HARVARD | BUSINESS | SCHOOL

Why Do Intermediaries Divert Search?

Andrei Hagiu
Bruno Jullien

Working Paper

08-010

Why Do Intermediaries Divert Search?*

Andrei Hagiu† and Bruno Jullien‡

First Draft: August 2007
This Draft: February 12th 2009

Abstract

We analyze the incentives to divert search for an information intermediary who enables buyers (consumers) to search affiliated sellers (stores). There are three motives for diverting search (i.e. inducing consumers to search more than they would like): i) trading off higher total consumer traffic for higher revenues per consumer visit; ii) reducing the variance of store profits when store affiliation decisions are endogenous; and iii) influencing stores' choices of strategic variables (e.g. pricing) once they have decided to affiliate. We show that search diversion remains a necessary strategic instrument for the intermediary even when the contracting space is significantly enriched: allowing the intermediary to charge consumers fixed fees, to offer them screening contracts, to subsidize search; allowing stores' strategic decisions to be contractible or controlled by the intermediary.

Keywords: Market Intermediation, Search, Two-Sided Markets, Platform Design.
JEL Classifications: L1, L2, L8

1 Introduction

The previous literature on market intermediation (e.g. Spulber (1996) and (2007)) as well as conventional wisdom presume that one of the most important functions of market intermediaries is to reduce search costs for the parties they serve and that they create more value the larger such cost reductions they generate. This would seem to be true of both traditional, brick-and-mortar intermediaries (e.g. retailers, shopping malls, brokers, magazines) and "new economy" ones (e.g. Amazon, eBay, Google, iTunes, Yahoo!, etc.). Many of these intermediaries seem however, through some aspects of their design, to do quite the opposite of reducing search costs - and purposefully rather than by accident. Popular magazines (e.g. Esquire, Vanity Fair, Vogue) make the (little) content they carry exceedingly difficult to find by interspersing it with lots of advertising and concealing the tables of contents. Shopping malls are designed to *maximize* the total distance

*The authors thank Stefan Behringer, Emilio Calvano, Jacques Crémer, Jim Dana, Bill Rogerson, Mike Whinston and Alexander White for helpful discussions and comments, as well as participants to the CSIO-IDEI conference, ISM, EARIE, WISE, Telecom ParisTech conference on ICT, and to seminars at Harvard Business School, London Business School, CREST-LEI, Brunel, University College London, Toulouse School of Economics, University Paris 1.

†Harvard University (HBS Strategy Unit), ahagiu@hbs.edu
‡Toulouse School of Economics (IDEI & GREMAQ), bjullien@cict.fr

consumers walk in the mall: anchor stores are usually located as far from each other as possible; the top of an escalator is placed at the opposite end of a floor from the bottom of another escalator; etc. Supermarkets stack the products they carry so that the most sought-after items (e.g. bread, diapers, milk and other essential staples) are situated at the back of the store (Petroski (2003). E-commerce platforms increasingly use various forms of recommender systems and contextual advertising in order to shift - more or less subtly - users' focus from the products they were *initially* looking for, towards "exploring" and "discovering" products they *might* be interested in - and eventually buy.

In this paper, we examine - theoretically - an intermediary's incentives *not* to optimize the search process by which consumers find the stores (sellers) that the intermediary provides access to. In our model, the intermediary has superior information about the match between consumers' preferences and the stores they can reach through that intermediary; consumers can only visit stores sequentially and each visit is costly. If the intermediary derives revenues from consumers' transactions with stores, then it may elect to *divert consumers' search,* i.e. direct them first to their *least* preferred stores. Importantly, diverting search is different than increasing *unitary* search costs: in our model (as in reality), intermediaries would always want to decrease consumers' unitary search costs (if they could).

Our model sheds light on how revenue structures determine intermediaries' strategic design of the information services provided to their customers. Consider the examples mentioned above. The reason for which magazines' layout is frustrating for at least some readers is that publishers make most of their revenues from advertisers, who do not want readers to be able to easily skip ads. Similarly, for supermarkets and other retailers, search diversion is driven by a business model which seeks to capitalize on consumers' impulse buys as they stroll past items they were not necessarily looking for in the first place. In the case of shopping malls, mall developers have always relied on a strategy of signing a few "anchor stores" (which are usually offered discounted rents as well as other inducements) to draw consumer traffic, and designing the mall such that the traffic would "spill over" to the other stores, which account for the bulk of developer profits.

These features of offline intermediaries carry over to their online counterparts. And the latter can arguably manipulate informational design in significantly subtler and more flexible ways in order to divert search. One prominent manifestation is the increasingly important role played by recommender systems: according to Forrester Research, consumers spent $220 billion online in 2006 and recommender systems could account for 10 to 30 percent of any online retailer's sales.[1] E-commerce platforms such as Amazon, Barnes and Noble, iTunes and Netflix have integrated recommender systems into their sites in order to extend consumers' visits and maximize their exposure to products they did not know they would be interested in, but that they might eventually purchase. This requires walking a fine line between providing relevant information and steering users towards "exploration". Furthermore, recommender systems themselves do not necessarily recommend the products that best match individual consumers' preferences *given* the information available. Instead, their algorithms oftentimes place a positive weight on business considerations

[1] "Click here for the upsell," *Business 2.0*, July 11th 2007.

which may have nothing to do with a user's tastes. For example, Netflix's recommender system takes into account inventory availability: when the movie that best fits the preferences of a given user is temporarily in limited stock (or out of stock), it automatically recommends a different movie that the user might like (Shih et al. (2007)).

Even search engines - which are arguably the most "unbiased" online information intermediaries - contain some element of search diversion. For example, the "objective" search results displayed in the middle of Google's search pages are surrounded by sponsored search results (on the right and on top), reflecting the inherent trade-off between what consumers want and where revenues come from. Consequently, the 11th search result (access to which requires users clicking to the second page - a step that the majority of users never take) is much harder to get to than the sponsored search results on the right of the first page, even though it might be more relevant to the user. Nor does Google provide an objective ranking of sponsored search results based on relevance - this would be at odds with the keyword auctions it runs for placing these results.

These examples raise several important questions about the design of information services by intermediaries. Under what conditions do intermediaries find it profitable *not* to maximize the effectiveness of their information service, i.e. to divert consumers' search process? What are the underlying motivations for diverting search? And if some form of search diversion is profit-maximizing, can it not be replicated by other contractual instruments that intermediaries might have at their disposition? In this paper we offer some answers to these questions.

We identify three motivations for search diversion by the intermediary. First and most fundamental, due to a failure of the Coase theorem, consumers do not internalize *ex-ante* all the externalities that their search activity generates. In particular, they do not account for the gains from trade bestowed on all of their potential trading partners - stores - when deciding to perform a search through the intermediary, which may lead to "insufficient" search. Since the intermediary derives revenues whenever consumers transact with stores, it has an incentive to introduce some noise in the search process (i.e. to divert search). In turn, consumers anticipate this and might be less likely to use the intermediary's service in the first place. Therefore, the intermediary has to trade off higher total consumer "traffic" against more searches per visitor.

The second motive for diverting consumer search emerges when stores' affiliation decisions with the intermediary are endogenized and the intermediary can commit to the design of its information service *before* (or at the same time as) setting affiliation fees for stores. In this context, search diversion reduces the profit differential between the more popular (infra-marginal) stores and the less popular (marginal) ones. It therefore increases the intermediary's profits when it cannot price discriminate among stores and the participation of the marginal stores is binding, or when it extracts a *higher* fraction of revenues from less popular stores relative to more popular stores (this could be because it has more bargaining power vis-a-vis less popular stores).

Third, an intermediary can use search diversion as an instrument to influence the strategic choices (pricing in particular) made by affiliated stores. Such indirect control is desirable for the intermediary since individual stores do not fully internalize the effect of their strategic decisions on total consumer demand for the intermediary's service. By altering the composition of the demand faced by each store, search diversion can force stores to lower their prices, thereby increasing the surplus left to consumers (even though search diversion by itself lowers consumer utility) and ultimately their traffic to the intermediary.

It is then natural to ask: couldn't the need for search diversion be eliminated by other pricing or contractual instruments that intermediaries might have at their disposition? We show that the answer to this question is generally no: the need to divert search survives even when the set of contracting instruments available to the intermediary is significantly enriched. Being able to charge consumers fixed fees for access, offer them screening contracts or subsidizing search reduce the need for such diversion but does not fully eliminate it. The only case in which we were able to fully eliminate the need for search diversion is that of an intermediary who bundles the information service with access to stores and can both charge access fees *and* subsidize search. Similarly, even if sellers' strategic decisions are fully contractible *ex-ante* (or if the intermediary is vertically integrated into stores), search diversion remains necessary in order to induce the "right" amount of consumer search.

Related literature

Our paper contributes both to the established economics literature on market intermediation (Biglaiser (1993), Gehrig (1993), Rubinstein and Wolinsky (1987), Spulber (1996), Rust and Hall (2001)) and to the more recent and quickly growing one on two-sided markets (Caillaud and Jullien (2003), Evans (2003), Rochet and Tirole (2003), Armstrong (2006), Hagiu (2006)). The former was mostly focused on traditional intermediaries, who buy and resell goods, while the latter was motivated by the rising importance of "new economy" intermediaries (called "two-sided platforms"), who connect buyers and sellers and provide matching, price discovery, certification, advertising and other informational services, without (usually) assuming full control over the transactions enabled.[2] In both of these strands of research however, intermediaries are presumed to create value by reducing search and/or transaction costs and the "technologies" which enable them to do so are taken as exogenously given.

To the best of our knowledge, ours is the first paper to study the design of information services by intermediaries and to show that their incentives with respect to search effectiveness are fundamentally driven by the structure of the revenues they derive from the parties they serve.[3] By contrast, most of the economics literature on two-sided markets to date has focused on the choice of pricing structures by two-sided platforms as a function of various industry factors - e.g. relative strengths of the indirect network effects on each side, relative demand elasticities - and has largely ignored two-sided platform design issues (one exception is Parker and Van Alstyne (2008), which focuses on platforms' choice of openness).

[2] Hagiu (2007) contains a unifying framework for analyzing these two forms of intermediation as two extremes along a continuum.

[3] There is a very recent literature focusing specifically on the design of search engines, which bears some aspects in common with our approach. See White (2008).

Since the design of an information service by an intermediary can be considered as a form of matching mechanism design, our work is also related to the literature on market design (surveyed in Roth (2002) and (2008), Roth and Sotomayor (1990)). There are three main differences between our paper and this literature. First, while the market design litterature attempts to derive efficient (from a total social welfare perspective) and/or stable matching mechanisms, we study information service design by a profit-maximizing platform. Second, "matches" in our model (i.e. consumers visiting stores) are not substitutes, as they are in the market design literature (e.g. new medical school graduates taking posts as interns or residents at hospitals). Third, we consider both non-price and price matching mechanisms with endogenous participation, and we investigate the possibility that the matching mechanism indirectly affects transfer prices within matched pairs, which to our knowledge is novel.[4]

The remainder of the paper is organized as follows. The next section explains how the main features of our model map to the real-world contexts to which it applies. In section 3 we lay out the formal baseline model, which captures the fundamental trade-off involved in diverting search by intermediaries: higher total consumer traffic vs. more searches per consumer visit. Section 4 analyzes the two other motives driving the intermediary's incentives to divert search: reducing the variance of store profits when stores' affiliation decisions are endogenous; and influencing stores' pricing decisions. In section 5 we analyze the robustness of the need to divert search to several enrichments of the contracting space. Section 6 contains a brief discussion of welfare implications and of the effects of introducing competition among intermediaries.

2 Preliminary considerations

We capture the relevant features of the information intermediaries mentioned above in a stylized setting with a monopoly intermediary, two stores (1 and 2) and a continuum of consumers. Each consumer has a high valuation for one store and a low valuation for the other store and can only visit the stores sequentially. Visits are based on the intermediary's recommendations and each visit incurs a search cost for the consumer. The focus of the model is on the intermediary's design of its information service. In our set-up, the intermediary would always want to design its service so as to minimize each consumer's search cost *per visit.* But it may *not* want to provide the most accurate recommendation, i.e. it may want to design its service so that each consumer engages in more searches than necessary. We will capture this aspect of design in the probability q with which the intermediary directs any given consumer to her high valuation store in the first round of search. *Search diversion* occurs when this probability is strictly below 1: a positive fraction of consumers are then directed first to their least preferred stores - they subsequently do also visit their most preferred stores, but they end up searching more than they would have liked to ex-ante. Consistent with the illustrations above, the intermediary is assumed to derive revenues r_i whenever

[4] Hatfield and Milgrom (2005) contains one of the first attempts to include contracts in matching games.

a consumers visits or conducts a transaction with store $i \in \{1, 2\}$. These revenues can be thought of as the compensation that the intermediary is able to extract in exchange for directing consumer traffic to the stores.[5] If the intermediary owns the stores, then r_i is simply equal to the profits made by store i per consumer visit.

Three observations regarding this basic setup are in order. First, our model is meant as a highly stylized abstraction of informational services offered by real world intermediaries. Precisely for that reason, we believe it applies quite broadly across the variety of illustrations we have mentioned in the introduction: Table 1 below summarizes the correspondence between the model features and the real-world applications we have in mind. In particular, the variable q synthesizes various design choices by the intermediary, some of which determine the effectiveness of search *directly* (e.g. the algorithms used by recommender systems), while others have an *indirect* effect (e.g. layout of shopping malls, retail stores or websites). Also, the table implicitly assumes (without loss of generality) that store 1 is more popular than store 2.

Intermediary	**Store 1**	**Store 2**	q	r_1 **and** r_2
Magazines (e.g. Esquire, Vanity Fair, Vogue)	Editorial content	Advertising	*Indirect*: organization of the magazine (e.g. q is lower when the table of contents is absent or hard to find)	$r_1 = 0$ and r_2 is the advertising rate per reader
Brick & mortar retailer (e.g. Apple Store, Target, Wal-Mart)	Essential staples	Less popular products	*Indirect*: store layout (e.g. q is lower when essential staples are at the back of the store)	Margins made on more various products (usually lower on essential staples)
Shopping Mall	Anchor stores	Other stores	*Indirect*: design and physical layout (e.g. q is lower when anchor stores are far from the main access points)	Rent plus percentage of sales revenues charged by mall developer to various stores (usually lower for anchor stores)
Search engines (e.g. Ask.com, Google, Microsoft Live Search)	Relevant search results	Sponsored search results	*Indirect*: layout of search results pages (e.g. q is lower when more space is dedicated to sponsored search links) *Direct*: relevance and ordering of search results	$r_1 = 0$ and r_2 is the "cost per click" or "cost per realized sale"
Online retailers (e.g. Amazon, iTunes) and portals (e.g. MSN, Yahoo!)	Most useful/popular products and content	Other products; advertising	*Indirect*: layout of web pages (e.g. q is lower if more space is dedicated to advertising and featured products)	r_1 is the margin made on useful/popular products (oftentimes $r_1 = 0$) r_2 is the advertising rate: per impression (CPM); per click (CPC); or per realized sale (CPS)
Recommender systems (e.g. Amazon, iTunes, Netflix, Pandora)	Products that best fit a consumer's preferences *conditional* on information available	Other products (that the consumer *might* be interested in)	*Direct*: match between user preferences and actual recommendations *conditional* on information available	Margins (or revenue shares) resulting from the sale of various products

Table 1

Second, note that a priori there could be several cases to analyze, depending on: i) whether the intermediary's information service to consumers is optional or "hardwired" into (bundled with)

[5] The r_i's can be either a percentage of store revenues or "per-click" (i.e. per customer visit) fees. For instance, shopping mall developers charge their retailer tenants a percentage of their sales, while magazines charge advertisers a fee based on expected readership and Internet portals charge advertisers a per-click fee. In our basic model, this distinction is immaterial for the results.

access to stores; ii) whether the intermediary derives revenues r_1 and r_2 whenever consumers visit stores or only whenever they visit stores *conditional on* having used the intermediary's information service. When an intermediary bundles its informational service with access to stores, the distinction in b) becomes irrelevant. Therefore, there are potentially three different types of intermediaries:

- Type (A) (also called "bottlenecks") offer the information service bundled with access to stores. This type includes magazines, retail stores, shopping malls and online portals who mix content with advertising on their pages - there is no way to access stores or content without navigating the layout of these intermediaries

- Type (B) (also called "pure recommenders") offer an optional information service *and* receive revenues r_i only when consumers use that service to access stores. This type includes pure recommender systems (e.g. ChoiceStream and CleverSet) who license their services to third-party online retailers (e.g. Blockbuster, Sephora, WineEnthusiast.com); search engines such as Google, since advertisers only pay Google when consumers "jump" to their sites directly from Google's search results page.

- Type (C) offer an optional informational service *and* receive revenues r_i whenever consumers access stores, regardless of whether they use its service or not. This type includes e-commerce sites which provide their own recommender systems (e.g. Amazon, iTunes, Netflix): consumers may or may not avail themselves of the availability of recommendations, but their purchase of any good results in revenues for the intermediary

These three types of intermediaries are formally equivalent in our model in sections 3 and 4 (i.e. their optimization problems in q are the same). The distinction will only matter when we allow intermediaries to charge fixed access fees to consumers in section 5.

Third, we will consider both exogenously given and endogenously determined revenues r_i from stores, but the real-world applications described above provide some justifications for the assumption that the intermediary takes r_1 and r_2 as exogenously given when optimizing over q. For brick-and-mortar intermediaries like shopping malls, the informational service is hardwired in the physical design, which is a longer-term and stickier decision than pricing and contracting with stores. In this case, (r_1, r_2) can be interpreted as forward-looking, expected values from the perspective of period 0, when the mall is being built - and when there is still significant uncertainty left regarding future costs and prices. For online intermediaries, quite the opposite is true: advertising and content supply contracts are stickier and longer-term than website design decisions: the latter can be adjusted by the intermediary almost as frequently as it desires. In this case, it is reasonable to assume that intermediaries make regular informational design decisions *taking as given* the structure of their revenue flows from various sources.

3 Basic Model

3.1 Set Up

A monopoly intermediary (or platform) allows a unit mass of consumers to access two affiliated stores, 1 and 2.

Consumers differ along two dimensions: preferences for stores and search costs. Along the first dimension, there are two types of consumers: type 1 consumers make up a fraction α of the population and derive *net* utilities u^H from visiting store 1 and u^L from visiting store 2, while type 2 consumers make up the remaining fraction $(1-\alpha)$ and derive net utilities u^H from visiting store 2 and u^L from visiting store 1,[6] where $0 < u^L < u^H < 1$ are exogenously given (we endogenize them in section 4.2). Although we investigate this possibility in a companion paper (Hagiu and Jullien [2008]), in this paper there is no substitutability nor complementarity between stores. Along the second dimension, consumers are differentiated in their unitary search cost c, which they incur whenever they visit a store. Consumers can only visit the two stores *sequentially* (they perform at most two rounds of search). We assume that c is distributed on $[0,1]$ according to a twice continuously differentiable cumulative distribution function F. The distribution of search costs F is independent of the distribution of types $(\alpha, 1-\alpha)$.

As mentioned above, the intermediary derives non-negative net revenues r_i whenever a consumer visits store $i \in \{1,2\}$. For now, r_i, $i = 1,2$ are fixed and exogenously given - for example, they could be the result of some prior bargaining game between the intermediary and the stores which we do not model. (In section 4.1 we endogenize the choice of r_i.)

The central assumption is that *prior to* making their first store visit and incurring the corresponding search costs, consumers do not know which store is which, but the intermediary does.[7] This informational advantage combined with the fact that store visits are costly allows the intermediary to provide a potentially valuable informational service by making a recommendation to consumers regarding which store to visit first. Consumers may choose to heed this recommendation or ignore it.

The intermediary's information service is defined by the probability $q \in (0,1)$ with which it directs *any* given consumer[8] to her preferred store (store i for type i) in the *first* round of search. Thus, the intermediary "diverts" a fraction $(1-q)$ of consumers, i.e. sends them to their less preferred store first. Once a consumer has visited and identified one store in the "first round" of search, she knows for sure the identity of the other store, although she needs to incur her search cost c again if she wants to visit it. Throughout the paper the focus will be on the intermediary's

[6] u^i should be interpreted as encompassing the utility of just "looking around" the store plus the expected utility of actually buying something, *net* of the price paid.

[7] The intermediary observes preference types, but not search costs. Given this assumption, in our model it does not matter whether consumers know their type or not.

[8] In most of the paper, the intermediary can set only one value q for all consumers (e.g. through an automated service). We relax this assumption in section 5.2 when we enable the intermediary to offer different "qualities of service" q and screen consumers.

choice of q.[9] We assume that q can be *costlessly* set to any value between 0 and 1 and that it is announced *publicly and credibly* by the intermediary. In particular, the timing in the baseline model is as follows: 1) the intermediary announces and credibly commits to q, the effectiveness of its information service; 2) consumers observe q, decide whether or not to use the intermediary's information service and engage in the search process.

3.2 Derivation of the intermediary's objective function

We start by deriving consumers' optimal search behavior. Consumers with $c \leq u^L$ (of both types) will visit both stores regardless of whether they use the intermediary's information service or not and no matter which store they visit in the first round of search. Their ex-ante utility is given by $u^H + u^L - 2c$, which is strictly positive.

Let us now look at consumers for whom $u^L < c \leq u^H$. If they find store 1 (yielding utility u^H) in the first round, they stop (a second search would cost c and yield $u^L < c$). If they find store 2 instead, they shop there and then do a second round of search in which they find store 1 with probability 1. There are however three possible, pure search strategies for such a consumer, aside from no participation, which yields 0 utility. First, she can follow the intermediary's recommendation, in which case her ex-ante utility is:

$$qu^H + (1-q)\left(u^L + u^H - c\right) - c = u^H + (1-q)\,u^L - (2-q)\,c \tag{1}$$

Second, she can "game" the intermediary's information service by always going to the store *not* recommended by the intermediary[10]. In this case she obtains $u^H + qu^L - (1+q)\,c$. Third, she can ignore the intermediary's recommendation altogether, in which case she finds her favorite store with probability $\frac{1}{2}$ in the first round and her ex-ante utility is $u^H + \frac{1}{2}u^L - \frac{3}{2}c$.

Therefore, the consumer will choose to come to the intermediary and rely on its information service (option 1) if and only if: i) $q \geq \frac{1}{2}$ and ii) $qu^H + (1-q)\left(u^L + u^H - c\right) - c \geq 0$. For the intermediary, setting $q < \frac{1}{2}$ is always weakly dominated by setting $q \geq \frac{1}{2}$,[11] therefore in all that follows we will always focus on the space $q \in \left[\frac{1}{2}, 1\right]$ - in fact we will be interested under what conditions $q^* < 1$. Inequality ii) (the consumer participation constraint) is equivalent to:

$$c \leq \frac{u^H + (1-q)\,u^L}{2-q} \equiv u\,(q) \tag{2}$$

where $u\,(q)$ is the average utility per search and is increasing in q. Then $F\,(u\,(q))$ is total consumer "traffic" to the intermediary (i.e. total demand for its service).

[9] Notice that the same q is offered to both consumer types. Analyzing the case when the intermediary can offer $q_1 \neq q_2$ to the two consumer types is straightforward and would yield any new insights. We will however consider screening schemes where consumers with different search costs select different q's.

[10] This strategy can alternatively be interpreted as the consumer misrepresenting her preferences, i.e. pretending she is of type 2 when she is in fact of type 1, or viceversa.

[11] If $q < \frac{1}{2}$ then all participating consumers with $c > u^L$ game the intermediary's service. The latter can then achieve at least the same demand and number of store visits with $q > \frac{1}{2}$.

The intermediary's profits are then:

$$\begin{aligned}\Pi^I(q) &= \underbrace{\alpha\left\{(r_1+r_2)F\left(u^L\right)+[r_1+(1-q)r_2]\left[F(u(q))-F\left(u^L\right)\right]\right\}}_{\text{revenues from type 1 consumers}} \\ &\quad+\underbrace{(1-\alpha)\left\{(r_1+r_2)F\left(u^L\right)+[r_2+(1-q)r_1]\left[F(u(q))-F\left(u^L\right)\right]\right\}}_{\text{revenues from type 2 consumers}}\end{aligned}$$

which can be written:

$$\Pi^I(q)=(\widetilde{r_1}+\widetilde{r_2})F\left(u^L\right)+[\widetilde{r_1}+(1-q)\widetilde{r_2}]\left[F(u(q))-F\left(u^L\right)\right] \tag{3}$$

where:

$$\widetilde{r_1}\equiv\alpha r_1+(1-\alpha)r_2 \quad\text{and}\quad \widetilde{r_2}\equiv\alpha r_2+(1-\alpha)r_1$$

are to be interpreted as the *average revenues* per consumer from the *favorite* store of consumers ($\widetilde{r_1}$) and the *average revenues* per consumer from the *less preferred* store of consumers ($\widetilde{r_2}$).

Expression (3) contains the basic trade-off involved in diverting search. On the one hand, a lower q increases the revenues from "accidental" shopping by each consumer at her less preferred store while searching for her favorite store. On the other hand, decreasing q also results in fewer shoppers using the intermediary's information service because they know it is not very reliable.

Note how diverting search differs from increasing search costs. If the intermediary had a technology that enabled it to reduce every consumer's unitary search cost from c to $c(1-\alpha)$, then the intermediary would choose the highest possible level of α (assuming 0 marginal cost of increasing α). By contrast, search diversion increases the number of searches performed by each consumer (and therefore *total* search costs).

If q were unobservable by consumers but the latter had rational expectations, then $q>\frac{1}{2}$ could not be sustainable as an equilibrium[12]. Therefore it is in the intermediary's best interest to publicly commit to q, which we have assumed above it can.

Note that our model could also be interpreted as a form of stochastic bundling[13]: $(1-q)$ would represent the probability with which a seller of goods 1 and 2 is bundling the two goods; $q=0$ would correspond to pure bundling. The possibility for consumers to search without taking into account the intermediary's recommendation is then akin to mixed bundling, in which the seller is also offering the two goods separately.

[12]Suppose $q^E>\frac{1}{2}$ were an equilibrium so that total consumer traffic is $F\left(u\left(q^E\right)\right)$. Then, given that consumers do not observe the actual q chosen by the intermediary, the latter has an incentive to deviate to $q<q^E$ in order to increase $(\widetilde{r_1}+(1-q)\widetilde{r_2})$.

[13]We are grateful to Michael Whinston for this observation.

3.3 Optimal search diversion

If the maximization of (3) over q has an interior solution - i.e. belonging to the interval $\left]\frac{1}{2},1\right[$ -, then the optimal level of search effectiveness q^* is determined by the first order condition:

$$-\widetilde{r_2}\left(F\left(u\left(q^*\right)\right)-F\left(u^L\right)\right)+\left[\widetilde{r_1}+\left(1-q^*\right)\widetilde{r_2}\right]\underbrace{\frac{u^H-u^L}{\left(2-q^*\right)^2}}_{u'(q^*)}f\left(u\left(q^*\right)\right)=0 \tag{4}$$

Thus, q^* only depends on $\widetilde{r_1}$ and $\widetilde{r_2}$ through $\frac{\widetilde{r_1}}{\widetilde{r_2}}$, which in turn only depends on α and $\frac{r_1}{r_2}$. To simplify the exposition - and without loss of substance - we make the following assumption (in the appendix we derive a sufficient condition for this assumption to hold):

Assumption 1 *The expression of intermediary profits (3) is quasi-concave in q on the interval $\left[\frac{1}{2},1\right]$.*

Under assumption 1, we can use the first order condition above to derive the necessary and sufficient condition for the optimal q^* to be less than 1 (i.e. for the intermediary to divert search):

$$\frac{\alpha r_1+(1-\alpha)\,r_2}{\alpha r_2+(1-\alpha)\,r_1}<\frac{F\left(u^H\right)-F\left(u^L\right)}{\left(u^H-u^L\right)f\left(u^H\right)} \tag{5}$$

This condition has a straightforward interpretation. At $q=1$, all consumers with $c\leq u^H$ use the intermediary's service and only those with $c\leq u^L$ visit both stores. We now ask whether the intermediary has any interest in slightly lowering the effectiveness of search by a very small amount ε. This has two effects on the intermediary's profits. The first effect is negative since the intermediary loses the traffic of the consumers with highest search costs. The absolute value of this effect is $\widetilde{r_1}\left[F\left(u^H\right)-F\left(u\left(1-\varepsilon\right)\right)\right]$, which is approximately equal to $\widetilde{r_1}\varepsilon\left(u^H-u^L\right)f\left(u^H\right)$. The second effect is positive since now a fraction ε of all (inframarginal) consumers with $c\in\left[u^L,u\left(1-\varepsilon\right)\right]$ also visit their less preferred store. The magnitude of this effect is approximately $\widetilde{r_2}\varepsilon\left[F\left(u^H\right)-F\left(u^L\right)\right]$. Comparing the two effects yields exactly condition (5) above.

The following proposition follows directly from (4) and (5):

Proposition 1 *a) The monopoly intermediary's optimal choice of search effectiveness q^* is increasing (decreasing) in $\frac{r_1}{r_2}$ for $\alpha\in\left[\frac{1}{2},1\right]$ (respectively $\alpha\in\left[0,\frac{1}{2}\right]$) and increasing (decreasing) in α for $r_1>r_2$ (respectively $r_1<r_2$)*

b) Under assumption 1, the intermediary is more likely to divert search when $\frac{r_1}{r_2}$ decreases (increases) for $\alpha\in\left[\frac{1}{2},1\right]$ (respectively for $\alpha\in\left[0,\frac{1}{2}\right]$) and less likely to divert search when α decreases (increases) for $r_1>r_2$ (respectively for $r_1<r_2$)

c) If F is concave and $\widetilde{r_1}<\widetilde{r_2}$ then $q^*=\frac{1}{2}$. If F is convex and $\widetilde{r_1}>\widetilde{r_2}$ then $q^*=1$.

d) The intermediary is more likely to divert search when u^H increases if and only if $\frac{F(u)-F\left(u^L\right)}{\left(u-u^L\right)f(u)}$ is increasing in u

e) The intermediary is more likely to divert search when u^L decreases if and only if $\frac{F\left(u^H\right)-F(u)}{(u^H-u)f(u^H)}$ is decreasing in u

Proof: c) is implied by (16) in the appendix, noting that $\frac{F(u)-F\left(u^L\right)}{(u-u^L)f(u)} > 1$ (< 1) if F is concave (respectively convex).

■

Results a) and b) reflect the basic mechanism at work in our model. As intuition might suggest, when there is a higher fraction of consumers who prefer store 1 ($\alpha > \frac{1}{2}$), the intermediary finds it profitable to *reduce* search diversion (or *increase* search effectiveness q^*) in response to an increase in revenues derived from store 1 *relative to* store 2 (and viceversa). The intermediary also finds it optimal to set q^* below 1 *less* often. This is because the average consumer prefers store 1, therefore if that store yields higher revenues relative to store 2, the intermediary has less incentive to divert the average consumer to store 2. One should then expect search effectiveness to be the worst in contexts in which $\frac{r_1}{r_2}$ is the lowest when $\alpha > \frac{1}{2}$. This would seem to be the case with popular magazines, if we interpret editorial content as store 1 and advertising as store 2. While there are presumably more readers who prefer content to advertising than viceversa, the revenues that magazines make from *content* are roughly 0, whereas most of the profits come from advertising. It is not surprising then that the organization of such magazines is very frustrating for readers - as anyone having tried to find an article in a fashion magazine can attest.

Note that in reality, intermediaries often make *less* money from more popular "stores" relative to less popular ones. Indeed, since popular stores (e.g. "anchor stores" for shopping malls) are critical for attracting consumer traffic, intermediaries may have to offer them special deals (i.e. charge low or even subsidized prices). Then, intermediaries' revenues will come primarily from the less popular stores, against which the intermediaries have more bargaining power. Our model predicts that search diversion will be particularly salient in these contexts and can be interpreted as a way to intensify the positive externality exerted by anchor stores on the other affiliated stores and thereby increase the latter's profits, which can then be extracted by the intermediary. This is consistent with the following quote from Elberse et al. (2007), which studies Roppongi Hills, Tokyo's most prominent real-estate complex[14]: "*To convey a feeling of exploration akin to that found in real, organic cities, the architects opted for a maze-like structure in which visitors and residents could wander around for hours, and "discover" new shops and restaurants along the way. The structure was thought to benefit some of the lesser-known shops and restaurants, but some corporate tenants were less pleased with the lack of clarity that, they complained, confused their prospective employees.*"

Result c) in Proposition 1 can be interpreted in the following way. If F is concave, then the distribution of search costs is concentrated around lower values of c, which means that diverting search causes relatively few consumers (the high search cost ones) to stop using the intermediary altogether. If in addition $\widetilde{r_1} < \widetilde{r_2}$, then the benefits of diverting search are so great relative to the

[14]Roppongi Hills is a 12-hectar mini-city encompassing retail space; restaurants and coffee shops; a large movie-theater; an outdoor arena; a television studio; a luxury hotel, and two residential buildings; as well as a 54-story office tower.

costs that it makes sense to offer the lowest level of search effectiveness possible - while maintaining consumer participation. Conversely, if F is convex, then the distribution is concentrated around the higher values of c, which means search diversion is quite costly; combined with $\widetilde{r_1} > \widetilde{r_2}$, this implies that it is never attractive to divert search.

Results d) and e) state that the effects of consumer valuations for their preferred store (u^H) and less preferred store (u^L) are ambiguous. The reason is that *increasing* u^H or *decreasing* u^L results in higher benefits of a small amount of search diversion (proportional to $\widetilde{r_2}\left[F\left(u^H\right)-F\left(u^L\right)\right]$), but may also increase the corresponding costs (which are proportional to $\widetilde{r_1}\left(u^H-u^L\right)f\left(u^H\right)$).[15]

4 Two additional motivations to divert search

In the basic model of the previous section, the intermediary's incentive to divert search was exclusively driven by the need to make consumers search more. This led to a fundamental trade-off between higher total traffic and higher revenues per consumer visit. In this section we identify two additional sources of incentives for an intermediary to divert search, both of which are driven by considerations on the store "side" of the market . The first one arises when stores' affiliation decisions with the intermediary are endogenous and the intermediary can charge stores in exchange for affiliation. The second additional incentive stems from the need to influence strategic choices (pricing in particular) made by stores affiliated with the intermediary.

In what follows, we isolate each of these two novel incentives in turn - needless to say, in reality they are most likely to coexist.

4.1 Endogenous store affiliation

The key modification that we make here relative to the basic model is to endogenize the affiliation of stores with the intermediary. We do so by introducing a stage right *after* the intermediary commits to q, in which the intermediary announces a per consumer visit fee r that stores have to pay in order to be accessible by the consumers who visit the intermediary (we continue to assume that the intermediary's information service is bundled with access to stores). For simplicity, we assume that the intermediary holds all the bargaining power, so that r is essentially a take-it-or-leave-it offer. However, the intermediary cannot price discriminate and therefore the fee r applies to both stores. We will briefly discuss below the effect of allowing the intermediary to charge fixed or other type of fees (e.g. percentage of sales).

The timing is then as follows: 1) the intermediary announces and credibly commits to q; 2) the intermediary announces the fee r that stores have to pay for each consumer visit they receive through the intermediary; 3) stores simultaneously decide whether or not to affiliate with the intermediary;

[15] In fact, decreasing u^L *always* increases the *costs* of search diversion because a lower u^L - all else equal - increases the number of high search cost consumers that stop using the intermediary's service in response to a small decrease in search effectiveness.

4) consumers observe q and store affiliations, decide whether or not to use the intermediary's informational service and engage in the search process.

Finally, we assume that for each store, affiliation with the intermediary requires a fixed investment $K > 0$ and that each store makes the same net profits π per consumer visit (no additional insight would be gained by allowing stores to have different π's here). Then store profits when both affiliate with the intermediary are (recall that $u(q) = \frac{u^H + (1-q)u^L}{2-q}$):

$$\begin{aligned}
\Pi_1^S &= (\pi - r)\left\{\alpha F(u(q)) + (1-\alpha)\left[qF\left(u^L\right) + (1-q)F(u(q))\right]\right\} - K \\
\Pi_2^S &= (\pi - r)\left\{(1-\alpha)F(u(q)) + \alpha\left[qF\left(u^L\right) + (1-q)F(u(q))\right]\right\} - K
\end{aligned}$$

We focus on the case $\alpha > \frac{1}{2}$ (i.e. store 1 is more popular than store 2), which implies $\Pi_1^S > \Pi_2^S$. To make things interesting we assume that the intermediary finds it profitable to induce both stores to affiliate, therefore it solves:

$$\max_{q,r} r\left[qF\left(u^L\right) + (2-q)F(u(q))\right] \quad \text{subject to } \Pi_2^S \geq 0$$

The constraint is binding, i.e. r is set so that $\Pi_2^S = 0$. Given this fee, it is always an equilibrium for both stores to accept the offer and affiliate with the intermediary.[16] Replacing the resulting r in the expression of platform profits, we obtain that the intermediary's optimal choice of q reduces to:

$$q_a^*(\alpha) \equiv \arg\max_q \left\{\left(\pi - \frac{K}{\alpha X(q) - (2\alpha - 1)F(u(q))}\right)X(q)\right\} \tag{6}$$

where $X(q) \equiv qF\left(u^L\right) + (2-q)F(u(q))$.

By contrast, recall from (3) that if the stores' affiliation with the intermediary is taken as given and the intermediary obtains the same variable fee r from both of them, its choice of q simply solves:

$$q_s^* \equiv \max_q X(q) \tag{7}$$

The following proposition is proven in the appendix:

Proposition 2 *If the two optimization programs in (6) and (7) are well-defined, then a*

[16] Both stores not accepting can also be an equilibrium. Indeed, if only store 1 enters then its profits are:

$$(\pi - r)\left[\alpha F\left(u^H\right) + (1-\alpha)F\left(u^L\right)\right] - K$$

and similarly if only store 2 enters replacing α by $(1-\alpha)$. Again, store 1's profits are higher hence the following condition is necessary and sufficient so that, given r set so that $\Pi_2^S(r) = 0$, neither store enters if the other one doesn't:

$$(\pi - r)\left[\alpha F\left(u^H\right) + (1-\alpha)F\left(u^L\right)\right] < K$$

which is equivalent to:

$$\alpha F\left(u^H\right) + (1-\alpha)F\left(u^L\right) < (1-\alpha)F(u(q)) + \alpha\left[qF\left(u^L\right) + (1-q)F(u(q))\right]$$

If this condition is satisfied then we assume the platform has the ability to coordinate stores on its most preferred equilibrium, i.e. the one where both stores enter.

monopoly intermediary chooses a lower level of search effectiveness when store affiliation is endogenous than when it is exogenously given, i.e. $q_a^*(\alpha) < q_s^*$ for all $\alpha > \frac{1}{2}$.

■

This result identifies a new source of incentives for which the intermediary might divert search even further relative to the baseline case. The added incentive comes from the need to reduce the profit differential between the two stores while still ensuring the participation of the less profitable store - store 2. To see this, note that the sum of stores' *gross* profits (before paying the royalty to the intermediary) is $\pi X(q) - 2K$, whereas the difference between these gross profits is $\pi (2\alpha - 1) q \left[F(u(q)) - F\left(u^L\right)\right]$, which is *strictly increasing* in q if $\alpha > \frac{1}{2}$. By contrast, note that $q_a^*\left(\frac{1}{2}\right) = q_s^*$, i.e. if the two stores are symmetric and therefore have the same profits, then there is no additional need for the intermediary to divert search.

Furthermore, if the intermediary could price discriminate among stores or charge two-part tariffs with a fixed fee R and a variable fee r, then it would be able to fully extract the sum of stores' profits. It would then solve $\max_q \{\pi X(q)\}$, which means there is no longer an additional incentive to divert search *relative to* the basic model. By contrast, when the intermediary is restricted to variable fees r and no price discrimination and the participation of the less popular store (store 2) is binding, it has to give up a portion of the profit differential between the two stores, so that its profit maximization place a higher weight on maximizing revenues coming from store 2, which leads to a lower level of search effectiveness q. It is straightforward to show that this result is unchanged if instead of charging variable fees r the platform could only charge fixed access fees R.[17]

The key insight is that the platform can use q to reduce the variance of store profits when store participation is endogenous and it cannot perfectly price discriminate among them.

4.2 Endogenous store prices

We now shift our focus from stores' decisions to affiliate with the intermediary to stores' strategic decisions *once* they have joined. For simplicity, throughout this subsection stores' affiliation with the intermediary is taken once again as exogenously given. Stores' strategic decisions affect consumers' *ex-ante* expected utility from using the intermediary's service and therefore total consumer demand for that service. Consequently, the intermediary would like to influence these decisions by stores. Although throughout this section we will focus on pricing decisions, it should be clear that the analysis is readily generalizable to any other choices made by stores that affect consumer utility and search decisions: advertising expenditures, store design, product quality, etc.

[17] In this case, the intermediary would solve $\max_{R,q} 2R$ subject to $\pi\left\{(1-\alpha)F(u(q)) + \alpha\left[qF\left(u^L\right) + (1-q)F(u(q))\right]\right\} - R \geq K$. This is equivalent to $\max_q \{2\pi[\alpha X(q) - (2\alpha - 1)F(u(q))]\}$. Since $F(u(q))$ is increasing in q, the solution to this optimization problem is also strictly lower than $q_s^* = \arg\max_q X(q)$.

4.2.1 Additional modelling ingredients

To analyze the effect of the intermediary's incentives to influence stores' pricing decisions we need to put more structure on our basic model. In particular, we endogenize consumers' utilities from visiting stores by allowing the latter to set prices. Consumers are still of two types, *equally* distributed in the population (i.e. $\alpha = \frac{1}{2}$). The valuations v for the product sold by store i of type $i \in \{1, 2\}$ consumers are distributed according to cumulative distribution function $G^H(.)$, while the valuations for store $j \neq i$ of type i consumers are distributed according to cumulative distribution function $G^L(.)$. G^H and G^L have continuous densities $g^H(.)$ and $g^L(.)$ respectively, and the same support $[0, V]$.

The key assumption is that $G^H(.)$ has a lower hazard rate than $G^L(.)$, $\frac{g^H(\pi)}{1-G^H(\pi)} < \frac{g^L(\pi)}{1-G^L(\pi)}$, and both hazard rates are monotone. This assumption ensures that profit functions are well behaved and implies that G^H *stochastically dominates* G^L, i.e. $G^H(v) < G^L(v)$ for all v on the support. This stochastic dominance captures the fact that consumers of type i prefer store i to store $j \neq i$; it also means that demand from type i consumers is less elastic at store i than at store j.

A consumer of type i derives utility $u^k(p) \equiv \int_{v \geq p} (v - p)\, dG^k(v)$ from store j when the latter charges a price p, where $k = H$ if $i = j$ and $k = L$ if $i \neq j$. Using integration by parts, we have:

$$u^H(p) = \int_{v \geq p} \left(1 - G^H(v)\right) dv \geq \int_{v \geq p} \left(1 - G^L(v)\right) dv = u^L(p)$$

For convenience, we also define:

$$R^H(p) \equiv p\left(1 - G^H(p)\right) \quad \text{and} \quad R^L(p) \equiv p\left(1 - G^L(p)\right)$$

the revenues that a store derives from consumers for whom it is the favorite (respectively less preferred) store. Note that $R^H(p) > R^L(p)$ for all p on the support. The assumption above implies that $R^H(.)$ and $R^L(.)$ are strictly concave, continuously differentiable functions and:

$$p^H \equiv \arg\max_p R^H(p) > \arg\max_p R^L(p) \equiv p^L$$

In principle, the intermediary could be receiving several types of variable fees from affiliated stores: "per click" (i.e. per consumer visit) fees, nominal royalties or a percentage of sales. However, the nature of the variable fees in place does not affect the substance of our result: we show this formally in the companion paper. Therefore, for concision, we focus here on the case in which the intermediary extracts an exogenously fixed per click fee r of each store's sales revenues (perhaps the outcome of some prior bargaining game that we do not model).

Store i's profits when it charges p_i and store $j \neq i$ charges p_j are then:

$$\Pi_i^S(p_i, p_j, q) = \left[R^H(p_i) - r\right] \underbrace{\frac{1}{2} F\left(u(p_i, p_j, q)\right)}_{\substack{\text{store } i\text{'s traffic by} \\ \text{type } i \text{ consumers}}} + \left[R^L(p_i) - r\right] \underbrace{\frac{1}{2}\left[qF\left(u^L(p_i)\right) + (1-q) F\left(u(p_j, p_i, q)\right)\right]}_{\substack{\text{store } i\text{'s traffic by} \\ \text{type } j \text{ consumers}}} \} \tag{8}$$

where $u(p_i, p_j, q) \equiv \frac{u^H(p_i) + (1-q)u^L(p_j)}{2-q}$ is the average utility per search with endogenous store prices.

The timing of decisions is as follows: 1) the intermediary publicly commits to q; 2) stores choose their prices p_1 and p_2 simultaneously and non-cooperatively; 3) consumers observe q and form rational expectations over store prices, which they do not observe prior to search; 4) consumers decide whether or not to use the intermediary's informational service, engage in the search process and make their purchase decisions when arriving at each store.

The key implication of this set-up is that the intermediary's choice of q influences stores' pricing decisions. Indeed, q affects not only total consumer traffic to the intermediary and at each individual store, but also the composition of that traffic for each store - high and low valuation consumers. Given the symmetry between stores, the equilibrium in stage 2) has both stores choose the same price $p_1 = p_2 = p$, so that the intermediary's profits are:

$$\Pi^I(p, q) \equiv r\left[qF\left(u^L(p)\right) + (2-q) F\left(u(p, p, q)\right)\right]$$

We can then write (from the perspective of stage 1)):

$$\frac{d\Pi^I}{dq} = \frac{\partial \Pi^I}{\partial q} + \frac{\partial \Pi^I}{\partial p} \frac{dp}{dq} \tag{9}$$

where $p(q)$ is the equilibrium (symmetric) price charged by stores in stage 2) as a function of q.

The assumption that consumers do not observe store prices prior to searching is made for analytical tractability[18]: it does not alter our main result. Even if consumers were able to observe store prices prior to searching, each store still does not internalize the effect of its individual pricing decision on traffic to the *other* store.

4.2.2 Optimal search diversion

First, note that if store prices were exogenously fixed (for instance in order to prevent arbitrage through some other distribution channels) and symmetric - $p_1 = p_2 = p$ -, then the condition for

[18] One could also justify this assumption by noting that in reality stores may have stochastic costs, which are not publicly observable. If the uncertainty over these costs is only realized after the intermediary's "design" decisions (q) are made, then it is not optimal for stores to set prices prior to the resolution of uncertainty. Consequently, at the time q is set and observed by consumers, the latter simply form expectations over store prices.

some search diversion to be optimal is $\frac{\partial \Pi^I}{\partial q}(p, q=1) < 0$, which can also be written:

$$1 \leq \frac{F\left(u^H(p)\right) - F\left(u^L(p)\right)}{\left(u^H(p) - u^L(p)\right) f\left(u^H(p)\right)} \tag{10}$$

This condition closely parallels (5) above with $r_1 = r_2$.

Relative to this benchmark, allowing stores to adjust their prices in response to the intermediary's choice of q will decrease (respectively increase) the optimal level of search effectiveness q if and only if $\frac{\partial \Pi^I}{\partial p}\frac{dp}{dq} < 0$ (respectively $\frac{\partial \Pi^I}{\partial p}\frac{dp}{dq} > 0$).

Denote by p^e the symmetric price that consumers expect to encounter at each store. Then store i's profits from the perspective of stage 2) are:

$$\left[R^H(p_i) - r\right]\frac{1}{2}F\left(u\left(p^e, p^e, q\right)\right) + R^L(p_i)\frac{1}{2}\left[qF\left(u^L\left(p^e\right)\right) + (1-q)F\left(u\left(p^e, p^e, q\right)\right)\right]$$

Since consumers do not observe store prices prior to search but form rational expectations which have to be fulfilled in equilibrium, the equilibrium price $p^*(q)$ in stage 2) is the solution to:

$$p^* = \arg\max_p \left\{R^H(p)F\left(u\left(p^*, p^*, q\right)\right) + R^L(p)\left[qF\left(u^L\left(p^*\right)\right) + (1-q)F\left(u\left(p^*, p^*, q\right)\right)\right]\right\} \tag{11}$$

Equation (11) implicitly defines $p^*(q)$, a function which the following lemma shows is increasing.

Lemma 1 *Assume that (11) defines a unique function $q \to p^*(q)$. Then this function is increasing in q.*

Proof In the appendix.

■

This result is key to understanding the intermediary's incentives to divert search. A lower q gives a higher weight to the low valuation consumers in the composition of demand faced by each store, which drives the equilibrium price down.[19]

Turning now to the intermediary's profits, we have (note that $u(p, p, 1) = u^H(p)$):

$$\frac{\partial \Pi^I}{\partial p}(q=1) = r\left[f\left(u^L\left(p^*(1)\right)\right)\frac{du^L}{dp}\left(p^*(1)\right) + f\left(u^H\left(p^*(1)\right)\right)\frac{du^H}{dp}\left(p^*(1)\right)\right] < 0$$

since $\frac{du^L}{dp} < 0$ and $\frac{du^H}{dp} < 0$. But $\frac{dp^*}{dq} > 0$ by lemma 1, therefore expression (9) allows us to conclude:

Proposition 3 *When stores can adjust their prices after observing the intermediary's choice of search effectiveness q, the intermediary is more likely to divert search (i.e. to set $q < 1$) relative to the case in which store prices are fixed.*

[19] This conclusion would be unchanged if we allowed for a more general formulation, in which stores set a single price for their sales through the intermediary, as well as for sales that occur through other channels (e.g. store fronts on other platforms). In that case, it is reasonable to expect the "responsiveness" of store prices to the level of search effectiveness ($\frac{dp^*}{dq}$) will be reduced, but still positive.

■

The main reason for which the intermediary has an additional incentive to further divert search is that each store fails to internalize the effect of its price on total traffic on the intermediary (including traffic at the other store). Thus, stores tend to charge prices which are too high relative to what the intermediary would like.[20] The latter can mitigate this problem by lowering q, which increases the elasticity of demand faced by each store.

5 Contractual extensions and robustness

Having identified three distinct reasons for which an intermediary may design its information service in order to divert users' search, we now ask the natural question: couldn't such an intermediary achieve the same objective if it were allowed to use richer pricing and contracting instruments - on the consumer or on the store side? In this section we show that the trade-off between higher total consumer demand for the service and higher revenues per consumer visit cannot in general be entirely resolved by the introduction of a variety of additional instruments - access fees charged to consumers, screening, search subsidies and contracting over store prices.

Throughout this section we will make the following additional assumption:

Assumption 2 *For all* $u \in \left[u^L, u^H\right]$, $\frac{\partial}{\partial u}\left[\frac{F(u)-F\left(u^L\right)}{(u-u^L)f(u)}\right] > 0$.

It is useful to note that assumption 2 implies $f'(u) > 0$ (i.e. F is concave) and $\frac{\partial}{\partial u}\left[\frac{F\left(u^H\right)-F(u)}{(u^H-u)f(u^H)}\right] < 0$ for all $u \in \left[u^L, u^H\right]$.

Assumption 2 helps streamline the analysis that follows (it essentially ensures the concavity of all objective functions we consider) without affecting the broader conclusions. In fact, it implies that, *considered separately*, access fees and search subsidies both reduce the need for search diversion. Even under this assumption however, the only case in which we are able to *entirely* eliminate search diversion is that of a bottleneck intermediary, which can *both* charge fixed access fees *and* subsidize consumers' second search.

5.1 Access fees charged to consumers

We start with the simplest additional pricing instrument: suppose that the intermediary can charge consumers a fixed fee $A \geq 0$ in exchange for access to its service.[21] While most shopping malls,

[20] In the companion paper we show that this result is robust when we change the nature of the variable fees received by the intermediary from stores from per-click (per visit) fees to per-sales fees.

[21] We could envision negative fees but then all consumers would join the platform and only those with a positive expected suprplus from search would engage in active search. The outcome would be the same as with $A = 0$ but with additional lump-sum payments to consumers.

retailers and online portals are free of access to consumers, some do charge access fees: examples include warehouse clubs (e.g. Costco and Sam's Club), which charge annual subscription fees; RealNetworks' Rhapsody music service, which charges users monthly subscription fees.

Here the distinction between the 3 intermediary types described in section 2 above matters. Indeed, the effects of charging positive access fees on consumer demand and intermediary profits depend on the alternatives available to consumers (can they access stores without using the informational service? does the intermediary get paid whenever consumers access stores or only when they do so after having used its informational service?).

First, consider an intermediary of type (A) (bottleneck): consumers must pay A to access stores, regardless of whether they actually use the information service or not. In this case, consumers with $c \leq u^L$ shop at both stores and derive net utility $u^H + u^L - 2c - A$. Therefore they use the intermediary's service if and only if $c \leq \frac{u^H + u^L - A}{2}$. Consumers with $c \geq u^L$ stop as soon as they find their favorite store: if $q \geq \frac{1}{2}$ then they use the intermediary's information service and their net utility from visiting is $u^H + (1-q)\, u^L - (2-q)\, c - A$. Therefore they visit if and only if $c \leq \frac{u^H + (1-q) u^L - A}{2-q} \equiv u(q, A)$. For large access fees, $A \geq u^H - u^L$, all consumers who visit the intermediary search twice (i.e. they all have $c \leq u^L$). Then the quality of search becomes irrelevant. So we focus on the case where a positive measure of consumers with $c > u^L$ participate, i.e. $0 < A < u^H - u^L$. The expression of the intermediary's profits is then:

$$\left(\widetilde{r}_1 + \widetilde{r}_2 + A\right) F\left(u^L\right) + \left(\widetilde{r}_1 + (1-q)\, \widetilde{r}_2 + A\right) \left[F\left(u\left(q, A\right)\right) - F\left(u^L\right)\right] \tag{12}$$

Second, consider an intermediary of type (B) (pure recommender). For any $A > 0$, consumers with $c \leq u^L$ do not use the intermediary's service because they always search both stores and therefore are indifferent to whether or not they get diverted during the first search. Consumers with $c > u^L$ attach a positive value to better information. They will use the recommender service only if $c \leq u(q, A)$. But in addition to this condition, their utility $u^H + (1-q)\, u^L - (2-q)\, c - A$ must also be larger than the value of their outside option (searching by themselves), which is now $u^H + \frac{1}{2} u^L - \frac{3}{2} c$ instead of 0. This condition is $c \geq u^L + \frac{2A}{2q-1}$. The demand for the intermediary's service is positive only if $A < \frac{2q-1}{3}\left(u^H - u^L\right)$, and in this case the expression of profits is:

$$\left(\widetilde{r}_1 + (1-q)\, \widetilde{r}_2 + A\right) \left[F\left(u\left(q, A\right)\right) - F\left(u^L + \frac{2A}{2q-1}\right)\right] \tag{13}$$

Third, consider an intermediary of type (C). Consumers' search decisions are the same as for a type (B) intermediary, but a type (C) intermediary gets paid more often, therefore its profits are:

$$\begin{aligned} &\left(\widetilde{r}_1 + \widetilde{r}_2\right) F\left(u^L\right) + \left(\widetilde{r}_1 + \frac{1}{2}\widetilde{r}_2\right) \left[F\left(u^L + \frac{2A}{2q-1}\right) - F\left(u^L\right)\right] \\ &+ \left(\widetilde{r}_1 + (1-q)\, \widetilde{r}_2 + A\right) \left[F\left(u\left(q, A\right)\right) - F\left(u^L + \frac{2A}{2q-1}\right)\right] \end{aligned} \tag{14}$$

In what follows, we will contrast the optimal search effectiveness choices of a type (A) and a type (B) intermediary. No additional insights are obtained by also including the analysis of type (C) intermediaries, therefore we omit it.

5.1.1 The bottleneck intermediary

Let $A^* \equiv \arg\max_A \ (\widetilde{r_1} + A) F\left(u^H - A\right)$ be the profit maximizing consumer access fee for a bottleneck intermediary *conditional on* choosing $q = 1$. Note that $A^* > 0$ if and only if $\widetilde{r_1} < \frac{F\left(u^H\right)}{f\left(u^H\right)}$. Since (12) is obtained from (3) simply replacing u^H by $u^H - A^*$ and $\widetilde{r_1}$ by $\widetilde{r_1} + A^*$, we can repeat the analysis in section 3.2 for a fixed value $A = A^*$ to obtain:

Proposition 4 *If* $\frac{F\left(u^L\right)}{f\left(u^L\right)} + u^L - u^H < \widetilde{r_1}$ *then a bottleneck intermediary who can charge fixed access fees to consumers will divert search by setting* $q^* < 1$ *if and only if:*

$$\frac{\widetilde{r_1} + A^*}{\widetilde{r_2}} < \frac{F\left(u^H - A^*\right) - F\left(u^L\right)}{\left(u^H - u^L - A^*\right) f\left(u^H - A^*\right)}$$

Proof The condition in the text of the proposition simply ensures that $A^* < u^H - u^L$; otherwise the intermediary does not attract any consumers with $c > u^L$ and therefore q is irrelevant.

■

Under assumption 2, allowing for access fees *reduces* the likelihood that the intermediary diverts search (cf. Proposition 1 d)). But provided that the fee A^* remains small, some search diversion still exists. This is not surprising: allowing the intermediary to charge access fees makes it place more weight on increasing consumer traffic relative to deriving more revenues per consumer visit, which is consistent with increasing the effectiveness of search.

5.1.2 The pure recommender

As soon as a pure recommender charges an arbitrarily small but positive access fee, it loses a positive amount of revenues because a positive but finite mass of consumers stop using the intermediary's service. Setting a positive access fee is profitable only if the revenues $\widetilde{r_1}$ and $\widetilde{r_2}$ are small or if the mass of consumers with low search cost is small. Suppose therefore that it is profitable to set a positive access fee.

As above we define $A^{**} \equiv \arg\max_A \ (\widetilde{r_1} + A)\left[F\left(u^H - A\right) - F\left(u^L + 2A\right)\right]$ as the profit maximizing access fee conditional on no search diversion, i.e. $q = 1$. Note that $A^{**} > 0$ if and only if $\widetilde{r_1} < \frac{F\left(u^H\right) - F\left(u^L\right)}{f\left(u^H\right) + 2f\left(u^L\right)}$. Moreover, $A^{**} < A^*$, which is not surprising: all other things equal, the fixed fee that a pure recommender can charge will generally be lower than what a bottleneck intermediary can charge since the latter controls and bundles access with information. Using the same analysis as above, we obtain:

Proposition 5 *A pure recommender who chargea fixed access fees to consumers will divert search by setting $q^* < 1$ if and only if:*

$$\frac{\widetilde{r_1} + A^{**}}{\widetilde{r_2}} < \frac{F\left(u^H - A^{**}\right) - F\left(u^L + 2A^{**}\right)}{\left(u^H - u^L - A^{**}\right) f\left(u^H - A^{**}\right) + 4A^{**} f\left(u^L + 2A^{**}\right)}$$

Proof The condition in the text of the proposition simply ensures that $0 < A^{**} < \frac{1}{3}\left(u^H - u^L\right)$. ■

Again, assumption 2 implies that the intermediary is less likely to divert search when it charges positive access fees (relative to the baseline case with no access fees). However, the need to divert search does not go away completely. Furthermore, we have the additional result:

Corollary *If assumption 2 holds, then a pure recommender will divert search more often than a bottleneck intermediary.*

■

This result is understood as follows. A pure recommender does not control nor does it get compensated for consumers' access to stores, therefore it has to charge lower access fees than the bottleneck intermediary all other things being equal. Therefore, the trade-off between higher revenues per consumer visit and higher realized consumer demand for the intermediary's service is more skewed in favor of the former in the case of a pure recommender, which means it has more incentives to divert search. By contrast, in the baseline case when access fees were not allowed, both types of intermediaries were choosing the exact same level of search effectiveness.

5.2 Screening of consumers

So far, we have considered the choice of q as an *ex-ante* technological choice that applies to all consumers. Suppose now that the intermediary is able to *both* charge consumer access fees *and* commit to individualized search qualities: in other words, it can use menus to screen between different types of consumers (according to their search costs). An example of a real world intermediary that uses such screening contracts is Pandora, an Internet radio and online music recommender, which offers two versions of its service: a free one supported by advertising and a fee-based subscription with no advertisements.

We assume that the intermediary can offer any level of search effectiveness q between $\frac{1}{2}$ and 1 and a tariff $A(q)$ (non-decreasing in q).[22] Consumers self-select by choosing q. Finally, just like in the previous subsection, we will distinguish between the two polar types of intermediaries: bottleneck and pure recommender.

Before analyzing this case, we should point out that this raises some issues on the store side of the market. If the intermediary can offer consumers different informational services at different

[22] If $A(q_1) > A(q_2)$ for some $q_1 < q_2$, no consumer would ever choose the option $(q_1, A(q_1))$.

access fees, it might also be able to charge stores different prices (i.e. the fees r_i) according to the levels of search effectiveness chosen by consumers. However this would require the ability to credibly verify the informational service used by a given consumer to find a given store. In what follows we assume that this is not feasible and that the intermediary bundles all informational services under the same tariff *for stores* (or simply that it owns the stores). Thus, we continue to take the revenues $\widetilde{r_1}$ and $\widetilde{r_2}$ as given and assume that they are independent of q.

First, the linearity of utility and of the profit function in q imply the following lemma (which we prove in the appendix):

Lemma 2 *For both a bottleneck and a pure recommender, the optimal screening contract consists in offering only the lowest ($q = \frac{1}{2}$) and the highest ($q = 1$) levels of search effectiveness (as opposed to a continuum).*[23]

■

Consequently, we can restrict attention to contracts that propose to choose between $q = \frac{1}{2}$ at price A_l and $q = 1$ at price $A_h > A_l$. The bottleneck intermediary can set $A_l > 0$, while the pure recommender *must* set $A_l = 0$ (otherwise no consumer will ever take the $\left(q = \frac{1}{2}, A_l\right)$ offer). Then, consumers with $c < u^H - A_h \equiv c_h$ are willing to pay for the "high-quality" (i.e. high search effectiveness) service, and consumers prefer the low quality if $c < c_l \equiv u^L + 2\left(A_h - A_l\right)$. For both types of intermediaries, all consumers with $c \leq c_h$ use the intermediary's service.[24]

The intermediary's profits can be written as:

$$\frac{1}{2}\widetilde{r_2}F\left(u^L\right) + \left(\frac{1}{2}\widetilde{r_2} + A_l - A_h\right)F\left(c_l\right) + \left(\widetilde{r_1} + A_h\right)F\left(c_h\right)$$

The optimal contract results in a positive measure of consumers being diverted if it is such that $c_l > u^L$ and involves some screening if it is such that $c_h > c_l$. Indeed, a contract resulting in $c_l = u^L$ would correspond to the highest quality of search only being offered, while a contracting resulting in $c_h = c_l$ would correspond to the lowest quality only being offered.

Consider first the case of a bottleneck intermediary, able to choose both A_l and A_h, or - equivalently - c_l and c_h. It maximizes:

$$\frac{1}{2}\widetilde{r_2}F\left(u^L\right) + \frac{1}{2}\left(\widetilde{r_2} + u^L - c_l\right)F\left(c_l\right) + \left(\widetilde{r_1} + u^H - c_h\right)F\left(c_h\right)$$

[23]The result in this lemma is due to the fact that with only two stores, consumers' expected utility is linear in q. If for instance there were more than two stores, only one of which yielded utility u^H rather than $u^{;L}$ then consumers would do more than two rounds of search with positive probability and their expected utility would be non-linear in q. In that case the optimal screening contract will also contain interior q's. We thank Stephane Behringer for this remark.

[24]In the case of a bottleneck, note that any consumer with $c < c_h$ can derive positive utility by taking the same offer as consumer c_h. In the case of a pure recommender, note in addition that the intermediary's $\left(q = \frac{1}{2}, 0\right)$ offer replicates the option of searching by herself for any consumer.

Optimizing over c_l and c_h (under assumption 2 the previous expression is concave in both c_l and c_h), we obtain:

Proposition 6 *A bottleneck intermediary who can offer screening contracts diverts a positive measure of consumers if and only if* $\widetilde{r}_2 > \frac{F(u^L)}{f(u^L)}$.

■

Consider now a pure recommender, who must set $A_l = 0$. It maximizes:

$$\frac{1}{2}\widetilde{r}_2 F\left(u^L\right) + \left(\frac{1}{2}\widetilde{r}_2 - A_h\right) F\left(u^L + 2A_h\right) + \left(\widetilde{r}_1 + A_h\right) F\left(u^H - A_h\right)$$

over A_h (again, assumption 2 guarantees concavity of the objective function). The intermediary offers a service with diverted search if and only if $A_h > 0$, which yields:

Proposition 7 *A pure recommender intermediary who can offer screening contracts diverts a positive measure of consumers if and only if* $\widetilde{r}_2 > \frac{F(u^L)}{f(u^L)} + \frac{\widetilde{r}_1 f(u^H) - F(u^H)}{f(u^L)}$.

■

First note that in order for the screening contract offered by a bottleneck to be well defined (in particular $c_h < u_h$), we must have $\widetilde{r}_1 f\left(u^H\right) < F\left(u^H\right)$. This implies that the pure recommender is more likely to divert a positive measure of consumers than a bottleneck when they can offer screening contracts. This result is consistent with the corollary to Proposition 5 above: the pure recommender resorts to search diversion more often than the bottleneck because it has less rent-extraction power through fixed fees.

5.3 Search subsidies

Since search diversion is essentially driven by the fact that consumers search too little from the intermediary's perspective, a third natural layer of contractual enrichment is to allow the intermediary to subsidize consumer search. For example, Microsoft's Live search cashback service pays consumers cash back on purchases made from certain advertisers conditional on using Live Search - Microsoft's search engine - to find the products.[25] Similarly, Amazon's Prime membership program offers users free (or discounted) shipping on select items in exchange for an annual membership fee of \$79.

Suppose first that the intermediary can only offer a subsidy $s \geq 0$ for each search performed by each consumer that uses its service. Then the intermediary's profits are (note that this expression is valid for all intermediary types since there are no access fees):

$$\left(\widetilde{r}_1 + \widetilde{r}_2 - 2s\right) F\left(u^L + s\right) + \left[\widetilde{r}_1 - s + (1-q)\left(\widetilde{r}_2 - s\right)\right]\left[F\left(u\left(q,s\right)\right) - F\left(u^L + s\right)\right]$$

[25] One of the authors has taken advantage of this offer and received 5% cash back on the purchase of a flat-screen television bought online from Circuit City (and found through Live Search).

where $u(q,s) \equiv \frac{u^H - s + (1-q)\left(u^L - s\right)}{2-q}$. The effect of the search subsidy is thus equivalent to replacing $\left(u^H, u^L, \widetilde{r}_1, \widetilde{r}_2\right)$ by $\left(u^H + s, u^L + s, \widetilde{r}_1 - s, \widetilde{r}_2 - s\right)$.

Denote by s^* the optimal search subsidy conditional on setting $q = 1$. Then $s^* > 0$ if and only if $\widetilde{r}_1 f\left(u^H\right) - F\left(u^H\right) + \widetilde{r}_2 f\left(u^L\right) - F\left(u^L\right) > 0$ and it is optimal to divert search ($q^* < 1$) if and only if:

$$\frac{\widetilde{r}_1 - s^*}{\widetilde{r}_2 - s^*} < \frac{F\left(u^H + s^*\right) - F\left(u^L + s^*\right)}{\left(u^H - u^L\right) f\left(u^H + s^*\right)}$$

Thus, the effect of being able to offer search subsidies to *both* searches is ambiguous: it could either increase or decrease the need to divert search.

A better alternative is then to only offer a subsidy to the second search. In this case, the intermediary's profits are:

$$\left(\widetilde{r}_1 + \widetilde{r}_2 - s\right) F\left(u^L + s\right) + \left[\widetilde{r}_1 + (1-q)\left(\widetilde{r}_2 - s\right)\right]\left[F\left(u(q,s)\right) - F\left(u^L + s\right)\right]$$

where $u(q,s) \equiv \frac{u^H + (1-q)\left(u^L - s\right)}{2-q}$.

Denote by s^* the optimal search subsidy conditional on setting $q = 1$. Then $s^* > 0$ if and only if $\widetilde{r}_2 f\left(u^L\right) > 0$, which is always true (not surprisingly, the intermediary would always find it profitable to subsidize the second search when there is no search diversion). The condition for search diversion is now:

$$\frac{\widetilde{r}_1}{\widetilde{r}_2 - s^*} < \frac{F\left(u^H\right) - F\left(u^L + s^*\right)}{\left(u^H - u^L - s^*\right) f\left(u^H\right)}$$

Thus, under assumption 2, being able to subsidize the second search only makes search diversion *less* likely. However, it may still not entirely eliminate the need to divert search.

Consequently, we now allow the intermediary to both charge an access fee A *and* subsidize the second search with $s \geq 0$. The respective expressions of a bottleneck intermediary's and of a pure recommender's profits are different. For the bottleneck, the expression of profits is:

$$\left(\widetilde{r}_1 + \widetilde{r}_2 + A - s\right) F\left(u^L + s\right) + \left[\widetilde{r}_1 + A + (1-q)\left(\widetilde{r}_2 - s\right)\right]\left[F\left(u(q,s,A)\right) - F\left(u^L + s\right)\right]$$

where $u(q,s,A) \equiv \frac{u^H - A + (1-q)\left(u^L - s\right)}{2-q}$. Proceeding in the same way as above, denote by s^* and A_1^* the optimal search subsidy and access fee conditional on setting $q = 1$. Then s^* is the solution to $\frac{F\left(u^L + s\right)}{f\left(u^L + s\right)} = r_2 - s$ and A^* is the solution to $\frac{F\left(u^H - A\right)}{f\left(u^H - A\right)} = r_1 + A$. Therefore, it is optimal to set $q^* < 1$ if and only if:

$$\frac{r_1 + A_1^*}{r_2 - s^*} < \frac{F\left(u^H - A_1^*\right) - F\left(u^L + s^*\right)}{\left(u^H - u^L - A_1^* - s^*\right) f\left(u^H - A_1^*\right)}$$

which is equivalent to:

$$\frac{F\left(u^H - A_1^*\right)}{F\left(u^L + s^*\right)} < \frac{F\left(u^H - A_1^*\right) - F\left(u^L + s^*\right)}{\left(u^H - u^L - A_1^* - s^*\right) f\left(u^L + s^*\right)}$$

But $u^H - A^* > u^L + s^*$ (otherwise the intermediary's service would not be used by consumers with $c > u^L + s^*$ and the level of search effectiveness would be irrelevant), therefore under assumption 2 (which implies that F is concave) the inequality above cannot hold and $q^* = 1$.

Proposition 8 *If the intermediary can implement access fees and search subsidies for the second search, then it will not divert search whenever Assumption 2 holds.*

■

The treatment of the pure recommender case is more cumbersome (we relegate it to the appendix), but most importantly, even under Assumption 2, access fees and search subsidies need *not* be sufficient for eliminating the need for search diversion, as in the bottleneck intermediary case. This is not all that surprising given that a pure recommender has more incentives to divert search than a bottleneck all other things equal.

5.4 Full ownership and control of stores by the intermediary

We have seen in section 4.1 that when store participation is endogenous, a richer set of pricing instruments (two part tariffs and/or the ability to freely price discriminate among stores) eliminates the need to divert search *more* than in the baseline model in order to reduce the profit differential between more popular and less popular stores. Similarly, a straightforward implication of the analysis in section 4.2 is that if the intermediary could contract upon or fully control store prices *and* publicly announce these prices to consumers before the latter engage in search, then the intermediary would no longer need to divert search further *relative to* the baseline model in order to influence stores' pricing decisions. Still, one may wonder whether it is possible to *entirely* eliminate the incentives to divert search by allowing the intermediary to acquire full ownership and control over stores.

We use the model from section 4.2 with elastic consumer demand for each store. We now allow the intermediary and the stores to contract on store prices *and* consumers to observe them prior to visiting the intermediary - this is the *least* favorable case for search diversion. The simplest contracting game has the intermediary make a take-it-or-leave-it offer to stores, which transfers complete ownership and control (in particular over pricing decisions) to the intermediary in exchange for a fixed payment. There is no longer any need to divert search in order to influence stores' pricing decisions and everything is as if the intermediary were vertically integrated into stores. Its profits are:

$$\Pi^I(p,q) = R^H(p)\,F(u(p,q)) + R^L(p)\left[qF\left(u^L(p)\right) + (1-q)\,F(u(p,q))\right] \tag{15}$$

where $u(p,q) \equiv \frac{u^H(p)+(1-q)u^L(p)}{2-q}$ and $u^k(p)$ as defined in 4.2.1 above $(k = L, H)$. Denote by p_1^* the optimal store price[26] conditional on setting $q = 1$, i.e. $p_1^* = \arg\max_p \left\{R^H(p)\,F\left(u^H(p)\right) + R^L(p)\,F\left(u^L(p)\right)\right\}$.

[26] By symmetry, the optimal price is the same for both stores.

The condition for the optimal q to be less than 1 is then $\frac{\partial \Pi^P}{\partial q}(p_1^*, 1) < 0$, which can be written as:

$$R^H(p_1^*) f\left(u^H(p_1^*)\right)\left[u^H(p_1^*) - u^L(p_1^*)\right] + R^L(p_1^*)\left[F\left(u^L(p_1^*)\right) - F\left(u^H(p_1^*)\right)\right] < 0$$

or:

$$\frac{R^H(p_1^*)}{R^L(p_1^*)} < \frac{F\left(u^L(p_1^*)\right) - F\left(u^H(p_1^*)\right)}{f\left(u^H(p_1^*)\right)\left[u^H(p_1^*) - u^L(p_1^*)\right]}$$

Thus, even when the store prices are contractible prior to consumers making their search decisions, the intermediary may still find it optimal to divert search in order to trade off higher traffic for higher revenues per consumer visit. This result extends the analysis of the basic model in section 3.2 (recall condition (5)) by endogenizing the utilities consumers derive from visiting each store.

Furthermore, recall that with $\alpha = \frac{1}{2}$, (5) was:

$$1 < \frac{F\left(u^L\right) - F\left(u^H\right)}{f\left(u^H\right)\left(u^H - u^L\right)}$$

Thus, since $R^H(p_1^*) > R^L(p_1^*)$, the incentives to divert search are now lower than in the basic model with two consumer types (everything else equal). The reason is that by owning stores and perfectly internalizing their profits, the intermediary's incentives are more aligned with those of the stores. This result helps explain why search diversion is a widespread feature, not just of two-sided platform-like intermediaries who do not control the pricing of the stores they provide access to (e.g. shopping malls), but also of intermediaries who have full control over product pricing and layout in stores. It is therefore not surprising that the classic supermarket tactic of placing essential staples as far from the entrance as possible has also been adopted by more modern retailers. For instance, Apple Stores place the iPods and iPhones - the most popular products - at the back or on the second floor, while the most prominent displays are dedicated to Macintosh computers.

6 Discussion

In the companion paper, we provide several variations of our basic model (exogenous revenues from consumer traffic, substitutability or complementarity between stores, stochastic and independent store values). These variations are meant to emphasize the general nature of the fundamental economic mechanism driving intermediaries' incentives to divert search that we have uncovered in this paper and to illustrate the flexibility of our basic model.

In the rest of this section, we briefly tackle two other issues: the welfare implications of search diversion and the effect of competition between intermediaries on the level of search effectiveness provided.

6.1 Welfare

To some extent, search diversion can be seen as an attempt by intermediaries to address the following "market failure": consumers do not internalize the benefits bestowed on their potential trading partners (stores) when they make their search decisions. This provides the first fundamental reason for which some search diversion may result in higher social welfare than none at all.

To be more precise, note that consumer surplus (and traffic) is increasing in the effectiveness of search. Indeed, the average utility per search $u(q)$ (cf. (2) above) is increasing in q and total consumer surplus can be written:

$$CS(q) = \int_0^{u^L} \left(u^H + u^L - 2c\right) f(c)\, dc + \int_{u^L}^{u(q)} \left(u^H + (1-q)\, u^L - (2-q)\, c\right) f(c)\, dc$$

where the first term (surplus of consumers with $c \leq u^L$) is independent of q and the second term is clearly increasing in q. Thus, from consumers' perspective, the monopoly level of search effectiveness q^* is always too low - their surplus is maximized for $q = 1$.

Consider now the simplest possible model of store profits: suppose store i's profits are constant per consumer visit and denoted by π_i, with $i \in \{1, 2\}$, while r_1 and r_2 are simply transfers from the stores to the intermediary ($r_i \leq \pi_i$). Then, total gross store surplus is:

$$SS(q) = (\widetilde{\pi_1} + \widetilde{\pi_2})\, F\left(u^L\right) + [\widetilde{\pi_1} + (1-q)\, \widetilde{\pi_2}] \left[F(u(q)) - F\left(u^L\right)\right]$$

where $\widetilde{\pi_1} = \alpha\pi_1 + (1-\alpha)\,\pi_2$ and $\widetilde{\pi_2} = \alpha\pi_2 + (1-\alpha)\,\pi_1$. The expression above is obtained from (3) simply by replacing $\widetilde{r_i}$ with $\widetilde{\pi_i}$. Note that when stores are fully owned by the intermediary, we have $\pi_i \equiv r_i$ and therefore the level of search effectiveness that maximizes total store surplus is exactly q^*. In this particular case, since $CS(.)$ is increasing, the level of search effectiveness q^W that maximizes total social welfare $W(q) = CS(q) + SS(q)$ is strictly higher than the monopoly choice q^* - but q^W may still be less than 1. In the general case, $r_i \leq \pi_i$ and the ratio $\frac{\widetilde{r_1}}{\widetilde{r_2}}$ can be either higher or lower than $\frac{\widetilde{\pi_1}}{\widetilde{\pi_2}}$[27], therefore q^W can be either higher or lower than q^*.

In fact, it is straightforward to show that some search diversion is socially optimal (i.e. $q_W < 1$) if and only if[28]:

$$\frac{\widetilde{\pi_1}}{\widetilde{\pi_2} + u^L - \frac{\int_{u^L}^{u^H} cf(c)dc}{F(u^H) - F(u^L)}} < \frac{F\left(u^H\right) - F\left(u^L\right)}{\left(u^H - u^L\right) f\left(u^H\right)}$$

The interpretation of this condition is similar to the one of (5), except that the social planner takes into account total store *and* consumer surplus for each consumer, as well as the search costs incurred. Thus, lowering the effectiveness of search by ε (very small) below 1 has two effects on total social welfare. The first one is negative and corresponds to the loss of the consumers with highest search costs. Its absolute value is $\left(\pi_1 + u^H\right) \left[F\left(u^H\right) - F(u(1-\varepsilon))\right] - \int_{u(1-\varepsilon)}^{u^H} cf(c)\, dc$, which is

[27] Recall that $\arg\max_q \Pi^I(q)$ only depends on $\frac{\widetilde{r_1}}{\widetilde{r_2}}$ and is increasing in this ratio and similarly for $\arg\max_q SS(q)$ and $\frac{\widetilde{\pi_1}}{\widetilde{\pi_2}}$.

[28] Assume for simplicity $\widetilde{\pi_2} + u^L \geq u^H$, which guarantees that both sides of the inequality are positive.

approximately equal to $\pi_1 \varepsilon \left(u^H - u^L\right) f\left(u^H\right)$. The second effect is positive and corresponds to the increase in total surplus generated by the fraction ε of consumers with $c \in \left[u^L, u\left(1-\varepsilon\right)\right]$ who now also visit store 2 ($\left(\pi_2 + u^L\right) \varepsilon \left[F\left(u^H\right) - F\left(u^L\right)\right]$), net of their additional search costs ($\varepsilon \int_{u^L}^{u(1-\varepsilon)} cf\left(c\right) dc$). Comparing the relative magnitudes of these two effects yields the condition above. Note in particular that the left-hand side can be higher or lower than the left-hand side of (5).

There are two other reasons for which some search diversion may increase social welfare relative to $q = 1$ - in fact it may even increase consumer surplus. First, as we have seen in section 4.1, search diversion reduces the difference in profits between more popular and less popular stores. Consequently, if store entry is endogenous and there are fixed costs of entry, then more search diversion may result in more stores entering the market - i.e. affiliating with a particular intermediary. This may increase consumer surplus to an extent which in principle might outweigh the direct negative effect of less effective search. Second, in relation to our model with endogenous store prices (section 4.2), more search diversion *decreases* store prices, which again has a countervailing positive effect on consumer surplus.

There are other potential social benefits of search diversion which are not directly captured in our model. An intriguing one is suggested by a recent study (Evans (2008)) of the impact of electronic publication on the way scholarly research is conducted. The key finding is that widespread electronic availability of journals and articles has made search "*too*" efficient, in the sense that scholars tend to "browse" and cite less and narrower material (the one closely related to the research at hand). By contrast, in the former print-only world, researchers were "forced" to browse more articles (perhaps less relevant a priori to what they were searching for in the first place), which may have resulted in a higher probability of novel and broader ideas. The author concludes that: "*This research ironically intimates that one of the chief values of print library research is poor indexing. Poor indexing—indexing by titles and authors, primarily within core journals— likely had unintended consequences that assisted the integration of science and scholarship. By drawing researchers through unrelated articles, print browsing and perusal may have facilitated broader comparisons and led researchers into the past.*"

We leave the formulation of a model that would explicitly capture this type of "serendipity" benefits of search diversion for future research.

6.2 Competition among intermediaries

For the purposes of this paper, we have chosen to focus on the choice of search effectiveness by a monopoly intermediary. If there are multiple intermediaries, one might be inclined to expect that the resulting level of search effectiveness to increase - perhaps all the way to 1. While a complete analysis of competing intermediaries is beyond the scope of this paper, a simple extension of our basic model suggests that this need not be the case.

To begin with, note that the intermediaries we analyze here act as "two-sided platforms" connecting consumers and stores. As mentioned earlier, consumers *always* prefer higher levels of search effectiveness (since $u(q)$ is increasing in q). However, stores' profits are generally maximized for $q < 1$. As a consequence, depending on the nature of competition between intermediaries - in particular on the intensity of competition for stores relative to the intensity of competition for consumers -, the resulting level of search effectiveness might place a higher weight on maximizing consumer surplus or stores' profits. In other words, given that two-sided platforms have to balance the interests of the two sides they serve, competition will tend to make the balance tilt in favor of the side that needs to be "courted" more assiduously by the intermediaries.

Suppose then that there are two stores - 1 and 2 - and two intermediaries - A and B - and consider two extreme cases. First, assume that both stores multihome, i.e. are accessible through both intermediaries. If intermediaries are undifferentiated from consumers' perspective and consumers only visit one intermediary, then intermediaries will focus their competitive efforts on the consumer side. They will set q to maximize consumer surplus, which leads to $q^* = 1$.[29]

Conversely, suppose now that stores only affiliate with one intermediary and that consumers can costlessly multihome, i.e. visit both intermediaries. Then competition will result in the maximization of store surplus, which generally requires $q^* < 1$. In fact, depending on how one models the bargaining game between stores and intermediaries, it is quite possible that the equilibrium level of search effectiveness with competing intermediaries might be lower than that chosen by a monopoly intermediary.

7 Conclusion

We have done two things in this paper. First, we have shown that an intermediary can have three fundamental motives for diverting search. The most basic one is in order to trade off higher consumer demand for its information service against higher revenues per consumer visit (making each consumer shop more). The second and third motives relate to the intermediary's incentives to control stores' profitability and strategic decisions. Whenever store affiliation with the intermediary is endogenous and the intermediary's profits place more weight on revenues extracted from marginal (less popular) stores relative to infra-marginal (more popular) stores, the intermediary has an incentive to divert search even more in order to reduce the differential in profits between marginal and infra-marginal stores. And once stores' affiliation decisions are made, the intermediary would like to induce each individual store to internalize the effect of its decisions such as pricing on total consumer demand as well as demand for the other affiliated stores - this can be achieved by using q to manipulate the composition of demand faced by each store.

Second, we have shown in section 5 that the need to divert search is quite resilient even when the contracting space and the set of pricing instruments available to the intermediary are significantly enriched. Such robustness suggests that search diversion is an essential strategic instrument

[29] With Hotelling differentiation among intermediaries and unit transport costs t, we would expect to find: $q^*(t) < 1$; $\frac{dq^*}{dt} < 0$.

for intermediaries and its use should be widespread - which is consistent with numerous casual observations.

There are a few promising extensions to explore. Analyzing the effect of competition among intermediaries on the effectiveness of the search service offered is the most immediate: we have provided some basic intuition for that in section 6.2, but a formal analysis is warranted. Secondly, one could look at a dynamic setting in which the quality of search (q) is not known to consumers *ex-ante*, but some of them are repeat visitors. In this case, the platform has to take into account the fact that consumers form expectations of q based on their past experience and use adequate decision rules to decide whether or not to visit again. Third, as suggested at the end of section 5.1, our model could be extended to encompass benefits of search diversion which appear in contexts such as dynamic problem-solving and scientific research.

8 References

References

[1] Avery, C., P. Resnick and R. Zeckhauser (1999) "The Market for Evaluations," *American Economic Review*, Vol. 89, No. 3., 564-584.

[2] Armstrong, M. (2006) "Competition in Two-Sided Markets," *Rand Journal of Economics*, Vol. 37 (3), 669-691.

[3] Biglaiser, G. (1993) "Middlemen As Experts," *Rand Journal of Economics*, Vol. 24, 212-223.

[4] Caillaud, B., and B. Jullien (2003) "Chicken and Egg: Competition Among Intermediation Service Providers," *Rand Journal of Economics,* 34(2), 309-328.

[5] Hervas-Drane, A. (2007) "Word of Mouth and Recommender Systems: A Theory of the Long Tail," working paper, Harvard Business School.

[6] Elberse, A., A. Hagiu and M. Egawa (2007) "Roppongi Hills: City Within A City," Harvard Business School Case Study 9-707-431.

[7] Ellison, G. and Ellison, S. F. (2005) "Search, Obfuscation, and Price Elasticities on the Internet," working paper, MIT and NBER.

[8] Evans, D. S. (2003) "The Antitrust Economics of Multi-Sided Platform Markets," *Yale Journal on Regulation,* 20 (2), 325-82.

[9] Evans, J. A. (2008) "Electronic Publication and the Narrowing of Science and Scholarship," *Science*, Vol. 321, 395-399.

[10] Gehrig, T. (1993) "Intermediation in Search Markets," *Journal of Economics and Management Strategy* Vol. 2, 97-120.

[11] Hagiu, A. (2006) "Pricing and Commitment by Two-Sided Platforms," *Rand Journal of Economics*, Vol. 37 (3), 720-737.

[12] Hagiu, A. (2007) "Merchant or Two-Sided Platform?" *Review of Network Economics*, Vol. 6(2), 115-133.

[13] Hagiu, A. and B. Jullien (2008) "Why Do Intermediaries Divert Search? - Companion Paper," Harvard Business School and Toulouse School of Economics working paper, available at http://www.people.hbs.edu/ahagiu/Hagiu%20Jullien%20companion%2012232008.pdf

[14] Hatfield, J. W. and P. R. Milgrom (2005) "Matching with Contracts," *American Economic Review*, Vol. 95 (4), 913-935.

[15] Parker, G. and M. W. Van Alstyne (2008) "Innovation, Openness and Platform Control," working paper, Tulane University, Boston University and MIT.

[16] Petroski, H. (2003) *Small Things Considered: Why There Is No Perfect Design*, New York: Knopf.

[17] Resnick, P. and H. R. Varian (1997) "Recommender Systems," *Communications of the ACM*, Vol. 40 (3).

[18] Rochet, J.-C., and J. Tirole (2003) "Platform Competition in Two-Sided Markets," *Journal of the European Economic Association*, Vol. 1 (4), 990-1029.

[19] Rochet, J.-C., and J. Tirole (2006) "Two-Sided Markets: Where We Stand," *Rand Journal of Economics*, Vol. 37 (3), 645-66.

[20] Roth, A. E. (2002) "The Economist as Engineer: Game Theory, Experimentation, and Computation as Tools for Design Economics," *Econometrica*, Vol. 70 (4), 1341-1378.

[21] Roth, A. E. (2008) "What have we learned from market design?" *Economic Journal*, Vol. 118, 285-310.

[22] Roth, A. E. and M. Sotomayor (1990) *Two-Sided Matching: A Study In Game-Theoretic Modeling and Analysis*, Cambridge: Cambridge University Press.

[23] Rubinstein, A. and A. Wolinsky (1987) "Middlemen," *Quarterly Journal of Economics*, Vol. 102, 581-593.

[24] Rust, J. and G. Hall (2003), "Middlemen vs. Market Makers: A Theory of Competitive Exchange," *Journal of Political Economy*, Vol. 111 (2), 353-403.

[25] Shih, W., S. Kaufman and D. Spinola (2007) "Netflix," Harvard Business School case study 9-607-138.

[26] Spulber, D. F. (1996) "Market Microstructure and Intermediation," *Journal of Economic Perspectives*, Vol. 10 (3), 135-152.

[27] Spulber, D. F. (2007) "Firms and Networks in Two-Sided Markets," *The Handbook of Economics and Information Systems*, Amsterdam: Elsevier.

[28] Stahl, B. (1988) "Bertrand Competition for Inputs and Walrasian Outcomes," *The American Economic Review*, Vol. 78, 189-201.

[29] White, A. (2008) "Search Engines: Left Side Quality versus Right Side Profits," working paper, Toulouse School of Economics.

9 Appendix

9.1 Concavity of the expression of intermediary profits

Given that $u(q)$ is strictly increasing and continuous in q, we can optimize the intermediary's profits over $u \equiv u(q) \in \left[\frac{2u^H+u^L}{3}, u^H\right]$ rather than over q. Expression (3) writes as:

$$(r_1 + r_2) F\left(u^L\right) + \left[r_1 - r_2 + r_2\left(\frac{u^H - u^L}{u - u^L}\right)\right]\left[F(u) - F\left(u^L\right)\right]$$

and its derivative with respect to u is:

$$\begin{aligned} & -r_2\frac{u^H - u^L}{\left(u - u^L\right)^2}\left[F(u) - F\left(u^L\right)\right] + \left[r_1 - r_2 + r_2\frac{u^H - u^L}{u - u^L}\right] f(u) \\ = & \left\{r_2\left[1 - \frac{F(u) - F\left(u^L\right)}{\left(u - u^L\right) f(u)}\right] + (r_1 - r_2)\frac{u - u^L}{u^H - u^L}\right\} f(u)\frac{u^H - u^L}{u - u^L} \end{aligned} \tag{16}$$

Thus, a sufficient condition for assumption 1 to hold is:

$$\frac{\partial}{\partial u}\left[\frac{F(u) - F\left(u^L\right)}{\left(u - u^L\right) f(u)}\right] > \left(\frac{\widetilde{r_1} - \widetilde{r_2}}{\widetilde{r_2}}\right) \times \frac{1}{u^H - u^L}$$

for all $u \in \left[\frac{2u^H+u^L}{3}, u^H\right]$. This condition guarantees that the term in-between curly brackets in (16) is decreasing in u, so that the objective function is concave. Imposing that this term is positive for $u = \frac{2u^H+u^L}{3}$ and negative for $u = u^H$ yields the sufficient conditions for the solution q^* to be interior (i.e.

$q^* \in \left]\frac{1}{2}, 1\right[$):

$$\frac{3}{2}\left[1-\frac{F\left(\frac{2u^H+u^L}{3}\right)-F\left(u^L\right)}{\frac{2}{3}\left(u^H-u^L\right)f\left(\frac{2u^H+u^L}{3}\right)}\right] > \frac{r_2-r_1}{r_2} > 1-\frac{F\left(u^H\right)-F\left(u^L\right)}{\left(u^H-u^L\right)f\left(u^H\right)}$$

■

9.2 Proof of Proposition 2

The optimization program (6) can be written as $\max_q \left\{\pi X(q) - G\left(\frac{F(u(q)}{X(q)}\right)\right\}$, where $G(t) \equiv \frac{K}{\alpha-(2\alpha-1)t}$ is an increasing function: $G'(.) > 0$. Define $\Phi(q) \equiv \pi X(q) - G\left(\frac{F(u(q))}{X(q)}\right)$. Then we have:

$$\begin{aligned}
\Phi'(q) &= \pi X'(q) - \frac{f(u(q))\,u'(q)\,X(q) - X'(q)\,F(u(q))}{(X(q))^2} G'\left(\frac{F(u(q))}{X(q)}\right) \\
&= X'(q)\left[\underbrace{\pi + \frac{F(u(q)}{(X(q))^2} G'\left(\frac{F(u(q))}{X(q)}\right)}_{>0}\right] - \underbrace{\frac{f(u(q))\,u'(q)}{X(q)} G'\left(\frac{F(u(q))}{X(q)}\right)}_{>0}
\end{aligned}$$

Thus we can conclude that:

$$X'(q) \leq 0 \Longrightarrow \Phi'(q) \leq 0$$

which implies that $q_a^*(\alpha) < q_s^*$ if the second order conditions are satisfied.

■

9.3 Proof of Lemma 2

Let $\beta(q,p) \equiv \frac{qF\left(u^L(p)\right)}{F\left(\frac{u^H(p)+(1-q)u^L(p)}{2-q}\right)} + (1-q)$. We then have:

$$\frac{\partial \beta}{\partial q} = -1 + \frac{F\left(u^L(p)\right)}{F\left(\frac{u^H(p)+(1-q)u^L(p)}{2-q}\right)} + q\frac{\partial}{\partial q}\left(\frac{F\left(u^L(p)\right)}{F\left(\frac{u^H(p)+(1-q)u^L(p)}{2-q}\right)}\right) < 0$$

for all p.

Let then $\phi(\beta) \equiv \arg\max_p R^H(p) + R^L(p)\beta$. Assumption 2 implies that $\phi(\beta) \in \left[p^L, p^H\right]$. On this range $R^L(.)$ is decreasing while $R^H(.)$ is increasing. In particular $\frac{\partial^2}{\partial\beta\partial p}\left(R^H(p) + R^L(p)\beta\right) = \frac{\partial}{\partial p}\left(R^L(p)\right) < 0$. This implies that $\phi(\beta)$ is decreasing.

Thus, (11) can then be rewritten as:

$$p^* = \phi\left(\beta\left(q, p^*\right)\right)$$

Using the implicit function theorem, we obtain:

$$\frac{dp^*}{dq} = \frac{\frac{d\phi}{d\beta}\frac{\partial\beta}{\partial q}}{1 - \frac{d\phi}{d\beta}\frac{\partial\beta}{\partial p}}$$

But the uniqueness condition for the process of tatonnement between consumer expectations and actual prices charged by the store, along with the fact that $\phi\left(\beta\left(q, p^L\right)\right) > p^L$ imply $1 > \left|\frac{d\phi}{d\beta}\frac{\partial\beta}{\partial p}\right|$. We can therefore conclude that $\frac{dp^*}{dq} > 0$.

■

9.4 Proof of Lemma 2

As mentioned in the text, the tariff $A(q)$ can be assumed to be non-decreasing without loss of generality. Denote by $q(c)$ the level of search effectiveness chosen by a consumer with search cost c. Following the revelation principle, denote by $(q(c), A(c))$ the search effectiveness and the access fee paid by consumer c, where $A(c) \equiv A(q(c))$.

Among the options offered by the intermediary, a consumer with $c \leq u^L$ prefers the one with the lowest tariff $A(q)$ as she is indifferent between all q: her utility from using the intermediary's service is $U(c) = u^H + u^L - A(q(c)) - 2c$. Thus, all consumers with $c \leq u^L$ prefer the lowest level of search efficiency offered, $q_L \geq \frac{1}{2}$.

9.4.1 The bottleneck intermediary

We assume it is optimal for the intermediary to induce a positive measure of consumers with $c > u^L$ to use its service.[30] The utility derived by such a consumer from using the intermediary's service is $U(c) = u^H + (1 - q(c))\, u^L - A(c) - (2 - q(c))\, c$. Since this utility is non-negative, it must be that $A(c) < u^H - u^L$. This implies that all consumers with $c' \leq u^L$ will also participate: they are indifferent between various q's and derive positive utility if they pay $A < u^H - u^L$. We can therefore restrict attention to $A(c) = A\left(u^L\right)$ for $c \leq u^L$ and $A(c) < u^H - u^L$ for $c > u^L$.

Denote by $\hat{c} > u^L$ the highest search cost of consumers that use the intermediary's service. Then all consumers with $c \leq \hat{c}$ use the intermediary's service and $U(\hat{c}) = 0$.

Incentive compatibility implies that $q(c)$ is non-decreasing and $U(c)$ is continuous and decreasing for $c \in [0, \hat{c}]$. Consequently, $U(c)$ is differentiable and $U'(c) = -(2 - q(c))$ for almost every $c \in \left[u^L, \hat{c}\right]$, so that $U(c) = \int_c^{\hat{c}} (2 - q(c'))\, dc'$ for all $c \in \left[u^L, \hat{c}\right]$.

The profit derived by the intermediary from consumer c is:

$$\begin{aligned} r_1 + r_2 + A\left(u^L\right) &= r_1 + r_2 + u^H - u^L - U\left(u^L\right) \quad \text{if} \quad c \leq u^L \\ r_1 + (1 - q(c))\, r_2 + A(c) &= r_1 + u^H + (1 - q(c))\left(r_2 + u^L\right) - (2 - q(c))\, c - U(c) \quad \text{if} \quad c > u^L. \end{aligned}$$

[30] If not, then the solution is to only offer one option with any quality q and $A = u^H - u^L$, attracting all consumers with $c \leq u^L$ and yielding profits of $\left(r_1 + r_2 + u^H - u^L\right) F\left(u^L\right)$.

The expression of intermediary profits is then:

$$\left(r_1 + r_2 + u^H - u^L\right) F\left(u^L\right) - \left(\int_{u^L}^{\hat{c}} (2 - q(c))\, dc\right) F\left(u^L\right)$$
$$+ \int_{u^L}^{\hat{c}} \left(r_1 + u^H + (1 - q(c))\left(r_2 + u^L\right) - (2 - q(c))\, c\right) f(c)\, dc$$
$$- \int_{u^L}^{\hat{c}} \left(\int_{c}^{\hat{c}} (2 - q(u))\, du\right) f(c)\, dc$$

which can be written as:

$$\left(r_1 + r_2 + u^H - u^L\right) F\left(u^L\right)$$
$$+ \int_{u^L}^{\hat{c}} \left\{ r_1 + u^H - \left(c + \frac{F(c)}{f(c)}\right) + (1 - q(c))\left[r_2 + u^L - \left(c + \frac{F(c)}{f(c)}\right)\right]\right\} dF(c)$$

Instead of optimizing over $\{q(c), A(c)\}_{c\in[0,u^H]}$, we optimize over $\{q(c)\}_{c\in[0,u^H]}$ and $\hat{c}$. Ignoring the monotonicity constraint, the optimal $q(.)$ is given by:

$$q(c) = \begin{cases} \frac{1}{2} & \text{if } \; r_2 + u^L > c + \frac{F(c)}{f(c)} \\ 1 & \text{if } \; r_2 + u^L \leq c + \frac{F(c)}{f(c)} \end{cases}$$

If $F(.)$ is concave then $c + \frac{F(c)}{f(c)}$ is increasing in c so that $q(c)$ as defined above is indeed increasing and therefore is the desired solution.

9.4.2 The pure recommender

In this case, a consumer c who uses the intermediary's service must derive at least $\hat{U}(c)$, where:

$$\hat{U}(c) = \begin{cases} u^H + u^L - 2c & \text{if} \quad c \leq u^L \\ u^H + \frac{1}{2}u^L - \frac{3}{2}c & \text{if} \quad u^L < c \leq \frac{2u^H + u^L}{3} \\ 0 & \text{if} \quad \frac{2u^H + u^L}{3} < c \end{cases}$$

is the utility derived from the consumer's best outside option. In particular, note that a pure recommender cannot charge a positive access fee to consumers with $c \leq u^L$. But the intermediary can always guarantee their participation by offering an option $q = \frac{1}{2}$ at a zero access price. This attracts consumers who would not use the service otherwise (it replicates the outside option of searching by themselves) and generates positive revenues for the intermediary, while not altering the choice of other consumers. It follows that all consumers with $c \leq \frac{2u^H + u^L}{3}$ use the intermediary's service and the optimal contract must have $A(c) = A\left(u^L\right) = 0$ for $c \leq u^L$.

Again, let:

$$U(c) = \begin{cases} u^H + u^L - 2c & \text{if} \quad c \leq u^L \\ u^H + (1 - q(c))\, u^L - (2 - q(c))\, c - A(c) & \text{if} \quad c \geq u^L \end{cases}$$

and denote by $\hat{c} \geq \frac{2u^H+u^L}{3}$ the highest search cost of consumers that use the intermediary's service. Then $U(\hat{c}) = 0$ and all consumers with $\frac{2u^H+u^L}{3} \leq c \leq \hat{c}$ use the intermediary's service.

Thus, all consumers with $c \leq \hat{c}$ use the intermediary's service and incentive compatibility implies that $q(c)$ is non-decreasing and $U(c)$ is decreasing and continuous, so that $U'(c)$ is defined for almost all c and $U'(c) = -(2 - q(c))$ for $c \geq u^L$.

The intermediary's profits can then be written (similarly to the bottleneck case):

$$\begin{aligned}
&(r_1 + r_2) F\left(u^L\right) \\
&+ \int_{u^L}^{\hat{c}} \left\{ r_1 + u^H - \left(c + \frac{F(c)}{f(c)}\right) + (1 - q(c)) \left[r_2 + u^L - \left(c + \frac{F(c)}{f(c)}\right)\right]\right\} dF(c)
\end{aligned}$$

The key difference is that we now have to maximize this expression subject to the constraint $A\left(u^L\right) = 0$, which is equivalent to $\int_{u^L}^{\hat{c}} (2 - q(c))\, dc = u^H - u^L$. Denoting by λ the multiplier on the constraint, we are maximizing:

$$\begin{aligned}
&(r_1 + r_2) F\left(u^L\right) \\
&+ \int_{u^L}^{\hat{c}} \left\{ r_1 + u^H - \left(c + \frac{F(c) - \lambda}{f(c)}\right) + (1 - q(c)) \left[r_2 + u^L - \left(c + \frac{F(c) - \lambda}{f(c)}\right)\right]\right\} dF(c)
\end{aligned}$$

over $\{q(.), \hat{c}\}$. The optimal contract is then given by:

$$q(c) = \begin{cases} \frac{1}{2} & \text{if} \quad r_2 + u^L > c + \frac{F(c)-\lambda}{f(c)} \\ 1 & \text{if} \quad r_2 + u^L \leq c + \frac{F(c)-\lambda}{f(c)} \end{cases}$$

■

9.5 Search subsidies and fixed fees with a pure recommender

The key difference relative to the bottleneck intermediary case is that the pure recommender's demand from consumers with $c \leq u^L + s$ depends on the relative values of A and s. Specifically, a consumer with $c \leq u^L$ will use the recommender's service if and only if $s \geq A$, while a consumer with $u^L < c \leq u^L + s$ will use it if and only if $c \leq u^L + 2(s - A)$.

We obtain the expression of the pure recommender's profits as a function of (A, s, q):

$$\begin{cases}
[\widetilde{r_1} + A + (1-q)(\widetilde{r_2} - s)]\left[F(u(q,s,A)) - F\left(u^L + \frac{2(A-(1-q)s)}{2q-1}\right)\right] & \text{if} \quad s < A \\
[\widetilde{r_1} + A + \widetilde{r_2} - s]\, F\left(u^L + 2(s-A)\right) & \\
+ [\widetilde{r_1} + A + (1-q)(\widetilde{r_2} - s)]\left[F(u(q,s,A)) - F\left(u^L + \frac{2(A-(1-q)s)}{2q-1}\right)\right] & \text{if} \quad A \leq s \leq 2A \\
[\widetilde{r_1} + A + \widetilde{r_2} - s]\, F\left(u^L + s\right) & \\
+ [\widetilde{r_1} + A + (1-q)(\widetilde{r_2} - s)]\left[F(u(q,s,A)) - F\left(u^L + s\right)\right] & \text{if} \quad s \geq 2A
\end{cases}$$

Denote by (s^*, A^*) the optimal subsidy and access fee conditional on $q = 1$. Then $s^* \geq A^*$ and:

- if $s^* < 2A^*$, the condition for $q < 1$ is:

$$\frac{\widetilde{r}_1 + A^*}{\widetilde{r}_2 - s^*} < \frac{F\left(u^H - A^*\right) - F\left(u^L + 2A^*\right)}{\left(u^H - u^L - A^* - s^*\right) f\left(u^H - A^*\right) + \left(4A^* - 2s^*\right) f\left(u^L + 2A^*\right)} \tag{17}$$

- if $s^* > 2A^*$, the condition for $q < 1$ is the same as for the bottleneck:

$$\frac{\widetilde{r}_1 + A^*}{\widetilde{r}_2 - s^*} < \frac{F\left(u^H - A^*\right) - F\left(u^L + s^*\right)}{\left(u^H - u^L - A^* - s^*\right) f\left(u^H - A^*\right)}$$

If $s^* > 2A^*$ then we end up with the same conditions as for the bottleneck, therefore we know that $q^* = 1$.

If $A^* < s^* < 2A^*$ then (s^*, A^*) must satisfy (17) and:

$$\begin{aligned} \widetilde{r}_1 + A^* &= \frac{F\left(u^H - A^*\right) - F\left(u^L + 2A^*\right)}{f\left(u^H - A^*\right) + 2f\left(u^L + 2A^*\right)} \\ \widetilde{r}_1 + A^* + \widetilde{r}_2 - s^* &= \frac{F\left(u^L + 2\left(s^* - A^*\right)\right)}{2f\left(u^L + 2\left(s^* - A^*\right)\right)} \end{aligned}$$

■