_{t-2}) - var\,(x_t - x_{t-1}) = \left(\frac{3-\rho_{x^*}^2}{1+\rho_{x^*}}\right) \rho_{x^*}\sigma_\epsilon^2 \qquad (66)
$$

So the key observation here is that the difference between the two variances is only a function of $\left(\frac{3-\rho_{x^*}^2}{1+\rho_{x^*}}\right) \rho_{x^*}\sigma_\epsilon^2$, but not the transitory shock σ_u. Therefore, having estimated ρ_{x^*}, I can simply back out σ_ϵ^2 from equation 66.

As discussed in the main text, if output indeed followed the above mentioned statistical process, then these two moments exactly identify ρ^z and σ^z.

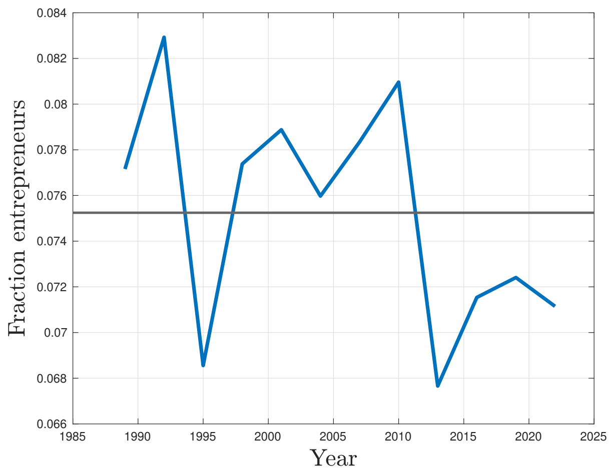
E.IV Additional Evidence For Calibration

1 Fraction of labor force that are entrepreneurs

Figure E10 plots the average fraction of the labor force that are entrepreneurs ("entrepreneurial rate"). Entrepreneurs are defined as self-employed individuals who own a business. The computations are derived using the Survey of Consumer Finances

(SCF). Rounded to two significant figures, the average entrepreneurial rate is around 7.5%, as depicted by the grey reference line in the figure.

Figure E10: Fraction of labor force that are entrepreneurs



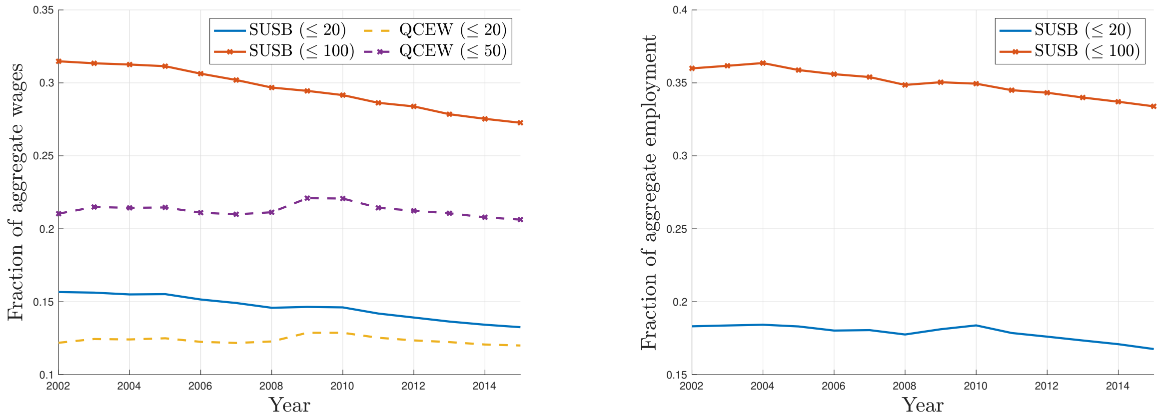
This figure plots the fraction of the labor force that are entrepreneurs. Grey solid reference line refers to the average over the entire sample. which is approximately 0.075, rounded to two significant figures.
Source: SCF.

2 Employment share of entrepreneurs

Figure E11 employment share in two ways. Panel A considers the aggregate wage bill or payroll accounted for by entrepreneurial firms. Panel B considers the total share of employment counts accounted for by entrepreneurial firms.

Unlike evidence from household or individual survey data, it is challenging to define an "entrepreneurial firm" using firm-level data alone. Unfortunately, firm-level data is the only source of information on employment counts in the United States. Given this limitation, I follow the literature by defining "entrepreneurial firms" as firms falling below some size threshold. It is common to choose size thresholds ranging under 20 employees to under 100 employees. The figures below consider all possible combinations from two data sets: the Statistics of U.S. Businesses (where I observe categories for under 20 and under 100), and the Quarterly Census of Employment and Wages (where I observe categories for under 20 and under 50). The figures show that the employment share ranges from a lower bound of around 12% to upper bound of around 36%. In the interest of being conservative, I therefore picked a target of 20% for my calibration exercise.

Figure E11: Employment share of entrepreneurs



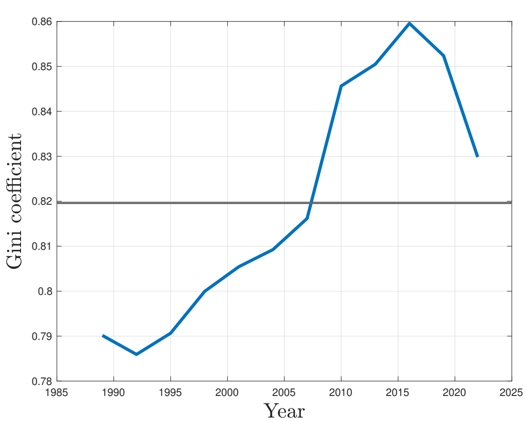
(a) Wage bill **(b)** Employment count

Panel A plots the fraction of aggregate payroll accounted for firms classified as "entrepreneur firms" as described in the text. Panel B plots the fraction of aggregate employment, in terms of number of workers, accounted for firms classified as "entrepreneur firms" as described in the text. Solid blue line and solid red line with crosses denote data from the Statistics of U.S. Businesses (SUSB). Dashed purple line and dashed purple line with crosses denote data from the Quarterly Census of Employment and Wages (QCEW).
Source: SUSB, QCEW.

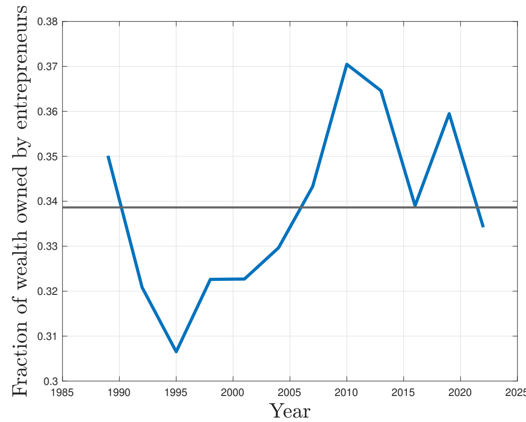
3 Wealth distribution

Figure E12 reports the Gini coefficient (Panel A) and total wealth in the economy owned by entrepreneurs (Panel B) for the years 1989 to 2022. The grey reference lines plot the average values for each relevant statistics. Rounded to two significant figures, the average Gini coefficient is approximately 0.82, and entrepreneur wealth share around 34%.

Figure E12: Gini coefficient and entrepreneur wealth share over time



(a) Gini coefficient



(b) Entrepreneur wealth share

Panel A plots the Gini coefficient from the years 1989 to 2022. Panel B plots the fraction of total wealth in the economy owned by entrepreneurs from the years 1989 to 2022. Grey reference lines plot the corresponding averages, which are 0.82 and 0.34 respectively, rounded to two significant figures
Source: SCF.

E.V Transition matrix of ARPK

Table E10 reports the full model-implied transition matrix for ARPK.

Table E10: Transition probabilities of ARPK in calibrated model.

		Quintile at $t+1$				
		1	2	3	4	5
Quintile at t	1	0.47	0.25	0.08	0.05	0.01
	2	0.21	0.20	0.28	0.10	0.07
	3	0.09	0.20	0.22	0.22	0.13
	4	0.04	0.19	0.09	0.38	0.15
	5	0.03	0.02	0.19	0.12	0.50

This table reports transition probabilities of ARPK under the benchmark calibration. Entries have been rounded to two decimal places for clarity of presentation.
Source. Author's calculations.

F Additional Material on Counterfactual Analyses

F.I Deriving the Decomposition of Excess Returns

The budget constraint can be written as

$$c = y + (1+r)b + (1-\tilde{\lambda})(1-\delta)k - (1-\tilde{\lambda})k' - b' - \tilde{f}_s k - \tilde{f}_d k \tag{67}$$

where $\tilde{\lambda} \equiv \lambda \times \mathcal{I}(k' < (1-\delta)k)$, $\tilde{f}_s \equiv f_s \times \mathcal{I}(k' > (1-\delta)k)$, and $\tilde{f}_d \equiv f_s \times \mathcal{I}(k' < (1-\delta)k)$.

Next, rewrite the model in terms of net liquid asset a, which I define as

$$a \equiv b + \hat{\phi}k, \tag{68}$$

where $\hat{\phi} \equiv \phi(1-\lambda)(1-\delta)$. Therefore, the collateral constraint can now be recast as $a' \geq 0$, and the budget constraint can be rewritten as

$$\begin{aligned} c &= y + (1+r)a + \left((1-\tilde{\lambda})(1-\delta) - (1+r)\hat{\phi}\right)k - (1-\tilde{\lambda}-\hat{\phi})k' - a' - \tilde{f}_s k - \tilde{f}_d k \\ &= y + (1+r)a - (r+\delta)k - \tilde{(}1-\delta)k + (1-\hat{\phi})(1+r)k - (1-\tilde{\lambda}-\hat{\phi})k' ... \\ &\quad ... - a' - \tilde{f}_s k - \tilde{f}_d k \end{aligned} \tag{69}$$

Combining the FOC and envelope conditions for a gives us

$$1 - \frac{\mu}{U_c} = \mathbb{E}\left[\beta \frac{U_{c'}}{U_c} | \theta, z\right](1+r), \tag{70}$$

where μ is the lagrange multiplier on the collateral constraint, and I will denote the stochastic discount factor $\beta\frac{U_{c'}}{U_c}$ by $\mathcal{M}_{s',s}$ going forward. Importantly, as can be seen from this equation, μ encodes *both* the impact of collateral constraints on investment, as well as the impact of borrowing constraints (as a result of the non-negativity constraint) on consumption smoothing.

Next, with an abuse of notation, the investment Euler equation can be written as

$$1-\tilde{\lambda}-\hat{\phi}=\mathbb{E}\left[\mathcal{M}_{s',s}\left(\frac{\partial y'}{\partial k'}-(r+\delta)-\tilde{\lambda}'(1-\delta)-\tilde{f}_s-\tilde{f}_d+(1-\hat{\phi})(1+r)\ \ldots\right.\right.$$

$$\left.\left.-\left(\frac{\partial\tilde{\lambda}'}{\partial k'}(1-\delta)+\frac{\partial\tilde{f}_s{}'}{\partial k'}+\frac{\partial\tilde{f}_d{}'}{\partial k'}\right)k'\right)|\theta,z\right]. \tag{71}$$

In turn, this same Euler equation can be re-written as,

$$\mathbb{E}\left[\mathcal{M}_{s',s}|\theta,z\right]\left(\frac{\partial y'}{\partial k'}-(r+\delta)\right)=\left(1-\mathbb{E}\left[\mathcal{M}_{s',s}|\theta,z\right]\right)(1+r)\left(1-\hat{\phi}\right)\ldots$$

$$\ldots-\tilde{\lambda}-\mathbb{E}\left[-\tilde{\lambda}'(1-\delta)-\tilde{f}_d{}'|\theta,z\right]-\mathbb{E}\left[-\tilde{f}_s{}'|\theta,z\right]-\ldots$$

$$\ldots cov_{\theta,z}\left(\mathcal{M}_{s',s},\frac{\partial y'}{\partial k'}\right)-cov_{\theta,z}\left(\mathcal{M}_{s',s},-\tilde{\lambda}'(1-\delta)-\tilde{f}_d{}'\right)-\ldots$$

$$\ldots cov_{\theta,z}\left(\mathcal{M}_{s',s},-\tilde{f}_s{}'\right)+\ldots$$

$$\ldots\mathbb{E}\left[\mathcal{M}_{s',s}\left(\frac{\partial\tilde{\lambda}'}{\partial k'}(1-\delta)+\frac{\partial\tilde{f}_s{}'}{\partial k'}+\frac{\partial\tilde{f}_d{}'}{\partial k'}\right)k'|\theta,z\right] \tag{72}$$

The left-hand side of the expression evaluates to the marginal product of capital next of the user cost, discounted back to current units using the individual's stochastic discount factor. Note that with complete markets (or risk-neutrality) and a frictionless economy, this term would evaluate to 0. However, with incomplete markets and the multiple frictions in the model, we would expect this term to be non-zero, that is, firms would exhibit non-zero average excess return.

The right-hand side decomposes the excess return into four main components. First, the term $\left(1-\mathbb{E}\left[\mathcal{M}_{s',s}|\theta,z\right]\right)(1+r)\left(1-\hat{\phi}\right)$ characterizes the impact of collateral constraints. As discussed before, the SDF encodes both the impact of borrowing constraints on consumption smoothing and investment. As a result, this term in the investment euler equation is further multiplied by $1-\hat{\phi}$, which isolates the role of investment. Therefore, it is possible, for instance, for an individual to be constrained due to consumption smoothing reasons ($\mathbb{E}\left[\mathcal{M}_{s',s}|\theta,z\right]<1$), but unconstrained due to investment reasons because all investment can be collateralized ($1-\hat{\phi}=0$).

The second set of terms, $-\tilde{\lambda}-\mathbb{E}\left[-\tilde{\lambda}'(1-\delta)-\tilde{f}_d{}'|\theta,z\right]-\mathbb{E}\left[-\tilde{f}_s{}'|\theta,z\right]$, captures the impact of the various capital adjustment frictions on investment decisions, effects that are relatively well understood in the early literature on investment adjustment costs (e.g., Caballero (1999)). The first term $\tilde{\lambda}$ is the transaction cost term, and captures the fact that when investment is negative, the value of capital is lower than its purchase price, leading the entrepreneur to tolerate a lower marginal product of capital in excess of the user cost. This is the usual under-disinvestment effect that arises in models of investment irreversibility. The second term, $\mathbb{E}\left[-\tilde{\lambda}'(1-\delta) - \tilde{f}_d{}'|\theta, z\right]$ captures the fact that capital is costly to downsize in the next period, which reduces its value from today's perspective. As such, as long as a firm expects to sell capital with non-zero probability in the next period, this term is negative, implying that the firm prefers to buy *less* capital relative to the frictionless economy. This is the usual under-investment effect that also arises in models of investment irreversibility. The last term refers to the fact that when a firms anticipates a non-zero probability of needing to expand in the next period, it internalizes that every additional unit of capital purchased today leads to a larger cost of adjustment the next period. This effect also leads to under-investment, and arises because the fixed investment cost scales with capital. Note that all of these effects appear when the market is complete or the firm is risk-neutral, and the effects are well understood, and therefore not a focus of my decomposition exercise.

The third set of terms, $cov_{\theta,z}\left(\mathcal{M}_{s',s}, \frac{\partial y'}{\partial k'}\right) - cov_{\theta,z}\left(\mathcal{M}_{s',s}, -\tilde{\lambda}'(1-\delta) - \tilde{f}_d{}'\right) - cov_{\theta,z}\left(\mathcal{M}_{s',s}, -\tilde{f}_s{}'\right)$ characterizes the role of risk. $cov_{\theta,z}\left(\mathcal{M}_{s',s}, \frac{\partial y'}{\partial k'}\right)$ captures the impact of productivity shocks on consumption growth, and as discussed in the main text, encodes risk arising from both income fluctuations and investment irreversibility. The second term $cov_{\theta,z}\left(\mathcal{M}_{s',s}, -\tilde{\lambda}'(1-\delta) - \tilde{f}_d{}'\right)$ captures the covariance of consumption growth with the value of capital when an individual sells capital. The covariance is negative, since the transaction cost and fixed costs are positive when the individual's consumption is low (and thus sells capital), and zero when consumption is high. Therefore, both the first and second term contributes positively to the excess return.

The last covariance term, $cov_{\theta,z}\left(\mathcal{M}_{s',s}, -\tilde{f}_s{}'\right)$, captures the covariance between the expected investment transaction cost to be paid and consumption growth. Note that this covariance is *positive*; when consumption is high, investment demand is high, and the fixed cost is more likely to be paid. As a result, the fixed investment cost introduces an insurance motive into the investment decision of the firm, and increases investment demand relative to a risk-neutral firm. However, this source of negative risk is quite small, so I do not focus on it.

Finally, the last set of terms $\mathbb{E}\left[\mathcal{M}_{s',s}\left(\frac{\partial \tilde{\lambda}'}{\partial k'}(1-\delta) + \frac{\partial \tilde{f}_s{}'}{\partial k'} + \frac{\partial \tilde{f}_d{}'}{\partial k'}\right)k'|\theta, z\right]$ captures the impact of an additional unit of capital on the probability that an individual would buy or sell capital the next period. In principle, this component also captures a dimension of liquidity risk. However, the issue at hand (and which makes this expression an abuse of notation) is that $\tilde{\lambda}$, $\tilde{f}_s$, and $\tilde{f}_d$ are not differential everywhere since they are indicator functions. As such, these terms are not numerically computable, and I ignore their impact when decomposing the sources of risk. That said, these terms are in principle computable if the model is extended to allow for additional stochasticity that smooths out the indicator function (e.g., preference shocks).

F.II Description of Model with Exogenous Partial Insurance

The model with exogenous partial insurance preserves most of the structure of the baseline model with one modification: the realized value of the entrepreneur's income depends on her previous period's productivity due to the risk sharing parameter ξ. In the interest of space, I will only present the relevant modification.

Let $\mathbf{s} = \{z_{-1}, z, \theta, k, a\}$ denote the vector of state variables for an entrepreneur. Following the main text, denote the following

$$\pi^{ex-post}(z_{-1}, z, k) = \xi y^{pool} + (1-\xi)\pi^*(z, k) \tag{73}$$

as the ex-post profits the entrepreneur receives. As discussed, if $\xi = 0$, there is no income risk-sharing (this is just the baseline model). If $\xi = 1$, then the entrepreneur fully shares all income risk with other entrepreneurs who had the same productivity draw and capital choices last period. Finally, any intermediate values for $\xi \in (0,1)$ relates to partial income risk sharing.

With this notation in hand, and using the same notation in the main text otherwise, the Bellman equation for the entrepreneur who continues in business is

$$\begin{aligned} V_{ee}(\mathbf{s}) &= \max_{k',b'} U(c) + \beta \mathbb{E}\left[V_e(\mathbf{s}') \,|\theta, z\right] \\ s.t. \quad & \hat{\pi} \equiv \pi^{ex-post}(\mathbf{s}) + (1+r)\, b - \mathcal{C}(k', k) \\ & k' > 0 \\ & c = \hat{\pi} - k' - b' \geq 0 \\ & b' \geq -\varphi(1-\lambda)(1-\delta_e)\, k' - \underline{b}, \end{aligned} \tag{74}$$

while the entrepreneur who exits entrepreneurship solves

$$
\begin{aligned}
V_{ew}(\mathbf{s}) &= \max_{b'} U(c) + \beta \mathbb{E}\left[V_w(z', \theta', b') \,|\theta, z\right] \\
s.t. \quad \hat{\pi} &\equiv \pi^{ex-post}(\mathbf{s}) + (1+r)\,b - \mathcal{C}(k', k) \\
k' &= 0 \\
c &= \hat{\pi} - k' - b' \geq 0 \\
b' &\geq -\underline{b},
\end{aligned} \tag{75}
$$

Importantly, notice that even for the case where $\xi = 1$, z remains a part of the entrepreneur's state vector. This is because z is used to forecast next-period productivity for continuing entrepreneurs, information which the entrepreneur uses to make her investment decision. In turn, given budget and borrowing constraints, this implies that her consumption policy function continues to depend on z, despite her facing no income risk.

F.III Further Discussion on Technicalities of Risk-Sharing

I now discuss the technicalities involved in my assumption of partial risk sharing. For exposition, I will use a simplified model without capital adjustment cost. This allows me to cleanly formalize the concepts in analytical form. Since the focus is on understanding the role of risk sharing, there is no loss of generality with this simplification.

We begin by implicitly defining the investment choice for a generic $\xi \in [0,1]$. Here, we have the Euler equation,

(76)

$$1 = \mathbb{E}\left[\mathcal{M}_{s',s}(\mathbf{s}', \mathbf{s})\left(\xi \frac{\partial y^{pool}(z, k')}{\partial k'} + (1-\xi)\left(\frac{\partial \pi(z', k')}{\partial k'}\right) + (1-\delta)\right) - u'(c)^{-1}\mu(\mathbf{s})|\mathbf{s}\right]$$

where $\mathcal{M}_{s',s} \equiv \frac{\beta u'(c')}{u'(c)}$ is the idiosyncratic stochastic discount factor, and μ is the Lagrange multiplier associated with the borrowing constraint, and therefore depends on the current period state vector $\mathbf{s}$.

We can further decompose the Euler equation as (suppressing the arguments into the various functions),

$$1+u'(c)^{-1}\mu = (1-\xi)\mathbb{E}\left[\mathcal{M}_{s',s}|\mathbf{s}\right]\mathbb{E}\left[\left(\frac{\partial\pi'}{\partial k'}+(1-\delta)\right)|\mathbf{s}\right]... \tag{77}$$
$$...+(1-\xi)\underbrace{cov_{\mathbf{s}}\left(\mathcal{M}_{s',s},\left(\frac{\partial\pi'}{\partial k'}+(1-\delta)\right)\right)}_{\text{covariance risk}}$$
$$...+\xi\mathbb{E}\left[\mathcal{M}_{s',s}|\mathbf{s}\right]\left(\frac{\partial y^{pool}}{\partial k'}+(1-\delta)\right)...$$
$$...+\xi\underbrace{cov_{\mathbf{s}}\left(\mathcal{M}_{s',s},\left(\frac{\partial y^{pool}}{\partial k'}+(1-\delta)\right)\right)}_{=0}$$

where I denote the covariance, conditional on today's state vector, as $cov_{\mathbf{s}}$.

I now briefly provide some economic intuition into the preceding terms. Beginning first with terms on the RHS of the equation, the first term is the usual risk-free discounted returns to capital; the second term is the usual conditional covariance risk term, and in the last line, since y^{pool} depends on only last period productivity and today's capital choice, it has no conditional covariance with the SDF.

Notably, the third term is unique to this model with partial insurance, and states that the individual chooses k' with the understanding that her investment determines her insurance pool. Critically, I allow for the entrepreneur to internalize that her capital choice determines her insurance pool. In this dimension, my model differs from Guvenen and Smith (2014) where risk is purely exogenous, and thus workers do not have to make such a choice. Note that by definition of the insurance pool (equation 52 in the main text), we have that

$$\frac{\partial y^{pool}}{\partial k'} = \mathbb{E}\left[\frac{\partial\pi'}{\partial k'}|\mathbf{s}\right]. \tag{78}$$

Consequently, the Euler equation can be further reduced to,

$$1+u'(c)^{-1}\mu(\mathbf{s}) = \mathbb{E}\left[\mathcal{M}_{s',s}|\mathbf{s}\right]\mathbb{E}\left[\left(\frac{\partial\pi'}{\partial k'}+(1-\delta)\right)|\mathbf{s}\right]... \tag{79}$$
$$...+(1-\xi)cov_{\mathbf{s}}\left(\mathcal{M}_{s',s},\left(\frac{\partial\pi'}{\partial k'}+(1-\delta)\right)\right).$$

It is now clear now partial insurance works in my model; with increasing insurance (i.e., $\uparrow \xi$), the exposure of the individual to the covariance risk term decreases. In the limit, it goes to 0. Note however that because of borrowing constraints that limit investment and consumption-smoothing, μ is still active for individuals who are liquidity-constrained (LHS of the equation), which leads to "underinvestment", and thus an amplification of the private equity premium.

Finally, for individuals who are not constrained, the Euler equation further reduces to

$$1 = \frac{1}{1+r}\mathbb{E}\left[\left(\frac{\partial \pi'}{\partial k'} + (1-\delta)\right)\right] + (1-\xi)cov_{\mathbf{s}}\left(\mathcal{M}_{s',s}, \left(\frac{\partial \pi'}{\partial k'} + (1-\delta)\right)\right) \tag{80}$$

where we see investment is dampened by covariance risk, but the effect gets weaker as ξ increases. At $\xi = 1$, this term disappears, and we return to the familiar investment Euler equation for a risk-neutral firm.

F.IV Computing the Shapley-Shorrocks decomposition

1 Motivation for use of the Shapley-Shorrocks decomposition

I wish to decompose the contribution of resale frictions, financial frictions, and production risk to productivity losses, relative to the allocation from the social planning problem. Furthermore, I wish to separately decompose their contributions through the extensive and intensive margin. I will now explain how I conduct this decomposition.

For now, for the ease of exposition, let's assume that there exist direct ways to "individually mute" each friction. Then, a standard approach would be to sequentially "mute" one friction at a time, and compute the impact of this marginal change on aggregate productivity. For instance, one might first eliminate production risk to compute the productivity cost of production risk, then further eliminate financial frictions to compute the productivity cost of financial frictions, and finally attribute the remainder of productivity loss to resale frictions.

This approach is not recommended–nor exactly possible–in my model for two reasons.

First, the naive sequential approach described above does not allow me to separate out the effect of a given friction on the extensive margin and intensive margin. Instead, such an approach would simultaneously confound effects on both the intensive and extensive margins.

In principle, one could "hold fix" the distribution of entrepreneur productivity (i.e., fix M and $\Lambda(z)$ to the distribution from the baseline) and conduct the above-mentioned sequential decomposition to obtain the effects of each friction on capital misallocation (i.e., evaluate how changing a friction changes $\Lambda(k|z)$); likewise, one could "hold fix" the distribution of capital (i.e., fix $\Lambda(k|z)$ to the distribution from the baseline) and conduct the above-mentioned sequential decomposition to obtain the effects of each friction on the extensive margin (i.e., evaluate how changing a friction changes M and $\Lambda(z)$).

However, the productivity cost of any friction on the intensive margin will in general also depend on the extensive margin, and vice versa. Therefore, using the above-mentioned approach would not lead to a proper decomposition; specifically, the productivity cost along the intensive and extensive margins, as computed using the sequential approach above, would in general not sum up to the *total* productivity loss (Section 4 below will present evidence for this).

Second, as I already showed evidence for in the main text, the frictions interact non-linearly with each other. Therefore, switching the sequential ordering of the decomposition will change the quantification of the contribution of each friction to productivity loss. For instance, as I already showed, the computed productivity loss due to production risk is *smaller* when financial frictions are active. Therefore, if we mute financial frictions first before computing the productivity loss due to production risk, we will end up with a larger estimate than in the case where we mute production risk without muting financial frictions.

2 Basic implementation of the Shapley-Shorrocks decomposition

My solution to this problem relies on a modification of the approach of Shapley-Shorrocks decomposition (Shorrocks, 2013). To build intuition for this decomposition, I first provide a simplified explanation of the method. Consider an outcome, Y, that is determined by two factors: X_1 and X_2. We can think of each factor as being either included ("on," represented by 1) or excluded ("off," represented by 0). My goal is to decompose the final value of Y into contributions from each factor. Let $Y(x_1, x_2)$ denote the outcome for a given state of the factors. The case where all factors are "off" is $Y(0, 0)$, while the case where all factors are "on" is $Y(1, 1)$. The goal is to decompose $Y(1, 1) - Y(0, 0)$ into its two components.

Consider now the contribution of X_1. I first compute the marginal impact of X_1 when it is added to *every possible counterfactual that excludes it*, that is:

1. When X_1 is added to a model where all factors are "off":
   The impact is $Y(1,0) - Y(0,0)$.

2. When X_1 is added to a model where X_2 is "on":
   The impact is $Y(1,1) - Y(0,1)$.

Then, the contribution of X_1 is computed as an average of these marginal effects. Finally, the contribution of X_2 is computed in an analogous way.

Importantly, because it is an average, the computed contribution does not depend on any single arbitrarily chosen order. For instance, in the case of X_1, it accounts for the fact that, in general, $Y(1,0) - Y(0,0) \neq Y(1,1) - Y(0,1)$ because the factors have interaction effects. In contrast, the standard sequential approach described above effectively assumes that $Y(1,0) - Y(0,0) = Y(1,1) - Y(0,1)$.

Crucially, this approach guarantees that the sum of the individual contributions add up to the total effect, since

$$\frac{1}{2}\left(Y(1,0) - Y(0,0) + Y(1,1) - Y(0,1)\right) + \frac{1}{2}\left(Y(0,1) - Y(0,0) + Y(1,1) - Y(1,0)\right)$$
$$... = Y(1,1) - Y(0,0).$$

In contrast, sequential approaches cannot guarantee that the individual contributions add up to the total effect, except for the case where there are no interaction effects.

3 Adaptation of the algorithm to my model

A limitation of the basic Shapley-Shorrocks decomposition approach is that it only allows me to compute the contribution of a given factor (e.g., X_1), but it does not account for models where the total contribution of a given factor can be further decomposed into additional margins. For instance, in my article, I want to compute the effect of production risk separately along the intensive and extensive margins. The naive implementation of the Shapley-Shorrocks decomposition would not allow me to do so, as it would only allow me to compute the total contribution of production risk along both margins (i.e., the naive implementation would allow me to compute the entries to the first row of Table 6, but not the remaining rows).

Therefore, I modified the basic Shapley-Shorrocks approach to address this limitation. For exposition purposes, let's continue to consider an outcome, Y, that is determined by two factors X_1 and X_2. However, I now extend the notation to allow for the fact that each factor contributes to Y through different margins, which I will denote as $Z(x_1, x_2)$. Notation-wise, let $Y(x_1, x_2|Z(x_1, x_2))$ denote the outcome for a given state of the factors. As before, $Y(1,1|Z(1,1)) - Y(0,0|Z(0,0))$ would give us the total effects.

The marginal impact of X_1, when it is added to *every possible counterfactual that excludes it*, is now:

1. $Y(1,0|Z(0,0)) - Y(0,0|Z(0,0))$.
2. $Y(1,1|Z(0,0)) - Y(0,1|Z(0,0))$.
3. $Y(1,0|Z(0,1)) - Y(0,0|Z(0,1))$.
4. $Y(1,1|Z(0,1)) - Y(0,1|Z(0,1))$.
5. $Y(1,0|Z(1,0)) - Y(0,0|Z(1,0))$.
6. $Y(1,1|Z(1,0)) - Y(0,1|Z(1,0))$.
7. $Y(1,0|Z(1,1)) - Y(0,0|Z(1,1))$.
8. $Y(1,1|Z(1,1)) - Y(0,1|Z(1,1))$.

Therefore, the extension to the basic decomposition is the inclusion of $Z(x_1, x_2)$, where I allow for the fact that the marginal contribution of X_1 is different when Z itself is different (e.g., $Y(1,0|Z(0,0)) - Y(0,0|Z(0,0)) \neq Y(1,0|Z(1,1)) - Y(0,0|Z(1,1))$).

Using this approach, when I want to compute the effect of each friction on capital misallocation, this would imply that Y (the outcome) is TFP computed using the relevant distribution of capital ($\Lambda(k|z)$), and Z (the conditioning factor) is the relevant measure of entrepreneurs and distribution of productivity (M and $\Lambda(z)$). Conversely, when I want to compute the effect of each friction on the extensive margin, this would imply that Y (the outcome) is TFP computed using the relevant measure of entrepreneurs and distribution of productivity (M and $\Lambda(z)$), and Z (the conditioning factor) is the relevant distribution of capital ($\Lambda(k|z)$). These combinations are what populates the entries into Table 7.

Finally, as discussed in the main text, it is not possible to "mute" the role of resale frictions explicitly (unlike financial frictions or production risk). Therefore, the effect of resale frictions is computed as a residual to the above decomposition (i.e., it is the component of $Y(1,1) - Y(0,0)$ that cannot be explained by financial frictions or production risk).

4 Illustration of importance of accounting for non-linear interaction

As an illustration of the importance of accounting for the non-linear interactions between the frictions and two margins of interest, I report the results of an alternative decomposition that does not account for interaction effects between the extensive and intensive margins (Table F11). In other words, I use the approach of the basic Shapley-Shorrocks decomposition as described in Section 2, but I do not modify the algorithm to account for interaction between the intensive and extensive margins. Instead, for computing capital misallocation, I simply fix the measure of entrepreneurs and distribution of productivity to that from the baseline economy. For computing productivity loss on the extensive margin, I fix the capital allocation to that of the baseline economy.

The first row of Table F11 repeats the headline results from the main text. The second row reports the degree of capital misallocation and its decomposition using the naive approach. The last row reports the degree of productivity loss on the extensive margin, and its decomposition using the naive approach.

We see that the naive approach departs from my preferred approach in three ways. First, in the first column, the sum of the productivity loss due to the intensive and extensive margins do not add up to the total effect. In particular, the effect is *larger* than the true total effect.

Second, we also see that the computed productivity losses for each margin is, in general, larger than their respective counterparts using my preferred approach.

Finally, financial frictions now account for a larger share of capital misallocation than resale frictions. Furthermore, production risk now also leads to capital misallocation, whereas it weakly reduces misallocation when the decomposition is done using my preferred approach.

Table F11: Decomposition of productivity losses (naive approach)

	Total	Inc. Risk	Fin. Fr.	Resale Fr.
Both margins	23.6	2.8	8.1	12.7
Intensive margin	11.0	0.4	5.4	5.2
Extensive margin	14.2	4.3	4.1	5.8

This table presents a decomposition of the losses to aggregate productivity due to the three sources of frictions in the model economy. The first row repeats the results from the main text. The next two rows present results using a naive implementation of the Shapley-Shorrocks decomposition. The values are reported in terms of percentages and rounded to one decimal place.
Source: Author's calculations

G Alternative Calibration

This section reports the calibration outcome for the alternative calibration whereby I additionally target the asymmetry in ARPK persistence.

Table G12: Model fit and parameters.

(a) Baseline Moments

Moment	**Data**	**Model**	**Moment**	**Data**	**Model**
Emp share of entrepreneurs	0.200	0.165	$Pr(\frac{i}{k} > 20\%)$	0.463	0.333
Fraction entrepreneurs	0.075	0.078	$\beta_{neg,arpk}$	-0.065	-0.000
Interest rate	0.100	0.102	$\mathbb{E}\left[\log \frac{k'}{k} \vert \text{neg spike}\right]$	-0.994	-1.203
Wealth Gini	0.820	0.811	Leverage	0.093	0.082
Fraction employers	0.580	0.552	$\Delta var(\Delta y)$	0.243	0.255
Exit rate	0.074	0.055	$\Delta\rho(y)$	0.956	0.904

(b) Parameters

Parameter	Value	Parameter	Value
μ^z	-1.503	λ	0.737
$\widetilde{\mu^z}$	-2.412	f_s	0.039
$\widetilde{\mu^\theta}$	-2.735	φ	0.456
ρ^z	0.960	$\bar{l}$	0.805
σ^z	0.318	β	0.895

(c) ARPK skewness and persistence

	Skewness	$Pr(1 \to 1)$	$Pr(5 \to 5)$	$Pr(5 \to 5)$ - $Pr(1 \to 1)$
Data	-0.301	0.584	0.480	-0.104
Baseline	-0.159	0.525	0.427	-0.099
$\lambda = 0$	0.202	0.401	0.473	0.072
$\varphi = 1$	-0.192	0.526	0.418	-0.108

This table reports the model fit for the internal calibration and the resulting parameters for the alternative model. Panel A reports the baseline targeted moments and corresponding moments from the alternative model. Panel B reports the parameter values. Panel C reports the skewness and transition probabilities for ARPK, in the data and the model. The last column of Panel C computes the difference between the probability of staying in the fifth quintile and staying in the first quintile. Where appropriate, all values have been rounded to three decimal places. *Source. KFS, author's calculations.*