SUPPLEMENTAL APPENDIX to "Bonus Question: How Does Flexible Incentive Pay Affect Unemployment Dynamics?"

A Analytical Results Appendix

This appendix provides additional analytical results and discussion. Subsection A.1 provides additional details and context for the proof of Theorem 1. It additionally offers an alternate proof to the one contained in the body of the text based on the First-Order Approach. Subsection A.2 extends the model to jointly study wage cyclicality from incentives and from bargaining or outside option fluctuations. Finally, subsection A.3 extends Theorem 1 to a setting with endogenous contractible separations.

A.1 Supplemental Details for Theorem 1 Proof

This subsection contains more details for the proof of Theorem 1. It first reproduces Theorem 4.13 of Bonnans and Shapiro (2000), which is the envelope theorem invoked in the proof. Then we provide a decomposition of the response of profits per worker in the dynamic model similar to equation (12) in the static model, which provides an additional interpretation to the proof strategy. Then we rewrite the firm's problem in terms of innovations to aggregate productivity ε to give additional context to equation (17) in the proof. Next, we provide low-level sufficient conditions for the space of feasible contracts satisfying the IC and PC conditions to be compact. Finally, we provide an alternative proof environment which employs a recursive formulation of the contracting problem and the common first-order approach (FoA) employed elsewhere in the literature (Farhi and Werning, 2013).

A.1.1 Statement of Theorem 4.13 of Bonnans and Shapiro (2000)

Consider the following optimization problem:

$$\min_{x \in \mathcal{X}} V(x, z) \quad \textit{subject to } x \in \Phi$$

where z is a member of a Banach space Z, $\mathcal{X}$ is a Hausdorff topological space, $\Phi \subset \mathcal{X}$ is nonempty and closed, and $V : \mathcal{X} \times Z \to \mathbb{R}$ is continuous. Let the value function be defined

as

$$J(z) \equiv \inf_{x \in \Phi(z)} V(x, z)$$

and the optimal control set be given by

$$\mathbf{\Gamma}^*(z) \equiv \arg\min_{x \in \Phi(z)} V(x, z).$$

Suppose that $z_0 \in Z$ *and*

1. *For all* $x \in \mathcal{X}$ *the function* $V(x, \cdot)$ *is Gateaux differentiable*
2. $V(x, z)$ *and its partial Fréchet derivative with respect to* z*, given by* $D_z V(x, z)$*, are continuous on* $\mathcal{X} \times Z$
3. *There exists* $M \in \mathbb{R}$ *and a compact set* $C \subset \mathcal{X}$ *such that for every* z *near* z_0 *the set* $A(z) \equiv \{x \in \Phi : V(x, z) \leq M\}$ *is non-empty and contained in* C*.*

Then the optimal value function $z(\cdot)$ *is Fréchet directionally differentiable at* z_0 *and*

$$J'(z_0, d) = \inf_{x \in \mathbf{\Gamma}^*(z_0)} D_z V(x, z_0)\, d,$$

where d *is the direction of the Fréchet derivative and* $J'(z_0, d)$ *is the Fréchet derivative of* J *with respect to* z *in that direction.*

This theorem provides conditions under which the total derivative of the value function with respect to some parameter z is equal to the partial derivative of the value function with respect to that parameter, taking the smallest product of the partial derivative and direction across the optimal control set. In our setting, $\mathcal{X}$ is the set of contracts, $x = (\mathbf{w}, \mathbf{a})$ and $\Phi(z)$ is the set of contracts $x \in \mathcal{X}$ that satisfy the incentive compatibility and participation constraints.

A.1.2 Decomposition of Profit Response in Dynamic Model

To study profits, we combine the IC and PC into a functional $G(\mathbf{w}, \mathbf{a})$, defined such that $G(\mathbf{w}, \mathbf{a}) \leq 0$ holds if and only if $(\mathbf{w}, \mathbf{a})$ is a feasible contract in $\mathcal{X}$ that satisfies the PC (9) and IC (8). Let $\lambda(z_0)$ denote the costate functional on these constraints, and let $\Phi \equiv \{(\mathbf{w}, \mathbf{a}) \in \mathcal{X} : G(\mathbf{w}, \mathbf{a}) \leq 0\}$ be the set of feasible contracts that satisfy the IC and PC constraints. We

write the value of a filled job using the functional Kuhn–Tucker Lagrangian:[15]

$$J(z_0) = V(\mathbf{w}^*, \mathbf{a}^*; z_0) - \langle G(\mathbf{w}^*, \mathbf{a}^*; z_0), \lambda^* \rangle \,, \tag{21}$$

where the star superscripts indicate values under the optimal contract at z_0. Then, we can decompose the response of firm profits to z_0, generalizing decomposition (10) from Section 2.1. The response of profits to aggregate shocks in the flexible incentive pay economy is

$$\frac{dJ(z_0)}{dz_0} = \underbrace{\frac{\partial}{\partial z_0} V(\mathbf{w}^*, \mathbf{a}^*; z_0)}_{\text{(A) direct productivity effect on profits}} - \underbrace{\left\langle \frac{\partial}{\partial z_0} G(\mathbf{w}^*, \mathbf{a}^*; z_0), \lambda^*(z_0) \right\rangle}_{\text{(B) direct effect on participation and incentives}} \tag{22}$$

$$+ \underbrace{\sum_{x \in \{\mathbf{w}^*, \mathbf{a}^*\}} \left[\partial_x V(\mathbf{w}^*, \mathbf{a}^*; z_0) - \langle \partial_x G(\mathbf{w}^*, \mathbf{a}^*; z_0), \lambda^*(z_0) \rangle\right] \cdot \frac{dx}{dz_0} - \left\langle G(\mathbf{w}^*, \mathbf{a}^*; z_0), \frac{d\lambda^*(z_0)}{dz_0} \right\rangle}_{\text{(C) indirect effects on optimal contract and costates}},$$

where ∂_x represents the vector of partial derivatives with respect to some variable x. The direct productivity effect (A) measures how shocks to initial productivity affect the expected present value of output in all periods, where the expectation conditions on initial productivity z_0 and contracted effort $\mathbf{a}^*$.

Term (B) captures the effects on the constraints. Since z_0 affects the constraints only indirectly through the contract $(\mathbf{w}, \mathbf{a})$ in our baseline model, there is no direct effect of z_0 on constraints.

The (C) term captures the effects that the shock has on profits through changes in the firm's choice variables. (C) has three pieces. First, the shock may shift the optimal contract's wage function $\mathbf{w}^*$. This is the wage effect: the wage paid for each future realization of η^t and z^t may differ for contracts signed at different initial aggregate productivity levels z_0. Second, the shock may shift the optimal contract's recommended effort function $\mathbf{a}^*$, which affects output. This is the incentive effect. Finally, the shock may shift the value of the costates on the participation and incentive constraints.

The proof of Theorem 1 effectively shows that this (C) term is zero along the optimal contract by applying an envelope theorem. It then argues that the (B) term is also zero under the baseline model. Thus only the direct effect remains in both flexible incentive pay and rigid wage economies. As discussed in section A.2, however, bargaining and outside option fluctuations will manifest as additional (B) terms, and thus will mute profits (and so market tightness and employment) responses to exogenous productivity shocks.

[15] The notation $\langle x, x^* \rangle$ denotes the value of the linear functional x^* at a point x. This notation is necessary because there is a continuum of constraints—see Section 3.1.1 of Golosov et al. (2016) for a formal definition of Lagrangians with this notation.

A.1.3 Firm Problem in Terms of Innovations

We now write the firm problem in terms of innovations ε, following the impulse response notation introduced in the main text. Specifically, we let the contracts be given by $(\mathbf{w}, \mathbf{a}) = \{w_t\left(\eta^t, \varepsilon^t; z_0\right), a_t\left(\eta^{t-1}, \varepsilon^t; z_0\right)\}_{t=0,\eta^t,\varepsilon^t}^{\infty}$ where $w_t\left(\eta^t, \varepsilon^t; z_0\right), a_t\left(\eta^{t-1}, \varepsilon^t; z_0\right)$ are continuous functions mapping from the history of idiosyncratic and aggregate shocks, and the initial state, to wages and effort. That is, contracts can depend on z_0 and a cumulative set of deviations from z_0. We use the fact that we consider impulse responses holding fixed a path of deviations to define the measure

$$\tilde{\pi}_t\left(\eta^t, \varepsilon^t | \mathbf{a}\left(z_0\right)\right) = \prod_{\tau=0}^{t} \pi_\tau\left(\eta_\tau | \eta^{\tau-1}, a^\tau\left(\eta^{\tau-1}, \varepsilon^\tau; z_0\right), \varepsilon^\tau\right) \pi_\tau\left(\varepsilon^\tau\right),$$

where the probability measure does not depend directly on z_0 because η^t is independent of z_0 by assumption, and ε^t does not depend on z_0 by our definition of an impulse response.

Thus, the firm's problem becomes

$$J\left(z_0\right) = \max_{\mathbf{w}(z_0), \mathbf{a}(z_0)} \sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^t \int\int \left(f\left(\mathbb{E}\left[z_t | z_0\right] + \varepsilon_t, \eta_t\right) - w_t(\eta^t, \varepsilon^t; z_0)\right) \tilde{\pi}_t\left(\eta^t, \varepsilon^t | \mathbf{a}\left(z_0\right)\right) d\eta^t d\varepsilon^t \tag{23}$$

subject to participation constraints

$$\begin{aligned} \sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^t \Bigg[\int\int u\Big(w_t(\eta^t, \varepsilon^t; z_0), a_t(\eta^{t-1}, \varepsilon^t; z_0)\Big) \tilde{\pi}_t\left(\eta^t, \varepsilon^t | \mathbf{a}\left(z_0\right)\right) d\eta^t d\varepsilon^t \\ + \beta s \int U(\mathbb{E}[z_{t+1} | z_0] + \varepsilon_t) \hat{\pi}_t(\varepsilon^{t+1}) d\varepsilon^{t+1} \Bigg] \geq \mathcal{E}\left(z_0\right) \end{aligned} \tag{24}$$

and incentive compatibility constraints

$$\begin{aligned} &\sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^t \Bigg[\int\int u\Big(w_t(\eta^t, \varepsilon^t; z_0), \tilde{a}_t(\eta^{t-1}, \varepsilon^t; z_0)\Big) \tilde{\pi}_t\left(\eta^t, \varepsilon^t | \tilde{\mathbf{a}}\right) d\eta^t d\varepsilon^t \Bigg] \\ &\leq \sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^t \Bigg[\int\int u\Big(w_t(\eta^t, \varepsilon^t; z_0), a_t(\eta^{t-1}, \varepsilon^t; z_0)\Big) \tilde{\pi}_t\left(\eta^t, \varepsilon^t |, \mathbf{a}\left(z_0\right)\right) d\eta^t d\varepsilon^t \Bigg] \end{aligned} \tag{25}$$

for all $\tilde{\mathbf{a}} \in \mathcal{X}$.

A.1.4 Sufficient Conditions for Compactness of Φ

This section provides two sets of sufficient conditions for Φ to be compact. The first condition is that η and z have finite support, and contracts last at least T periods for T finite. The second is that contracts are continuous and twice differentiable in their arguments with uniformly bounded first and second derivatives and that T is finite.

Lemma 2. *If η and z have finite support and the time horizon is finite, then the feasible set of contracts Φ is compact.*

Proof. Suppose $\eta_t \in \{\eta_1, \ldots, \eta_N\}$ and $z_t \in \{z_1, \ldots, z_M\}$ have finite support and T is finite. We will show that the space of $[w, a]$ functions of (η, z) that are IC and PC is compact.

Consider a sequence of functions $[w_n, a_n]$ that are IC and satisfy the PC. The sequence $[w_n(\eta_1, z_1), a_n(\eta_1, z_1)]$ takes values in a compact set. Therefore, it has a subsequence that converges. Call it $[w_{\phi_{1,1}(n)}(\eta_1, z_1), a_{\phi_{1,1}(n)}(\eta_1, z_1)]$. Now apply the same reasoning to the sequence $[w_{\phi_{1,1}(n)}(\eta_1, z_2), a_{\phi_{1,1}(n)}(\eta_1, z_2)]$; similarly, it is in a compact set, so it has a subsequence that converges.

Through this diagonal argument, we construct $[w_{\phi_{1,1}\circ\phi_{1,2}\cdots\circ\phi_{N,M}(n)}, a_{\phi_{1,1}\circ\phi_{1,2}\cdots\circ\phi_{N,M}(n)}]$: a sub-sequence of *functions* that converges. Now we need to show that the limiting function is also in the set, that is, it is IC and satisfies PC.

The PC is a closed inequality involving continuous functions. Due to the continuity of the involved functions, the limit of any sequence of functions satisfying the PC will also satisfy it. Analogously, for the incentive compatibility constraint (IC), consider fixing an action $\tilde{a}(\cdot)$. Any sequence of $[w, a]$ satisfying the IC inequality for $\tilde{a}(\cdot)$, by the continuity of the functions involved, will satisfy it at the limit. Since this applies for all $\tilde{a}(\cdot)$, the limiting function must be IC. We began with an arbitrary sequence of $[w, a]$ that are both IC and PC, and we have shown that it has a subsequence converging to a limit that is also IC and PC. Therefore, the space of mechanisms is compact.

We can now employ a standard envelope theorem in this case, given that the choice set is compact and Corollary 4 of Milgrom and Segal (2002) applies.

□

Lemma 3. *The set of feasible contracts that satisfy the IC constraints, Φ, is compact if contracts are restricted to being continuous and twice differentiable in their arguments $\{\eta^t, z^t\}$, with uniformly bounded first and second derivatives and the horizon T is finite.*

Proof. We will show that Φ is equicontinuous.[16] Let $\Xi = [\underline{\eta}, \bar{\eta}]$ be the set of possible values

[16] Φ is said to be equicontinuous at a point $x \in \Xi^T \times Z^T$ if, for every $\epsilon > 0$, there exists a $\delta > 0$ such that for every function f in Φ and every point y in $\Xi^T \times Z^T$, if $||x - y|| < \delta$ then $||f(x) - f(y)||_{C^1} < \epsilon$, where $||\cdot||_{C^1}$ is the C^1 norm.

for η. Consider a set of functions that are continuously differentiable on $[0,1]$ and such that both the functions and their first and second derivatives are uniformly bounded. This means there exists some real number M such that for every function f in the set and every $x \in \Xi^T \times Z^T, ||f(x)|| \leq M$ and for its Jacobian $||Df(x)|| \leq M$, where $||\cdot||$ is the Euclidian norm in $\Xi^T \times Z^T$.

Given $\epsilon > 0$, choose $\delta = \epsilon/2M$. Then for any function f in Φ and any points x and y in $\Xi^T \times Z^T$ such that $||x-y|| < \delta$, by the mean value theorem, we have $||f(x)-f(y)||_\infty = |Df(c)| \cdot ||x-y||$ for some c in the line $xt + (1-t)y$, $t \in [0;1]$. Since $|Df(c)| \leq M$ and $|x-y| < \delta = \epsilon/2M$, we get $||f(x)-f(y)||_\infty < \epsilon/2$.

Similarly, we can apply the mean value theorem to the Jacobian of f, and since the second derivatives are bounded, an analogous argument to that above yields $||Df(x)-Df(y)||_\infty < \epsilon/2$. Therefore $||f(x)-f(y)||_{C^1} \equiv ||f(x)-f(y)||_\infty + ||Df(x)-Df(y)||_\infty < \epsilon$ and we have shown that Φ is equicontinuous. By the Ascoli Theorem, any sequence in Φ thus has a subsequence that converges. Therefore, Φ is compact. □

A.1.5 Proof Using First-Order Approach and Recursive Formulation

We now pursue the First-Order Approach (FoA) to prove Theorem 1. To make progress, we must introduce one additional assumption guaranteeing that the so-called FoA offers a valid solution to the contracting problem:

Assumption 3. *The set of feasible contracts $(\mathbf{w}, \mathbf{a}) \in \mathcal{X}$ is compact and convex. Assume standard Inada conditions on utility, $\lim_{c \to \underline{w}} u_c(c,a) = \lim_{a \to \bar{a}} u_c(c,a) = \infty$ and $\lim_{c \to \bar{w}} u_c(c,a) = \lim_{a \to \underline{a}} u_c(c,a) = 0$, that the worker's optimal effort choices are determined by the first-order condition to problem (8) and that the density of η_t can be expressed as*

$$\pi_t\left(\eta_t | \eta^{t-1}, a^t\right) = \pi_t\left(\eta_t | \eta_{t-1}, a_t\right).$$

Under this assumption, the incentive compatibility constraint may be written as the first-order condition to the worker's problem, and the firm's contracting problem may be expressed recursively.

We now show how to apply an envelope theorem to the flexible incentive pay problem under this stronger assumption. This proof is clarifying because the approach is closer to standard practice, and it will also be useful because it derives results that are necessary for Proposition 4. Therefore, for this subsection, we make both Assumption 1 from the main text and 3.

The application of the envelope theorem proceeds in three steps in this environment. First, we derive a first-order approach to simplify incentive constraints into local incentive

constraints as in Farhi and Werning (2013) or Pavan et al. (2014). Then, we develop a recursive formulation of the problem following the steps in Ndiaye (2020). Finally, we use these constructions to prove our main theorem.

Step 1: First-Order Approach The first-order condition for a_t in the worker's problem (8) given a contract is

$$0 = \int \int \left[u_a\Big(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t) \Big) \tilde{\pi}_t \left(\eta^t, z^t | z_0, \mathbf{a} \right) + u\Big(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t) \Big) \frac{\partial}{\partial a_t} \tilde{\pi}_t \left(\eta^t, z^t | z_0, \mathbf{a} \right) \right] d\eta^t dz^t$$

Note that this holds for every t and realization of z^t. Thus, one can remove the outer integral to write first-order incentive constraints as

$$\int \left[u_a\Big(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t) \Big) \pi_t \left(\eta_t | z^t, \eta^{t-1}, \mathbf{a} \right) + u\Big(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t) \Big) \frac{\partial}{\partial a_t} \pi_t \left(\eta_t | z^t, \eta^{t-1}, \mathbf{a} \right) \right] d\eta_t = 0$$

Step 2: Recursive Formulation We will work with the relaxed problem and develop a recursive formulation of the firm's problem. Notationally, let the value of some variable X in the period t problem be given by X, the value of X in $t-1$ be given as X_-, and the value of X in $t+1$ be given by X'. Suppressing explicit dependence of the problem on initial productivity z_0 for notational convenience, the recursive formulation of the firm's problem is then (we now drop the history dependence with the assumption that the process for η is a Markov process):

$$\begin{aligned} J(v_-, \eta_-, z_-, t) = \max_{a(\eta_-, z), w(\eta, z), v(\eta, z)} \int \int \Big[& f(\eta, z) - w(\eta, z) \\ & + \beta (1-s) J\left(v(\eta, z), \eta, z, t+1 \right) \Big] \pi\left(\eta | z, \eta_-, a(\eta_-, z) \right) \hat{\pi}(z | z_-) d\eta dz \end{aligned} \tag{26}$$

subject to the following constraints:

$$\omega(\eta, z) = \; u\left(w(\eta, z), a(\eta_-, z) \right) + \beta \left[(1-s) v(\eta, z) + s \int U\left(z' \right) \hat{\pi}\left(z' | z \right) dz' \right] \tag{27}$$

for all η and z realizations,

$$[\lambda]: \quad v_- \leq \int \int \omega(\eta, z) \pi\left(\eta | z, \eta_-, a(\eta_-, z) \right) \hat{\pi}(z | z_-) d\eta dz, \tag{28}$$

and the first-order incentive constraints:

$$\int \left[u_a\Big(w(\eta, z), a(\eta_-, z) \Big) \pi\left(\eta | z, \eta_-, a \right) + u\Big(w(\eta, z), a(\eta_-, z) \Big) \frac{\partial}{\partial a} \pi\left(\eta | z, \eta_-, a \right) \right] d\eta = 0. \tag{29}$$

The firm begins period t knowing the prior realization of shocks z_- and η_- and inherits a utility it must promise to the worker over the remaining life of the contract, which we denote v_-. The firm's flow profits are the expected output $f(\eta, z)$ minus their expected wage payments $w(\eta, z)$. Firms additionally receive a continuation value with probability $1 - s$, which they discount at rate β. The firm maximizes the sum of flow profits and continuation values by choosing the suggested effort and wage functions for every realization of η and z, as well as a function for the next period's promised utility to the worker $v(\eta, z)$, subject to some constraints that we now describe.

The worker's value under the contract given a realization (η, z) is given by $\omega(\eta, z)$, defined in equation (27). It is equal to the worker's flow utility $u(w(\eta, z), a(\eta_-, z)$ plus a continuation value. With probability s, the match dissolves, and the worker receives the value of unemployment. With probability $1 - s$, the match survives, and the worker receives $v(\eta, z)$.

The recursive version of the participation constraint states that the worker's expected value under the contract must be at least the value promised to them v, and is given by equation (28). Note that v_- in the initial period of the match maps to the utility promised to the worker overall $\mathcal{B}$ in the non-recursive formulation of the problem. For periods after the start of the contract, equation (28) may be interpreted as a promise-keeping constraint. Equation (29) is the relaxed incentive constraint described above.

Let the Lagrangian of the recursive problem be defined by $\int\int \mathcal{L}(\cdot) d\eta dz$ for

$$\begin{aligned}\mathcal{L} \equiv & [f(\eta, z) - w(\eta, z; z_0)] \pi(\eta|z, \eta_-, a(\eta_-, z)) \hat{\pi}(z|z_-) \\ & + \beta(1-s) \Big[J(v(\eta, z), \eta, z, t+1) \Big] \pi(\eta|z, \eta_-, a(\eta_-, z)) \hat{\pi}(z|z_-) \\ & - \lambda [v_- - \omega(\eta, z) \pi(\eta|z, \eta_-, a(\eta_-, z)) \hat{\pi}(z|z_-)] \\ & - \gamma(z) \left[u_a\Big(w(\eta, z), a(\eta_-, z)\Big) \pi(\eta|z, \eta_-, a) + u\Big(w(\eta, z), a(\eta_-, z)\Big) \frac{\partial}{\partial a} \pi(\eta|z, \eta_-, a) \right],\end{aligned} \tag{30}$$

where λ is the Lagrange multiplier on the participation constraint and $\gamma(z)$ is the multiplier on the incentive constraint given aggregate productivity z. Again, we suppress dependence on z_0, but the firm's choice variables and the distribution of z and η may all depend on z_0.

Next, we introduce the change of variable with the notation $z_t = \mathbb{E}[z_t|z_0] + \varepsilon_t$, where by definition, ε_t is the cumulative innovation to the process for z between 0 and t and ε_0 is

known to be 0. One can write the Lagrangian as:

$$\begin{aligned}\mathcal{L} &= [f\left(\eta, \mathbb{E}[z|z_0] + \varepsilon\right) - w\left(\eta, \varepsilon\right)]\pi\left(\eta|\varepsilon, \eta_-, a(\eta_-, \varepsilon)\right)\hat{\pi}(\varepsilon|\varepsilon_-) \\ &+ \beta\left(1-s\right)\Big[J\left(v(\eta,\varepsilon), \eta, \varepsilon, t+1\right)\Big]\pi\left(\eta|\varepsilon, \eta_-, a(\eta_-, \varepsilon)\right)\hat{\pi}(\varepsilon|\varepsilon_-) \\ &- \lambda[v_- - \omega(\eta,\varepsilon)\pi\left(\eta|\varepsilon, \eta_-, a(\eta_-, \varepsilon)\right)\hat{\pi}(\varepsilon|\varepsilon_-)] \\ &- \gamma(\varepsilon)\left[u_a\Big(w(\eta,\varepsilon), a(\eta_-,\varepsilon)\Big)\pi\left(\eta|\varepsilon, \eta_-, a\right) + u\Big(w(\eta,\varepsilon), a(\eta_-,\varepsilon)\Big)\frac{\partial}{\partial a}\pi\left(\eta|\varepsilon, \eta_-, a\right)\right]\end{aligned} \tag{31}$$

Step 3: Envelope Theorem We seek to apply Theorem 1 of Marimon and Werner (2021), which relies on the following technical assumptions.

Technical Assumptions:

TA1. The set $\mathcal{X}$ of feasible allocations is convex, and f, u, π, u_a, and π_a are continuous functions of $\{a, w, z_0\}$

TA2. The constraint set $\mathcal{G}(z_0) = \{(\mathbf{w}, \mathbf{a}) \in \mathcal{X} : G(\mathbf{w}, \mathbf{a}; z_0) \leq 0\}$ is compact for every $z \in Z$, a neighborhood of z_0, and there exists a contract $(\mathbf{w}, \mathbf{a})$ such that the participation constraint (28) is slack.

TA3. The set of optimal contracts is non-empty.

We argue these conditions apply in our setting. $\mathcal{X}$ is convex as the product of segments. Under Assumption 1, $\mathcal{X}$ is compact. Then the constraint set $\mathcal{G}(z_0)$ is a closed subset of a compact and so is compact. What's more, there exists a contract such that the participation constraint is slack since, for every z_0 and promised utility v_-, there exists a feasible continuation value and effort $\omega(\eta, z), a(\eta_-, z)$ that yield strictly higher utility than v_-: that is inequality (28) is strict. Finally, since from Assumption 1, $\mathcal{X}$ is compact and non-empty, the set of optimizers of our continuous objective (i.e., the set of optimal contracts) is non-empty.[17]

One can now apply the envelope theorem of Marimon and Werner (2021) to argue that the derivative of the value function with respect to all variables the firm chooses and costates – a^*, w^*, v^*, λ^*, and γ^* – sum to zero. Therefore, differentiating the Lagrangian (31) with

[17]This envelope theorem is better suited for our purposes than Corollary 5 of Milgrom and Segal (2002) since it does not require compactness assumptions on the support of the shocks.

respect to z_0 and substituting in for $\omega(\eta, \varepsilon)$ yields the right-hand derivative:

$$\frac{\partial J(v_-, \eta_-, z_-, t)}{\partial z_0^+} = \sup_{(\mathbf{w}^*, \mathbf{a}^*) \in \mathbf{\Gamma}^*(z_0)} \int\int \frac{\partial}{\partial z_0}[f(\eta, \mathbb{E}[z|z_0] + \varepsilon)] \pi(\eta|\varepsilon, \eta_-, a^*(\eta_-, \varepsilon)) \hat{\pi}(\varepsilon|\varepsilon_-) d\eta d\varepsilon \tag{32}$$

$$+ \beta(1-s) \int\int \frac{\partial}{\partial z_0^+} \Big[J(v^*(\eta, \varepsilon), \eta, \varepsilon, t+1) \Big] \pi(\eta|\varepsilon, \eta_-, a^*) \hat{\pi}(\varepsilon|\varepsilon_-) d\eta d\varepsilon$$

$$+ \beta s \lambda^*(\eta_-, z_-) \int\int \frac{\partial}{\partial z_0} U(\mathbb{E}[z'|z_0] + \varepsilon') \hat{\pi}(\varepsilon'|\varepsilon) \hat{\pi}(\varepsilon|\varepsilon_-) d\varepsilon' d\varepsilon.$$

This is a refinement of a recursive version of equation (22): the first-order impact of aggregate productivity on the value of a filled job is given by the sum of the direct effect on the firm's flow and continuation values, plus the direct effect on the constraints. Two terms are missing from the fuller decomposition in equation (22). First, the "B-term" features no direct effect on incentive constraints. This arises from the assumption that the distribution of η and ε do not directly depend on z_0. Second, the "C-term" – the indirect effect on firm value that arises from changes in the contracted wages or effort – does not appear because we have applied the envelope theorem of Marimon and Werner (2021).

We can write explicitly the sequence of participation constraints from time 0 as:

$$\begin{aligned} \lambda_-(z_0): &\qquad \mathcal{E}(z_0) \leq v \\ \lambda_{t-1}(\eta^{t-1}, z^{t-1}): &\qquad v_{t-1}(\eta^{t-1}, z^{t-1}) \leq \int\int \omega(\eta^t, z^t) \pi(\eta_t|z^t, \eta^{t-1}, a(\eta^{t-1}, z^t)) \hat{\pi}(z_t|z^{t-1}) d\eta_t dz_t, \ \forall t \geq 1. \end{aligned}$$

The corresponding sequential participation constraints are:

$$[\lambda_-(z_0)]: \quad \mathcal{B}(z_0) \leq \sum_{t=0}^{\infty} (\beta(1-s))^t \Bigg[\int\int u\Big(w_t(\eta^t, \varepsilon^t; z_0), a_t(\eta^{t-1}, \varepsilon^t; z_0) \Big) \tilde{\pi}_t\left(\eta^t, \varepsilon^t | \mathbf{a}(z_0)\right) d\eta^t d\varepsilon^t$$

$$+ \beta s \int U(\mathbb{E}[z_{t+1}|z_0] + \varepsilon_{t+1}) \hat{\pi}_{t+1}\left(\varepsilon^{t+1}\right) d\varepsilon^{t+1} \Bigg]$$

$$[\lambda_\tau(\eta^\tau, \varepsilon^\tau; z_0)]: \quad \sum_{t=\tau+1}^{\infty} (\beta(1-s))^{t-\tau-1} \Bigg[\int\int u\Big(w_t(\eta^t, \varepsilon^t; z_0), a_t(\eta^{t-1}, \varepsilon^t; z_0) \Big) \tilde{\pi}_t\left(\eta^t, \varepsilon^t | \mathbf{a}(z_0)\right) d\eta^t d\varepsilon^t$$

$$+ \beta s \int U(\mathbb{E}[z_{t+1}|z_0] + \varepsilon_{t+1}) \hat{\pi}_{t+1}\left(\varepsilon^{t+1}\right) d\varepsilon^{t+1} \Bigg] \geq v_\tau(\eta^\tau, \varepsilon^\tau; z_0), \quad \forall \tau = 0, \ldots, +\infty \tag{33}$$

Now we apply the envelope theorem to the problem recursively, replacing $\mathcal{E}$ with its equilib-

rium value $\mathcal{B}$ to obtain

$$\frac{\partial J}{\partial z_0} = \sup_{\{w^*,a^*\}\in\mathbf{\Gamma}^*(z_0)} \sum_{t=0}^{+\infty} \Big[\int\int (\beta(1-s))^t \frac{\partial f\left(\mathbb{E}[z_t|z_0]+\varepsilon_t,\eta_t\right)}{\partial z_0} \tilde{\pi}_t\left(\eta^t,\varepsilon^t|\mathbf{a}(z_0)\right) d\eta^t d\varepsilon^t \quad (34)$$

$$- \lambda_-(z_0)\Big[\frac{\partial \mathcal{B}(z_0)}{\partial z_0} - \beta s \sum_{t=1}^{+\infty} \int\int (\beta(1-s))^{t-1} \frac{\partial U\left(\mathbb{E}[z_t|z_0]+\varepsilon_t\right)}{\partial z_0} \hat{\pi}\left(\varepsilon^t\right) d\varepsilon^t\Big]$$

$$+ \sum_{\tau=0}^{\infty} \int\int \lambda_\tau(\eta^\tau,\varepsilon^\tau;z_0)\beta s\Big[\sum_{t=\tau+2}^{+\infty} \int\int (\beta(1-s))^{t-\tau-2} \frac{\partial U\left(\mathbb{E}[z_t|z_0]+\varepsilon_t\right)}{\partial z_0} \hat{\pi}\left(\varepsilon^t|\varepsilon^\tau\right) d\varepsilon^t\Big] \times$$

$$\tilde{\pi}_\tau\left(\eta^\tau,\varepsilon^\tau|\mathbf{a}(z_0)\right) d\eta^\tau d\varepsilon^\tau$$

When the outside option of the worker is acyclical and TIOLI, we have:

$$\frac{\partial J(z_0)}{\partial z_0} = \sup_{\{w^*,a^*\}\in\mathbf{\Gamma}^*(z_0)} \sum_{t=0}^{\infty} (\beta(1-s))^t \left[\int\int \frac{\partial f\left(\mathbb{E}[z_t|z_0]+\varepsilon_t,\eta_t\right)}{\partial z_0} \tilde{\pi}_t\left(\eta^t,\varepsilon^t|\mathbf{a}^*(z_0)\right) d\eta^t d\varepsilon^t\right]. \quad (35)$$

The preceding equation is the same as equation (18) from the Appendix, the previous application of the envelope theorem. Therefore, the same manipulations performed at the end of the Appendix under Assumption 2 yield the market tightness dynamics in Theorem 1. □

A.2 Non-incentive reasons for wage cyclicality

To derive Theorem 1, we assumed that workers' promised utility is acyclical. This assumption provides the clean benchmark in which all wage cyclicality in the model arises due to incentives. We now relax this assumption to consider non-incentive reasons for wage cyclicality. To do so, we assume that the ex ante utility promised to the worker $\mathcal{E}(z_0)$ is given by a reduced-form function $\mathcal{B}(z_0)$: firms commit to providing workers with utility $\mathcal{B}(z_0)$ over the contract.[18] Common bargaining protocols in the labor search literature implicitly define different functions for $\mathcal{B}(z_0)$. For instance, if firms make take-it-or-leave-it offers to workers, the value of employment is equal to the value of nonemployment: $\mathcal{B}(z_0) = \sum_t \beta^t \mathbb{E}[\xi(z_t)|z_0]$, where $\xi(z_t)$ is the flow value of unemployment. This nests the case in which unemployment benefits or the opportunity cost of unemployment is procyclical (Hagedorn et al., 2013; Chodorow-Reich and Karabarbounis, 2016; Mitman and Rabinovich, 2019). Nash bargaining also implicitly defines an increasing function for $\mathcal{B}(z_0)$, as we prove below. Our formulation

[18] See Blanchard and Galí (2010) and Michaillat (2012) for an implementation of this approach in search models without effort.

also evokes a notion of unemployment as a "worker discipline device" (Shapiro and Stiglitz, 1984): if the value of employment is low because unemployment at present or in the future is costly, workers will offer higher effort at lower wages.

The reduced-form approach has two advantages. First, our conclusions about the role of bargaining and outside options will be robust to a specific protocol. Second, we can tractably incorporate bargaining into dynamic incentive contract models. The disadvantage of the approach is that $\mathcal{B}(z_0)$ is a reduced-form object, which is not invariant to changes in the primitives of the environment.

Let $\mathcal{W}(z_0)$ denote the present discounted value of wage payments under the optimal wage contract:

$$\mathcal{W}(z_0) \equiv \sum_{t=0}^{\infty} (\beta(1-s))^t \int w_t^*(\eta^t, z^t) \tilde{\pi}_t(\eta^t, z^t | z_0, \mathbf{a}^*(z_0)) d\eta^t dz^t.$$

Likewise, Let $\mathcal{Y}(\mathbf{a}^*(z_0), z_0)$ denote the expected present discounted value of output from a match given the optimal effort function $\mathbf{a}^*(z_0)$:

$$\mathcal{Y}(\mathbf{a}^*(z_0), z_0) \equiv \sum_{t=0}^{\infty} (\beta(1-s))^t \int \int f(z_t, \eta_t) \tilde{\pi}_t(\eta^t, z^t | z_0, \mathbf{a}^*(z_0)) d\eta^t dz^t.$$

Denote as $\tilde{\mathcal{B}}(z)$ the ex ante utility promised to the worker at the start of the contract, net of her continuation value with regard to separation into unemployment: $\tilde{\mathcal{B}}(z) \equiv \mathcal{B}(z) - \sum_{t=0}^{\infty} (\beta(1-s))^t \beta s \mathbb{E}[U(z_{t+1}) | z_0 = z]$. Fluctuations in $\tilde{\mathcal{B}}(z)$ capture variations in workers' ex ante utility due to, for instance, bargaining power or changes in worker outside options. Finally, define *non-incentive wage cyclicality (NWC)* as wage movements over and above output movements driven by effort fluctuations

$$\frac{\partial \mathcal{W}^{\text{non-incentive}}(z_0)}{\partial z_0} \equiv \frac{d\mathcal{W}(z_0)}{dz_0} - \partial_{\mathbf{a}} \mathcal{Y}(\mathbf{a}^*(z_0); z_0) \frac{d\mathbf{a}^*}{dz_0}. \tag{36}$$

Note that Theorem 1 states that non-incentive wage cyclicality is zero under Assumption 1 when $\tilde{B}(z_0)$ is acyclical; this is the sense in which $\frac{\partial \mathcal{W}^{\text{non-incentive}}(z_0)}{\partial z_0}$ is not related to incentives.

Characterizing the response of market tightness to productivity becomes more difficult when $\tilde{\mathcal{B}}(z)$ is nonconstant, as the set of contracts satisfying the participation constraint now moves directly with z_0. We therefore must maintain Assumption 3 in Appendix Section A.1.5 so that the FoA offers a valid solution to the contracting problem. This then yields the following proposition:

Proposition 4. *Assume that Assumption 1 hold. The impulse response of market tightness*

to aggregate shocks in the flexible incentive pay economy is

$$d\ln\theta_0 = \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \frac{\partial}{\partial \ln z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\left(z_0\right)\right] - \frac{\partial \mathcal{W}^{non\text{-}incentive}(z_0)}{\partial \ln z_0}}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \mathbb{E}\left[f(z_t,\eta_t) - w_t^*(z_0)|z_0,\mathbf{a}^*\left(z_0\right)\right]} d\ln z_0, \tag{37}$$

where $\partial W^{non-incentive}(z_0)/\partial \ln z_0$ *is defined in equation* (36)*. Moreover, if Assumption* 3 *additionally holds, then*

$$\frac{\partial \mathcal{W}^{non\text{-}incentive}\left(z_0\right)}{\partial \ln z_0} > 0 \quad \Longleftrightarrow \quad \tilde{\mathcal{B}}'\left(z_0\right) > 0;$$

i.e., Non-Incentive Wage Cyclicality is positive if and only if ex ante utility is procyclical.

Equation (37) shows that Non-Incentive Wage Cyclicality (NWC) appears alongside the direct productivity effect. Therefore, when NWC is high, the impulse response of tightness is small – wage cyclicality arising for non-incentive reasons dampen employment responses to shocks as in the standard model. The proposition also shows that what we have defined as NWC corresponds to the cyclicality of workers' ex ante utility—NWC is positive if and only if the utility promised to workers at the start of a contract is procyclical.

A.2.1 Proof of Proposition 4

First, we derive equation (37). Differentiating the value of a filled job $J(z_0)$ with respect to z_0 yields the following expression:

$$\frac{dJ\left(z_0\right)}{dz_0} = \frac{\partial \mathcal{Y}\left(\mathbf{a}^*\left(z_0\right);z_0\right)}{\partial z_0} - \left(\frac{d\mathcal{W}\left(z_0\right)}{dz_0} - \partial_{\mathbf{a}}\mathcal{Y}\left(\mathbf{a}^*\left(z_0\right);z_0\right)\frac{d\mathbf{a}^*}{dz_0}\right). \tag{38}$$

and from equation (36) we have

$$\frac{\partial \mathcal{W}^{\text{non-incentive}}\left(z_0\right)}{\partial z_0} \equiv \frac{d\mathcal{W}\left(z_0\right)}{dz_0} - \partial_{\mathbf{a}}\mathcal{Y}\left(\mathbf{a}^*\left(z_0\right);z_0\right)\frac{d\mathbf{a}^*}{dz_0}.$$

The preceding two equations imply

$$\begin{aligned}\frac{dJ\left(z_0\right)}{dz_0} &= \frac{\partial \mathcal{Y}\left(\mathbf{a}^*\left(z_0\right);z_0\right)}{\partial z_0} - \frac{\partial \mathcal{W}^{\text{non-incentive}}\left(z_0\right)}{\partial z_0} \\ &= \sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \frac{\partial}{\partial z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right] - \frac{\partial \mathcal{W}^{\text{non-incentive}}\left(z_0\right)}{\partial z_0}\end{aligned}$$

$$\implies \frac{d\ln J\left(z_0\right)}{d\ln z_0} = \frac{z_0\left(\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \frac{\partial}{\partial z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right] - \frac{\partial \mathcal{W}^{\text{non-incentive}}(z_0)}{\partial z_0}\right)}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \mathbb{E}\left[f(z_t,\eta_t) - w_t^*|z_0,\mathbf{a}^*\right]}$$
$$= \frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \frac{\partial}{\partial \ln z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right] - \frac{\partial \mathcal{W}^{\text{non-incentive}}(z_0)}{\partial \ln z_0}}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \mathbb{E}\left[f(z_t,\eta_t) - w_t^*|z_0,\mathbf{a}^*\right]},$$

which is equation (37).

We now prove the "moreover" statement that NWC is positive if and only if the promised utility is procyclical. The derivation makes use of equation (34) derived in Section A.1.5. Suppose the optimal contract features optimal choices for wages and effort, which are in the interior of Φ, which is true under the Inada conditions made in Assumption 3. Then the additional Lagrangian terms after time zero are non-binding, and equation (34) becomes

$$\frac{\partial J}{\partial z_0^+} = \sup_{\{w^*,a^*\}\in\mathbf{\Gamma}^*(z_0)} \sum_{t=0}^{\infty}\Big[\int\int \left(\beta\left(1-s\right)\right)^t \frac{\partial f\left(\mathbb{E}[z_t|z_0]+\varepsilon_t,\eta_t\right)}{\partial z_0}\tilde{\pi}_t\left(\eta^t,\varepsilon^t|\mathbf{a}^*\left(z_0\right)\right)d\eta^t d\varepsilon^t \quad (39)$$
$$-\lambda_{PC}^*(z_0)\Big[\frac{\partial \mathcal{B}(z_0)}{\partial z_0} - \beta s\sum_{t=1}^{+\infty}\int\int\left(\beta\left(1-s\right)\right)^{t-1}\frac{\partial U\left(\mathbb{E}[z_t|z_0]+\varepsilon_t\right)}{\partial z_0}\hat{\pi}\left(\varepsilon^t\right)d\varepsilon^t\Big]$$

Under Assumption 1, $\mathcal{X}$ is compact, and so the supremum is achieved at a contract $\{w^*,a^*\}\in\mathbf{\Gamma}^*(z_0)$. Evaluated at that optimum and comparing equation (37) with equation (39) yields

$$\lambda_{PC}^*(z_0)\left[\frac{\partial\tilde{\mathcal{B}}(z_0)}{\partial z_0}\right] = \frac{\partial \mathcal{W}^{non-incentive}(z_0)}{\partial z_0}.$$

Finally, $\lambda_{PC}^*(z_0)>0$ because the participation constraint must bind on the optimal contract. It immediately follows that $\partial\mathcal{W}^{non-incentive}(z_0)/\partial z_0 > 0$ if and only if $\partial\tilde{\mathcal{B}}(z_0)/\partial z_0 > 0$. □

A.2.2 Implicit Definition of $\mathcal{B}(z_0)$ under Nash Bargaining

This subsection shows that Nash bargaining implicitly defines a functional form for $\mathcal{B}(z_0)$. Suppose that the firm and worker engage in generalized Nash bargaining over the surplus of the match, and φ is the firm's bargaining power. Firms and workers take as given the utility that workers would receive were they to match with another firm next period $\mathcal{E}(z)$. Promised utility $\mathcal{B}\left(z_0\right)$ is implicitly defined by

$$\mathcal{B}\left(z_0\right) = \arg\max_{\overline{\mathcal{B}}} J\left(z_0,\overline{\mathcal{B}}\right)^{\varphi}\left(\overline{\mathcal{B}}-U\left(z_0\right)\right)^{1-\varphi}.$$

Here, as in the main text, $U\left(z_0\right)$ is the value of unemployment at time 0. $J\left(z_0,\overline{\mathcal{B}}\right)$ is defined by equations (7)-(9) in the main text, replacing $\mathcal{E}\left(z_0\right)$ with $\overline{\mathcal{B}}$ in equation (9). Therefore,

$\mathcal{B}(z_0)$ is the solution to the standard Nash bargaining problem, albeit in an environment with dynamic incentive pay. The solution is

$$\varphi \frac{\frac{\partial J(z_0, \mathcal{B}(z_0))}{\partial \mathcal{B}}}{J(z_0, \mathcal{B}(z_0))} + \frac{(1-\varphi)}{\mathcal{B}(z_0) - U(z_0)} = 0. \tag{40}$$

Note that when a firm and worker bargain, they take the expected outcome of a worker bargaining with other firms as given. Thus $U(z_0)$ does not itself depend directly on $\mathcal{B}(z_0)$. Therefore, equation (40) implicitly characterizes a particular choice for $\mathcal{B}(z_0)$ from the Nash bargain.

A.3 Endogenous Separations and Limited Worker Commitment

This section introduces efficient endogenous separations and limited worker commitment into the baseline environment. To economize, we only discuss the parts of the model that change due to efficient separations or limited worker commitment; otherwise, the model is the same as the flexible incentive pay economy of the main text.

A.3.1 Economic Environment

Labor Market As in the baseline model of the main text, a large measure of risk-neutral firms match with unemployed workers according to a frictional matching technology. Fluctuations are driven by aggregate productivity z_t, and there is free entry to vacancy posting at a constant flow cost κ, as in the main text.

At the end of period $t-1$ an endogenous fraction s_t of workers separate from employment and enter unemployment. The unemployed search for new jobs, so u_t evolves as

$$u_t = u_{t-1} + s_t(1 - u_{t-1}) - \phi(\theta_{t-1})u_{t-1}(1 - s_t). \tag{41}$$

Preferences and Consumption Workers' preferences are identical to the model of the main text.

Firms and Wage Setting The value of a firm of posting a vacancy at time 0 is

$$\Pi_0 = q(\theta_0)\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(\prod_{j=1}^{t}(1 - s_j)\right)(f(z_t, \eta_t) - w_t)\right] - \kappa, \tag{42}$$

where $\mathbb{E}$ conditions on the firm's information set at time 0 prior to meeting a worker. A vacancy is filled with probability $q(\theta)$. If a firm meets a worker, its value is the expected

present value of the difference between production and wage payments, discounted by the firm's discount factor β as well as separation risk. Here, $\prod_{j=1}^{t}(1-s_j)$ is the endogenous probability that a match survives until period t, which cumulates the probability $1-s_j$ that a match survives period j. We entertain two possibilities for wage setting.

Flexible Incentive Pay Economy As in the main text, the firm observes realizations of both z_t and η_t, but does not observe worker's effort. When a firm and a worker meet, the firm offers the worker a contract to incentivize effort and maximize firm value. The innovation of this section is that the firm now has the additional option to vary the probability that the match separates in each date and state. For instance, if the expected present value of profits has turned negative, the firm may choose to terminate the contract. Thus the contract may be summarized by functions $w_t(\eta^t, z^t) \in [\underline{w}, \bar{w}]$, $a_t(\eta^{t-1}, z^t) \in [\underline{a}, \bar{a}]$ and a separation probability $s_t(\eta^t, z^t) \in [0,1]$ for all t and all realizations of η^t and z^t. Let $(\mathbf{w}, \mathbf{a}, \mathbf{s})$ denote a contract, with $\mathbf{w} \equiv \{w_t(\eta^t, z^t)\}_{t=0,\eta^t,z^t}^{\infty}$, $\mathbf{a} \equiv \{a_t(\eta^{t-1}, z^t)\}_{t=0,\eta^{t-1},z^t}^{\infty}$ and $\mathbf{s} \equiv \{s_t(\eta^t, z^t)\}_{t=0,\eta^t,z^t}^{\infty}$. Let $\mathcal{X}$ denote the space of possible contracts.

Value of a Filled Vacancy. Under the contract $(\mathbf{w}, \mathbf{a}, \mathbf{s})$, and initial productivity z_0, the firm's expected present value of profits from posting a vacancy is

$$V(\mathbf{w}, \mathbf{a}, \mathbf{s}; z_0) = \sum_{t=0}^{\infty} \int \int \beta^t \mathcal{S}_t\left(\eta^t, z^t\right) \left(f(z_t, \eta_t) - w_t(\eta^t, z^t)\right) \tilde{\pi}_t\left(\eta^t, z^t | z_0, \mathbf{a}\right) d\eta^t dz^t, \quad (43)$$

where $\mathcal{S}_t(\eta^t, z^t) \equiv \prod_{j=1}^{t}\left(1 - s_{t-j}\left(\eta^{t-j}, z^{t-j}\right)\right)$ is the probability that a match survives the sequence η^t, z^t; and $\tilde{\pi}_t(\eta^t, z^t | z_0, \mathbf{a})$ is the probability of observing a realization of η^t and z^t given the initial z_0 and the contracted effort function $\mathbf{a}$, as in the main text. The risk-neutral firm discounts period t profits by the economy-wide discount rate β^t and the probability $\mathcal{S}_t(\eta^t, z^t)$ that the match survives t periods.

The contract maximizes the value of a filled vacancy

$$J(z_0) = \max_{\{w_t(\eta^t,z^t), a_t(\eta^{t-1},z^t), s_t(\eta^t,z^t)\}_{t=0,\eta^t,z^t \in \mathcal{X}}^{\infty}} V(\mathbf{w}, \mathbf{a}, \mathbf{s}; z_0) \quad (44)$$

subject to the incentive compatibility and participation constraints described below, as well as a new set of constraints that captures limited commitment by the worker.

Incentive Constraints. The incentive compatibility condition is similar to the main

text but now accounts for endogenous separation risk

$$[\mathbf{IC}] : \mathbf{a} \in \underset{\{\tilde{a}_t(\eta^{t-1},z^t)\}_{t=0,\eta^t,z^t}^{\infty}}{\operatorname{argmax}} \sum_{t=0}^{\infty}\Bigg[\int\int \beta^t \mathcal{S}_t\left(\eta^t,z^t\right)\Bigg[u\Bigg(w_t(\eta^t,z^t),\tilde{a}_t(\eta^{t-1},z^t)\Bigg)-\Psi(s\left(\eta^t,z^t\right)) \\ +\beta s\left(\eta^t,z^t\right)\int U\left(z_{t+1}\right)\pi\left(z_{t+1}|z_t\right)dz^{t+1}\Bigg]\tilde{\pi}_t\left(\eta^t,z^t|z_0,\tilde{\mathbf{a}}\right)d\eta^t dz^t, \tag{45}$$

where $\Psi(s_j)$ represents a convex utility cost to the worker of searching for a new job.

Participation Constraint. Likewise, the participation constraint must also account for separation risk and becomes

$$[\mathbf{PC}] : \sum_{t=0}^{\infty}\Bigg[\int\int \beta^t \mathcal{S}_t\left(\eta^t,z^t\right)\Bigg[u\Bigg(w_t(\eta^t,z^t),\tilde{a}_t(\eta^{t-1},z^t)\Bigg)-\Psi(s\left(\eta^t,z^t\right)) \\ +\beta s\left(\eta^t,z^t\right)\int U\left(z_{t+1}\right)\pi\left(z_{t+1}|z_t\right)dz_{t+1}\Bigg]\tilde{\pi}_t\left(\eta^t,z^t|z_0,\tilde{\mathbf{a}}\right)d\eta^t dz^t\Bigg] \geq \mathcal{E}\left(z_0\right). \tag{46}$$

Limited Commitment. Limited commitment and endogenous separations mean that after any history η^τ, z^τ the worker must rather stay in the match than separate, leading to a constraint that for each η^τ, z^τ:

$$[\mathbf{ES}] : \sum_{t=\tau}^{\infty}\mathbb{E}\Bigg[\beta^{t-\tau}\mathcal{S}_t^\tau\left(\eta^t,z^t\right)\Bigg[u\Bigg(w_t(\eta^t,z^t),a_t(\eta^{t-1},z^t)\Bigg)-\Psi(s\left(\eta^t,z^t\right)) \\ +\beta s\left(\eta^t,z^t\right)\mathbb{E}[U\left(z_{t+1}\right)|z_t]\Bigg]\Bigg|\eta^\tau,z^\tau\Bigg] \geq U\left(z_\tau\right), \tag{47}$$

where $\mathcal{S}_t^\tau$ is the survival probability after time τ, $\mathcal{S}_t^\tau\left(\eta^t,z^t\right) \equiv \prod_{j=\tau+1}^{t}\left(1-s_{t+\tau+1-j}\left(\eta^{t+\tau+1-j},z^{t+\tau+1-j}\right)\right)$.

Bargaining and ex ante utility. To close the flexible incentive pay economy, we again assume ex ante utility $\mathcal{E}(z_0)$ is given by a reduced-form "bargaining schedule" $\mathcal{B}(z_0)$ as in Section A.2.

Rigid Wage Economy The rigid wage economy is identical to the rigid wage economy of the main text, including the assumption of an exogenous separation rate s.

Equilibrium Given initial unemployment u_0 and a stochastic process $\{z_t,\eta_t\}_{t=0}^{\infty}$, an equilibrium is a collection of stochastic processes $\{\theta_t,u_t\}_{t=0}^{\infty}$ and functions $J(z),U(z),\mathcal{E}(z)$, and $(\mathbf{w},\mathbf{a},\mathbf{s})$ such that for all firms: (i) θ_t satisfies the free entry condition so that Π_t, given in equation (42), is equal to 0 for all t; (ii) u_t satisfies the law of motion for unemployment

(41); (iii) wage, effort, and separation functions $(\mathbf{w}, \mathbf{a}, \mathbf{s})$ satisfy the flexible incentive pay economy equations (44)-(47), or $w_t = \bar{w}$, $a_t = \bar{a}$ and $s_t = s$ in the rigid wage economy; (iv) the value of unemployment $U(z)$ is defined in the same way as the main text; (v) the value of employment is defined the same way as the main text for the in the rigid wage economy, or $\mathcal{E}(z) = \mathcal{B}(z)$ in the flexible incentive pay economy; and (vi) the value of a filled vacancy $J(z)$ is given by (44) in the flexible incentive pay economy or the same way as the main text for the rigid wage economy.

A.3.2 Equivalence of Rigid and Incentive Pay with Endogenous Separations

This subsection shows that, without bargaining power or fluctuations in outside options, the first-order response of market tightness is the same in the rigid wage economy and the flexible incentive pay economy with endogenous separations. For simplicity, we make the same assumptions as the main text, such as studying impulse responses in a neighborhood of the non-stochastic steady state.

Proposition 5. *Maintain Assumptions 1 and 2, and assume that the firm makes take it or leave it offers to workers, the flow value of unemployment is constant, and the optimal contract is unique. Then, the impact elasticity of market tightness to shocks to z_0 is*

$$\frac{d \ln \theta_0}{d \ln z_0} = \frac{1}{\bar{\nu}} \frac{1}{1 - \Lambda} \tag{48}$$

where Λ is the steady state labor share defined as

$$\Lambda \equiv \frac{\sum_{t=0}^{\infty} \mathbb{E} \beta^t \prod_{j=1}^{t} \left(1 - s_j^*\right) w_t^*}{\sum_{t=0}^{\infty} \mathbb{E} \beta^t \prod_{j=1}^{t} \left(1 - s_j^*\right) f\left(z^{SS}, \eta_t\right)},$$

s_j^* and w_t^* denote choices of separations and wages along the optimal contract, where the expectation $\mathbb{E}$ is evaluated along the optimal contract, and z^{SS} is the value of z_t at the aggregate steady state.

This theorem shows that the flexible incentive pay economy with endogenous separations has an equivalent response of tightness on impact to the rigid wage economy of the main text. Note that the dynamics of the rigid wage economy are still given by equation (15). Therefore, incentive wage cyclicality does not affect the impact response of tightness with endogenous separations as long as the flexible incentive pay economy and the rigid wage economy are calibrated to the same steady state labor share. In the incentive pay economy

with endogenous separations, the labor share depends on the optimal choice of separation rates, as well as the factors from the model of the main text, such as wages and effort.

We stress that this result leads to equivalence for impact elasticities (Pissarides, 2009). In general, the response of unemployment to productivity shocks after impact will be different in the rigid wage and flexible incentive pay economies because endogenous separations lead to additional dynamics of unemployment after the impact of the shock.

Intuitively, in the model with efficient endogenous separations, separations are an additional choice which the firm can optimize over. However, changes in the optimal separation choice after TFP shocks have no first-order effect on profits—just as neither changes in optimally chosen effort nor wages affect profits. Likewise, the optimal contract ensures that workers do not wish to leave the match. Reoptimizations by the worker as aggregate conditions change do not affect profits. This logic is again due to the envelope theorem.

A.3.3 Proof of Proposition 5

The free entry condition in the flexible incentive pay economy is

$$\frac{\kappa}{q(\theta)} = J(z_0),$$

where $J(z_0)$ is defined in equation (44). Taking derivatives and rearranging implies

$$\begin{aligned}\frac{d\ln\theta_0}{d\ln z_0} &= \frac{1}{\nu_0}\frac{d\ln J(z_0)}{d\ln z_0} \\ &= \frac{1}{\nu_0}\frac{z_0}{J(z_0)}\frac{dJ(z_0)}{dz_0}.\end{aligned} \tag{49}$$

With Ψ convex, the optimal separation rates s_j^* will be interior. Under the assumptions of the proposition, z_0 does not enter either the incentive constraints, the participation constraint, or the limited commitment constraints directly. Therefore we have

$$\begin{aligned}\frac{dJ(z_0)}{dz_0} = \frac{\partial J(z_0)}{\partial z_0} &= \frac{\partial}{\partial z_0}\sum_{t=0}^{\infty}\mathbb{E}\beta^t\prod_{j=1}^{t}\left(1-s_j^*\right)\left(f(z_t,\eta_t)-w_t\right) \\ &= \sum_{t=0}^{\infty}\mathbb{E}\beta^t\prod_{j=1}^{t}\left(1-s_j^*\right)\left(f_z(z_t,\eta_t)\right),\end{aligned} \tag{50}$$

where the first equality invokes the envelope theorem, using the same argument as the main Appendix and also using our assumption of a unique optimal contract in order to dispense with a sup operator; the second equality rewrites the definition of profits from equation (43)

using the notation from the theorem and exploits that terms involving the participation, incentive, or limited commitment constraints vanish because z_0 does not enter them directly; and the final equality uses that z_t is a random walk.

Substituting in equations (49) and (50) implies

$$\begin{aligned}\frac{d\ln\theta_0}{d\ln z_0} &= \frac{1}{\nu_0}\frac{z_0\sum_{t=0}^{\infty}\mathbb{E}\beta^t\Pi_{j=1}^{t}\left(1-s_j^*\right)\left(f_z(z_t,\eta_t)\right)}{\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^{t}\left(1-s_j^*\right)\left(f\left(z_t,\eta_t\right)-w_t^*\right)\right]}\\ &= \frac{1}{\nu_0}\frac{z^{SS}\sum_{t=0}^{\infty}\mathbb{E}\beta^t\Pi_{j=1}^{t}\left(1-s_j^*\right)\left(f_z(z^{SS},\eta_t)\right)}{\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^{t}\left(1-s_j^*\right)\left(f\left(z^{SS},\eta_t\right)-w_t^*\right)\right]}\\ &= \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}\mathbb{E}\beta^t\Pi_{j=1}^{t}\left(1-s_j^*\right)\left(f(z^{SS},\eta_t)\right)}{\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^{t}\left(1-s_j^*\right)\left(f\left(z^{SS},\eta_t\right)-w_t^*\right)\right]}\\ &= \frac{1}{\nu_0}\frac{1}{1-\frac{\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^{t}\left(1-s_j^*\right)w_t^*\right]}{\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^{t}\left(1-s_j^*\right)(f(z^{SS},\eta_t))\right]}},\end{aligned}$$

where we use the assumption of an aggregate steady state at $z = z^{SS}$. □

B Numerical Appendix

This appendix studies a quantitative version of our flexible incentive pay economy. Appendix Section B.1 calibrates the model. Appendix Section B.2 uses the model for three purposes. First, we try to disentangle the share of total wage cyclicality that arises for non-incentive versus incentive reasons. Second, we show that thanks in part to incentive pay, our model can quantitatively match unemployment and productivity dynamics. Third, we numerically establish that the equivalence between rigid wage and incentive pay economies holds globally, for large productivity shocks. Appendix Section B.3 includes additional details on our solution algorithm.

B.1 Numerical Analysis: Calibration

Parameterizing the model. We parameterize the production function, utility function, ex ante utility, and information structure of the flexible incentive pay model of Section 1 following Edmans et al. (2012), allowing for cyclical ex ante utility as in section A.2.

Production function. The firm's production function is $y = z(a + \eta)$. Idiosyncratic profit shocks η are assumed to be i.i.d. over time and across individuals and normally distributed with zero mean and standard deviation σ_η. σ_η determines the extent to which

firms can infer workers' effort, which is key for incentive pay.[19]

Preferences. We assume that workers' utility function is given by $u(c, a) \equiv \ln c - \frac{a^{1+1/\epsilon}}{1+1/\epsilon}$. ϵ governs the Frisch elasticity of effort, which determines how costly effort is to workers.

Information structure. We assume that η is not observed to the firm; it therefore plays the role of $\tilde{\eta}$ in the static limit studied in section 2.1 as firms can infer $a + \eta$ each period when they observe y. We make the "effort after noise" assumption as in Edmans et al. (2012): workers observe the idiosyncratic profit shock η before making an effort choice. Thus, there is an incentive compatibility constraint for each value of η. Following Edmans et al. (2012), we assume that a unique level of effort $a(z^t)$ is implemented regardless of the idiosyncratic shock η.[20] However, effort varies with the history of aggregate productivity z^t.

Ex ante utility. We assume that firms make take-it-or-leave-it offers to workers who face procyclical unemployment benefits. Workers' flow unemployment benefits take the form $b(z) = \gamma z^{\chi}$. Here, γ specifies the level of unemployment benefits when $z = 1$, while χ determines the elasticity of unemployment benefits to aggregate productivity. This specification is a log-linear approximation of any differentiable $\mathcal{B}(z)$ function, including models in which workers and firms bargain over ex ante utility at the start of the contract. However, this specification is numerically tractable in that it abstracts from complications of bargaining and ensures that unemployed workers' value is given by the present discounted value of expected unemployment benefits. The parameter χ represents all the reasons why the utility promised to workers at the beginning of an employment relationship may be cyclical—such as fluctuations in either the worker's outside option (changes in the value of unemployment) or her inside option (bargained utility)—and determines NWC.

We now characterize the behavior of wages under the optimal contract following Edmans et al. (2012), which will be useful for motivating our calibration strategy.

Proposition 6. *The earnings schedule in the optimal contract satisfies the following difference equation (given initial productivity z_0):*

$$\ln(w_t(\eta^t, z^t)) = \ln(w_{t-1}(\eta^{t-1}, z^{t-1})) + \psi h'(a_t)\eta_t - \frac{1}{2}(\psi h'(a_t)\sigma_\eta)^2, \tag{51}$$

where $\psi = 1 - \beta(1 - s)$ and $w_{-1}(z_0)$, which initializes the difference equation, is defined in the proof of the proposition.

[19] In reality, idiosyncratic shocks are likely persistent. Unfortunately, solving dynamic incentive contract models with persistent shocks is not currently computationally feasible.

[20] This assumption differs from the setup of the model of Section 1 and is for computational tractability. As Edmans et al. (2012) discuss, without this assumption, a closed form for the optimal contract is unavailable, which prohibits simulating the model.

The proof closely follows those of Edmans et al. (2012) and Doligalski et al. (2023).[21] Equation (51) characterizes wage growth. The pass-through of idiosyncratic shocks to wages, $\psi h'(a_t)\eta_t$, corresponds to incentives. If the marginal disutility of effort h' is high, there must be a high pass-through from η to wages to induce workers to supply the optimal effort level. To satisfy dynamic incentives, the pass-through of idiosyncratic productivity shocks to wages is scaled down by a quantity ψ that reflects discounting. Exponentiating equation (51), one observes that wages are a random walk: the expectation of wages in period $t+h$ is equal to the level of wages in period t. The random walk property is a consequence of the inverse Euler equation (Rogerson, 1985). Thus, rescaled expected wages at the start of the job, $w_{-1}(z_0)/\psi$, equal the expected present discounted value (EPDV) of wage payments.

Effort is determined by the first-order condition of the firm's maximization problem:

$$J(z_0) = \max_{\{a_{it}, w_{-1}(z_0)\}} \mathbb{E}_0 \left[\sum_t z_t(a_{it} + \eta_{it}) \right] - \frac{w_{-1}(z_0)}{\psi},$$

subject to the participation constraint, where the worker's expected utility under the contract can be solved for using equation (51). The optimal effort schedule, given an initial z_0 and current z_t, thus satisfies

$$a\left(z_t; z_0\right) = \left[\frac{z a\left(z_t; z_0\right)}{w_{-1}(z_0)} - \frac{\psi}{\epsilon} \left(h'\left(a\left(z; z_0\right)\right) \sigma_\eta \right)^2 \right]^{\frac{\epsilon}{1+\epsilon}}.$$

Free entry into vacancy posting requires that EPDV of profits, $Y(z_0) - w_{-1}(z_0)/\psi$, equals the expected cost of filling a vacancy $\kappa/q(\theta)$, where $Y(z_0)$ is the EPDV of output. This implies that $w_{-1}(z_0) = \psi(Y(z_0) - \kappa/q(\theta(z_0)))$.

Calibration: Separating bargaining and outside options from incentives. Our goal is to infer the role of bargaining and outside options versus that of incentives in determining wage cyclicality. We disentangle these forces with two sets of moments: the cyclicality of the wage for new hires, which informs non-incentive wages, and the pass-through of idiosyncratic firm output shocks into wages as well as the variance of workers' wage growth, both of which inform incentives. That is, wage fluctuations after the start of the match inform incentives, while wage fluctuations at the start of the contract inform NWC.

We calibrate the parameters of the labor search block largely following the standard practice of Shimer (2005) and Petrosky-Nadeau and Zhang (2017).[22] Productivity is assumed

[21] We make three advances relative to Edmans et al. (2012): we introduce aggregate risk z_t, allow for pro-cyclical ex ante worker utility, and develop a global solution algorithm to efficiently simulate the model with labor market search.

[22] These parameters are the discount rate, the vacancy creation cost, the matching function, and the

to follow an AR(1) process in logs, with autocorrelation parameter ρ_z, innovation $\zeta_t \sim \mathcal{N}(0, \sigma_z^2)$, and mean μ_z. We discretize the AR(1) productivity process for $\ln z_t$ onto a finite grid: $z \in \mathcal{Z} = [\underline{z}, ..., \overline{z}]$ following Rouwenhorst (1995). We set the number of gridpoints to 13. We normalize μ_z such that $\mathbb{E}[z_t] = 1$. To account for the effects of effort fluctuations on productivity, we calibrate our monthly process for z such that the log of the quarterly average of z_t matches the autocorrelation and standard deviation of the quarterly log TFP series described in Fernald (2014), which accounts for variable capacity utilization in labor. We view the TFP series net of variable capacity utilization as a reasonable proxy for exogenous productivity, as labor utilization is a concept highly related to effort.[23] This procedure implies a monthly autocorrelation $\rho_z = 0.965$ and standard deviation of shocks $\sigma_z = 0.0056$.[24]

This leaves four parameters to internally calibrate: the variance of noise σ_η, the level and cyclicality of ex ante utility χ and γ, and the effort elasticity ϵ. We target the variance of incumbent wage growth, the pass-through of firm shocks into wages, the cyclicality of new hire wages, and the average unemployment rate. While we estimate all parameters jointly, these moments have intuitive mappings to particular parameters, which we explore below.

First, the variance of wage growth naturally informs the variance of idiosyncratic profit shocks σ_η. To see this, note that rearranging equation (51) shows that the monthly wage growth of job-stayers is given by $\Delta \ln w_t = \psi h'(a_t)\eta_t - 1/2 \left(\psi h'(a_t)\sigma_\eta\right)^2$. At an aggregate non-stochastic steady state, $a_t = a^{SS}$, for example, the cross-sectional variance of wage growth is given by $Var(\Delta \ln w) = \psi^2 h'(a^{SS})^2 \sigma_\eta^2$, which is closely tied to the value of σ_η. The firm provides *intertemporal incentives* by exposing the worker to wage-growth risk as in Sannikov (2008). We target a standard deviation of year-over-year wage growth of job-stayers of 0.064 as measured by Grigsby et al. (2021), where we calculate year-over-year wage growth in the model with stochastic z_t by iterating on equation (51) for job-stayers.[25]

Second, the pass-through of firm-specific shocks to wages is informative of whether incentives are high powered within the contract, as in classic theories of moral hazard. In particular, this pass-through helps us identify the parameter governing the disutility of effort ϵ. In our model, the expected pass-through from idiosyncratic output shocks to the wages of job-stayers is given by $\mathbb{E}\left[\partial \ln w / \partial \ln y\right] = \mathbb{E}\left[\psi h'(a)(a + \eta)\right]$, which is directly affected by $h'(a)$. The firm provides *intratemporal incentives* with the pass-through of output to wages. Intuitively, if $h'(a)$ is high, then workers would prefer not to supply more effort,

separation rate. We discuss the details in Appendix Section B.3.

[23] Basu and Kimball (1997) find that variable capacity utilization explains approximately 40–60% of fluctuations in unadjusted TFP and that capacity utilization is procyclical.

[24] We HP-filter the TFP data and model-simulated series with a smoothing parameter of $\lambda = 10^5$, following Shimer (2005), which removes a very low-frequency trend.

[25] Hours are observable and thus contractible. We therefore consider earnings per hour— including base pay, bonuses, and overtime—to be the correct empirical counterpart of w_t.

so the firm must make wages highly dependent on output to incentivize effort.

Pass-through of firm-level output shocks to wages is evidence that firm-specific labor supply curves are upward-sloping. This is commonly interpreted as evidence that firms have some degree of monopsony in the labor market (Manning, 2003; Card et al., 2018; Berger et al., 2022). In the textbook monopsony model, a firm chooses a labor input internalizing the labor supply curve it faces. The reason that firm-level shocks passthrough to wages is that the firm must trade up its labor supply curve in order to increase its desired labor input; since the curve is upward-sloping, the firm must raise wages to increase labor input.

These economics are nearly identical to those in the incentive pay model. Here, the firm-specific labor supply curve is given by the incentive compatibility and participation constraints. The passthrough informs the power of incentives precisely because it reveals the slope of the firm-specific labor supply curve; that is, how much must the firm raise wages in order to receive one more unit of the labor input: effort. Therefore, the reason why firms pass through output shocks to incumbent workers' wages is the same in both the monopsony and incentive models. If "labor input" in the monopsony model occurs on the intensive margin of output per worker — such as hours or effort movements — then there is little distinction between the monopsony and incentive models. Indeed, passthrough reflects exactly the extent to which wage movements are offset by movements in output per worker in both models. Thus the passthrough moment remains an appropriate choice for our purposes.

However, if the labor input in the monopsony model occurs on the extensive margin by hiring workers, then any passthrough to incumbent workers' wages would constitute a rent those workers receive. Some of the passthrough in the data may reflect these kinds of considerations; we therefore target a low estimate of the passthrough from the literature.

A large literature seeks to estimate the pass-through to job-stayers' wages of firm-specific profitability shocks; Card et al. (2018) provides a comprehensive survey. We target an average pass-through of firm-level output shocks to wages of 0.039, the value estimated in Martins (2009), which is on the low end of the range reported by Card et al. (2018), which spans from 0.020 to 0.290. Our targeting of a low pass-through value is likely to be a conservative choice, as low pass-through suggests that incentives are not high powered and therefore are a relatively unimportant determinant of wage variation.

Third, we identify γ, which pins down the level of unemployment benefits from the average level of unemployment. Average unemployment is determined by workers' job finding rates, which in turn are determined by expected profits per worker. γ directly influences expected profits because it governs workers' value of unemployment and shifts the level of the required wage payments to workers. We target an average unemployment rate of 6%, consistent with average U.S. unemployment between 1951 and 2019.

Fourth, we target the cyclicality of new hire wages to inform the cyclicality of nonemployment benefits χ. Conditional on the parameters governing incentives, the cyclicality of new hire wages is highly informative of χ. Intuitively, if the worker's outside option is highly procyclical, so too is her promised utility, and thus, so too will be the present value of her wages. Since wages are a random walk in the optimal contract by equation (51), the cyclicality of new hire wages strongly informs the cyclicality of the present value of wages. We target a semielasticity of new hire wages to unemployment of -1, which is at the high end of the range found by Grigsby et al. (2021) and Hazell and Taska (2022), and explore the robustness of our findings to this choice.

Our model links ex post wage pass-through to incentives and not to Nash bargaining. In the face of this particular concern, we target a conservative value of pass-through. Moreover, there is empirical evidence that pass-through is procyclical (Chan et al., 2023), which is consistent with our model and inconsistent with pass-through representing Nash bargaining.

The intuition for our estimation is as follows. The cyclicality of new hire wages is highly informative about the cyclicality of the present discounted value of wage payments over the life of a match, because log wages are a random walk on the optimal contract. It then remains to determine the share of wage cyclicality that is matched with movements in effort. Two parameters are key for this. First, σ_η governs the degree to which effort can be inferred from realized output: if $\sigma_\eta = 0$, then output directly corresponds to an effort movement, while if $\sigma_\eta \to \infty$, effort has no impact on idiosyncratic output. This parameter is thus crucial for the degree to which effort must be incentivized rather than enforced. As discussed above, the variance of wage growth on the job is highly informative for this parameter. Second, ε governs the cost of incentive provision on the margin. If ε is high, then workers strongly dislike providing additional effort on the margin. As a result, the firm must increase expected wages a great deal in order to incentivize worker effort. As discussed above, the passthrough of idiosyncratic shocks to wages is informative about this parameter. Therefore, three moments—new hire wage cyclicality, on-the-job variance of wage growth, and the passthrough of idiosyncratic shocks to wages—principally determine the key parameters for the share of wage cyclicality due to incentives (σ_η, ε and χ).

Calibration results. Table B1 summarizes our calibration, while Table B2 examines the implications for various moments. We estimate that the elasticity of the disutility of effort ϵ is 3. Note that standard estimates of micro labor supply elasticities, such as those computed by Chetty (2012), consider how hours vary with wages. Since hours are observable and contractible by the firm, the lower elasticities of hours need not have any relationship with the elasticity of unobservable effort. Intuitively, one might expect the elasticity of effort to

Table B1: Calibrated parameter values

Parameter	Description	Value	Source/Target
Externally Calibrated			
β	Discount Rate	0.997	Petrosky-Nadeau and Zhang (2017)
κ	Vacancy Creation Cost	0.450	Petrosky-Nadeau and Zhang (2017)
s	Separation Rate	0.031	CPS E, U Stocks (following Shimer (2005))
ρ	Autocorrelation: Agg. Productivity	0.965	Autocorrelation: Fernald (2014) TFP
σ_z	Cond. S.D. of Agg. Productivity	0.006	Uncond. S.D.: Fernald (2014) TFP
Internally Calibrated			
γ	Level: Unemployment Benefits	0.453	Average Unemployment Rate
ϵ	Elasticity: Disutility of Effort	2.999	Pass-Through: Profits to Wages
σ_η	S.D.: Idiosyncratic Profit η	0.538	S.D.: Job-Stayer Log Wage Growth
χ	Cyclicality: Promised Utility to Worker	0.462	New Hire Wage Cyclicality

be larger than that of hours: while many jobs have a fixed number of hours over which the worker has little control, workers may be able to adjust unobserved effort more elastically.

We find the level of unemployment benefits γ to be 0.45. This value is close to the value chosen by Shimer (2005) to match the replacement rate of unemployment benefits (0.4).[26]

We estimate the standard deviation of idiosyncratic profit shocks to be $\sigma_\eta = 0.54$, similar to estimates in other labor search calibrations with idiosyncratic shocks (e.g., Schaal, 2017). This, coupled with a sizable elasticity of effort, suggests that incentive provision is a relatively important consideration for the firm. We estimate the cyclicality of flow unemployment benefits χ to be 0.46, implying moderately procyclical promised utility to the worker.[27]

Table B2 compares key moments in both the data (Column (1)) and calibrated model (Column (2)). The top panel reports the moments that we target in the estimation. The model is able to fit the targeted moments very well. Most notably, we match the cyclicality of new hire wages almost exactly and, if anything, underestimate the pass-through of firm shocks to wages, suggesting that our estimate of the importance of incentives for wage cyclicality is likely a lower bound on its true importance.

The bottom panel of the table shows that the model generates approximately half of the unconditional volatility of aggregate unemployment observed in the data. One should expect a quantitatively successful calibration to only match half of the unemployment volatility. The reason is that, as Hall and Milgrom (2008) point out, in the data not all unemployment fluctuations correlate with output per worker fluctuations. In both our model and the standard Diamond-Mortensen-Pissarides model, all unemployment fluctuations correlate with output

[26]Note, however, that unemployed workers do not need to supply effort in this model, which increases the effective flow unemployment value.

[27]Chodorow-Reich and Karabarbounis (2016) estimate $\chi \approx 0.8$; however, the value of unemployment in our model is different from theirs because workers supply effort and do not have access to financial assets.

Table B2: Model fit to data moments

Moment	Description	Data (1)	Model (2)
Targeted			
$d\mathbb{E}[\ln w_0]/du$	Cyclicality of new hire wages	-1.000	-1.001
$\mathbb{E}[\partial \ln w_{it}/\partial \ln y_{it}]$	Within-job pass-through of idiosyncratic shock	0.039	0.037
std$(\Delta \ln w_{it})$	std(ln wage growth for job-stayers)	0.064	0.064
$\bar{u}_t$	Mean unemployment	0.060	0.060
Untargeted			
std $(\ln u_t)$	Volatility of unemployment (quarterly)	0.203	0.101
Incentive share	Share of wage cyclicality due to incentives	–	0.469

per worker fluctuations. Other shocks must account for the remaining unemployment fluctuations. As such, Hall and Milgrom (2008) argues that one should evaluate whether the model can explain the portion of unemployment volatility that, in the data, correlates with output per worker. Hall and Milgrom (2008) calculate that the volatility of the component of unemployment that correlates with output per worker is 0.68 percentage points. Our model predicts that the volatility of unemployment is $0.101 \times 6\% = 0.61pp$, though our sample is somewhat different from Hall and Milgrom (2008). Therefore, even though our main focus is the impulse response of unemployment, our calibrated model does match unconditional unemployment fluctuations reasonably well. Matching the micro moments of wage adjustment therefore generates significant unemployment volatility, the reasons for which we will discuss shortly.

B.2 Numerical Results

Incentive wage cyclicality. Now, we discuss our key numerical result: the model suggests that a significant share of wage cyclicality is due to incentives. As a result, unemployment responds strongly to business cycle shocks despite wages being relatively procyclical.

The model calibration reveals in the final row of Table B2 that approximately 47% of the total wage cyclicality is due to incentives. This may seem large. Non–base compensation, which may be associated with incentives, is relatively small for most workers. However, what matters for wage cyclicality is whether the *marginal* dollar of wages paid is due to incentives or bargaining and outside options. If, for instance, 2% of compensation is incentive pay in the steady state but only incentive pay is cut in response to output shocks, then the share of wage cyclicality attributable to incentives is 100%. Further, base wages may embed

Table B3: Model moments: Alternative calibrations

	Model: Source of wage flexibility			
Moment	(1) Incentives + Bargaining	(2) Incentives	(3) Bargaining	(4) Bargaining: $\partial\mathbb{E}[\ln w_0]/\partial u = -0.53$
$d\mathbb{E}[\ln w_0]/du$	-1.00	-0.59	-1.00	-0.53
$d\ln\theta_0/d\ln z_0$	13.59	18.10	10.27	13.31
$\text{std}(\ln u_t)$	0.10	0.15	0.07	0.10
$\mathcal{W}_0/\mathcal{Y}_0$	0.96	0.96	0.96	0.96
$d\ln\mathcal{W}_0/d\ln z_0$	0.44	0.38	0.31	0.23
$d\ln\mathcal{Y}_0/d\ln z_0$	0.70	0.87	0.50	0.50
NWC share	0.53	0.00	1.00	1.00

Notes: New hire wage cyclicality is targeted, while the second set of moments is untargeted. Column (1) is our baseline model. Column (2) sets $\chi = 0$ and does not target the cyclicality of new hire wages. Columns (3) and (4) fix effort $a = 1$, set wages to be constant within the contract, and do not target the standard deviation of wage growth or the pass-through. Column (4) targets a cyclicality of new hire wages of -0.46. The standard deviation of log unemployment is computed at the quarterly frequency. For the untargeted moments, x_0 denotes the value of variable x, evaluated at $\ln z = \mu_z$. $\mathcal{W}$ and $\mathcal{Y}$ refer to the expected present value of wage payments and output, respectively. "NWC share" is the share of wage cyclicality not accounted for by incentives, also evaluated at $\ln z = \mu_z$.

some incentive components if workers can be promoted after good performance; indeed, promotions appear to be quite cyclical in the data (Méndez and Sepúlveda, 2012; Wilson, 1996; Addison et al., 2014). These kinds of incentives can explain why incentive wages may be quite cyclical even if measured bonuses are relatively acyclical.[28]

Because NWC is relatively small, the impulse response of unemployment to business cycle shocks is relatively large. Table 3 reports a number of additional features of our model calibrated in a variety of ways. Column (1) reproduces the baseline calibration as in Table B2. The impulse response of market tightness to business cycle shocks is in the second row. Market tightness responds greatly to exogenous productivity shocks: the elasticity of market tightness to aggregate productivity is 13.6.

This occurs despite total wages being quite procyclical. The conditional elasticity of the present value of expected wage payments with respect to productivity is 0.44. However, as we have discussed in previous sections, the stabilizing effect on unemployment of procyclical wages is offset by the amplifying effect of effort and incentives. Because of incentives, the impulse response of the present value of output, $\mathcal{Y}_0$, to TFP shocks is a relatively large value of 0.70. As a result, profit fluctuations—and thus market tightness and employment

[28]There is debate about how cyclical bonuses are in the data. Grigsby et al. (2021) find very limited bonuses cyclicality in recent administrative payroll data, while Swanson (2007) and Shin and Solon (2007) both find that non-based pay is quite cyclical in longer-run survey data. We choose not to target measured cyclicality of bonuses both due to this empirical debate, and because bonuses need not reflect incentives; see, for instance, Oyer (2004) for a theory in which bonuses do not have strong incentive effects.

fluctuations—are large despite the procyclicality of wages.[29] The model implies a labor share (defined as $\mathcal{W}_0/\mathcal{Y}_0$) of 0.96, in line with, for instance, Hall (2005).[30]

To emphasize the role of incentive wage cyclicality, we consider versions of our model that load all wage cyclicality in the data onto either incentives or bargaining and outside options. We present the calibration with only incentives in Column (2), which leads to a large impulse response of tightness in row 2.[31] Nevertheless, the incentives-only model still generates large wage cyclicality in row 1. This is a manifestation of our analytical results in a globally solved model. Column (3) presents a version of the model without incentives and with only bargaining, where the impulse response of tightness is relatively small, reflecting the dampening effect of NWC.[32]

The distinction between bargaining and incentives also matters for unemployment volatility. If all wage cyclicality were due to bargaining, then the standard deviation of log unemployment would be 0.07. Therefore if all wage cyclicality is due to bargaining, unemployment volatility is 30% lower than in the baseline calibration, which we have argued is reasonably consistent with the data.

Overall, in our calibration, incentive pay appears to be important. With this feature, one can simultaneously match a relatively high estimate of wage cyclicality, alongside volatile unemployment. We stress that other mechanisms can also match both of these facts, such as a small "fundamental surplus" (Ljungqvist and Sargent, 2017).

Calibrating simpler models. We argue that the simple version of our model in which all wage cyclicality is due to bargaining should target a new hire wage cyclicality given only by NWC—that is, a calibration in which wages are less procyclical than in the data. To illustrate the point numerically, we recalibrate the bargaining-only version of the model targeting a new hire wage cyclicality of -0.53, which is what we previously inferred to be non-incentive wage cyclicality.[33] Column (4) of Table B3 presents the results of this exercise.

The numerical results show that to produce the correct impulse response of market tightness in the simple model with only bargaining, calibrating to target NWC is crucial. When calibrated to NWC, the bargaining-only model features an elasticity of market tightness to

[29] We caution that this statistic does not have an easy-to-measure empirical counterpart, given the challenge of measuring either TFP shocks or the present value of output well.

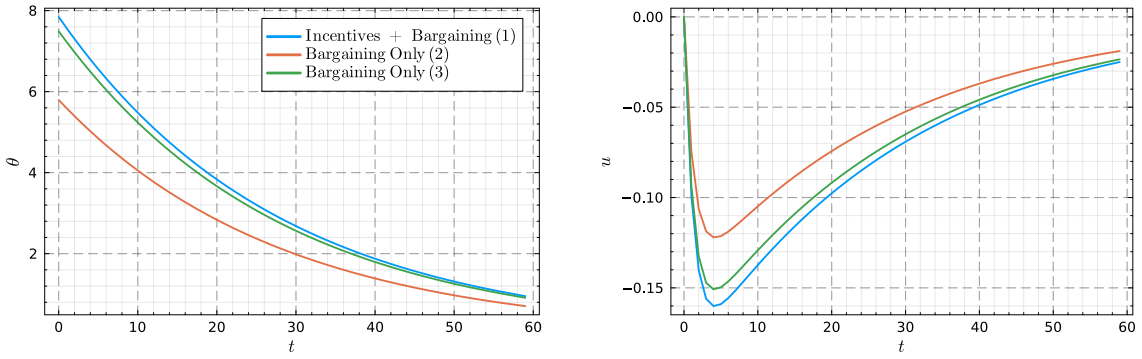
[30] Since our model does not have capital, the labor share corresponds to the labor share of payroll and rents from search frictions in the labor market, excluding capital (Pissarides, 2000).

[31] This calibration assumes that the cyclicality of ex ante utility is zero and do not target wage cyclicality.

[32] This calibration turns off incentives by setting the variance of the idiosyncratic profitability shocks to $\sigma_\eta = 0$, exogenously fixing effort $a = 1$, setting $\epsilon = 1$, and setting wages to be fixed within a contract. We attribute all wage cyclicality in the data to the cyclicality of promised utility governed by χ.

[33] We normalize $\epsilon = 1$ for this exercise and solve for fixed wages within the contract. We also drop the standard deviation of log wage growth and the average pass-through of firm shocks as targeted moments.

Figure B3: Impulse response to shock to z_0 with bargaining-only and incentive pay models



Panel A: Tightness θ_t Panel B: Unemployment u_t

Notes: The figure shows impulse responses for five years after a one-standard-deviation shock to z_0. In Panel A, θ_t is shown in percentage deviations from the steady state. In Panel B, u_t is shown as deviations away from the steady state in percentage points. All models are calibrated as in B3.

exogenous shocks that is nearly identical (13.3) to that in the full model (13.6). Furthermore, both models generate an unconditional standard deviation of log unemployment rates of 0.10. The similar dynamics arise because the two models imply similar ex ante utility cyclicality even though overall wage flexibility is different: the simple bargaining-only model of column (4) estimates an elasticity of unemployment benefits $\chi = 0.46$, nearly identical to that found under the full model.

Figure B3 plots the impulse of market tightness (Panel A) and unemployment (Panel B) in response to a one-standard-deviation increase in aggregate productivity z, which decays according to an AR(1).[34] The blue line is the response in the full model with both incentives and bargaining. The red line is the response in the bargaining-only model calibrated to the full wage cyclicality in the data. The green line is the response in the bargaining-only model calibrated to our estimate of NWC in the data. The response of both market tightness and unemployment is approximately 25% less pronounced in the bargaining-only model than in the full model with incentives and bargaining. However, the impulse responses of both tightness and unemployment are nearly identical in the full model and the bargaining-only model calibrated to relatively rigid wages.

Robustness. The key numerical result of this section is that a significant share of wage cyclicality is due to incentives, leading to volatile unemployment dynamics despite wages being relatively procyclical.

[34]Further details on the construction of these impulse responses are described in Section B.3.5.

Table B4: Alternative calibration strategies: Internally calibrated parameters

	$\partial\mathbb{E}[\ln w_0]/du$ target				Internal Calibration: ALP	
Parameter	-0.5	-0.75	-1.25	-1.5	Incentives + Bargaining	Bargaining
σ_η	0.536	0.537	0.538	0.538	0.533	0.000*
χ	0.180	0.344	0.547	0.610	0.480	0.609
γ	0.475	0.452	0.453	0.453	0.585	0.584
ϵ	2.325	3.000	2.999	3.000	0.966	1.000*
ρ_z	0.965*	0.965*	0.965*	0.965*	0.990	0.969
σ_z	0.006*	0.006*	0.006*	0.006*	0.003	0.006

Notes: Table reports estimated parameters for our alternative calibration strategies. The first four columns change the target of new hire wage cyclicality for our full model with both incentives and bargaining. The final two columns internally calibrate the exogenous productivity process to match moments of measured labor productivity under our full model and model with only bargaining. Asterisks indicate fixed parameters.

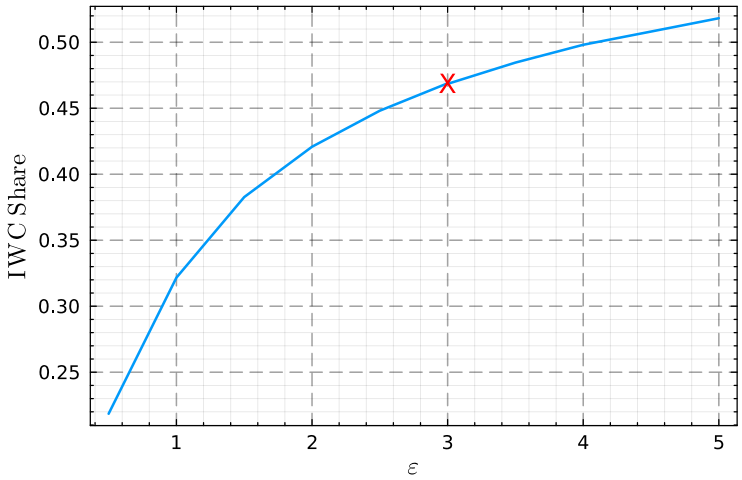
Table B5: Varying cyclicality of new hire wages: Simulated model moments

	Model: $\partial\mathbb{E}[\ln w_0]/du$ target			
Moment	-0.50	-0.75	-1.25	-1.50
$d\mathbb{E}[\ln w_0]/du$	-0.50	-0.75	-1.25	-1.50
std($\ln u_t$)	0.16	0.13	0.09	0.07
$d\ln\theta_0/d\ln z_0$	18.18	15.79	11.83	10.44
Incentive Wage Cyclicality share	0.76	0.59	0.39	0.33
Incentive Wage Cyclicality	-0.38	-0.44	-0.48	-0.49

Notes: New hire wage cyclicality is targeted, while the second set of moments are untargeted. std($\ln u_t$) is the unconditional standard deviation of the log of the quarterly average of the monthly unemployment rate, HP-filtered with smoothing parameter $\lambda = 10^5$. x_0 denotes the value of variable x, evaluated at $\ln z = \mu_z$. Incentive Wage Cyclicality share is the share of wage cyclicality that is due to incentives, also evaluated at $\ln z = \mu_z$. Incentive Wage Cyclicality is computed as $\mathbb{E}[\ln w_0]/du$ × Incentive Wage Cyclicality share.

Tables B4 and B5 report the estimated parameters and model-implied moments, respectively, when we target different values of wage cyclicality ranging from -0.5 to -1.5. Each column corresponds to a recalibration of the model. We find that the share of wage cyclicality attributable to incentives declines as we increase the target estimate of cyclicality of new hire wages. However, Incentive Wage Cyclicality is always large and relatively stable between -0.38 and -0.49.

Figure B5: Incentive wage cyclicality share for different calibrations of ϵ



Notes: Figure reports the estimated share of wage cyclicality due to incentives at $\ln z_0 = \mu_z$ as we vary the disutility of effort ϵ. To produce this figure, we first impose a value of ϵ and then recalibrate our model to match all four of our baseline calibration targets.

To account for uncertainty in our pass-through target, Figure B5 reports the estimated value of the incentive wage cyclicality share for various imposed values of ϵ, allowing all other parameters to be recalibrated. The X on the plot reports our baseline estimate for ϵ. The estimated share of wage cyclicality due to incentives is increasing in ϵ, rising to 52% for $\epsilon = 5$ and falling to 22% for $\epsilon = 0.5$.

One concern with our calibration is as follows. Our empirical measure of productivity, from Fernald (2014), may not be meaningful in this environment. The measure of productivity accounts for variable capacity utilization. However in our model, one requires a measure of productivity that accounts for worker effort, which is different from capacity utilization. Therefore, we adopt a second calibration strategy. We internally calibrate the exogenous productivity process in our incentive pay model to match moments of average labor productivity (ALP) in the data.[35] Tables B4 and B6 report the estimated parameters and model-implied moments, respectively. Calibrated thus, the model infers that incentives account for about 35% of overall wage cyclicality and a large impulse response of market tightness to exogenous productivity shocks. Moreover the model predicts log unemployment volatility of 0.09, compared with log unemployment volatility in the data of 0.2. As we have discussed, an appropriate calibration should explain roughly half of the unemployment volatility in the data.[36]

[35]Here, ALP is the seasonally adjusted, quarterly average output per hour for all workers in the nonfarm business sector, as reported by the BLS.

[36]In Appendix Tables B4 and B6, we also recalibrate the bargaining-only model to target average labor productivity. The bargaining-only model continues to have a significantly smaller impulse response of

Table B6: Internally calibrating labor productivity process: simulated model moments

		Model: Source of wage flexibility	
Moment	Data	Incentives + Bargaining	Bargaining
ρ_p	0.89	0.87	0.88
σ_p	0.02	0.02	0.02
$\text{std}(\ln u_t)$	0.20	0.09	0.08
$d \ln \theta_0 / d \ln z_0$	-	18.94	10.78
$\mathcal{W}_0 / \mathcal{Y}_0$	-	0.96	0.96
$d \ln \mathcal{W}_0 / d \ln z_0$	-	0.58	0.33
$d \ln \mathcal{Y}_0 / d \ln z_0$	-	0.99	0.54
Incentive share	-	0.35	0.00

Notes: New hire wage cyclicality is targeted, while the second set of moments are untargeted. $\text{std}(\ln u_t)$ is the unconditional standard deviation of the log of the quarterly average of the monthly unemployment rate, HP-filtered with smoothing parameter $\lambda = 10^5$. ρ_p and σ_p are the autocorrelation and unconditional variance of the log of quarterly average labor productivity, HP-filtered with smoothing parameter $\lambda = 10^5$. x_0 denotes the value of variable x, evaluated at $\ln z = \mu_z$.

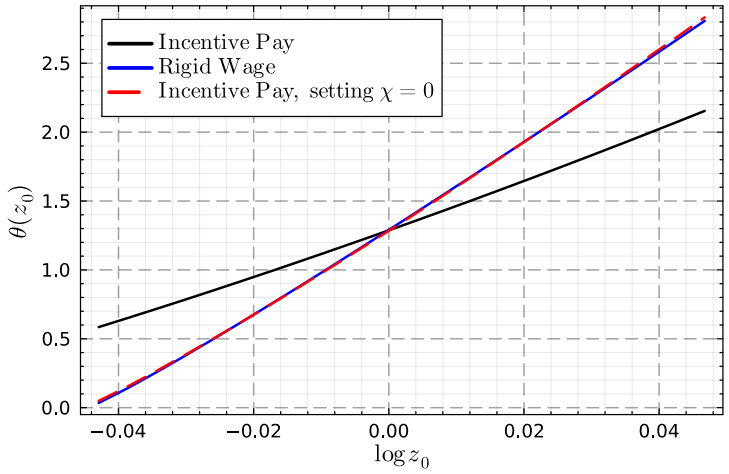
Taking stock, we find that a relatively large share of wage cyclicality in the data is attributable to incentives despite our conservative calibration. Therefore, our model generates a large impulse response of unemployment despite the cyclicality of wages.

Global Equivalence. Our theoretical result is first-order in nature: it shows that the response of market tightness to exogenous productivity shocks is the locally same in the flexible incentive pay model with acyclical promised utility as in an appropriately-calibrated model with rigid wages and effort. Figure B6 studies the extent to which this holds globally in the quantitative model. It plots movements in market tightness θ as aggregate productivity z_0 varies for up to 5% aggregate TFP shocks.

The figure contains three lines. The red line reports the relationship between θ and z_0 for the flexible incentive pay model, calibrated exactly as in Table B1 only with $\chi = 0$ so that there is no non-incentive wage cyclicality. The blue line plots this relationship in the rigid wage model, calibrated such that the present value of wages and output are the same in the rigid wage and incentive pay models at $z_0 = 1$. This calibration ensures that the blue and red lines intersect at $\log z_0 = 0$ by construction, since the labor share at that point is given by 0.96 in both economies. The content of our theorem is that the blue and red lines lie on top of each other, not just at $\log z_0 = 0$, but also for movements in z_0 of at least 5%; that is, the flexible incentive pay and rigid wage models generate the same movements in θ (and

tightness than does the full model and requires exogenous productivity shocks to be approximately twice as volatile as in the full model to match output fluctuations.

Figure B6: Tightness Fluctuations in Quantitative Model



Notes: Figure plots market tightness θ as a function of aggregate productivity z_0 in the quantitative model model. The black line considers the calibrated flexible incentive pay economy with non-zero non-incentive wage cyclicality. Calibration follows Table B1. The red line considers the flexible incentive pay economy without non-incentive reasons for wage cyclicality, generated by setting $\chi = 0$. The blue line plots considers the rigid wage economy, calibrated to have the same present value of wages and output as the flexible incentive pay economy for $z_0 = 1$.

thus unemployment) even for large shocks to aggregate productivity. Theorem 1 numerically appears to not only hold locally, but for larger shocks as well.

As a point of comparison, the black line on Figure B6 reports the response of θ to z_0 for the full model with non-zero non-incentive wage cyclicality (that is, $\chi = 0.46$ as in Table B1). The plot makes clear that non-incentive wage cyclicality dampens the relationship between market tightness θ and exogenous productivity shocks $\log z_0$, as suggested by Proposition 4.

B.3 Numerical Details

We calibrate the model such that t represents a month. Specifically, we set the discount rate β to $0.99^{1/3}$, the vacancy creation cost to 0.45 and employ a matching function given by $m(u, v) = uv(u^\iota + v^\iota)^{-1/\iota}$ so that $q(\theta) = (1 + \theta^\iota)^{1/\iota}$, which is bounded between 0 and 1. We set $\iota = 0.9$ by nonlinear least squares to match the empirical relationship between aggregate market tightness and job-finding rates. We measure empirical labor market tightness as job openings from the Job Openings and Labor Turnover Survey divided by household unemployment in the Current Population Survey (CPS). We measure empirical job-finding rates over the same time period that JOLTS data is available (2000-2019). We set the exogenous separation rate $s = 0.031$ to the average monthly separation rate from 1951 to 2019. Job-finding rates and separation rates are computed using data on short-term unemployment

and employment from the CPS, following Shimer (2005). These values of β and s imply that the pass-through parameter $\psi = 1 - \beta(1-s)$ equals 0.034.

We now rewrite some of the key equations in our numerical model recursively, given the Markovian structure for productivity. Let $\pi(z'|z)$ denote the probability of aggregate productivity transitioning from z to z'. Let $\tilde{Y}(z; z_0)$ denote the EPDV of future output, conditional on initial productivity z_0, effort schedule $a(z; z_0)$, and current productivity z:

$$\tilde{Y}(z; z_0) = z a(z; z_0) + \sum_{z' \in \mathcal{Z}} \beta (1-s) \tilde{Y}(z'; z_0)) \pi(z'|z).$$

It follows that $Y(z_0) = \tilde{Y}(z_0; z_0)$. Recall that the optimal effort depends on z_0 through $w_{-1}(z_0)$, which is an equilibrium object in our model. Define the worker's expected present discounted utility from starting work at z_0, $\tilde{\mathcal{E}}(z_0)$, taking as given the effort schedule $a(\cdot; z_0)$ and the wage schedule $w(\cdot; z_0)$ defined in Proposition 6:

$$\tilde{\mathcal{E}}(z_0) = \frac{1}{\psi} \ln w_{-1}(z_0) + \mathbb{E}_z \Bigg[- \sum_{t=0}^{\infty} [\beta(1-s)]^t \frac{1}{\psi} \frac{1}{2} (\psi h'(a(z_t; z_0)) \sigma_\eta)^2 - \sum_{t=0}^{\infty} [\beta(1-s)]^t h(a(z_t; z_0)) + \sum_{t=0}^{\infty} [\beta(1-s)]^t \beta s \omega(z_{t+1}) \Bigg],$$

where

$$\omega(z) = \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \ln b(z_t) \mid z_0 = z \right].$$

We can write the term in brackets recursively as $W(z_0; z_0)$, where

$$W(z; z_0) = -\frac{1}{\psi} \frac{1}{2} (\psi h'(a(z; z_0)) \sigma_\eta)^2 - h(a(z; z_0)) + \sum_{z' \in \mathcal{Z}} \beta s \omega(z') \pi(z'|z_0) + \sum_{z' \in \mathcal{Z}} \beta (1-s) W(z'; z_0) \pi(z'|z_0).$$

B.3.1 Algorithm to solve for the optimal contract, given z_0

Fix an initial $z_0 \in \mathcal{Z}$. To solve for the optimal contract beginning at z_0, we perform a bisection search over the job-filling rate $w_{-1}(z_0)$. Let n index iterations over our guess of $w_{-1}(z_0)$. Then, for a given $w_{-1}^n(z_0)$, we can solve for the optimal effort schedule $a^n(z; z_0)$. With values of $a^n(z; z_0)$ and $w_{-1}^n(z_0)$, we can recursively solve for the EPDV of the utility offered by the contract $\mathcal{E}^n(z_0)$. We then check whether $\mathcal{E}^n(z_0) = \omega(z_0)$, and accordingly update the lower and upper bounds for w_{-1} in the next iteration, $\underline{w}_{-1}^{n+1}$ and $\bar{w}_{-1}^{n+1}$, respectively. We continue this process until convergence of $w_{-1}(z_0)$. We solve for the implied $\theta(z_0)$ from the free entry

condition. Below, we describe the algorithm in further detail.

1. Set $n = 1$. Set $0 < \underline{w}^n < \bar{w}^n$.

2. Set $w^n(z_0) = \frac{1}{2}(\underline{w}^n + \bar{w}^n)$.

3. Solve numerically for effort as $a^n(z; z_0) = \tilde{a}(z, w_{-1}^n(z_0))$, where

$$\tilde{a}\left(z, w_{-1}\right) = \left[\frac{z\tilde{a}}{\psi\left(w_{-1}\right)} - \frac{\psi}{\epsilon}\left(h'\left(\tilde{a}\right)\sigma_\eta\right)^2\right]^{\frac{\epsilon}{1+\epsilon}}.^{37}$$

4. Set $j = 1$. Make initial guess for $W^{j,n}\left(z; z_0\right)$.

5. Update $W^{j+1,n}(\cdot; z_0)$ as

$$W^{j+1,n}\left(z; z_0\right) = -\frac{1}{\psi}\frac{1}{2}(\psi h'(a^n(z))\sigma_\eta)^2 - h(a^n(z)) + \sum_{z' \in \mathcal{Z}} \beta s \omega\left(z'\right)\pi\left(z'|z\right) + \sum_{z' \in \mathcal{Z}} \beta\left(1-s\right) W^{j,n}\left(z'; z_0\right)\pi\left(z'|z\right)$$

6. Repeat (8) until $\|W^{j,n+1}(\cdot; z_0) - W^{j,n}(\cdot; z_0)\| < \delta_2$ for some small tolerance $\delta_1 > 0$. Define $\mathcal{E}^n(z_0) = \frac{1}{\psi}\ln w_{-1}^n(z_0) + W^{j,n}(z_0; z_0)$.

7. If $\mathcal{E}^n(z_0) > \omega(z_0)$ then set $\bar{w}^{n+1} = w^n(z_0)$. If $\mathcal{E}^n(z_0) < \omega(z_0)$, then set $\underline{w}^{n+1} = w^n(z_0)$. Note that $\omega(z_0)$ can be computed by a simple value function iteration.

8. Repeat steps (2)-(7) until $|\mathcal{E}^n(z_0) - \omega(z_0)| < \delta_2$ or $|w_{-1}^{n+1}(z_0) - w_{-1}^n(z_0)| < \delta_2$ for some small tolerance $\delta_3 > 0$ to obtain $w_{-1}(z_0)$. Set $w_{-1} = w_{-1}^n$ and $a(z; z_0) = a^n(z; z_0)$.

9. Set $k = 1$. Make initial guess for $Y^k(z; z_0)$ for $z \in \mathcal{Z}$.

10. Update $Y^{k+1}(\cdot; z_0)$ as

$$Y^{k+1}\left(z; z_0\right) = z a(z; z_0) + \sum_{z' \in \mathcal{Z}} \beta\left(1-s\right) Y^k\left(z'; z_0\right)\pi\left(z'|z\right).$$

11. Repeat (4) until $\|Y^{k+1}\left(\cdot; z_0\right) - Y^k\left(\cdot; z_0\right)\| < \delta_3$ for some small tolerance $\delta_3 > 0$. Set $Y(z_0) = Y^n(z_0; z_0)$.

12. Define $J^*(z_0) = Y(z_0) - w_{-1}(z_0)/\psi$. Define $J(z_0) = \max\{J^*(z_0), 0\}$. Define $q(z_0) = \min\{\kappa/J(z_0), 1\}$. Define $\theta(z_0) = q^{-1}(q^n(z_0))$, where $q(\theta) = \frac{1}{(1+\theta^\iota)^{1/\iota}}$.

[37] We restrict attention to positive solutions of this implicit equation. We verify uniqueness of the obtained solution at our estimated parameters.

We repeat this procedure for all values of $z_0 \in \mathcal{Z}$ to obtain the equilbrium objects $\theta(z_0)$, $Y(z_0)$, $w_{-1}(z_0)$, and $a(z; z_0)$.

B.3.2 Details on Simulation

Our set of targeted moments includes two moments that depend on within-contract, idiosyncratic realizations: the standard deviation of annual (YoY) wage growth ($\text{std}(\Delta \ln w_{it})$) and the average pass-through from idiosyncratic shocks to firm profits to wages ($\mathbb{E}[{}^{\partial \ln w_{it}}/_{\partial \ln y_{it}}]$), and two moments which can be computed from aggregate time series simulated in the model: the cyclicality of new hire wages (${}^{\partial \mathbb{E}[\ln w_0]}/_{\partial u}$) and average unemployment ($\bar{u}_t$). To compute these moments for a given set of parameters $\Omega \coloneqq \{\epsilon, \sigma_\eta, \chi, \gamma\}$, we solve the model for each initial $z_0 \in \mathcal{Z}$ following the procedure outlined in Section B.3.1 to obtain $a(\cdot|z_0)$, $w_{-1}(z_0)$, and $\theta(z_0)$. We then simulate the economy with aggregate shocks and compute moments.

Simulating $\text{std}(\Delta \ln w_{it})$ and $\mathbb{E}[{}^{\partial \ln w_{it}}/_{\partial \ln y_{it}}]$. We simulate a panel of $I = 10,000$ idiosyncratic η_{it} shocks of length $T = 828 + 500$ (and one sequence of aggregate z_t shocks of length T). For each period t and worker i, we simulate separations and job-finding shocks consistent with the exogenous probability of separation s and endogenous job-finding probability $\phi(\theta(z_t))$. All workers are employed at the beginning of $t = 0$. During job spells and given realizations of z_t and η_{it}, we can compute log wages and individual pass-through according to the equations derived in Section B.1. For job spells that last at least 13 months, we can compute YoY log wage growth as $\ln w_{i,t+12} - \ln w_{it}$ (for each year of employment). We can compute the pass-through from idiosyncratic output shocks to the wages $\partial \ln w_{it} / \partial \ln y_{it}$ as $\psi h'(a_{it})(a_{it} + \eta_{it})$. We discard the first $t_{\text{burn-in}} = 500$ periods as a burn-in period. We then compute the pooled variance of YoY log wage growth and the average pass-through across all periods of employment for $t \geq t_{\text{burn-in}}$.[38] Our simulation procedure captures composition effects of initial z_0 on the employment contracts.

Simulating ${}^{d\mathbb{E}[\ln w_0]}/_{du}$ and $\bar{u}_t$. We simulate 10,000 z_t sequences of length $T = 828$ periods (with an additional burn-in period of length 500 periods). For each z_t path, we compute the path for unemployment as

$$u_{t+1} = u_t + s(1 - u_t) - \phi(\theta(z_t))u_t(1 - s)$$

[38] We condition on positive output when computing the average pass-through, since individual output in our model can be negative.

given initial condition $u_0 = 0.06$. The expected log wage of new hire wages in our model is

$$\mathbb{E}_{\eta_{it}}[\ln w_0(z_t)] = \ln w_{-1}(z_t) - \frac{1}{2}(\psi h'(a(z_t|z_t))\sigma_\eta)^2.$$

We compute $\bar{u}_t$ as the average unemployment u_t for $t \geq t_{\text{burn-in}}$. We measure $d\mathbb{E}[\ln w_0]/du$ in the model by running an OLS regression of $\mathbb{E}[\ln w_0](z_t)$ on u_t and a constant in the simulated data for $t \geq t_{\text{burn-in}}$. We report cross-simulation averages for both moments.

Simulating ρ_p and σ_p We simulate a panel of $I = 10,000$ workers of length $T = 828 + 500$ (and 25 sequences of aggregate z_t shocks of length T). For each period t and worker i, we simulate separations and job-finding shocks consistent with the exogenous probability of separation s and endogenous job-finding probability $\phi(\theta(z_t))$. We compute average labor productivity as the average output among employed workers within a quarter. We report averages of the unconditional standard deviation and autocorrelation of average labor productivity across the 25 simulations. This simulation procedure captures composition effects of initial z_0 on the employment contracts.

B.3.3 Estimation Algorithm

We implement a version of the Tik-Tak algorithm, a multi-start global optimization algorithm, as described by Arnoud et al. (2019), to minimize the following objective function

$$J(\Omega) = (\tilde{m}(\Omega) - m)'W(m)(\tilde{m}(\Omega) - m),$$

where Ω is a vector of the parameters to be estimated, $\tilde{m}(\Omega)$ is a vector of the targeted moments computed using the model simulated data given the parameter vector Ω, and m is the vector of targeted empirical moments. We set the weight matrix W to $W_{j,j} = 1/m_j^2$ for each targeted moment j (and 0, otherwise). Thus, the objective function to minimize is the sum of squared percentage differences between simulated and empirical moments to account for differences in scale between the targeted moments. We have experimented with different derivative-free local optimization algorithms, such as BOBYQA and the Nelder-Mead Simplex Algorithm, for the local optimization step. All estimation results correspond to solutions obtained using a combination of the Nelder-Mead Simplex Algorithm and BOBYQA algorithm with 1,000 initial points. We implement a pre-testing stage to detect promising regions of the parameter space by evaluating the objective function at a large number of initial points drawn from Sobol sequences; we use the 1,000 points that yield the lowest values of the objective function as the initial points in the global search iterations.

Technical detail on computing the new hire wage cyclicality For some parameter values in the calibration, $q(\theta(z_0))$ may hit its upper bound of 1. When $q(\theta(z_0)) = 1$, the implied new hire wage that we compute for z_0 would thus not be an observed wage, as $\phi(\theta(z_0)) = 0$. The other wage moments do not pose this challenge because that we simulate employment spells in accordance with the endogenous job-finding probabilities.

To address this discrepancy with the data, we penalize parameters for which $q(\theta(z_0)) = 1$ for values of $\ln z_0$ within three unconditional standard deviations of μ_z and truncate the simulation to fall within this range.[39] We do not penalize violations for lower values of z_0 as the probability of reaching them is low, and it is also reasonable to expect that the $q(\theta(z_0)) \approx 1$ for some very low z_0. We have explored alternative approaches, with the results being largely consistent across different approaches.

B.3.4 Calculating Incentive Wage Cyclicality

Non-incentive wage cyclicality reflects movements in the promised utility of workers. For a given calibration, we calculate how the value of a filled job moves with exogenous productivity $dJ(z_0)/dz_0$. The "direct effect" of z_0 on the expected present discounted value of profits per worker, given the AR(1) process for $\ln z$, can be approximated as

$$\frac{dJ(z_0)}{dz_0}^{Direct} = \sum_{t=0}^{\infty} (\beta(1-s))^t \frac{\mathbb{E}_0[a^*(z_t)\rho^t z_t]}{z_0}.$$

That is, the direct effect is the effect that z has on profits holding fixed the optimally contracted choice of effort and wages. We calculate NWC as

$$\text{NWC}(z_0) = \frac{dJ(z_0)}{dz_0} - \frac{dJ(z_0)}{dz_0}^{Direct}.$$

The share of wage fluctuations attributable to incentives is then the negative of one minus NWC(z_0) divided by the cyclicality of the expected present discounted value of wage payments $^{d\mathcal{W}^*(z_0)}/_{dz_0}$.

B.3.5 Construction of Impulse Responses

We compute the impulse response to a one (conditional) standard deviation (σ_z) shock to $\ln z_0$ in an economy that is at an aggregate non-stochastic steady state. We construct nonlinear perfect foresight impulse responses to a one-time shock to productivity at time 0

[39] We still solve the model on a wider grid to obtain a more precise solution.

that decays at rate ρ_z. We define the non-stochastic steady state of $\log z$ to be 0, dropping the normalization of μ_z that ensures $\mathbb{E}[z_t] = 1$ given that $\mu_z \approx 0$.

We first solve for the non-stochastic steady state of the model, where $z_t = z_{ss} = 1$, $\theta_t = \theta(z_{ss})$, and $u_t = u_{ss} = \frac{s}{s+\phi(\theta(z_{ss}))(1-s)}$ for all t. We next solve for the path of $\theta_t(\{z_s\}_{s\geq t})$, given a sequence of shocks $\{z_t\}$. Finally, we solve for the path of unemployment u_t, given the path of θ_t, setting $u_0 = u_{ss}$.[40] We construct these impulse responses in a finite horizon contract setting and set the length of the contract, T, to be 240 model periods (20 years), which is a close approximation to the infinite horizon contract environment.[41]

References

Addison, John T., Orgul Demet Ozturk, and Si Wang, "Job promotion in midcareer: gender, recession, and "crowding"," *U.S. Bureau of Labor Statistics Monthly Labor Review*, 2014.

Arnoud, Antoine, Fatih Guvenen, and Tatjana Kleineberg, "Benchmarking Global Optimizers," 2019.

Basu, Susanto and Miles Kimball, "Cyclical productivity with unobserved input variation," 1997.

Berger, David, Kyle Herkenhoff, and Simon Mongey, "Labor Market Power," *American Economic Review*, April 2022, *112* (4), 1147–93.

Blanchard, Olivier and Jordi Galí, "Labor markets and monetary policy: A new keynesian model with unemployment," *American economic journal: macroeconomics*, 2010, *2* (2), 1–30.

Bonnans, J. Frédéric and Alexander Shapiro, *Perturbation Analysis of Optimization Problems*, 1 ed., Springer, 2000.

Card, David, Ana Rute Cardoso, Joerg Heining, and Patrick Kline, "Firms and Labor Market Inequality: Evidence and Some Theory," *Journal of Labor Economics*, 2018, *36* (S1), S13–S70.

Chan, Mons, Sergio Salgado, and Ming Xu, "Heterogeneous Passthrough from TFP to Wages," Technical Report, Working Paper 2023.

Chetty, Raj, "Bounds on Elasticities with Optimization Frictions: A Synthesis of Micro and Macro Evidence on Labor Supply," *Econometrica*, 2012, *80* (3), 969–1018.

Chodorow-Reich, Gabriel and Loukas Karabarbounis, "The cyclicality of the opportunity cost of employment," *Journal of Political Economy*, 2016, *124* (6), 1563–1618.

[40] The calibration was done for the infinite horizon contract environment and targeted the stochastic mean of unemployment rather than steady state unemployment rate as implied in a non-stochastic model. Therefore, the steady state across the models need not be the same, although they are very close in practice.

[41] There is an additional term in the law of motion for unemployment in the finite horizon contract setting because workers separate with probability one after they have completed their contract without experiencing a separation shock. However, the measure of workers that do not separate by time T is essentially zero, given that $T = 240$. Therefore, we ignore this inflow into unemployment in this numerical exercise.

Doligalski, Paweł, Abdoulaye Ndiaye, and Nicolas Werquin, "Redistribution with performance pay," *Journal of Political Economy Macroeconomics*, 2023, *1* (2), 000–000.

Edmans, Alex, Xavier Gabaix, Tomasz Sadzik, and Yuliy Sannikov, "Dynamic CEO Compensation," *Journal of Finance*, oct 2012, *67* (5), 1603–1647.

Farhi, Emmanuel and Ivan Werning, "Insurance and taxation over the life cycle," *Review of Economic Studies*, 2013, *80* (2), 596–635.

Fernald, John G., "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity," *Federal Reserve Bank of San Francisco Working Paper Series*, 2014, (2012-19).

Golosov, Mikhail, Aleh Tsyvinski, and Nicolas Werquin, "Recursive contracts and endogenously incomplete markets," in "Handbook of Macroeconomics," Vol. 2, Elsevier, 2016, pp. 725–841.

Grigsby, John, Erik Hurst, and Ahu Yildirmaz, "Aggregate Nominal Wage Adjustments: New Evidence from Administrative Payroll Data," *American Economic Review*, 2021, *111* (2), 428–471.

Hagedorn, Marcus, Fatih Karahan, Iourii Manovskii, and Kurt Mitman, "Unemployment benefits and unemployment in the great recession: the role of macro effects," Technical Report, National Bureau of Economic Research 2013.

Hall, Robert E., "Employment Fluctuations with Equilibrium Wage Stickiness," *The American Economic Review*, 2005, *95* (1), 50–65.

Hall, Robert E and Paul R Milgrom, "The Limited Influence of Unemployment on the Wage Bargain," *American Economic Review*, 2008, *98* (4), 1653–1674.

Hazell, Jonathon and Bledi Taska, "Downward Rigidity in the Wage for New Hires," *MIT Working Paper*, 2022.

Ljungqvist, Lars and Thomas J Sargent, "The fundamental surplus," *American Economic Review*, 2017, *107* (9), 2630–65.

Manning, Alan, *Monopsony in Motion: Imperfect Competition in Labor Markets*, Princeton University Press, 2003.

Marimon, Ramon and Jan Werner, "The envelope theorem, Euler and Bellman equations, without differentiability," *Journal of Economic Theory*, 2021, *196*, 105309.

Martins, Pedro S., "Rent Sharing Before and After the Wage Bill," *Applied Economics*, 2009, *41* (17), 2133–51.

Méndez, Fabio and Facundo Sepúlveda, "The cyclicality of skill acquisition: evidence from panel data," *American Economic Journal: Macroeconomics*, 2012, *4* (3), 128–152.

Michaillat, Pascal, "Do matching frictions explain unemployment? Not in bad times," *American Economic Review*, 2012, *102* (4), 1721–1750.

Milgrom, Paul and Ilya Segal, "Envelope theorems for arbitrary choice sets," *Econometrica*, 2002, *70* (2), 583–601.

Mitman, Kurt and Stanislav Rabinovich, "Do Unemployment Benefit Extensions Explain the Emergence of Jobless Recoveries?," 2019.

Ndiaye, Abdoulaye, "Flexible Retirement and Optimal Taxation," *SSRN Electronic Journal*,

2020.

Oyer, Paul, "Why Do Firms Use Incentives That Have No Incentive Effects?," *Journal of Finance*, 2004, *59* (4), 1619–1649.

Pavan, Alessandro, Ilya Segal, and Juuso Toikka, "Dynamic mechanism design: A myersonian approach," *Econometrica*, 2014, *82* (2), 601–653.

Petrosky-Nadeau, Nicolas and Lu Zhang, "Solving the Diamond-Mortensen-Pissarides model accurately," *Quantitative Economics*, jul 2017, *8* (2), 611–650.

Pissarides, Christopher A, *Equilibrium unemployment theory*, MIT press, 2000.

_ , "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?," *Econometrica*, 2009, *77* (5), 1339–1369.

Rogerson, William P., "The First-Order Approach to Principal-Agent Problems," *Econometrica*, 1985, *53* (6), 1357–1367.

Rouwenhorst, K Geert, "10 Asset Pricing Implications of Equilibrium Business Cycle Models," in "Frontiers of business cycle research," Princeton University Press, 1995, pp. 294–330.

Sannikov, Yuliy, "A continuous-time version of the principal-agent problem," *The Review of Economic Studies*, 2008, *75* (3), 957–984.

Schaal, Edouard, "Uncertainty and unemployment," *Econometrica*, 2017, *85* (6), 1675–1721.

Shapiro, Carl and Joseph E Stiglitz, "Equilibrium unemployment as a worker discipline device," *The American economic review*, 1984, *74* (3), 433–444.

Shimer, Robert, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, March 2005, *95* (1), 25–49.

Shin, Donggyun and Gary Solon, "New Evidence on Real Wage Cyclicality Within Employer-Employee Matches," *Scottish Journal of Political Economy*, 2007, *54* (5), 648–660.

Swanson, Eric T, "Real wage cyclicality in the Panel Study of Income Dynamics," *Scottish Journal of Political Economy*, 2007, *54* (5), 617–647.

Wilson, Beth Anne, "Movements of Wages over the Business Cycle: An Intra-Firm View," *Federal Reserve Board Staff Reports*, 1996.