# On the possibility of political change

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**Abstract**

We revisit the study of voting over education subsidies where poor individuals may be excluded and the rich may chose private alternatives. Introducing a new way of analyzing this type of problem we show that under realistic assumptions there are typically multiple equilibria. Shifts between these can only happen through "shocks to policy", not through gradual change. The reason for this is that even in the presence of a "Condorcet winner" there are typically also "local equilibria" which turn out to not only defeat neighboring policies but a broad range of alternatives. When introducing costs of changing policy these can become stable outcomes implying, for example, that identical countries or regions with different starting points could end up with completely different redistributive systems. Outcomes change in intuitive ways with the parameters and several insights with respect to the possibilities of political change seem general for problems of redistribution with excludability.

*Keywords:* political economy, political equilibrium, voting, redistribution, education subsidies, local equilibrium, non-median voter equilibrium

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"Economic technocrats [.] never have the luxury of a clean slate on which to put up their [.] economic policy framework. Each new chief executive, and each new finance minister inherits the patchwork quilt put together by their predecessors. Like it or not, each new administration will end up adding its own new patches to this quilt."

A. Harberger (1999)

"There is enormous inertia - a tyranny of the status quo - in private and especially governmental arrangements."

M. Friedman (1962/1982)

# 1 Introduction

In standard majority rule problems of redistribution individuals with income below the median gain from redistribution while those above lose. This splits the population into two opposing halves, making the voter with median-income decisive. There are, however, a number of well-known extensions of such problems where this conflict between the rich and the poor is not necessarily sustained. If, for example, a poor individual must reach some minimum level of income to benefit from redistribution, he may dislike small tax increases at low levels, but favour them given that the level is sufficiently high. This, in turn, may affect the possibility to evoke any of the median-voter theorems and also the existence of equilibria.[1] A classic situation of this kind is voting over school subsidies (e.g. Barzel, 1973; Stiglitz 1974; Glomm and Ravikumar, 1992; Fernandez and Rogerson, 1995; Epple and Romano, 1998). For low levels of taxation, subsidies may not be sufficient to enable the poorest to attend school and, hence, they do not gain anything from the transfer system. This means that they - at this level - oppose taxes in coalition with the rich. However, at higher tax rates transfers could become larger and education accessible for the poor who now may change their attitude to further tax increases. In a situation where transfers

[1] In these situations individual preferences are not necessarily single-peaked, nor single-crossing and, hence, the conditions for using either version of the median voter theorem are not always met. See Black (1948) and Downs (1957) on single-peaked preferences, and Roberts (1977), Grandmont (1978), Rothstein (1990, 1991) on various versions of restricting preference ordering. The two versions are unified in Gans and Smart, 1996. See Austen-Smith and Banks (1999) and Persson and Tabellini (2000) for overviews.

are so high that everyone attends school and hence gets the subsidy, the political conflict returns to the standard rich-against-poor situation.

Even though it is the standard example, voting over education subsidies is far from the only situation where this can happen. Consider, for example, subsidies to the arts: at low levels of taxation the subsidies may be so small that the poor do not consume the cultural good and hence would oppose its subsidization (again, at that level). If taxes are higher, however, the poor may start to consume the cultural good and may consequently change their mind regarding it being subsidized.[2] Other examples include public health care, housing and more generally, the provision of public goods with the possible exclusion of some groups depending on the subsidy level (e.g. Besley and Coate, 1991; Epple and Romano 1996a and 1996b; Gouveia (1997); Blomquist and Christiansen (1999); Epple and Romano; 2003).[3]

In this paper, we revisit this kind of problem in the context of voting over education subsidies using a new approach. While most previous work make simplifying assumptions so as to enable the use of the median voter theorem, we develop a method that does not require it's applicability.[4] This is, of course, desirable in itself but more importantly this also allows us to gain a number of new insights. We find that a typical situation has *multiple local equilibria*; one with zero tax and no subsidies; one with a low tax rate, a large number of individuals still outside the schooling system and many in private schooling; and yet another one with a high tax rate, everyone attending school, and few choosing the private alternative. Clearly, only one of these outcomes can be the Condorcet winner (the global equilibrium), and usually (though not always) this turns out to be the high tax equilibrium. In a standard majority rule situation the Condorcet winner is, of course, the expected outcome. However, when introducing small, realistic changes to the standard game, such as costs associated with altering the incumbent policy (borne either by the candidates or the electorate) each local equilibrium can become a stable outcome, that is the

[2] While Austen-Smith (2003) studies a different aspect of this type of problem (the choice of subsidies over direct transfers) the motivating example is the subsidization of the arts.

[3] As pointed out by Besley and Coate (1991) in an early paper on this topic, one should note that de jure universal subsidization schemes are not necessarily de facto universal. Exclusion may, however, be a result of different factors operating at both ends of the distribution. The poor can be excluded due to credit constrints or due to the good being indivisible in such a way that they do not wish to consume the smallest available amount, but it can also be the rich who opt out of the sysem if the quality of the publically provided good is not sufficiently high and there are private alternatives.

[4] In some cases this may not be very restrictive, indeed, part of the contribution of previous work has been to show the applicability of the median voter theorem to problems where it may at first seem problematic. However, it does not cover all relevant cases. As shown by Fernandez and Rogerson (1995) - which solves for all outcomes including non-median voter outcomes - there are several possible equilibria that are disregarded if the problem is restricted.

(Nash) equilibrium policy, in a two candidate majority rule game.

The fact that a local equilibrium (or indeed any outcome) can remain the equilibrium if switching costs are high enough is clearly not surprising in itself. Our main contribution lies in that we develop a simple way of studying precisely *how* stable local equilibria are under various parameter assumption (such as relative cost of public and private schooling and the initial income distribution). As we will show it turns out that local equilibria typically do not only defeat neighboring policies, but often a broad range of policies around itself. We refer to such a range as a *stable region* since the local equilibrium prevails as long as competition is limited to this region. Under realistic assumptions, we find the low tax equilibrium to have a very broad stable region - in our benchmark case defeating all tax rates between zero and 68 percent. Hence, relatively small costs - or other plausible reasons for why competition might not effectively be over the entire space of alternatives - are sufficient to make it the stable outcome under majority rule, even though the high tax-transfer state is the Condorcet-winner.[5] Figure 1 illustrates this outcome. There are two local equilibria, i.e. tax rates which are majority preferred to small changes from that policy, and one Condorcet winner, i.e. a tax rate which is majority preferred to all other tax rates. One of the local equilibria is at zero and one at the tax rate 0.14. The local equilibrium at zero, it turns out, has a relatively small stable region and is defeated by tax rates above 0.07, but, more interestingly, the local equilibrium at 0.14 defeats all tax rates in the range 0 to 0.68. Under such circumstances it is not difficult to envision situations where 0.14 could remain the popular choice, even if there are tax rates which are majority preferred to it.

In general, if an economy starts out in a local equilibrium, the relationship between costs of changing policy and the size of the stable region, determines whether change will occur or not. This means, for example, that two otherwise identical economies could exhibit very different transfer systems as a result of different initial policies. It is also the case that the only way to shift from a local equilibrium to another is by a "jump" in policy, while gradual change is not possible. With respect to this, our method allows for an evaluation of exactly how small (or large) perturbations of the existing situation must be for political change to take place. Finally, it turns out that the stable regions as well as the political

[5] It is important to note that we never restrict the competition to any subset of policies. In our examples, it is always the consequence of a (rational) choice made by the competing candidates.

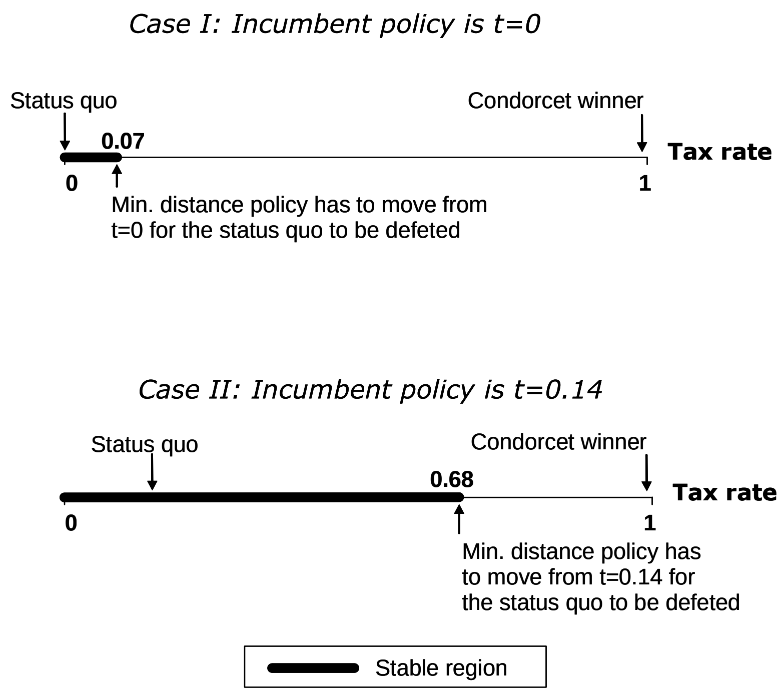


Figure 1: Different status quo policies and their respective stable regions.

coalitions, change in economically intuitive ways with the parameters.

The awareness of potential problems with majority rule equilibrium in standard redistributive situations go back to classics such as Bowen (1944) and Musgrave (1959, Chapter 6).[6] One partial solution to the potential problem of non-existence of voting equilibria in these cases, has been to note that when the problem is one dimensional there will, under very general assumptions, at least be local equilibria.[7] This was discussed in the context of schooling by Stiglitz (1974) and used by Klevorick and Kramer (1973) in a study of social choice on pollution management. However, in their classic textbook *Public Economics,* Atkinson and Stiglitz (1980) deemed the local equilibrium concept unsatisfactory. As they put it: "whether it provides a persuasive resolution to the "majority-voting paradox" depends on the extent to which choices are limited to small perturbations of the existing situation" (p. 307). As noted

[6]Bowen (1944) is particularly interesting as this is an early recognition of the fact that education is an example of a social good which is not equally beneficial to everyone. He also points out the difference between voting over preferred levels of public goods and "voting on increments to existing outputs". Again he points to schooling as an example where individuals typically vote "not on how much of the good they prefer, but rather on whether or not they wish a given increment of decrement to the quantity already provided" (p.40). Finally Bowen also mentions the potential problems with strategic voting (though he does not use the term).

[7]E.g. Theorem 2 in Klevorick and Kramer (1973). See also Kats and Nitzan (1977) on the relations between global and local equilibria.

above, the standard approach has since been to restrict the problem to situations where existence of (global) equilibria can be guaranteed using some version of the median voter theorem.

In this paper, we instead argue that small plausible changes of the standard game can be sufficient to make the previously discarded local equilibrium concept relevant as an equilibrium outcome, even though agents are not myopic or otherwise restricted to small perturbations of the current situation. Rather than only considering policies that defeat *all* other alternatives (i.e. the Condorcet winner), we first find all local equilibria and then focus on studying how far policy must shift for the local equilibrium to be defeated (in the process we, of course, also find the global equilibrium, if one exists).[8] Our analysis partly rely on numerical methods, but it is throughout supported by straight forward intuitive analytical expressions.[9]

The remainder of the paper is structured as follows: In Section 2 we apply the method to a problem of majority rule decisions over school subsidies where credit constraints for investing in education can be binding. For simplicity and ease of reference the model is close to that in Fernandez and Rogerson (1995) with the main difference being that we also allow public and private schooling to coexist.[10] In Section 3 we define relevant political equilibria (in particular global and local equilibria) and explicitly give examples of majority rule games where our approach is relevant. In Section 4 we solve the problem and show how the outcome varies with the parameters of the problem. In particular we show how the stable regions around local equilibria change with the costs of schooling and the initial distribution of income. In Section 5 we discuss some more general insights that can be drawn from our example and, finally, Section 6 concludes the paper.

[8] While local equilibria have not recently been considered in this context it appears in work on related topics such as Alós-Ferrer and Ania (2001) and Crémer and Palfrey (2002).

[9] The first steps of our numerical analysis are similar to the computational model developed by Epple and Romano (1996b). Their main purpose was, however, quite different from ours. They focused on finding the (global) equilibrium (if it exists) for situations where the median voter theorem could not be applied. They also used it to evaluate the effect of vouchers for private education. They did not, however, consider local equilibria and consequently they did not consider the possibility of multiple equilibria or "stable regions".

[10] Another difference is that we use a continuous distribution of individuals rather than three discreate groups of varying size.

## 2 A simple model of educational choice

Consider an economy with a continuum of individuals who differ in initial income $y_i$. The distribution of income is given by a cumulative distribution function, $F(y)$, with a corresponding probability density function denoted $f(y)$, assumed to be continuous and positive over it's support $[0, \infty)$. The number of individuals is normalized to one and hence, aggregate income $Y = \int_0^\infty y f(y)\, dy$ is equal to average income.

Individuals are assumed to live for two periods. For simplicity utility is linear in income and there is no discounting between periods.[11] In the first period each individual has to decide whether to invest in public, private or no schooling. Investing in public schooling has the fixed cost $E$, while investing in private schooling has the fixed cost $P$, with $P > E > 0$.[12] The return to schooling is realized in the second period, and given by $g(y_i)$, and $h(y_i)$ for public and private schooling respectively. Those who do not attend school get the same income as in the first period, $y_i$. Furthermore, it is assumed that $h(y_i) - P > g(y_i) - E > y_i$ for all $i$, which implies that all individuals prefer private education to public education, and that all individuals prefer public education to no education. However, in the absence of perfect credit markets (and without special government intervention) individuals would sort into three groups depending on whether they can afford private schooling, public schooling, or no schooling at all.[13]

To enable schooling for a larger share of the population a uniform tax, $\tau \in [0, 1)$, chosen in a majority rule election, is raised to subsidize public education. Specifically, we assume that agents who choose public schooling get a subsidy, $s(\tau)$, while those who choose no schooling and private schooling respectively get no subsidy.[14]

Given initial income, individual utility (over both periods) is given by

[11] These simplifications are not in anyway necessary for the application of our method. It is straight forward to use it with any well-behaved utility function. We only choose the simplest possible setting for expositional purposes. Also, discounting can be thought of as included in the payoff functions below.

[12] These costs can be thought of as timecosts, foregone income or acctual schooling fees or some combination of these. We assume that the costs can be treated as one fixed, lump-sum cost.

[13] Note that what we call "absence of government intervention" does not imply that there is no government, and in particular it does not imply the absence of public schooling. It should only be taken to mean that at this point there is no specific subsidy to those attending the public school.

[14] We could generalize this to a situation where individuals who choose private school get at least a fraction $\delta s(t), \delta \in [0, 1]$.To simplify the exposition of our method and avoid carrying a number of possible cases throughout the analysis we simplify the setting to one where $\delta = 0$.

$$u_i = \begin{cases} (1-\tau)\, y_i + y_i & \text{if no schooling} \\ (1-\tau)\, y_i + s(\tau) - E + g(y_i) & \text{if public schooling} \\ (1-\tau)\, y_i - P + h(y_i) & \text{if private schooling} \end{cases} \tag{1}$$

Assuming that the ordinal relationship between the different choices is not affected by the subsidy, i.e. that, $h(y_i) - P > g(y_i) - E + s(\tau) > y_i$ for all $i$ and all $\tau$, the population will split into three groups defined by two thresholds.[15] If an individual has an income equal to or above the critical income

$$y^*(\tau) = \begin{cases} 0 & \text{if } E \leq s \\ \frac{E-s(\tau)}{1-\tau} & \text{if } E > s \end{cases}, \tag{2}$$

he will chose public schooling, unless he has an income higher than

$$y^{**} = \frac{P}{1-\tau}, \tag{3}$$

in which case he chooses private schooling.

Denoting the share of the population in private school by

$$N_{priv} = \int_{y^{**}}^{\infty} f(y)\, dy = 1 - F\left(y^{**}(t)\right),$$

and the share investing in public schooling

$$N_{pub} = \int_{y^{*}}^{y^{**}} f(y)\, dy = F(y^{**}(t)) - F(y^{*}(t)),$$

the share unable to afford schooling is given by $1 - N_{pub} - N_{priv}$. The size of the subsidy $s(\tau)$, which goes only to those in public school, depends on the tax rate $\tau$, but also on the number of individuals optimally choosing public schooling. Assuming a balanced budget

[15] The case which we assume away is the possibility that there are those who could pay for the private education with the higher return, but who would abstain from this so as to get the subsidy in period one. This can happen in general but, as we will confirm, does not happen for any relevant cases in our example. It would also be possible to solve for these situations but it would add a number of cases without any additional insights.

$$s(\tau) = \frac{\tau Y}{N_{pub}} = \frac{\tau Y}{F(\frac{P}{1-\tau}) - F(\frac{E-s(\tau)}{1-\tau})} \tag{4}$$

(or zero if no one chooses public schooling).[16]

Given the critical income levels above, we can define the utility function in terms of these threshold and as a function of the tax rate.[17] Differentiating (1) with respect to the tax rate gives

$$\frac{\partial u_i}{\partial \tau} = \begin{cases} -y_i & \text{if } y_i < y^* \\ -y_i + s'(\tau) & \text{if } y^* \leq y_i \leq y^{**} \\ -y_i & \text{if } y_i \geq y^{**} \end{cases} \tag{5}$$

The interpretation of these expressions are straightforward: Individuals who cannot afford the investment despite the subsidy, would oppose a marginal increases in the tax rate, since this will only lead to an increase of their tax payment ($-y_i$). The same is true for those who chose the private alternative since they also pay taxes and get nothing in return. Those who chose public schooling, on the other hand, also experience an increase in their tax burden, but at the same time they get an increased subsidy (as long as $s'(\tau) > 0$).[18] Within this group, the preference toward a marginal increase depends on the relative size of the increase of the subsidy compared to the increased individual tax burden. Given the obvious monotonicity of this relation, the share of the population in favour of a marginal increase at $\tau$ would be those with income $y_i \in (y^*, \widehat{y})$, where $\widehat{y}$ is the income of an indifferent agent given by $\widehat{y} = s'(\tau)$, given that $\widehat{y} \in (y^*, y^{**})$.[19]

The above does not say anything about individual preferences over (all) tax rates. It is simply a description of how the population would be split into different groups in favor of, or opposed to, a marginal change of the tax rate (evaluated at any tax rate). Nevertheless, this way of describing the

[16] This implicit function can be shown to have a fix-point (the proof has been placed below in connection to where we solve the model and find the equilibria).

[17] One may note that already at this point is it possible to solve for an individuals preferred tax rate numerically as the utility of each individual can be evaluated at all tax rates. Based on this one could also numerically check for majority preferred tax rates and "solve" the problem. We firmly belive that going further in analytical terms adds insights which otherwise would be hard to reach.

[18] Obviously, if $s'(t) < 0$, no one is in favor of a marginal increase of the tax rate.

[19] We can disregard the weight of those who shift between groups (i.e. shift from not being able to afford schooling, to investing in public school, or shift between public and private school) as their weight goes to zero for an infinitesimally small change of the tax rate. We will discuss this in more detail below.

marginal (or local) preferences of the population is, as we will show, useful when studying how the population is divided in their views of different tax rates and in finding local as well as global political equilibria.

### 2.1 Potential problems with the existence of majority rule equilibrium

Given the individual alternatives described above, what level of taxation would be chosen in a majority rule election? What would be the political equilibrium? The answer to this depends on the exact political game. We will consider four variations of standard Downsian competition below. There are however some things we can note without specifying the precise rules of political competition. First, the shifts in how an individual evaluates a marginal change of the tax rate (depending on her choice of education) imply that there may be problems with applying either version of the median voter theorems (in the Appendix we show this in more detail).[20]

Regardless of the applicability of the median voter theorems there is no ambiguity in terms of how individuals evaluate *marginal changes* of the tax rate. At any tax rate $\tau$, we can find the share of the population in favour and opposed to a marginal change in the tax rate $\tau \pm \epsilon$, respectively. Furthermore, we can define a tax rate as a *local equilibrium* if it is majority preferred to its neighboring tax rates $\tau \pm \epsilon$ (in the case of the possible corner solutions only one neighboring point is relevant, with an obvious corresponding definition).[21] We now make the following observations regarding the local equilibria and their relationship to the global equilibrium (i.e. the Condorcet winner) if it exists:

**Lemma 1** *For a tax rate to be a global equilibrium it must also be a local equilibrium (while the reverse is obviously not true).*

**Lemma 2** *If a local equilibrium tax rate also defeats all other tax rates, then this local equilibrium is also a global equilibrium.*

[20] As mentioned in the introduction, one may distinguish between the theorem relying on preferences being single peaked and the theorem which requires single crossing (shown to be equivalent to order-restriction in Gans and Smart, 1996).

[21] See Kramer and Klevorick (1973) and (1974), Kats and Nitzan (1976), or more recently Alós-Ferrer and Ania (2001) or Crémer and Palfrey (2001) for more rigorous definitions of local equilibrium in majority rule games. As shown by Kramer and Klevorick (1973) a local equilibrium will exist in one dimensional settings under very general circumstances.

**Lemma 3** *If no local equilibrium tax rate is majority preferred to all other tax rates, then there exists no global equilibrium.*

As we can construct a function for the *total support for a marginal increase* over the whole policy space (the relevant space here being the one dimensional set of alternatives $\tau \in [0;1)$) we can use this information to determine in which direction a majority would like to push the tax rate at any point (given that we only consider marginal changes).[22] In the problem above, the fraction of the population that favours an increase of the tax rate at any $\tau$ is simply:

$$H(\tau) = \begin{cases} 0 & \text{if } \widehat{y} < y^* \\ \displaystyle\int_{y^*}^{\widehat{y}} f(y)dy & \text{if } \widehat{y} \in (y^*, y^{**}) \end{cases}$$

For this characterization to be correct we must show that the critical incomes are continuos in $\tau$, that is

$$\lim_{\epsilon \to 0} y^*(\tau) = y^*(\tau + \epsilon) \tag{6}$$

$$\lim_{\epsilon \to 0} \widehat{y}(\tau) = \widehat{y}(\tau + \epsilon)$$

$$\lim_{\epsilon \to 0} y^{**}(\tau) = y^{**}(\tau + \epsilon) \tag{7}$$

which in turn depends on the properties of $s(\tau)$. The limit for investing in private schooling is given by $y^{**} = \frac{P}{1-\tau}$ and is obviously continuous in $\tau$. The critical income for investing in public schooling, $y^*(\tau)$ is given by (2) above and the continuity of this depends on the continuity of $s(\tau)$, and as $\widehat{y}(\tau) = s'(\tau)$ we must also show that the function $s(\tau)$ is continuously differentiable for all critical incomes to be continuous in the tax rate. Using the implicit function theorem we can show that $s(\tau)$ exists and is a continuously differentiable function with a unique solution for every $\tau$.

**Proof.** See Appendix A. ■

[22] Not including the tax rate one is just for convenience so as to avoid carrying additional notation as $\tau = 1$ creats conditions where terms are undifined due to division by zero. There is, however, no ambiguity in the support at this point.

# 3 Finding equilibria and stable regions

To explicitly solve the model we need to choose values for the parameters (schooling costs and returns to schooling) and the initial income distribution. We do not set out to calibrate the model to any specific country or situation, but instead chose reasonable values to illustrate a possible outcome, and then (in Section 4) study how the equilibria and stable regions move with the parameters. Below we analyze which policy that will be announced by the office-seeking politicians in four examples of majority rule competition.

## 3.1 Parameterization

First, a distribution function for pre-tax income must be chosen. The model can be solved for any continuous and well-behaved initial income distribution. The specific shape of the distribution function will however, be an important determinant of the political support for redistribution. Second, the costs of investing in public and private schooling respectively (i.e., $E$ and $P$) must be chosen. Third, we must specify the functions for the return to public and private schooling, i.e., $g(y_i)$ and $h(y_i)$.

To approximate this, we assume pre-tax income to be Weibull distributed with parameters $(b = 100, c = 1.4)$, which generates a Gini Coefficient of 0.39.[23] The fixed cost of investing in public and private schooling may respectively be expressed as shares of the average income:

$$E = \mu Y, \qquad P = \varphi E \qquad \mu, \varphi \in \mathbb{R}^+, \quad \varphi > 1. \tag{8}$$

In the benchmark calibration, we set $\mu = 0.5$ and $\varphi = 2.8$, but again, we will solve for equilibria over a wide range of values.[24] The return to public and private schooling finally, are set to $h(y_i) = E + \alpha y_i$ and $g(y_i) = P + \beta y_i$, with $\alpha = 2$ and $\beta = 3$.[25]

[23] The average market (pre-tax) income Gini is 0.39 according to data from the Luxembourg Income Study (based on 34 observations taken from the OECD Economic Studies, 1997). The *two-parameter* Weibull *pdf* is given by $p(y) = \left(\frac{y}{b}\right)^c \exp\left[-\left(\frac{y}{b}\right)^c\right]$, where $y > 0, b > 0$ and $c > 0$. The parameter $b$ is just a scale parameter, and $c$ is a shape parameter determining the degree of inequality.

[24] A discussion of the benchmark calibration can be found in the Appendix.

[25] Note that the exact functional forms of these functions are not important for the political outcomes locally, since neither $y^*, y^{**}$ or $\widehat{y}$ depend on these functions. However, they do enter the utility functions and will therefore play a role in the comparisons between alternatives where individuals choose different (or no) schooling between the respective policies being compared. Hence, they affect the global equilibrium as well as the stability regions.

### 3.2 The Total Support for Changing the Policy

Given the chosen parameters we can now determine $H(\tau)$, which shows how the total support for a local increase of the tax rate varies with the tax rate. This is illustrated in Figure 2. Recall that when $H(\tau)$ is above the 0.5-line, a majority is in favour of a marginal increase of the tax rate, and vice versa for points below the 0.5-line. Starting from the left in Figure 2, i.e. at $\tau = 0$, we see that only around 43 percent prefers a marginally higher tax rate to the zero tax rate, but as we move to the right, the aggregate support for a local increase of the tax rate first increases to 57 percent at the tax rate 11 percent, and then decreases and reaches a minimum around 37 percent at the tax rate 40 percent and finally increases again to a maximum at the tax rate 1.

For interior tax rates, only points where $H(\tau)$ cuts the 0.5 line from above (reading left to right) are local equilibria. Since there is only one such point - at the tax rate $\tau = 0.1429$ - this is the only interior local equilibrium. There are also two tax rates for which $H(\tau)$ cuts the 0.5 line from below, i.e., at $\tau = 0.05$ and $\tau = 0.49$. Clearly these tax rates never constitute local equilibria since they are defeated by *both* their neighboring tax rates. Note finally that both corners also are local equilibria, since a majority of the population is against a local increase at $\tau = 0$, while a majority of the population is in favour of an increase at $\tau = 1$. Hence, to sum up, there are three local equilibria, at $\tau = 0, \tau = 0.1429$ and at $\tau = 1$, and from Lemma 1 above it immediately follows these are also the only candidates for being the global equilibrium (the Condorcet winner) since no other tax rate is even a local equilibrium.[26]

To understand what drives the changes in $H(\tau)$ it is useful to look at the division of the population at each local equilibrium, shown in Figure 3 below. The top panel shows the situation at $\tau = 0$. First, there are those at the lower end of the distribution (with $y \in [0, y^*]$) who can not afford education at this point. They gain nothing from a marginal change in the subsidy which they do not get and consequently oppose marginal increases of the tax rate (again, note that the weight of those who shift between groups as a consequence of a marginal change is zero). Second, those with $y \in [y^*, \widehat{y}]$ consists of those who at this point gain from increased taxes as they get a subsidy which is larger than their tax payment, while

[26] Recall that there are no disincentive effects from taxation in the model. Hence, in a purely redistributive setting as long as the median is poorer than the mean there will always be a majority in favor of more redistribution (given that everyone participates and gets the subsidy). Introducing effects which bound the maximum away from one is straight forward and does not change the qualitative results.

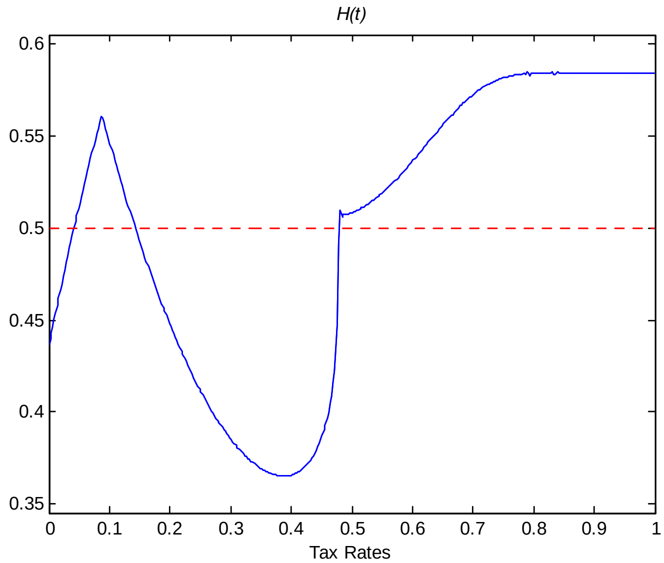


Figure 2: The Aggregate Support for a Marginal Increase in the Tax Rate

the third group are those at the top of the distribution who, at this point, pay more than they get and consequently oppose a tax increase.[27] As the tax rate increases the lower threshold $y^*$ falls since the subsidy enables more low income individuals to choose schooling and at the same time some individuals with higher income switch to public schooling. Initially the increased support for further tax increases, hence, comes from both ends of the distribution sharply increasing $H(\tau)$ to above 0.5. Just below $\tau = 0.1$ the limit given by $\widehat{y}$ starts to fall (and it is separated from $y^{**}$) as some individuals (in the upper end of the distribution) who still choose public schooling pay more in taxes than they receive in transfers (the rate of increase in $s(\tau)$ decreases due to the initial inflow of individuals at both ends of the distribution). This effect now dominates the continued growth in support of higher taxes at the lower end where increased subsidies enable more and more individuals to invest in education and consequently $H(\tau)$ now decreases.

The middle panel shows how the composition has changed at the local equilibrium $\tau = 0.1429$. As at the starting point $\tau = 0$ there is a coalition between the poorest and the rich who oppose a marginal increase of the tax rate. However, the share at the lower end is now smaller than before as more individuals can afford education and the group of high income people consists of those who attend public school but

[27] As $\widehat{y} = \min[s'(\tau); y^{**}]$ this initially binds and is given by $y^{**}$ and therefore this group is composed entirely by those who choose private schooling. When the tax rate increases so does $y^{**} = P/(1-\tau)$ and at least initially this means that the upper cut-off also increases.

receive a subsidy lower than their tax payment as well as those who attend private school and therefore get no subsidy.

The continued changes in $H(\tau)$ depend on the relative size of the changes at the thresholds $y^*$ and $\widehat{y}$. Typically increased taxes cause both to fall, increasing the support for further tax increases at the lower end and decreasing it at the upper end.[28] Depending on the underlying distribution either one of these may dominate.

As the tax rate approaches one we have a situation where the subsidy approaches a number greater than the average income $Y$ (since the tax proceeds are divided by less than everybody when some chose private schooling). Given the usual assumption of a distribution where the mean is larger than the median income (and the cost of public schooling is lower than mean income) there will be a majority in favour of tax increases at this point. This is illustrated in the bottom panel showing the division at $\tau = 1$ where everyone receives the subsidy and everyone with an endowment below the mean wants to maximize redistribution.

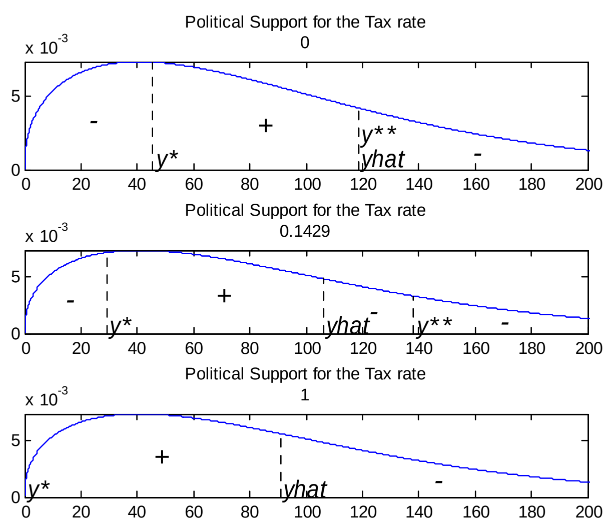


Figure 3: Political Coalitions that Favor a Marginal Increase of the Tax Rate.

[28] There are, however, also situations (for other parametrizatioions) where the crowding effect dominates and $s(\tau)$ actually falls. In these cases clearly no one supports marginal tax increases.

## 3.3 Equilibria and stable regions

Figures 2 and 3 are both silent on which (if any) of the three local equilibria that is the global equilibrium. To determine this, we compare the three candidates to (a dense grid of) all other tax rates. Doing this, we find $\tau = 1$ to be the global equilibrium (Condorcet winner). This could be the final solution to the problem - and indeed, is the solution in some situations. However, we can also note that in this search process, we get information about exactly how each local equilibrium compares to all other tax rates. This allows us to create what we call *stable regions* around each local equilibrium, that is the range of tax rates defeated by the local equilibrium tax rate in a pair-wise competition (hence, making it "stable" as long as it is not faced with competition outside of this range). Fig.4 shows the results from such a comparison for $\tau = 0$ and $\tau = 0.1429$, with the stars indicating the *stable region*. The tax rates without stars indicates the policies that defeat the respective local equilibrium in a majority vote. The top panel shows that $\tau = 0$ is not very stable since it is defeated by any tax rate higher than 7 percent. However, this is not the case for the local equilibrium $\tau = 0.1429$, which is majority preferred to all tax rates between zero and $\tau = 0.68$. As we will see below, these stable regions might be interesting in a number of situations.

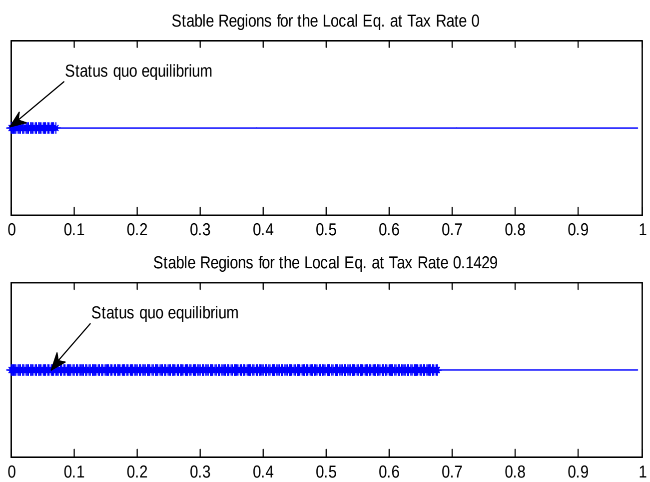


Figure 4: Stability Properties for the Local Equilibria

In a standard setting of Downsian competition the global equilibrium is the obvious outcome. There are, however, plausible modifications of such a game which lead to situations where this is not necessarily the case. We therefore introduce the following concept:

**Definition 1** *The **stable outcome** (or Nash equilibrium) of a two candidate majority rule election game is a tax rate chosen by both candidates from which none of them has an incentive do deviate.*

In the following section we consider four different specifications of the political game that illustrate how the global equilibrium (or Condorcet winner) does not always coincide with the stable outcome, and in particular, why local equilibria - depending on the stable regions - are likely to be the stable outcome.

# 4 Majority rule political games

## 4.1 Benchmark cases

We begin by considering two polar extremes as benchmarks. First, standard Downsian competition where two parties (costlessly) choose any tax rate as their platform and the party which gets the majority of the votes wins the election, Second, a situation where parties only can suggest alternatives which constitute a small change from some predetermined status quo.

### 4.1.1 Downsian competition

In this case the relevant political equilibrium is the *majority voting equilibrium* and the policy outcome will be the "Condorcet winner". In terms of the method above the $H(\tau)$-function provides information on which tax rates that are local equilibria (and hence candidates for being the global equilibrium). Since we assume that there are no cost associated either with campaigning or changing the policy, both parties will, in the example above, announce the tax rate $\tau = 1$ as their platform. Hence, the stable outcome is the global equilibrium. The local equilibria are not really interesting in this game, but merely a step on the way to finding the global equilibrium (if it exists).[29]

[29] If there is no global equilibrium the situation is one usually thought of as a case of "policy cycling". Roine (2006) however presents some arguments for local equilibria in Downsian competition in the absence of a Condorcet winner.

## 4.2 Myopic political competition

Consider now instead the polar extreme where the candidates for some reason only can alter the policy in small steps. Under this assumption the $H(\tau)$-function also reveals the transition path to the equilibrium. Which of the local equilibria that actually becomes the stable outcome will depend on the initial tax rate. Fig. 2 shows that if the economy starts out with a tax rate in the interval $\tau = (0.05, 0.49)$, it will eventually end up at the local equilibrium $\tau = 0.1429$, whereas if the initial tax rate is below 0.05, it will converge to the tax rate $\tau = 0$. If the initial tax rate is above 0.49 finally, the policy will instead be pushed toward 1.

It is important to note that in this type of game the restriction on the candidates alternatives is very strict. Even if one thinks that it is reasonable to limit the politicians possibility to suggest very large moves from the starting point (at least without incurring any costs) this is not ideally captured. The marginal conditions behind the $H(\tau)$-function only evaluate each tax rate against $\varepsilon-$changes and can not in itself say anything about the aggregate preference for a slightly larger (but still small) move. As was noted in the introduction, this is likely to be the reason for why local equilibria in majority voting games have been dismissed in the literature.

## 4.3 Downsian competition with costs of altering the policy

Now consider instead a standard Downsian competition game between two parties starting at some status quo position, but where we add the reasonable assumption that there are costs involved in shifting policy (or altering policy platform). The outcome of the game will then typically come to depend on the *stable regions* around the local equilibria. The set up depends on what we assume about the costs of moving and in particular on whether these costs are borne by the candidates or by the voters. We analyze both cases in turn.

### 4.3.1 Costs Borne by the Candidates

Consider the following reduced form of the problem facing each candidate at the status quo: If the candidate sticks to the status quo policy and the other candidate does the same they both have a 50/50 chance of winning. If one candidate moves to a policy which is majority preferred to the status quo and the other one does not, the candidate that moves wins with certainty but bears a cost, $c$, of moving. Finally, if both move (to the same policy) they are back to a situation where both have a 50 percent chance of winning, but now both of them will have to bear the cost of moving.[30] Normalizing the payoff from winning to one, this game has the following normal form:

$$\begin{array}{ccc} & \text{Move} & \text{Stay} \\ \text{Move} & (0.5-c;\, 0.5-c) & (1-c;\, 0) \\ \text{Stay} & (0;\, 1-c) & (0.5;\, 0.5) \end{array} \cdot \tag{9}$$

Assume now that the economy starts out at a local equilibrium, and that the parties move simultaneously in a one-shoot game.[31] Obviously, if the initial tax rate is $\tau = 1$, the economy stays there, but if we start at any of the other local equilibria the stable outcome clearly depends on the cost. More precisely, if $c < 0.5$, moving is a strictly dominant strategy but if instead $c > 0.5$ staying with the status quo is optimal (with obvious indifference at $c = 0.5$). If we, which we think is realistic, assume that the cost of moving is increasing in the distance moved, this implies a negative relation between the size of the stable region and the "cost per unit moved" needed to make a local equilibrium the stable outcome of the game.[32] In other words, the broader is the stable region, the further the parties must move from the status quo to get a majority of the votes, making the local equilibrium more robust. To see this explicitly, assume that the candidates face the following simple cost function when moving

$$c\,(\Delta\tau) = \xi\Delta\tau, \tag{10}$$

[30] To illustrate our point we assume a one-shot game where we only consider two possible actions; "Move" (implicitly to the nearest point which defeats the status quo) or "Stay" (at the status quo). In a full version of the game there are of course an infinite number of possible moves. Some, such as moving to policies which are defeated by the status quo, are not interesting but others may be. However, as our aim here is to give conditions for when both candidates would chose to stick to the status quo this simplified version is sufficient.

[31] For simplicity, we assume that the starting point is a local equilibrium. This is, however, not very restrictive since a policy which is not a local equilibrium can be defeted by an infinitesimally small move (which, if costs are related to distance moved, has a very low $c$). In fact, if we only require the cost of moving to be an increasing continous function $c$ of the distance moved ($\Delta\tau$) such that $c(0) = 0$, $\{stay; stay\}$ can never be the equilibrium.

[32] An example of a reason for why large moves should be more costly is that it is likely to be more costly for parties to communicate major changes in their programs compared to small ones.

where $\Delta\tau$ is the Euclidian distance between the status quo policy and the policy the candidate is moving to. If we now compare the two local equilibria $\tau = 0$ and $\tau = 0.1429$, the marginal cost needed to make each of them the stable outcome is more than 8 times higher for the zero tax equilibrium than the equilibrium with $\tau = 0.1429$.[33] This is expected since the former equilibrium has a very short stable region, whereas the latter already without costs defeats more than 2/3 of all tax rates. Note that the tax rate $\tau = 1$ is still the Condorcet winner, but that the stable outcome of the game can differ from that, depending on the starting point and the cost of altering the platform. This simple example illustrates that the relevance of a local equilibrium policy as a plausible outcome depends crucially on precisely *how* locally stable it is.

#### 4.3.2 Transition costs borne by the voters

As another illustration, we consider the case where transition costs are incurred by the citizens.[34] Specifically, assume that the transition cost function is still given by (10), but that the cost of moving just a marginal unit is zero (i.e., $c(\epsilon) = 0$) and that the transition is financed with a tax on the income in period 2. These assumptions are sufficient to make sure that the $H(\tau)$-function in Fig. 3 remains unchanged.[35] Voters face the exact same problem *locally*, but the comparison of the equilibria to all other tax rates is different, due to the transition costs.

As in the previous section, if the economy starts out with a tax rate $\tau = 1$, it stays there and the stable outcome is $\tau = 1$. If, however, the economy starts out with any other tax rate, the stable outcome could again differ from $\tau = 1$. It is of course possible to consider a number of different games but what we are interested in here is to illustrate the relation between the stable region and the transition cost. In particular, we ask the following question: given that the economy starts out in a local equilibrium (that is not also the global equilibrium), how large must the transition cost be for this to be the stable outcome. Furthermore, we want to get a sense of whether the order of magnitude of the cost is such that this is a realistic case. With knowledge of the stable regions we can calculate the minimum cost required

[33] The total cost is not really interesting since payoffs have been normalized.

[34] These might be administration costs, or simply costs associated with expanding the system.

[35] This assumption can easily be relaxed. If there was a positive cost for a marginal move this would simply shift the $H(\tau)$-function as there would be an additional cost (which would be a fixed number for the $\varepsilon-$sized move) to every individuals utility function. It is also straightforward to show that the tax rate needed to finance the transition is $\tau^t = \xi\Delta\tau/Y$.

to make the respective local equilibria stable. We can then express these as fractions of total income to get a sense of their size. The results for such an exercise in the above benchmark problem are presented in Table 1.

Table 1: Transition Cost Needed to Make the Local Equilibrium Stable

| LOCAL EQUILIBRIUM | $\tau = 0$ | $\tau = 0.1429$ |
|---|---|---|
| Min. Transition Cost for Stability (% of Tot. Inc.) | 17.64% | 2.2% |

Consequently, if the economy starts out at the local equilibrium of $\tau = 0.1429$ and the total cost of moving to a majority preferred policy is larger than 2.2 percent of total income, the status quo remains the stable outcome.[36] However, if the initial tax rate is $\tau = 0$, the total transition cost needed to make it the stable outcome is more than 17.64 percent of GDP. These results are similar to the previous case in the sense that transition costs must be eight times higher to make the zero-tax equilibrium stable compared to the interior equilibrium ($\tau = 0.1429$). Also, the general point is again that a relatively small transition costs may be sufficient to make a local equilibrium, with a broad stable region around it, the stable outcome.

# 5 Outcomes under varying assumptions

Having established the potential importance of not just the global equilibrium tax rate but also the local equilibria and their respective stable regions we now move on to study how the possible outcomes depend on the parameters. More precisely, we will illustrate how the outcomes change over ranges of different starting values of one parameter at a time, keeping the others at their benchmark values. The main reason for these exercises is that looking at different outcomes help us understand the underlying mechanisms of the changing political support in various dimensions. Of particular interest is, of course, to see if broad stable regions are "rare" and only occur under specific parameter constellations or whether they seem to be typical for this type of problem.

[36] It is of course possible to be more general and compute the transition cost needed to make $\tau = 0.1429$ the stable outcome given that the economy starts out without any public subsidies to schooling.

## 5.1 The cost of public schooling

Figure 5 shows the different equilibria and their stable regions as we change the cost of public education (expressed as a fraction of total income and keeping the cost of private schooling fixed at the benchmark level). Note that the three panels in figure 5 illustrate outcomes that coexist but have been separated so as to make interpretation easier. Each panel shows the range over which there is a local equilibrium (solid lines) as well as how far policy must move from the local equilibrium for an alternative to be majority preferred (stars).

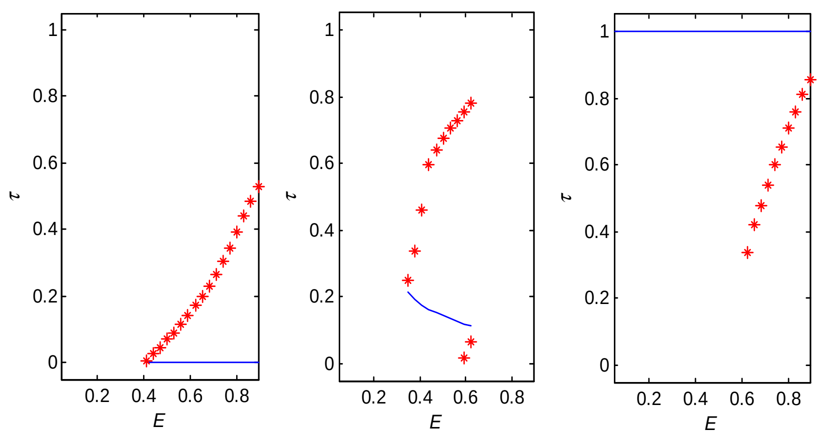


Figure 5: Local Equilibria and Stability Regions as Functions of the Cost of Schooling (Gini=0.39)

Provided that public schooling is relatively inexpensive the high tax rate is the only equilibrium. Even without a subsidy a large share of the population can afford schooling and a majority favours it being subsidized. However, as the cost of public schooling goes up, the share of the population at the bottom of the distribution who can not afford it (unless it is sufficiently subsidized) grows. At some point they form a locally stable coalition with the rich in favour of a low interior tax rate. Increasing the cost a little more also introduces zero as a local equilibrium.[37] The stable regions around these respective local equilibria

[37] At first glance the graph may look contradictory as the interior tax rate is stable inside the zero tax rate's region of stability. However, this is perfectly possible as the starting point for the comparision is each local equilibrium respectively. In the region where the interior equilibrium is inside the zero tax rate's stability region, there are points which are defeated by both the zero and the interior local equilibria depending on where we start. However, by definition, the zero tax rate would defeat the interior local equilibrium in this region. This can also be seen by the fact that when the local equilibrium interior tax rate crosses the border of the zero tax rate's stability region, the interior is no longer stable agains zero (it's stability region starts moving up from zero).

increases as the cost of public schooling goes up. Since the minimum subsidy needed for the poor to benefit goes up as public schooling becomes more expensive, only a large enough subsidy will enable the population at the bottom of the distribution to participate and benefit. In the range where the cost of public schooling is approximately 0.4-0.6 times that of private, the high tax rate is the global equilibrium but at the same time the low tax rate (and zero in some range) is majority preferred to moderate, or even high levels of redistribution, as the cost of public schooling increases. As it reaches about 0.6 of private, there is no global equilibrium and the zero tax outcome becomes increasingly stable as the share of the population at the bottom of the distribution is excluded from education with or without the subsidy and consequently vote against it.

These findings are interesting in light of the discussion in Fernandez and Rogerson (1995), p. 260, where it is conjectured that rich individuals would support increasing the "height" of the barrier to education. Figure 5 illustrates precisely how the stable region of the zero tax equilibrium increases as the barrier to education goes up. A related interpretation lies in the comparison between rich and poor countries (rich and poor in the sense that the cost of public schooling is a smaller or larger share of the average income). Our results suggest that even in a setting where this kind of redistributive system is majority preferred (globally), it is less likely that the country would end up with this system the poorer is the country. Furthermore, if change is to take place this must be drastic rather than gradual. These conclusions could not be reached in a setting where the focus was on the global equilibrium only.

## 5.2 Changing the initial distribution

Another obvious question is how the results change with initial income inequality. To illustrate the most important effects, figures 6 and 7 show replications of figure 5 above, but with the difference that the initial distribution in the first figure has a Gini coefficient of 0.35 (i.e. a more equal distribution) and the other has a Gini coefficient of 0.45 (that is, a more unequal distribution compared to the figure above). Starting first with comparing the possibility of a zero tax outcome, it seems that the lower the Gini is, the less likely is a situation where a majority favors no system at all. The reason for this is that increasing inequality puts more weight on both ends of the distribution thereby increasing the likelihood of an ends-against-the-middle equilibrium. This effect is interesting as it goes in the opposite direction of the standard effect where increasing inequality leads to higher demand for redistribution. Here being poor potentially means not having access to the redistributive system at all, and higher inequality, ceteris paribus, increases the likelihood of the poor opposing redistribution. There also seems to be an effect making the low interior equilibrium more likely when the initial distribution is more even. As a larger share of the population is part of the middle-class the likelihood of *some* redistribution increases.

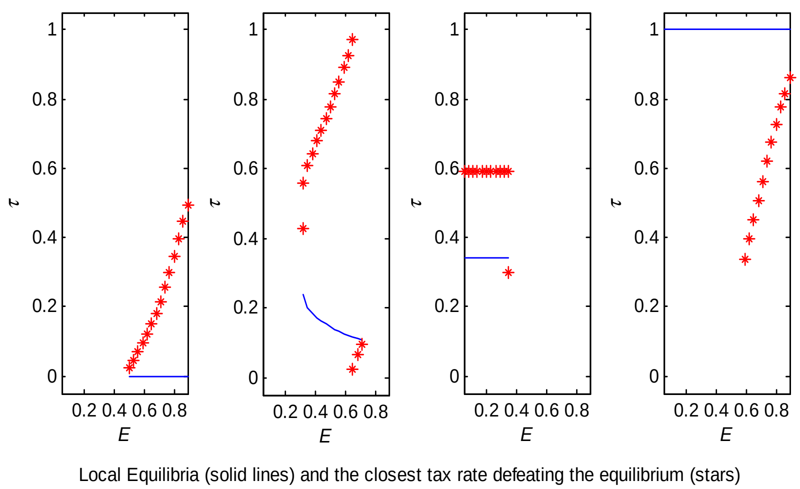


Figure 6: Local Equilibria and Stability Regions as Functions of the Cost of Schooling (Gini=0.35)

It hence seems as if an uneven initial distribution increases the likelihood of an extreme outcome, either in the form of a collapse toward complete redistribution as soon as schooling becomes affordable for the poor, or a collapse toward no redistribution at all when schooling is too expensive unless the redistributive system is sufficiently large. When the initial distribution is more even, this gradual move between the extremes involves a range of schooling costs where the middle-class - which in this case is a larger fraction of the population - gains the most from limited redistribution, leading to a majority favoring an interior tax rate.

This confirms the finding in Fernandez and Rogerson (1995) that an ends-against-the-middle equilibrium, where the poor are excluded, is more likely the higher is the initial income inequality. But our way of studying the problem also allows us to see how the underlying support for the different local equilibria

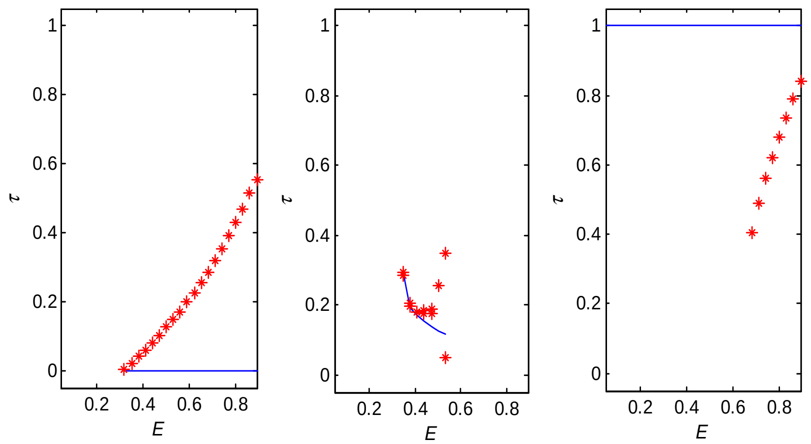


Figure 7: Local Equilibria and Stability Regions for Different Initial Distributions (Gini=0.45)

change gradually and, for a broad range of schooling costs, we would argue that multiple equilibria are plausible. We can also see the support for the local equilibria when there is no globally stable outcome which enables us to say more than that the outcome is indeterminate.

# 6 Discussion and general insights

The model of voting over education subsidies is an example of a reoccurring problem in public economics. The common feature is that individuals shift between participating and not participating in some redistributive scheme as the policy changes, which in turn typically lead to difficulties when trying to determine the political support for redistribution. Often - as in the above - the problem involves an investment of some kind which is partially subsidized by a universal tax, and where poor individuals can not (or choose not to) invest if the subsidy is too small. However, similar problems may also arise when studying consumption problems where a good is not consumed at all by poor individuals unless it is sufficiently subsidized ("culture goods" are the standard example). Furthermore, even though it is most common that the exclusion happens for the poor and on the side of receiving the subsidy, it is also possible to envision situations where individuals "exit" the system on the side of paying the tax while still receiving subsidies.[38]

If one insists that there is only one possible majority rule equilibrium in this type of problem (if an equilibrium exists) the method exemplified above can solve the problem in the sense that it finds the global equilibrium tax rate (the Condorcet-winner).[39] However, if one is prepared to consider settings where, for example, there are costs involved in shifting the policy, the stable regions around the local equilibria introduces the possibility to consider multiple equilibria.[40]

Considering first what we have learnt about the specific problem of majority decisions over education subsidies it is illustrative to, once again, compare our results to those in Fernandez and Rogerson (1995), who study essentially the same problem as the one above. They reach three main conclusions: First, they show that there can be equilibria where transfers go from lower income groups to higher income groups (that is the case when the poor are excluded from the benefits). Second, they find that this kind of situation becomes more likely the higher is the initial inequality of income, and third, they show that wealthier individuals may gain from higher costs of education as this may enable them to exclude poorer individuals from the redistributive system. Our analysis reaches the same conclusions. However, our way of studying the problem introducesa number of additional insights, which also seem to have more general implications.

First, we show that when introducing costs of altering the status quo policy in the political game, the possibility of multiple equilibria arises. Depending on the starting point (or on small initial differences) the stable outcome can be very different even if the situation is such that there is only one global equilibrium. In terms of the example above, two otherwise equal countries (or regions) could based on, for example, differences in the cost of changing the policy end up choosing either a low tax equilibrium with limited subsidies and a large share of the population excluded from education (an outcome which is not a global equilibrium but which may never the less be stable), or a high tax equilibrium with full subsidies and everyone attending school. Similar situations can be envisioned for countries with different initial income

[38] Roine (2006a) is an example where fix-cost investments in tax avoidance can cause violations of conditions required for the median voter theorems to hold. In this paper the rich and the poor may in equilibrium favor increased redistribution.

[39] This is in it self important as the alternative has previously been to simplyfy the problem in various ways.

[40] The other obvious way we see to introduce differences between moves close to the status quo and moves far from it is informational aspects. These may of course be linked to costs as overcoming information problems can be seen as possible but costly.

distribution, starting at different status quo policies, or with different entry barriers for participating. In all these cases it is worth noting that the way in which the stable regions around local equilibria change correspond to the basic insights above. As inequality increases, or the entry barriers become higher, the stable region around a low tax equilibrium increases making such an outcome more plausible. However, we can also see more detailed aspects such as the increased possibility of an interior low tax equilibrium for a more equal initial distributions, and an increased likelihood of extreme outcomes when the initial inequality is greater. These possibilities arise in a range of parameters where a tax rate of one is the global equilibrium and hence the only considered outcome in previous work.

Second, our way of studying the problem also illustrates why shifts in policy must sometimes be drastic rather than gradual if they are to happen at all. If a redistributive system, with possible exclusion, is to be introduced it is likely that it must immediately have some minimum size. If it does not, there can be a majority who prefers the zero tax rate since too many of the poor would oppose the introduction. Similarly shifting from a situation with a low tax rate and limited redistribution to a higher tax-transfer state may also only be possible through a very large shift as all intermediate states would be defeated by the smaller system in a pair-wise competition. It is worth noting that the stable region of the low tax rate in the example above had a range of up to over 50 percentage points. This means that a very dramatic shift would be necessary for a shift to occur.[41]

Relating to the opening quotes above, assuming that policy must change in relation to a status quo and that moving away from this initial state involves some cost that increases with the distance moved, our analysis has shown how a local equilibrium status quo can remain the equilibrium outcome without invoking any myopia on anyone's part. Furthermore, we have also shown that if change is to happen it must often be radical. Attempts at small changes would be defeated by the "tyrannical status quo" even if the incumbent policy it is not the Condorcet winner. While we have only exemplified the mechanisms behind these outcomes we believe that the insights are more general and useful for understanding aspects of political change.

[41] A third point is to note that the method gives a "continuous picture" of political support for different policies ranging from local to global equilibrium. The aggregation of the support for marginal (local) change over the policy space gives a map of the direction in which the policy would move starting at any point. The stability regions around each local equilibrium indicates precisely how far policy would have to move for change to be majority preferred, and finally, the last step in the procedure finds the global equilibrium (if it exists).

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# A Failure to satisfy the conditions of the median voter theorem(s)

The failure to satisfy single peaked preferences over tax rates is easily illustrated. As taxes increase an individual who is too poor to make the investment will only pay increasing taxes with decreasing utility as a consequence. At some point, however, the size of the subsidy can be large enough to enable the poor individual to make the investment leading utility to (possibly) increase. Figure 8a shows an individual with an initial income (endowment) $y < y^*$ for all $\tau < \tau'$. At $\tau'$ the subsidy becomes just large enough for $(1-\tau')y + s(\tau') = E$ which allows the individual with initial income $y$ to invest and (under a certain subsidy function) experience increasing utility for higher tax rates. Similarily we can illustrate how individual indifference curves (in the tax-subsidy space) can be non-single crossing. Figure 8b illustrates the fact that an individual who at tax rate $\tau$ requires at least $s(\tau)$ to make the investment has an indifference curve which crosses the indifference curve of a richer person, who invests regardless of the subsidy, twice. The fact that preferences are not necessarily single peaked nor single crossing means that none of the median voter theorems can be applied in general.[42]

# B Continuity of $y^*(\tau), y^{**}(\tau)$ and $\widehat{y}(\tau)$

To be able to construct the function

$$H(\tau) = \begin{cases} 0 & \text{if } \widehat{y} < y^* \\ \displaystyle\int_{y^*}^{\widehat{y}} f(y)dy & \text{if } \widehat{y} \in (y^*, y^{**}) \end{cases}$$

[42] The exact form of the utility function, as well as the indifference curves, of course depend on how subsidies evolve over tax rates. For violation of single-peaked preferences to occur in this type of setting, what is needed is that there be some segment of the policy where the poor person has decreasing utility from increasing the subsidy because they do not (can not) participate, and that in some other segment where they do participate, they gain from redistribution. For single-crossing to be violated what is needed is that poor individuals, who normally have indifference curves with a smaller slope than richer once (as they require smaller subsidies to be indifferent to a certain tax increase), for low enough tax rates can not participate in the redistributive system and, hence have vertical indifference curves at this point.

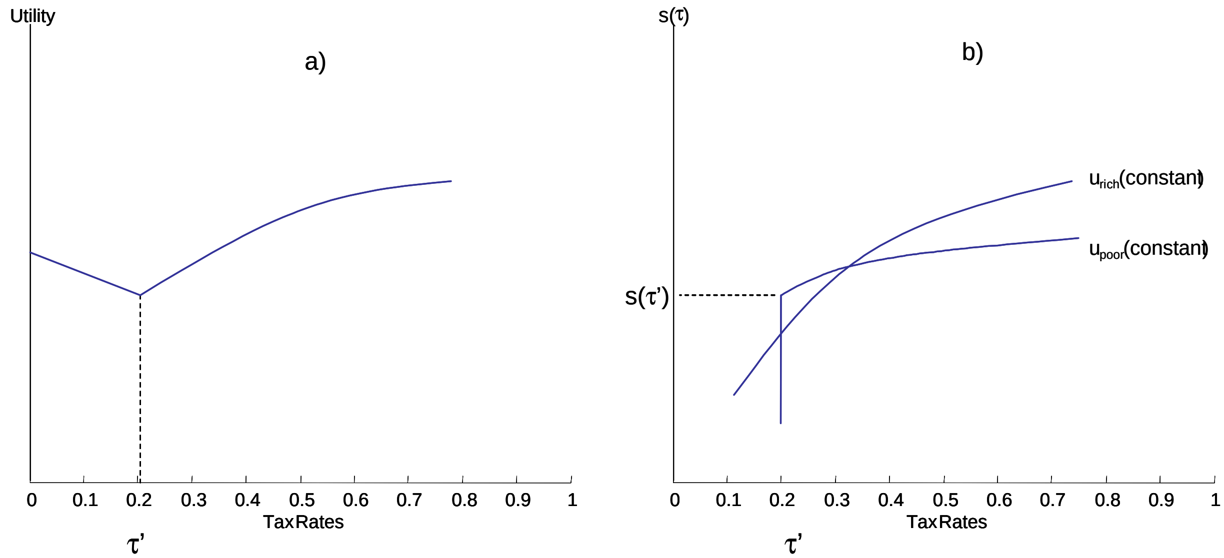


Figure 8: Illustration of failure to satisfy conditions for the Median Voter Theorems.

we need the critical incomes to be continuos in $\tau$, that is

$$\lim_{\epsilon \to 0} y^*(\tau) = y^*(\tau+\epsilon) \tag{11}$$

$$\lim_{\epsilon \to 0} \widehat{y}(\tau) = \widehat{y}(\tau+\epsilon)$$

$$\lim_{\epsilon \to 0} y^{**}(\tau) = y^{**}(\tau+\epsilon). \tag{12}$$

The limit for investing in private schooling is given by $y^{**} = \frac{P}{1-\tau}$ and is obviously contious in $\tau$, $\widehat{y}(\tau)$ is continous if $s(\tau)$ is continously differentiable and since $\lim\limits_{s \to E} \frac{E-s(\tau)}{1-\tau} = 0$,

$$y^*(\tau) = \begin{cases} 0 & \text{if } E \leq s \\ \frac{E-s(\tau)}{1-\tau} & \text{if } E > s \end{cases}$$

is continuous in $\tau$ if $s$ is continuous in $\tau$. The proof hence concernes the properties of $s$.

Using the implicit function theorem we can show that $s(\tau)$ exists and is a continously differentiable function with a unique solution for every $\tau$.

**Proof.** We want to prove that for the implicit function $s(\tau)\left[1 - F\left(\frac{E-s(\tau)}{1-\tau}\right)\right] - \tau Y = 0$ there exists a unique continously differentiable function $s$ which has a solution for every $\tau$. Consider the continously differentiable function $\Phi(x^o, \tau^o) = \tau Y$ where

$$\Phi(x^o, \tau^o) = x\left[1 - F\left(\frac{E-x}{1-\tau}\right)\right].$$

Taking the derivative w.r.t $x$ (at $x^o$ with $\tau^o$ fixed) gives

$$\frac{\partial \Phi}{\partial x} = \left[1 - F\left(\frac{E-x}{1-\tau}\right)\right] + \frac{x}{1-\tau} \times f\left(\frac{E-x}{1-\tau}\right)$$

which is always positive (for the relevant domains of $x, \tau$ and $E$ and given the functions $F$ and $f$) and hence is a bijection. By the implicit function theorem we then know that there exists a neighborhood of $\tau$ and a unique continously differentiable function $s$ such that $s(\tau^o) = x^o$ and $\Phi(s(\tau), \tau) = 0\ \forall \tau$. ■

# C The Benchmark Calibration

To calibrate $\mu$ in (10),' we note that the fixed cost of investing in public schooling $E$ may be decomposed into direct and indirect costs associated with schooling. Direct costs includes tuition and non-tuition spending, such as other school fees, textbooks, supplementary study guides, uniforms, writing supplies, transportation etc. Indirect costs on the other hand, include the value of lost labor income, as well as the economic value of all the unpaid work related to schooling, that parents and community members may carry out.[43]

The direct costs for schooling may be very large. Bray (1999) report for instance that household expenditure on primary (public) education per child in Cambodia is up to 20 percent of the household income. In addition to these direct costs, there are other large costs associated with schooling that are met by community financing and government subsidies. Bray argues that the situation is similar for a number of other developing countries. Taking this as a benchmark of what the direct cost of schooling would be to the household in the absence of donations and subsidies, we set the direct cost of schooling

[43] For example the time parents spend on helping their kids with homework and transportation to school related activities. Parents and community members may also be asked to provide labor and/or materials for construction and maintenance of the school.

to be 25 percent of average income.

To this direct cost, the indirect cost of schooling, i.e., the economic value of foregone opportunities of schooling must be added. There do not really seem to exist any available studies quantifying the total indirect cost of schooling. Bray (1999) report the value of lost labor income of attaining primary education in Cambodia to be of almost the same magnitude as the direct cost. In a broad sense, we are considering both primary and secondary education and the value of labor generally gets larger as agents get older. Moreover, to the value of lost labor income, the value of all unpaid work related to schooling carried out by parents and community members should be added. Taking this into account and lacking other estimates, we set the indirect cost to be as large as the direct cost, generating a total fixed cost of investing in public schooling of 50 percent of average income, i.e., $\mu = 0.5$.

Now consider the parameter $\varphi$. Unfortunately, there are not many available studies on relative costs in private and public schools. One exception is Tsang (2002), who compares the costs of public and private schools in developing countries. He report that the direct private cost, i.e., the cost that households have to pay up front to be allowed to enrol the school, is between 1.83 and 8.02, times higher for private than for public schools. Lacking other estimates, we set $\varphi = 2.6$.

Finally, we need to specify the returns to schooling. We have assumed that the return to private schooling is higher than the return to public schooling, i.e., that $h(y_i) > g(y_i)$, $\forall y_i$. In our calibration, we set the returns to $E + \alpha y_i$ and $P + \beta y_i$ for public and private schooling respectively, with $\beta > \alpha > 1$. Note that the exact functional forms of these functions are not important for the political outcomes locally, since neither $y^*, y^{**}$ or $\widehat{y}$ depend upon these functions. However, they do enter the utility functions and will therefore play a role in the comparisons between alternatives where individuals choose different (or no) schooling between the respective policies being compared.

The empirical evidence supporting the assumption that agents from private schools would perform better than agents from public schools is somewhat weak, but the assumption at least seem to have some empirical support (Rouse, 1998 and Long, 2004).[44]

[44] Generally there is a substantial selection problem involved when trying to estimate the relative return to private and public schooling.

# D Numerical Computation of the Equilibrium

1. Set up a grid for policy $[\tau_1, ..., \tau_J]$.

2. At each grid point solve for $y_j^*$, the subsidy $s_j = \tau_j Y/N_j$ and the share of the population that participates $N_j = \int_{y_j^*}^{\infty} f(y)\, dy$; $j = 1, ..., J$

3. Approximate the functions $y^*(\tau), s(\tau)$ and $N(\tau)$. We use cubic splines

4. At each grid point, compute $\widehat{y}_j = ds(\tau_j)\, d\tau$, and $H_j = \int_{y_j^*}^{\widehat{y}_j} f(y)\, dy$; $j = 1, ..., J$

5. Approximate the functions $\widehat{y}(\tau)$ and $H(\tau)$. Again, we use cubic splines

6. Find all equilibria, i.e., find $H(\tau) = 0.5$ and cuts the 0.5 line from above. Also check corners.

7. Compare these candidates to a dense grid of all other tax rates and report the result.