When Does Labor Scarcity Encourage Innovation? Strongly Labor Saving Technological Progress

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October 2008.

Abstract

This paper studies the conditions under which the scarcity of a factor (in particular, labor) encourages technological progress and technology adoption. In standard endogenous growth models, which feature a strong scale effect, an increase in the supply of labor encourages technological progress. In contrast, the famous Habakkuk hypothesis in economic history claims that technological progress was more rapid in 19th-century United States than in Britain because of labor scarcity in the former country. Similar ideas are often suggested as possible reasons for why high wages might have encouraged rapid adoption of certain technologies in continental Europe over the past several decades. I present a general framework for the analysis of these questions. I define the concept of strongly labor saving technological change as follows. Suppose that the aggregate production function of the economy is supermodular in a vector of technologies denoted by θ. Technological progress is strongly labor saving if the production function exhibits decreasing differences in θ and labor. Conversely, technological progress is strongly labor complementary if the production function exhibits increasing differences in θ and labor. The main result of the paper shows that labor scarcity will encourage technological advances if technological progress is strongly labor saving. In contrast, labor scarcity will discourage technological advances if technological progress is strongly labor complementary. I provide examples of environments in which technological progress can be strongly labor saving and also show that such a result is not possible in certain canonical models. These results clarify the conditions under which labor scarcity and high wages encourage technological progress and the reason why such results were obtained or conjectured in certain settings but do not always apply in many models used in the growth literature.

JEL Classification: O30, O31, O33, C65.
Keywords: Habakkuk hypothesis, high wages, innovation, labor scarcity, technological change.

Very Very Preliminary. Please Do Not Circulate.

1 Introduction

There is widespread consensus that technological differences are a central determinant of productivity differences across firms, regions, and nations. Despite this consensus, determinants of technological progress and adoption of new technologies are poorly understood. A basic question concerns the relationship between factor endowments and technology. For example, whether the scarcity of a factor, and the high factor prices that this leads to, will induce technological progress. There is currently no comprehensive answer to this question, though a large literature develops conjectures on this topic. In his pioneering work, *The Theory of Wages*, John Hicks was one of the first economists to consider this possibility and argued:

> "A change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind—directed to economizing the use of a factor which has become relatively expensive..." (1932, p. 124).

Similarly, the famous Habakkuk hypothesis in economic history, proposed by Habakkuk (1962), claims that technological progress was more rapid in 19th-century United States than in Britain because of labor scarcity in the former country, which acted as a powerful inducement for mechanization, for the adoption of labor-saving technologies, and more broadly for innovation. For example, Habakkuk quotes from Pelling:

> "... it was scarcity of labor 'which laid the foundation for the future continuous progress of American industry, by obliging manufacturers to take every opportunity of installing new types of labor-saving machinery.' " (1962, p. 6),

and continues

> "It seems obvious— it certainly seemed so to contemporaries— that the dearness and inelasticity of American, compared with British, labour gave the American entrepreneur ... a greater inducement than his British counterpart to replace labour by machines." (1962, p. 17).

Mantoux's (1961) classic history of the Industrial Revolution also echoes the same theme, emphasizing how the changes in labor costs in spinning and weaving have been a major impetus to innovation in these industries. Robert Allen (2008) extends and strengthens this argument, and proposes that the relatively high wages in 18th-century Britain were the main driver of the Industrial Revolution. Allen, for example, starts his book with a quote from T. Bentley in 1780 explaining the economic dynamism of British industrial centers:

> "... Nottingham, Leicester, Birmingham, Sheffield etc. must long ago have given up all hopes of foreign commerce, if they had not been constantly counteracting the advancing price of manual labor, by adopting every ingenious improvement the human mind could invent."

Similar ideas are often suggested as possible reasons why high wages, for example induced by minimum wages or other regulations, might have encouraged faster adoption of certain technologies, particularly those complementary to unskilled labor, in continental Europe (see, among others, Beaudry and Collard, 2002, Acemoglu 2003, Alesina and Zeira, 2006).

In contrast to these conjectures on how scarcity might lead to more rapid innovation, the standard endogenous growth models make the opposite prediction. These models typically have a single factor of production, labor, and exhibit strong scale effect. An increase in the size of the labor force will induce more rapid technological progress, and either increase the growth rate (in the first-generation models, such as Romer, 1986, 1990, Segerstrom, Anant and Dinopoulos, 1990, Aghion and Howitt, 1992, Grossman and Helpman, 1991), or the level of output (in the semi-endogenous growth models such as Jones, 1995, Young, 1998, Howitt, 1999). In addition, models based on the neoclassical growth model, with new technologies embodied in capital goods, also predict that labor scarcity or high wages should discourage the adoption of new technologies.[1]

The directed technological change literature (e.g., Acemoglu, 1998, 2002, 2007) provides a characterization of how the bias of technology will change in response to changes in the supplies of factors. In particular, Acemoglu (2007) establishes that, under weak regularity conditions, an increase in the supply of a factor always induces a change in technology *biased* towards that factor.[2] This result implies that labor scarcity will lead to technological changes biased against labor. Nevertheless, these results do not address the question of whether the scarcity of a factor will lead to *faster technological progress* or change technology in a way that increases the level of output in the economy. Reviewing the intuition for previous results is useful to explain why this is. In particular, suppose that technology can be represented by a single variable θ and suppose that θ and labor, L, are complements. Then, an increase in the supply of labor induces an increase in θ, and because of θ-L complementarity, this induced change is biased towards labor. Conversely, if θ and L are substitutes, an increase in the supply of labor

[1] See Ricardo (1951) for an early statement of this view.

[2] A change in technology is biased towards a factor if it increases the marginal product of this factor at given factor proportions. Conversely, a change in technology is biased against a factor if it reduces its marginal product at given factor proportions.

leads to a decline in θ, and now because of θ-L substitutability, the decrease in θ increases the marginal product of labor and is again biased towards this factor. This discussion not only illustrates the robust logic of equilibrium bias in response to changes in supply, but it also makes it clear that the induced changes in technology, though always biased towards the more abundant factor, could take the form of more or less advanced technologies; in other words, they may correspond to the choice of equilibrium technologies that increase or reduce output.[3]

The question of whether scarcity of a particular factor of production, in particular, labor, spurs further innovation and adoption of technologies increasing output is as important as the implications of these induced changes on the bias of technology. This paper investigates the impact of labor scarcity on technological advances and innovation. I present a general framework and a comprehensive answer to this question. The key is the concept of *strongly factor (labor) saving technological change*.[4] Suppose aggregate output (or net output) can be expressed as a function $Y(L, Z, \theta)$, where L denotes labor, Z is a vector of other factors of production, and θ is a vector of technologies. Suppose that Y is *supermodular* in θ, so that changes in two components of the vector θ do not offset each other. Suppose also that an increase in (any component of) θ increases output. Then technological progress is strongly labor saving if $Y(L, Z, \theta)$ exhibits *decreasing differences* in θ and L. Intuitively, this means that technological progress reduces the marginal product of labor. Conversely, we say that technological progress is *strongly labor complementary* if $Y(L, Z, \theta)$ exhibits *increasing differences* in θ and L. The main result of the paper shows that labor scarcity encourages technological progress if technological change is strongly labor saving. In contrast, when technological change is strongly labor complementary, then labor scarcity discourages technological progress.

This result can be interpreted both as a positive and a negative one. On the positive side, it characterizes a wide range of economic environments where labor scarcity can act as a force towards innovation and technological progress, as claimed in various previous historical and economic analyses. On the negative side, most models we use in the growth literature exhibit increasing rather than decreasing differences between technology and labor. Moreover, the intuition that technological change has been the key driving force of the secular increase in wages also suggests that increasing differences may be more likely than decreasing differences.[5]

[3] In a dynamic framework, changes towards less advanced technologies would typically take the form of a slowdown in the adoption and invention of new, more advanced technologies.

[4] The adjective "strongly" is added here, since the term "labor saving" is often used for many different contexts and in most of these cases, the "decreasing differences" conditions here are not satisfied.

[5] Nevertheless, it should be noted that in the context of a dynamic model, it is possible for the past technological changes to increase wage levels, while current technology adoption decisions, at the margin, reduce the marginal product of labor. This is illustrated by the dynamic model presented in subsection 5.1.

To highlight what decreasing differences between technology and labor means in specific settings, I consider several different environments and production functions, and discuss when the decreasing differences condition is satisfied. An important class of models where technological change can be strongly labor saving is developed by Champernowne (1963), Zeira (1998, 2006), Hellwig and Irmen (2001) and Alesina and Zeira (2006). In these models, technological change takes the form of machines replacing tasks previously performed by labor. I show that there is indeed a tendency of technology to be strongly labor saving in these models, though there are also competing effects that need to be incorporated into the analysis.

Most of the analysis focuses on the implications of labor scarcity on technology choices. These results are directly applicable to the question of the impact of wage push on technology choices in the context of a competitive labor market. In particular, in this context, a minimum wage above the equilibrium wage is equivalent to a decline in labor supply.[6] Nevertheless, I also show the conditions under which the implications of labor scarcity and wage push can be very different—particularly because the long-run relationship between labor supply and wages could be upward sloping owing to general equilibrium technology effects.

Even though the investigation here is motivated by technological change and the study of economic growth, the economic environment I use to investigate these questions is static. A static framework is useful because it enables us to remove functional form restrictions that would be necessary to generate endogenous growth and allows the appropriate level of generality to clarify the conditions for labor scarcity to encourage innovation and technology adoption. This framework is based on Acemoglu (2007). Section 2 adapts this framework for the current paper and also contains a review of the results on equilibrium bias that are useful for later sections. The main results are contained in Section 3. These results are applied to a number of familiar models in Section 4. Section 5 discusses various extensions. Section 6 concludes.

2 The Basic Environments and Review

This section is based on, reviews, and extends some of the results in Acemoglu (2007). Its inclusion is necessary for the development of the main results in Section 3, but since the results are already present in previous work, I omit many details. Consider a static economy consisting of a unique final good and $N+1$ factors of production. The first factor of production

[6] The implications of "wage push" in noncompetitive labor markets are more complex and depend on the specific aspects of labor market imperfections and institutions. For example, Acemoglu (2003) shows that wage push resulting from a minimum wage or other labor market regulations can encourage technology adoption when there is wage bargaining and Karen sharing.

is labor, denoted by L, and the rest are denoted by the vector $Z=(Z_1,...,Z_N)$ and stand for land, capital, and other human or nonhuman factors. All agents' preferences are defined over the consumption of the final good. To start with, let us assume that all factors are supplied inelastically, with supplies denoted by $\bar{L} \in \mathbb{R}_+$ and $\bar{Z} \in \mathbb{R}_+^N$. Throughout I will focus on comparative statics with respect to changes in the supply of labor, while holding the supply of other factors, Z, constant at some level $\bar{Z}$ (though, clearly, mathematically there is nothing special about labor).[7] The economy consists of a continuum of firms (final good producers) denoted by the set $\mathcal{F}$, each with an identical production function. Without loss of any generality let us normalize the measure of $\mathcal{F}$, $|\mathcal{F}|$, to 1. The price of the final good is also normalized to 1.

I first describe technology choice in four different economic environments. These are:

1. Economy D (for *decentralized*) is a decentralized competitive economy in which technologies are chosen by firms themselves. In this economy, technology choice can be interpreted as choice of just another set of factors and the entire analysis can be conducted in terms of technology adoption.

2. Economy E (for *externality*) is identical to Economy D, except for a technological externality as in Romer (1986).

3. Economy M (for *monopoly*) will be the main environment used for much of the analysis in the remainder of the paper. In this economy, technologies are created and supplied by a profit-maximizing monopolist.

4. Economy O (for *oligopoly*) features a set of oligopolistically (or monopolistically) competitive firms supplying technologies the rest of the economy. After introducing the main results for Economy M, I then generalize them to this environment.

2.1 Economy D—Decentralized Equilibrium

In the first environment, Economy D, all markets are competitive and technology is decided by each firm separately. This environment is introduced as a benchmark.

Each firm $i \in \mathcal{F}$ has access to a production function

$$y^i = G(L^i, Z^i, \theta^i), \tag{1}$$

[7] Endogenous responses of the supply of labor and other factors, such as capital, are discussed in subsection 5.2.

where $L^i \in \mathcal{L} \subset \mathbb{R}_+$, $Z^i \in \mathcal{Z} \subset \mathbb{R}_+^N$ and $\theta^i \in \Theta \subset \mathbb{R}^K$ is the measure of technology. I use lower case y^i to denote output, since Y will be defined as net aggregate output below. The production function G is assumed to be twice continuously differentiable in (L^i, Z^i, θ^i) on its domain $\mathcal{L} \times \mathcal{Z} \times \Theta$. The cost of technology $\theta \in \Theta$ in terms of final goods is $C(\theta)$. The function $C(\theta)$ can also be taken to be twice continuously differentiable. For now, we do not need to assume that $C(\theta)$ is increasing.

Each final good producer maximizes profits, or in other words, solves the following problem:

$$\max_{L^i \in \mathcal{L}, Z^i \in \mathcal{Z}, \theta^i \in \Theta} \pi(L^i, Z^i, \theta^i) = G(L^i, Z^i, \theta^i) - w_L L^i - \sum_{j=1}^{N} w_{Zj} Z_j^i - C\left(\theta^i\right), \tag{2}$$

where w_L is the wage rate and w_{Zj} is the price of factor Z_j for $j = 1, ..., N$, all taken as given by the firm. The vector of prices for factors Z is denoted by w_Z. Since there is a total supply $\bar{L}$ of labor and a total supply $\bar{Z}_j$ of Z_j, market clearing requires

$$\int_{i \in \mathcal{F}} L^i di \leq \bar{L} \text{ and } \int_{i \in \mathcal{F}} Z_j^i di \leq \bar{Z}_j \text{ for } j = 1, ..., N. \tag{3}$$

Definition 1 An equilibrium in Economy D is a set of decisions $\left\{L^i, Z^i, \theta^i\right\}_{i \in \mathcal{F}}$ and factor prices (w_L, w_Z) such that $\left\{L^i, Z^i, \theta^i\right\}_{i \in \mathcal{F}}$ solve (2) given prices (w_L, w_Z) and (3) holds.

I refer to any θ^i that is part of the set of equilibrium allocations, $\left\{L^i, Z^i, \theta^i\right\}_{i \in \mathcal{F}}$, as *equilibrium technology*. For notational convenience let us define the "net production function":

$$F(L^i, Z^i, \theta^i) \equiv G(L^i, Z^i, \theta^i) - C\left(\theta^i\right). \tag{4}$$

Assumption 1 Either $F(L^i, Z^i, \theta^i)$ is jointly strictly concave in (L^i, Z^i, θ^i) and increasing in (L^i, Z^i), and $\mathcal{L}$, $\mathcal{Z}$ and Θ are convex; or $F(L^i, Z^i, \theta^i)$ is increasing in (L^i, Z^i) and exhibits constant returns to scale in (L^i, Z^i, θ^i).

Assumption 1 is restrictive, since it requires concavity (strict concavity or constant returns to scale) jointly in the factors of production and technology. Such an assumption is necessary for a competitive equilibrium to exist; the other economic environments considered below will relax this assumption.

Proposition 1 Suppose Assumption 1 holds. Then any equilibrium technology θ in Economy D is a solution to

$$\max_{\theta' \in \Theta} F(\bar{L}, \bar{Z}, \theta'), \tag{5}$$

and any solution to this problem is an equilibrium technology.

Proposition 1 implies that to analyze equilibrium technology choices, we can simply focus on a simple maximization problem. An important implication of this proposition is that the equilibrium is a Pareto optimum (and vice versa) and corresponds to a maximum of F in the entire vector $\left(L^i, Z^i, \theta^i\right)$.

It is also straightforward to see that equilibrium factor prices are equal to the marginal products of the G or the F functions. That is, the wage rate is $w_L = \partial G(\bar{L}, \bar{Z}, \theta)/\partial L = \partial F(\bar{L}, \bar{Z}, \theta)/\partial L$ and the prices of other factors are given by $w_{Zj} = \partial G(\bar{L}, \bar{Z}, \theta)/\partial Z_j = \partial F(\bar{L}, \bar{Z}, \theta)/\partial Z_j$ for $j = 1, ..., N$, where θ is the equilibrium technology choice (and where the second set of equalities follow in view of equation (4)).

An important implication of (5) should be emphasized at this point. Since equilibrium technology is a maximizer of $F(\bar{L}, \bar{Z}, \theta)$, this implies that any induced small change in technology, θ, cannot be construed as "technological progress," since it will have no effect on net output at the starting factor proportions. In particular, for future use, let us introduce the notation $Y\left(\bar{L}, \bar{Z}, \theta\right)$ to denote net output in the economy with factor supplies given by $\bar{L}$ and $\bar{Z}$. Clearly, in Economy D, $Y\left(\bar{L}, \bar{Z}, \theta\right) \equiv F\left(\bar{L}, \bar{Z}, \theta\right) \equiv G\left(\bar{L}, \bar{Z}, \theta\right) - C\left(\theta\right)$. Then, assuming that the equilibrium technology θ^* is differentiable in $\bar{L}$, the change in net output in response to a change in the supply of labor, $\bar{L}$, can be written as

$$\frac{dY(\bar{L}, \bar{Z}, \theta^*)}{d\bar{L}} = \frac{\partial Y(\bar{L}, \bar{Z}, \theta^*)}{\partial \bar{L}} + \frac{\partial Y(\bar{L}, \bar{Z}, \theta^*)}{\partial \theta} \frac{d\theta^*}{d\bar{L}}, \tag{6}$$

where the second term is the induced technology effect. When this term is strictly negative, then a decrease in labor supply (labor scarcity) will have induced a change in technology that increases output—that is, a "technological advance". However, by the envelope theorem, this second term is equal to zero, since θ^* is a solution to (5). Therefore, there is no effect on net output through induced technological changes and no possibility of induced technological progress because of labor scarcity in this environment.

2.2 Economy E—Decentralized Equilibrium with Externalities

The discussion at the end of the previous subsection indicated why Economy D does not enable a systematic study of the relationship between labor scarcity and technological progress. For this reason, I will use Economy M, introduced in the next subsection, as the baseline environment for analysis. This section proposes an alternative, which is in many ways closer to Economy D, which can also be used for such an analysis. In particular, let us follow Romer (1986) and suppose that technology choices create positive externalities on other firms. In particular, suppose that output of producer i is now given by

$$y^i = G(L^i, Z^i, \theta^i, \bar{\theta}), \tag{7}$$

where $\bar{\theta}$ is some aggregate of the technology choices of all other firms in the economy. For simplicity, we can take $\bar{\theta}$ to be the sum of all firms' technologies. In particular, if θ is a K-dimensional vector, then $\bar{\theta}_k = \int_{i\in\mathcal{F}} \theta_k^i di$ for each component of the vector (i.e., for $k = 1, 2, ..., K$). The rest of the assumptions are the same as before (in particular, with G being jointly concave in L^i, Z^i and θ^i) and a slightly modified version of Proposition 1 can be obtained. Let us first note that the maximization problem of each firm now becomes

$$\max_{Z^i\in L, Z^i\in\mathcal{Z}, \theta_i\in\Theta} \pi(L^i, Z^i, \theta^i, \bar{\theta}) = G(L^i, Z^i, \theta^i, \bar{\theta}) - w_L L^i - \sum_{j=1}^{N} w_{Zj} Z_j^i - C\left(\theta^i\right), \tag{8}$$

and under the same assumptions as above, each firm will hire the same amount of all factors, so in equilibrium, $L^i = \bar{L}$ and $Z^i = \bar{Z}$ for all $i \in \mathcal{F}$. Then the following proposition characterizes equilibrium technology.

Proposition 2 Equilibrium technology in Economy E is a solution to the following fixed point problem:

$$\theta \in \arg\max_{\theta'\in\Theta} G(\bar{L}, \bar{Z}, \theta', \bar{\theta} = \theta) - C\left(\theta'\right). \tag{9}$$

Even though this is a fixed point problem, its structure is very similar to (5) and it can be used in the same way for our analysis. However, crucially, the envelope theorem type reasoning no longer applies to the equivalent of equation (6). In particular, let us define net output again as $Y(\bar{L}, \bar{Z}, \theta) \equiv G(\bar{L}, \bar{Z}, \theta, \theta) - C(\theta)$. Then once again assuming differentiability, we have

$$\frac{dY(\bar{L}, \bar{Z}, \theta^*)}{d\bar{L}} = \frac{\partial Y(\bar{L}, \bar{Z}, \theta^*)}{\partial \bar{L}} + \frac{\partial Y(\bar{L}, \bar{Z}, \theta^*)}{\partial \theta} \frac{d\theta^*}{d\bar{L}},$$

but now second term is not equal to zero. In particular, if $C(\theta)$ is increasing in θ and if externalities are positive (that is, if G is increasing in the vector $\bar{\theta}$), then $\partial Y/\partial\theta$ will be positive and induced increases θ will raise output and thus correspond to induced technological advances.

2.3 Economy M—Monopoly Equilibrium

The main environment used for the analysis in this paper features a monopolist supplying technologies to final good producers. There is a unique final good and each firm has access to the production function

$$y^i = \alpha^{-\alpha}(1-\alpha)^{-1} G(L^i, Z^i, \theta^i)^{\alpha} q\left(\theta^i\right)^{1-\alpha}, \tag{10}$$

with $\alpha \in (0,1)$. This is similar to (1), except that $G(L^i, Z^i, \theta^i)$ is now a subcomponent of the production function, which depends on θ^i, the technology used by the firm. The subcomponent G needs to be combined with an intermediate good embodying technology θ^i, denoted by $q(\theta^i)$—conditioned on θ^i to emphasize that it embodies technology θ^i. This intermediate good is supplied by the monopolist. The term $\alpha^{-\alpha}(1-\alpha)^{-1}$ is a convenient normalization. The more important role of the parameter α is in determining the elasticity of aggregate output to the component G. The higher is α, the more responsive is output to G (and the less responsive it is to the quantity of intermediate goods).

This production structure is similar to models of endogenous technology (e.g., Romer, 1990, Grossman and Helpman, 1991, Aghion and Howitt, 1992), but is somewhat more general since it does not impose that technology necessarily takes a factor-augmenting form. I continue to assume that $L^i \in \mathcal{L} \subset \mathbb{R}_+, Z^i \in \mathcal{Z} \subset \mathbb{R}_+^N$, that G is twice continuously differentiable in (L^i, Z^i, θ^i), and that $C(\theta)$ is convex and twice continuously differentiable. Moreover, for the remainder of the analysis, let us also assume that $C(\theta)$ is increasing in θ (that is, it is increasing in each component of the vector θ), so that higher θ can be interpreted as "more advanced" technology.

The monopolist can create technology θ at cost $C(\theta)$ from the technology menu. Once θ is created, the technology monopolist can produce the intermediate good embodying technology θ at constant per unit cost normalized to $1-\alpha$ unit of the final good (this is also a convenient normalization). It can then set a (linear) price per unit of the intermediate good of type θ, denoted by χ.

All factor markets are again competitive, and each firm takes the available technology, θ, and the price of the intermediate good embodying this technology, χ, as given and maximizes

$$\max_{\substack{L^i \in \mathcal{L}, Z^i \in \mathcal{Z}, \\ q(\theta) \geq 0}} \pi(L^i, Z^i, q(\theta) \mid \theta, \chi) = \alpha^{-\alpha}(1-\alpha)^{-1} G(L^i, Z^i, \theta)^{\alpha} q(\theta)^{1-\alpha} - w_L L^i - \sum_{j=1}^{N} w_{Zj} Z_j^i - \chi q(\theta), \tag{11}$$

which gives the following simple inverse demand for intermediates of type θ as a function of its price, χ, and the factor employment levels of the firm as

$$q^i(\theta, \chi, L^i, Z^i) = \alpha^{-1} G(L^i, Z^i, \theta) \chi^{-1/\alpha}. \tag{12}$$

The problem of the monopolist is to maximize its profits:

$$\max_{\theta, \chi, [q^i(\theta, \chi, L^i, Z^i)]_{i \in \mathcal{F}}} \Pi = (\chi - (1-\alpha)) \int_{i \in \mathcal{F}} q^i(\theta, \chi, L^i, Z^i)\, di - C(\theta) \tag{13}$$

subject to (12). Therefore, an equilibrium in this economy can be defined as:

Definition 2 An equilibrium in Economy M is a set of firm decisions $\left\{L^i, Z^i, q^i\left(\theta, \chi, L^i, Z^i\right)\right\}_{i\in\mathcal{F}}$, technology choice θ, and factor prices (w_L, w_Z) such that $\left\{L^i, Z^i, q^i\left(\theta, \chi, L^i, Z^i\right)\right\}_{i\in\mathcal{F}}$ solve (11) given (w_L, w_Z) and technology θ, (3) holds, and the technology choice and pricing decision for the monopolist, (θ, χ), maximize (13) subject to (12).

This definition emphasizes that factor demands and technology are decided by different agents (the former by the final good producers, the latter by the technology monopolist), which is an important feature both theoretically and as a representation of how technology is determined in practice. Since factor demands and technology are decided by different agents, Assumption 1 can now be relaxed and replaced by the following.

Assumption 2 Either $G(L^i, Z^i, \theta^i)$ is jointly strictly concave and increasing in (L^i, Z^i) and $\mathcal{L}$ and $\mathcal{Z}$ are convex; or $G(L^i, Z^i, \theta^i)$ is increasing and exhibits constant returns to scale in (L^i, Z^i).

This assumption no longer requires joint concavity of the function G in L^i, Z^i and θ, but only in L^i and Z^i.

To characterize the equilibrium, note that (12) defines a constant elasticity demand curve, so the profit-maximizing price of the monopolist is given by the standard monopoly markup over marginal cost and is equal to $\chi = 1$. Consequently, $q^i(\theta) = q^i\left(\theta, \chi = 1, \bar{L}, \bar{Z}\right) = \alpha^{-1}G(\bar{L}, \bar{Z}, \theta)$ for all $i \in \mathcal{F}$. Substituting this into (13), the profits and the maximization problem of the monopolist can be expressed as

$$\max_{\theta\in\Theta} \Pi(\theta) = G(\bar{L}, \bar{Z}, \theta) - C(\theta). \tag{14}$$

Thus we have established the following proposition.

Proposition 3 Suppose Assumption 2 holds. Then any equilibrium technology θ in Economy M is a solution to

$$\max_{\theta'\in\Theta} F(\bar{L}, \bar{Z}, \theta') \equiv G(\bar{L}, \bar{Z}, \theta') - C\left(\theta'\right) \tag{15}$$

and any solution to this problem is an equilibrium technology.

This proposition shows that equilibrium technology in Economy M is a solution to a problem identical to that in Economy D, that of maximizing $F(\bar{L}, \bar{Z}, \theta) \equiv G(\bar{L}, \bar{Z}, \theta) - C(\theta)$ as in (4). Naturally, the presence of the monopoly markup introduces distortions in the equilibrium. These distortions are important in ensuring that equilibrium technology is not exactly at the level that maximizes net output. In particular, let us use the fact that the profit-maximizing monopoly price is $\chi = 1$ and substitute (12) into the production function (10), and then subtract the cost of technology choice, $C(\theta)$, and the cost of production of the machines, $(1-\alpha)\alpha^{-1}G(L^i, Z^i, \theta)$, from gross output. This gives net output in this economy as

$$Y\left(\bar{L}, \bar{Z}, \theta\right) \equiv \frac{2-\alpha}{1-\alpha} G\left(\bar{L}, \bar{Z}, \theta\right) - C(\theta). \tag{16}$$

Clearly, since the coefficient in front of $G\left(\bar{L}, \bar{Z}, \theta\right)$ is strictly greater than one, as in Economy E, $Y\left(\bar{L}, \bar{Z}, \theta\right)$ will be increasing in θ in the neighborhood of θ^* that is a solution to (15) (recall that $C(\theta)$ is increasing in θ).

Finally, it can be verified that in this economy equilibrium factor prices are given by $w_L = (1-\alpha)^{-1} \partial G(\bar{L}, \bar{Z}, \theta)/\partial L$ and $w_{Zj} = (1-\alpha)^{-1} \partial G(\bar{L}, \bar{Z}, \theta)/\partial Z_j$, which are proportional to the derivatives of the G or the F function defined in (4) as well as the derivatives of the net output function Y defined in (16).

In what follows, I take Economy M as the baseline.

2.4 Economy O—Oligopoly Equilibrium

It is also straightforward to extend the environment in the previous subsection so that technologies are supplied by a number of competing (oligopolistic) firms rather than a monopolist. Let θ^i be the vector $\theta^i \equiv \left(\theta^i_1, ..., \theta^i_S\right)$, and suppose that output is now given by

$$y^i = \alpha^{-\alpha}(1-\alpha)^{-1} G(L^i, Z^i, \theta^i)^{\alpha} \sum_{s=1}^{S} q_s\left(\theta^i_s\right)^{1-\alpha}, \tag{17}$$

where $\theta^i_s \in \Theta_s \subset \mathbb{R}^{K_s}$ is a technology supplied by technology producer $s = 1, ..., S$, and $q_s\left(\theta^i_s\right)$ is an intermediate good (or machine) produced and sold by technology producer s, which embodies technology θ^i_s. Factor markets are again competitive, and a maximization problem similar to (11) gives the inverse demand functions for intermediates as

$$q^i_s\left(\theta, \chi_s, L^i, Z^i\right) = \alpha^{-1} G(L^i, Z^i, \theta) \chi_s^{-1/\alpha}, \tag{18}$$

where χ_s is the price charged for intermediate good $q_s\left(\theta^i_s\right)$ by technology producer $s = 1, ..., S$.

Let the cost of creating technology θ_s be $C_s(\theta_s)$ for $s = 1, ..., S$, convex, and strictly increasing. The cost of producing each unit of any intermediate good is again normalized to $1-\alpha$.

Definition 3 An equilibrium in Economy O is a set of firm decisions $\left\{L^i, Z^i, \left[q^i_s\left(\theta, \chi_s, L^i, Z^i\right)\right]_{s=1}^{S}\right\}_{i \in \mathcal{F}}$, technology choices $(\theta_1, ..., \theta_S)$, and factor prices (w_L, w_Z)

such that $\left\{L^i, Z^i, \left[q_s^i\left(\theta, \chi_s, L^i, Z^i\right)\right]_{s=1}^S\right\}_{i\in\mathcal{F}}$ maximize firm profits given (w_L, w_Z) and the technology vector $(\theta_1, ..., \theta_S)$, (3) holds, and the technology choice and pricing decision for technology producer $s = 1, ..., S$, (θ_s, χ_s), maximize its profits subject to (18).

The profit maximization problem of each technology producer is similar to (13) and implies a profit-maximizing price for intermediate goods equal to $\chi_s = 1$ for any $\theta_s \in \Theta_s$ and each $s = 1, ..., S$. Consequently, with the same steps as in the previous subsection, each technology producer will solve the problem:

$$\max_{\theta_s\in\Theta_s} \Pi_s\left(\theta_s\right) = G(\bar{L}, \bar{Z}, \theta_1, ..., \theta_s, ..., \theta_S) - C_s\left(\theta_s\right). \tag{19}$$

This argument establishes the following proposition:

Proposition 4 Suppose Assumption 2 holds. Then any equilibrium technology in Economy O is a vector $(\theta_1^*, ..., \theta_S^*)$ such that θ_s^* is solution to

$$\max_{\theta_s\in\Theta_s} G(\bar{L}, \bar{Z}, \theta_1^*, ..., \theta_s, ..., \theta_S^*) - C_s\left(\theta_s\right)$$

for each $s = 1, ..., S$, and any such vector gives an equilibrium technology.

This proposition shows that the equilibrium corresponds to a Nash equilibrium and thus, as in Economy E, it is given by a fixed point problem. Nevertheless, this has little effect on the results below and all of the results stated in this paper hold for this oligopolistic environment.

It is also worth noting that the special case where $\partial^2 G/\partial\theta_s\partial\theta_{s'} = 0$ for all s and s' is identical to the product variety models of Romer (1990) and Grossman and Helpman (1991), and in this case, the equilibrium can again be represented as a solution to a unique maximization problem, i.e., that of maximizing $G(\bar{L}, \bar{Z}, \theta_1, ..., \theta_s, ..., \theta_S) - \sum_{s=1}^S C_s\left(\theta_s\right)$. Finally, note also that, with a slight modification, this environment can also embed monopolistic competition, where the number of firms is endogenous and determined by zero profit condition (the technology choice of non-active firms will be equal to zero in this case, and the equilibrium problem will be $\max_{\theta_s\in\Theta_s} G(\bar{L}, \bar{Z}, \theta_1^*, ..., \theta_s, ..., \theta_{S'}^*, 0, ..., 0) - C_s\left(\theta_s\right)$ for $1 \leq s \leq S'$, with S' determined endogenously in equilibrium.

2.5 Review of Previous Results

Let us also briefly review the previous results concerning the bias of technology in response to changes in factor supplies. These results apply to all of the environments discussed so far, though for concreteness, the reader may wish to consider Economy M. Further details and the proofs of these results can be found in Acemoglu (2007). Recall that θ is a K-dimensional vector and let us denote the equilibrium technology at factor supplies $\left(\bar{L}, \bar{Z}\right) \in \mathcal{L} \times \mathcal{Z}$ by $\theta^* \left(\bar{L}, \bar{Z}\right)$. Moreover, to simplify the discussion here, it is also useful to impose the following Inada-type assumption:

Assumption 3 Θ is a convex subset of $\mathbb{R}^K$, $C(\theta)$ is twice continuously differentiable and convex in θ on Θ. Moreover, for each $k = 1, 2, ..., K$, we have

$$\frac{\partial C(\theta_k, \theta_{-k})}{\partial \theta_k} > 0, \lim_{\theta_k \to 0} \frac{\partial C(\theta_k, \theta_{-k})}{\partial \theta_k} = 0 \text{ and } \lim_{\theta_k \to \infty} \frac{\partial C(\theta_k, \theta_{-k})}{\partial \theta_k} = \infty \text{ for all } \theta_{-k},$$

and

$$\frac{\partial G\left(\bar{L}, \bar{Z}, \theta_k, \theta_{-k}\right)}{\partial \theta_k} \in (0, +\infty) \text{ for all } \theta_{-k} \text{ and for all } \left(\bar{L}, \bar{Z}\right) \in \mathcal{L} \times \mathcal{Z}.$$

This assumption is not necessary for the results and is adopted to simplify the notation (and in fact Theorem 2 is stated without this assumption). The only important part of this assumption, that C is increasing, was already mentioned above and will ensure that we can think of higher θ as corresponding to "more advanced" technology.

In addition, let us suppose for now that $\partial \theta_k^* / \partial L$ exists at $\left(\bar{L}, \bar{Z}\right)$ for all $k = 1, ..., K$. Then we say that there is weak (absolute) equilibrium bias at $\left(\bar{L}, \bar{Z}, \theta^* \left(\bar{L}, \bar{Z}\right)\right)$ if

$$\sum_{k=1}^{K} \frac{\partial w_L}{\partial \theta_k} \frac{\partial \theta_k^*}{\partial L} \geq 0. \tag{20}$$

In other words, there is weak equilibrium bias if the induced change in technology resulting from an increase in labor supply increases the marginal product of labor at the starting factor proportions. The next result shows that an increase in labor supply (or the supply of any other factor) will always induce weak equilibrium bias (provided that $\partial \theta_j^* / \partial L$ exists). Conversely, it also shows that labor scarcity (here corresponding to a decline in the supply of labor) will induce technological changes that are biased against labor.

Theorem 1 Suppose that Assumption 3 holds and that $F(L, Z, \theta)$ is twice continuously differentiable in (Z, θ). Let the equilibrium technology at factor supplies $\left(\bar{L}, \bar{Z}\right)$ be $\theta^* \left(\bar{L}, \bar{Z}\right)$ and assume that $\partial \theta_k^* / \partial L$ exists at $\left(\bar{L}, \bar{Z}\right)$ for all $k = 1, ..., K$. Then, there is *weak absolute equilibrium bias* at all $\left(\bar{L}, \bar{Z}\right) \in \mathcal{L} \times \mathcal{Z}$, i.e.,

$$\sum_{k=1}^{K} \frac{\partial w_L}{\partial \theta_k} \frac{\partial \theta_k^*}{\partial L} \geq 0 \text{ for all } \left(\bar{L}, \bar{Z}\right) \in \mathcal{L} \times \mathcal{Z}, \tag{21}$$

with strict inequality if $\partial \theta_k^* / \partial L \neq 0$ for some $k = 1, ..., K$.

Note also that while this result and all of the other theorems below are stated for changes in L, these changes are also equivalent to changes in factor proportions provided that the G function (or equivalently the F function) is homothetic in L and Z and, as assumed here, the supplies of the other factors are held constant. The case in which some of these supplies may also endogenously respond to the change in $\bar{L}$ or to wage push is discussed in subsection 5.2.

The next theorem extends the results to cases in which there are large changes in supplies. For this result and for what will come in the next section, recall that given a vector $x = (x_1, ..., x_n)$ in $\mathbb{R}^n$, twice continuously differentiable function $f(x)$ is *supermodular* on X if and only if $\partial^2 f(x)/\partial x_i \partial x_{i'} \geq 0$ for all $x \in X$ and for all $i \neq i'$. In addition, a function $f(x,t)$ defined on $X \times T$ (where $X \subset \mathbb{R}^n$ and $T \subset \mathbb{R}^m$) has *increasing differences* (strict increasing differences) in (x,t), if for all $t'' > t$, $f(x,t'') - f(x,t)$ is nondecreasing (increasing) in x. The concept of *decreasing differences* is defined similarly and requires $f(x,t'') - f(x,t)$ to be nonincreasing.

Theorem 2 Suppose that $\Theta \subset \mathbb{R}^K$, let $\bar{\mathcal{L}}$ be the convex hull of $\mathcal{L}$, let $\theta^*(\bar{L}, \bar{Z})$ be the equilibrium technology at factor proportions $(\bar{L}, \bar{Z})$, and suppose that $F(L, Z, \theta)$ is continuously differentiable in L, supermodular in θ on Θ for all $L \in \bar{\mathcal{L}}$ and $Z \in \mathcal{Z}$, and exhibits strictly increasing differences in (L, θ) on $\bar{\mathcal{L}} \times \Theta$ for all $Z \in \mathcal{Z}$, then there is *global absolute equilibrium bias*. That is, for any $\bar{L}', \bar{L} \in \mathcal{L}$, $\bar{L}' \geq \bar{L}$ implies

$$\theta^*(\bar{L}', \bar{Z}) \geq \theta^*(\bar{L}, \bar{Z}) \text{ for all } \bar{Z} \in \mathcal{Z},$$

and

$$w_L\left(\tilde{L}, \bar{Z}, \theta^*(\bar{L}', \bar{Z})\right) \geq w_L\left(\tilde{L}, \bar{Z}, \theta^*(\bar{L}, \bar{Z})\right) \text{ for all } \tilde{L} \in \mathcal{L} \text{ and } \bar{Z} \in \mathcal{Z}, \tag{22}$$

with strict inequality if $\theta^*(\bar{L}', \bar{Z}) \neq \theta^*(\bar{L}, \bar{Z})$.

One reason for mentioning this theorem is to emphasize the role of supermodularity. Without supermodularity, a result of this type is not possible because changes in different components of technology θ would not be mutually reinforcing and thus counterintuitive results are possible. Clearly, when θ is single dimensional as most neoclassical and endogenous growth models assume, the supermodularity condition is trivially satisfied.

Finally, the next theorem is a somewhat striking result, which shows how general equilibrium technology choices can make the long run relationship between the supply of factor and its price increasing. It will play two important roles in our analysis. First, it clarifies the conditions under which labor scarcity and wage push can be analyzed in a unified manner.

Second, it will later allow us to discuss how, under certain conditions, the implications of wage push are different from labor scarcity.

Let us first define strong equilibrium bias. We say that there is *strong (absolute) equilibrium bias* at $(\bar{L}, \bar{Z}) \in \mathcal{L} \times \mathcal{Z}$ if

$$\frac{dw_L}{dL} = \frac{\partial w_L}{\partial L} + \sum_{k=1}^{K} \frac{\partial w_L}{\partial \theta_k} \frac{\partial \theta_k^*}{\partial L} > 0.$$

Clearly, here dw_L/dL denotes the total derivative, while $\partial w_L/\partial L$ denotes the partial derivative holding $\theta = \theta^*(\bar{L}, \bar{Z})$. Recall also that if F is jointly concave in (L, θ) at $(L, \theta^*(\bar{L}, \bar{Z}))$, its Hessian with respect to (L, θ), $\nabla^2 F_{(L,\theta)(L,\theta)}$, is negative semi-definite at this point (though negative semi-definiteness is not sufficient for local joint concavity).

Theorem 3 Suppose that Assumption 3 holds and that F is twice continuously differentiable in (L, θ). Let $\theta^*(\bar{L}, \bar{Z})$ be the equilibrium technology at factor supplies $(\bar{L}, \bar{Z})$ and assume that $\partial \theta_k^*(\bar{L}, \bar{Z})/\partial L$ exists at $(\bar{L}, \bar{Z})$ for all $k = 1, ..., K$. Then there is *strong absolute equilibrium bias* at $(\bar{L}, \bar{Z})$ if and only if $F(L, Z, \theta)$'s Hessian in (L, θ), $\nabla^2 F_{(L,\theta)(L,\theta)}$, *is not negative semi-definite* at $(\bar{L}, \bar{Z})$.

A number of implications of this theorem are worth noting. First, in contrast to basic producer theory, where all demand curves are downward sloping, this result shows that endogenous technology choices in general equilibrium can easily lead to upward sloping demand curves for factors. In particular, the condition that $\nabla^2 F_{(L,\theta)(L,\theta)}$ is not negative semi-definite is not very restrictive, since technology and factors are being chosen by different agents (monopolists or oligopolists on the one hand and final good producers on the other). However, this result also highlights that there cannot be strong bias in a fully competitive economy such as Economy D, because equilibrium existence in Economy D imposes convexity of the aggregate production possibilities set and thus ensures that $\nabla^2 F_{(L,\theta)(L,\theta)}$ must be negative semi-definite. Most importantly for our purposes here, this theorem implies that when $\nabla^2 F_{(L,\theta)(L,\theta)}$ is negative semi-definite, then the equilibrium relationship (taking into account the endogeneity of technology) between wage and labor supply can be expressed by a diminishing function $w_L^*(L)$. We will make use of this feature in the next section.

What do these results imply about the impact of labor scarcity on technological progress. The answer is almost nothing. These results imply that when labor becomes more scarce, either because labor supply declines or because some regulation increases wages above market clearing and we move along the curve $w_L^*(L)$ to an employment level below $\bar{L}$, technology will change in a way that is biased against labor. But as already discussed in the Introduction, this can take the form of "technological regress" (meaning technology is now less of a contributing factor to output) or "technological advance" (change in technology increasing output further). In a dynamic framework, these changes could take the form of the economy foregoing some of the technological advances that it would have made otherwise or making further advances, contributing to growth (see Acemoglu, 2002). Therefore, these results results are not informative on this important problem.

The next section shows what types of this general results are available on the effect of changes in the supply of labor (and its price) on technology.

3 Labor Scarcity and Technological Progress

In this section, I present the main results of the current paper as well as a number of relevant extensions.

3.1 Main Results

Let us first focus on Economy M. Recall that $\Theta \subset \mathbb{R}^K$, so that θ is a K-dimensional vector and equilibrium technology choice can be represented as the solution to the problem

$$\max_{\theta \in \Theta} F\left(\bar{L}, \bar{Z}, \theta\right) \equiv G\left(\bar{L}, \bar{Z}, \theta\right) - C\left(\theta\right), \tag{23}$$

where from Assumption 3, $C(\theta)$ is convex, twice differentiable and strictly increasing in θ (in particular with $\partial C(\theta)/\partial\theta_k > 0$ for each $k = 1, 2, ..., K$, and for all $\theta \in \Theta$). In addition, net output $Y\left(\bar{L}, \bar{Z}, \theta\right)$ is given by (16). Since all these functions are differentiable, an (interior) equilibrium $\theta^*\left(\bar{L}, \bar{Z}\right)$ satisfies

$$\frac{\partial F\left(\bar{L}, \bar{Z}, \theta^*\left(\bar{L}, \bar{Z}\right)\right)}{\partial \theta_k} = 0 \text{ for } k = 1, 2, ..., K. \tag{24}$$

Then (16) together with the fact that $C(\theta)$ is strictly increasing implies that

$$\frac{\partial Y\left(\bar{L}, \bar{Z}, \theta^*\left(\bar{L}, \bar{Z}\right)\right)}{\partial \theta_k} > 0 \text{ for } k = 1, 2, ..., K. \tag{25}$$

In light of this, we say that there is technological progress if θ increases (meaning that each component of the vector θ increases or remains constant).[8]

The key concepts of strongly labor (or more generally factor) saving technology and strongly labor complementary technology are now introduced in the next definition.

[8] We only use Assumption 3 for ensuring that (16) holds. None of the other results in this section required equilibrium technology to be interior or to be differentiable. Thus this assumption could be dispensed with if we were to adopt a weaker notion of " technology advances". Nevertheless, using this assumption simplifies the notation and the discussion.

Definition 4 Technological progress is *strongly labor saving* at $L \in \mathcal{L}$ and $Z \in \mathcal{Z}$ if there exist open neighborhoods $\mathcal{B}_L$ and $\mathcal{B}_Z$ of $\bar{L}$ and $\bar{Z}$ in $\mathcal{L}$ and $\mathcal{Z}$ such that $G(L, Z, \theta)$ exhibits decreasing differences in (L, θ) on $\mathcal{B}_L \times \mathcal{B}_Z \times \Theta$. Conversely, technological progress is *strongly labor complementary* at $\bar{L} \in \mathcal{L}$ and $\bar{Z} \in \mathcal{Z}$ if there exist open neighborhoods $\mathcal{B}_L$ and $\mathcal{B}_Z$ of $\bar{L}$ and $\bar{Z}$ in $\mathcal{L}$ and $\mathcal{Z}$ such that $G(L, Z, \theta)$ exhibits increasing differences in (L, θ) on $\mathcal{B}_L \times \mathcal{B}_Z \times \Theta$.

Theorem 4 Consider Economy M and suppose that $G(L, Z, \theta)$ is supermodular in L and θ on $\mathcal{L} \times \mathcal{Z} \times \Theta$ and that $C(\theta)$ is strictly increasing in θ. Then labor scarcity will induce technological advance (increase θ) if technology progress is strongly labor saving and will discourage technological advance if technological progress is strongly labor complementary.

Proof. The fact that $C(\theta)$ is strictly increasing in θ together with equation (25) implies that technological advance corresponds to a change in technology from θ' to $\theta'' \geq \theta'$. The function G is supermodular in θ by assumption. Clearly, it is also supermodular in $-\theta$. Moreover, when technological progress is strongly labor saving, G exhibits decreasing differences in L and θ in the neighborhood of $\bar{L}$, $\bar{Z}$, and thus increasing differences in L and $-\theta$. Then Theorem 2.8.1 from Topkis (1998) applied to the maximization problem (23) implies that $\theta^*\left(\bar{L}, \bar{Z}\right)$ is decreasing in L in the neighborhood of $\bar{L}$ and $\bar{Z}$. This yield the first part of the desired result. Conversely, when G exhibits increasing differences in L and θ, $\theta^*\left(\bar{L}, \bar{Z}\right)$ is increasing in L in the neighborhood of $\bar{L}$ and $\bar{Z}$, and therefore, labor scarcity reduces θ, corresponding to technological regress or to a discouragement to technological advances. ■

Though simple, this theorem provides a fairly complete characterization of the conditions under which labor scarcity and wage push (in competitive factor markets) will lead to further technological adoption or technological progress (and innovation). The only cases that are not covered by the theorem are those where G is not supermodular in θ and those when G exhibits neither increasing differences nor decreasing differences. Without supermodularity, the "direct effect" of labor scarcity on each technology component would be positive, but because of lack of supermodularity, the advance in one component may then induce an even larger deterioration in some other component, thus a precise result becomes impossible. When G exhibits neither increasing or decreasing differences, then a change in labor supply $\bar{L}$ will affect different components of technology in different directions and we cannot reach unambiguous conclusion about the overall effect. Clearly, when θ is single dimensional, the supermodularity condition is automatically satisfied and G must exhibit either increasing or decreasing differences in the neighborhood of $\bar{L}$ and $\bar{Z}$.

Another potential shortcoming of this analysis is that the environment is static. Although this makes the results not readily generalizable to a dynamic framework, it is clear that there are multiple ways of extending this framework to a dynamic environment and the main forces will continue to apply in this case (see subsection 5.1 for an illustration of this point using an extension to a growth model). Moreover, the advantage of the static environment is that it enables to develop these results at a fairly high level of generality, without being forced to make functional form assumptions in order to ensure balanced growth or some other notion of a well-defined dynamic equilibrium.

3.2 Further Results

The results of Theorem 4 apply to Economy M. They can be generalized to Economies E and O. Here I show how this can be done for Economy O. The next theorem is a direct analog of 4 and holds without any further assumptions, except that now equilibrium technology corresponds to the Nash equilibrium of the game among oligopolist technology suppliers and multiple equilibria, and thus multiple equilibrium technologies, are possible. As is well known (e.g., Milgrom and Roberts, 1994, Topkis, 1998), when there are multiple equilibria, we can typically only provide unambiguous comparative statics for extremal equilibria. These extremal equilibria in the present context correspond to the greatest and smallest equilibrium technologies, θ' and θ'' (meaning that if there exists another equilibrium technology, $\tilde{\theta}$, we must have $\theta' \geq \tilde{\theta} \geq \theta''$). In this light, a technological advance now refers to an increase in the greatest and the smallest equilibrium technologies.

Theorem 5 Consider Economy O and suppose that $G(L, Z, \theta)$ is supermodular in L and θ on $\mathcal{L} \times \mathcal{Z} \times \Theta$ and that $C(\theta)$ is strictly increasing in θ. Suppose that technological progress is strong the labor saving. Then labor scarcity will induce technological advances in the sense that the greatest and the smallest equilibrium technologies θ' and θ'' both increase. If technological progress progress is strongly labor complementary, then labor scarcity will discourage technological advances in the sense that the greatest and the smallest equilibrium technologies θ' and θ'' both decrease.

Proof. In Economy O, the equilibrium is given by Proposition 4 and corresponds to a Nash equilibrium of a game among the S oligopolist technology suppliers. Inspection of the corresponding profit functions of each oligopolist shows that when G is supermodular in θ, this is a supermodular game. Then, when G exhibits increasing differences in L and θ, the payoff of each oligopolist exhibits increasing differences in its own strategies and L. Then, Theorem 4.2.2 from Topkis (1998) implies that the greatest and least equilibria of this game will increase when $\bar{L}$ increases. This establishes the second part of the theorem. The first part follows with the same argument, using $-\theta$ instead of θ, when technological progress is strongly labor saving. ■

This theorem therefore shows that essentially the same result holds when we consider Economy O which features furthergame-theoretic interactions among the oligopolists. Similarly, the same result can be generalized to Economy E, with a minimal additional assumption on the interaction between own technology and the technological externality.

However, interestingly and importantly, the results do not apply to Economy D. The main reason for this is that, as already discussed in the previous section, in the neighborhood of an equilibrium in Economy D, there is no meaningful notion of "induced technological advance". Any small change in θ will have second-order effects on net output and any non-small change in θ will reduce net output at the starting factor proportions (at $\bar{L}$ and $\bar{Z}$) since $\theta^* \left(\bar{L}, \bar{Z}\right)$ already maximizes output at these factor proportions.

Finally, Theorem 4 does not specify what types of production functions lead to technological progress being strongly labor saving. This will be discussed in the next section. Before this discussion, however, let us see how the same results also hold when we consider wage push rather than a change in labor supply.

3.3 Implications of Wage Push

Let us consider the same environments as in the previous subsections, but we will also suppose that $\nabla^2 F_{(L,\theta)(L,\theta)}$ is negative semi-definite at the factor supplies $\left(\bar{L}, \bar{Z}\right)$, so that, from Theorem 3, the (endogenous-technology) relationship between labor supply and wage is given by a decreasing function $w_L^*(L)$. This implies that we can equivalently talk of a decrease in labor supply (corresponding to labor becoming more "scarce") or a "wage push," where a wage above the market clearing level is imposed. In this light, we can generally think of equilibrium employment as $L^e = \min\left\{(w_L^*)^{-1}(w_L^e), \bar{L}\right\}$, where w_L^e is the equilibrium wage rate, either determined in competitive labor markets or imposed by regulation. Under these assumptions, all of the results presented in this section continued to hold. This is stated in the next corollary.

Corollary 1 Suppose that $\nabla^2 F_{(L,\theta)(L,\theta)}$ is negative semi-definite that the factor supplies $\left(\bar{L}, \bar{Z}\right)$. Then under the same assumptions as in Theorems 4 and 5, a minimum wage above the market clearing wage level induces technological advances when technological progress is strongly labor saving and discourages technological advances when technological progress is strongly labor complementary.

Proof. This result follows immediately from Theorems 4 and 5 combined with the observation that, when $\nabla^2 F_{(L,\theta)(L,\theta)}$ is negative semi-definite, a wage above the market clearing level is equivalent to a decline in employment. ■

While this result shows that wage push can induce technological advances, it should be noted that even when this is the case, net output may decline because of the reduction in employment. Nevertheless, it is also possible to construct examples where, even though employment declines, overall output increases by more. Consider the following example, which illustrates both this possibility and also gives a simple instance where technological progress is strongly labor saving.

Example 1 Suppose that the G function takes the form

$$G(L, Z, \theta) = 3\theta Z^{1/3} + 3(1-\theta) L^{1/3},$$

and the cost of technology creation is $C(\theta) = 3\theta^2/2$. Let us normalize the supply of the Z factor to $\bar{Z} = 1$ and denote labor supply by $\bar{L}$. Suppose to start with that equilibrium wages will be given by marginal product. Equilibrium technology is then given by

$$\theta^*\left(\bar{L}\right) = 1 - \bar{L}^{1/3}.$$

Equilibrium wage is given by the marginal product of labor at labor supply $\bar{L}$ and technology θ:

$$w\left(\bar{L}, \theta\right) = (1-\theta) \bar{L}^{-2/3}.$$

To obtain the endogenous-technology relationship between labor supply and wages, we substitute for $\theta^*\left(\bar{L}\right)$ into this wage expression and obtain

$$w\left(\bar{L}, \theta^*(L)\right) = \bar{L}^{-1/3}.$$

This shows that there is a decreasing relationship between labor supply and wages.

Suppose that labor supply $\bar{L}$ is equal to $1/84$. In that case, the equilibrium wage will be 4. Next consider a minimum wage imposed that $\bar{w} = 5$. Since final good producers take prices as given, they have to be along their (endogenous-technology) labor demands, this implies that employment will fall to $L^e = 1/125$. Without "wage push," technology was $\theta^*\left(\bar{L}\right) = 3/4$, whereas after the minimum wage, we have $\theta^*(L^e) = 4/5$, which illustrates the induced technology adoption/innovation effect of wage push.

Does wage push increase overall output? This example can also be used to answer this question. Recall that net output is equal to $Y(L, Z, \theta) \equiv (2-\alpha)/(1-\alpha)\, G(L, Z, \theta) - C(\theta)$.

It can be verified that for α close to 0, wage push reduces net output, however for sufficiently high α, net output increases despite the decline in employment.

The close association between labor scarcity and wage push relies on the assumption that $\nabla^2 F_{(L,\theta)(L,\theta)}$ is negative semi-definite, so that the endogenous-technology demand curves are downward sloping. When this is not the case, wage push can have richer effects and this is discussed in Section 5.

4 When is Technological Progress Strongly Labor Saving?

In this section, I investigate the conditions under which, in a range of standard models, technological progress is strongly labor saving. The results show that it is possible to construct a rich set of economies in which this is the case, though in most models commonly used in macroeconomics and economic growth, technological progress turns out not to be strongly labor saving. Throughout, I provide examples in which technology can be represented by a single-dimensional variable. This is to simplify the expressions and communicate the basic ideas in the most transparent manner. As the analysis in the previous sections illustrated, none of the results require technology to be single dimensional.

4.1 Cobb-Douglas Production Functions with Harrod Neutral Technology

As a first example, suppose that the function G and thus the aggregate production function of the economy takes a "Cobb-Douglas form" with Harrod technology. In particular, let us write this function as

$$G(L, Z, \theta) = H(Z)(\theta L)^{\alpha},$$

where $H : \mathbb{R}_+^N \to \mathbb{R}_+$, and thus aggregate net output is given by

$$Y(L, Z, \theta) = \frac{2-\alpha}{1-\alpha} H(Z)(\theta L)^{\alpha},$$

where $\alpha \in (0, 1)$ is the parameter of the production function in (10), measuring the elasticity of aggregate output to the subcomponent G. It is straightforward to verify that the cross-partial of G with respect to L and θ in this case is

$$G_{L\theta}(L, Z, \theta) = \alpha^2 H(Z)(\theta L)^{\alpha-1} > 0.$$

Therefore, technological progress is always strongly labor complementary in this case and labor scarcity or wage push will necessarily discourage technological advances.

It is also straightforward to verify that the same conclusion holds if the production function is modified to

$$G(L, Z, \theta) = H(Z, \theta) L^{\alpha}.$$

In this case, the assumptions imposed so far imply that H must be strictly increasing in θ. Supposing that it is also differentiable, we have

$$G_{L\theta}(L, Z, \theta) = \alpha H_{\theta}(Z, \theta) L^{\alpha-1} > 0,$$

so that the same conclusion is reached.

4.2 Changes in Substitution Patterns

Technological change that alters the substitution patterns across factors often turns out to be strongly labor saving. Our discussion below will illustrate this in detail, but a simple example based on Cobb-Douglas production functions illustrates the intuition. Suppose that the function G takes the form

$$G(L, Z, \theta) = A(\theta) H(Z) L^{1-\theta},$$

where $A(\theta)$ is a strictly increasing and differentiable function. From Proposition 3, equilibrium technology will satisfy

$$A'(\theta^*) H(\bar{Z}) \bar{L}^{1-\theta^*} - A(\theta^*) H(\bar{Z}) \bar{L}^{1-\theta^*} \ln \bar{L} = C'(\theta^*). \tag{26}$$

Therefore, whether we have strongly labor saving technological progress depends on the sign of

$$G_{L\theta}(\bar{L}, \bar{Z}, \theta^*) = \left[(1-\theta^*) A'(\theta^*) - (1-\theta^*) A(\theta^*) \ln \bar{L} - A(\theta^*)\right] H(\bar{Z}) \bar{L}^{-\theta^*}.$$

It is clear that by modifying the function $C(\theta)$ and the level of labor supply and the economy, $\bar{L}$, this expression can be made positive or negative. In particular, suppose that in equilibrium $C'(\theta^*) \approx \epsilon \bar{L}$ (where ϵ the function C is chosen appropriately for this to be the case in equilibrium). Then using (26), we can write

$$G_{L\theta}(\bar{L}, \bar{Z}, \theta^*) = (1-\theta^*) \epsilon - A(\theta^*) H(\bar{Z}) \bar{L}^{-\theta^*},$$

which will be negative for ϵ small enough.

4.3 Constant Elasticity of Substitution With Factor Augmenting Technological Change

Let us next turn to constant elasticity of substitution (CES) production functions, which are also commonly used in the macroeconomics literature and allow technological change to affect the extent of substitution across factors to a limited extent. To simplify the discussion, let us continue to focus on cases in which technology is represented by a single-dimensional variable, θ, and also suppose that there is only one other factor of production, for example land or capital, and thus Z is also single dimensional. This means that we have to distinguish between two cases, one in which θ "augments" Z and one in which θ "augments" labor. Let us start with the former. The G function can then be written as

$$G(L, Z, \theta) = \left[(1-\eta)(\theta Z)^{\frac{\sigma-1}{\sigma}} + \eta L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\gamma\sigma}{\sigma-1}},$$

for $\eta \in (0,1)$, and once again, net output is equal to the same expression multiplied by $(2-\alpha)/(1-\alpha)$.

Straightforward differentiation then gives

$$G_{L\theta}(L, Z, \theta) = \frac{\gamma\sigma + 1 - \sigma}{\sigma}\gamma\eta(1-\eta) Z^{\frac{\sigma-1}{\sigma}} (\theta L)^{-\frac{1}{\sigma}} \left[(1-\eta)(\theta Z)^{\frac{\sigma-1}{\sigma}} + \eta L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\gamma\sigma}{\sigma-1}-2}.$$

This expression shows that technological progress will be strongly labor complementary (i.e., $G_{L\theta} > 0$) if either of the following two conditions are satisfied:

1. $\gamma = 1$ (constant returns to scale)

2. $\sigma \leq 1$ (gross complements).

Therefore, in this case for technological progress to be strongly labor saving we would need both $\gamma < 1$ and $\sigma > 1$ (and in fact both of them sufficiently so) so that the following condition is satisfied.

$$1 - \gamma > \frac{1}{\sigma}. \tag{27}$$

In fact, this result can be generalized as to any G that is homothetic in Z and L and incorporates technology θ in a Z-augmenting form, that is, to a G function of the following form:

$$G(L, Z, \theta) = \tilde{G}(\theta Z, L),$$

where $\tilde{G}$ is homothetic. In this case, (27) is again necessary and sufficient for technological progress to be strongly labor saving, with γ corresponding to the local degree of homogeneity of $\tilde{G}$ and σ corresponding to the local elasticity of substitution (both "local" qualifiers are added, since these need not be constant).

This result shows that with Z-augmenting technology, constant returns to scale is sufficient to rule out strongly labor saving technological progress. In addition, in this case need a high elasticity of substitution. Since θ is augmenting the other factor, Z, a high elasticity of substitution corresponds to technology "substituting" for tasks performed by labor. This intuition will exhibit itself somewhat differently next, when we turn to the CES production function with labor-augmenting technology.

With labor-augmenting technology, the G function takes the form

$$G(L, Z, \theta) = \left[(1-\eta) Z^{\frac{\sigma-1}{\sigma}} + \eta (\theta L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\gamma\sigma}{\sigma-1}}.$$

Straightforward differentiation now gives $G_L(L, Z, \theta) = \gamma\eta\theta^{\frac{\sigma-1}{\sigma}} L^{-\frac{1}{\sigma}} \left[(1-\eta) Z^{\frac{\sigma-1}{\sigma}} + \eta (\theta L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\gamma\sigma}{\sigma-1}-1}$ and therefore

$$\begin{aligned}
G_{L\theta}(L, Z, \theta) &= \frac{\gamma\sigma + 1 - \sigma}{\sigma}\gamma\eta^2 (\theta L)^{\frac{\sigma-2}{\sigma}} \left[(1-\eta)(\theta Z)^{\frac{\sigma-1}{\sigma}} + \eta L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\gamma\sigma}{\sigma-1}-2} \\
&\quad + \gamma\eta\frac{\sigma-1}{\sigma}(\theta L)^{-\frac{1}{\sigma}} \left[(1-\eta) Z^{\frac{\sigma-1}{\sigma}} + \eta (\theta L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\gamma\sigma}{\sigma-1}-1} \\
&= \left\{\frac{\gamma\sigma + 1 - \sigma}{\sigma}\eta (\theta L)^{\frac{\sigma-1}{\sigma}} + \frac{\sigma-1}{\sigma}\left[(1-\eta) Z^{\frac{\sigma-1}{\sigma}} + \eta (\theta L)^{\frac{\sigma-1}{\sigma}}\right]\right\} \\
&\quad \times\gamma\eta (\theta L)^{-\frac{1}{\sigma}} \left[(1-\eta) Z^{\frac{\sigma-1}{\sigma}} + \eta (\theta L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\gamma\sigma}{\sigma-1}-2} \\
&= \left\{\gamma\eta (\theta L)^{\frac{\sigma-1}{\sigma}} + \frac{\sigma-1}{\sigma}(1-\eta) Z^{\frac{\sigma-1}{\sigma}}\right\} \\
&\quad \times\gamma\eta (\theta L)^{-\frac{1}{\sigma}} \left[(1-\eta) Z^{\frac{\sigma-1}{\sigma}} + \eta (\theta L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\gamma\sigma}{\sigma-1}-2}
\end{aligned}$$

Now defining the relative labor share as

$$s_L \equiv \frac{w_L L}{w_Z Z} = \frac{\eta (\theta L)^{\frac{\sigma-1}{\sigma}}}{(1-\eta) Z^{\frac{\sigma-1}{\sigma}}} > 0,$$

the condition that $G_{L\theta} < 0$ is equivalent to

$$s_L < \frac{1-\sigma}{\sigma\gamma}. \tag{28}$$

As with condition (27), (28) is more likely to be satisfied, and technological change is more likely to be strong the labor saving, when γ is smaller and thus there are strong decreasing returns. However, now technology cannot be strongly labor saving when $\sigma \geq 1$, which is the opposite of the restriction on the elasticity of substitution in the case when θ augments

Z. Nevertheless, this is not unintuitive. When θ augments Z, a high degree of substitution between technology and labor requires a high elasticity of substitution, in particular, $\sigma > 1$. In contrast, when θ augments labor, a high degree of substitution between technology and labor corresponds to $\sigma < 1$.

Once again this result can be extended to a general homothetic functions with labor-augmenting technology, in particular, to the case where the G function takes the form

$$G(L, Z, \theta) = \tilde{G}(Z, \theta L).$$

This discussion highlights that with the most common production functions in macroeconomics, Cobb-Douglas and constant elasticity of substitution, technological progress tends to be strongly labor complementary rather than strongly labor saving. Nevertheless, the latter possibility is not ruled out, at least when technological change affects the patterns of substitution. We next turn to a setup where technological change more explicitly replaces labor.

4.4 Machines Replacing Labor

Models in which technological change is caused or accompanied by machines replacing human labor have been proposed by Champernowne (1963), Zeira (1998, 2006), Hellwig and Irmen (2001) and Alesina and Zeira (2006). Let us consider a setup building on and generalizing the paper by Zeira (1998). I first describe a fully competitive economy and explain why we need to depart from this towards an environment such as Economy M.

Aggregate output is given by

$$y = \left[\int_0^1 y(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $y(\nu)$ is intermediate good of type ν produced as

$$y(\nu) = \begin{cases} \frac{k(\nu)}{\eta(\nu)} & \text{if } \nu \text{ uses new "technology"} \\ \frac{l(\nu)}{\beta(\nu)} & \text{if } \nu \text{ uses old "technology"}, \end{cases}$$

where both $\eta(\nu)$ and $\beta(\nu)$ are assumed to be continuous functions and goods are ordered such that $\eta(\nu)$ is a strictly increasing function, $\beta(\nu)$ is decreasing. In addition, $k(\nu)$ denotes capital used in the production of intermediate ν. I use capital as the other factor of production here to maximize similarity with Zeira (1998).

Firms are competitive and can choose which product to produce with the new technology and which one with the old technology. Total labor supply is $\bar{L}$. For now, let us also suppose that capital is supplied inelastically, with total supply given by $\bar{K}$.

Let the price of the final good be normalized to 1 and that of each intermediate good be $p(\nu)$. We write $n(\nu) = 1$ if ν is using the new technology. Clearly, $n(\nu) = 1$ whenever

$$R\eta(\nu) < w\beta(\nu),$$

where w is the wage rate and R is the endogenously determined rate of return on capital. Let

$$\gamma(\nu) \equiv \frac{\eta(\nu)}{\beta(\nu)},$$

which is strictly increasing by assumption. In fact, this is all that is required, so $\eta(\nu)$ could be decreasing or $\beta(\nu)$ could be increasing as long as $\gamma(\nu)$ is strictly increasing.

In the competitive equilibrium, we will have θ^* such that

$$\theta^* = \gamma^{-1}\left(\frac{w}{R}\right),$$

so that

$$n(\nu) = 1 \text{ for all } \nu \leq \theta^*.$$

Since γ is increasing, its inverse is also increasing, so a higher wage to rental rate ratio encourages higher levels of θ^*. This effect is highlighted and exploited in Zeira (1998).

Let us now see that this is indeed related to decreasing differences. With the same reasoning, suppose that

$$n(\nu) = 1 \text{ for all } \nu \leq \theta$$

for some $\theta \in (0, 1)$ (since, clearly, in any equilibrium or optimal allocation, this type of "single crossing" must hold). Then, prices of intermediates must satisfy

$$p(\nu) = \begin{cases} \eta(\nu) R & \text{if } \nu \leq \theta \\ \beta(\nu) w & \text{if } \nu > \theta \end{cases}.$$

Therefore, profit maximization of final good producers is

$$\max_{[y(\nu)]_{\nu\in[0,1]}} \left[\int_0^1 y(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right]^{\frac{\varepsilon}{\varepsilon-1}} - R\int_0^\theta \eta(\nu) y(\nu) d\nu - w\int_\theta^1 \beta(\nu) y(\nu) d\nu,$$

which gives the following simple solution

$$y(\nu) = \begin{cases} (\eta(\nu) R)^{-\varepsilon} Y & \text{if } \nu \leq \theta \\ (\beta(\nu) w)^{-\varepsilon} Y & \text{if } \nu > \theta \end{cases}$$

Now market clearing for capital implies

$$\begin{aligned} \int_0^\theta k(\nu) d\nu &= \int_0^\theta \eta(\nu) y(\nu) d\nu \\ &= \int_0^\theta \eta(\nu)^{1-\varepsilon} R^{-\varepsilon} Y d\nu = \bar{K} \end{aligned}$$

and similarly, market clearing for labor gives

$$\int_{\theta}^{1} \beta(\nu)^{1-\varepsilon} w^{-\varepsilon} Y d\nu = \bar{L}.$$

Let us define

$$A(\theta) \equiv \int_{0}^{\theta} \eta(\nu)^{1-\varepsilon} d\nu \text{ and } B(\theta) \equiv \int_{\theta}^{1} \beta(\nu)^{1-\varepsilon} d\nu. \tag{29}$$

Then the market clearing conditions can be expressed as

$$R^{1-\varepsilon} = \left(\frac{Y}{K} A(\theta)\right)^{\frac{1-\varepsilon}{\varepsilon}} \text{ and } w^{1-\varepsilon} = \left(\frac{Y}{L} B(\theta)\right)^{\frac{1-\varepsilon}{\varepsilon}}, \tag{30}$$

and using (30), we can write aggregate output (and aggregate net output) as

$$\begin{aligned} Y &= \left[\int_{0}^{\theta} \left((\eta(\nu) R)^{-\varepsilon} Y\right)^{\frac{\varepsilon-1}{\varepsilon}} d\nu + \int_{\theta}^{1} \left(\beta(\nu) w^{-\varepsilon} Y\right)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= \left[A(\theta)^{\frac{1}{\varepsilon}} K^{\frac{\varepsilon-1}{\varepsilon}} + B(\theta)^{\frac{1}{\varepsilon}} L^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \end{aligned} \tag{31}$$

Equation (31) gives a simple expression for aggregate output as a function of the threshold sector θ. It can be verified that Y exhibits decreasing differences in L and θ in the competitive equilibrium. In particular, equilibrium technology in this case will satisfy

$$\frac{\partial Y}{\partial \theta} = (\varepsilon - 1)\left[\eta(\theta^*)^{1-\varepsilon} A(\theta^*)^{\frac{1-\varepsilon}{\varepsilon}} K^{\frac{\varepsilon-1}{\varepsilon}} - \beta(\theta^*)^{1-\varepsilon} B(\theta^*)^{\frac{1-\varepsilon}{\varepsilon}} L^{\frac{\varepsilon-1}{\varepsilon}}\right] Y^{\frac{1}{\varepsilon}} = 0.$$

Since the term in square brackets must be equal to zero, we must have

$$\frac{\partial^2 Y}{\partial \theta \partial L} = -\frac{(\varepsilon-1)^2}{\varepsilon} \beta(\theta^*)^{1-\varepsilon} B(\theta^*)^{\frac{1-\varepsilon}{\varepsilon}} L^{-\frac{1}{\varepsilon}} Y^{\frac{1}{\varepsilon}} < 0.$$

This argument suggests why there is an intimate connection between machines replacing labor and technological progress being strongly labor saving. However, Theorem 4 does not apply to this economy because we are in a fully competitive environment, and thus $\partial Y/\partial\theta = 0$ in equilibrium, and therefore, induced changes in technology do not correspond to "technological advances".

Hence, we must consider a version of the current environment corresponding to Economy M or O. To do this, let us suppose that

$$G(L, Z, \theta) = \left[A(\theta)^{\frac{1}{\varepsilon}} K^{\frac{\varepsilon-1}{\varepsilon}} + B(\theta)^{\frac{1}{\varepsilon}} L^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

with cost $C(\theta)$, $\varepsilon > 1$, and $A(\theta)$ and $B(\theta)$ defined as in (29). The fact that $\alpha > 0$ in these economies ensures that an increase in θ indeed corresponds to a technological advance.

Therefore, we only have to check whether technological progress is strongly labor saving, or whether G exhibits decreasing differences in L and θ. Straightforward differentiation and some manipulation imply that $G_{L\theta}$ is proportional to

$$-\beta\left(\theta^*\right)^{1-\varepsilon} B\left(\theta^*\right)^{\frac{1-\varepsilon}{\varepsilon}} L^{\frac{\varepsilon-1}{\varepsilon}} G\left(L, Z, \theta\right)^{\frac{1}{\varepsilon}} + \left(2-\alpha\right) C'\left(\theta^*\right) S_L,$$

with $S_L \equiv w_L L / \left[\left(2-\alpha\right) G\left(L, Z, \theta\right) / \left(1-\alpha\right)\right]$ is the labor share of income. This expression will be negative when $C'\left(\theta^*\right)$ is small or when the share of labor is small. But without specifying further functional forms, we cannot give primitive conditions for this to be the case. Instead, technological progress that is strongly labor saving obtains easily if we consider a slight variation on this baseline model, where the constant returns to scale assumption has been relaxed, so that the G function takes the form

$$G\left(L, Z, \theta\right) = \left[A\left(\theta\right)^{\frac{1}{\varepsilon}} K^{\frac{\varepsilon-1}{\varepsilon}} + B\left(\theta\right)^{\frac{1}{\varepsilon}} L^{\frac{\varepsilon-1}{\varepsilon}}\right],$$

then it can be verified that

$$G_{L\theta} = -\frac{\varepsilon-1}{\varepsilon^2}\beta\left(\theta^*\right)^{1-\varepsilon} B\left(\theta^*\right)^{\frac{1-\varepsilon}{\varepsilon}} L^{-\frac{1}{\varepsilon}} < 0,$$

so that technology is always strongly labor saving and a decrease in $\bar{L}$ will induce technological advances.

Therefore, models where technological progress is caused or accompanied by machines replacing human labor create a tendency for strongly labor saving technological progress. This is intuitive, since the process of machines replacing labor is closely connected to new technology substituting for and saving on labor.

5 Extensions and Discussion

In this section, I discuss a number of issues raised by the analysis so far. First, I show that technological progress being strongly labor saving does not contradict the positive impact of secular technological changes on wages. Second, I show how endogenous factor supplies can be incorporated into this framework. Finally, I discuss how wage push can lead to very different results than labor scarcity when the endogenous-technology demand curve for labor is upward sloping (in line with the conditions provided in Theorem 3).

5.1 Technological Change and Wage Increases

One objection to the plausibility of strongly labor saving technological progress is that the growth process is accompanied by a steady increase in the wage rate, while strongly labor saving technological progress implies that further technological advances will tend to reduce the marginal product of labor. In this subsection, I show that a simple dynamic extension allows technological change to increase wages while still maintaining technological progress as strongly labor saving.

My purpose here is not to develop a full dynamic general equilibrium model. For this reason, I use a slight variant of Economy E and a very simple demographic structure to communicate the main ideas. The exact form of the production function is motivated by the models in which machines replace labor such as those discussed in subsection 4.4, though various different alternative formulations could also have been used to obtain similar results.

Suppose that the economy is in discrete time and runs to infinite horizon. It is inhabited by one-period lived individuals, each operating a firm. Therefore, each firm maximizes static profits. The total measure of individuals and firms is normalized 1. There are two factors of production, L and Z, both inelastically supplied in each period with supplies equal to $\bar{L}$ and $\bar{Z}$. Past technology choices create an externality similar to that in Economy E. In particular, suppose that all firms are fully competitive and the production function of each at time t is firm

$$y_t^i\left(L_t^i, Z_t^i, \theta_t^i\right) = \bar{A}_t\left[\left(\theta_t^i\right)^{1+\gamma}\left(Z_t^i\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(1-\theta_t^i\right)^{1+\gamma}\left(L_t^i\right)^{\frac{\varepsilon-1}{\varepsilon}}\right], \tag{32}$$

where $\gamma < 0$ and we assume that ε is sufficiently close to 1, so that (32) is jointly concave in L, Z and θ. Finally, let us also assume that $\bar{Z} < 2^{\gamma\varepsilon/(\varepsilon-1)}\bar{L}$. This production function implies that higher θ will correspond to substituting factor Z, which may be capital or other human or nonhuman factors, for tasks performed by labor.

In line with Proposition 2, an equilibrium technology $\theta^*\left(\bar{L}, \bar{Z}\right)$ will be independent of $\bar{A}_t$ and will satisfy

$$\frac{\partial Y_t^i\left(\bar{L}, \bar{Z}, \theta^*\left(\bar{L}, \bar{Z}\right)\right)}{\partial \theta} = 0,$$

or in other words

$$\theta^*\left(\bar{L}, \bar{Z}\right)^{\gamma}\bar{Z}^{\frac{\varepsilon-1}{\varepsilon}} = \left(1-\theta^*\left(\bar{L}, \bar{Z}\right)\right)^{\gamma}\bar{L}^{\frac{\varepsilon-1}{\varepsilon}},$$

or

$$\theta^*\left(\bar{L}, \bar{Z}\right) = \frac{1}{1+\left(\bar{Z}/\bar{L}\right)^{\frac{\varepsilon-1}{\gamma\varepsilon}}}.$$

The assumption that $\bar{Z} < 2^{\gamma\varepsilon/(\varepsilon-1)}\bar{L}$ ensures that $\theta^*\left(\bar{L}, \bar{Z}\right) \in (0,1)$. Moreover, since $\gamma < 0$ and $\varepsilon > 1$, $\theta^*\left(\bar{L}, \bar{Z}\right)$ is decreasing in $\bar{L}$, so labor scarcity increases $\theta^*\left(\bar{L}, \bar{Z}\right)$.

Next, suppose that the intertemporal externalities take the form

$$\bar{A}_t = \left(1+g\left(\bar{\theta}_{t-1}\right)\right)\bar{A}_{t-1},$$

where g is an increasing function and $\bar{\theta}_t \equiv \int_{i \in \mathcal{F}} \theta_t^i di$ is the average technology choice of firms at time t. This form of intertemporal technological externalities is similar to that in Romer (1986) and may result from the fact that past efforts to substitute machines or capital for labor advance (and also build upon) the knowledge stock of the economy.

The important implication of the simple economy for our purposes is that higher equilibrium level of $\theta^* \left(\bar{L}, \bar{Z}\right)$ will lead to faster growth of output and wages, even though at the margin, labor scarcity increases $\theta^* \left(\bar{L}, \bar{Z}\right)$ and substitutes for tasks previously performed by labor. In particular, it can be easily verified that an increase in $\bar{Z}$ will also increase $\theta^* \left(\bar{L}, \bar{Z}\right)$. The immediate impact of this will be to reduce the level of wages, but this change will also increase the rate of wage and output growth. This result highlights that in a dynamic framework with strongly labor saving technological progress, the short-run and long-run impacts of technological advances on wages will typically differ.

This analysis therefore shows that there is no tension, in a dynamic economy, between technological changes leading to a secular increase in wages and technological progress being strongly labor saving.

5.2 Endogenous Factor Supplies

To highlight the new results of the framework presented in this paper, the analysis so far has treated the supply of all factors as exogenous. This ignores both the response of labor supply to changes in wages and also the adjustment of other factors, such as capital, to changes in factor supplies or labor market regulations that create wage push. From the analysis leading to Corollary 1, it is clear that none of the results will affected if $\nabla^2 F_{(L,\theta)(L,\theta)}$ is negative semi-definite, the supply of other factors are fixed and the supply of labor is given by $L^s(w_L)$, with an increasing function L^s. In particular, in this case, we can study the impact of a shift in labor supply from $L^s(w_L)$ to $\tilde{L}^s(w_L)$, where $\tilde{L}^s(w_L) < L^s(w_L)$. Clearly, since $\nabla^2 F_{(L,\theta)(L,\theta)}$ is negative semi-definite, the endogenous-technology relationship between employment and wages is decreasing. Thus a leftwards shift of the labor supply schedule to $L^s(w_L)$ to $\tilde{L}^s(w_L)$ will reduce employment and increase wages. The implications for technology are determined by whether technological progress is strongly labor saving or strongly labor complementary. How the analysis changes when $\nabla^2 F_{(L,\theta)(L,\theta)}$ is not negative semi-definite is discussed in the next subsection.

Endogenizing the supply of other factors also has important and interesting implications on the analysis. In particular, we can imagine a situation in which one of the other factors of production is capital, which is elastically supplied, at least in the long run. In this case, a change in labor supply will change technology in such a way that the rental rate of capital remains constant. This implies that the supply of capital will change endogenously and thus the overall impact on technology will be a combination of the direct effect of labor supply and the indirect effect working through the induced changes in the capital stock of the economy.

Let us briefly see how this affects the analysis in the context of the model of machines replacing labor discussed in the previous section. The following example considers a version of that framework with perfectly elastic supply of capital.

Example 2 Consider the perfectly competitive economy, with machines replacing labor, discussed in subsection 4.4. Instead of the constant supply of capital $\bar{K}$, suppose that there is a perfectly elastic supply of capital, which ensures that the rental rate of capital remains constant at some $\bar{R}$. The market clearing condition for capital from (30) still applies, but now it determines the equilibrium level of capital stock such that the rental rate is equal to $\bar{R}$. Taking this into account, equation (31) is modified and can be written as

$$\begin{aligned}\tilde{Y} &= \left[\bar{R}^{1-\varepsilon} Y^{\frac{\varepsilon-1}{\varepsilon}} A(\theta)^{\frac{1}{\varepsilon}} + B(\theta)^{\frac{1}{\varepsilon}} L^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= \left[\frac{B(\theta)^{\frac{1}{\varepsilon}} L^{\frac{\varepsilon-1}{\varepsilon}}}{1-\bar{R}^{1-\varepsilon} A(\theta)^{\frac{1}{\varepsilon}}}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \end{aligned} \tag{33}$$

This expression is similar to (31), though the exact impacts of θ on output and the marginal product of labor are naturally different. One can also develop a similar expression for Economy M, where θ is determined by a profit-maximizing monopolist rather than competitive firms.

When capital is perfectly elastic, induced technological advances may become possible in a perfectly competitive economy. In particular, if all competitive firms took into account that the rental rate would immediately adjust, then the choice of θ by each firm would maximize (33), and in that case, we would again have $\partial\tilde{Y}/\partial\theta = 0$ in equilibrium and thus induced technological advances would not be possible. However, each firm may choose its technology θ for a given level of capital stock, K, and then in general equilibrium, the capital stock may adjust slowly to restore the rental rate of capital to $\bar{R}$. In that case, the equilibrium technology choice would set $\partial Y/\partial\theta = 0$, where Y is given by (31) with the current level of capital stock, K. In this case, the impact of marginal changes in technology on "long-run" output $\tilde{Y}$ need not be zero, and we can have $\partial\tilde{Y}/\partial\theta > 0$. This is in fact closer to the setup in Zeira (1998), where capital stock adjusts slowly in a dynamic growth model.

When we use Economy M instead for modeling technological change, whether the capital stock is perfectly elastic does not have any major effects on the analysis and does not change the general insights on when we should expect labor scarcity to encourage technological change (though of course the exact conditions change since aggregate output will be given by an expression like (33) instead of (31)).

Overall, the previous example illustrates that the framework and developed here can be easily adapted to a situation in which the supply of other factors change endogenously and this will not affect the general theoretical insights.

5.3 Wage Push vs. Labor Scarcity

The close connection between wage push and labor scarcity highlighted in Corollary 1 is broken when $\nabla^2 F_{(L,\theta)(L,\theta)}$ is not negative semi-definite. In this case, the endogenous-technology demand curve is upward sloping and thus a decrease in labor supply reduces wages, whereas an increase in labor supply increases wages. This becomes particularly interesting when labor supply is endogenous as discussed in the previous subsection. In this case, multiple equilibria, characterized by different levels of labor supply, technology and wages, become possible. The next example illustrates this possibility using a simple extension of Example 1.

Example 3 Suppose that the G function takes a form very similar to that in Example 1, except a slight variation in exponents. In particular,

$$G(L, Z, \theta) = \frac{3}{2}\theta Z^{2/3} + \frac{3}{2}(1-\theta) L^{2/3},$$

and the cost of technology creation is $C(\theta) = 3\theta^2/4$. It can now be verified that $\nabla^2 F_{(L,\theta)(L,\theta)}$ is no longer negative semi-definite (where as it was in Example 1). Therefore, from Theorem 3, we expect the endogenous-technology relationship between employment (labor supply) and wage to be increasing. We will now see how this interacts with endogenous labor supply.

Let us again normalize the supply of the Z factor to $\bar{Z} = 1$ and denote employment by L^e. Equilibrium technology is then satisfies

$$\theta^*(L^e) = 1 - (L^e)^{2/3}.$$

Equilibrium wage is then given by

$$w(L^e, \theta) = (1-\theta)(L^e)^{-1/3}$$

for a given level of technology θ and once we take into account the response of θ to employment L^e, we have

$$w(L^e, \theta^*(L^e)) = (L^e)^{1/3}, \tag{34}$$

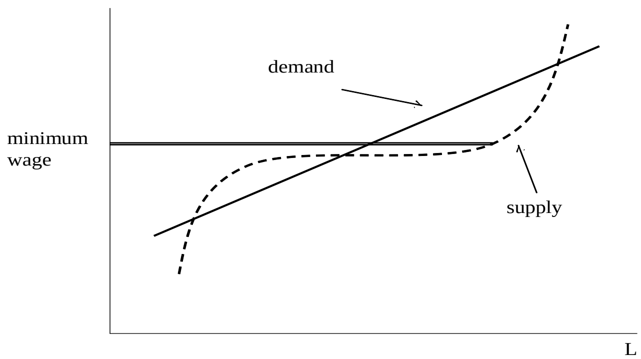

Figure 1: Multiple Equilibria

which illustrates the potentially upward-sloping endogenous-technology relationship between employment and wages shown more generally in Theorem 3. Now suppose that labor supply is also responsive to wage and takes the form

$$L^s(w) = 6w^2 - 11w + 6.$$

Now combining this supply relationship with (34), we find that there are three equilibrium wages, with different levels of labor supply and technology, $w = 1$, 2 and 3. Moreover, technology is most advanced and labor supply is highest at $w = 3$.

Next consider a minimum wage between 2 and 3. This will typically destroy the first two equilibria. Thus the implications of "wage push" are potentially very different when may destroy the other equilibria. Figure 1 illustrates the situation diagrammatically. The minimum wage indeed destroys the equilibria at $w = 1$ and $w = 2$. Nevertheless,some caution is also necessary. In certain situations it can also introduce an extreme no-activity equilibrium, where there is no employment, or it may make such a no-activity equilibrium, which may have already existed, more likely. When we are in Economy M, such a no-activity equilibrium does not exist, because the monopolist acts as a "Stackleberg leader" and chooses the technology anticipating employment. However, in Economy O, such a no-activity equilibrium may arise if a high level of minimum wages imposed.

6 Conclusion

This paper studied the conditions under which the scarcity of a factor (in particular, labor) encourages technological progress and technology adoption. Despite a large literature on endogenous technological change and technology adoption, we do not yet have a comprehensive theoretical or empirical understanding of the determinants of innovation, technological progress, and technology adoption. Most importantly, how factor proportions, for example, abundance or scarcity of labor, affect technology is poorly understood.

In standard endogenous growth models, which feature a strong scale effect, an increase in the supply of a factor encourages technological progress. In contrast, the famous Habakkuk hypothesis in economic history claims that technological progress was more rapid in 19th-century United States than in Britain because of labor scarcity in the former country. Similar ideas are often suggested as possible reasons for why high wages might have encouraged more rapid adoption of certain technologies in continental Europe than in the United States over the past several decades.

I presented a general framework for the analysis of these questions. As shown in Acemoglu (2007), this general framework leads to a systematic characterization of how the bias of technology responds to changes in factor supplies. However, I show that the implications for whether changes in technology correspond to "technological advances", increasing output (at given factor proportions) are ambiguous. The main results in the paper characterize the conditions under which labor scarcity induces technological advances and the conditions under which it discourages technological progress.

The concepts of strongly labor saving and strongly labor complementary technological change play a central role in the analysis. Suppose that the aggregate production function of the economy is supermodular in a vector of technologies denoted by θ. Technological progress is strongly labor saving if the production function exhibits decreasing differences in θ and labor. Conversely, technological progress is strongly labor complementary if the production function exhibits The main result of the paper shows that labor scarcity induces technological advances if technological progress is strongly labor saving. In contrast, labor scarcity discourages technological advances if technological progress is strongly labor complementary. I also show that, under some additional conditions, wage push—an increase in wage levels above the competitive equilibrium—has similar effects to labor scarcity.

I also provided examples of environments in which technological progress can be strongly labor saving and also showed that such a result is not possible in certain canonical models.

These results clarify the conditions under which labor scarcity and high wages are likely to encourage innovation and technological progress and the reason why such results were obtained or conjectured in certain settings but do not always apply in many models used in the growth literature.

While the theoretical analysis provided here clarifies the conditions under which labor scarcity and wage push may induce innovation and technology adoption, the conditions highlighted by the analysis may or may not hold depending on the specific application. This suggests that empirical evidence is necessary for shedding light on when we may expect different levels of factor and labor supplies to lead to different technological tendencies. Acemoglu and Finkelstein (2008), for example, show that the Prospective Payment System reform of Medicare in the United States, which increased the labor costs of hospitals with a significant share of Medicare patients, appears to have induced significant technology adoption in the affected hospitals. In a different context, Lewis (2007) shows that the skill mix in US metropolitan areas appears to have an important effect on the choice of technology of manufacturing firms. Further research on this type might be shed more systematic light on the empirical conditions under which we may expect high labor costs and labor scarcity to be an inducement, rather than a deterrent, to technology adoption and innovation.

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