AGENCY PROBLEMS AND ENDOGENOUS ECONOMIC FLUCTUATIONS

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Abstract. This paper proposes a theory of investment fluctuations where the source of the oscillating dynamics is an agency problem between financiers and entrepreneurs. A central tenet of the theory is that investment decisions depend upon entrepreneurs' initiative to select investment projects *ex-ante,* and financiers' incentive to control entrepreneurs *ex-post.* Too much control discourages entrepreneurial incentive to initiate new investment, while too little control jeopardizes its productivity. This initiative-control trade-off generates investment dynamics that mimic those of a standard credit frictions model, in which more entrepreneurial net worth leads to higher investment. The same trade-off is capable of generating endogenous reversal of investment booms, induced by an ongoing deterioration of project profitability. Investment fluctuations may take place even though no external shocks hit the economy, and even though agents are perfectly rational.

JEL:E 32, E 24; *Keywords*: Credit market imperfections; Double moral hazard; Business cycles; Endogenous fluctuations.

1. Introduction

Starting with the seminal contributions of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), a large theoretical literature in macroeconomics has studied how credit market imperfections shape investment and output dynamics. At the heart of this literature is the inverse relationship between firms' financial assets and the agency costs of investment. In the presence of adverse selection or moral hazard problems, firms' debt capacity is constrained by the level of assets that can be pledged to outside lenders. An adverse shock that worsens financial conditions may therefore generate a negative spiral whereby low profits reduce debt capacity and hence investment, which further reduces profit, exacerbating the initial negative shock. This mechanism, known as the credit multiplier, or the

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financial accelerator, has been extremely influential in explaining how relatively small and temporary exogenous shocks to the economy may be amplified and become persistent.[1]

A salient feature of models featuring a credit multiplier is that agency costs are more severe in recessions than in booms, precisely because agency problems are inversely related to firms' net worth, which is procyclical. While in recessions a firm's ability to finance productive investment is constrained by its balance sheet, financial frictions are mitigated in booms as improved financial conditions mitigate the agency cost of investment finance. In the absence of exogenous shocks that impair balance sheets, these models are therefore unable to explain why periods of expansion may sow the seeds for future recessions.

The purpose of this paper is to present a model where agency problems in the credit market are a source of endogenous business fluctuations, rather than being a mere source of propagation of exogenous shocks. The key assumption of the model is that the profitability of investment projects depends upon the joint effort of investors and entrepreneurs. Entrepreneurs need to exert effort in acquiring information about project characteristics. Investors need to control the selection of projects, ruling out those that, for example, confer private benefits to the entrepreneur at the expense of cash flows.[2] It is shown that entrepreneurs and investors' incentives vary over the cycle in a way that endogenous reversal of booms may take place, even though no external shocks hit the economy, and even though agents are perfectly rational.

More specifically, the paper proposes the following mechanism. An entrepreneur needs to borrow funds from competitive investors to start one of several different potential investment projects. Projects differ in terms of verifiable cash flows and non-verifiable private benefits. The entrepreneur may receive non-transferable private benefits from operating or managing a project, but these private benefits reduce the project's profitability. This generates a basic conflict of interest with the investor since the entrepreneur would like to undertake projects with some private benefits, even if this comes at the cost of lower cash flows. In contrast, the investor can only put her hands on the verifiable cash-flows and thus prefers

[1]For recent surveys of this literature, see Bernanke, Gertler and Gilchrist (1996, 1999).

[2]Underlying this assumption is the idea that bank-like financial intermediaries play a dual role in lending relationships, by limiting entrepreneurs' moral hazard through adequate control and valuations of alternative investment projects, and by assisting entrepreneurs in setting up their business by means of specialized expertise. See Diamond (1991), Besanko and Kanatas (1993) and Holmstrom and Tirole (1997), among others, for papers that emphasize the role of banks in limiting entrepreneurial moral hazard, and Manove, Pagano and Padilla (2001) and Inderst and Mueller (2004) for the idea that because of their expertise, banks often play an essential role in assisting entrepreneurs. Empirical support for the fact that banks provide special services to the entrepreneur, not available to other lenders, can be found in James (1987), Billet Flanery and Garfinkel (1995) and Thakor (1996). The importance of specialized screening and monitoring abilities as well as superior knowledge in some sectors of the economy is also stressed, with reference to venture capitalists, by Gompers and Lerner (1999), Casamatta (2003) and Kaplan and Stromberg (2004), to cite a few.

to finance projects that maximize the size of cash flows or minimize the extent of private benefits.

It is assumed that before proposing a project to an investor, the entrepreneur engages in a costly process of project evaluation. This enables him to understand the project's characteristics and to pick the most preferred one. After the entrepreneur's proposal is made, the investor has the option to exercise some control that gives her the right to influence the course of action before a project gets started. Interference in the implementation of the project is value enhancing because it forces the entrepreneur to give more weight to cash flows and less weight to private benefits. Too much interference, however, comes at the cost of destroying private benefits, which, in turn, dilutes the entrepreneur's ex-ante incentive to evaluate projects. Thus, although excess control guarantees that only high cash flow projects get their way, it also stifles the entrepreneur's initiative to propose projects. This interplay between investor control and entrepreneurial effort is the key determinant of investment fluctuations in this economy.

The driving force of the analysis is that neither entrepreneurial effort nor investor control are contractible. This implies that both parties' acquisition of information is endogenous and affected by the relative costs and incentives. Under the assumption of perfect competition in the credit market, the investor's incentives to interfere in the entrepreneur's selection of projects uniquely depend on her desire to break even. Monitoring incentives are high if financial exposure in the entrepreneur's project is large and low when exposure is small. Therefore, when the entrepreneur has low wealth and must rely extensively on outside funds, the investor scrupulously controls the entrepreneur's selection of projects and endorses only projects that maximize cash flows. By contrast, when the entrepreneur's net worth increases and he needs to borrow less, the investor's incentives to engage in monitoring activity are blunted, since she needs to be compensated less for her investment. A wealthier entrepreneur, therefore, acquires independence from the investor and eventually undertakes projects with lower profitability but higher private benefits — as long as the value of these private benefits is higher than the residual share of cash flows he can pocket after having repaid the investor.

If such a mechanism is embedded in a simple dynamic model with overlapping generations, interesting endogenous investment dynamics arise. During boom times, when entrepreneurs can supply a large fraction of the initial investment, the incentives of investors to control project characteristics are weak. Since investor control is valuable, the undesired effect is that the average project productivity in the economy deteriorates. Moreover, reduced investor control has the additional effect of inducing entrepreneurs to propose those (low-productivity-high-private-benefit) projects that would hardly pass investors' evaluation test in periods of "normal" control activity. Thus, at the peak of an economic boom, less and less productive projects get funded, which paves the way for a subsequent downturn. The opposite effect occurs in "bad times": ruthless cash-flow maximization by investors improves the average productivity of projects, promoting a new period of expansion. Tight investor control, however, comes at the cost of reducing entrepreneurs' incentives to evaluate projects. Thus only a few projects are proposed in downturns, but those undertaken are very profitable.

In the mechanics of the model, the condition under which fluctuations arise takes a very simple form: the cost of control for the investor (or the degree of the agency problem) is neither too high nor too low. Under this condition, the economy either converges to its steady state in an oscillatory manner, or never reaches the steady state and keeps on cycling between periods of boom and recession. Conversely, if the cost of control is too high (or the agency problem is large), the economy does not experience instability but monotonically converges to a stable steady state, featuring low investment.

Overall, the agency problem emphasized in this paper, and its variation over the cycle, has two main implications. First, it suggests that increased firm internal finance may reduce rather than increase economic efficiency. This implication is in line with Jensen's theory of "free cash flows", but stands in contrast with a standard model of credit frictions where more borrower net worth reduces agency costs and therefore restores efficiency. In the story of this paper, better balance sheets are not necessarily associated with more efficient modes of production or allocation of resources, since lower investor control impairs project profitability. Second, exogenous shocks to the economy may be dampened rather than amplified. This is another point of divergence with standard models based on the credit multiplier. The reason why credit markets act as dampeners of shocks in this economy is easy to explain. A positive shock to firm's net worth relaxes investor incentives to control activity and less and less profitable investments are financed, shortening the boom. A similar but opposite mechanism occurs after a negative shock to firm net worth. Whether the financial sector acts in dampening or amplifying exogenous shocks still remains an open question in the literature.

The model I propose is not only able to generate endogenous fluctuations in business investments. It also captures salient features of investment dynamics and lending patterns. For example, the model is consistent with the finding that firms' investment is highly dependent on internal funds (see Hubbard for an extensive survey, 1998). In the model, this dependence arises since low net worth triggers investor control and depresses entrepreneurial effort, limiting the total amount of investment undertaken. Another implication of the model is that only productive projects are financed in bad times. This prediction is in line with findings documenting a clear tendency of banks to extend credit only to "good" borrowers during periods of slumps. Such "flight to quality" (see Bernanke, Gertler and Gilchrist, 1996) is commonly interpreted as evidence that firms with weak financial conditions are more likely to be credit rationed in recessions than in booms. Finally, the model captures the importance of lending practices for investment dynamics. For the US, for example, Asea and Blomberg (1998) find that bank lending standards are countercyclical, and that lending to risky and less productive borrowers increases in good times but decreases in bad times.

The rest of the paper proceeds as follows. Section 2 provides a review of the related literature. Section 3 sets up the basic model, studies in a static set-up the main trade-off that arises in the investor-entrepreneur relationship and fleshes out the main macroeconomic implications. Section 4 embeds the static analysis in a general equilibrium OLG framework in order to study some dynamics and to show under which conditions endogenous fluctuations may emerge. Section 5 discusses some of the key assumptions of the model. Section 6 concludes.

2. **Related Literature**

This paper is related to different strands of literature. First of all, it builds on the insights of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) that credit market frictions affect investment and output dynamics. It goes beyond those papers, however, in examining how agency costs in credit markets may be a potential source of endogenous investment fluctuations, and not only a source of amplification and persistence of exogenous shocks.

In this respect, the paper is close to the recent contributions of Aghion, Banerjee and Piketty (1999) and Matsuyama (2004, 2007). These papers emphasize the role of a pecuniary externality arising in an economy where credit markets are not perfect and borrowers' net worth mitigates credit frictions. In Aghion *et al.*, the pecuniary externality comes from the general equilibrium effects of the interest rate.[3] In Matsuyama, it comes from the selection of investment projects that generate different demand spillover in the economy. My contribution is related to both papers. As in Aghion *et al.*, there is a separation between lenders and borrowers, in the sense that not everyone in the economy is in the position to run investment projects, and, as in Matsuyama, entrepreneurs have access to projects with different productivity. It differs, however, from both contributions since it stresses the role played by the financial intermediaries in permitting less productive investments to get funded during boom periods. It proposes therefore a mechanism that is potentially more suitable for empirical tests.[4]

One prediction emanating from my model is that during periods of low economic activity investors evaluate projects scrupulously, forcing entrepreneurs to shift to more efficient modes of production. This prediction is related to the "pit-stop" view of recessions, according to which recessions encourage agents to engage in activities that contribute to future productivity instead of engaging in production, because the return to the latter declines in recessions (Davis and Haltiwanger, 1990, Aghion and Saint Paul, 1998, Hall, 2000). In my paper, recessions are times

[3]A similar mechanism, operating through the endogenous movements in the price of productive inputs, is explored in Aghion, Bacchetta and Banerjee (2003).

[4]Suarez and Sussman (1997), Azariadis and Smith (1997) and Siconolfi and Reichlin (2004) have also examined the importance of credit market frictions for endogenous fluctuations. In these models, however, borrowers' net worth has no role to play. Another recent example is Martin (2006). In his model borrower's net worth affect investment but it is counter-cyclical, a feature not consistent with the empirical evidence.

when productivity-improving activities are undertaken because of the strengthened investors' incentives to finance only productive projects.

Within the large literature on macroeconomic fluctuations, this paper is also related to the theory of endogenous business cycles (see Boldrin and Woodford (1990) for a comprehensive survey). It differs, however, from most of the papers in this literature because cycles do not originate from special assumptions on preferences and technology that generate a strong income effect or a low substitutability between inputs in the production function. Cycles arise only from credit market imperfections, which affect indirectly the overall level of productivity in the economy.

This paper is also related to several contributions in the banking literature. The role played by the investor in my model is, for example, similar to the one emphasized by Holmström and Tirole (1997). In their model, investor interference is meant to eliminate non-verifiable entrepreneurial benefits. In my paper, investor control is intended to limit the entrepreneurial waste *and* increase project profitability. Related are also the papers of Rajan and Winton (1995) and Manove, Padilla and Pagano (2001) which explore bank incentives to monitor entrepreneurs ex-post or screen them ex-ante, when debt is collateralized. In Rajan and Winton (1995), more collateral increases the incentive of banks to monitor entrepreneurs, whenever collateral value is sensitive to borrower behavior. In Manove *et al.*, (2001), collateral and screening are substitutes because more collateral protects the lenders against the potential risk of default. As in Manove *et al.*., I exploit the fact that collateral and control are substitutes but, in contrast to their paper, I also examine the implications of investor interference on entrepreneurial initiative. Related are also the contributions of Thakor (1996) and Ruckes (2004) who point to the screening and monitoring activity of banks as independent sources of credit and investment cycles through their endogenous effect on the pool of borrowers.

In closing this review, it is worth mentioning that the idea that too much investor control is detrimental for entrepreneurial initiative is inspired by the formal versus real authority analysis of Aghion and Tirole (1997).[5] Also, the emphasis on the varying degree of investor control on entrepreneurial activity, depending on the state of the firms' balance sheet, is reminiscent of the analysis of Aghion and Bolton (1992) where the optimal balance of control between investors and entrepreneurs is state contingent: the entrepreneur should have control rights in good states when his actions do not compromise the return to the investor; the investor should have control rights in bad states since private benefits are less important relative to cash flows.

[5]See Burkart *et al.* (1997) for an application of the control-initative trade-off to corporate finance.

3. **The Basic Model**

This section considers the basic agency problem between an individual firm and a single investor in a partial equilibrium. It is meant to illustrate the main tensions that arise between the parties and indicate their implications for business cycles.

3.1. **Technology, Information Structure and Payoffs.**

The economy has two agents, an entrepreneur and a deep pocket investor, and lasts for two periods, $t = 1, 2$. In the first period, investment decisions are made and financial contracts are signed. In the second period, investment returns are realized and claims settled. Both agents have linear utility in period 2 consumption and the entrepreneur is protected by limited liability. A single good is used for both consumption and investment. The entrepreneur has an endowment w of this good which can be either stored or invested. Storage has a gross return of r units of output per unit of input. The investment technology, instead, yields a random payoff that depends on actions taken by the entrepreneur and the investor.

3.1.1. *Project Types.*

The entrepreneur has access to $J = \{G, B, U\}$ *a-priori* identical projects.[6] Each project J involves a set up cost of $1 > w$ unit of goods, and is characterized by a *verifiable* cash flow Π_J and a non-*verifiable* private benefit b_J for the entrepreneur. While profits and private benefits differ among projects, projects all look *ex-ante* identical and therefore cannot be distinguished from each other without proper investigation. In what follows, it is assumed that the entrepreneur has access to a costly evaluation technology that allows him to discern the project characteristics. Of the J projects, only G and B are "relevant", meaning that they yield non-negative cash flows and/or private benefits. The remaining project, U, entails a large negative payoff for the entrepreneur. This assumption implies that it is never optimal for the entrepreneur to select a project at random.

More specifically, the J projects have the following payoffs:

	G	B	U
Private Benefits	0	b	$-\infty$
Cash Flows	Π	0	Π

Project G generates more cash flows than project B, but offers no private benefits to the entrepreneur. The congruence of the objectives between the entrepreneur and the investor depends on how much cash flow needs to be shared among the two parties. Private benefits, in fact, are not transferable and pertain to the entrepreneur only. If b is higher than the fraction of Π that the entrepreneur can pocket, after repaying the investor, the entrepreneur will prefer implementing project B rather than G. On the other hand, because the investor can appropriate cash flows only, she will prefer to see project G implemented. In the current set up,

[6] Adopting the entertaining terminology of Matsuyama (2004), G, B, and U, stand for Good, Bad and Ugly.

project choice is not verifiable so that no contract can specify either compensation schemes for the entrepreneur based on project selection or investor control.[7] I assume that the payoffs associated with the different projects satisfy the following assumption:

Assumption 1: $\Pi > b \geq r$

3.1.2. *Information and Control.* The information structure is as follows. Since project payoffs are *ex-ante* unknown, the entrepreneur has to acquire information before suggesting one project to the investor. At private convex cost, $c_e(e)$, he learns the payoffs of all possible projects with probability e. With probability $1-e$, he learns nothing and still views the projects as identical. In this last instance, he simply puts his wealth in the storage technology, that guarantees the safe gross return, r. For the entrepreneur, selecting a project at random would not be optimal, given the large negative payoff associated with the U project. With probability e, the entrepreneur discovers the projects' characteristics, discards the U project and approaches the investor to borrow $1-w$.

Depending on the amount of credit that needs to be extended, and hence on the risk of receiving due repayment, the investor chooses how much control to exercise on the entrepreneur's selection of projects. I assume the investor can interfere at a private convex cost $c_m(m)$. By interfering, she limits the entrepreneur's selection of projects. With intensity $m \in (0,1)$, the investor forces the entrepreneur to pick the project that maximizes cash flows, leaving the entrepreneur the freedom to consume $(1-m)b$ of private benefits.

Remark 1. The investor's control on the implementation of projects can be given a much broader interpretation than the one of mere interference. For example, the investor may have access to a screening technology that allows her to receive a signal over the type of projects proposed by the entrepreneur. If the signal is informative, the investor understands the project's characteristics and dictates the type of project that the entrepreneur must run. If the signal is not informative, she does not understand the project type, and rubber-stamps the project proposal that can be G or B with, say, equal probability (as long as the payoff satisfies her break-even constraint). In an alternative interpretation, the control of the investor can be thought of as assistance to the entrepreneur during the phase of planning and implementation of the project. Too little assistance results in poor cash flow

[7] At the cost of more involved algebra, one could alternatively assume that project G produces no private benefits and a stochastic cash flow with probability p, while project B produces private benefits b with certainty and cash flows Π with probability $q < p$. If these probabilities of success or failure are independent of entrepreneurial effort and the entrepreneur's evaluation effort continues to be non-contractible, the analysis conducted below would not be affected at all, with the only difference that the investor would now monitor in proportion to the riskiness of his financial claim, equal to the difference of payoffs between the two projects. In the current set up the investor's payoff if the B project is implemented is zero; therefore her degree of control would always be higher than in the modified set up. If anything, then, the modified set up would strengthen the results presented below.

performance and high consumption of perks. Both interpretations are consistent with the interference formulation, since interference has the dual effect of limiting inefficiencies associated with the consumption of private benefits and improving project cash flows.

Remark 2. Rather than just effort devoted to project evaluation, the entrepreneur's effort may be interpreted as effort to set up a particular business plan, or even more generally a non-contractible firm-specific investment that increases or reduces firm value. What is essential is that some of these actions are socially sub-optimal, though individually optimal. Similarly, private benefits do not need to be interpreted as consumption of perks, diversion of resources or personal satisfaction. They can also be interpreted as the negative of the private cost that the entrepreneur has to pay for adopting new technologies (such as effort to get properly trained, reorganize the firm or retrain workers). What is crucial is that one dollar of private benefits reduces firm value by more than a dollar.

Remark 3. The problem between the investor and the entrepreneur could be interpreted as arising in the course of an ongoing relationship, rather than upon first contact. The latter interpretation is preferred in order to emphasize the consequences of ex-ante selection of projects. What is essential is that the investor has the opportunity to stop some actions through interference.

3.1.3. *Contracts and Timing.* It is assumed that there is a large supply of outside financiers. The resulting competition gives all the ex-ante bargaining power to the entrepreneur so that the investor's optimal decision to exercise control is taken to maximize the entrepreneur's ex-ante expected utility, subject to her break-even constraint.

The entrepreneur has an endowment w of consumption goods. To activate a project, he needs to pay a fixed cost $1 > w$ and thus needs to borrow at least $1-w$ from the investor. By assumption, the investor has to be repaid out of Π, and to attract the investor, the entrepreneur offers her a share α of Π.

The relationship between the two parties is described by the following game, summarized in Figure 1. At stage 0, the entrepreneur exerts an evaluation effort. After evaluation takes place, he decides whether to proceed. If he does not proceed, he simply stores his endowment. If he chooses to proceed, the entrepreneur contacts the investor and offers a contract. A contract specifies how much each side should invest and how much each party should be repaid out of the project's outcome. Without loss of generality, I restrict attention to one arrangement where: (1) the entrepreneur invests all its funds, w, while the investor puts up the balance, $1-w$; (2) the investor is paid a fraction $0 < \alpha < 1$ of the verifiable cash flow Π, whereas the entrepreneur keeps the difference.[8] At stage 1, after the contract is signed, and before the project is implemented, the investor chooses his monitoring intensity

[8]Because the projects' outcome has a two-point distribution – success or failure – this contract can be interpreted as either a debt or an equity contract.

as a function of its overall exposure towards the entrepreneur. At stage 2, the project's payoff is realized.[9]

$t = 0$	$t = 1/2$	$t = 1$	$t = 2$
E exerts effort e	Financing occurs and contract terms defined α	I chooses monitoring intensity m	Π and/or b realized.

Figure 1

3.1.4. *Payoffs.* Under the assumption that the investor receives a share α of the verifiable cash flow Π, a conflict of interest arises between the investor and the entrepreneur whenever $(1-\alpha)\Pi < b$ for some $0 < \alpha < 1$. In this case, the entrepreneur's preferred project is B, while the investor prefers project G.

With the proposed timing, and the assumption of universal risk neutrality, the entrepreneur's and the investor's ex-ante expected utilities are:

$$u_E = (1-e)rw + e\left[m(1-\alpha)\Pi + (1-m)\max\left\{(1-\alpha)\Pi, b\right\}\right] - c_e e^2/2, \quad (3.1)$$

$$u_I = (1-e)r(1-w) + e\left\{m\alpha\Pi + (1-m)\times 0 - c_m m^2/2\right\}. \quad (3.2)$$

For the entrepreneur, the first term is the return from storing his endowment, if his evaluation of the projects is not successful. The second term is the expected payoff of undertaking the project. When the investor monitors with intensity m, the entrepreneur receives a fraction $m(1-\alpha)$ of Π, but can still reap a portion $(1-m)$ of the available private benefits b, insofar as $(1-\alpha)\Pi < b$ for some α (to be determined in equilibrium). Finally, the third term is the entrepreneur's cost of evaluating the projects. For the investor, her payoff is equal to the return on the storage technology if, with probability $1-e$, she is not approached by the entrepreneur. Conditional on e, instead, the investor's payoff depends on the amount of cash flows that she receives by interfering with intensity m, net of monitoring costs.

The two payoff functions highlight the different roles played by e and m in this framework. In both (3.1) and (3.2) e is crucial since it affects the overall size of the return to both parties. More generally, e can be interpreted as determining the overall level of investment of this economy. The "nature" of this investment, in turn, depends on m. Projects produce more cash flow for high m values. In contrast, for low m, the entrepreneur can reap some non transferable output in the

[9]The process of information acquisition for the investor and the entrepreneur could be simultaneous, rather than sequential. This modification would not bring additional substantive issues into the analysis but would lead to no-closed-form solutions. Moreover, nothing would change if the monitoring intensity is chosen after entrepreneurial effort but before the financing stage. What is important is that the evaluation cost for the entrepreneur occurs ex-ante, and control rights are given to the investor ex-post so that her interference has an adverse effect on the entrepreneur's ex-ante incentives to evaluate projects.

form of private benefits. e and m are therefore complementary: e determines "the size of the pie", m affects "the way the pie is distributed". Without e no output is produced, without m no cash flow is generated. Because of the non contractibility of the two actions, e and m will be chosen by each of the two parties to maximize their own utility, and given the conflict of interest between the two, e and m will, in general, be *strategic substitutes*. The implication is that the size of the investment and the amount of cash flows generated cannot, in general, be jointly maximized.

3.2. **First Best.** The natural benchmark is the case where the conflict of interest is absent. This case arises if $(1-\alpha)\Pi > b$, i.e., the monetary incentives for the entrepreneur are powerful enough that he forgoes private benefits and always prefers to maximize cash flows.[10] If condition $(1-\alpha)\Pi > b$ holds, the entrepreneur chooses e to maximize the expected second period consumption from implementing project G:

$$\max_{e} \quad u_E = (1-e)rw + e(1-\alpha)\Pi - c_e e^2/2$$

subject to the investor break-even constraint:

$$(1-e)r(1-w) + e\alpha\Pi \geq r(1-w).$$

It then immediately follows that the first-best is achieved by setting

$$e^{fb} = \min\left\{\frac{\Pi - r}{c_e}, 1\right\}. \tag{3.3}$$

In words, the level of effort is constant and independent of the entrepreneur's wealth.[11]

In a second-best world, entrepreneurial effort will be lower, because absent monitoring, the entrepreneur will just implement the project with private benefits, forcing the rational investor to monitor. This results in higher debt repayment than in the first best scenario, given that the entrepreneur now needs to compensate the investor for the opportunity cost of funds *and* the cost of monitoring. Because the repayment is higher, and control reduces the size of private benefits, the entrepreneur does not appropriate the full return of his evaluation effort, and thus supplies less effort than in the first best. This inefficiency stems from the inability of the entrepreneur to commit not to undertake any projects with private benefits.

[10]Obviously, another possible interpretation of the first best is when the project choice is contractible, or the entrepreneur is not wealth-constrained so that he can finance the project himself. In the latter case the entrepreneur does not need to share any part of the project's cash flow with the outside investor. Given the assumption that $b < \Pi$, the entrepreneur always chooses the G project and the conflict of interest does not arise.

[11]The entrepreneur always prefers evaluating the project to storing his wealth straight away, whenever $u_E(e^{fb}) \geq rw$ holds. This condition is satisfied if $\Pi \geq r$, which is always true by Assumption 1.

In this first best scenario, monitoring is always zero, since the entrepreneur always chooses project G and thus the investor does not need to monitor.

3.3. **The Optimal Contract.** I now consider the case where the level of private benefits is high enough for a conflict of interest to exist between the parties, i.e., $(1-\alpha)\Pi > b$. To determine equilibrium effort and monitoring, and hence the optimal selection of the investment project, the following problem must be solved:

$$\max_{e,\,\alpha}\ (1-e)rw+e\left\{m^*(1-\alpha)\Pi+(1-m^*)b\right\}-c_e\frac{e^2}{2} \tag{3.4}$$

subject to,

$$m^* = \arg\max_m e^*\left\{m\alpha\Pi+(1-m)\times 0-c_m\frac{m^2}{2}\right\} \tag{3.5}$$

$$e^*\left\{m^*\alpha\Pi+(1-m^*)\times 0-c_m\frac{m^{*2}}{2}\right\}\geq e^*r(1-w) \tag{3.6}$$

$$0\leq\alpha\leq 1. \tag{3.7}$$

$$(1-e^*)rw+e^*\left\{m^*(1-\alpha)\Pi+(1-m^*)b\right\}-c_e\frac{e^{*2}}{2}\geq rw. \tag{3.8}$$

Equation (3.4) is the entrepreneur's expected utility, equal to his gross gain from evaluating the project, less his expected obligation to the investor and the evaluation cost. This objective is maximized with respect to the level of effort and the fraction of project outcome to be shared with the investor, subject to the investor incentive compatibility constraints (3.5) and her break-even condition (3.6). Equation (3.7) is a feasibility constraint, which requires that the investor cannot appropriate more than the entire cash flow of the project, ensuring limited liability for the entrepreneur. Finally, (3.8) is the participation constraint for the entrepreneur, stating that at the equilibrium level of effort, e^*, and monitoring, m^*, his utility of evaluating and undertaking the project is larger than the utility of storing his wealth straight away.

3.3.1. *The Basic Trade-off.* The basic trade-off underlying the entrepreneur-investor relationship follows directly from inspection of the two parties' reaction curves:

$$e^*=\min\left\{\frac{b-rw-m^*(b-(1-\alpha)\Pi)}{c_e},1\right\} \tag{3.9}$$

and

$$m^*=\min\left\{\frac{\alpha\Pi}{c_m},1\right\}. \tag{3.10}$$

Equation (3.9) indicates that the entrepreneur's effort to become informed (i.e., his *initiative*) increases with the size of the private benefits and decreases with the opportunity cost of investing funds in the project, rw, and with the evaluation cost, c_e. Moreover, e falls with the likelihood of having to lose control over the choice of the project, m^*. For a given m^*, effort is also lower the higher the share of the final output that needs to be given to the outside investor $\alpha\Pi$. Equation (3.10) suggests, instead, that the investor's incentive to monitor (i.e., the degree of *interference* on the entrepreneurial project choice) increases monotonically with her share in the project's revenue and decreases with the cost of monitoring.[12]

The incentive compatibility conditions for effort and monitoring identify the crucial tension between the investor and the entrepreneur. Specifically, when the entrepreneur borrows money, he needs to share part of the project's income, $\alpha\Pi$, with the outside investor. When this share is high, two forces affect the entrepreneur's incentives. The first is a traditional one in a principal-agent relationship: for a given level of monitoring, a lower share of income that accrues to the entrepreneur increases the conflict of interest with the investor – the discrepancy between b and $(1-\alpha)\Pi$ – and thus, reduces the incentives of the entrepreneur to select the G project. This force is mitigated when more entrepreneurial wealth is invested in the project. With more wealth at stake, the entrepreneur internalizes the consequences of his actions mitigating the agency problem with the investor and *a fortiori* the incentives of the latter to control the entrepreneur's actions. The second force is more specific to the current set-up and operates through the investor's control. When the share of income that must be given to the investor is high, she has a large incentive to monitor entrepreneur's selection of projects. More interference, however, destroys private benefits, reducing the entrepreneur's incentives to evaluate projects ex ante. Extensive recourse to external financing has therefore two negative effects on entrepreneur effort but also a positive effect on project value, as monitoring increases the likelihood of generating cash flows. Thus, in the current setting, and contrary to the traditional literature on investment in the presence of agency problems, projects financed by external capital may be more profitable than projects relying more on internal finance, exactly because investor's control limits private benefits and increases cash flows.

The fact that, in the presence of a conflict of interest, i.e. $b > (1-\alpha)\Pi$, the entrepreneur's reaction curve is downward sloping with respect to m^*, implies that the investor refrains from exerting maximum investigation as this worsens the entrepreneur's initiative. The crucial feature of this model that effort and monitoring are substitutes, differs substantially from that arising in a set-up with ex-post entrepreneurial moral hazard (as in Holmström and Tirole (1997)) or in a monitoring model with costly state verification (as in Bernanke and Gertler (1989)). In those models, the entrepreneur's unobservable actions occur after financing takes place and limit the size of future cash flows that can be pledged to the investor. To limit the moral hazard problem, the investor can monitor, and in equilibrium more control implies more entrepreneurial effort. In the set up of this paper, instead, more control reduces entrepreneurial incentives to exert ex-ante effort, and thus to undertake new investment.

[12]Notice that the investor reaction curve is independent of e because she monitors after the entrepreneur has made his proposal.

3.3.2. *Equilibrium Outcomes.* To understand the implications of non contractible effort and control, it is useful to start by substituting the equilibrium level of monitoring, m^*, into the investor's break-even condition which, in equilibrium, must be binding. The equilibrium terms at which lending occurs, i.e., the portion of cash flow that the investor requires to be willing to participate in the contract, is given by:

$$\alpha = \frac{\sqrt{2r(1-w)c_m}}{\Pi}. \tag{3.11}$$

Inspection of (3.11) reveals that, *ceteris paribus*, α decreases with borrower wealth. Hence, as the entrepreneur supplies a larger fraction of the initial investment, the investor requires a smaller fraction of the project cash flows. Given that the optimal level of monitoring is monotone in α (see (3.10)) it follows that as entrepreneur's net worth increases, investor control falls, and given (3.9) the entrepreneur's effort increases.

From the entrepreneur's reaction function, however, e^* and m^* are inversely related if and only if $b > (1-\alpha)\Pi$ or $\alpha > \overline{\alpha} \equiv 1 - b/\Pi$. Using (3.11), this condition amounts to saying that effort and monitoring are strategic substitutes, whenever the level of net worth is below the threshold $\overline{w}$:

$$w \leq 1 - \frac{(\Pi - b)^2}{2rc_m} \equiv \overline{w}. \tag{3.12}$$

In the other case, where the entrepreneur is sufficiently wealthy, i.e. $w > \overline{w}$, or $\alpha < \overline{\alpha} \equiv 1 - b/\Pi$, the share of the project payoff that accrues to the entrepreneur, $1-\alpha$, is large enough that he values cash flow more than private benefits. Therefore, for $w > \overline{w}$ we are back to the first-best case, where only high cash flows projects are selected by the entrepreneur and the conflict of interest is mitigated.

For given profitability, the contract feasibility constraint (3.7) also determines a lower bound on entrepreneur's wealth below which no profitable transactions take place. Specifically, equations (3.11) and (3.7) imply that entrepreneurs must put a minimum level of wealth into the project to credibly offer a repayment α to the investor. This minimum level of wealth is given by:

$$\alpha \leq 1 \iff w \geq 1 - \frac{\Pi^2}{2rc_m} \equiv \underline{w} \tag{3.13}$$

that is larger than or equal to zero for a value of Π such that $\Pi \leq \sqrt{2rc_m}$, which from now on, it is assumed to hold :

Assumption 2: $\sqrt{2rc_m} \geq \Pi$

Finally, the participation constraint of the entrepreneur determines the minimum level of wealth, $\widetilde{w}$, above which he is willing to undertake a costly process of project evaluation.[13] Defining with $\widehat{w} = \max\{\underline{w}, \widetilde{w}\}$, these results can be summarized in the following

Lemma 1. *For given parameters* (Π, b, r, c_m, c_e) *satisfying Assumptions 1 and 2, there exist two cut-off values* $\widehat{w}$ *and* $\overline{w}$, *with* $\widehat{w} < \overline{w}$ *such that:*

1. If $0 \leq w \leq \widehat{w}$ the entrepreneur either has insufficient wealth to undertake the project or does not wish to invest at all. In this case, $e = m = 0$.
2. If $\widehat{w} < w < \overline{w}$, the project is funded and the equilibrium levels of effort and monitoring are given by (3.9) and (3.10), respectively.
3. If $w \geq \overline{w}$, the conflict of interest between the parties vanishes. The optimal level of effort is given by (3.3), and both investment and productivity are constant.

Lemma 1 suggests that the strategic interaction between investor and entrepreneur actions is relevant only for levels of entrepreneurial net worth in an intermediate range. In this range, the two actions are strategic substitutes, implying that the overall amount of investment and its productivity cannot be maximized jointly. To further explore the implications of Lemma 1 it is instructive to define

$$p = em\Pi \tag{3.14}$$

as the level of productive investment, measured by the amount of output that can be shared between the entrepreneur and the investor, and

$$y = em\Pi + e(1-m)b \tag{3.15}$$

as the total output in the economy, comprising both the part of output that can be shared $em\Pi$ and the part that accrues to the entrepreneur, in the form of private benefits, $e(1-m)b$.

With these definitions in mind, the main result of this section can be stated in the following proposition.

---

[13] The minimum level of wealth, $\widetilde{w}$, above which the entrepreneur's partecipation constraint holds, is given by

$$e^*\left\{m^*(1-\alpha)\Pi + (1-m^*)b\right\} + (1-e^*)r\widetilde{w} - c_e\frac{e^{*2}}{2} \geq r\widetilde{w}.$$

After substituting for the equilibrium values of m^*, e^* and α, the expression above holds whenever $e(\widetilde{w}) \geq 0$ or

$$e(\widetilde{w}) = \frac{(b-r) + (\Pi - b)\sqrt{\frac{2r(1-\widetilde{w})}{c_m}} - r(1-\widetilde{w})}{c_e} \geq 0$$

Unfortunately, the above expression does not have an explicit solution, unless $b = r$. It is easily shown, however, that $e(\overline{w}) > 0$ for the parameter values satisfying Assumption 1 and, at $\underline{w}$, it may be that $e(\underline{w}) < 0$, if $\Pi < 2b$ and $(b-r) + (\frac{\Pi}{2} - b)\frac{\Pi}{c_m} < 0$. However, since $e(\widetilde{w})$ is an increasing and concave function of w, and reaches a maximum at $\overline{w}$, there exists a $\widetilde{w} \geq \underline{w}$ such that $e(\widetilde{w}) = 0$, and the entrepreneur with $w < \widetilde{w}$ prefers not to borrow. $\widetilde{w}$ is identical to $\underline{w}$ if $b = r$ and $\Pi = 2b$.

Proposition 1. *When $w \in (\widehat{w}, \overline{w})$, the entrepreneur's effort and the investor's control are strictly positive. As w increases in this range, overall output, y, increases monotonically. Moreover, there is a threshold level of net worth, $w^* \in (\widehat{w}, \overline{w})$, such that productive investment, p, rises for $w < w^*$ and falls for $w > w^*$.*

Proof. See Appendix. □

The mechanism behind Proposition 1 is easy to state. When the entrepreneur is eligible for financing, but has little wealth, the investor's financial exposure is high and she must appropriate a high fraction of the project return to ensure non-negative profits. Given the underlying conflict of interest with the entrepreneur, the investor needs to monitor to ensure that only projects that generate high cash flows are undertaken. Therefore, at low levels of net worth, high investor control helps to improve the project's profitability, for a given level of entrepreneurial effort. Excess monitoring, however, increases interference, reducing the entrepreneur's initiative. It follows that at low levels of net worth, the amount of investment undertaken is low, but the overall productivity is increasing in monitoring. The beneficial effect of high control on productivity vanishes, however, when net worth further increases in the range $\widehat{w} < w < \overline{w}$. The less the exposure of the investor (i.e. higher w) and thus the higher the fraction returns that goes to the entrepreneur, the more likely the investor will go along with the entrepreneur's proposal. As a consequence, investor control falls and entrepreneur effort rises. The overall effect is that as w increases, entrepreneurial investment goes up and, with it, the amount of entrepreneurial waste, at the expense of investment productivity. Eventually, as the level of wealth surpasses the threshold w^*, productivity starts falling until w approaches $\overline{w}$. At this point, the conflict of interest vanishes and investment and its productivity depend only on entrepreneurial effort, given by (3.3).

3.3.3. *A Numerical Example.* To gain further insights into the potential dynamic implications of the model, it is useful to present a simple numerical example. I set $\Pi = 2$, $b = r = 1$, $c_m = c_m = 2.22$ and compute the equilibrium value of α that satisfies the investor's break-even constraint, using equation (3.11) and alternative values of w in $[0, 1)$.

Figure 2 depicts the impact of the equilibrium effort and monitoring on the total amount of output y, given by (3.15), and the productive investment, p, given by (3.14). Both variables are plotted against the level of net worth. As discussed above, the relationship between these two variables and entrepreneurial wealth, w is non-monotonic. For the parameter values used in this example, below a critical level of net worth, $\underline{w} \approx 0.09$, the entrepreneur optimally decides to exert no effort, so that no investment takes place. For an intermediate range of the net worth, $0.09 < w < 0.6$, output and productive investment increase monotonically. In this range, investor control intensity falls gradually, while the entrepreneurial effort increases steadily. Initially, the increase in e, is enough to compensate for the fall in m, so that y and p, rise in tandem, though at a decreasing rate. As

w increases further, i.e., $w > 0.6$, the control exerted by the investor is so low that only non-productive projects are financed. Thus, for high values of w, but not too high —that is before the conflict of interest vanishes $\overline{w} \approx 0.78$— a rising entrepreneurial effort and a falling investor control lead to higher investment and less productive investment.

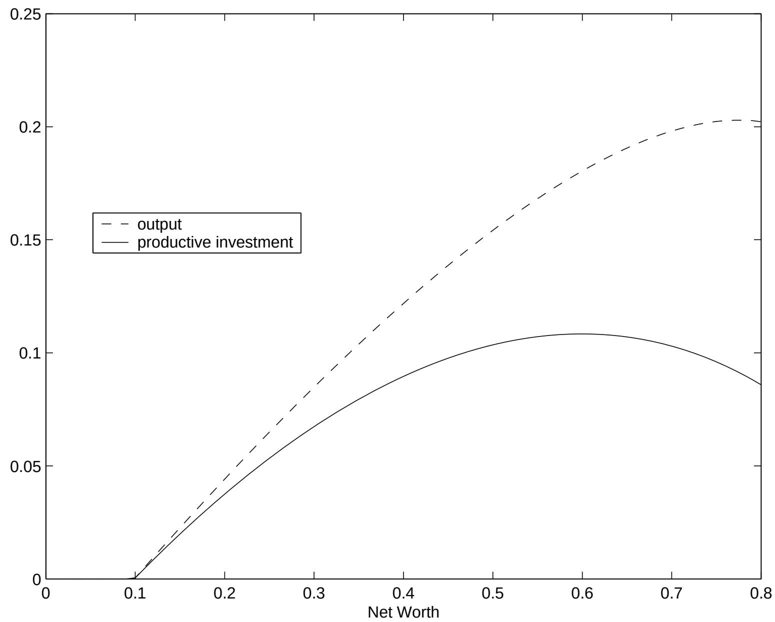

Figure 2: Output and Productivity

These comparative statics should be contrasted with those arising from a standard investment model with financial frictions (as, for example, in Bernanke and Gertler, 1989). In that model, an increase in borrower net worth reduces the agency costs and raises investment and output. This pattern also arises in the current setting, as the total level of investment is increasing in w. In this model, however, an additional effect comes into play once entrepreneur net worth becomes sufficiently high: a wealthy entrepreneur gains independence from the investor and less productive projects get funded. In other words, while the current setting has the same implications on the overall level of investment as in a standard model with credit frictions, it also has something to say about the level of productive investment in the economy. Specifically, the productivity of the investment is increasing at low levels of wealth through the value enhancing effect of investor control, and deteriorates when the level of net worth surpasses a certain threshold. Once more, this effect arises because the investor reduces interference with the entrepreneur's selection of projects.

4. **Dynamics**

To endogenize the evolution of borrower net worth and thus, the time path of monitoring and effort costs, this section embeds the static analysis presented above into a dynamic model with overlapping generations. The framework is a modified version of the OLG model of Diamond (1965) with agents living for two periods.[14] The dynamic analysis will affect only the development of entrepreneurial net worth but not the financial relationship between borrowers and lenders. The two parties continue to be related by a financial contract that lasts only for one period. Their non-cooperative actions, however, affect the amount of capital that can be brought to the next period in the final good sector and thus, the wealth of future generations.

4.1. **The Model.**

4.1.1. *Agents, Preferences and Endowments.* The economy is populated by an infinite sequence of overlapping generations of agents. Each generation lives for two periods and consists of a continuum of agents with unit mass. Agents are risk neutral, endowed with a fixed amount of labor, L, and care only about second period consumption, net of effort costs. Within each generation agents are heterogenous. An exogenous fraction, η, are entrepreneurs, with access to an investment technology, to be described below. The remaining fraction, $1-\eta$, of agents have no entrepreneurial ability and will be referred to as lenders or investors.

4.1.2. *Technology.* The production side of the economy consists of a single final good sector and a continuum of intermediate good sectors. The final good sector produces a consumption good by means of a Cobb-Douglas production function,

$$Y_t = A_t K_t^{\beta} L_t^{1-\beta}$$

where K_t is capital, L_t labor and A_t a scale parameter which, as in Romer (1986), depends on the aggregate stock of capital in the economy (the effects of which are not internalized by individual firms):

$$A_t = K_t^{\gamma} \quad \text{with} \quad \gamma = 1-\beta.$$

Perfect competition in this sector implies that the price of capital and labor are, respectively,

$$\rho_t = \beta \tag{4.1}$$

and

$$w_t = (1-\beta)k_t = w(k_t), \tag{4.2}$$

where $k_t = K_t/L_t$ denotes the capital-labor ratio. These two equations imply zero profits for all firms producing $y_t = Y_t/L_t$ and indicate that as the stock of capital in the economy, k, expands, wage income, $w(k_t)$, increases, while the price of

[14]A similar framework is used in Bernanke and Gertler (1989), Azariadis and Smith (1997) and Matsuyama (2004). As in these papers, the "period" is supposed to represent the length of a typical financial contract, rather than a generation of individuals.

capital remains constant.[15] For convenience, I normalize the economy-wide labor endowment L to unity so that per capita and aggregate quantities are the same.

In this final good sector, labor is supplied inelastically by the young agents of period t, at no utility cost. Their wage income, w_t, is then used to finance consumption in period $t+1$, so that the total level of saving in this economy is also equal to w_t.[16] Young entrepreneurs have two saving options: their wage income can be saved in a storage technology, which has a non-stochastic gross return of r units of consumption goods, or it can be used to partially finance an investment project that transforms consumption goods into capital goods. Young lenders, on the other hand, can finance their $t+1$ consumption by lending their wage income to entrepreneurs or by saving through a storage technology.

The capital stock used in the final good sector comes from an intermediate-capital-producing sector operated by young entrepreneurs. The intermediate sector transforms, without using labor, consumption goods of time $t-1$ into capital goods available for use at time t. More precisely, capital produced by young entrepreneurs at the end of time $t-1$ is sold at the beginning of period t to the final good sector, at price ρ_t, and, for simplicity, fully depreciates after use.[17]

4.1.3. *The Intermediate Sector and the Credit Market.* The intermediate sector works in the same way as in the static model discussed in the previous section. Young entrepreneurs have access to the three types of investment technologies, G, B and U, with the qualification that project G now produces capital goods only, while project B generates consumption goods for the entrepreneur. Therefore, the agency problem that arises between entrepreneurs and investors determines the amount of capital that can be brought forward to the next period, k_{t+1}, and hence, the wage income of future young agents (see equation (4.2)). The feedback from k_{t+1} to w_{t+1} is the crucial link of the dynamics of this economy.

It is convenient to think of the borrowing-lending relationship as occurring through financial intermediaries that accept deposits, extend loans and exercise control on entrepreneurs. This way, lenders and entrepreneurs with a negative evaluation of projects, allocate their wealth between deposits with financial intermediaries and the storage technology. Entrepreneurs that successfully evaluate their projects enter instead in a financial arrangement with an intermediary which lasts for one period only.

[15]Aghion *et al.* (1999) use a similar assumption on the production function, with the intent of fixing the wage rate and allowing the interest rate to fluctuate.

[16]The fact that agents care only about second period consumption allows us to focus on the interaction between investors and entrepreneurs, without also having to worry about the consumption-saving decisions of old and young.

[17]The assumption that capital depreciates fully after use ensures that at each point in time investment is equal to the capital stock. It also allows me to ignore the additional complication of formalizing the capital resale market.

The final assumption is that the supply of funds available in the economy is larger than the maximum amount of funds demanded by the entrepreneurs[18]

$$w_t > \eta e_t.$$

Hence, storage is always used in equilibrium, and its return pins down the interest rate in the economy.[19] The fact that the supply of loanable funds is perfectly elastic implies that all actions in this economy come from investment demand. The supply side of the credit matters only in terms of investors' incentives to monitor entrepreneurs. This set up may therefore be considered as characterizing an economy where availability of loanable funds is not a problem. Instead, it is the investor's incentive to control entrepreneurial behavior that shapes investment and its productivity.

4.1.4. *Payoff Structure.* Following the same steps as in Section 3, the entrepreneur's and lender's expected utility, are given by

$$u_E = (1-e_t)rw_t + e_t\left[m_t(1-\alpha_t)\Pi\beta + (1-m_t)\max\{(1-\alpha_t)\Pi\beta, b\}\right] - c_e\frac{e_t^2}{2}, \quad (4.3)$$

and

$$u_I = e_t\left\{m_t\alpha_t\Pi\beta + (1-m_t)\times 0 - c_m\frac{m_t^2}{2}\right\}, \quad (4.4)$$

which are the equivalents of (3.1) and (3.2), with the difference that $\Pi\beta$ is now the consumption value of the capital goods produced by the G project. Notice that in this formulation, private benefits and the costs of evaluation and control are expressed in terms of consumption goods.

4.2. **First-Best.** The benchmark case arises, again, when $b < (1-\alpha_t)\Pi\beta$. Simple maximization of (4.3), subject to the intermediary break-even constraint,

$$\alpha_t\Pi\beta = r(1-w_t),$$

gives

$$e_t^{fb} = \min\left\{\frac{\Pi\beta - r}{c_e}, 1\right\}. \quad (4.5)$$

This equation parallels equation (3.3).

[18]In each period, the amount of loanable funds is equal to the wealth in the hands of lenders, $(1-\eta)w_t$, plus the wealth of those entrepreneurs who, with probability $1-e_t$, decide not to go ahead with the project, $\eta(1-e_t)w_t$. The funds demanded are $\eta e_t(1-w_t)$, corresponding to the fraction of entrepreneurs who decide to go ahead with the project times the amount of consumption goods $(1-w_t)$ they need to borrow to start the investment.

[19]The assumption that the supply of investment funds is perfectly elastic follows Bernanke and Gertler (1989). Aghion et. al. (1999) and Matsuyama (2004), show instead that endogenous movements in the interest rate can give rise to non linear investment dynamics, when credit market are not frictionless. This paper complements the work of these authors by showing how endogenous cycles can be obtained even if the interest rate is constant.

In this first-best scenario, the dynamic equilibrium of the economy is trivial. At any point in time t, the total investment is

$$i_t = e_t^{fb} \times \eta,$$

and assuming an interior solution for e_t^{fb}, the per capita capital stock available in the next period is

$$k_{t+1} = i_t \times \Pi = \frac{(\Pi\beta - r)}{c_e}\eta\Pi, \tag{4.6}$$

which is independent of period-t state variables. Hence, in an economy — free of agency problems — the level of capital is constant over time and there is a unique stable steady state to which the economy converges in one period.[20]

4.3. **Equilibrium with Agency Problems.** When agency costs are re-introduced, the amount of capital crucially depends on the way saving is allocated across the two technologies, G and B. In particular, the equilibrium wage, given the inherited capital stock k_t,

$$w_t = w(k_t)$$

and the equilibrium level of effort and control, given the current wage w_t,

$$e_t = e(w_t) \text{ and } m_t = m(w_t)$$

determine the production of new capital k_{t+1}

$$k_{t+1} = K(e(w(k_t)), m(w(k_t))), \tag{4.7}$$

with $K_e > 0$ and $K_m > 0$. Therefore, even though the supply of credit is perfectly elastic, the amount of capital that can be brought forward to the next period is now indirectly dependent on the total amount of savings $w(k_t)$, through its impact on $e(w(k_t))$ and $m(w(k_t))$.

In (4.7) k_{t+1} is increasing with respect to both e_t and m_t, but, as shown in the previous section, while e_t is an increasing function of $w(k_t)$, m_t is decreasing in $w(k_t)$. As a consequence, the accumulation path of capital:

$$\frac{dk_{t+1}}{dk_t} = \left(\underset{+}{\frac{\partial K}{\partial e}} \underset{+}{\frac{\partial e}{\partial w}} + \underset{+}{\frac{\partial K}{\partial m}} \underset{-}{\frac{\partial m}{\partial w}} \right) \underset{+}{\frac{\partial w}{\partial k_t}} \tag{4.8}$$

may be non-monotonic in $w(k_t)$.

[20]As in Bernanke and Gertler (1989), the fact that the frictionless economy does not have any dynamics is due to the assumption that the supply of funds is perfectly elastic with respect to the interest rate. In Bernanke and Gertler, the introduction of information asymmetries generates a demand for investment that is persistent and dependent of entrepreneur internal funds. In the current set up the introduction of agency costs generates not only persistence, but also instability in investment dynamics.

4.3.1. *Investment Dynamics.* To gain further insights into the dynamics implied by the difference equation (4.7), I repeat the steps of Section 3.3, under the technical requirement

Assumption 3. $\sqrt{2rc_m} \geq 2(\Pi\beta - b)$

and the convenient normalization[21]

Assumption 4. $b = r$.

The following Lemma, which parallels Lemma 1 of Section 3, characterizes investment dynamics for different levels of entrepreneurial wealth

Lemma 2. *For parameters values* (Π, β, b, r, c_m), *satisfying Assumptions 3 and 4, there exist two cut-off values* $\widetilde{w} \equiv 1 - \frac{2(\Pi\beta - b)^2}{rc_m}$ *and* $\overline{w} \equiv 1 - \frac{(\Pi\beta - b)^2}{2rc_m}$, *with* $\widetilde{w} < \overline{w}$ *such that:*

1. If $0 \leq w_t \leq \widetilde{w}$, the entrepreneur does not wish to invest. In this case $e(w_t) = m(w_t) = 0$.
2. If $\widetilde{w} < w_t < \overline{w}$, investment takes place and the equilibrium levels of monitoring and effort are given by:
$$m(w_t) = \sqrt{2r(1 - w_t)/c_m} > 0$$
$$e(w_t) = (m_t(\Pi\beta - b) - r(1 - w_t))/c_e > 0.$$
3. If $w_t \geq \overline{w}$, the conflict of interest between the parties vanishes and the optimal level of effort, $e(w_t)$, is given by (4.5), while monitoring, $m(w_t) = 0$.

Proof. See the Appendix □

Therefore, per-capita investment depends on the interaction of effort and control only for intermediate ranges of wealth $w_t \in (\widetilde{w}, \overline{w})$ and is given by:

$$i = \eta \left[\underbrace{e_t(m_t(w_t), w_t) \times m_t(w_t) \times \Pi}_{k_{t+1}} + \underbrace{(1 - m_t(w_t)) \times e_t(m_t(w_t), w_t) \times b}_{b_{t+1}} \right] \quad (4.9)$$

In (4.9) the first term represents the amount of capital goods available for final good production in $t + 1$, while the second term is the amount of consumption goods available to entrepreneurs for consumption in period $t + 1$, which is larger the lower is investor control.

For ease of exposition, and given the one-to-one mapping between wage and capital implied by equation (4.2), it is convenient to formulate the law of motion of capital, k_t, in terms of the equilibrium wage, w_t :

[21]This normalization is made to obtain explicit solutions.

$$w_{t+1} = \Phi(w_t) = \begin{cases} 0 & \text{if } \ w_t < \widetilde{w}_t \\ \phi(w_t) & \text{if } \ \widetilde{w}_t \le w_t \le \overline{w}_t \\ w^{fb} & \text{if } \ w_t > \overline{w}_t \end{cases} \tag{4.10}$$

where

$$w^{fb} = \lambda\delta, \tag{4.11}$$

is the first-best level of wage, obtained by replacing (4.2) into (4.6), while

$$\phi(w_t) = (1 - w_t)\left[\frac{2r}{c_m}\lambda - r\sqrt{\frac{2r}{c_m}(1 - w_t)}\right]\delta, \tag{4.12}$$

is obtained by substituting the equilibrium level of control and effort (given in Lemma 2) into the first term of (4.9).

In the expressions above

$$\lambda = \Pi\beta - b \tag{4.13}$$

is the surplus of producing capital relative to private benefits, and is therefore a measure of the severity of the agency problem[22], and

$$\delta = \frac{\eta\Pi(1-\beta)}{c_e}, \tag{4.14}$$

measures the fraction, $(1 - \beta)$, of the total amount of new capital, $\eta\Pi$, that is distributed in the form of wage, weighted by the cost of effort.

To solve for the equilibrium trajectory of the economy, the mapping $w_{t+1} = \Phi(w_t)$ can be applied iteratively, for any initial condition, w_0. However, since $\Phi(w_t)$ depends on the shape of the non linear function $\phi(w_t)$, it is essential to spell out its basic property.

Lemma 3. *The map $\phi(w_t)$ is unimodal with a critical point at $w^* \equiv 1 - \frac{8\lambda^2}{9rc_m} \in (\widetilde{w}, \overline{w})$ and maximum value $\phi(w^*) = \frac{16}{27}\frac{\lambda^3}{c_m^2}\delta$. Moreover, if*

$$c_m < r\delta\lambda, \tag{C1}$$

the mapping $\phi(w)$ has, at most, one interior steady state.

Proof. See the Appendix. □

Lemma 3 implies that the dynamics of $\Phi(w_t)$ is non monotonic for $w_t \in (\widetilde{w}_t, \overline{w}_t)$. The mapping $w_{t+1} = \Phi(w_t)$ also implies that the dynamic system (4.10) admits at most two steady states. A trivial one, w^{fb}, when $w_t > \overline{w}$, and a second one if the map $\phi(w)$ crosses the 45^o degree line at $\widetilde{w} \le w_t \le \overline{w}$. Unfortunately, in this intermediate range, the steady state, w^{ss}, of $w_{t+1} = \phi(w_t)$, if it exists, does not have a closed form solution. To characterize its stability and the dynamic trajectories

[22] By Assumption 3, λ is also equivalent to $(\Pi\beta - r)$, i.e. the surplus of productive capital relative to storage. Both expressions measure the degree of idle saving in the economy, i.e. saving not put into the productive investment activity.

in its neighborhood, it is therefore necessary to consider different possible cases, depending on parameter values. To ensure that the mapping $\Phi(w_t)$ maps $(\widetilde{w}, \delta\lambda)$ into itself, I also impose an additional condition

Assumption 5. $\frac{4}{3\sqrt{3}}\lambda \leq c_m$,

which requires that the maximal level of wealth $\phi(w^*)$, attainable in the presence of the agency problem, is less than or equal to the first-best level $w^{fb} = \delta\lambda$. Assumptions 3 and 5 and condition (C1) in Lemma 3, require that the following restrictions hold on the cost of monitoring

$$\max\left\{\frac{2\lambda^2}{r}, \frac{4}{3\sqrt{3}}\lambda\right\} \leq c_m < r\delta\lambda.$$[23]

Without loss of generality, it is convenient to assume that $\lambda > 2r/3\sqrt{3}$ so that the above condition is rewritten as

$$\frac{2\lambda^2}{r} \leq c_m < r\delta\lambda, \tag{4.15}$$

which, in turn, together with (C1), requires that

$$\frac{2r}{3\sqrt{3}} \leq \lambda \leq \frac{r^2\delta}{2}. \tag{4.16}$$

4.3.2. *Dynamic Analysis.* Figures 3a-3c depict four different cases consistent with the restrictions implied by (4.15) and (4.16). The first case, shown in Figure 3a, arises when the mapping $\phi(w)$ satisfies the condition:

$$w^* > \phi(w^*)$$

which can be more explicitly rewritten as

$$1 - \frac{8\lambda^2}{9rc_m} > \frac{16}{27}\frac{\lambda^3}{c_m^2}\delta \tag{4.17}$$

[23] If c_m were higher than $r\delta\lambda$, the slope of the map would never be larger than one at $\widetilde{w}$ and thus, the only (trivial) steady state of the dynamics would be $w^{ss} = \widetilde{w}$. If c_m were lower than $4\lambda/(3\sqrt{3})$ then $\phi(w^*)$ would be higher than $\delta\lambda$ and $\Phi(w)$ could not map the interval $[\widetilde{w}, \delta\lambda]$ into itself. Finally, if c_m were lower than $2\lambda^2/r$, $\widetilde{w} = 1 - \frac{2\lambda^2}{rc_m}$ would be negative.

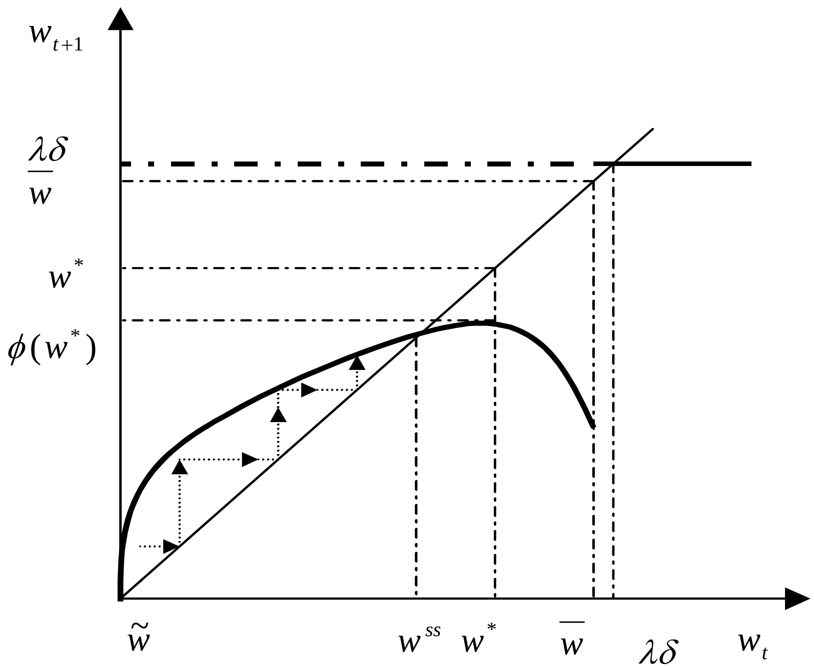

Figure 3a

Under this configuration of parameters, the dynamics are monotonic. Starting from any initial value, w_0, the economy gradually converges to a (low) steady state $w_1^{ss} = \overline{w}$. For a given level of the agency problem, $\lambda = \Pi\beta - b$, condition (4.17) is satisfied, if the cost of monitoring, c_m, is sufficiently high. The reason why a high monitoring cost leads to well behaved dynamics and a low steady state is quite intuitive. If the cost of borrowing is high, entrepreneurs' incentives to undertake investment projects are low, since the minimum level of wealth $\widetilde{w}$ above which an entrepreneur is willing to exert positive effort becomes large. As a result, few projects are undertaken limiting the total wealth that can be accumulated over time. Thus, when c_m is sufficiently high, the agency problem between intermediaries and entrepreneurs constrains investment dynamics exactly as in a model with standard credit market imperfections. For given c_m, condition (4.17) is also met when $\lambda = \Pi\beta - b$ is sufficiently low. A low λ occurs if the productivity of the investment, Π, is low or the amount of private benefits b is large. In both cases the agency problem is relatively more severe limiting the level of wealth that can be accumulated over time. In other words, under this parameter configuration entrepreneurs never become rich enough to "escape" from investor control. This process of "controlled" investment leads to a stable, though low, steady state.

Figures 3b-3c show the dynamic of wage accumulation when

$$w^* < \phi(w^*).$$

Unlike the previous case, the resulting dynamics can be of different types. Figures 3b(1) and 3b(2) display the case where

$$w^* < \phi(w^*) \text{ and } \overline{w} > \phi(\overline{w})$$

or

$$1 - \frac{8\lambda^2}{9rc_m} < \frac{16}{27}\frac{\lambda^3}{c_m^2}\delta \text{ and } 1 - \frac{\lambda^2}{2rc_m} > \frac{\lambda^3}{2c_m^2}\delta. \tag{4.18}$$

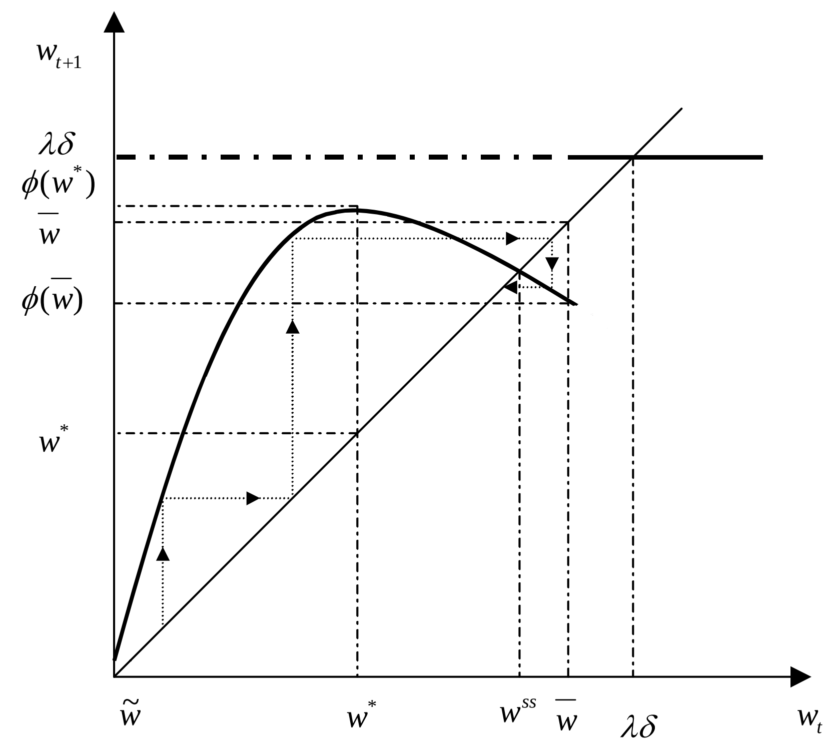

Figure 3b(1)

Under condition (4.18), the map $\phi(w)$ intersects with the 45° line at the downward sloping part, so that the dynamics around the steady state may be oscillatory but stable, in the sense that the economy eventually converges to the steady state, possibly after several periods of fluctuations. This is the case depicted in Figure 3b(1). Under the same conditions prevailing in (4.18), however, the dynamic can also be unstable with the economy moving back and forth between booms and recessions, as shown in Figure 3b(2). In that figure, the interval $[w^*, \overline{w}]$ is a trapping region, i.e. once the economy eventually enters this region it will never leave.[24]

[24]Ideally, to examine for which configuration of parameters this case actually arises, one should check the sufficient condition that the slope of the function $\phi(w)$ at the steady state is such that $|\phi'(w^{ss})| > 1$. Unfortunately, this characterization is not feasible given that the steady state of the mapping, $w_{t+1} = \phi(w_t)$, cannot be explicitly derived. In principle, this case may in fact never arise. A necessary condition for ruling this possibility out is that the slope of the map $\phi(w)$ at

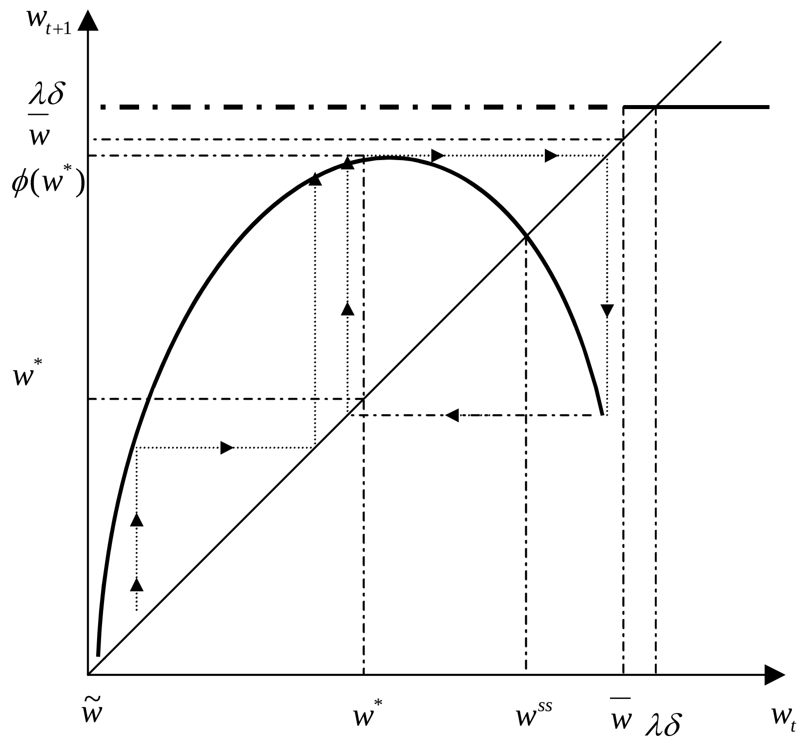

Figure 3b(2)

Condition (4.18) holds under two conditions: 1) for a given λ, the cost of monitoring is sufficiently low (so that $w^* < \phi(w^*)$) but not too low (so that $\overline{w} > \phi(\overline{w})$), and 2) for a given c_m, λ is sufficiently high but not too high. The reason instability arises in this economy is quite simple. For a given, λ, if the cost of monitoring is low, investor control is high, forcing the selection of productive projects that contribute to increase the level of wealth in the economy. At the same time, since c_m is not very high, an increasing amount of resources remain in the hands of the entrepreneurs, leading to further accumulation of wealth. As next period wealth rises, lenders' financial exposure shrinks and monitoring intensity falls. Entrepreneurs eventually gain independence from investors and have the option to finance projects involving private-benefits, which reduces the amount of capital available for next period production. Hence, capital stock falls, the wealth of future generations deteriorates and the cycle starts all over again.

the point $\overline{w}$, is larger than one, in absolute value. Simple algebra shows that

$$\left|\phi'(\overline{w})\right| > 1 \text{ iff } c_m < \frac{r\delta\lambda}{2}$$

which is evidently possible, given that $\frac{2\lambda^2}{r} < c_m < \frac{r\delta\lambda}{2}$ requires that $\lambda < \frac{r^2\delta}{4}$ which is compatible with (4.16). Therefore, limit-cycles cannot be excluded with certainty, but only remain a possibility.

Similarly, for a given c_m, a high λ, but not too high to blunt the agency problem, initially leads to fast accumulation of capital – given the high degree of investor control at low level of wealth and the fact that high λ reflects a high project return, Π. As the amount of wealth accumulated increases, a fall in investor control permits that resources are put to less than optimal use, generating less wealth for future generations of entrepreneurs and, hence, initiating a period of slump.

The final case is depicted in Figure 3c. For this case to arise it must be that

$$w^* < \phi(w^*) \text{ and } \overline{w} < \phi(\overline{w}),$$

or,

$$1 - \frac{8\lambda^2}{9rc_m} < \frac{16}{27}\frac{\lambda^3}{c_m^2}\delta \text{ and } 1 - \frac{\lambda^2}{2rc_m} < \frac{\lambda^3}{2c_m^2}\delta. \tag{4.19}$$

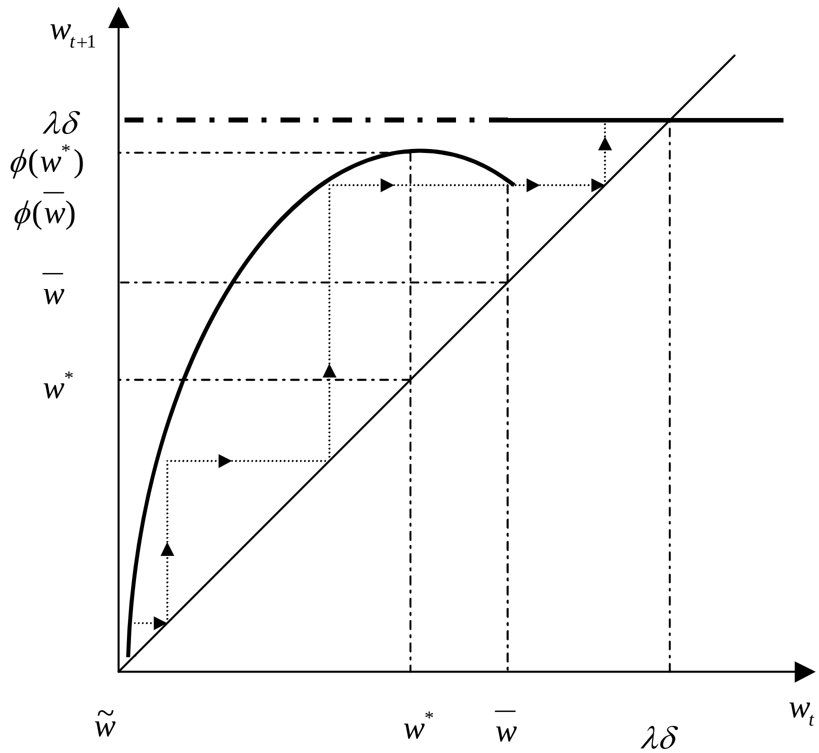

Figure 3c

This restriction is essentially stronger than (4.18), since it requires a lower c_m and/or higher λ. When (4.19) holds, the dynamics always converge to the first best. The reason is intuitive. If c_m is very low, monitoring is high, and the process of capital accumulation fast. Moreover, a very low c_m reduces $\overline{w}$ and thus the range of wealth below which the conflict of interest between entrepreneurs and investors is active. Hence, entrepreneurs internalize more the consequence of choosing projects that generate capital goods and the dynamic eventually converges to that of an economy without agency problems. Similar effects arise when λ is very high, i.e., the agency problem is unimportant or the project return is high. Entrepreneurs have more to gain from undertaking projects generating capital goods and therefore, the drop in wealth in the interval $(w^*, \overline{w})$ is not so large as to generate changes in the dynamics of wealth formation.

The following proposition summarizes the central results of this section .

Proposition 2. Assume condition (4.15) and (4.16) hold. Then there exist two cutoffs $\underline{c}_m < \overline{c}_m$, such that for any λ:

- **a.** If $c_m > \overline{c}_m$, the dynamic of w_t, converges monotonically to a low stable steady state.
- **b.** If $\underline{c}_m < c_m < \overline{c}_m$, the dynamic of w_t either has locally oscillatory convergence to a unique steady state, or equilibrium trajectories that are trapped in the interval $[w^*, \overline{w}]$.
- **c.** If $c_m < \underline{c}_m$, the dynamic of w_t converges to the first best equilibrium.

Moreover, there exist two cutoffs, $\underline{\lambda} < \overline{\lambda}$, such that for any c_m :

- **d.** If $\lambda < \underline{\lambda}$ the dynamic is as in **a**.
- **e.** If $\underline{\lambda} < \lambda < \overline{\lambda}$ the dynamic is as in **b**.
- **f.** If $\lambda > \overline{\lambda}$, the dynamic is as in **c**.

Proof See the Appendix.

4.3.3. *Discussion.* Proposition 2 says that the double incentive problem emphasized in this paper may lead to instability and fluctuations depending on the costs of monitoring c_m and the degree of agency problem, λ. For given initial conditions small changes in c_m and λ can, therefore, lead to different dynamic patterns. Consider, for example, the case where c_m is related to the characteristics of investment technologies, so that the cost of monitoring is larger for, say, new technologies than for more mature ones. In this case Proposition 2 suggests that it is only once the properties of these technologies become properly understood that instability may arise in the economy. Alternatively, if one is willing to assume that the magnitude of the monitoring costs mirrors the stage of the financial development, then the analysis above suggests that economies with less developed financial markets are not necessarily prone to fluctuations, whereas small improvements in credit markets might lead to instabilities. In Proposition 2, λ also plays a crucial role. Since λ commoves with Π, small shocks to the productivity of investment projects may initiate different dynamics. If λ is low, negative shocks to Π become persistent and low investment leads to further lower activity. On the other hand, starting from a low λ, small but positive shocks may lead to complicated dynamics and instability. If one considers positive shocks to Π as initiated by the adoption of new technologies, or more generally by exogenous technology shocks, the economy may experience periods of fluctuations, unless the increase in projects' productivity is large.

4.4. **Empirical Predictions.** Having discussed the static and dynamic implications of the interplay between entrepreneurial effort and investor control, I am now in a position to evaluate some of the predictions of the model. The analysis in Section 3 has two main comparative static results, both stemming from the fact that more investor control reduces entrepreneurial incentives. The first prediction is that entrepreneurs with low net worth undertake few investment projects. The second is that firms with high leverage invest less. Both predictions stand close to the findings emerging from the large empirical literature on credit frictions and firm investment. That investment is sensitive to cash flow (holding constant investment opportunities) and that large debt burdens prevent firms from raising additional funds are, in fact, two robust results of this literature (see, for example, the surveys of Bernanke, Gertler and Gilchrist, 1996 or Stein, 2004).

The static model has the further implication that investors lend more easily if the project is a good one or the entrepreneur can supply a large fraction of the initial investment. This prediction is in agreement with "received wisdom" in the banking industry and has large empirical support (see, for example, Gorton and Winton, 2004). Moreover, the emphasis that investor control affects entrepreneurs' incentives also has implications on how this mechanism varies across financial systems and, within country, across industries. It suggests, for example, that in industries with more pledgeable assets, investment profitability should be lower and/or the amount of private benefits enjoyed by entrepreneurs larger than in industries with less tangible capital. Similarly, it suggests that profitability should vary with respect to the lender's ability to monitor entrepreneurial activity. Unfortunately, I am not aware of any systematic empirical study relating investor control to investment profitability. There is, instead, some evidence that agency costs are lower when financing occurs through banks acting in their role of delegated monitoring on behalf of other shareholders (see Ang, Cole and Lin, 2000).

Several interesting predictions also come out of the dynamic model of Section 4. First of all, it suggests that investment dynamics are non-linear in the state of the economy. This is in agreement with the fact that changes in internal finance affect firms' investment more when the economy is deeper in recession (see Bernanke, Gertler and Gilchrist, 1996). In the model, this occurs because at a low level of net worth, a small increase in firm internal funds permits the entrepreneur to gain some independence from the investor and thus increase investment. This effect is, however, smaller if the degree of external interference is low, since in this situation, the entrepreneur effort becomes less and less dependent on internal funds. Second, the varying intensity at which the investor exercises control on the selection of projects, suggests that lending standards shape investment dynamics. This is consistent with the anecdotal evidence reported in Rajan (1994) according to which lending standards (i.e. criteria by which banks determine and rank loan applicants) are relaxed in booms and tightened in recessions. It is also in line with the findings of Asea and Blomberg (1998) and Lown and Morgan (2004) that in the U.S., bank lending standards are important for aggregate economic activity.

Moreover, it is related to the phenomenon of "flight to quality" (Bernanke, Gertler and Gilchrist, 1996) i.e. the tendency of lenders to favor, in recession, borrowers that are less likely to default.[25]

A final prediction worth mentioning is that exogenous shocks to firm net worth may be dampened rather than amplified. This fact stands in sharp contrast with that arising in a standard model based on the credit multiplier. The reason why this occurs rests, once again, on the crucial role played by the investor in selecting entrepreneur projects. If a recessionary shock arises, for example, stringent control only allows productive projects to get funding and negative shocks are quickly stabilized. Empirical evidence that the credit market acts in dampening or amplifying shocks is unfortunately scarce. In the literature, there is only microevidence that firms' investment decisions are affected by credit frictions, but no evidence that these frictions actually matter for aggregate dynamics (see Bacchetta and Caminal, 2001).

5. Robustness

The results of this paper are obtained in a highly simplified representation of the economy. It is therefore worth discussing some of its modelling assumptions.

1. Perhaps the assumption driving most of the results is that entrepreneurs are short lived. By adopting investment projects that generate only private benefits entrepreneurs do not internalize the consequences of their choice on the funds available for future investments. If entrepreneurs were long-lived, boom periods would last longer and the amplitude of fluctuations would be reduced, though not eliminated. In this modified set up, the concept of borrower net worth would have to be extended to include current and future firm's expected cash flows. This extension, however, would not invalidate the logic of the model, insofar as lenders' expectations on firms' future net worth are persistent. If firms' profits are high today and expected to remain high in the future, lenders will reduce monitoring intensity in a manner similar to what is discussed above. Moreover, in a repeated interaction, the lender break-even constraint needs to be satisfied over a longer horizon rather than period by period, which may further weaken banks' incentives to monitor. This effect is obviously counterbalanced by the fact that long-lived entrepreneurs internalize the consequences of project choice to a larger extent. Allowing for long-lived entrepreneurs, however, is not an easy task, given that the contracting problem between lenders and borrowers would be one with repeated-double-moral-hazard.[26]

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[25] In the literature, flight to quality is indeed meant to represent a phenomenon of "flight to safety", since the focus is on borrowers that can pledge more or less collateral. Though this paper does not distinguish between safe and risky borrowers, its central premise is that less efficient projects are more likely to fail to repay the lenders. In this sense safety and quality go hand in hand.

[26] The difficulty associated with having long-lived entrepreneurs is also recognized in Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999). In their models, entrepreneurs have a *de facto* finite horizon, since they face a constant probability of surviving until the next

2. The fiction that entrepreneurs enjoy private benefits was used to create a conflict of interest between investors and entrepreneurs. As discussed in the text, several interesting interpretations can be given to private benefits. The most preferred one, however, is that entrepreneurs are "conservative", in the sense of preferring to delay the adoption of new technologies, because they must otherwise incur a private effort cost. Obviously, there may be different explanations why non-profit maximizing entrepreneurs can survive in a market economy. A first reason may be due to the market structure: in a less competitive environment, there is larger scope for entrepreneurial slack. For example, in the OLG model of Section 4, one could assume that producers of intermediate goods are protected by monopoly rights on their innovation and thus have the opportunity to "buy time" for the adoption of new technologies, without being threatened by solvency constraints. Alternatively, as discussed in the text, there is an agency problem between intermediate producers and outside financiers since, for example, the adoption of new technologies is non contractible. In both interpretations projects involving private benefits are less productive and, as a consequence, contribute to deteriorating the overall productivity in the economy.

3. Central to the results is the premise that investor control as well as the entrepreneurial effort are essential to generate cash flows, and that entrepreneurial incentive to exert effort is adversely affected by investor control. These assumptions are responsible for the unimodal shape of the investment dynamics of Sections 3 and 4. Alternatively, one may assume that investors enhance profit maximization directly and independently of the entrepreneur effort by posing, for example, that cash flows are generated with probability $(e+m)$ rather than em, as assumed in the text. Such a formulation would obviously kill the possibility that the investment dynamics are hump shaped and hence, that endogenous fluctuations may arise. However, this would only capture one aspect of the problem, namely that the investor assists the entrepreneur in his venture, but not the role of the investor as a monitor who destroys private benefits. The multiplicative formulation, instead, ensures that entrepreneur's preferences are congruent with the advising role of the investor if private benefits are small, while they are dissonant with the investor if private benefits are large. More generally, one could consider the case where the probability of productive output is $p(e,m) = \left[\gamma e^k + (1-\gamma) m^k\right]^{\frac{1}{k}}$ with $\kappa \leq 1$ and $\gamma \in (0,1)$. In this case, the two actions are perfect substitutes if $\kappa = 1$, and complements otherwise. To generate endogenous fluctuations, it would be necessary for κ to be sufficiently low.

4. The model relies on the presumption that the entrepreneur pays the evaluation costs first and lending occurs only afterwards. In the alternative case when

period. This assumption circumvents the problem arising from the repeated interaction between lenders and borrowers and allows them to consider the case of financial contracts that last one period only. The assumption that entrepreneurs die with some probability is also introduced to preclude the possibility that entrepreneurs accumulate enough wealth to be able to self-finance investment, thus eliminating the importance of financial frictions.

financing occurs before moral hazard, the zero profit condition for the investor would depend on the entrepreneurial effort. This would complicate the expression defining the fraction of profit that must be given to the investor, as well as her optimal degree of control. However, the intuition that the investor monitors less when the entrepreneur puts more of his wealth into the project would continue to hold, so that the essential results of this paper would be unaffected.

5. In the OLG model of Section 4, the marginal productivity of capital is assumed constant. This assumption rules out the possibility that entrepreneur's effort also depends on the future price of capital. Though realistic, such an extension might give rise to multiple equilibria, in which the optimal effort supply of an entrepreneur at time t would directly depend on the degree of investor control, and indirectly on the effort decision of the other entrepreneurs, given that the actions of these entrepreneurs affect the amount of capital available in the economy at $t+1$ and hence, its price. Extending the model in this direction would certainly add another interesting element to the dynamics, but this would be unrelated to the strategic interaction between investors and entrepreneurs, the main focus of this paper.

6. Throughout the analysis, we have maintained the assumption of a credit market where the supply of credit is infinitely elastic and hence the interest rate is constant. This is a reasonable assumption if the economy under consideration is small and open. Allowing for the interest rate to vary, however, would not invalidate the results provided the supply, and not only the demand, of credit increases as entrepreneur's net worth improves and the economy expands. This effect is excluded in Aghion *et al.* (1999) and Matsuyama (2004), given their assumption of infinitely inelastic supply of credit.

6. **Conclusion**

This paper has offered a preliminary investigation of the link between credit market frictions and endogenous cycles. An extensive literature in macroeconomics has studied the relation between entrepreneurial net worth and firm investment to explain the persistence and amplification of small shocks to the economy. Little attention has been paid to the possibility that credit frictions generate instability and endogenous fluctuations. To highlight this connection, this paper has proposed a mechanism based on the joint interaction of borrowers' and lenders' incentives.

Starting with the premise that the profitability of investment projects depends on the joint non-contractible actions of investors and entrepreneurs, the paper illustrates how borrowers' and investors' incentives may vary over the cycle. The model has been set-up in such a way that the entrepreneurial initiative is essential for selecting investment projects and that investor control is crucial for selecting only profitable projects. Since there is a basic conflict of interest between the entrepreneur and the investor over the selection of projects, too much control discourages entrepreneurial incentive to initiate investment projects, while too little control jeopardizes their productivity. I have shown how this trade-off between entrepreneurial initiative and investor control can generate investment dynamics that mimic those of a standard model with credit frictions, in which more entrepreneurial net worth leads to higher investment. However, I have also shown that the same trade-off is capable of generating endogenous fluctuations, induced by an ongoing deterioration of project profitability. In particular when embedded in a dynamic model with overlapping generations it is possible to derive a simple condition for endogenous cycles. The condition is that the cost of monitoring for the investor (or the degree of the agency problem) is neither too high nor too low. Under this condition the economy either converges to its steady state in an oscillatory manner, or never reaches the steady state and keeps on cycling between periods of boom and recession.

Business cycles are inherently complex and the agency problem in this paper is certainly much too simple to do full justice to reality. Many important issues deserve a more careful analysis in future research. First, while entrepreneurial private benefits are the source of divergent interests between firms and investors, the nature of these private benefits has been left unspecified. Modelling these benefits in greater detail can open the way to more elaborate theories, with more convincing explanations why entrepreneurs may prefer the adoption of less productive investment technologies at different stages of the economic cycle. Second, the overlapping generation framework is a useful device to single out the dynamic consequences of the agency problem between lenders and borrowers. A framework richer in dynamics and in the details of private benefits may, however, lead to more interesting insights into the source of business fluctuations. At the moment, the framework is too stylized to permit meaningful quantitative analysis and evaluate the importance of the mechanism emphasized in this paper. These extensions are left to future research.

I have also neglected the normative question of what government policy could do to minimize fluctuations. In the dynamic version of the model, however, fluctuations are an efficient equilibrium outcome. This is the case, even though low productivity projects impair future generation net worth. In fact, when entrepreneurs select bad projects, they still maximize their utility while keeping the investor on her break-even constraint. Room for government intervention is therefore limited to the case where the welfare of future generations is also taken into account. In this case the planner could restore efficiency by taxing rich young entrepreneurs so that their independence from investor control would never be gained. Alternatively, the planner could tax investor revenues so as to induce more intensive investor monitoring, and prevent entrepreneur from selecting bad projects. The exact details of these policy option, however, are intricate and left to future work.

Finally, although the main focus of this paper has been on business cycle implications, the agency problem between entrepreneurs and investors also has interesting cross-sectional and cross-country predictions. For example, a prediction coming out of the model is that the profitability of investment should vary with respect to the lender's ability to monitor entrepreneurial activity. Countries differ extensively in terms of how their financing system works. Bank-based financial systems require a very direct control over the borrower. Similarly, a direct link between investors and entrepreneurs exists in systems where financing occurs through venture capitalists. Conversely, in markets relying more on arm's length financing, control is less direct. Insofar as banks and stock markets are fundamentally different in the way they process information and control borrowers, this paper has potentially something to say about the possibility that market-based rather than bank-based economies are more prone to instabilities. A careful empirical examination of these empirical predictions is also left to future work.

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Appendix

Proof of Proposition 1. (i) Proof that $m > 0$ and $e > 0$ in $w \in (\widehat{w}, \overline{w})$ where $\widehat{w} = \max\{\underline{w}, \widetilde{w}\}$:
The equilibrium value of monitoring, after substituting in the value of α given by (3.11), is

$$m = \frac{\alpha\Pi}{c_m} = \sqrt{\frac{2r(1-w)}{c_m}} = \sqrt{\sigma(1-w)} \text{ where } \sigma = \frac{2r}{c_m}.$$

m is a decreasing function of w, in the interval $(\widehat{w}, \overline{w})$, and at $\underline{w} = 1 - \frac{\Pi^2}{2rc_m}$, and $\overline{w} = 1 - \frac{(\Pi-b)^2}{2rc_m}$ it is such that

$$0 < \frac{\Pi - b}{c_m} = m(\overline{w}) < m(\underline{w}) = \frac{\Pi}{c_m}.$$

The equilibrium value of effort is given by,

$$\begin{aligned} e(w) &= \frac{(b - rw + m(\Pi - b) - m\alpha\Pi)}{c_e} \\ &= \frac{((b-r) + (\Pi - b)\sqrt{\sigma(1-w)} - r(1-w))}{c_e} \end{aligned}$$

Under Assumption 1, $e(w)$ is a concave and increasing function of w in $(\widehat{w}, \overline{w})$:

$$e'(w) = r - \frac{(\Pi - b)\sqrt{2r/c_m}}{2\sqrt{1-w}}, \quad e'(\underline{w}) > 0, \quad e'(\overline{w}) = 0$$

$$e''(w) = -\frac{(\Pi - b)r^2/c_m^2}{((1-w)2r/c_m)^{3/2}} < 0.$$

Moreover under Assumption 1 $e(\underline{w}) \geq 0$ if $\Pi \geq 2b$, $e(\widetilde{w}) \geq 0$ by definition, and at $\overline{w}$

$$e(\overline{w}) = \frac{(b-r)}{c_e} + \frac{(\Pi - b)^2}{2c_m c_e} > 0.$$

(ii) Proof that p is concave in $(\widehat{w}, \overline{w})$, and has a maximum at $w^* \in (\widehat{w}, \overline{w})$:
The level of productive investment is measured by:

$$p(e(w), m(w)) = em\Pi$$

After replacing the equilibrium value of $m(w)$ and $e(w)$:

$$p(w) = \frac{\Pi}{c_e}\left((\Pi - b)\sigma(1-w) + (b-r)\sqrt{\sigma(1-w)} - r(1-w)\sqrt{\sigma(1-w)}\right)$$

which is convenient to rewrite as

$$p(w) = \frac{\Pi}{c_e}\left(\lambda\sigma(1-w) + \kappa\sqrt{\sigma(1-w)} - r(1-w)\sqrt{\sigma(1-w)}\right) \qquad \text{(A1)}$$

where, by Assumption 1

$$\lambda = (\Pi - b) > 0 \text{ and } \kappa = (b - r) \geq 0.$$

Equation (A1) is increasing in $[0, w^*]$ and decreasing in $[w^*, \overline{w}]$, where w^* is the root of:

$$\frac{\partial p}{\partial w} = \frac{\Pi\sigma}{c_e}\left[\frac{3}{4}c_m\sqrt{\sigma(1-w)} - \left(\lambda + \frac{\kappa}{2\sqrt{\sigma(1-w)}}\right)\right] = 0$$

i.e.:

$$w^* = \frac{9c_m^2\sigma^2 - 6c_m\sigma\kappa - 8\sigma\lambda^2 - 4\sqrt{2}\sqrt{2c_m\sigma^2\kappa\lambda^2 + 2\sigma^2\lambda^4}}{9c_m^2\sigma^2} \equiv w^*(\underset{+}{c_m}, \underset{+}{r}, \underset{+}{b}, \underset{-}{\Pi})$$

$p(w)$ is also a concave function in $w \in (0,1)$ since

$$\frac{\partial^2 p}{\partial w^2} = -\left(\frac{3c_m\sigma}{8\sqrt{\sigma(1-w)}} + \frac{\kappa}{4\left(\sigma(1-w)\right)^{3/2}}\right) < 0$$

I now show that for parameter values satisfying Assumptions A1 and A2,

$$\widehat{w} < w^* < \overline{w}$$

(iia) Proof that $w^* < \overline{w}$ **:**
Remember that $\overline{w} = 1 - \frac{\lambda^2}{2rc_m}$. Hence,

$$w^* < \overline{w} = 1 - \frac{\lambda^2}{2rc_m}$$

if and only if

$$\frac{6c_m\sigma\kappa + 8\sigma\lambda^2 + 4\sqrt{2}\sqrt{2c_m\sigma^2\kappa\lambda^2 + 2\sigma^2\lambda^4}}{9c_m^2\sigma^2} > \frac{\lambda^2}{2rc_m}$$

or

$$6c_m\sigma\kappa + 4\sqrt{2}\sqrt{2c_m\sigma^2\kappa\lambda^2 + 2\sigma^2\lambda^4} > \lambda^2\sigma$$

which is always true, because

$$4\sqrt{2}\sqrt{2c_m\sigma^2\kappa\lambda^2 + 2\sigma^2\lambda^4} > \lambda^2\sigma$$

(iib) Proof that $w^* > \widehat{w} = \max\{\underline{w}, \widetilde{w}\}$ **:**
It suffices to prove that $w^* > \widetilde{w}$, since by Assumption 1, $\widetilde{w} \geq \underline{w}$.
$\widetilde{w}$ is the root of the entrepreneur's participation constraint:

$$(b - r) + (\Pi - b)\sqrt{\frac{2r(1-\widetilde{w})}{c_m}} - r(1-\widetilde{w}) = 0.$$

or

$$\kappa + \lambda\sqrt{\sigma(1-\widetilde{w})} - \frac{c_m\sigma}{2}(1-\widetilde{w}) = 0$$

which has as the solution:

$$\widetilde{w} = \frac{c_m^2\sigma^2 - 2c_m\sigma\kappa - 2\sigma\lambda^2 - 2\sqrt{2c_m\sigma^2\kappa\lambda^2 + \sigma^2\lambda^4}}{c_m^2\sigma^2} \equiv \widetilde{w}(\underset{+}{c_m}, \underset{+}{r}, \underset{+}{b}, \underset{-}{\Pi})$$

Hence, $\widetilde{w} < w^*$ iff

$$\frac{2c_m\sigma\kappa + 2\sigma\lambda^2 + 2\sqrt{2c_m\sigma^2\kappa\lambda^2 + \sigma^2\lambda^4}}{c_m^2\sigma^2} > \frac{6c_m\sigma\kappa + 8\sigma\lambda^2 + 4\sqrt{2}\sqrt{2c_m\sigma^2\kappa\lambda^2 + 2\sigma^2\lambda^4}}{9c_m^2\sigma^2}$$

or

$$6c_m\sigma\kappa + 5\sigma\lambda^2 + 9\sqrt{2c_m\sigma^2\kappa\lambda^2 + \sigma^2\lambda^4} > 2\sqrt{2}\sqrt{2c_m\sigma^2\kappa\lambda^2 + 2\sigma^2\lambda^4}$$

which holds true because $\kappa > 0$, $\lambda > 0$ and

$$9\sqrt{2c_m\sigma^2\kappa\lambda^2 + \sigma^2\lambda^4} > 2\sqrt{2}\sqrt{2c_m\sigma^2\kappa\lambda^2 + 2\sigma^2\lambda^4}.$$

(iii) Proof that $i = em\Pi + e(1-m)b$ is monotonically increasing in $w \in (\widehat{w}, \overline{w})$
It follows directly from points (i) and (ii).

Proof of Lemma 2. In the presence of an agency problem, i.e.

$$b > (1-\alpha_t)\Pi\beta \tag{A2}$$

the optimal level of investor control and entrepreneurial effort are obtained by maximizing

$$u_E = (1-e_t)rw_t + e_t\left[m_t(1-\alpha_t)\Pi\beta + (1-m_t)b\right] - c_e\frac{e_t^2}{2}, \tag{A3}$$

and

$$u_I = e_t\left\{m_t\alpha_t\Pi\beta + (1-m_t)\times 0 - c_m\frac{m_t^2}{2}\right\},$$

and are given by:

$$m_t = \min\left\{\frac{\alpha_t\Pi\beta}{c_m}, 1\right\}, \tag{A4}$$

and

$$e_t = \min\left\{\frac{(b - rw_t) - m_t(b - (1-\alpha_t)\Pi\beta)}{c_e}, 1\right\}, \tag{A5}$$

where α_t is pinned down by the intermediary break-even condition,

$$\left\{m_t\alpha_t\Pi\beta + (1-m_t)\times 0 - c_m\frac{m_t^2}{2}\right\} = r(1-w_t)$$

or

$$\alpha_t(w_t) = \frac{\sqrt{2r(1-w_t)c_m}}{\Pi\beta} \leq 1. \tag{A6}$$

Replacing (A6) in (A4) and (A5), and assuming interior solutions, the equilibrium values of $m_t(w_t)$ and $e_t(m(w_t), w_t)$ can be conveniently rewritten as

$$m_t(w_t) = \sqrt{\sigma(1-w_t)} > 0$$

and

$$e_t(m_t(w_t), w_t) = \frac{m_t\lambda - r(1-w_t)}{c_e} > 0$$

where $\sigma = \frac{2r}{c_m}$, and $\lambda = \Pi\beta - b > 0$. In order to have positive investment entrepreneur's effort must be positive, which occurs when the level of net worth is not too low,

$$w_t \geq 1 - \frac{2\lambda^2}{rc_m} \equiv \widetilde{w}.$$

Moreover, the agency problem between the investor and the entrepreneur exists, if entrepreneur net worth is not too high,

$$w_t \leq 1 - \frac{\lambda^2}{2rc_m} \equiv \overline{w}, \tag{A7}$$

where (A7) is obtained using (A2) and (A6). Assumption 3 ensures that $\widetilde{w} > 0$ and $\overline{w} < 1$. The conflict of interest vanishes when

$$b < (1-\alpha_t)\Pi\beta, \tag{A8}$$

or, using the expressions above, when $w_t > \overline{w}$. When (A8) holds, maximization of (A3) leads to (4.5) in the text.

Proof Lemma 3. The map

$$\phi(w_t) = (1-w_t)\left[\sigma\lambda - r\sqrt{\sigma(1-w_t)}\right]\delta, \tag{A9}$$

is zero at $\widetilde{w}_t = 1 - \frac{2\lambda^2}{rc_m}$,

$$\phi(\widetilde{w}_t) = \frac{2\lambda^2}{rc_m}\left[\frac{2r}{c_m}\lambda - r\sqrt{\frac{2r}{c_m}\frac{2\lambda^2}{rc_m}}\right]\delta = 0.$$

Moreover, its first derivative

$$\phi'(w_t) = \left[\sqrt{(1-w_t)}\frac{3r\sigma}{2\sqrt{\sigma}} - \sigma\lambda\right]\delta,$$

evaluated at $\widetilde{w}_t$

$$\phi'(\widetilde{w}_t) = \frac{r\lambda\delta}{c_m},$$

is larger than one if

$$c_m < r\lambda\delta,$$

which is condition (C1) in the Lemma. Hence, under this parameter restriction the maps start at zero at $\widetilde{w}_t$ with a slope larger than one. Simple differentiation of (A9) gives $w^* = 1 - \frac{8\lambda^2}{9rc_m}$ as its critical point. $\phi(w_t)$ is strictly increasing for $w_t < w^*$ and strictly decreasing for $w_t > w^*$. At the maximum,

$$\phi(w^*) = \frac{8\lambda^2}{9rc_m}\left[\frac{2r}{c_m}\lambda - r\sqrt{\frac{2r}{c_m}\frac{8\lambda^2}{9rc_m}}\right]\delta = \frac{16}{27}\frac{\lambda^3}{c_m^2}\delta.$$

The existence of at most one steady-state of the map $w_{t+1} = \phi(w_t)$ in the range $(\widetilde{w}, \overline{w})$ is guaranteed by (C1) and the fact the the function is single peaked. A necessary and sufficient condition for the existence of a steady state is that $\phi(\overline{w}) < \overline{w}$, or

$$\frac{\lambda^3}{2c_m^2}\delta \leq 1 - \frac{\lambda^2}{2rc_m}.$$

Proof Proposition 2. c_m and λ must satisfy conditions (4.15) and (4.16) which are rewritten for convenience:

$$\frac{2\lambda^2}{r} \leq c_m \leq r\delta\lambda \tag{A10}$$

$$\frac{2r}{3\sqrt{3}} \leq \lambda \leq \frac{r^2\delta}{2}. \tag{A11}$$

(i) Proof of (a)-(c).

Condition (**a**), or equivalently Figure 3a, obtains if

$$1 - \frac{8\lambda^2}{9rc_m} > \frac{16}{27}\frac{\lambda^3}{c_m^2}\delta,$$

which can be rewritten as

$$c_m^2 - \frac{8}{9}\frac{\lambda^2}{r}c_m - \frac{16}{27}\lambda^3\delta > 0. \tag{A12}$$

Disregarding the negative root, the solution of (A12) is given by

$$c_m > \overline{c}_m,$$

where

$$\overline{c}_m = \frac{4}{9r}\left(\lambda^2 + \lambda^{3/2}\sqrt{\lambda + 8\delta r^2}\right). \tag{A13}$$

Notice that

$$\overline{c}_m < r\delta\lambda \quad \text{for} \quad \lambda < \frac{81}{200}r^2\delta,$$

which is compatible with (A11).

Condition (**b**) or Figure 3b(1) or 3b(2), obtains if

$$1 - \frac{8\lambda^2}{9rc_m} > \frac{16}{27}\frac{\lambda^3}{c_m^2}\delta \quad \text{or} \quad c_m < \overline{c}_m \tag{A14}$$

and

$$1 - \frac{\lambda^2}{2rc_m} > \frac{\lambda^3}{2c_m^2}\delta$$

which can be rewritten as

$$c_m^2 - \frac{\lambda^2}{2r}c_m - \frac{\lambda^3\delta}{2} > 0. \tag{A15}$$

Disregarding the negative root, the solution to (A14) and (A15) is given by

$$\underline{c}_m < c_m < \overline{c}_m,$$

where

$$\underline{c}_m = \frac{1}{4r}\left(\lambda^2 + \lambda^{3/2}\sqrt{\lambda + 8\delta r^2}\right) < \overline{c}_m. \tag{A16}$$

Condition (**c**), or Figure 3c, is obtained if

$$1 - \frac{8\lambda^2}{9rc_m} > \frac{16}{27}\frac{\lambda^3}{c_m^2}\delta \quad \text{or} \quad c_m < \overline{c}_m$$

and

$$1 - \frac{\lambda^2}{2rc_m} < \frac{\lambda^3}{2c_m^2}\delta \quad \text{or} \quad c_m < \underline{c}_m$$

Hence condition (c) holds if,

$$\frac{2\lambda^2}{r} < c_m < \frac{1}{4r}\left(\lambda^2 + \lambda^{3/2}\sqrt{\lambda + 8\delta r^2}\right),$$

where $\frac{2\lambda^2}{r}$ is the lower bound in (A10).
Notice that

$$\frac{2\lambda^2}{r} < \frac{1}{4r}\left(\lambda^2 + \lambda^{3/2}\sqrt{\lambda + 8\delta r^2}\right) \quad \text{for} \quad \lambda < \frac{r^2\delta}{6}$$

which is compatible with (A11).

(ii) Proof of (d)-(f).

Case (**d**) holds when (A12) is satisfied which, rewritten in terms of λ, gives

$$\lambda^3 + \frac{3}{2}\frac{c_m}{\delta r}\lambda^2 - \frac{27}{16}\frac{c_m^2}{\delta} < 0. \tag{A17}$$

Similarly, case (**e**) holds when (A14)

$$\lambda^3 + \frac{3}{2}\frac{c_m}{\delta r}\lambda^2 - \frac{27}{16}\frac{c_m^2}{\delta} > 0$$

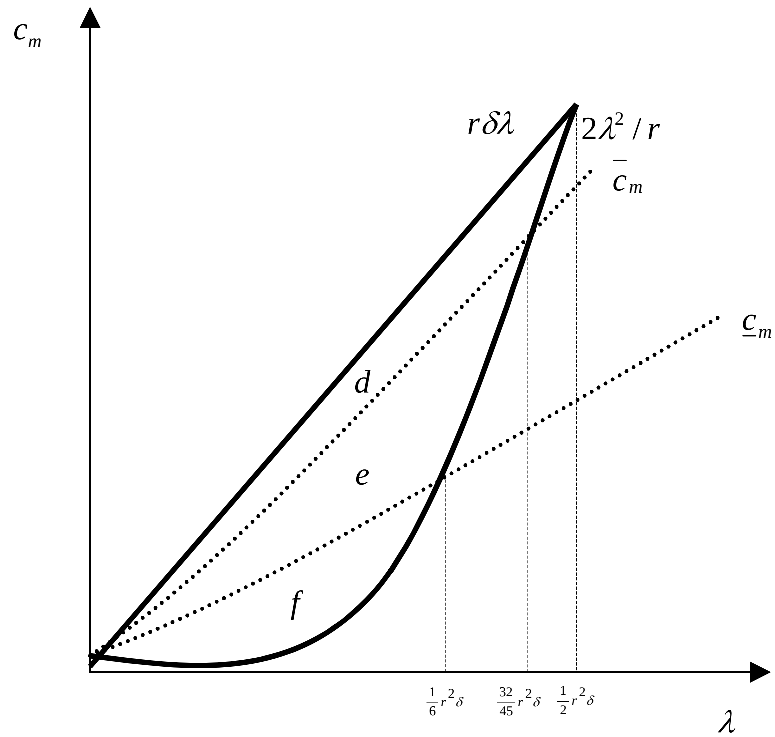
and (A15)

$$\lambda^3 + \frac{c_m}{\delta r}\lambda^2 - 2\frac{c_m^2}{\delta} < 0 \tag{A18}$$

hold.

Since it is not possible to obtain explicit solutions of (A17) and (A18), it is useful to consider the following graph, which plots c_m as a function of λ, for a given δ and r. The two tick lines refer to the upper and lower bounds on c_m, given by (A10). The two dotted lines refer to (A13) and (A16). As can be seen from the picture, fixing c_m in the admissible range, and increasing λ, the economy moves from region (d) corresponding to Figure 3a (or case (d) in the Proposition) to region (e), corresponding to Figures 3b(1)-3b(2) (or case (e) in the Proposition), eventually reaching region (f) discussed in Figure 3c (or case (f) in the Proposition).

In closing, notice that using the same Figure, fixing λ and moving c_m gives an alternative representation of the results regarding the parameter c_m. That is, for a given λ, as c_m decreases, the economy moves from region (d) (or case (a) in the proposition) to region (e) (or case (b) in the proposition) and eventually region (f) (or case (c) in the proposition).



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