The Dynamics of Climate Agreements

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Abstract

I study dynamic private provision of public goods (or bads) when agents (or countries) can invest in cost-reducing technologies and sign incomplete contracts. The model leads to a dynamic common pool problem that is more severe than its static counter-part. Nevertheless, a sequence of short-term agreements on contribution levels makes everyone *worse off* since countries invest less when they anticipate future negotiations. Long-term agreements induce countries to invest more. The best agreement is more demanding if the time horizon of the agreement is short and the externality from investing large (e.g., if the patent system is weak). If investments can be subsidized, the subsidy should be larger if the agreement is short-lasting. The *first best* can always be implemented by long-term agreements with renegotiations. The results have implications for the optimal design of climate treaties and they hold whether permits are tradable, non-tradable or if instead emission taxes are used.

Key words: Dynamic private provision of public goods, dynamic common pool problems, dynamic hold-up problems, time horizon of agreements, renegotiation design, climate change and climate agreements

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1. Introduction

This paper studies dynamic private provision of public goods when the agents can invest in cost-reducing technologies. The unique Markov-perfect equilibrium is compared to situations where the agents can contract on provision levels but not on investment levels, and the optimal contract is derived.

While the model fits many contexts with private provision of public goods, climate change is a particularly important application. Environmental agreements (e.g. the Kyoto protocol) are specifying pollution levels but not investments in technology. They typically have a limited time horizon, and future commitments remain to be negotiated. To fix ideas, I therefore call the agents "countries", and I focus on a public bad instead of a public good (a public bad can easily be reformulated to a public good). The public bad is the stock of greenhouse gases, and all countries suffer from a cost that is a convex function of the pollution stock level. At the same time, each country finds it costly to reduce its own emission level. This creates a common-pool problem that is dynamic since pollution cumulates over time. In addition, I let the countries invest in technology. A country's investment increases its stock of technology, which may be interpreted as abatement technology (alternatively, it can be interpreted as its renewable energy sources). There might also be an externality from a country's investment, since other countries may be able to simply copy some of the generated ideas.

In the business-as-usual equilibrium, countries act non-cooperatively at all stages. If one country happens to pollute a lot, the other countries are, in the future, induced to pollute less since the problem is then more severe. At the same time, they find it optimal to invest more in technology, to be able to afford the anticipated reduction in emission. If a country invests a lot in abatement technology, on the other hand, everyone understands that this country is polluting less in the future, and the other countries find it optimal to increase their emission levels as well as reduce their investments in abatement technology. Anticipating these effects, a country is induced to pollute more and invest less than it would in a static model (or in the open-loop equilibrium). Thus, the dynamic common pool problem is more severe than its static counterpart.

Nevertheless, short-term agreements make everyone *worse off*. The reason is that a hold up problem is created when the countries negotiate emission levels: If one country has a large stock of technology, it can reduce its emission level fairly cheaply, and the other countries will demand that it bears the lion's share of the burden when emissions are reduced. Anticipating this, countries invest less when negotiations are anticipated, and this makes everyone worse off, particularly if the time horizon of an agreement is short and the number of countries large.

The hold-up problem may be mitigated by long-term agreements, if commitments are determined before the countries invest in technology. Then, a country cannot be hold up if it invests a lot in technology - at least not as long as the agreement lasts. Thus, countries invest more when agreements are long-lasting. Nevertheless, countries are likely to invest too little, also under long-term agreements, if (i) the externality is positive and large and (ii) the agreement is not lasting forever (since countries then anticipate that investments harm their future bargaining position). To encourage countries to invest more, the best long-term agreement is more ambitious (i.e., the emission levels are lower) if the externality is positive and large and the time horizon of the agreement short.

But a long-term agreement is not optimal ex post, once the investments are sunk and the state of the world realized. It may thus be tempting for the countries to renegotiate the agreement at that stage. By renegotiating the initial agreement, emission levels are negotiated to the ex post optimal level. The role of the initial agreement is then only to affect the incentives to invest. And, the more ambitious is the initial agreement, the more the countries invest. When the initial agreement is very ambitious, countries with poor technology has a bad bargaining position since they are going to be "desperate" when renegotiating the initial, ambitious, agreement. It is then the high-tech countries that are going to get the better deal. Anticipating this, countries invest more in technology, particularly if the initial agreement is very ambitious. Since investments are particularly beneficial if the externality is large and the time horizon short (i.e., when countries otherwise under-invest), the agreement should be more ambitious in these circumstances.

In sum, the analysis generates several lessons for the design of contracts in a dynamic setting. Short-term agreements can actually be worse than no agreement at all, and long-term agreements should be more ambitious than what is optimal ex post, particularly if the externality is large and the time horizon of the agreement short. Carefully designed long-term agreements with renegotiation implement the first best emission levels as well as investments.

The paper is organized as follows. After presenting a linear-quadratic model in the next section, Section 3 solves the model under four scenarios: (i) business as usual (no negotiations), (ii) short-term agreements (negotiations that take place after investments are chosen), (iii) long-term agreements (negotiations take place before the investment stage), and (iv) long-term agreements with renegotiation. Section 4 shows that the main result continues to hold if (i) the countries can patent and trade technologies and R&D can be subsidized; (ii) whether side transfers are feasible or not in the negotiations, and whether non-tradable quotas are replaced by tradable permits or emission taxes in the negotiations; and (iii) if the utility function is general (and not necessarily linear-quadratic). Related literature is reviewed in Section 5, while the final section concludes.

2. The Linear-Quadratic Model

This section presents a model where n agents over time contribute to the public good and invest in technology. The purpose of the technology is to reduce the cost of providing public goods in the future. There may be technological spillovers, such that one agent may be able to learn and benefit from the other agents' investments. This section presents the model, while the next studies various contracting possibilities for the agents. I then assume that the agents can contract on the provision of public good, but not on how much each of them is supposed to invest. Private investments are observable but not verifiable, in line with the contracting literature.

Many types of public good provision can be captured by the model. To fix ideas, I will use climate change as the driving example. I will thus refer to the agents as "countries", the public good (or its negative counterpart; the public bad) as the stock of greenhouse gases, and the contributions as emissions.

The public bad is represented by the stock G, i.e., the stock of "greenhouse gases". G

can be interpreted as the stock of CO2 beyond what would be the natural level. Since the natural level is thus $G = 0$, there is a tendency of reverting to 0 for any given size of G, and I let d_G measure the fraction of G that "depreciates" every period. G may increase, nevertheless, if a country i selects a positive emission level, $g_i > 0$:

$$G = (1 - d_G)\, G_{-} + \sum_i g_i + \theta \tag{2.1}$$

G_{-} represents the stock of greenhouse gases in the previous period (this way, I do not need subscripts for periods). Parameter θ is random, somehow capturing the uncertainty related to global warming. The main impact of θ is to make the marginal cost of adding emission random. θ is arbitrary distributed with the mean 0 and variance σ^2. Although θ is distributed iid across periods, the impact of θ is long-lasting: in line with (2.1), its effect depreciates at the rate d_G.

The other type of stock in the model is technology. For each country i, R_i measures its technology stock. Technology depreciates over time at the rate d_R, but it may increase if country i invests. Let r_i measure the amount of resources (or private good) that country i invests or spends on R&D in the current period.

When one country invests, other countries may benefit as well. R&D is a creative process and the ideas that are generated can be used also in other countries, although the environment there may differ somewhat. I let $e > 0$ measure this externality, while b measures the impact of i's investments on i's own stock of technology. As long as the externality is not complete, $b > e$. In sum, the technology stocks follow a dynamic path given by:

$$R_i = (1 - d_R)\, R_{i,-} + br_i + e \sum_{j \neq i} r_j. \tag{2.2}$$

There are several interpretations of R_i that are consistent with the model. For example, R_i may measure country i's abatement technology, i.e., how much of its emission i can costlessly clean. If energy production, y_i, is generally polluting, the emission of country i is given by:

$$g_i = y_i - R_i.$$

Alternatively, R_i may measure the effectiveness of country i's windmill park (or renewable energy sources). If the windmill park can generate R_i units of energy, the total amount of energy produced is given by $y_i = g_i + R_i$, if the alternative to windmills is to use fossil fuel. Of course, y_i can measure the general industrial production instead of energy in particular.

In each period, a country (i) suffers from the stock of greenhouse gases, (ii) benefits from consuming energy y_i, and (iii) pays the cost of investing in technology. I assume utilities are quadratic in the first two terms, but linear in the investment costs. Formally, i's utility in a period is given by:

$$u_i = -\frac{c}{2}G^2 - \frac{v}{2}\left(\overline{y} - y_i\right)^2 - kr_i, \tag{2.3}$$

where $c > 0$ measures the cost of greenhouse gases, $\overline{y}$ is the bliss point for energy production, $v > 0$ represents the importance of energy and $k > 0$ is the unit cost when investing in technology.

Since there are many periods, country i ultimately cares about the present-discounted value of all future utilities. So, if δ represents the discount factor, i's objective is to maximize

$$U_i = \sum_{\tau=t}^{\infty} u_{i,\tau}\delta^{\tau-t} = u_i + V\left(G, R_1, R_2, ...R_n\right),$$

where $V\left(.\right)$ is a country's continuation value as measured at the end of each period. Again, subscripts denoting period t is skipped.

The timing of the model is the following: The investment stages and the pollution stages alternate over time. Somewhat arbitrary, I define "a period" to be such that the countries first (simultaneously) invest in technology, thereafter they (simultaneously) decide how much to pollute. In between these two stages, the parameter θ is realized. Information is symmetric at all stages.

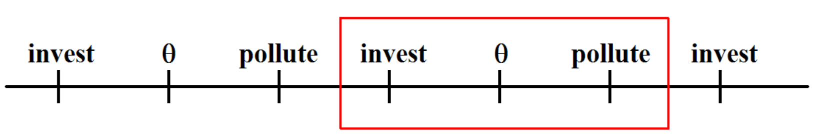

Figure 1: The definiton of "a period"

This model is used below to study the impact of environmental agreements and negotiations. Before polluting, the countries may get together to negotiate a climate agreement where they all commit to pollute less. I only allow the countries to negotiate this period's emission levels (or, stated differently, the length of the period represents the length of the agreement). All the countries have the same bargaining power and I study the outcome if each country gets $1/n$ of the bargaining surplus (this follows, for example, if using the Nash Bargaining Solution). The countries cannot negotiate the investments in technology, however, perhaps because these investments are harder to verify and monitor. To simplify further, side transfers are feasible at the negotiation stage, and negotiated emission quotas are not tradable across the countries. In this game, I am looking for a stationary Markov-Perfect Equilibrium (MPE) as defined by Maskin and Tirole (2001). Assuming that $V(.)$ is continuously differentiable, this equilibrium turns out to be unique for each of the situations studied below: Business as usual, short-term agreements, long-term agreements, and long-term agreements with renegotiations.

While the next section solves this simple model, Section 4 shows that the main results hold if (i) technologies can be patented and traded, (ii) side payments cannot be used, (iii) the utility function is more general (and not necessarily linear-quadratic). Section 4 also shows that the results are similar if the political instrument is tradable permits (instead of non-tradable quotas) or an emission tax.

3. Solutions

This section solves the game above under various scenarios for when the countries may negotiate. First, I assume negotiations never take place. The second subsection let negotiations take place after the investment stage; the third permits negotiations only before the investments; while the fourth subsection allows negotiations before as well as after investments are made. For each scenario, the countries negotiate the current period's emission levels only. This is actually not a severe constraint, since the length of a period is not specified and it can be arbitrarily long (by letting $\delta \to 0$).

3.1. Business as Usual

Suppose there is never any coordination or negotiations between the countries. At every stage, the countries make their decisions non-cooperatively. In this section, I solve each period by backwards induction, taking the continuation value function $V(.)$ as given.

The pollution stage should be solved for first. Since the technologies are given, at this stage, choosing g_i is equivalent to choosing y_i. Country i's first-order condition becomes:

$$\begin{aligned} 0 &= -cG + v\left(\overline{y} - y_i\right) - V_G \Rightarrow \\ y_i &= \overline{y} - \frac{cG + V_G}{v}, \end{aligned}$$

where $V_G \equiv \partial V / \partial G$. Intuitively, i pollutes less if c and G are large, since the problem is then more severe.

By (2.1), G is itself a function of the y_is, and solving for these gives:

$$\begin{aligned} G^{bau} &= \frac{nv\overline{y} - nV_G + v\left(\left(1 - d_G\right)G_- + \theta - R\right)}{nc + v} \text{ and} \\ y_i^{bau} &= \frac{v\overline{y} - V_G - c\left(\left(1 - d_G\right)G_- + \theta - R\right)}{nc + v}, \text{ where} \\ R &= \sum R_j. \end{aligned}$$

Consistent with my remark above (where I took G as given), y_i is now smaller if G_- is large, since that makes the problem more severe. Moreover, y_i is large if R_j is small, no matter j. The reason is that if technology stocks are large, pollution is going to be less for a given set of y_j, and country i can enjoy some more energy without suffering terribly from the greenhouse gases. Since $g_i = y_i - R_i$, we can write

$$g_i^{bau} = \frac{v\overline{y} - V_G - c\left(\left(1 - d_G\right)G_- + \theta - \sum_{j \neq i} R_j\right)}{nc + v} - \left(1 - \frac{c}{nc + v}\right)R_i. \tag{3.1}$$

Thus, i pollutes more if its own technology is good, since it can then consume a lot of energy without having to pollute. Symmetrically, j pollutes less if R_j is large, and this allows i, $i \neq j$, to increase its own emission level, since the problem is then less severe. This is why g_i increases in R_j, $j \neq i$.

At the investment stage, i takes all this into account. It understands that if it invests and makes R_i large, it can pollute less, and G is going to be less, as well. However, the other countries are going to find it optimal to increase their emissions, so parts of the gain is crowded out. As shown in the Appendix, the equilibrium R&D level is given by:

$$r_i^{bau} = \frac{(1-d_G)\,G_- - (1-d_R)\,R_-}{nB} + \frac{\overline{y}}{B} - \frac{V_G}{vB} - \frac{(k-V_R)\,(v+nc)^2}{cvnB\,(v+c)} + \frac{V_G\,(nc+v)}{cvnB} \tag{3.2}$$

where $B \equiv \partial R/\partial r_i = b + (n-1)\,e$.

Having solved for the investment levels, we can calculate u_i and recursively derive $V(.)$. As shown in the appendix,

$$\begin{aligned} \partial V/\partial G &= -\frac{\delta d_R k}{Bn} \\ \partial V/\partial R_j &= \frac{\delta\,(1-d_R)\,k}{Bn} \forall j \in \{1, ...n\}, \end{aligned} \tag{3.3}$$

and $V(.)$ is thus uniquely defined, assuming it is continuously differentiable. That $\partial V/\partial R_i = \partial V/\partial R_j \forall i,j$ shows that the stock of technology, R, is like a public good benefitting everyone, no matter who actually owns it. The reason is that the countries' energy production is going to be the same for all countries, not matter whether they have different technologies, and the impact of the technologies is thus only to reduce the emission levels, to the benefit of everyone. For this reason, equilibrium investment levels depend only on the total value it generates, B, and not on the private benefit b in particular.

Proposition 1: *(i) If R_i is large, country i pollutes less while country j, $j \neq i$, pollutes more. (ii) In equilibrium, i is polluting according to (3.1) and investing according to (3.2).*

This is a "dynamic common pool problem" where each country's contribution (or investment) is strategically distorted in order to affect the other countries' emissions and investments. Compared to the static common pool problem (or the open loop equilibrium where each country commits to all future of g_i and r_i), countries pollute too much, invest too little, and receives a lower utility.

3.2. Short-term Agreements

This subsection analyzes "short-term agreements". The label could alternatively be "spot contracts", since I am assuming that the countries, just before polluting non-cooperatively, get together and negotiate an emission-vector that is better for everyone. Since the technologies are fixed, at this point in time, negotiating g_i is equivalent to negotiating y_i. Notice that the countries have identical preferences when it comes to y_i. When technologies are sunk (even if technology stocks may differ across the countries), the bargaining game is perfectly symmetric when considering the y_is. Thus, the bargaining solution is simply that all y_is are set at the socially optimal level:

$$\begin{aligned} 0 &= -cnG + v\left(\overline{y} - y_i\right) - nV_G \Rightarrow \\ y_i &= \overline{y} - \frac{cnG + nV_G}{v}. \end{aligned}$$

Since G is, by (2.1), a function of the y_is, we can write:

$$\begin{aligned} G^{st} &= \frac{nv\overline{y} - n^2 V_G + v\left(\left(1-d_G\right)G_- + \theta - R\right)}{n^2 c + v}, \\ y_i^{st} &= \frac{v\overline{y} - nV_G - nc\left(\left(1-d_G\right)G_- + \theta - R\right)}{n^2 c + v}, \\ g_i^*\left(\mathbf{R}\right) &= \frac{v\overline{y} - nV_G - nc\left(\left(1-d_G\right)G_- + \theta - \sum_{j\neq i} R_j\right)}{n^2 c + v} - \left(1 - \frac{nc}{n^2 c + v}\right) R_i, \quad (3.4) \end{aligned}$$

where $\mathbf{R}= (R_1, ...R_n)$ and $g_i^*(\mathbf{R})$ is the optimal (as well as the equilibrium) pollution level given the vector of technologies. Clearly, i pollutes less if R_i is large, since i then can enjoy a large y_i without having to pollute, and its marginal benefit of polluting is then smaller. The other countries take advantage of this fact, and require that i reduces its pollution. This means that if $\sum_{j\neq i} R_j$ is large, the other countries are polluting less, the marginal cost of polluting one more unit is small, and i is able to negotiate a larger pollution permit than if $\sum_{j\neq i} R_j$ were small. Thus, while i pollutes less when R_i is large, in the bargaining equilibrium, $j \neq i$ pollutes more. This is exactly as in the business-as-usual scenario, although for different reasons (since this is a bargaining outcome).

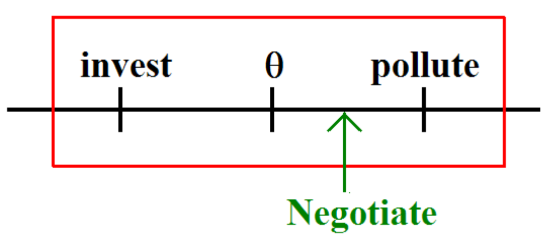

Figure 2: The timing for "short-term agreements"

Of course, i anticipates all this at the investments stage. It understands that if it invests a lot, it will be hold up by the other countries and forced to pollute less. They know that i is, in the end, going to accept such a request, since it does not find it too costly to reduce its pollution level. Anticipating this, investments are reduced. Appendix derives the equilibrium R&D level:

$$r^{st} = \frac{(1-d_G)\,G_- - (1-d_R)\,R_-}{nB} - \frac{(k-V_R)\,(n^2c+v)}{cvnB} + \frac{\overline{y}}{B} + \frac{V_G}{ncB}. \tag{3.5}$$

Clearly, i invests more if G_- is large, because the problem is then large, and if R_- is small, since the existing technology is then poor.

Again, we can derive the utilities and the continuation value function. This turns out to be given by (3.3), also in this case. This does *not* mean that the continuation values are identical in the two situations, only that the marginal benefit w.r.t. G and R are the same whether the countries negotiate short-term agreements or if they do not.

Proposition 2: *(i) If R_i is large, country i pollutes less while country j, $j \neq i$, pollutes more. (ii) In equilibrium, i is polluting according to (3.4) and investing according to (3.5).*

Note that all the comparative static is similar to the business-as-usual situation: For example, a country is polluting less if it has invested a lot (part (i) of Proposition 1 and 2 are identical), and it invests more if G_- is large and R_- small. However, the levels of emission and R&D are different in the two cases. Since (3.3) holds in both cases, it is easy to compare the two cases.

3.2.1. Are short-term agreements worse than no agreement?

Since V_G and V_R are the same whether short-term agreements are negotiated or not, the equilibrium in this period is unrelated to whether there will be an agreement in the next period. Thus, it is enough to compare the utilities in one particular period to conclude whether short-term agreements are better than business as usual in that (or any) period: We do not need to make statements *conditional* on whether there will be agreements also in the future. The comparison itself is undertaken in the Appendix.

Proposition 3: *Relative to the business as usual, a short-term agreement (i) reduces pollution, (ii) reduces R&D levels, and (iii) is beneficial if and only if (3.6) holds.*

$$\begin{aligned} g^{st} &< g^{bau} \\ r^{st} &< r^{bau} \\ \left(1-\frac{1}{n}\right)^2-\left(\frac{1-\delta\left(1-d_R\right)}{n}\right)^2 &< \frac{(v+c)\left(\sigma vcB/k\right)^2}{\left(n^2c+v\right)\left(nc+v\right)^2} \end{aligned} \tag{3.6}$$

.Part (i) is obvious: The entire point of the agreement is to reduce emission. Part (ii), on the other hand, is disappointing. Instead of encouraging the countries to invest, they are actually investing *less* when an agreement is expected. The reason is the following: The hold-up problem, when negotiations are anticipated, is exactly as strong as the crowding-out problem in the business-as-usual scenario: In either case, each country only enjoys $1/n$ of the total benefits generated by its investments. In addition, when an agreement is expected country i understands that the problem will be "taken care off", to some extent, since emission levels are going to be reduced. This implies that the marginal benefit of further reductions decline, and it is marginally less important to invest in technologies that would reduce future pollution. Hence, each country invests less.

Since investments decrease under short-term agreements, it may not be a surprise that utilities can decrease as well. This is the case, in particular, if each period is quite short. Then, δ is likely to be large, d_R is likely to be small, and so is the uncertainty from one period to the next (i.e., $\sigma \approx 0$). All changes make (3.6) less likely to hold. Moreover, (3.6) is less likely to hold if n is large, since then the under-investment problem is large, it is very important to increase investments, and this can be done by having the business-as-usual situation instead of a short-term agreement.

Thus, unless (3.6) holds, the countries would be better off if they could commit not to negotiate short-term agreements. One way of committing may be to commit to emission levels in advance, without allowing renegotiation after the investment stage. This is the scenario I turn to next.

3.3. Long-term Agreements

The hold-up problem in the previous subsection appeared because the g_is were negotiated after investments were sunk. Suppose, instead, that the countries negotiate the g_is before the investment stage. This can be done, for example, by having a long-term agreement, where the current period's g_is are fixed already when countries invest in technologies. For real-world long-term agreements, it is indeed reasonable that future commitments are made for such a long time horizon that a country is able to invest between the time at which the promises were made, and the time at which the last promise is supposed to be kept.

Analyzing multi-period agreements turn out to be quite difficult in the present model, unfortunately. However, a sense of "long-term agreements" is still possible to study, even within each of the model's periods, if we simply assume that the countries negotiate this period's g_is in the beginning of the period, before investments are made. While these agreements only last one period, they are indeed "longer" than the short-term agreements studied above. Moreover, each period can be quite long in the model, since we have not specified whether the discount factor, for example, is large or small.

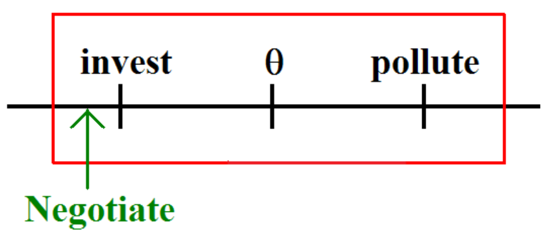

Figure 3: The timing for "long-term agreements"

In the model, the timing is now reversed: The countries first negotiate the g_i^{lt}s, then investments are chosen. When solving the model by backwards induction, we thus have to start with the investment stage, taking this period's g_i^{lt}s as given. This gives the first-order condition for country i's r_i:

$$\begin{aligned} 0 &= vb\left(\overline{y} - g_i^{lt} - R_i\right) - k + V_R \Rightarrow \\ R_i &= \overline{y} - g_i^{lt} - \left(k - V_R\right)/vb. \end{aligned} \tag{3.7}$$

Obviously, a country wants a larger stock of technology if its quota, g_i^{lt}, is small, since otherwise it would find it very costly to comply.

Together with (2.2), (3.7) gives the countries' investment levels as a function of the negotiated pollution levels. Taking these into account, it is straightforward to calculate the bargaining outcome w.r.t. the g_i^{lt}s. This is done in the Appendix, but the result is presented here:

Proposition 4: *The bargaining outcome is given by (3.8). Thus, the quotas are smaller if the externality is large and if the time horizon of the agreement is short.*

$$g_i^{lt} = \mathrm{E}g_i^* \left(\mathbf{R}^{lt}\right) - \frac{k\,(n-1)}{B\,(n^2c+v)} \left(\frac{e}{b} + \frac{\delta\,(1-d_R)}{n} \left(1-\frac{e}{b}\right)\right), \tag{3.8}$$

where $\mathrm{E}g_i^*\,(.)$, defined by (3.4), is the expected optimal pollution level ex post (after r_i and θ are realized) given the equilibrium technology vector. Thus, $g_i^{lt} < \mathrm{E}g^* \left(r^{lt}\right)$ unless $e = \delta\,(1-d_R) = 0$.

If $e = \delta\,(1-d_R) = 0$, then $g_i^{lt} = \mathrm{E}g^* \left(\mathbf{R}^{lt}\right)$, meaning that the commitments under the long-term agreement should be equal to the expected optimal pollution levels. Since, in this case, there are no externalities, and the countries are not concerned with how their technologies affect their future bargaining power, investments are first best.

However, if $e > 0$, there are externalities related to the investments, and a country is likely to under-invest. To encourage countries to invest closer to the optimal level, the optimal long-term agreement specifies commitments that are tougher than what is likely going to be optimal ex post. With tough commitments, each country is induced to invest more, and that is good when the externalities are positive. The larger are these externalities, the more ambitious the agreement should be (in that the g_i^{lt}s should be smaller).

If $\delta\,(1-d_R)$ is large, countries are likely to under-invest even if $e = 0$. The reason is that the countries do not fully take into account the total value of increasing R for the future. If, in the next period, R_i is large, then country i is again going to get a smaller pollution quota, and the other countries are going to pollute more. A country is more concerned with this future hold-up problem if δ is large, i.e., the future is close, and if d_R is small, since then today's investments have a large impact on tomorrow's technology stock. In words, the agreement should be more ambitious if (i) there are large technological spillovers and (ii) if the time horizon of the agreement is short.

3.4. Long-term Agreements with Renegotiation

Long-term agreements, such they are defined above, is never first-best. First, the commitments are made before one knows the severity of the problem (determined by θ). Second, the optimal long-term agreement (3.8) specifies pollution levels that are less than what is likely going to be optimal ex post, since the emission levels are trading off ex post optimality with ex ante incentives. The countries may thus be tempted to renegotiate the treaty, after θ and the investments are realized. What happens if such renegotiation is possible and allowed? Does it ruin the long-term agreement's intention of encouraging the countries to invest more?

To solve for this case, we must specify the default outcome: What happens if the renegotiation game breaks down? If, then, the outcome is the non-cooperative outcome, the bargaining outcome is going to be exactly the "short-term agreements" studied above, and so are the incentives to invest. Suppose, instead, the default outcome is the initial, long-term agreement.

In each period, the following events unfold. First, the countries negotiate the initial commitments g_i^{de} (called the "default outcome"). Thereafter, the countries invest and θ is realized. Before carrying out their commitments, however, the countries get together and (re)negotiate another set of emission levels, g_i^{re}, where everyone understands that if this renegotiation game fails, the outcome is the default outcome agreed to initially.

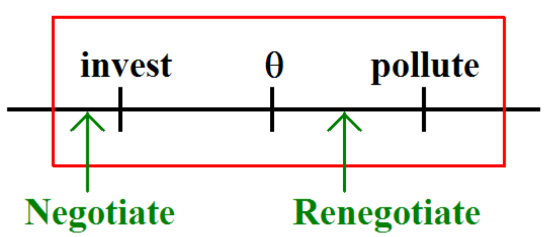

Figure 4: The timing when renegotiation is possible

When renegotiating the emissions, the countries find it optimal to pollute just as they would under short-term agreements (taking technology stocks as given), so $g_i^{re}(\mathbf{R}) = g_i^*(\mathbf{R})$. Anticipating that they will be able to renegotiate the agreement, aren't the countries induced to invest just as little as they did under short-term agreements?

The answer is no. When investing, the countries do, indeed, recognize that, after renegotiation, they will be able to pollute as they would do under a short-term agreement. But in the short-term-scenario, it was the countries with the poorest technology that got the better deal, since these countries were quite happy with the default outcome (i.e., the non-cooperative outcome) where they were able to pollute unconstrained. This makes the "technology-losers" reluctant to negotiate an agreement, giving them a better bargaining position. However, things are quite different when renegotiating a very ambitious agreement. Then, the technology-losers are desperate to reach a new agreement, replacing the very expensive commitments. These countries thus have a poor bargaining position, and they are, in equilibrium, going to get a quite bad deal (where they must pay the other countries to sign the renegotiated treaty). Fearing this situation, the countries are induced to invest more, particularly if the default pollution levels, the g_i^{de}s, are small.

Technically, each country can expect to enjoy the utility it would under the default agreement, plus $1/n$ of the benefit from renegotiation. When it chooses investments, it will thus seek to maximize

$$U_i^{de} + \frac{1}{n}\left(\sum_j U_j^{re} - \sum_j U_j^{de}\right).$$

All this will be taken into account when specifying the initial agreement, the g_i^{de}s. The more ambitious is this agreement, the more the countries invest. This is attractive, particularly in situations where the countries otherwise are tempted to under-invest, i.e., if the externality e is large, and if $\delta(1 - d_R)$ is large, since then the countries fear that more technology today hurts their bargaining position in the near future. Thus, the agreement should be more ambitious if e and $\delta(1 - d_R)$ are large. By setting the g_i^{de}s carefully, the first-best technology levels, $\mathbf{R}^*$, are induced. The final pollution levels are not going to be distorted, since the renegotiation stage ensures that, ex post, emission levels are set optimally. Thus, such an agreement implements the first best investment levels as well as the first best pollution level. Appendix derives the optimal contract.

Proposition 5: *With renegotiation, the initial agreement (3.9) implements first best investments as well as emissions. Thus, the initial agreement should be more ambitious if its time horizon is short and the R&D spillovers large.*

$$g_i^{de} = \mathrm{E}g_i^*(\mathbf{R}^*) - \frac{k}{Bv}\left[\delta(1-d_R) + \frac{en}{b-e}\right] \tag{3.9}$$

4. Robustness

The analysis above has relied on a number of strong assumptions. This section discusses how some of them can be relaxed. The proofs of the following propositions are similar to the analogical proofs above, and thus omitted (but they are available upon request).

4.1. Patents and R&D Subsidizes

Above, I simply assumed that a fraction of the innovation in one country, e/b, was free to copy for another country. Thus, there was no trade in technological products, and no transfers when one country copied another's idea.

To discuss intellectual property rights, suppose that a country can purchase or pay for the remaining fraction, $(b-e)/b$. Then, $e<b$ measures the weakness of the patent system, or the fraction of an innovation that cannot be protected. Let the timing, at the investment stage, be the following: First, each country chooses r_i. While innovator i's technology stock immediately increases by br_i, the technology stock of another country increases by er_i, following i's investment. However, a country j, $j \neq i$, can pay i to license it the patent, and thus be able to increase R_j by the full amount, br_j. At this stage, all the investments are sunk and there will be a market price for such licenses that clear the market. Clearly, the price is lower if e is large, and if the r_is are large.

This model can be solved just the same way as that above. The value of owning technology determines the price of licensing, and the price, in turn, determines the incentives to invest in R&D. Just as before, it may be reasonable that the countries' investment levels are unverifiable. But the trade in technology is certainly verifiable, and that can make it possible to e.g. subsidize internationally trade in technologies. Let s measure the subsidy, as a fraction of the numerical value of the transaction, paid by other countries when j pays to learn i's innovation. It does not matter whether this subsidy goes to the buyer or the seller (the price will adjust, of course, but not the transaction or the investment levels). Let s be given. Qualitatively, the model gives the same results as those above. In particular, the first best can be achieved by renegotiation if the initial, long-term agreement, is given by:

$$\begin{aligned} g_i^{de} &= \mathrm{E}g_i^*(\mathbf{R}^*) - \frac{k}{bnv}\left[\delta(1-d_R) + \frac{n(1-z)}{z(n-1)}\right], \text{ where} \\ z &\equiv (1+s)(1-e/(b-e)). \end{aligned} \tag{4.1}$$

Proposition 6: *If $e < b$ measures the weakness of the patent system and s the international subsidy when patents are licensed, the first best can be achieved by the initial agreement (4.1) and renegotiation.*

Just as before, the agreement should be more ambitious if the "externality is large", which in this case means that the patent system is weak. Then, countries tend to underinvest, and the commitments should be tougher to encourage more R&D. Moreover, the agreement should be more ambitious if the subsidy level is small, for the same reason.

Equation (4.1) can also tell us about the optimal subsidy level, s, for a given level of e and g_i^{de}. Obviously, the subsidy level should be larger if e is large, to compensate for the spillover that is created. In addition, s should be larger if δ is large, i.e. if the agreement is quite "short-term", since then investments tend to be too low, unless such subsidizes are in place. Finally, s should be larger if the g_i^{de}s are small, since that, too, lead to little investments.

4.2. Transfers, Taxes and Tradable Permits

In Section 2, I started out by assuming that the quotas cannot be traded, and that side transfers can be used when the countries (re)negotiate the agreement. Both assumptions can be relaxed, without changing any of the results. The reason for making the assumptions, in the first place, was only to simplify the proofs and the discussions.

Proposition 7: *If side transfers were not available in the (re)negotiations, all the results above continue to hold.*

A shallow intuition for this result is that, since the model is symmetric, no side transfers are going to take place in equilibrium, anyway. However, when considering whether to deviate, it is important to know that a deviation would be harmful when the effects on the side transfers are taken into account. What, then, if transfers are not possible? If they are not, a country can always pay in "relative contributions" what it cannot pay in direct transfers: Instead of a payment from i to j, i can simply reduce its emission level relative to j. Such a transfer "in kind" plays just the same role as a monetary transfer at the margin in the symmetric equilibrium (then, the deadweight costs of a marginal increase in "transfer in kind" is zero).

For related reasons, the results above continue to hold if the quotas were tradable. The shallow intuition is that there will be no trade in permits anyway, in equilibrium, and, thus, it does not matter whether such trade is allowed. However, when considering whether to deviate and invest more or less in technology, then the value of investing more, when quotas are not tradable, is given by the country's own value of increasing y_i. When permits are tradable, the value of more technology is given by the equilibrium permit price. In the symmetric equilibrium, however, these two are equal and the equilibrium investment level is the same, as well. Because of this, all the results are exactly the same if permits are tradable.

Proposition 8: *If the pollution permits were tradable, all the results above continue to hold.*

Tradable permits and non-tradable quotas are just two possible political instruments. Another popular instrument is Pigou taxes. Suppose that t_i is the tax that country i must pay, for each unit it pollutes. Suppose all the tax revenues are redistributed equally on all the countries (if country i's payment were given back to i, the tax would of course have no effect). Instead of negotiating the emission levels, let now countries negotiate the taxes. The analysis would be different, but similar, to that above. A country with good technology can expect to pay little in taxes, and the negotiated t_i is then going to be larger for this country. Anticipating this, countries are discouraged to invest, just as before. The first-best can be achieved, however, if renegotiation is possible when the initial, long-term, agreement is given by:

$$t_i^{de} = \mathrm{E}t_i^* + \frac{k}{bn}\left(\delta\left(1-d_R\right)+\frac{n(1-z)}{z\left(N-1\right)}\right), \tag{4.2}$$

where $\mathrm{E}t_i^*$ is the expected optimal emission tax ex post.

Proposition 9: *Suppose the countries negotiated Pigou taxes (that are redistributed evenly) and not emission levels. Abatement and investment levels are all first best if renegotiation is possible and the initial agreement is given by (4.2).*

Interestingly, the comparative static is just as before: The initial agreement should be more ambitious (in that the tax should be lower) if the externality is large (or patent protection weak), the subsidy small and the agreement is short-lasting (in that δ is large). After these high taxes have induced countries to invest a lot, fearing that they otherwise would have a bad bargaining position, the countries renegotiate to a set of taxes that are smaller, and optimal.

4.3. A More General Utility Function

By assuming quadratic utility functions and a linear cost of investing in R&D, all the above results could be analytically derived. The main result of this paper, however, does not rely on these functional forms. Suppose that a country's one-period utility is measured by:

$$u_i = c\left(G\right) + v\left(y_i\right) - k\left(r_i\right), \tag{4.3}$$

where $v\left(.\right)$ is increasing and concave while $c\left(.\right)$ and $k\left(.\right)$ are increasing and convex. If renegotiation is possible, the first best is implemented if the initial agreement specifies the g_i^{de}s such that:

$$v'\left(g_i^{de}+R_i^*\right) = \mathrm{E}v'\left(g_i^*+R_i^*\right) + \frac{k'\left(r_i^*\right)}{bn}\left(\delta\left(1-d_R\right)+\frac{n(1-z)}{z\left(n-1\right)}\right) \tag{4.4}$$

Proposition 10: *If utility is given by (4.3) rather than (2.3), both emission and investment levels are first best if renegotiation is possible and the initial agreement is given by (4.4).*

The comparative static is exactly the same as before. If δ is large and z small, the right-hand side of (4.4) is large and the first best is thus achieved if the left-hand side is large as well, which implies that g_i^{de} must be small relative to g_i^*. Naturally, (4.1) is a special case of (4.4), and it is straightforward to rewrite (4.1) to take the form of (4.4). Proposition 10 shows that the insight of the linear-quadratic case holds also for general utility functions, at least when it comes to the optimal agreement under renegotiation. The proof follows the same lines as for Proposition 5, and is thus omitted.

5. A (preliminary) Literature Review

[This review is preliminary and suggestions are welcome.]

The paper is related to several strands of literature. For the situation without an agreement, the model is one of *dynamic private provision of public goods*. Related papers typically conclude, as I do, that the public good is under-provided, whether it is related to abatements (Dutta and Radner, 2005), saving fish (Levhari and Mirman, 1980), reducing the deficit (Velasco, 2000), or contributing to discrete public projects (Marx and Matthews, 2000). Fehrstman and Nitzan (1991) present a linear-quadratic model where there is under-provision of public goods if agents use linear strategies, but multiple equilibria exist (and some of them are almost efficient) if strategies can be discontinuous (Wirl, 1996, Dockner and Sorger, 1996, Sorger, 1998). Partly to avoid such multiplicity, this paper focuses on continuous strategies, and the equilibrium is then unique. My contribution to this literature is to add contracts and the possibility to invest in technologies.

Since I allow for investments, the paper borrows from the literature on capital accumulation in firms. Firms tend to over-accumulate capital in order to discourage the competitors from investing or producing (Reynolds, 1987, Maskin and Tirole, 1987, Saloner, 1987). My model can, in fact, be reformulated to capture such competition between firms. Unlike most of this literature (exceptions are Fershtman and Pakes, 2000, Gatsios and Karp, 1992), I allow for the possibility to contract.

However, since I do *not* allow the countries to contract on investment levels, the model is part of the literature on incomplete contracts, following Hart and Moore (1988). Since the model is dynamic, the time horizon is naturally important. Harris and Holmstrom (1987) show how the optimal time horizon solves the trade-off between mitigating the hold-up problem (by having a longer-term contract) and allowing flexibility (by a short-term contract). When renegotiation is possible, Guriev and Kvasov (2005) find that an initial long-term agreement between a buyer and a seller is necessary to increase relation-specific investments but the time horizon should not be too long, in their paper, since that would lead to over-investments. While Guriev and Kvasov (2005) arrive at the first best by specifying the time horizon, Section 3.4 in the present paper specifies the pollution levels in the default agreement. This is more similar to Edlin and Reichelstein (1996), who study the default quantity that should be traded to induce optimal investments. Also they find that the agreement should be more "ambitious" (in that the traded quantity should be larger) if the externality is large, but Che and Hausch (1999) find that if the externality is very large, no contracts can help to solve the under-investment problem (Segal and Whinston, 2002, generalize these models). My finding is similar, although the setting is very different (since my model has an arbitrary number of agents and an infinite number of periods). In addition, the present paper proves that the agreement should be more ambitious if it is short-lasting. The idea of using the initial contract as an incentive device when renegotiation is allowed goes back to Chung (1991) and Aghion, Dewatripont and Rey (1994).

The paper is certainly relevant for the design of climate treaties (for some alternatives, see Aldy and Stavins, 2007). There is a large literature on environmental agreements (Barrett, 2005, and Kolstad and Toman, 2005, provide some overview), but most of the papers study models that are static or with two periods. My contribution to this literature is to show (i) how countries may under-invest when negotiations are anticipated, and (ii) the implications for the optimal environmental agreements. Thus, my conclusion that short-term agreements are bad is at odds with most of the literature: Karp and Zhao (2008), for example, propose agreements of 10-year length to ensure flexibility (without discussing the hold-up problem). Moreover, I am not aware of any paper in this literature discussing renegotiation-design. On the other hand, many papers focus on participation and coalition sizes, and how they depend on e.g. uncertainty (Burger and Kolstad, 2008) and the type of the agreement (Barrett and Stavins, 2003), or how incomplete participation affects R&D levels (Golombek and Hoel, 2004). Participation plays no role in the present paper, however, since I assume that all countries negotiate the agreement. Allowing countries to opt out, before the negotiations start, is thus a natural next step for the present model.

6. Conclusions

This paper presents a dynamic model where a number of agents contribute to a public bad as well as invest in technology. The investments affect the future costs of contributing, and there may be externalities from the investments. The model fits many types of private provision to public goods; climate change is a leading example. The larger is a country's stock of abatement technology, the more the other countries choose to pollute. Moreover, the more one country pollutes, the less the other countries pollute, and the more they invest in R&D. Both effects induce countries to pollute more and invest less than they would have done in a one-shot model (or in the open loop equilibrium). In this setting, I study agreements or contracts between the countries. I assume that the countries can negotiate and commit to future pollution levels, while they cannot contract on investment levels.

First, I find that a sequence of short-term agreements is *worse* than business as usual. At the negotiation stage, a country with good technology is going to be hold up by the other countries, demanding that the country reduces its pollution by a lot (since it can afford doing so). Anticipating this, countries invest less when negotiations are anticipated, and the countries may thus be better off if no such agreements were taking place.

Second, "long-term agreements" should be more ambitious (and specify lower emission levels) if its time horizon is relatively short and the externality from investing positive and large. The hold-up problem does not arise before the agreement expires, so countries tend to under-invest if the time horizon of the agreement is short. To encourage investments, the agreement should be more ambitious than what is optimal ex post, particularly if the time horizon is short and the externality (from investing in R&D) large, since then countries are otherwise under-investing. Therefore, if R&D can be subsidized, the subsidy should be larger if the time horizon is short.

Third, the first best can be implemented for all investment and emission levels if renegotiation is possible. Renegotiation ensures that the emission levels are ex post optimal, and the role of the initial agreement is only to affect incentives. Again, the agreement should be more ambitious if its time horizon is short and the externality large. The results hold no matter whether side transfers are feasible in the negotiations, whether permits are tradable or not, or whether an emission tax is negotiated instead of permits.

The results have important implications for any future climate agreement. If the time horizon of an agreement is short, it should be more demanding and R&D should be subsidized by more. A long-term agreements induces more R&D and is better, particularly when patents are imperfectly enforced. Flexibility can be ensured if the agreement is renegotiated.

While this paper establishes some benchmark results, it is only a small step towards a better understanding of good environmental agreements. I have assumed that countries are homogenous, able to commit to future pollution levels, there is no private information and everyone participate in the negotiations. Relaxing these assumptions is certainly the next thing to do!

7. Appendix - Solving the Model

7.1. Business as usual

To simplify, I have used the symbols $m = V_G$, $m' = V_{R_i}$, $R \equiv \sum_j R_j$, $R' = R_- (1 - d_R)$ and $G' = G_- (1 - d_G)$. Moreover, I have written $V(.)$ as a function of G and $R \equiv \sum_j R_j$ only, and not the individual R_is, anticipating that $V(.)$ is, indeed, going to be a function of R_i only to the extent it is reflected by R. At the pollution stage, each country's first-order conditions is (when choosing y_i):

$$
\begin{aligned}
0 &= -cG + v\overline{y} - vy_i - m \Rightarrow y_i = \overline{y} - \frac{m + cG}{v} \\
G &= \sum_j (y_j - R_j) + G' + \theta = G' + \theta + n\left(\overline{y} - \frac{m + cG}{v}\right) - \sum R_j \Rightarrow \\
G &= \frac{nv\overline{y} - nm + v(G' + \theta - R)}{nc + v} \Rightarrow \qquad (7.1) \\
y_i &= \overline{y} - \frac{m}{v} - \frac{c}{v}\left(\frac{nv\overline{y} - nm + v(G' + \theta - R)}{nc + v}\right) = \frac{v\overline{y} - m - c(G' + \theta - R)}{nc + v} \Rightarrow \\
g_i &= y_i - R_i = \frac{v\overline{y} - m - c\left(G' + \theta - \sum_{j \neq i} R_j\right)}{nc + v} - \frac{R_i(nc + v - c)}{nc + v}.
\end{aligned}
$$

Interrim utility (after investments are sunk) can be written as:

$$
\begin{aligned}
w_i^{bau} &\equiv -v(\overline{y} - y_i)^2/2 - cG^2/2 + V(G, R) \\
&= -c(1 + c/v)G^2/2 - Gmc/v + \frac{(v\overline{y})^2 - m^2}{2v} + V(G, R). \text{ Thus,} \\
\partial w_i^{bau}/\partial R_j &= c(1 + c/v)G\left(\frac{v}{nc + v}\right) + \frac{vm(1 + c/v)}{nc + v} + m' \qquad (7.2)
\end{aligned}
$$

At the investment stage, each country sets $k/B =$E$\partial w_i^{bau}/\partial R$, recognizing that one unit of investment increases R by $B \equiv b + (n - 1)e$ units. From (7.2):

$$
\begin{aligned}
k/B &= c\text{E}G\left(\frac{c + v}{nc + v}\right) + \frac{m(v + c)}{nc + v} + m', \text{ while from (7.1):} \\
\text{E}G &= \frac{nv\overline{y} - nm + v(G' - R)}{nc + v}, \text{ implying} \\
R &= G' - \frac{k(v + nc)^2}{cvB(v + c)} + \overline{y}n + \frac{m'(v + nc)^2}{cv(v + c)} + \frac{m}{c}, \text{ or} \\
r_i nB &= -R' + G' - \frac{k(v + nc)^2}{cvB(v + c)} + \overline{y}n + \frac{m'(v + nc)^2}{cv(v + c)} + \frac{m}{c}.
\end{aligned}
$$

We can now write:

$$\begin{aligned}
y &= \overline{y} - \frac{(k/B - m')(v + nc)}{v(v + c)} - \frac{\theta vc}{v(nc + v)} \\
\mathrm{E}G &= \frac{k(nc + v)}{cB(c + v)} - \frac{m}{c} - \frac{m'(nc + v)}{c(c + v)} \\
G &= \mathrm{E}G + \frac{\theta v}{nc + v}. \\
\mathrm{E}G^2 &= (\mathrm{E}G)^2 + \left(\frac{\sigma v}{nc + v}\right)^2,
\end{aligned}$$

which is helpful when calculating utility, which becomes:

$$\begin{aligned}
u_i &= -\frac{c}{2}\left(\frac{k(nc + v)}{cB(c + v)} - \frac{m}{c} - \frac{m'(nc + v)}{c(c + v)} + \frac{\theta v}{nc + v}\right)^2 - \frac{v}{2}\left(\frac{(k/B - m')(v + nc)}{v(v + c)} + \frac{\theta vc}{v(nc + v)}\right)^2 \\
&\quad - \frac{k}{nB}\left(-R' + G' - \frac{k(v + nc)^2}{cvB(v + c)} + \overline{y}n + \frac{m'(v + nc)^2}{cv(v + c)} + \frac{m}{c}\right)
\end{aligned}$$

So,

$$\begin{aligned}
\mathrm{E}u_i &= -\frac{c}{2}\left(\frac{k(nc + v)}{cB(c + v)} - \frac{m}{c} - \frac{m'(nc + v)}{c(c + v)}\right)^2 - \frac{v}{2}\left(\frac{(k/B - m')(v + nc)}{v(v + c)}\right)^2 \\
&\quad - \frac{k}{nB}\left(-R' + G' - \frac{k(v + nc)^2}{cvB(v + c)} + \overline{y}n + \frac{m'(v + nc)^2}{cv(v + c)} + \frac{m}{c}\right) - \frac{cv(c + v)\sigma^2}{2(nc + v)}.
\end{aligned}$$

Notice that

$$\begin{aligned}
V_-(G_-, R_-) &= \delta u(G_-, R_-) + \delta V(G, R), \text{ so} \qquad (7.3) \\
\partial V_-/\partial G_- &= -\delta\frac{k}{Bn} + \delta V_G(\partial G/\partial G_-) + \delta V_R(\partial R/\partial G_-) \\
&= -\delta\frac{k}{Bn} + \delta V_R \text{ and} \\
\partial V_-/\partial R_- &= \delta\frac{k(1 - d_R)}{Bn} + \delta V_G(\partial G/\partial R_-) + \delta V_R(\partial R/\partial R_-) \\
&= \frac{\delta k(1 - d_R)}{Bn}.
\end{aligned}$$

Thus, $\partial V/\partial R = \partial V_-/\partial R_-$ and

$$\partial V_-/\partial G_- = -\frac{\delta k d_R}{Bn}.$$

Hence, we can write

$$V(.) = \frac{\delta k(1 - d_R)}{Bn}R - \frac{\delta k d_R}{Bn}G, \qquad (7.4)$$

plus some constant. The $n + 1$ stocks can thus be represented by one state parameter.

7.2. Short-term agreements

When choosing y_i in negotiations, notice that the countries have the same preferences over y_i. At the bargaining stage, the continuation value is:

$$-\frac{c}{2}G^2-\frac{v}{2}\left(\overline{y}-y_i\right)^2+V\left(G,R\right).$$

If bargaining fails, continuation values are also independent of R_i, for R given. Thus, the problem is symmetric (even if R_is should differ) when negotiating y_i, and if assuming that each country gets an equal share of the bargaining surplus (which would be the case under Nash bargaining solution, for example), the outcome is simply that all y_is are the same, and they are such that u_i is maximized:

$$\begin{aligned}
0 &= -ncG+v\overline{y}-vy_i-nm\Rightarrow y_i=\overline{y}-\frac{nm+ncG}{v} \\
G &= \sum_j\left(y_j-R_j\right)+G'+\theta=G'+\theta+n\left(\overline{y}-\frac{nm+ncG}{v}\right)-R\Rightarrow \\
G &= \frac{nv\overline{y}-n^2m+v\left(G'+\theta-R\right)}{n^2c+v}\Rightarrow \\
y_i &= \overline{y}-\frac{nm}{v}-\frac{nc}{v}\left(\frac{nv\overline{y}-n^2m+v\left(G'+\theta-R\right)}{n^2c+v}\right) \\
&= \frac{v\overline{y}-nm-nc\left(G'+\theta-R\right)}{n^2c+v}\Rightarrow \\
g_i &= \frac{v\overline{y}-nm-nc\left(G'+\theta-R\right)}{n^2c+v}-R_i. \qquad (7.5)
\end{aligned}$$

Interrim utility is

$$\begin{aligned}
w_i^{st} &= -\frac{c}{2}G^2-\frac{v}{2}\left(\frac{nm+ncG}{v}\right)^2+V\left(G,R\right),\text{ so} \\
\partial w_i^{st}/\partial R_j &= \left(cG+\frac{nc\left(nm+ncG\right)}{v}\right)\left(\frac{v}{n^2c+v}\right)+m\left(\frac{v}{n^2c+v}\right)+m' \\
&= cG+m+m'.
\end{aligned}$$

So, a country invests until the marginal costs of investment is

$$\begin{aligned}
k/B &= \mathrm{E}cG + m + m' \Rightarrow \mathrm{E}G = \frac{k}{Bc} - \frac{m+m'}{c} \Rightarrow \\
R &= G' + n\left(\overline{y} - \frac{nm + nc\mathrm{E}G}{v}\right) - \mathrm{E}G = G' + n\overline{y} - \frac{n^2 m}{v} - \left(\frac{n^2 c + v}{v}\right)\left(\frac{k}{Bc} - \frac{m+m'}{c}\right) \Rightarrow \\
rnB + R' &= G' + n\overline{y} - \frac{m}{c} - \left(\frac{n^2 c + v}{v}\right)\left(\frac{k}{Bc} + \frac{m'}{c}\right) \text{ and} \\
G &= \frac{k}{Bc} - \frac{m+m'}{c} + \frac{v\theta}{n^2 c + v} \\
\overline{y} - y_i &= \frac{nm}{v} + \frac{nc}{v}\left(\frac{k}{Bc} - \frac{m+m'}{c} + \frac{v\theta}{n^2 c + v}\right) = \frac{n}{v}\left(\frac{k}{B} - m' + \frac{vc\theta}{n^2 c + v}\right)
\end{aligned}$$

Similarly to before, we can write

$$\begin{aligned}
u_i^{st} &= -\frac{c}{2}G^2 - \frac{v}{2}\left(\overline{y} - y_i\right)^2 - kr_i \\
&= -\frac{c}{2}\left(\frac{k}{Bc} - \frac{m+m'}{c} + \frac{\theta v}{n^2 c + v}\right)^2 - \frac{n^2}{2v}\left(\frac{k}{B} - m' + \frac{\theta vc}{n^2 c + v}\right)^2 - kr_i
\end{aligned}$$

$$\begin{aligned}
\mathrm{E}u_i^{st} = & -\frac{c}{2}\left(\frac{k}{Bc} - \frac{m+m'}{c}\right)^2 - \frac{n^2}{2v}\left(\frac{k}{B} - m'\right)^2 \\
& -\frac{k}{nB}\left(G' - R' + n\overline{y} + \frac{m}{c} - \left(\frac{n^2 c + v}{v}\right)\left(\frac{k}{Bc} - \frac{m'}{c}\right)\right) + \frac{\sigma^2 vc}{2}.
\end{aligned}$$

The argument following (7.3) continues to hold, leading to (7.4).

7.2.1. A comparison

First, notice that

$$\begin{aligned}
R^{bau} &= G' - \frac{k\left(v + nc\right)^2}{cvB\left(v + c\right)} + \overline{y}n + \frac{m'\left(v + nc\right)^2}{cv\left(v + c\right)} + \frac{m}{c} \text{ and} \\
R^{st} &= G' + n\overline{y} - \frac{m}{c} - \left(\frac{n^2 c + v}{v}\right)\left(\frac{k}{Bc} - \frac{m'}{c}\right) \text{ so} \\
R^{bau} - R^{st} &= -\frac{k\left(v + nc\right)^2}{cvB\left(v + c\right)} + \frac{m'\left(v + nc\right)^2}{cv\left(v + c\right)} + \left(\frac{n^2 c + v}{v}\right)\left(\frac{k}{Bc} - \frac{m'}{c}\right) \\
&= \frac{-1}{cv\left(v + c\right)}\left(\left(v + nc\right)^2 - \left(n^2 c + v\right)\left(v + c\right)\right)\left(\frac{k}{B} - \frac{\delta k\left(1 - d_R\right)}{Bn}\right) > 0.
\end{aligned}$$

Comparing the utilities (after a lot of algebra) leads to the following condition. $U^{st} > U^{bau}$ if and only if

$$\left(1 - \frac{1}{n}\right)^2 - \left(\frac{1 - \delta\left(1 - d_R\right)}{n}\right)^2 < \frac{\left(v + c\right)\left(\sigma vcB/k\right)^2}{\left(n^2 c + v\right)\left(nc + v\right)^2}.$$

7.3. Long-term Agreements

Again, we solve each period by backwards induction, taking the continuation value $V(.)$ as given. After the R&D stage, a country's interrim utility is given by:

$$w_i^{lt} = -\frac{c}{2}\left(G' + \theta + \sum g_i^{lt}\right)^2 - \frac{v}{2}\left(\overline{y} - g_i - R_i\right)^2 + V(.) .$$

The first-order condition, when choosing r_i, is therefore:

$$\begin{aligned} 0 &= v\left(\overline{y} - g_i - R_i\right) b + m'B - k \Rightarrow \\ R_i &= \overline{y} - g_i - \frac{k - m'B}{vb} \\ R_i' + br_i + \sum_{j \neq i} er_j &= \overline{y} - g_i - \frac{k - m'B}{vb}, \end{aligned}$$

where I have used $R_i' \equiv R_{i,-}(1 - d_R)$. In equilibrium, $R_i' + g_i$ is going to be identical across countries, implying that all r_is will be identical (and equal to, say, r):

$$Br = \overline{y} - (g_i + R_i') - \frac{k - m'B}{vb}.$$

This taken into account, a country's expected utility before the investment stage (but after the initial commitments are made) is given by:

$$-\frac{c}{2}\left(G' + \sum g_i^{lt}\right)^2 - \frac{v}{2}\left(\frac{k - m'B}{vb}\right)^2 - \frac{k}{B}\left(\overline{y} - (g_i + R_i') - \frac{k - m'B}{vb}\right) + V(.) - \frac{c\sigma^2}{2}.$$

Negotiating the g_is is equivalent to negotiating the $(g_i + R_i')$s, and all countries have symmetric and identical preferences over these terms. Thus, the $(g_i + R_i')$s are going to be equal in equilibrium, and they are going to be chosen such that u_i is maximized. This gives the first-order condition for increasing all the g_is is:

$$\begin{aligned} 0 &= -cn\left(G' + \sum g_i^{lt}\right) + \frac{k}{B} - nm - nm' \Rightarrow \\ \sum g_i^{lt} &= \frac{k}{Bcn} - \frac{m}{c} - \frac{m'}{c} - G' \Rightarrow \\ g_i^{lt} &= \frac{k}{Bcn^2} - \frac{m}{cn} - \frac{m'}{cn} - \frac{G'}{n} + (R' - R_i') . \end{aligned}$$

This can be compared to the emission levels that would be optimal (to negotiate to) ex post, after θ (and the investments) are realized. These emission levels are given by

(7.5) above, where we have to substitute for the technology levels under the long-term agreement:

$$G' + \sum g_i + \theta = \frac{nv\overline{y} - n^2 m + v\left(G' + \theta - R\right)}{n^2 c + v}$$

$$\begin{aligned}
\mathrm{E}g_i^* &= \frac{v\overline{y} - nm + v\left(G' - R^{lt}\right)/n}{n^2 c + v} - G'/n \\
&= \frac{v\overline{y} - nm - ncG'}{n^2 c + v} - \frac{v}{n^2 c + v}\left(\overline{y} - g_i^{lt} - \frac{k - m'B}{vb}\right) \\
&= \frac{\left(k - m'B\right)/b - nm - ncG'}{n^2 c + v} + \frac{vg_i^{lt}}{n^2 c + v}.
\end{aligned}$$

Thus,

$$\begin{aligned}
\left(\mathrm{E}g_i^* - g_i^{lt}\right)\left(n^2 c + v\right) &= \left(\frac{k - m'B}{b}\right) - nm - ncG' - n^2 c g_i^{lt} \\
&= \left(\frac{k - m'B}{b}\right) - nm - ncG' - n^2 c\left(\frac{k}{Bcn^2} - \frac{m}{cn} - \frac{m'}{cn} - \frac{G'}{n}\right) \\
&= \left(\frac{k - m'B}{b}\right) - n^2 c\left(\frac{k}{Bcn^2} - \frac{m'}{cn}\right).
\end{aligned}$$

Calculating u_i gives $m' = \delta k\left(1 - d_R\right)/Bn$, just as before. Anticipating and substituting this, we get:

$$\begin{aligned}
\left(\mathrm{E}g_i^* - g_i^{lt}\right)\left(n^2 c + v\right)B/k &= B\left(\frac{1 - \delta\left(1 - d_R\right)/n}{b}\right) - 1 + \delta\left(1 - d_R\right) \\
&= \left(b + (n-1)e\right)\left(\frac{1 - \delta\left(1 - d_R\right)/n}{b}\right) - 1 + \delta\left(1 - d_R\right) \\
&= (n-1)e\left(\frac{1 - \delta\left(1 - d_R\right)/n}{b}\right) + \delta\left(1 - d_R\right)\left(1 - 1/n\right) \\
&= \frac{(n-1)e}{b} + \delta\left(1 - d_R\right)\left(1 - 1/n\right)\left(1 - \frac{e}{b}\right).
\end{aligned}$$

7.4. Long-term Agreements with Renegotiation

In each period, the timing is as follows: First, the countries negotiate a set of emission levels, g_i^{de}. Thereafter, countries invest, θ is realized, and the countries renegotiate or negotiate another set of emission levels, g_i^{re}. Finally, countries pollute their allowed levels, and utilities are realized. If the renegotiations fail, the initial agreement is the default outcome. If the initial negotiation fails, the non-cooperative equilibrium is the outcome. As always, each period is solved by backwards induction.

Under the default outcome, a country's (interrim) utility is:

$$w_i^{de} = -\frac{c}{2}\left(G' + \theta + \sum g_j^{de}\right)^2 - \frac{v}{2}\left(\overline{y} - g_i^{re} - R_i\right)^2 + V\left(.\right).$$

The sum of the utilities after renegotiation is (optimal) as after a short-term agreement is negotiated

$$\begin{aligned}
\sum \frac{w_i^{st}}{n} &= -\frac{c}{2}G^2 - \frac{v}{2}\left(\frac{nm + ncG}{v}\right)^2 + V\left(G, R\right), \text{ where} \\
G &= \frac{nv\overline{y} - n^2 m + v\left(G' + \theta - R\right)}{n^2 c + v}.
\end{aligned}$$

From the envelope theorem,

$$\partial\left(\sum \frac{w_i^{st}}{n}\right)/\partial R_i = \left(m + cG\right) + m',$$

and the optimal investment level is given by

$$\begin{aligned}
k/B &= mn + nc\mathrm{E}G + nm' \Rightarrow \\
\mathrm{E}G &= G' + \sum g_j^* = \frac{k}{Bnc} - \frac{m + m'}{c} \Rightarrow \\
R^* &= G' + n\overline{y} - \frac{n^2 m}{v} - \left(\frac{n^2 c + v}{v}\right)\left(\frac{k}{Bnc} - \frac{m + m'}{c}\right).
\end{aligned} \tag{7.6}$$

Since i gets $1/n$ of the renegotiation-surplus, in addition to its default utility, i's utility can be written as:

$$w_i^{de} + \frac{1}{n}\sum_j\left(w_i^{st} - w_i^{de}\right).$$

Maximizing this expression w.r.t. r_i gives the equilibrium first-order condition

$$0 = bv\left(\overline{y} - g_i^{de} - R_i\right) + Bm' - B\left(\frac{v\left(\overline{y} - g_i^{de} - R_i\right)}{n} + m'\right) - k + \frac{B}{n}\partial\left(\sum w_i^{st}\right)/\partial R$$

If investments are first-best, the last term is equal to k/n. Substituting this (hoping that equilibrium investments are first-best), we get

$$\begin{aligned}
0 &= v\left(\overline{y} - g_i^{de} - R_i\right)\left(b - B/n\right) - k\left(1 - 1/n\right) \Rightarrow \\
R_i^{de} &= \overline{y} - g_i^{de} - \frac{k\left(1 - 1/n\right)}{v\left(b - B/n\right)}.
\end{aligned}$$

Investments are first best, indeed, if $nR_i^{de} = R^*$, requiring

$$\begin{aligned} n\overline{y} - ng_i^{de} - \frac{k(n-1)}{v(b-B/n)} &= G' + n\overline{y} + \frac{m}{c} - \left(\frac{n^2c+v}{v}\right)\left(\frac{k}{Bnc} - \frac{m'}{c}\right) \Rightarrow \\ ng_i^{de} &= -\frac{k(n-1)}{v(b-B/n)} - G' - \frac{m+m'}{c} - \frac{m'n^2}{v} + \left(\frac{n^2c+v}{v}\right)\frac{k}{Bnc}. \end{aligned}$$

On the other hand, the expected optimal pollution level is given by (7.6)

$$ng_i^* = \frac{k}{Bnc} - \frac{m+m'}{c} - G', \text{ so}$$

$$ng_i^* - ng_i^{de} = \frac{k(n-1)}{v(b-B/n)} + \frac{m'n^2}{v} - \frac{kn}{Bv}.$$

Again, anticipating $m' = \delta k(1-d_R)/Bn$, we can write

$$\begin{aligned} \left(g_i^* - g_i^{de}\right)\frac{Bv}{k} &= \delta(1-d_R) + \frac{(n-1)}{(bn/B-1)} - 1 \\ &= \delta(1-d_R) + \frac{n(1-b/B)}{bn/B-1} \end{aligned}$$

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