

Women's Labor Supply Incentives and Old-Age Income Redistribution

Appendix for Online Publication

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A Building Intuition: Analytical Derivations

Let us assume a household who starts her economic life at age 20 and lives for J years. The household can work until she enters retirement at age j_r . Labor productivity is risky in this model, but survival is certain. This means that the individual doesn't know her exact wage path $\{w_j\}_{j=20}^{j_r-1}$ in advance. The household maximizes the intertemporal utility function

$$U = E_{20} \left[\sum_{j=20}^J c_j - \sum_{j=1}^{j_r-1} v(\ell_j) \right],$$

where $\ell_j \in \{0, 1\}$ denotes the labor supply decision at age j . For simplicity, we assume that utility is linear in consumption.¹ $v(\ell)$ indicates the disutility of labor with $v'(\ell) > 0$ and $v''(\ell) > 0$. There is no discounting.

The household earns gross labor income $y_j = w_j \ell_j$. From this income, she has to pay contributions at rate τ_p to the pension system. In reward for her contributions, the individual receives a pension payment at retirement, which is calculated from

$$p = \kappa \times F(y_{20}, y_{21}, \dots, y_{j_r-1}).$$

¹In our full quantitative simulation model, we relax this assumption and let there be curvature in both utility of consumption and disutility of labor.

The intertemporal budget constraint of the household consequently reads

$$\begin{aligned} E_{20} \left[\sum_{j=20}^J c_j \right] &= E_{20} \left[\sum_{j=20}^{j_r-1} (1 - \tau_p) w_j \ell_j + \sum_{j=j_r}^J \kappa \times F(y_{20}, y_{21}, \dots, y_{j_r-1}) \right] \\ &= E_{20} \left[\sum_{j=20}^{j_r-1} (1 - \tau_p) w_j \ell_j + \kappa \times [J - j_r + 1] \times F(w_{20} \ell_{20}, w_{21} \ell_{21}, \dots, w_{j_r-1} \ell_{j_r-1}) \right], \end{aligned}$$

where we assumed (again for simplicity) that the interest rate is equal to $r = 0$.

Combining the budget constraint and the household utility function, the household's optimization problem reads

$$\begin{aligned} \max_{\ell_{20}, \ell_{21}, \dots, \ell_{j_r-1}} E_{20} &\left[\sum_{j=20}^{j_r-1} (1 - \tau_p) w_j \ell_j + \kappa \times [J - j_r + 1] \right. \\ &\left. \times F(w_{20} \ell_{20}, w_{21} \ell_{21}, \dots, w_{j_r-1} \ell_{j_r-1}) - \sum_{j=20}^{j_r-1} v(\ell_j) \right]. \end{aligned}$$

If labor supply was a continuous choice variable, the first order condition with respect to labor supply in period j would read

$$v'(\ell_j) = (1 - \tau_p) w_j + \kappa \times [J - j_r + 1] \times w_j \times E_j \left[\frac{\partial F(y_{20}, \dots, w_j \ell_j, \dots, y_{j_r-1})}{\partial (w_j \ell_j)} \right].$$

In a discrete choice framework, where the household can only decide between $\ell_j = 0$ and $\ell_j = 1$, the discrete version of this equation reads

$$\begin{aligned} v(1) - v(0) &= (1 - \tau_p) w_j + \kappa \times [J - j_r + 1] \\ &\times E_j \left[[F(y_{20}, \dots, w_j, \dots, y_{j_r-1}) - F(y_{20}, \dots, 0, \dots, y_{j_r-1})] \right]. \end{aligned}$$

It is hence the effective net return to working

$$\begin{aligned} w_{j,net} &= (1 - \tau_p) w_j + \kappa \times [J - j_r + 1] \\ &\times E_j \left[\underbrace{[F(y_{20}, \dots, w_j, \dots, y_{j_r-1}) - F(y_{20}, \dots, 0, \dots, y_{j_r-1})]}_{\text{change in earnings record when working at age } j} \right]. \end{aligned}$$

that is the prime determinant of the individual's labor force participation decisions.

Proportional system Under a proportional pension system, we have

$$F(y_{20}, y_{21}, \dots, y_{j_r-1}) = \frac{\sum_{j=20}^{j_r-1} y_j}{j_r - 20}.$$

Consequently, we get

$$\begin{aligned} E_j [F(y_{20}, \dots, w_j, \dots, y_{j_r-1}) - F(y_{20}, \dots, 0, \dots, y_{j_r-1})] \\ = E_j \left[\sum_{i=20, i \neq j}^{j_r-1} \frac{y_i}{j_r - 20} \right] + \frac{w_j}{j_r - 20} - E_j \left[\sum_{i=20, i \neq j}^{j_r-1} \frac{y_i}{j_r - 20} \right] = \frac{w_j}{j_r - 20}. \end{aligned}$$

The net return to working at age j under such a system therefore is

$$w_{j,net}^{PR} = (1 - \tau_p)w_j + \kappa \times [J - j_r + 1] \times \frac{w_j}{j_r - 20}.$$

Lifetime-Earnings-Based Progressive Pensions In a lifetime-earnings-based progressive pension we have

$$F(y_{20}, y_{21}, \dots, y_{j_r-1}) = f \left(\frac{\sum_{j=20}^{j_r-1} y_j}{j_r - 20} \right).$$

Therefore we get

$$\begin{aligned} E_j [F(y_{20}, \dots, w_j, \dots, y_{j_r-1}) - F(y_{20}, \dots, 0, \dots, y_{j_r-1})] \\ = E_j \left[f \left(\sum_{i=20, i \neq j}^{j_r-1} \frac{y_i}{j_r - 20} + \frac{w_j}{j_r - 20} \right) - f \left(\sum_{i=20, i \neq j}^{j_r-1} \frac{y_i}{j_r - 20} \right) \right]. \end{aligned}$$

Since $f(\cdot)$ is a (piecewise) linear function, we can immediately write

$$\begin{aligned} E_j \left[f \left(\sum_{i=20, i \neq j}^{j_r-1} \frac{y_i}{j_r - 20} + \frac{w_j}{j_r - 20} \right) - f \left(\sum_{i=20, i \neq j}^{j_r-1} \frac{y_i}{j_r - 20} \right) \right] \\ = E_j \left[f' \left(\frac{\sum_{i=20}^{j_r-1} y_i}{j_r - 20} \right) \right] \cdot \frac{w_j}{j_r - 20}. \end{aligned}$$

This equation results from a first-order Taylor approximation, which is exact for linear functions. Note that, strictly speaking, this equality doesn't hold in the rare case that

$$\sum_{i=20, i \neq j}^{j_r-1} \frac{y_i}{j_r - 20} < b\bar{y} \quad \text{and} \quad \sum_{i=20, i \neq j}^{j_r-1} \frac{y_i}{j_r - 20} + \frac{w_j}{j_r - 20} > b\bar{y}.$$

But for illustrative purposes, our approximation suffices. The net return to working at a given age j reads

$$w_{j,net}^{LE} = (1 - \tau_p)w_j + \kappa \times [J - j_r + 1] \times \frac{w_j}{j_r - 20} \times E_j \left[f' \left(\frac{\sum_{j=20}^{j_r-1} y_j}{j_r - 20} \right) \right].$$

Annual-Earnings-Based Progressive Pensions In an annual-earnings-based progressive pension we have

$$F(y_{20}, y_{21}, \dots, y_{j_r-1}) = \frac{\sum_{j=20}^{j_r-1} f(y_j)}{j_r - 20}.$$

We consequently have

$$\begin{aligned} E_j [F(y_{20}, \dots, w_j, \dots, y_{j_r-1}) - F(y_{20}, \dots, 0, \dots, y_{j_r-1})] \\ = E_j \left[\sum_{i=20, i \neq j}^{j_r-1} \frac{f(y_i)}{j_r - 20} \right] + \frac{f(w_j)}{j_r - 20} - E_j \left[\sum_{i=20, i \neq j}^{j_r-1} \frac{f(y_i)}{j_r - 20} \right] = \frac{f(w_j)}{j_r - 20} \end{aligned}$$

and the net return to working becomes

$$w_{j,net}^{AE} = (1 - \tau_p)w_j + \kappa \times [J - j_r + 1] \times \frac{w_j}{j_r - 20} \times \frac{f(w_j)}{w_j}.$$

B Simulation Model: Computational Details

In this Appendix, we describe further details of the quantitative simulation model. We first show the first order conditions of the household optimization problem. We then describe the stationary recursive competitive equilibrium of the economy and show how to compute the invariant measure of households. Finally, we briefly describe the computational algorithm.

B.1 The Single Household: First Order Conditions

In the following, we derive the solution of the single household's problem ($i = s$) in an economy with a proportional pension system. Since labor supply constitutes a discrete choice, we can not formulate a first-order condition for labor supply. Instead, we solve the problem in two steps. First, we assume the household had already made a labor supply choice ℓ . Conditional on this labor supply decision, we determine the optimal consumption-saving decision by solving the conditional optimization problem $\tilde{v}(\mathbf{x}_g, \emptyset, \ell)$.

Consumption-savings choice: The dynamic household optimization problem reads

$$\begin{aligned} \tilde{v}(\mathbf{x}_g, \emptyset, \ell) = \max_{c, a^+} \frac{\left[\frac{c}{v(i, k)} \right]^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} - \nu \frac{[\zeta(g, i, k) + \ell]^{1 + \frac{1}{\chi_g}}}{1 + \frac{1}{\chi_g}} - \xi \times \mathbb{1}_{\ell > 0} \\ + \beta \psi(j + 1, g, i) E \left[v(\mathbf{x}_g^+, \emptyset) \middle| \mathbf{x}_g, \ell \right] \end{aligned}$$

with $\mathbf{x}_g = (j, g, i, k, e, \eta, h, \xi, a, d)$ and $\mathbf{x}_g^+ = (j + 1, g, i, k^+, e, \eta^+, h^+, \xi^+, a^+, d^+)$. Households maximize their conditional utility with respect to the budget constraint

$$(1 + \tau_c) \times c + a^+ + T_p(y, y_{mini}) + T(y, 0, p, s) = (1 + r)a + y + y_{mini} + p + t(i, k) + b. \quad (1)$$

The first-order conditions of the household then read

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c} &= \left[\frac{c}{v(i, k)} \right]^{-\frac{1}{\sigma}} - \mu(1 + \tau_c)v(i, k) = 0 \\ \frac{\partial \mathcal{L}}{\partial a^+} &= -\mu + \beta\psi(j + 1, g, i)E \left[v_a(\mathbf{x}_g^+, \emptyset) \mid \mathbf{x}_g, \ell \right] = 0, \end{aligned}$$

where μ is the multiplier on the budget constraint in the Lagrangian $\mathcal{L}$. Using the envelope theorem, we immediately obtain

$$v_a(\mathbf{x}_g^+, \emptyset) = (1 + r)\mu^+.$$

The Euler equation then reads

$$\frac{\left[\frac{c}{v(i, k)} \right]^{-\frac{1}{\sigma}}}{(1 + \tau_c) \times v(i, k)} = (1 + r)\beta\psi(j + 1, g, i)E \left[\frac{\left[\frac{c^+(\mathbf{x}_g^+, \emptyset)}{v(i, k^+)} \right]^{-\frac{1}{\sigma}}}{(1 + \tau_c) \times v(i, k^+)} \mid \mathbf{x}_g, \ell \right]. \quad (2)$$

The Euler equation (2) and the budget constraint (1) define the optimal level of total household consumption $\tilde{c}(\mathbf{x}_g, \emptyset, \ell)$, savings $\tilde{a}^+(\mathbf{x}_g, \emptyset, \ell)$ and the utility value $\tilde{v}(\mathbf{x}_g, \emptyset, \ell)$ conditional on a certain labor supply decision $\ell \in \{0, \ell_{full}\}$ for single men and $\ell \in \{0, \ell_{mini}, \ell_{part}, \ell_{full}\}$ for female households.² Furthermore, the law of motion for lifetime insured earnings

$$\tilde{d}^+(\mathbf{x}_g, \emptyset, \ell) = e + y_{ins}(\mathbf{x}_g, \emptyset, \ell) \quad (3)$$

determines next period's lifetime insured earnings.

Labour supply decision: Given the utility $\tilde{v}(\mathbf{x}_g, \emptyset, \ell)$ for every possible $\ell \leq h$, the utility maximizing labor supply decision is

$$\ell(\mathbf{x}_g, \emptyset) = \arg \max \tilde{v}(\mathbf{x}_g, \emptyset, \ell),$$

²Note that the female household might be constraint in her labor choice set, i.e. $\ell \leq h$ must hold.

and the policy functions can be immediately inferred from the conditional policy functions derived in the previous step.

B.2 The Couples Household: First Order Conditions

The solution of the couple household's problem ($i = c$) is quite similar. We first solve the consumption-saving problem conditional on the labor supply choices $\ell_f \in \{0, \ell_{mini}, \ell_{part}, \ell_{full}\}$ and $\ell_m \in \{0, \ell_{full}\}$. For every possible and valid (ℓ_f, ℓ_m) combination we solve the optimization problem

$$\begin{aligned} \tilde{v}(\mathbf{x}_f, \mathbf{x}_m, \ell_f, \ell_m) = \max_{\substack{c, a^+, \\ d_m^+, d_f^+}} & \left[\frac{\left[\frac{c}{v(i, k)} \right]^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} - \nu_f \frac{(\zeta(f, i, k) + \ell_f)^{1 + \frac{1}{\chi_f}}}{1 + \frac{1}{\chi_f}} - \xi \times \mathbb{1}_{\ell_f > 0} \right. \\ & \left. + \frac{\left[\frac{c}{v(i, k)} \right]^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} - \nu_m \frac{(\zeta(m, i, k) + \ell_m)^{1 + \frac{1}{\chi_m}}}{1 + \frac{1}{\chi_m}} - \xi \times \mathbb{1}_{\ell_m > 0} \right] \\ & + \beta \psi(j + 1, g, i) E \left[v(\mathbf{x}_f^+, \mathbf{x}_m^+) \mid \mathbf{x}_f, \mathbf{x}_m, \ell_f, \ell_m \right] \end{aligned}$$

with

$$\begin{aligned} \mathbf{x}_f &= (j, f, i, k, e_f, \eta_f, h, \xi, a, d_f), \\ \mathbf{x}_m &= (j, m, i, k, e_m, \eta_m, h, \xi, a, d_m), \\ \mathbf{x}_f^+ &= (j + 1, f, i, k^+, e_f, \eta_f^+, h^+, \xi^+, a^+, d_f^+), \text{ and} \\ \mathbf{x}_m^+ &= (j + 1, m, i, k^+, e_m, \eta_m^+, h^+, \xi^+, a^+, d_m^+). \end{aligned}$$

Couples maximize their utility subject to the borrowing constraint $a^+ \geq 0$, and the budget constraint

$$\begin{aligned} (1 + \tau_c) \times c + a^+ + T_p(y_m, y_{mini, m}) + T_p(y_f, y_{mini, f}) + T(y_m, y_f, p, i) \\ = (1 + r)a + y + y_{mini} + p + t(i, k) + b. \end{aligned} \quad (4)$$

The first-order conditions read³

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial c} &= \left[\frac{c}{v(i, k)} \right]^{-\frac{1}{\sigma}} - \mu(1 + \tau_c)v(i, k) = 0 \\ \frac{\partial \mathcal{L}}{\partial a^+} &= -\mu + \beta\psi(j + 1, g, i)E \left[v_a(\mathbf{x}_f^+, \mathbf{x}_m^+) \mid \mathbf{x}_f, \mathbf{x}_m, \ell_f, \ell_m \right] = 0,\end{aligned}$$

where μ is the multiplier on the budget constraint in the Lagrangian $\mathcal{L}$. Using the envelope theorem, we obtain

$$v_a(\mathbf{x}_f^+, \mathbf{x}_m^+) = (1 + r)\mu^+.$$

The Euler equation then reads

$$\frac{\left[\frac{c}{v(i, k)} \right]^{-\frac{1}{\sigma}}}{(1 + \tau_c) \times v(i, k)} = (1 + r)\beta\psi(j + 1, g, i)E \left[\frac{\left[\frac{c(\mathbf{x}_f^+, \mathbf{x}_m^+)}{v(i, k^+)} \right]^{-\frac{1}{\sigma}}}{(1 + \tau_c) \times v(i, k^+)} \mid \mathbf{x}_f, \mathbf{x}_m, \ell_f, \ell_m \right]. \quad (5)$$

The Euler equation (5) and the budget constraint (4) define the optimal level of total household consumption $\tilde{c}(\mathbf{x}_f, \mathbf{x}_m, \ell_f, \ell_m)$, savings $\tilde{a}^+(\mathbf{x}_f, \mathbf{x}_m, \ell_f, \ell_m)$ and the utility value $\tilde{v}(\mathbf{x}_f, \mathbf{x}_m, \ell_f, \ell_m)$ conditional on a certain labor supply decision (ℓ_f, ℓ_m) . Furthermore, the laws of motion for lifetime insured earnings

$$\tilde{d}_g^+(\mathbf{x}_f, \mathbf{x}_m, \ell_f, \ell_m) = d_g + y_{ins, g}(\mathbf{x}_f, \mathbf{x}_m, \ell_f, \ell_m) \quad (6)$$

determine next period's lifetime insured earnings for each partner.

Labour supply decision: Given the conditional utility $\tilde{v}(\mathbf{x}_f, \mathbf{x}_m, \ell_f, \ell_m)$ for every possible (ℓ_f, ℓ_m) combination with $\ell_f \leq h$, the household chooses the utility maximizing labor supply combination

$$\left(\ell_f(\mathbf{x}_f, \mathbf{x}_m), \ell_m(\mathbf{x}_f, \mathbf{x}_m) \right) = \arg \max \tilde{v}(\mathbf{x}_f, \mathbf{x}_m, \ell_f, \ell_m),$$

and the policy functions can be immediately inferred from the conditional policy functions derived in the previous step.

³Note that we deliberately let couples share consumption equally across partners. This is however not restrictive, since we assume couples to maximize joint utility with equal weights. An equal consumption distribution would therefore be the optimal outcome of an optimization problem in which partners could decide how to distribution consumption across partners.

B.3 Stationary Recursive Competitive Equilibrium

Definition 1. Given an international interest rate $\bar{r}$, government expenditures G , a consumption tax rate τ_c , a progressive tax system $T(\cdot)$ as well as a characterization of the pension system $\{\tau_p, \varrho, \kappa\}$, a stationary recursive equilibrium is a collection of value and policy functions $\{v, c, \ell, a^+, d^+\}$ for single and $\{v, c, \ell_f, \ell_m, a^+, d_f^+, d_m^+\}$ for couple households, optimal production inputs $\{K, L\}$, accidental bequests $\{b_j\}_{j=1}^J$, a net foreign asset position and a trade balance $\{Q, TB\}$ as well as factor prices $\{r, w\}$ that satisfy

1. (Household Optimization) Given prices and characteristics of the tax and pension system, the value function v satisfies the Bellman equation (10) for singles and (11) for couples together with the budget constraint (9), the accumulation equations for pension claims (14) and (15), the borrowing constraint $a^+ \geq 0$ and the laws of motion for productivity risk, career choice and fertility. c, ℓ, a^+ and d^+ are the associated policy functions for singles and $c, \ell_f, \ell_m, a^+, d_f^+, d_m^+$ are the policy functions for couples.
2. (Firm Optimization) Given the international interest rate $\bar{r}$ as well as the wage rate w , firms employ capital and labor according to the demand functions

$$\bar{r} = \Omega\alpha \left(\frac{L}{K}\right)^{1-\alpha} - \delta \quad \text{and} \quad w = \Omega(1-\alpha) \left(\frac{K}{L}\right)^\alpha.$$

Aggregate output is calculated from (12) and the capital stock evolves according to (13).

3. (Government Constraints) The budget constraint of the pension system

$$\underbrace{\int \mathbb{1}_{j \geq j_r} \times [\mathbb{1}_{i=s} \times p(d) + \mathbb{1}_{i=c} \times [p(d_f) + p(d_m)]] d\Phi}_{\text{total pension claims}} = \underbrace{\int \mathbb{1}_{i=s} \times T_p(y_g, y_{\min i, g}) + \mathbb{1}_{i=c} \times [T_p(y_f, y_{\min i, f}) + T_p(y_m, y_{\min i, m})] d\Phi(\cdot, \cdot)}_{\text{total contributions}} \quad (7)$$

and that of the tax system

$$\tau_c \times C + \int T(y_m, y_f, p, i) d\Phi(\cdot, \cdot) = G + \int t(i, k) d\Phi(\cdot, \cdot) \quad (8)$$

hold. Accidental bequests are calculated from

$$b_j = \frac{\int \frac{1-\psi(j, g, i)}{\psi(j, g, i)} \times (1+r)a d\Phi(\cdot, \cdot)}{\int \mathbb{1}_{j < j_R} d\Phi(\cdot, \cdot)} \quad \text{if } j < j_R. \quad (9)$$

4. (Market Clearing:)

(a) The labor market clears:

$$wL = \int \mathbb{1}_{i=s} \times y + \mathbb{1}_{i=c} \times [y_f + y_m] d\Phi(\cdot, \cdot)$$

(b) The capital market clears:

$$K + Q = \int a d\Phi(\cdot, \cdot)$$

(c) The balance of payments identity is satisfied:

$$TB = -\bar{r}Q$$

(d) The goods market clears:

$$Y = \int c d\Phi(\cdot, \cdot) + \delta K + G + TB.$$

5. (Consistency of Probability Measure Φ) The invariant probability measure is consistent with the population structure of the economy, with the exogenous processes of labor productivity η , labor flexibility h and fertility k , and the household policy functions a^+ and d^+ or d_f^+ and d_m^+ , respectively. A formal definition of the probability measures is provided in Appendix B.4.

B.4 The Measure of Households

The population consists of women and men who form couple and single households. At age 20, each individual draws a gender g from the distribution $\phi_g(g)$ and, conditional on gender, is assigned a marital status $i \in \{s, c\}$ according to the probability $\phi_i(g, i)$. In order for each woman to be matched to a suitable man, we require that

$$\phi_g(f) \times \phi_i(f, c) = \phi_g(m) \times \phi_i(m, c).$$

Couple households At age 20, couple households draw one of four possible education level combinations (e_f, e_m) from the joint distribution $\phi_{e,c}(e_f, e_m)$. Conditional on the their education level, each partner draws an initial labor productivity η_g from the invariant distribution $\phi_\eta(e, \eta)$ of the process for η . The household enters the economy without kids $k = 0$, with the full labor choice set $h = \ell_{full}$, with zero assets $a = 0$ and with zero insured lifetime earnings $d_m = d_f = 0$. The realization of ξ follows a log-normal distribution $\pi_\xi(\xi)$ with mean μ_ξ and variance σ_ξ^2 . Thus, the measure of couple households

with characteristics

$$\mathbf{X}_g = (\{20\}, \{g\}, \{c\}, \{0\}, \{e_g\}, \{\eta_g\}\{\ell_{full}\}, \{\xi\}, \{0\}, \{0\})$$

is constructed as

$$\Phi(\mathbf{X}_f, \mathbf{X}_m) = \phi_g(f) \times \phi_i(f, c) \times \phi_{e,c}(e_f, e_m) \times \phi_\eta(e_f, \eta_f) \times \phi_\eta(e_m, \eta_m) \times \pi_\xi(\xi),$$

and zero otherwise.

We can then construct the probability measure for all ages $j > 1$. For all Borel sets of assets $\mathcal{A}$ and insured lifetime earnings of the wife $\mathcal{D}_f$ and the husband $\mathcal{D}_m$ we have

$$\begin{aligned} \Phi(\mathbf{X}_f^+, \mathbf{X}_m^+) &= \psi(j+1, f, c) \times \pi_k(k^+ | j, f, c, k) \\ &\times \pi_\eta(\eta_f^+ | e_f, \eta_f) \times \pi_\eta(\eta_m^+ | e_m, \eta_m) \times \pi_h(h^+ | f, h, \ell_f) \times \pi_\xi(\xi) \\ &\times \int \mathbb{1}_{\{a^+(\mathbf{x}_f, \mathbf{x}_m) \in \mathcal{A}\}} \times \mathbb{1}_{\{d_f^+(\mathbf{x}_f, \mathbf{x}_m) \in \mathcal{D}_f\}} \times \mathbb{1}_{\{d_m^+(\mathbf{x}_f, \mathbf{x}_m) \in \mathcal{D}_m\}} \Phi(d\mathbf{X}_f, d\mathbf{X}_m), \end{aligned}$$

where

$$\begin{aligned} \mathbf{X}_g^+ &= (\{j+1\}, \{g\}, \{c\}, \{k^+\}, \{e_g\}, \{\eta_g^+\}\{h^+\}, \{\xi^+\}, \mathcal{A}, \mathcal{D}_g) \quad \text{and} \\ d\mathbf{X}_g &= (\{j\}, \{g\}, \{c\}, \{k\}, \{e_g\}, \{\eta_g\}\{h\}, \{\xi\}, da, dd_g) \end{aligned}$$

The integral is the measure of assets a and lifetime insured earnings d_m and d_f today such that for fixed $(\mathbf{x}_f, \mathbf{x}_m)$, the optimal choice today of assets for tomorrow $a^+(\mathbf{x}_f, \mathbf{x}_m)$ lies in $\mathcal{A}$ and the optimal choice today of lifetime insured earnings for tomorrow $d_m^+(\mathbf{x}_f, \mathbf{x}_m)$ and $d_f^+(\mathbf{x}_f, \mathbf{x}_m)$ lie in $\mathcal{D}_m$ and $\mathcal{D}_f$, respectively.

Single households Next, we construct the measure of single households. Single households draw their education level $e \in \{hs, co\}$, from the probability distribution $\phi_{e,s}(g, e)$. Conditional on the education level, households draw an initial labor productivity η from the distribution $\phi_\eta(e, \eta)$, see above. Households enter the economy without kids, with the full labor choice set $h = \ell_{full}$, with zero assets and with zero lifetime insured earnings. Thus, the measure of single households with characteristics

$$\mathbf{X}_g = (\{20\}, \{g\}, \{s\}, \{0\}, \{e\}, \{\eta\}, \{\ell_{full}\}, \{\xi\}, \{0\}, \{0\})$$

is constructed as

$$\Phi(\mathbf{X}_g, \emptyset) = \phi_g(g) \times \phi_i(g, s) \times \phi_{e,s}(g, e) \times \phi_\eta(e, \eta) \times \pi_\xi(\xi),$$

and zero otherwise.

We can then construct the probability measure for all ages $j > 1$. For all Borel sets of assets $\mathcal{A}$ and lifetime insured earnings $\mathcal{D}$ we have

$$\begin{aligned}\Phi(\mathbf{X}_{\mathbf{g}}^+, \emptyset) &= \psi(j+1, g, s) \times \pi_k(k^+ | j, g, s, k) \times \pi_\eta(\eta^+ | e, \eta) \times \pi_h(h^+ | g, h, \ell) \\ &\quad \times \pi_\xi(\xi) \times \int \mathbb{1}_{\{a^+(\mathbf{x}_{\mathbf{g}}, \emptyset) \in \mathcal{A}\}} \times \mathbb{1}_{\{d^+(\mathbf{x}_{\mathbf{g}}, \emptyset) \in \mathcal{D}\}} \Phi(d\mathbf{X}_{\mathbf{g}}, \emptyset),\end{aligned}$$

where

$$\begin{aligned}\mathbf{X}_{\mathbf{g}}^+ &= (\{j+1\}, \{g\}, \{c\}, \{k^+\}, \{e\}, \{\eta^+\}, \{h^+\}, \{\xi^+\}, \mathcal{A}, \mathcal{D}) \quad \text{and} \\ d\mathbf{X}_{\mathbf{g}} &= (\{j\}, \{g\}, \{c\}, \{k\}, \{e\}, \{\eta\}, \{h\}, \{\xi\}, da, dd).\end{aligned}$$

The integral is the measure of assets a and lifetime insured earnings d today such that, for fixed $\mathbf{x}_{\mathbf{g}}$, the optimal choice today of assets for tomorrow $a^+(\mathbf{x}_{\mathbf{g}}, \emptyset)$ lies in $\mathcal{A}$ and the optimal choice today of lifetime insured earnings for tomorrow $d^+(\mathbf{x}_{\mathbf{g}}, \emptyset)$ lies in $\mathcal{D}$.

B.5 Computational Algorithm

Following Kindermann et al. (2020), we solve the model in two steps. First, we apply the method of endogenous grid points to solve the household's consumption-savings problem. We can then compute policy functions $c(\mathbf{x}_{\mathbf{g}}, \emptyset)$, $\ell(\mathbf{x}_{\mathbf{g}}, \emptyset)$, $a^+(\mathbf{x}_{\mathbf{g}}, \emptyset)$, $d^+(\mathbf{x}_{\mathbf{g}}, \emptyset)$ for single households and $c(\mathbf{x}_{\mathbf{f}}, \mathbf{x}_{\mathbf{m}})$, $\ell_f(\mathbf{x}_{\mathbf{f}}, \mathbf{x}_{\mathbf{m}})$, $\ell_m(\mathbf{x}_{\mathbf{f}}, \mathbf{x}_{\mathbf{m}})$, $a^+(\mathbf{x}_{\mathbf{f}}, \mathbf{x}_{\mathbf{m}})$, $d_f^+(\mathbf{x}_{\mathbf{f}}, \mathbf{x}_{\mathbf{m}})$, $d_m^+(\mathbf{x}_{\mathbf{f}}, \mathbf{x}_{\mathbf{m}})$ for couple households as well as the value function $v(\cdot, \cdot)$. Second, we determine equilibrium quantities and prices using a standard rootfinding method.

B.5.1 Computation of Policy and Value Functions

This section presents the method for computing the policy and value functions of single households using the method of endogenous gridpoints. The solution method for couple households is technically identical. The state of a single household is given by $\mathbf{x}_{\mathbf{g}} = (j, g, e, k, \eta, h, \xi, a, d)$. To solve the model on a computer, we start with discretizing the continuous elements a, d, η . We use routines provided by the toolbox that accompanies Fehr and Kindermann (2018).

- We specify the asset grid $\hat{\mathcal{A}} = \{\hat{a}_0, \dots, \hat{a}_{40}\}$ as nodes with growing distance on the interval $[\bar{a}_l, \bar{a}_u]$. In particular, we let

$$\hat{a}_k = \bar{a}_l + \frac{\bar{a}_u - \bar{a}_l}{(1 + g_a)^{40} - 1} \times [(1 + g_a)^k - 1] \text{ for } k = 0, 1, \dots, 40.$$

The lower limit of the asset grid is $\bar{a}_l = 0$, the upper limit of the asset grid is $\bar{a}_u = 50$, the growth rate of gridpoints is $g_a = 0.14$.

- We specify the grid for insured lifetime earnings $\hat{\mathcal{D}} = \{\hat{d}_0, \dots, \hat{d}_{12}\}$ as a grid on the

interval $[0, 2]$ with equally spaced nodes.

- We approximate the stochastic process of the AR(1) labor productivity process η by a discrete Markov chain. We use the Rouwenhorst method to discretize the stochastic process $\hat{\mathcal{E}} = \{\hat{\eta}_1, \dots, \hat{\eta}_5\}$ and to determine a transition matrix

$$\pi_\eta(\eta^+|\eta) = \begin{bmatrix} \pi_{11} & \pi_{12} & \dots & \pi_{15} \\ \pi_{21} & \pi_{22} & \dots & \pi_{25} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{51} & \pi_{52} & \dots & \pi_{55} \end{bmatrix}. \quad (10)$$

The policy and value functions can now be solved via backward induction. In the last possible age J , the household will not work⁴ and not save, but will consume all remaining resources. This determines the policy functions

$$c(\hat{\mathbf{x}}_{\mathbf{g}}, \emptyset) = \frac{(1+r)\hat{a} + p(\hat{d}) - T(0, 0, \hat{d}, s) + b}{(1+\tau_c)}, \quad l(\hat{\mathbf{x}}_{\mathbf{g}}, \emptyset) = 0, \quad a^+(\hat{\mathbf{x}}_{\mathbf{g}}, \emptyset) = 0$$

and the value function

$$v(\hat{\mathbf{x}}_{\mathbf{g}}, \emptyset) = \frac{\left[\frac{c(\hat{\mathbf{x}}_{\mathbf{g}}, \emptyset)}{v(i, k)} \right]^{1-\sigma}}{1-\sigma}$$

for all discretized elements of the state space $\hat{\mathbf{x}}_{\mathbf{g}}$.

With the final period policy functions and value function at hand, we can iterate backwards over ages to determine the full history of household decisions. We now only describe the procedure for working-age single households. The optimization problem as well as the first-order conditions for these households are discussed in Appendix B.1.

We now apply the method of endogenous gridpoints. We use the exogenous grid $\hat{\mathcal{A}} = \{\hat{a}_0, \dots, \hat{a}_{40}\}$ to indicate the remainder of assets into the next period, i.e. $a^+ = \hat{a}_k$. For each state $\check{\mathbf{x}}_{\mathbf{g}} = (j, g, i, k, e, \eta, h, \xi, \mathbf{a}^+, \hat{d})$ and possible labor choice $\ell \in \{0, \ell_{mini}, \ell_{part}, \ell_{full}\}$:

1. we first determine

$$d^+ = \frac{(j-1)d}{j} + \frac{y_{ins}}{j}$$

2. given a^+ and d^+ we calculate $\check{c}(\check{\mathbf{x}}_{\mathbf{g}}, \emptyset, \ell)$ from the Euler Equation (2)

3. with $\check{c}(\check{\mathbf{x}}_{\mathbf{g}}, \emptyset, \ell)$ and a given ℓ , we use the budget constraint (1) to get $\check{a}(\check{\mathbf{x}}_{\mathbf{g}}, \emptyset, \ell)$.

⁴Remember, the compulsory retirement age is j_r .

Once $\check{c}(\mathbf{x}_g, \emptyset, \ell)$ and $\check{a}(\mathbf{x}_g, \emptyset, \ell)$ are solved, we can interpolate along a to obtain the policy functions $\tilde{c}(\mathbf{x}_g, \emptyset, \ell)$ and $\tilde{a}^+(\mathbf{x}_g, \emptyset, \ell)$ as well as the value function $\tilde{v}(\mathbf{x}_g, \emptyset, \ell)$ for each today's asset value $\hat{a}_k$ and insured lifetime earnings $\hat{d}_v$ by piecewise linear interpolation.

In case the asset restriction $a^+ \geq 0$ is binding, we extend the interpolation data by another point of value 0 on the left and determine the policy and value functions at this point. We assume the household consumes all available resources and has no savings left over for tomorrow.

B.5.2 A Long-Run Equilibrium of the Macroeconomy

We model a small open economy, hence prices r and w are fixed. In order to determine aggregate quantities and policy parameters in the initial equilibrium ($t = 0$) we need to determine the following four variables numerically:

- the government budget balancing consumption tax rate τ_c that satisfies equation (8)
- the pension replacement rate κ that balances pension contributions and pension payments as outlined in equation (7)
- average earnings $\bar{y}_t$ of the employed population⁵
- aggregate bequests $\bar{B}$, which immediately allows us to compute cohort bequests $\{b_j\}_{j=1}^J$, see equation (9).

Once a guess of these four variables is available, we can use the following algorithm to compute the remainder individual and aggregate variables of the economy:

1. We solve the household optimization problem using the guesses for $\tau_c, \kappa, \bar{y}, \bar{B}$ and determine the measure of households.
2. We compute aggregate quantities $\{L, K, Q, A, TB, Y, C, G, I, \Omega, B\}$ from individual decisions and the measure of household and determine the gap $D = Y - C - I - G$ between demand and supply.

We determine the four central parameters $(\tau_c, \kappa, \bar{y}_t, \bar{B})$ by means of a quasi-Newton rootfinding method. The method receives an initial guess of these variables and updates them in each iteration step using the (numerical) Jacobian of the determining equation system. The iteration process stops when the government and the pension budget are in equilibrium and the model implied average earning and aggregate bequest equal the guess provided by the method.

⁵This is an important parameter, as it determines the pension contribution cap, pension payments and earnings of the low-earning group.

C The Calibration Process in Detail

This appendix discusses our choices of functional forms and parameters in detail. We pay particular attention to demographics and the labor market characteristics of different types of households. We parameterize the model using data and calibration targets from the German economy, which currently features a proportional pension system. Germany therefore serves as a good benchmark for reforms that aim at introducing progressivity into the pension formula.⁶ Our base year is 2017, in which the average earnings, the empirical counterpart to average earnings $\bar{y}$ in our model, amounted to EUR 37,000, see DRV Bund (2020). Our parameterization process is a two-step procedure: We try to identify as many parameters in the model as possible to which we can directly assign values either by estimating them from data or by using direct estimates from the literature. We then use the remaining set of parameters to calibrate the model to data moments, which we again derived from micro and macro data of the German economy. We provide a summary table of all parameters at the end of this appendix.

C.1 Demographics

We let households start their economic life at the age of 20 and allow for a maximum life span of 99 years. The mandatory retirement age j_r is 64, which equals the average retirement age of the German regular retirement population in 2017, see DRV Bund (2019). We use data from the age cohorts 35-49 of the 2017 German Microcensus⁷ to estimate the following demographic parameters:

1. 50.78% of individuals in the sample are male, 33.06% of them have a college education, and 67.54% of them live in a couple household.⁸
2. The proportion of women is 49.22% and 27.76% of them have a college education.
3. We also examine patterns of assortative mating and find that 85.69% of non-college educated men are married to a non-college educated woman. 54.81% of college-educated men are married to a college-educated woman.

Informed by these empirical facts, we set the demographic parameters of the model as follows:

1. A share $\phi_g(m) = 0.5078$ of each new-born cohort are men and $\phi_i(m, c) = 0.6754$ of them are married, the remainder is single. For women, these shares are $\phi_g(f) = 0.4922$ and $\phi_i(f, c) = 0.69680$.

⁶U.S. Social Security would not make the best possible starting point, as it redistributes heavily towards single-earner married couples through spousal and survivor benefits, see for example Kaygusuz (2015). The German system has survivor benefits, too, but no spousal transfers.

⁷See RDC of the Federal Statistical Office and Statistical Offices of the Federal States (2021)

⁸This includes individuals who live in a couple household but are not formally married.

2. The education distribution within couples is given by

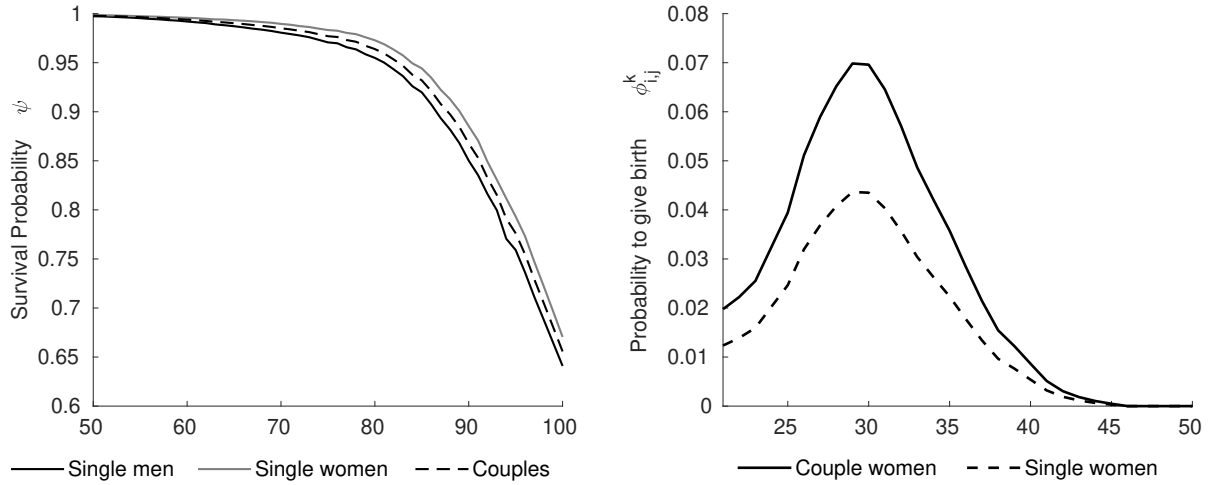
$$\phi_{e,c}(e_f, e_m) = \begin{bmatrix} \phi_e^c(hs, hs) = 0.5736 & \phi_e^c(hs, co) = 0.1494 \\ \phi_e^c(co, hs) = 0.0958 & \phi_e^c(co, co) = 0.1812 \end{bmatrix}, \quad (11)$$

where $\phi_e^c(hs, hs)$ indicates that both partners are non-college educated and $\phi_e^c(co, co)$ that both are college educated. A single woman is college-educated with likelihood $\phi_{e,s}(g, co) = 0.2790$ and a single men with likelihood $\phi_{e,s}(m, co) = 0.3306$.

This calibration strategy provides us with a model consistent measure of households that matches the empirical targets.

Retirement and Survival We extract the 2017 annual life tables for men and women from the Human Mortality Database (2020) to calculate average survival probabilities $\psi(j, g, i)$ by gender. For couples, we simply take the average of the survival probabilities for both genders, i.e. $\psi(j, g, c) = 0.5 \times (\psi(j, f, s) + \psi(j, m, s))$. Single men die at age 79.5 on average, single women at age 84.1 and couples at age 81.7. The left panel of Figure C.1 shows the respective survival probability profiles starting from age 50.

Figure C.1: Survival probability and probability to give birth by age



Source: left: Human Mortality Database (2020), right: German Microcensus 2017, Eurostat (2023).

C.2 The Structure of the Labor Market

A crucial element for the analysis of any fiscal reform, and pension reforms in particular, is a model that can replicate empirical labor supply patterns across demographic groups and over the life cycle. Labor supply choices are to a large degree determined by labor productivity and preferences. To discipline our choices of the various model parameters that can impact individual labor supply decisions, we proceed as follows: We first estimate average labor productivity profiles and risk processes for men from administrative data of

the German pension insurance system. These estimates deliver the general productivity profiles $z(j, e, \eta)$ for workers at different ages j , education levels e and productivity shocks η . We assume that women share the general productivity process with men, but on top they are exposed to a gender wage gap w_{gap} as well as a motherhood wage penalty $w_{pnty}(k)$, which we estimate from data by Schrenker and Zucco (2020).

After all these model parameters are set from direct empirical estimates, we calibrate the remaining parameters of the model so as to tackle empirical statistics on labor supply from the German 2017 Microcensus shown in Table C.1. The data show quite distinct patterns of labor supply across genders, age groups and family status. Labor supply at both the extensive and the intensive margin is, for example, much smaller for older men and women as well as for women that live in a marriage. Single women, on the other hand, work on a full-time job more often than married women. While of course, in a non-linear model, every parameter causes changes in several variables, we will try to identify the data moments onto which a parameter or a set of parameters has the most direct impact. At the end of this appendix, we provide a detailed table of exogenously set and endogenously calibrated parameters as well as their data sources or calibration targets, respectively.

Table C.1: Benchmark: labor supply

	Women				Men	
	not empl	minijob	PT	FT	not empl	FT
Young: Ages 25-44						
Data	27.86	5.94	26.09	40.11	13.30	86.70
Model	23.16	6.22	23.98	46.64	12.45	87.55
Older: Ages 45-63						
Data	25.09	8.47	31.95	34.50	19.64	80.36
Model	25.53	7.34	33.61	33.52	19.85	80.15
Singles: Ages 25-63						
Data	22.98	4.03	21.66	51.33	23.73	76.27
Model	22.60	0.01	17.21	60.19	19.34	80.66
Couples: Ages 25-63						
Data	27.71	8.73	32.55	31.01	12.73	87.27
Model	25.04	9.72	33.58	31.67	14.38	85.62

Source: German Microcensus (2017), own calculations.

C.2.1 Labor Hours Choices

Labor supply ℓ is modeled as discrete choice, see the discussion in Section II.B. According to the German Microcensus, full-time employees of ages 25-63 work an average of 40.3 hours per week, whereas part-time employees work 21.0 hours per week. Assuming a maximum time endowment of 100 hours per week (the equivalent to $\ell = 1$) for an individual,⁹ we set $\ell_{full} = 0.403$ and $\ell_{part} = 0.210$. For minijobs, we finally set $\ell_{mini} = 0.100$ paying tribute to the fact that those jobs are typically low-hours marginal types of employment.

C.2.2 General Labor Productivity

All individuals of an education level e share a common deterministic age-specific labor productivity profile $\theta(j, e)$. We estimate these profiles using administrative data from the German public pension insurance (Deutsche Rentenversicherung). The dataset contains detailed information on the monthly history of pension claims earned by each individual as well as the sources these claim were derived from. Pension claims are in general a good indicator for estimating earnings processes, as they are proportional to individual earnings. See Appendix D for details on the data.

Using these data, we estimate the following flexible functional form for the education-specific age-productivity profiles:

$$\theta(j, e) = b_0(e) + b_1(e) \times \hat{j} + b_2(e) \times \hat{j}^2 + b_3(e) \times \hat{j}^3 \quad \text{with } \hat{j} = \frac{\min(j, j_M(e))}{10}. \quad (12)$$

This form is flexible enough to capture both a hump-shaped ($j_M(e) = \infty$) and a stagnating ($j_M(e) < j_R$) life-cycle labor productivity profile. Note that in the case of a stagnating profile, labor productivity is constant from age $j_M(e)$ onward. The upper part of Table C.2 shows the estimation results, the left panel of Figure 3 visualizes the estimated labor productivity profiles as well as their empirical counterpart.

Taking residual earnings, we estimate the education-specific parameters of an AR(1)-process for log-productivity as specified in (7). The results are shown in the lower panel of Table C.2. The life-cycle average labor productivity profiles exhibit the usual shape, where earnings rise over time and especially so for college graduates. The processes for labor productivity risk are highly persistent, with a somewhat smaller persistence for high-school workers and a larger persistence for college graduates. The overall unconditional process variance ranges at around 28 to 30 log-points.

According to the German tax law the earnings-threshold for minijobs is 450 Euros per month. Since not every minijob worker earns the maximum amount, we assume minijob

⁹Out of a total of 168 hours per week this means that 49 are reserved for sleeping and another 19 for eating, hygiene, etc.

Table C.2: Parameter values of labor productivity profiles and risk

	High school $e = hs$	College $e = co$
Intercept $b_0(e)$	-2.0732	-17.2099
Linear age term $b_1(e)$	0.7833	11.8163
Quadratic age term $b_2(e)$	-0.0572	-2.6345
Cubic age term $b_3(e)$	-0.0026	0.1984
Stagnation threshold $j_M(e)$	∞	44.26
Autocorrelation $\rho(e)$	0.9300	0.9900
Innovation variance $\sigma_\varepsilon^2(e)$	0.0372	0.0059
Unconditional variance $\frac{\hat{\sigma}_\varepsilon^2(e)}{1-\hat{\rho}(e)^2}$	0.2756	0.2983

earnings of 400 Euros per month that corresponds to 4800 Euros annually or $\bar{y}_{mini} = 0.1297 \times \bar{y}$.

C.3 Preferences and the Budget Constraint

Utility is additively separable in consumption c and labor supply ℓ , see again (8). Utility from consumption features constant absolute risk aversion σ , utility from labor a constant but gender-specific Frisch elasticity χ_g . On top of disutility from working, parents have to bear time for childcare amounting to $\zeta(g, i, k)$, see the discussion below. Finally, participation in the labor market is costly to individuals. Specifically, when choosing labor hours greater than zero, a worker has to pay the participation utility cost ξ . We assume that ξ is drawn at the household-level, meaning that it is common to married couples, but iid across households and across time and independent of individual labor productivity. We let ξ follow a log-normal distribution with mean μ_ξ and variance σ_ξ^2 .

Preference parameters We assign a value of 2.0 to risk aversion σ , a choice quite typical for the heterogeneous agent macroeconomics literature though at the lower end of values that generate an extensive desire for redistribution.¹⁰ The empirical literature has pointed to the fact that Frisch elasticities differ significantly between men and women, see for example Keane (2011). Consistent with this evidence, we chose values of $\chi_m = 0.4$ for men and $\chi_w = 0.75$ for women. After making these data-based choices, we are left with the set $(\beta, \nu_m, \nu_f, \mu_\xi, \sigma_\xi^2)$ of parameters that we need to calibrate. We choose the time discount factor $\beta = 0.9785$ so that all capital is entirely absorbed by private savings in the initial equilibrium, and net foreign assets as well as the trade balance are zero. We

¹⁰In this model, σ fulfils two roles as it defines both the coefficient of relative risk aversion and, through its inverse, the intertemporal elasticity of substitution. Estimates for the latter typically range between values of 1 and 3, whereas risk aversion can be quite high and well beyond values of 10 when estimated from individual financial choices, see for example Vissing-Jørgensen and Attanasio (2003).

then jointly calibrate $\nu_m = 70.0$, $\mu_\xi = 1.65$ and $\sigma_\xi^2 = 2.5$ to match the participation rates of men across demographic groups in Table C.1. Finally we set $\nu_f = 22.0$ to achieve an overall good divide between minijob, part-time and full-time work for women.

Time costs of children Children need to be taken care of, which we model as time costs when they are present in the household, i.e. child care costs depend on $k \in \{0, 1, 2, 3\}$. Single mothers have to bear the entire time cost of children on their own. We hence calibrate the time cost of young children $\zeta(f, s, 1) = 1.5$ and older children $\zeta(f, s, 2) = 0.25$ to match the labor supply patterns of both single women as well as young women in Table C.1. Married mother exhibit a somewhat different labor supply pattern over the life cycle, which might be either due to the fact that they live in a two earner household or may result from partners partially sharing the cost of raising children. In order to match the empirical labor supply profiles of married women, too, we assume that the overall time costs of raising children are the same in single and married couple families, but that fathers take a certain (small) share of these costs. This leads us to $\zeta(f, s, 1) = 1.20$ and $\zeta(m, s, 1) = 0.30$ for young children as well as $\zeta(f, s, 2) = 0.1875$ and $\zeta(m, s, 1) = 0.0625$ for older children.

Labor hours flexibility Finally, we pay tribute to the fact that many mothers continue to work part-time, even when their children have already left the household. Our model would not be able to adequately replicate this pattern without restrictions on mother's flexibility in choosing labor hours. The state $h \in \{\ell_{part}, \ell_{full}\}$ refers to a woman's choice set for labor hours. The transition matrix $\pi_h(h^+|g, h, \ell)$ governs the transition between these states and is conditional on the current labor hours choice ℓ . For women who work full time, we assume that the transition matrix is the identity matrix, meaning that they will not be at the risk of facing labor hours restrictions in the next period. Women who do not work full-time transition from $h = \ell_{full}$ into the state $h = \ell_{part}$ with a likelihood of 0.95. Once in this state, they come back to $h = \ell_{full}$ with an annual probability of 0.15. The average duration of a period of labor hours inflexibility is therefore 6.67 years.

C.4 Technology

We choose a depreciation rate of $\delta = 0.07$, which leads to a realistic investment to output ratio of 21%, see German Statistical Office (2020). We set the capital share in production at $\alpha = 0.3$ to obtain a capital-to-output ratio of three and normalize the technology level Ω such that the wage rate per efficiency unit of labor w is equal to 1. Finally, we assume an international interest rate of $\bar{r} = 0.03$, which constitutes as mix between the (in 2017) very low interest rates on deposits and long-run investment opportunities that offer higher returns.

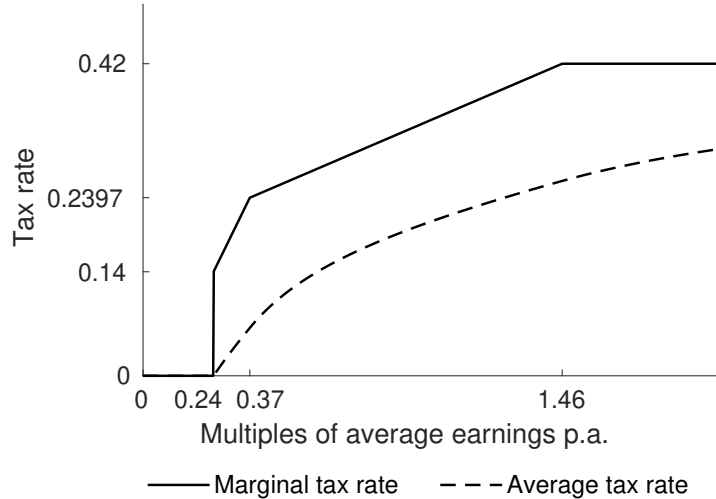
C.5 The Public Pension System

The pension system runs on a pay-as-you-go basis. This means that in equilibrium, total annual pension contributions need to be equal to the total amount of annual pension payments. We fix the pension contribution rate at its statutory rate of $\tau_p = 0.187$ in 2017. The accrual rate for minijob earnings is $\varrho = 0.80$. These choices result in a gross pension replacement of $\kappa = 0.41$, which is similar to the German gross replacement rate for the mean earner as reported by OECD (2021).

C.6 The Tax System and Government Expenditure

The government raises proportional taxes on consumption and progressive taxes on labor earnings to cover government expenditures. We employ the 2017 statutory German progressive income tax code as depicted in Figure C.2. Recall that couples enjoy a tax advantage in the form of income splitting, see (17). We set the proportional consumption tax rate at $\tau_c = 0.16$ to balance the fiscal budget. This untargeted tax rate is in line with the German VAT tax. Although consumption goods are generally taxed at a rate of 19%, many goods (such as food, books and newspapers) are taxed at a lower rate. We fix (wasteful) government consumption at 19% of GDP in the benchmark economy, see German Statistical Office (2020).

Figure C.2: Labor tax schedule



The government provides two direct transfer programs to support families with children. In 2017, parents received a child benefit of EUR 192 per child and month. Moreover, we let the government pay additional tax financed child support payments to single mothers,

which mimic both alimony payments but also subsistence transfers in the real world. We set these monthly child support payments to EUR 576 per child.¹¹ With average labor earnings of 37,000 Euros, the child transfer function consequently reads

$$t(i, k) = \begin{cases} 0.1245 \times \bar{y} & \text{if } k \in \{1, 2\} \text{ and } i = c \\ 0.4980 \times \bar{y} & \text{if } k \in \{1, 2\} \text{ and } i = s \\ 0 & \text{otherwise.} \end{cases}$$

In addition to direct transfers, mothers are compensated for foregone pension contributions while raising children. For each child, they are credited a pension top-up that is equal to the pension top-up of an average earner. In order to account for this in the model, we set the child pension credit to $p_{child}(k = 1) = \bar{y}$ and to zero otherwise, as the two children borne to a mother remain small ($k = 1$) for an average of six years.

C.7 Parameter Values - Overview

Table C.3 summarizes calibrated parameters as well as their targets. Table C.4 summarizes all model parameters that were taken directly from the literature or estimated directly from data.

D Datawork

The productivity profiles in this paper are based on administrative data from the German Pension Insurance. In particular we use the 2017 wave of the scientific use-file of the Versichertenkontenstichprobe (FDZ-RV, 2017) that contains monthly earnings data of 69,520 insured individuals. This is about 0.18% of the actively insured population.¹² We will limit our attention to the male sample population between the ages of 25 and 60, for which we have information on the level of education. We consider observations for the years 2000 to 2016. Our measure of monthly labor earnings comprises income from regular work and short-term unemployment (up to one year). We consider short-term unemployment as an employment type, since individuals are productive when searching for a new job. We sum up monthly earnings observations to construct an annual earnings measure for each individual.

We use the same dataset, data cleaning process and estimation strategy as outlined in detail in Kindermann and Pueschel (2023). However, our approach differs with respect to low earnings individuals. Kindermann and Pueschel (2023) define a low-earnings group

¹¹In Germany, alimony payments depend on individual income and the age of the child. The so-called "Duesseldorfer Table" specifies the exact amount, see OLG Düsseldorf (2017). For monthly net earnings between 1,500-5,100 Euro, monthly payments amount 360-736 Euro in 2017.

¹²The German pension scheme covered of 38 million actively insured individuals in 2017.

Table C.3: Summary of endogenous model parameters

	Value	Target
To target labor supply		Data from Table C.1
Disutility labor ν_f	22.0000	
Disutility labor ν_m	70.0000	
Disutility empl. mean μ_ξ	1.6500	
Disutility empl. var σ_ξ^2	2.5000	
Transition prob. $\pi_h(h_{part} f, h_{full}, \ell < \ell_{full})$	0.9500	
Transition prob. $\pi_h(h_{full} f, h_{part}, \ell < \ell_{full})$	0.1500	
Time costs of children		
- single mothers, $k = 1, \zeta(f, s, 1)$	1.5000	
- single mothers, $k = 2, \zeta(f, s, 2)$	0.2500	
- couple mothers, $k = 1, \zeta(f, c, 1)$	1.2000	
- couple mothers, $k = 2, \zeta(f, c, 2)$	0.1875	
- couple fathers, $k = 1, \zeta(m, c, 1)$	0.3000	
- couple fathers, $k = 2, \zeta(m, c, 2)$	0.0625	
- all parents, $k = 3, \zeta(g, i, 3)$	0.0000	
Others		
Depreciation rate δ	0.0700	Investment/output: 21.0
Technology level Ω	0.9232	Wage per efficiency unit $w = 1$
Discount factor β	0.9785	Closed economy: NFA = 0.00
Consumption tax rate τ_c	0.1617	Government budget balance
Replacement rate κ	0.4133	Pension budget balance

that is excluded from the sample. We, instead, simply exclude minijob worker from the sample, as we also treat them separately in the model. Hence, we use the empirical earnings profiles to calibrate the productivity processes for full time- and part-time worker in our model.

D.1 Statistical Model

We describe the earnings dynamics of the sample by a standard AR(1) process in logs. We therefore split the sample according to an individuals' education level $e \in \{hs, co\}$. $e = hs$ summarizes all individuals with high school education, while $e = co$ indicates the college educated workforce. For each education group, we estimate the statistical model

$$\log(y_{iejt}) = \kappa(t, e) + \theta(j, e) + \eta_{iejt} \quad \text{with} \quad \eta_{iejt} = \rho(e)\eta_{iej-1, t-1} + \varepsilon_{iejt}, \quad (13)$$

for labor earnings y_{iejt} of an individual i with education e at age j in year t . $\kappa_{t,e}$ is a year fixed effect that controls for earnings changes along the business cycle. $\theta(j, e)$ is an age fixed effect that informs us about the age-earnings relationship. The noise term ε_{iejt} is assumed to follow a normal distribution with mean 0. Furthermore, we let the stochastic

Table C.4: Summary of exogenous or estimated model parameters

	Value (m/f)	Source
Demographics		
Max. age J	99	Assumption
Survival probabilities $\psi(j, g, i)$	Figure C.1	Human Mortality Database (2020)
Gender distribution $\phi_g(g)$	0.5078/0.4922	German Microcensus 2017
Share college educated $\phi_e(g)$	0.3306/0.2776	German Microcensus 2017
Share married $\phi_i(g)$	0.6754/0.6968	German Microcensus 2017
Share mother (singles) $\phi_k(s)$	0.4753	German Microcensus 2017
Share mother (married) $\phi_k(c)$	0.8002	German Microcensus 2017
Retirement age j_r	64	DRV Bund (2019)
Labor market		
Weekly hours full-time ℓ_{full}	0.4030	German Microcensus 2017
Weekly hours part-time ℓ_{part}	0.2100	German Microcensus 2017
Weekly hours minijob ℓ_{mini}	0.1000	Minijob Law 2017
Minijob earnings $\bar{y}_{mini}$	$0.1297 \times \bar{y}$	Minijob Law 2017
Others		
Child tranfers $t(s, k \in \{1, 2\})$	$0.4980 \times \bar{y}$	OLG Düsseldorf (2017)
Child tranfers $t(c, k \in \{1, 2\})$	$0.1245 \times \bar{y}$	German Statistical Office (2020)
Pension top-up $p_{child}(1)$	$\bar{y}$	DRV Bund (2020)
Returns to scale $v(i, k)$		New OECD equiv. scale
Pension contribution rate τ_p	0.1870	DRV Bund (2020)
Pension share minijobs ϱ	0.8021	DRV Bund (2020)
International interest rate $\bar{r}$	0.0300	
Capital share in production α	0.3000	
Intert. elast. of substitution σ	2.0000	Heathcote et al. (2014)
Frisch elast. of labor supply χ_g	0.4000/0.7500	Keane (2011)

process start from its long-run variance $\sigma^2(e)$. This means that

$$\varepsilon_{iejt} \sim N(0, \sigma_\varepsilon^2(e)) \quad \text{and} \quad \eta_{ie20t} \sim N(0, \sigma^2(e)) \quad \text{with} \quad \sigma^2(e) = \frac{\sigma_\varepsilon^2(e)}{1 - \rho(e)^2}.$$

We use a generalized method of moments estimator to determine the parameters of this model. We thereby control for the fact that the data are top-coded.

D.2 Moment Conditions and Estimation

To estimate the statistical model in (13) with our data, we have to determine a total of 110 parameters:

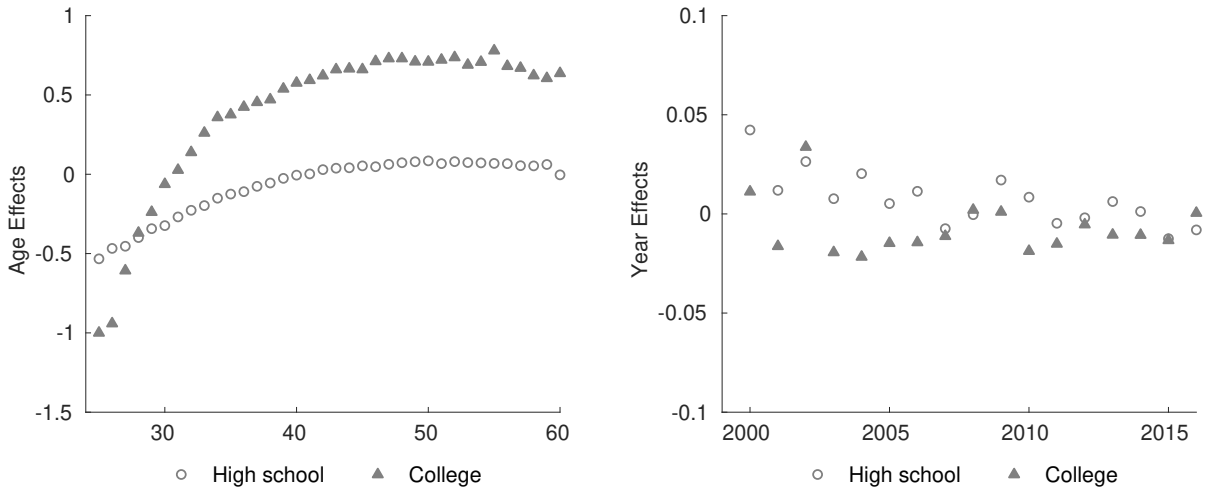
1. 34 year fixed effects $\kappa(t, e)$ for the years 2000 to 2016 and the education levels $e \in \{hs, co\}$;
2. 72 age fixed effects $\theta(j, e)$ for the ages 25 to 60 for each education level e ;

3. the two unconditional variances $\sigma^2(e)$;
4. the two autocorrelation parameters $\rho(e)$.

In order to estimate these parameters, we use the labor earnings data y_{iejt}^p to calculate the empirical moments that correspond to the means E_{ejt} , censoring shares P_{ejt} , variances Var_{ejt} and covariances Cov_{ejt} for each education level e , age j and year t . We use these empirical moments to calculate a residual sum of squares measure. We use a diagonal weighting matrix that has the inverse of the squared standard errors of the empirical moments on the diagonal. To minimize the residual sum of squares and account for multiple local minima, we use the method of simulated annealing, see Du and Swamy (2016). We estimate parameters separately for each education level e .

The results of this estimation process are quite standard in the sense that the estimates exhibit typical life cycle labor earnings profiles, a significant college wage premium as well as a high auto-correlation of earnings, see left panel of Figure D.1. We will use these estimates as prime inputs into the calibration of our quantitative model. Yet, as the statistical model describes labor earnings and not labor productivity, we can not use the estimated parameters as direct inputs. The left panel of Figure D.1 visualizes the point estimates of the age fixed effects by education level. Up to the age of 45, earnings steeply

Figure D.1: Age fixed-effects and year fixed-effects

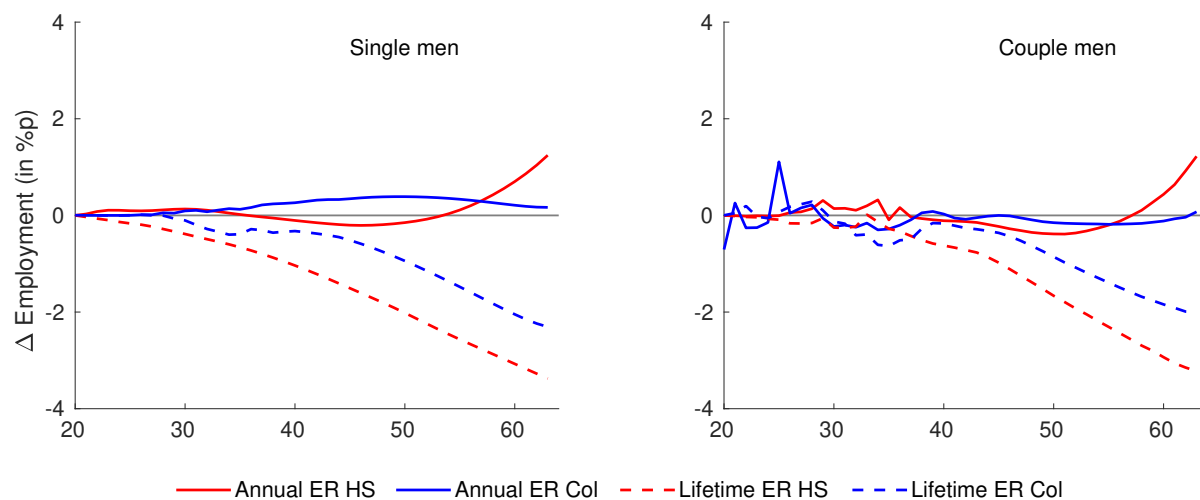


increase for both education groups, especially so for the college educated. Afterwards, they stagnate or decline slightly for the rest of an individual's working life. This shape of life cycle earnings is quite common in the empirical literature and has been found for other countries as well, see for example Heckman et al. (1998) or Casanova (2013). The college-wage premium implied by these profiles is equal to 60 percent, which is in line with empirical findings (OECD, 2016). The right panel of the figure shows the year fixed

effects. These are generally small relative to the age effects and exhibit some cyclical dynamics. Table C.2 summarizes the estimation results for the residual earnings process.

E Further Simulation Results

Figure E.1: Changes in employment rates men



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