Currency Misalignments and Optimal Monetary Policy: A Reexamination

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Abstract

This paper examines optimal monetary policy in an open-economy two-country model with sticky prices. We show that currency misalignments are inefficient and lower world welfare. We find that optimal policy must target not only inflation and the output gap, but also the currency misalignment. However the interest rate reaction function that supports this targeting rule may involve only the CPI inflation rate. This result illustrates how examination of "instrument rules" may hide important trade-offs facing policymakers that are incorporated in "targeting rules". The model is a modified version of Clarida, Gali, and Gertler's (JME, 2002). The key change is that we allow pricing to market or local-currency pricing and consider the policy implications of currency misalignments. Besides highlighting the importance of the currency misalignment, our model also gives a rationale for targeting CPI, rather than PPI, inflation.

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Exchange rates among the large economies have fluctuated dramatically over the past 30 years. The dollar/euro exchange rate has experienced swings of greater than 60%, and even the Canadian dollar/U.S. dollar has risen and fallen by more than 35% in the past decade, but inflation rates in these countries have differed by only a percentage point or two per year. Should these exchange rate movements be a concern for policymakers? Would it not be better for policymakers to focus on output and inflation and let a freely floating exchange rate settle at a market determined level?

It is widely understood that purchasing power parity does not hold in the short run. Empirical evidence points to the possibility of "local-currency pricing" (LCP) or "pricing to market".[1] That is, exporting firms may price discriminate among markets, and/or set prices in the buyers' currencies. A currency could be overvalued if the consumer price level is higher at home than abroad when compared in a common currency, or undervalued if the relative price level is lower at home. Currency misalignments can be very large even in advanced economies.

There is frequent public discussion of the importance of controlling currency misalignments. For example, on November 3, 2008, Robert Rubin (former U.S. Secretary of the Treasury) and Jared Bernstein (of the Economic Policy Institute) co-authored an op-ed piece in the New York Times that argued, "Public policy…has been seriously deficient [because of] false choices, grounded in ideology" (Rubin and Bernstein, 2008.) One of the principles they argue that all should agree upon is "we need to work with other countries toward equilibrium exchange rates." Yet there is little support in the modern New Keynesian literature on monetary policy for the notion that central banks should target exchange rates. Specifically, if policymakers are already optimally responding to inflation and the output gap, is there any reason to pay attention to exchange-rate misalignments?

Our answer is yes. In a simple, familiar framework, this paper draws out the implications for monetary policy when currency misalignments are possible. Currency misalignments lead to inefficient allocations for reasons that are analogous to the problems with inflation in a world of staggered price setting. When there are currency misalignments, households in the Home and Foreign countries may pay different prices for the identical good. A basic tenet of economics is that violations of the law of one price are inefficient – if the good's marginal cost is the same irrespective of where the good is sold, it is not efficient to sell the good at different prices. We

[1] Many studies have found evidence of violations of the law of one price for consumer prices. Two prominent studies are Engel (1999) and Atkeson and Burstein (2008). The literature is voluminous – these two papers contain many relevant citations.

find that currency misalignments lead to a reduction in world welfare and that optimal monetary policy trades off this currency misalignment with inflation and output goals.

These currency misalignments arise even when foreign exchange markets are efficient. That is, the currency misalignment distortion that concerns policymakers arises in the goods market – from price setting – and not in the foreign exchange market. The model of this paper determines the foreign exchange rate in an efficient currency market as a function of fundamental economic variables.

Clarida, Gali, and Gertler (2002), Gali and Monacelli (2005), and others have emphasized the important role of exchange rate adjustment in a model that assumes firms set prices in their own currency (PCP, for "producer-currency pricing".) In the PCP framework, a change in the nominal exchange rate automatically translates into a change in the price of imported goods. The exchange rate immediately changes the relative price of imported to local goods, and so plays an important role in achieving nearly efficient outcomes. Why, then, is it optimal to target currency misalignments when there is pricing to market? In this setting, the exchange rate does not play role of automatically adjusting relative prices facing households. The effect is much weaker because imported goods prices are set by the producer and do not respond automatically to exchange rate changes. On the other hand, if prices are sticky in the importer's currency, the change in the exchange rate can lead to inefficient movements in the price of the same good sold in different countries. Hence there is a need to target currency misalignments.[2]

To understand the contribution of this paper, it is helpful to place it relative to three sets of papers:

1. Clarida, Gali, and Gertler (2002) develop what is probably the canonical model for open-economy monetary policy analysis in the New Keynesian framework. Their two-country model assumes PCP, and that Home and Foreign households have identical preferences. These two assumptions lead to the conclusion that purchasing power parity holds at all times – the consumption real exchange rate is constant.[3]

This paper introduces local-currency pricing into CGG's model. We derive simple rules for monetary policy that are similar to CGG's. While the model is not rich relative to sophisticated models in the literature (models that introduce capital, working capital, capacity utilization, habits in preferences, etc.), the simple model is helpful for developing intuition because the model can be solved analytically, an explicit second-order approximation to the

[2] This does not, however, imply that fixed exchange rates are optimal.

[3] Benigno and Benigno (2003, 2006) are important contributions that use models similar to Clarida, Gali, and Gertler's but consider optimal policy when the optimal subsidies to deal with monopoly distortions are not present in steady state.

policymaker's loss function can be derived, explicit "target criteria" for policy can be derived, and explicit interest rate reaction functions can be derived.

The paper also allows Home and Foreign households to have different preferences. They can exhibit a home bias in preferences – a larger weight on goods produced in a household's country of residence.[4] This generalization does not change the optimal target criteria at all in the CGG framework, but as we now explain, is helpful in developing a realistic LCP model.

2. Devereux and Engel (2003) explicitly examine optimal monetary policy in a two-country framework with LCP. Corsetti and Pesenti (2005) extend the analysis in several directions. However, neither of these studies is suited toward answering the question posed above: is currency misalignment a separate concern of monetary policy, or will the optimal exchange-rate behavior be achieved through a policy that considers inflation and the output gap?

These models have a couple of crucial assumptions that make them unsuited to answering this question. First, like CGG, they assume identical preferences in both countries. This assumption leads to the outcome that currency misalignments are the only source of CPI inflation differences between the two countries in the LCP framework. Eliminating inflation differences eliminates currency misalignments and vice-versa.[5]

Second, price stickiness is the only distortion in the economy in these papers. In contrast, CGG introduce "cost-push shocks", so that policymakers face a tradeoff between the goals of zero inflation and zero output gap. In Devereux and Engel (2003), the optimal monetary policy under LCP sets inflation to zero in each country, thus eliminating any currency misalignment.

By introducing home bias in preferences, the tight link between relative inflation rates and currency misalignments is broken. A more realistic model for inflation results – one in which relative CPI inflation rates depend not only on currency misalignments, but also on the internal relative price of imported to domestically-produced goods. Moreover, we follow CGG in allowing for cost-push shocks.[6]

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[4] de Paoli (2009) allows for home bias in preferences in a small open economy model. There is home bias in the sense that while the country is small, the limit of the ratio of expenditure share on home goods to population share is not equal to one. Faia and Monacelli (2008) examine optimal monetary policy in a small open economy model with home bias, using a Ramsey style analysis. Pappa (2004) considers a two-country model with home bias. However, the second-order approximation to the welfare function is expressed in terms of deviations of consumption from its efficient level, rather than in terms of the output gap, so the analysis is not strictly comparable to ours. See Woodford (2003) for a discussion of why we approximate in terms of the output gap rather than consumption.

[5] See Duarte and Obstfeld (2008), who emphasize this point.

[6] The contribution of Sutherland (2005) merits attention. His two-country model allows for imperfect pass-through, and for differences in Home and Foreign preferences. His model is static, and he derives a welfare function in which the variance of the exchange rate appears. However, the other terms in the welfare function are prices, so it is not clear how this function relates to standard quadratic approximations that

This paper also derives optimal policy in a framework that is consonant with the bulk of New Keynesian models of monetary policy analysis. Devereux and Engel (2003) and Corsetti and Pesenti (2005) assume price setting is synchronized, with prices set one period in advance.[7] Here we adopt the standard Calvo price-setting technology, which allows for asynchronized price setting. This change is important, because it emphasizes the point that the cost of inflation under sticky is misaligned relative prices. Also, the previous papers assumed that the money supply was the instrument of monetary policy. This paper follows CGG and most of the modern literature in assuming that the policymakers directly control the nominal interest rate in each country.[8]

3. There are many papers that numerically solve rich open economy models, and examine optimal policy. Some of these papers allow for local currency pricing. Many of those papers are in the framework of a small open economy, and so do not specifically account for the global misallocation of resources that occurs with currency misalignments.[9] Moreover, many use ad hoc welfare criteria for the policymaker or approximations that are not strictly derived from household welfare.[10] One of the main contributions of this paper is to derive the role of the currency misalignment in the policymaker's loss function.

Some papers have considered whether it is beneficial to augment the interest rate reaction function of central banks with an exchange-rate variable.[11] They ask the question: if the Taylor rule has the interest rate reacting to inflation and the output gap, is there any gain from adding the exchange rate? Typically these studies find little or no evidence of welfare gains from adding the exchange rate to the Taylor rule.

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involve output gaps and inflation levels. Moreover, Sutherland does not derive optimal monetary policy in his framework.

[7] A sophisticated extension of this work is the recent paper by Corsetti, Dedola, and Leduc (2007). That paper extends earlier work in several dimensions, including staggered price setting. But it does not directly address the issue of whether currency misalignments belong in the targeting rule along with output gaps and inflation.

[8] While the model of this paper adheres strictly to the set-up of CGG, changing only the assumptions of identical preferences and LCP instead of PCP price setting, the model is very similar to that of Benigno's (2004). Woodford (2007) also considers the LCP version of CGG (though not for optimal monetary policy analysis) and makes the connection to Benigno's paper. Monacelli (2005) considers a small-open economy model with local-currency pricing, and examines optimal monetary policy using an ad hoc welfare criterion.

[9] See, for example, Kollmann (2002), Smets and Wouters (2002), Ambler, Dib, and Rebei (2004), and Adolfson, et. al. (2008). See also Leith and Wren-Lewis (2006) who examine a small-open economy model with non-traded goods (but with PCP for export pricing.)

[10] For example, Smets and Wouters (2002), Ambler, Dib, and Rebei (2004), and Adolfson, et. al. (2008).

[11] In a small open economy, see Kollmann (2002) and Leitemo and Soderstrom (2005). In a two-country model, see Wang (2009).

The question posed this way is misleading. To understand this point, it is helpful first to return to the optimal policy analysis in CGG. That paper finds (recall, under their assumption of PCP) that optimal monetary policy can be characterized by a pair of "target criteria" or "targeting rule": $\tilde{y}_t + \xi\pi_{Ht} = 0$ and $\tilde{y}_t^* + \xi\pi_{Ft}^* = 0$. In these equations, $\tilde{y}_t$ refers to the output gap of the Home country – the percentage difference between the actual output level and its efficient level. π_{Ht} is the producer-price inflation rate in the Home country. Analogously, $\tilde{y}_t^*$ is the Foreign output gap and π_{Ft}^* the Foreign PPI inflation rate.[12] These equations describe the optimal tradeoff of the output gap and inflation for the policymaker. It is desirable to allow inflation to be positive if the output gap is negative, for example. CGG then derive interest rate rules that will deliver these optimal policy tradeoffs. They find interest rate reaction functions (assuming discretionary policy) given by: $r_t = \overline{rr}_t + b\pi_{Ht}$ and $r_t^* = \overline{rr}_t^* + b\pi_{Ft}^*$. r_t is the Home nominal interest rate, and $\overline{rr}_t$ is the "Wicksellian" or efficient real interest rate. The response of the interest rate to inflation, *b*, is a function of model parameters.[13]

The key point to be made here is that CGG's model shows that optimal policy must trade off the inflation and output goals of the central bank. But the optimal interest rate reaction function does *not* necessarily include the output gap. That is, adding the output gap to the interest rate rule that already includes inflation will not improve welfare. Focusing on the "instrument rule" does not reveal the role of the output gap that is apparent in the "targeting rule" in the terminology of Svensson (1999, 2002).

An analogous situation arises in the LCP model concerning currency misalignments. We can characterize the "target criteria" in this model with two rules, as in the CGG model. The first is $\tilde{y}_t + \tilde{y}_t^* + \xi(\pi_t + \pi_t^*) = 0$. This rule, at first glance, appears to be simply the sum of the two "target criteria" in the CGG model. It is, except that the inflation rates that appear in this tradeoff (π_t and π_t^*) are CPI inflation rates, rather than PPI as in CGG's model. The second target criterion is $\frac{1}{\sigma}\tilde{q}_t + \xi(\pi_t - \pi_t^*) = 0$, where q_t is the real exchange rate (defined as Foreign prices relative to Home prices expressed in a common currency.) $\tilde{q}_t$ is the deviation of the real exchange rate from its efficient level. We define the parameters in this equation below. The important point is that the tradeoff described here relates real exchange rates and relative CPI inflation rates. For example, even if inflation is low in the Home country relative to the Foreign

[12] ξ is a preference parameter defined below.

[13] Specifically, we show below that $b = \rho + (1-\rho)\sigma\xi$. We define the parameters below.

country, optimal policy may under some circumstances still call for a tightening of the monetary policy stance in the Home country if the Home currency is sufficiently undervalued.

Like CGG, we can derive the optimal interest rate rules that support these targeting rules. We find these interest rate reaction functions are $r_t = \overline{rr}_t + b\pi_t$ and $r_t^* = \overline{rr}_t^* + b\pi_t^*$. They are identical to the ones derived in CGG (the parameter *b* is the same), except they target CPI inflation rather than PPI inflation as in CGG. The conclusion is that while the target criteria include currency misalignments, the currency misalignment is not in the optimal interest rate reaction function. If we focus on only the latter, we miss this tradeoff the policymaker faces.

Previous studies have found little welfare gain from adding an exchange rate variable to the Taylor rule. Properly speaking, these studies examine the effects of simple targeting rules under commitment. Our results describe the welfare function, the target criteria and the optimal interest rate reaction functions under discretionary policymaking. But our results here suggest that even if there is no role for the currency misalignment in a simple targeting rule, exchange rate concerns may still be important in terms of welfare. This point is brought out in the context of a relatively simple model that can be solved analytically (with approximations), but is obscured in larger models that are solved numerically.[14]

We proceed in two steps. After setting out the objectives of households and firms, the production functions, and the market structure, we derive a global loss function for cooperative monetary policymakers. We can derive the period loss function without making any assumptions about how goods prices or set or how wages are set.[15] We find that in addition to squares and cross-products of Home and Foreign output gaps, and the cross-sectional dispersion of goods prices within each country, the loss also depends on the squared currency misalignment. This loss function evaluates the welfare costs arising because firms set different prices in the Home and Foreign country (assuming the costs of selling the good in both countries are identical), and does not depend on whether the price differences arise from local-currency price stickiness, from price discrimination, or for some other reason.

We then follow CGG and assume a Calvo mechanism for price setting. However, we allow for possibility of local-currency pricing. As in CGG, we derive optimal policy under discretion. We obtain both targeting rules and instrument rules. We consider only optimal

[14] Coenen, Lombardo, Smets, and Straub (2007) examine optimal monetary policy in a two-country model that exhibits incomplete pass-through. However, the numerical analysis does not allow the reader to see explicitly the role of currency misalignments.

[15] Except that we do assume that all households (which are identical) set the same wage. As we note later, this rules out a model of staggered wage setting such as in Erceg, Henderson, and Levin (2000), though a generalization to encompass that case would be straightforward.

cooperative policy. Our goal is to quantify the global loss from currency misalignments, which we can see by deriving the loss function for a policymaker that aims to maximize the sum of utilities of Home and Foreign households. Practically speaking, international agreements that prohibit currency manipulation may mean that the currency misalignment can only be addressed in a cooperative environment.[16] That is, it seems likely that if central banks are going to move toward policies that explicitly target exchange rates, they will do so cooperatively.

1. The Model

The model we examine is nearly identical to CGG's. We consider two countries of equal size, while CGG allow the population of the countries to be different. Since the population size plays no real role in their analysis, we simplify along this dimension. But we make two significant generalizations. First, we allow for different preferences in the two countries. Home agents may put a higher weight in utility on goods produced in the Home country. Home households put a weight of $\frac{\nu}{2}$ on Home goods and $1-\frac{\nu}{2}$ on Foreign goods (and vice-versa for Foreign households.) This is a popular assumption in the open-economy macroeconomics literature, and can be considered as a short-cut way of modeling "openness". That is, a less open country puts less weight on consumption of imported goods, and in the limit the economy becomes closed if it imports no goods. The second major change we allow, as already noted, is we allow for goods to be sold at different prices in the Home and Foreign countries.

The model assumes two countries, each inhabited with a continuum of households, normalized to a total of one in each country. Households have utility over consumption of goods and disutility from provision of labor services. In each country, there is a continuum of goods produced, each by a monopolist. Households supply labor to firms located within their own country, and get utility from all goods produced in both countries. Each household is a monopolistic supplier of a unique type of labor to firms within its country. We assume that there is trade in a complete set of nominally-denominated contingent claims

Monopolistic firms produce output using only labor, subject to technology shocks.

At this stage, we will not make any assumptions on how wages are set by monopolistic households or prices are set by monopolistic firms. In particular, prices and wages may be sticky, and there may be LCP or PCP for firms. We derive the period loss function for the policymaker, which expresses the loss (relative to the efficient outcome) in terms of within-country and

[16] For example, the IMF Articles of Agreement state that member countries shall "avoid manipulating exchange rates ... to prevent effective balance of payments adjustment or to gain an unfair competitive advantage over other members." See Staiger and Sykes (2008).

international price misalignments and output gaps. This loss function applies under various assumptions about how prices are actually set, and so is more general than the policy rules we subsequently derive which depend on the specifics of price and wage setting.

All households within a country are identical. We will assume that in each period, their labor supplies are identical. This assumption rules out staggered wage setting as in Erceg, Henderson, and Levin (2000), because in that model there will be dispersion in labor input across households that arises from the dispersion in wages set. Our set-up is consistent with sticky wages, but not wage dispersion. However, it is entirely straightforward to generalize the loss functions we derive to allow for wage dispersion following the steps in Erceg, Levin, and Henderson. We do not do that because we want our model to be directly comparable to CGG.

1.a. *Households*

The representative household in the home country maximizes

(1) $$U_t(h) = \mathrm{E}_t\left\{\sum_{j=0}^{\infty}\beta^j\left[\frac{1}{1-\sigma}C_{t+j}(h)^{1-\sigma} - \frac{1}{1+\phi}N_{t+j}(h)^{1+\phi}\right]\right\},\ \sigma>0, \phi\geq 0$$

$C_t(h)$ is the consumption aggregate. We assume Cobb-Douglas preferences:

(2) $$C_t(h) = \left(C_{Ht}(h)\right)^{\frac{\nu}{2}}\left(C_{Ft}(h)\right)^{1-\frac{\nu}{2}},\ 0\leq\nu\leq 2 .$$

If $\nu=1$, Home and Foreign preferences are identical as in CGG. There is home bias in preferences when $\nu>1$.

In turn, $C_{Ht}(h)$ and $C_{Ft}(h)$ are CES aggregates over a continuum of goods produced in each country:

(3) $$C_{Ht}(h) = \left(\int_0^1 C_{Ht}(h,f)^{\frac{\xi-1}{\xi}}df\right)^{\frac{\xi}{\xi-1}} \quad \text{and} \quad C_{Ft}(h) = \left(\int_0^1 C_{Ft}(h,f)^{\frac{\xi-1}{\xi}}df\right)^{\frac{\xi}{\xi-1}} .$$

$N_t(h)$ is an aggregate of the labor services that the household sells to each of a continuum of firms located in the home country:

(4) $$N_t(h) = \int_0^1 N_t(h,f)df .$$

Households receive wage income, $W_t(h)N_t(h)$, aggregate profits from home firms, Γ_t. They pay lump-sum taxes each period, T_t. Each household can trade in a complete market in contingent claims (arbitrarily) denominated in the home currency. The budget constraint is given by:

(5) $$P_t C_t(h) + \sum_{\nabla^{t+1} \in \Omega_{t+1}} Z(\nabla^{t+1} \mid \nabla^t) D(h, \nabla^{t+1}) = W_t(h) N_t(h) + \Gamma_t - T_t + D(h, \nabla^t),$$

where $D(h,\nabla^t)$ represents household h's payoffs on state-contingent claims for state ∇^t. $Z(\nabla^{t+1} \mid \nabla^t)$ is the price of a claim that pays one dollar in state ∇^{t+1}, conditional on state ∇^t occurring at time *t*.

In this equation, P_t is the exact price index for consumption, given by:

(6) $$P_t = k^{-1} P_{Ht}^{\nu/2} P_{Ft}^{1-(\nu/2)}, \quad k = (1-(\nu/2))^{1-(\nu/2)} (\nu/2)^{\nu/2}.$$

P_{Ht} is the Home-currency price of the Home aggregate good and P_{Ft} is the Home currency price of the Foreign aggregate good. Equation (6) follows from cost minimization. Also, from cost minimization, P_{Ht} and P_{Ft} are the usual CES aggregates over prices of individual varieties, *f*:

(7) $$P_{Ht} = \left(\int_0^1 P_{Ht}(f)^{1-\xi} df \right)^{\frac{1}{1-\xi}}, \text{ and } P_{Ft} = \left(\int_0^1 P_{Ft}(f)^{1-\xi} df \right)^{\frac{1}{1-\xi}}.$$

Foreign households have analogous preferences and face an analogous budget constraint.

Because all Home households are identical, we can drop the index for the household and use the fact that aggregate per capita consumption of each good is equal to the consumption of each good by each household. The first-order conditions for consumption are given by:

(8) $$P_{Ht} C_{Ht} = \frac{\nu}{2} P_t C_t,$$

(9) $$P_{Ft} C_{Ft} = \left(1 - \frac{\nu}{2}\right) P_t C_t,$$

(10) $$C_{Ht}(f) = \left(\frac{P_{Ht}(f)}{P_{Ht}} \right)^{-\xi} C_{Ht} \quad \text{and} \quad C_{Ft}(f) = \left(\frac{P_{Ft}(f)}{P_{Ft}} \right)^{-\xi} C_{Ft},$$

(11) $$\beta \left(C(\nabla^{t+1}) / C(\nabla^t) \right)^{-\sigma} (P_t / P_{t+1}) = \ddot{Z}(\nabla^{t+1} \mid \nabla^t).$$

In equation (11), we explicitly use an index for the state at time *t* for the purpose of clarity. $\ddot{Z}(\nabla^{t+1} \mid \nabla^t)$ is the normalized price of the state contingent claim. That is, it is defined as $Z(\nabla^{t+1} \mid \nabla^t)$ divided by the probability of state ∇^{t+1} conditional on state ∇^t.

Note that the sum of $Z(\nabla^{t+1} \mid \nabla^t)$ across all possible states at time $t + 1$ must equal $1/R_t$, where R_t denotes the gross nominal yield on a one-period non-state-contingent bond. Therefore, taking a probability-weighted sum across all states of equation (11), we have the familiar Euler equation:

(12) $$\beta R_t \mathrm{E}_t \left[\left(C(\nabla^{t+1}) / C(\nabla^t) \right)^{-\sigma} (P_t / P_{t+1}) \right] = 1.$$

Analogous equations hold for Foreign households. Since contingent claims are (arbitrarily) denominated in Home currency, the first-order condition for Foreign households that is analogous to equation (11) is:

(13) $\beta\left(C^*(\nabla^{t+1})/C^*(\nabla^t)\right)^{-\sigma}(E_tP_t^*/E_{t+1}P_{t+1}^*)=\ddot{Z}(\nabla^{t+1}\,|\,\nabla^t)$.

Here, E_t refers to the home currency price of foreign currency exchange rate.[17]

As noted above, we will assume at this stage that labor input of all households is the same, so $N_t = N_t(h)$.

1.b. *Firms*

Each Home good, $Y_t(f)$ is made according to a production function that is linear in the labor input. These are given by:

(14) $Y_t(f) = A_tN_t(f)$.

Note that the productivity shock, A_t, is common to all firms in the Home country. $N_t(f)$ is a CES composite of individual home-country household labor, given by:

(15) $$N_t(f)=\left(\int_0^1 N_t(h,f)^{\frac{\eta_t-1}{\eta_t}}dh\right)^{\frac{\eta_t}{\eta_t-1}},$$

where the technology parameter, η_t, is stochastic and common to all Home firms.

Profits are given by:

(16) $\Gamma_t(f) = P_{Ht}(f)C_{Ht}(f)+E_tP_{Ht}^*(f)C_{Ht}^*(f)-(1-\tau_t)W_tN_t(f)$.

In this equation, $P_{Ht}(f)$ is the home-currency price of the good when it is sold in the Home country. $P_{Ht}^*(f)$ is the foreign-currency price of the good when it is sold in the Foreign country. $C_{Ht}(f)$ is aggregate sales of the good in the home country:

(17) $C_{Ht}(f)=\int_0^1 C_{Ht}(h,f)dh$.

$C_{Ht}^*(f)$ is defined analogously. It follows that $Y_t(f) = C_{Ht}(f)+C_{Ht}^*(f)$.

[17] We offer an apology to the reader here. We want to stick to CGG's notation, who use e_t for the log of the nominal exchange rate. Consistency requires us to use E_t to refer to the level of the nominal exchange rate, so we have used the distinct but similar notation E_t to be the conditional expectation operator.

There are analogous equations for $Y_t^*(f)$, with the foreign productivity shock given by A_t^*, the foreign technology parameter shock given by η_t^*, and foreign subsidy given by τ_t^*.

1.c *Equilibrium*

Goods market clearing conditions in the Home and Foreign country are given by:

(18) $$Y_t = C_{Ht} + C_{Ht}^* = \frac{\nu}{2}\frac{P_t C_t}{P_{Ht}} + \left(1-\frac{\nu}{2}\right)\frac{P_t^* C_t^*}{P_{Ht}^*} = k^{-1}\left(\frac{\nu}{2}S_t^{1-(\nu/2)}C_t + \left(1-\frac{\nu}{2}\right)(S_t^*)^{-\nu/2}C_t^*\right),$$

(19) $$Y_t^* = C_{Ft} + C_{Ft}^* = \left(1-\frac{\nu}{2}\right)\frac{P_t C_t}{P_{Ft}} + \frac{\nu}{2}\frac{P_t^* C_t^*}{P_{Ft}^*} = k^{-1}\left(\frac{\nu}{2}(S_t^*)^{1-(\nu/2)}C_t^* + \left(1-\frac{\nu}{2}\right)S_t^{-\nu/2}C_t\right).$$

We have used S_t and S_t^* to represent the price of imported to locally-produced goods in the Home and Foreign countries, respectively:

(20) $$S_t = P_{Ft} / P_{Ht},$$

(21) $$S_t^* = P_{Ht}^* / P_{Ft}^*.$$

Equations (11) and (13) give us the familiar condition that arises in open-economy models with a complete set of state-contingent claims when PPP does not hold:

(22) $$\left(\frac{C_t}{C_t^*}\right)^{\sigma} = \frac{E_t P_t^*}{P_t} = \frac{E_t P_{Ht}^*}{P_{Ht}}(S_t^*)^{-\nu/2}S_t^{(\nu/2)-1}.$$

Total employment is determined by output in each industry:

(23) $$N_t = \int_0^1 N_t(f)df = A_t^{-1}\int_0^1 Y_t(f)df = A_t^{-1}\left(C_{Ht}V_{Ht} + C_{Ht}^* V_{Ht}^*\right).$$

where

(24) $$V_{Ht} \equiv \int_0^1 \left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} df\ , \quad \text{and} \quad V_{Ht}^* \equiv \int_0^1 \left(\frac{P_{Ht}^*(f)}{P_{Ht}^*}\right)^{-\xi} df\ .$$

2. Log-linearized Model

In this section, we present some log-linear approximations to the models presented above. The full set of log-linearized equations appears in Appendix A. Our approach to the optimal policy decision is to consider a second-order approximation of the welfare function around the efficient steady state. The derivation of the loss function itself requires a second-order approximation of the utility function itself, but in the course of the derivation will actually require second-order approximations to some of the equations of the model. However, for many purposes, the first-order approximations are useful: the constraints in the optimization problem need only be approximated to the first order; the optimality conditions for monetary policy – the "target criteria" – are linear; and, we can analyze the dynamics under the optimal policy in the linearized model.

In our notation, lower case letters refer to the deviation of the log of the corresponding upper case from steady state.

If firms set the same price for Home and Foreign consumers, then $s_t^* = -s_t$. We will assume, to a first order, $s_t^* = -s_t$ even if firms set different prices in the two countries. That is, for the aggregate price indexes, $p_{Ft} - p_{Ht} = p_{Ft}^* - p_{Ht}^*$, so relative prices are the same in the Home and Foreign countries. This relationship will turn out to hold in our Calvo pricing model under LCP, when the frequency of price adjustment is identical in the two countries.

We define the log of the deviation from the law of one price as:

(25) $$\Delta_t \equiv e_t + p_{Ht}^* - p_{Ht}.$$

Because $s_t^* = -s_t$, the law of one price deviation is the same for both goods: $\Delta_t \equiv e_t + p_{Ft}^* - p_{Ft}$.

The market-clearing conditions, (18) and (19) are approximated as:

(26) $$y_t = \frac{\nu(2-\nu)}{2} s_t + \frac{\nu}{2} c_t + \frac{2-\nu}{2} c_t^*,$$

(27) $$y_t^* = \frac{-\nu(2-\nu)}{2} s_t + \frac{\nu}{2} c_t^* + \frac{2-\nu}{2} c_t.$$

The condition arising from complete markets that equates the marginal utility of nominal wealth for Home and Foreign households, equation (22), is given by:

(28) $$\sigma c_t - \sigma c_t^* = \Delta_t + (\nu - 1) s_t.$$

We can express c_t, c_t^*, and s_t in terms of y_t, y_t^* and Δ_t:

(29) $$c_t = \frac{D + \nu - 1}{2D} y_t + \frac{D - (\nu - 1)}{2D} y_t^* + \frac{\nu(2-\nu)}{2D} \Delta_t$$

(30) $$c_t^* = \frac{D + \nu - 1}{2D} y_t^* + \frac{D - (\nu - 1)}{2D} y_t - \frac{\nu(2-\nu)}{2D} \Delta_t$$

(31) $$s_t = \frac{\sigma}{D}(y_t - y_t^*) - \frac{(\nu - 1)}{D} \Delta_t,$$

where $D \equiv \sigma\nu(2-\nu) + (\nu - 1)^2$.

The model is closed and solutions for the endogenous variables can be derived once policy rules are determined. We turn to consideration of optimal monetary policy.

3. Loss Functions and Optimal Policy

We derive the loss function for the cooperative monetary policy problem. The loss function is derived from a second-order approximation to households' utility functions. Loss is measured relative to the efficient allocations.

The policymaker wishes to minimize

(32) $$\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \mathrm{X}_{t+j} \ .$$

This loss function is derived from household's utility, given in Equation (1). The period loss, X_{t+j}, represents the difference between the maximum utility achievable under efficient allocations and the utility of the market-determined levels of consumption and leisure.

We derive the loss function for the cooperative policymaker which is the relevant criterion for evaluating world welfare. We aim to highlight the global inefficiency that arises from currency misalignments. For that reason, we set aside the difficult issues involved with deriving the loss function for a non-cooperative policymaker and defining a non-cooperative policy game. In practice WTO and IMF rules as well as implicit rules of neighborliness may prohibit this type of policy.[18] Major central banks have generally been unwilling to announce explicit targets for exchange rates without full cooperation of their partners. Even if the cooperative policy analysis is not a realistic description of actual policy decision-making, the welfare function is a measure of what could be achieved under cooperation.

We note for future work three technical problems that arise in examining non-cooperative policy. None of these problems crop up in CGG because of a special set of assumptions in that model: the law of one price holds; Home and Foreign households have identical preferences; no preference shocks; and, preferences over Home and Foreign aggregates are Cobb-Douglas. The first problem is that (22), which assumes an equal distribution of wealth between Home and Foreign households, does not necessarily hold under off-equilibrium policy alternatives (see the discussion in Devereux and Engel, 2003.) Second, in the LCP model, we do not have $S_t^{*-1} = S_t$, except to a first-order log-linear approximation when we assume equal speeds of price adjustment for all goods.[19] The third is that optimal state-dependent policy reaction functions in each country require the policymaker to adjust both the Home and Foreign interest rate to Home and Foreign variables, while in the CGG model the policy reactions neatly dichotomize: the Home

[18] See Staiger and Sykes (2008) for a discussion of how some have interpreted WTO rules as prohibiting "currency manipulation", and the relationship between IMF rules prohibiting exchange rate manipulation and WTO rules prohibiting certain export subsidies.
[19] See Benigno (2004) and Woodford (2007) on this point.

policymaker sets Home PPI inflation and the Home output gap taking the Foreign policy choices as given, and vice-versa for the Foreign policymaker.

Appendix B shows the steps for deriving the loss function when there are no currency misalignments, but with home bias in preferences. We use a superscript ~ to denote the deviation of a variable from the value it would take under an efficient allocation given the sequence of exogenous shocks that have hit the economy. For example, $\tilde{y}_t$ is the Home output gap (in logs). It is worth pointing out one aspect of the derivation. In closed economy models with no investment or government, consumption equals output. That is an exact relationship, and therefore the deviation of consumption from the efficient level equals the deviation of output from the efficient level to any order of approximation: $\tilde{c}_t = \tilde{y}_t$. In the open economy, the relationship is not as simple. When preferences of Home and Foreign agents are identical, and markets are complete, then the consumption aggregates in Home and Foreign are always equal (up to a constant of proportionality equal to relative wealth.) But that is not true when preferences are not the same. Equation (22) shows that we do not have $C_t = C_t^*$ under complete markets, even if the law of one price holds for both goods. Because of this, we do not have $\tilde{c}_t + \tilde{c}_t^*$ equal to $\tilde{y}_t + \tilde{y}_t^*$, except to a first-order approximation. Since we are using a second-order approximation of the utility function, we need to account for the effect of different preferences (or the effects of the terms of trade) in translating consumption gaps into output gaps.

We find:

$$(33) \qquad \mathrm{X}_t = -\left(\frac{\nu(\nu-2)\sigma(1-\sigma)}{4D}\right)(\tilde{y}_t - \tilde{y}_t^*)^2 + \left(\frac{\sigma+\phi}{2}\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) + \frac{\xi}{2}\left(\sigma_{p_H t}^2 + \sigma_{p_F^* t}^2\right).$$

The period loss depends on the squared output gap in each country, as well as the squared difference in the output gaps. The terms $\sigma_{p_H t}^2$ and $\sigma_{p_F^* t}^2$ represent the cross-sectional variance of prices of Home goods and Foreign goods, respectively. (Recall $D \equiv \sigma\nu(2-\nu) + (\nu-1)^2$.)

Appendix B also shows the derivation of the loss function in the more general case in which currency misalignments are possible. Two aspects of the derivation merit attention. First, in examining the first-order dynamics of the model, we can make use of the first-order approximation $s_t^* = -s_t$. That is an exact equation when the law of one price holds, but we do not require that this relationship hold to a second-order approximation. The derivation of the loss function must take this into account. The second point to note is that, as is standard in this class of models, price dispersion leads to inefficient use of labor. But, to a second-order approximation, this loss depends only on the cross-section variances of p_{Ht}, p^*_{Ht}, p_{Ft}, and p^*_{Ft}, and not their comovements (which would play a role in a third-order approximation.)

The loss function is given by:

$$\text{(34)}\qquad \begin{aligned} X_t = &-\left(\frac{\nu(\nu-2)\sigma(1-\sigma)}{4D}\right)(\tilde{y}_t-\tilde{y}^*_t)^2+\left(\frac{\sigma+\phi}{2}\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)+\left(\frac{\nu(2-\nu)}{4D}\right)\Delta_t^2 \\ &+\frac{\xi}{2}\left[\frac{\nu}{2}\sigma^2_{p_H t}+\frac{2-\nu}{2}\sigma^2_{p_H * t}+\frac{\nu}{2}\sigma^2_{p_F * t}+\frac{2-\nu}{2}\sigma^2_{p_F t}\right] \end{aligned}.$$

It is important to recognize that the loss function derived here, as well as the loss function derived previously (equation (33)) do not depend on how prices are set – indeed whether prices are sticky or not. The loss function here generalizes (33) to the case in which there are deviations from the law of one price, so that $\Delta_t \neq 0$. This can be seen by directly comparing (33) to (34). In (34), $\sigma^2_{p_H t}$ is the cross-sectional variance of Home goods prices in the Home country, $\sigma^2_{p_H * t}$ is the cross-sectional variance of Home goods prices in the Foreign country, etc. If there is no currency misalignment, then $\Delta_t = 0$, so $p_{Ht}(f) = p^*_{Ht}(f) + e_t$ and $p_{Ft}(f) = p^*_{Ft}(f) + e_t$ for each firm *f*. In that case, $\sigma^2_{p_H t} = \sigma^2_{p_H * t}$ and $\sigma^2_{p_F t} = \sigma^2_{p_F * t}$ because the exchange rate does not affect the cross-sectional variance of prices. If we have $\Delta_t = 0$, $\sigma^2_{p_H t} = \sigma^2_{p_H * t}$, and $\sigma^2_{p_F t} = \sigma^2_{p_F * t}$, then (34) reduces to (33).

Why does the currency misalignment appear in the loss function? That is, if both Home and Foreign output gaps are zero, and all inflation rates are zero, what problem does a misaligned currency cause? From equation (31), if the currency is misaligned, then internal relative prices (s_t) must also differ from their efficient level if the output gap is zero. For example, suppose $\Delta_t > 0$, which from (31) implies we must have $\tilde{s}_t < 0$ if both output gaps are eliminated. On the one hand, $\Delta_t > 0$ tends to lead to overall consumption in Home to be high relative to Foreign consumption (equations (29)-(30).) That occurs because financial markets pay off to Home residents when their currency is weak. But Home residents have a home bias for Home goods. That would lead to overproduction in the Home country, were it not for relative price adjustments – which is why $\tilde{s}_t < 0$.

We highlight the fact that the loss functions are derived without specific assumptions about price setting not to give a false patina of generality to the result, but to emphasize that the loss in welfare arises not specifically from price stickiness but from prices that do not deliver the efficient allocations. Of course it is our specific assumptions of nominal price and wage setting that give rise to the internal and external price misalignments in this model, and indeed monetary policy would be ineffective if there were no nominal price or wage stickiness. But one could imagine especially a number of mechanisms that give rise to deviations from the law of one price, because the literature has produced a number of models based both on nominal stickiness and real factors. In the next section, we modify the CGG model in the simplest way – allowing local-currency pricing instead of producer-currency pricing – to examine further the implications of currency misalignments.

4. Price and Wage Setting

We now introduce our models of price and wage setting. We follow CGG in assuming wages are set flexibly by monopolistic suppliers of labor, but goods prices are sticky. Wages adjust continuously, but households exploit their monopoly power by setting a wage that incorporates a mark-up over their utility cost of work.

Government is assumed to have only limited fiscal instruments. The government can set a constant output subsidy rate for monopolistic firms, which will achieve an efficient allocation in the non-stochastic steady state. But unfortunately, the mark-up charged by workers is time-varying because the elasticity of demand for their labor services is assumed to follow a stochastic process. These shocks are sometimes labeled “cost-push” shocks, and give rise to the well-known tradeoff in CGG’s work between controlling inflation and achieving a zero output gap.

Households are monopolistic suppliers of their unique form of labor services. Household *h* faces demand for its labor services given by:

(35) $$N_t(h) = \left(\frac{W_t(h)}{W_t} \right)^{-\eta_t} N_t \,,$$

where

(36) $$W_t = \left(\int_0^1 W_t(h)^{1-\eta_t}\, dh \right)^{\frac{1}{1-\eta_t}} .$$

The first-order condition for household *h*’s choice of labor supply is given by:

(37) $$\frac{W_t(h)}{P_t} = (1+\mu_t^W)(C_t(h))^{\sigma}(N_t(h))^{\phi} \text{, where } \mu_t^W \equiv \frac{1}{\eta_t - 1} .$$

The optimal wage set by the household is a time-varying mark-up over the marginal disutility of work (expressed in consumption units.)

Because all households are identical, we have $W_t = W_t(h)$ and $N_t = N_t(h)$.

Since all households are identical, we have from equation (37):

(38) $$W_t / P_{Ht} = (1+\mu_t^W) C_t^{\sigma} N_t^{\phi} S_t^{1-(v/2)} .$$

We will consider three different scenarios for firm behavior. In the first, prices can be adjusted freely. In the second, the PCP scenario that CGG analyze, firms set prices in their own country's currency and face a Calvo pricing technology. In the third, when firms are allowed to change prices according to the Calvo pricing rule, they set a price in their own currency for sales in their own country and a price in the other country's currency for exports. This is the LCP scenario.

We adopt the following notation. For any variable K_t:

$\dot{K}_t$ is the value under flexible prices.

$\bar{K}_t$ is the value of variables under globally efficient allocations. In other words, this is the value for variables if prices were flexible, and optimal subsidies to monopolistic suppliers of labor and monopolistic producers of goods were in place. This includes a time-varying subsidy to suppliers of labor to offset the time-varying mark-up in wages in equation (37).

Flexible Prices

Home firms maximize profits given by equation (16), subject to the demand curve (10). They optimally set prices as a mark-up over marginal cost:

(39) $\dot{P}_{Ht}(f) = \dot{E}_t \dot{P}^*_{Ht}(f) = (1-\tau_t)(1+\mu^P)\dot{W}_t / A_t$, where $\mu^P \equiv \dfrac{1}{\xi - 1}$.

When optimal subsidies are in place:

(40) $\bar{P}_{Ht}(f) = \bar{E}_t \bar{P}^*_{Ht}(f) = \bar{W}_t / A_t$.

From (37), (39), and (40), it is apparent that the optimal subsidy satisfies

(41) $(1-\tau_t)(1+\mu^P)(1+\mu^W_t) = 1$.

Note from (39) that all flexible price firms are identical and set the same price. Because the demand functions of Foreign residents have the same elasticity of demand for Home goods as Home residents, firms set the same price for sale abroad:

(42) $E_t \dot{P}^*_{Ht} = \dot{P}_{Ht}$ and $E_t \bar{P}^*_{Ht} = \bar{P}_{Ht}$

From (41), using (38), we have:

(43) $\dot{P}_{Ht} = \dot{E}_t \dot{P}^*_{Ht} = \dot{W}_t / ((1+\mu^W)A_t)$ and $\dot{P}^*_{Ft} = \dot{E}_t^{-1} \dot{P}_{Ft} = \dot{W}^*_t / ((1+\mu^{*W})A^*_t)$.

We can conclude:

(44) $\dot{S}^{*-1}_t = \dot{S}_t$.

Because $\dot{P}_{Ht}(f)$ is identical for all firms, (23) collapses to

(45) $\dot{Y}_t = A_t \dot{N}_t$.

PCP

We assume a standard Calvo pricing technology. A given firm may reset its prices with probability $1-\theta$ each period. We assume that when the firm resets its price, it will be able to reset its prices for sales in both markets. The PCP firm sets both prices in its own currency – that is, the Home firm sets both $P_{Ht}(f)$ and $P_{Ht}^{**}(f) \equiv E_t P_{Ht}^{*}(f)$ in Home currency. (As will become apparent, the firm optimally chooses the same price for both markets, $P_{Ht}(f) = P_{Ht}^{**}(f)$.)

The firm's objective is to maximize its value, which is equal to the value at state-contingent prices of its entire stream of dividends. Given equation (11), it is apparent that the firm that selects its prices at time *t*, chooses its reset prices, $P_{Ht}^{0}(f)$ and $P_{Ht}^{0**}(f)$, to maximize

(46) $$\mathrm{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left[P_{Ht}^{0}(f) C_{Ht+j}(f) + P_{Ht}^{0**} C_{Ht+j}^{*}(f) - (1-\tau_t) W_{t+j} N_{t+j}(f) \right],$$

subject to the sequence of demand curves given by equation (10) and the corresponding Foreign demand equation for Home goods. In this equation, we define

(47) $$Q_{t,t+j} \equiv \beta^j \left(C_{t+j} / C_t \right)^{-\sigma} (P_t / P_{t+j}).$$

The solution for the optimal price for the Home firm for sale in the Home country is given by:

(48) $$P_{Ht}^{0}(z) = \frac{\xi}{\xi-1} \frac{\mathrm{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} (1-\tau_t) W_{t+j} P_{Ht+j}^{\xi} C_{Ht+j} / A_{t+j}}{\mathrm{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} P_{Ht+j}^{\xi} C_{Ht+j}}.$$

For sale in the foreign market, we have:

(49) $$P_{Ht}^{0**}(z) = \frac{\xi}{\xi-1} \frac{\mathrm{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} (1-\tau_t) W_{t+j} (E_{t+j} P_{Ht+j}^{*})^{\xi} C_{Ht+j}^{*} / A_{t+j}}{\mathrm{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} (E_{t+j} P_{Ht+j}^{*})^{\xi} C_{Ht+j}^{*}}.$$

Under the Calvo price setting mechanism, a fraction θ of prices remain unchanged from the previous period. From equation (7), we can write:

(50) $$P_{Ht} = \left[\theta (P_{Ht-1})^{1-\xi} + (1-\theta)(P_{Ht}^{0})^{1-\xi} \right]^{1/(1-\xi)},$$

(51) $$P_{Ht}^{*} = \left[\theta (P_{Ht-1}^{*})^{1-\xi} + (1-\theta)(P_{Ht}^{0*})^{1-\xi} \right]^{1/(1-\xi)}$$

Taking equations (48), (49), (50), and (51), we see that the law of one price holds under PCP. That is, $P_{Ht}(f) = E_t P_{Ht}^{*}(f)$ for all *f*, hence $P_{Ht} = E_t P_{Ht}^{*}$. Hence, under PCP we have $S_t^{*-1} = S_t$.

LCP

The same environment as the PCP case holds, with the sole exception that the firm sets its price for export in the importer's currency rather than its own currency when it is allowed to reset prices. The Home firm, for example, sets $P_{Ht}^{*}(f)$ in Foreign currency. The firm that can reset its price at time *t* chooses its reset prices, $P_{Ht}^{0}(f)$ and $P_{Ht}^{0*}(f)$, to maximize

(52) $$\mathrm{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left[P_{Ht}^{0}(z) C_{Ht+j}(f) + E_t P_{Ht}^{0*} C_{Ht+j}^{*}(f) - (1-\tau_t) W_{t+j} N_{t+j}(f) \right].$$

The solution for $P_{Ht}^{0}(z)$ is identical to (48). We find for export prices,

(53) $$P_{Ht}^{0*}(z) = \frac{\xi}{\xi - 1} \frac{\mathrm{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} (1-\tau_t) W_{t+j} (P_{Ht+j}^{*})^{\xi} C_{Ht+j}^{*} / A_{t+j}}{\mathrm{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} E_{t+j} (P_{Ht+j}^{*})^{\xi} C_{Ht+j}^{*}}.$$

Equations (50) and (51) hold in the LCP case as well. However, the law of one price does not hold.

Subsidies

As in CGG, we will assume that subsidies to monopolists are not set at their optimal level except in steady-state. That is, instead of the efficient subsidy given in equation (41), we have:

(54) $$(1-\tau)(1+\mu^{P})(1+\mu^{W}) = 1.$$

Here, μ^{W} is the steady-state level of μ_t^{W}. We have dropped the time subscript on the subsidy rate τ_t because it is not time-varying.

5. Log-linearized Phillips Curves

Under PCP, we can derive a New Keynesian Phillips curve for an open economy:

$$\pi_{Ht} = \delta(\tilde{w}_t - \tilde{p}_{Ht}) + \beta \mathrm{E}_t \pi_{Ht+1},$$

or:

(55) $$\pi_{Ht} = \delta\left[\left(\frac{\sigma(1+D)}{2D} + \phi\right)\tilde{y}_t + \frac{\sigma(D-1)}{2D}\tilde{y}_t^{*}\right] + \beta \mathrm{E}_t \pi_{Ht+1} + u_t,$$

where $\delta \equiv (1-\theta)(1-\beta\theta)/\theta$, and $u_t \equiv \delta \mu_t^{W}$.

Similarly for foreign producer-price inflation, we have:

(56) $$\pi_{Ft}^{*} = \delta\left[\left(\frac{\sigma(1+D)}{2D} + \phi\right)\tilde{y}_t^{*} + \frac{\sigma(D-1)}{2D}\tilde{y}_t\right] + \beta \mathrm{E}_t \pi_{Ft+1}^{*} + u_t^{*}.$$

In the LCP model, the law of one price deviation is not zero. We have:

(57) $$\pi_{Ht} = \delta\left[\left(\frac{\sigma(1+D)}{2D}+\phi\right)\tilde{y}_t + \frac{\sigma(D-1)}{2D}\tilde{y}_t^* + \frac{D-(\nu-1)}{2D}\Delta_t\right] + \beta E_t\pi_{Ht+1} + u_t\ ,$$

(58) $$\pi_{Ft}^* = \delta\left[\left(\frac{\sigma(1+D)}{2D}+\phi\right)\tilde{y}_t^* + \frac{\sigma(D-1)}{2D}\tilde{y}_t + \frac{\nu-1-D}{2D}\Delta_t\right] + \beta E_t\pi_{Ft+1}^* + u_t^*\ .$$

There are also price adjustment equations for the local prices of imported goods:

(59) $$\pi_{Ht}^* = \delta\left[\left(\frac{\sigma(1+D)}{2D}+\phi\right)\tilde{y}_t + \frac{\sigma(D-1)}{2D}\tilde{y}_t^* - \left(\frac{D+\nu-1}{2D}\right)\Delta_t\right] + \beta E_t\pi_{Ht+1}^* + u_t\ .$$

(60) $$\pi_{Ft} = \delta\left[\left(\frac{\sigma(1+D)}{2D}+\phi\right)\tilde{y}_t^* + \frac{\sigma(D-1)}{2D}\tilde{y}_t + \frac{D+\nu-1}{2D}\Delta_t\right] + \beta E_t\pi_{Ft+1} + u_t^*\ .$$

From (57)-(58) and (59)-(60), we see $\pi_{Ft} - \pi_{Ht} = \pi_{Ft}^* - \pi_{Ht}^*$. Assuming a symmetric initial condition, so that $p_{F0} - p_{H0} = p_{F0}^* - p_{H0}^*$, we conclude $s_t^* = -s_t$ as we noted above. That is, the relative price of Foreign to Home goods is the same in both countries. We emphasize that this is true in general for a first-order approximation.

The policymaker has home and foreign nominal interest rates as instruments. As is standard in the literature, we can model the policymaker as directly choosing output gaps, inflation levels, and (in the LCP case) deviations from the law of one price, subject to constraints. From the first-order conditions, we can back out the optimal choice of nominal interest rates using a log-linearized version of equation (12) and its foreign counterpart, given by:

(61) $$r_t - E_t\pi_{t+1} = \sigma(E_t c_{t+1} - c_t)\ ,$$

(62) $$r_t^* - E_t\pi_{t+1}^* = \sigma(E_t c_{t+1}^* - c_t^*)\ .$$

In these equations, π_t and π_t^* refer to Home and Foreign consumer price inflation, respectively:

(63) $$\pi_t = \frac{\nu}{2}\pi_{Ht} + \frac{2-\nu}{2}\pi_{Ft}\ ,$$

(64) $$\pi_t^* = \frac{\nu}{2}\pi_{Ft}^* + \frac{2-\nu}{2}\pi_{Ht}^*\ .$$

6. <u>Optimal Policy under PCP</u>

As is familiar in the New Keynesian models with Calvo price adjustment, the loss function can be rewritten in the form:

$$E_t\sum_{j=0}^{\infty}\beta^j X_{t+j} = E_t\sum_{j=0}^{\infty}\beta^j \Psi_{t+j}$$

where:

(65) $$\Psi_t \propto -\frac{\nu(2-\nu)\sigma(\sigma-1)}{4D}(\tilde{y}_t - \tilde{y}_t^*)^2 + \left(\frac{\sigma+\phi}{2}\right)((\tilde{y}_t)^2 + (\tilde{y}_t^*)^2) + \frac{\xi}{2\delta}\left((\pi_{Ht})^2 + (\pi_{Ft}^*)^2\right).$$

This loss function extends the one derived in CGG to the case of home bias in preferences (i.e., $\nu \geq 1$ rather than $\nu = 1$.)

The policymaker chooses values for $\tilde{y}_t$, $\tilde{y}_t^*, \pi_{Ht}$ and π_{Ft}^* to minimize the loss, subject to the Phillips curves (55) and (56). Under discretion, the policymaker takes past values of $\tilde{y}_t$, $\tilde{y}_t^*, \pi_{Ht}$ and π_{Ft}^* as given, and also does not make plans for future values of these variables understanding that future incarnations of the policymaker can alter any given plan. The policymaker at time *t* cannot influence $\mathrm{E}_t\pi_{Ht+1}$ and $\mathrm{E}_t\pi_{Ft+1}^*$ because future inflation levels are chosen by future policymakers and there are no endogenous state variables that can limit the paths of future inflation levels. Hence, the policymaker's problem is essentially a static one – to maximize (65) subject to (55) and (56), taking $\mathrm{E}_t\pi_{Ht+1}$ and $\mathrm{E}_t\pi_{Ft+1}^*$ as given.

Even though we have introduced home bias in consumption, the optimal policy rules are the same as in CGG. The first-order conditions are given by:

(66) $$\tilde{y}_t + \tilde{y}_t^* + \xi(\pi_{Ht} + \pi_{Ft}^*) = 0$$

(67) $$\tilde{y}_t - \tilde{y}_t^* + \xi(\pi_{Ht} - \pi_{Ft}^*) = 0.$$

These two "target criteria" can be rewritten as:

(68) $$\tilde{y}_t + \xi\pi_{Ht} = 0, \quad \text{and} \quad \tilde{y}_t^* + \xi\pi_{Ft}^* = 0.$$

The criteria given in (68) are identical to those that arise in the closed-economy version of this model. There is a tradeoff between the goals of eliminating the output gap and driving inflation to zero, and the elasticity of substitution among goods produced in the country determines the weights given to output gaps and inflation.

It is worth emphasizing that equation (68) indicates the optimal policy entails a tradeoff between the output gap and the *producer price* inflation level. In a closed economy with no intermediate goods, there is no distinction between producer and consumer prices. But in an open economy there is an important distinction. The policies described in (68) imply that policymakers should not give any weight to inflation of imported goods. In conjunction with the Phillips curves, (55) and (56), equation (68) allows us to solve for the Home and Foreign output gaps and π_{Ht} and π_{Ft}^* as functions of current and expected future cost-push shocks, u_t and u_t^*. With the output gap determined by optimal policy, the terms of trade must adjust to insure goods market clearing. But the terms of trade adjust freely in the PCP world because nominal exchange rate changes translate directly into import price changes. In essence, the import sector is like a flexible-price sector, so policymakers can ignore inflation in that sector, as in Aoki (2001).

7. Optimal Policy under LCP

The transformed loss function, Ψ_t, can be written as:

$$\text{(69)}\quad \begin{aligned}\Psi_t \propto & -\frac{\nu(2-\nu)\sigma(\sigma-1)}{4D}(\tilde{y}_t-\tilde{y}_t^*)^2+\left(\frac{\sigma+\phi}{2}\right)((\tilde{y}_t)^2+(\tilde{y}_t^*)^2)+\left(\frac{\nu(2-\nu)}{4D}\right)\Delta_t^2 \\ & +\frac{\xi}{2\delta}\left(\frac{\nu}{2}(\pi_{Ht})^2+\frac{2-\nu}{2}(\pi_{Ft})^2+\frac{\nu}{2}(\pi_{Ft}^*)^2+\frac{2-\nu}{2}(\pi_{Ht}^*)^2\right)\end{aligned}.$$

The loss function is similar to the one under PCP. The main point to highlight is that squared deviations from the law of one price matter for welfare, as well as output gaps and inflation rates. Deviations from the law of one price are distortionary and are a separate source of loss in the LCP model.

The policymaker under discretion seeks to minimize the loss subject to the constraints of the Phillips curves, (57)-(58) and (59)-(60). There is an additional constraint in the LCP model. Note that $\pi_{Ft}-\pi_{Ht}=s_t-s_{t-1}$. But from equation (31), we have

$$\text{(70)}\quad s_t-s_{t-1}=\frac{\sigma}{D}\left(y_t-y_t^*-(y_{t-1}-y_{t-1}^*)\right)-\frac{(\nu-1)}{D}(\Delta_t-\Delta_{t-1}).$$

Using (70) in conjunction with (57) and (60), we derive:

$$\text{(71)}\quad \begin{aligned}&\frac{\sigma}{D}\left(y_t-y_t^*-(y_{t-1}-y_{t-1}^*)\right)-\frac{(\nu-1)}{D}(\Delta_t-\Delta_{t-1})= \\ &\delta\left[\left(\frac{\sigma}{D}+\phi\right)(\tilde{y}_t-\tilde{y}_t^*)-\frac{(\nu-1)}{2D}\Delta_t\right]+\beta E_t(\pi_{Ht+1}-\pi_{Ft+1})+u_t-u_t^*\end{aligned}.$$

This constraint arises in the LCP model but not in the PCP model precisely because import prices are sticky and subject to a Calvo price-adjustment mechanism, rather than free to respond via nominal exchange-rate changes.

Another contrast with the PCP model is that there are four sticky prices in the LCP model, so non-zero inflation rates for each of the four matter for welfare. Indeed, we note that we can rewrite the loss function as:

$$\text{(72)}\quad \begin{aligned}\Psi_t \propto & -\frac{\nu(2-\nu)\sigma(\sigma-1)}{4D}(\tilde{y}_t-\tilde{y}_t^*)^2+\left(\frac{\sigma+\phi}{2}\right)((\tilde{y}_t)^2+(\tilde{y}_t^*)^2)+\left(\frac{\nu(2-\nu)}{4D}\right)\Delta_t^2 \\ & +\frac{\xi}{2\delta}\left((\pi_t)^2+(\pi_t^*)^2+\frac{\nu(2-\nu)}{2}(s_t-s_{t-1})^2\right)\end{aligned}.$$

Under this formulation, the loss function is seen to depend on the aggregate CPI inflation rates, π_t and π_t^*, and the change in the terms of trade, $s_t - s_{t-1}$, rather than the four individual inflation rates given in equation (69). This formulation is particularly useful when we consider a simplification below, under which $s_t - s_{t-1}$ is independent of policy.

The LCP optimization problem under discretion becomes very messy and difficult because of the additional constraint given by equation (71), because there now are endogenous state variables. The choices of Home output gap relative to the Foreign output gap and the deviation from the law of one price puts constraints on the evolution of future output gaps, inflation rates and deviations from the law of one price. In the LCP case, the dynamic game between current and future policymakers is non-trivial.

But inspection of equation (71) reveals a special case in which the policy decision under uncertainty can be settled under the same simple conditions as in the PCP model. When $\phi = 0$, so utility is quasi-linear in labor, equation (71) simplifies considerably:

(73) $$s_t - s_{t-1} = -\delta\tilde{s}_t + \beta \mathrm{E}_t(s_{t+1} - s_t) + u_t - u_t^*.$$

With $\phi = 0$, we have $\tilde{s}_t = s_t - \bar{s}_t = s_t - (a_t - a_t^*)$. So we can write (73) as a second-order expectational difference equation:

(74) $$s_t = \frac{1}{1+\delta+\beta}s_{t-1} + \frac{\beta}{1+\delta+\beta}\mathrm{E}_t s_{t+1} + \frac{1}{1+\delta+\beta}\vartheta_t,$$

where $\vartheta_t = \delta(a_t - a_t^*) + u_t - u_t^* = \delta\dot{s}_t$.

Equation (74) determines the evolution of s_t independent of policy choices. So while s_t is a state variable, it is not endogenous for the policymaker. One nice thing about considering this special case is that the parameter ϕ does not appear in either the target criteria or the optimal interest rate rule in the CGG model, so we can compare the criteria and rules directly to the LCP model. We also note that Devereux and Engel (2003) make the same assumption on preferences.

Using (72), in this case the loss function can be simplified to:

(75) $$\Psi_t \propto \frac{-\sigma}{4D}(\tilde{y}_t^R)^2 - \frac{\sigma}{4}(\tilde{y}_t^W)^2 - \left(\frac{\nu(2-\nu)}{4D}\right)\Delta_t^2 - \frac{\xi}{4\delta}\left((\pi_t^R)^2 + (\pi_t^W)^2 + \nu(2-\nu)(s_t - s_{t-1})^2\right)$$

where the R superscript represents Home relative to Foreign. That is, $\pi_t^R = \pi_t - \pi_t^*$, $u_t^R = u_t - u_t^*$, etc. Likewise, the W superscript refers to the sum of Home and Foreign variables: $\pi_t^W = \pi_t + \pi_t^*$, $u_t^W = u_t + u_t^*$, etc. Since $s_t - s_{t-1}$ is independent of policy, we can express the policymaker's problem as choosing relative and world output gaps, $\tilde{y}_t^R$ and $\tilde{y}_t^W$, relative and world CPI inflation rates, π_t^R and π_t^W, and the currency misalignment, Δ_t to maximize (75) subject to the "gap" version of equation (31) and the linear combination of the Phillips curves that give us equations for CPI inflation in each country (which are derived here under the assumption that $\phi = 0$):

(76) $$\tilde{s}_t = \frac{\sigma}{D}(\tilde{y}_t - \tilde{y}_t^*) - \frac{(\nu-1)}{D}\Delta_t$$

(77) $$\pi_t^R = \delta\left[\frac{\sigma(\nu-1)}{D}\tilde{y}_t^R + \frac{\sigma\nu(2-\nu)}{D}\Delta_t\right] + \beta \mathrm{E}_t \pi_{t+1}^R + (\nu-1)u_t^R$$

(78) $$\pi_t^W = \delta\sigma\tilde{y}_t^W + \beta \mathrm{E}_t \pi_{t+1}^W + u_t^W$$

The first condition seems quite similar to the first condition in the PCP case, (66):

(79) $$\tilde{y}_t^W + \xi\pi_t^W = 0.$$

The condition calls for a tradeoff between the world output gap and the world inflation rate, just as in the PCP case. But there is a key difference – here in the LCP model, it is the CPI, not the PPI, inflation rates that enter into the policymaker's tradeoff.

The second condition can be written as:

(80) $$\frac{1}{\sigma}\tilde{q}_t + \xi\pi_t^R = 0.$$

Here, q_t is the consumption real exchange rate, defined as:

(81) $$q_t = e_t + p_t^* - p_t.$$

$\tilde{q}_t$ is the deviation of the real exchange rate from its efficient level, and we have used the relationship:

(82) $$\frac{1}{\sigma}\tilde{q}_t = \frac{\nu-1}{D}\tilde{y}_t^R + \frac{\nu(2-\nu)}{D}\Delta_t.$$

Equation (80) represents the second of the target criteria as a tradeoff between misaligned real exchange rates and relative CPI inflation rates. In the LCP model, where exchange-rate misalignments are possible, we can see from (82) this optimal policy involves trading off relative output gaps, relative CPI inflation rates, and the currency misalignment.

Optimal Policy under PCP vs. LCP

It is helpful to compare the target criteria under PCP and LCP. We will compare conditions (66) and (67) under PCP to conditions (79) and (80) under LCP.

First, compare (66) to (79). Both involve the tradeoff between the world output gap and world inflation. But under PCP, producer price inflation appears in the tradeoff. However, world producer price inflation is equal to world consumer price inflation under PCP. To see this,

(83) $$\pi_t^W = \frac{\nu}{2}\pi_{Ht} + \frac{2-\nu}{2}\pi_{Ft} + \frac{\nu}{2}\pi_{Ft}^* + \frac{2-\nu}{2}\pi_{Ht}^* = \pi_{Ht} + \pi_{Ft}^* .$$

The second equality holds because the relative prices are equal in Home and Foreign under PCP (and, for that matter, to a first-order approximation under LCP) so $\pi_{Ht}^* = \pi_{Ft}^* + \pi_{Ht} - \pi_{Ft}$.

This tradeoff is the exact analogy to the closed economy tradeoff between the output gap and inflation, and the intuition of that tradeoff is well understood. On the one hand, with asynchronized price setting, inflation leads to misalignment of relative prices, so any non-zero level of inflation is distortionary. On the other hand, because the monopoly power of labor is time-varying due to the time-varying elasticity of labor demand, output levels can be inefficiently low or high even when inflation is zero. Conditions (66) or (79) describe the terms of that tradeoff. Inflation is more costly when the higher is the elasticity of substitution among varieties of goods, ξ , because a higher elasticity will imply greater resource misallocation when there is inflation.

The difference in optimal policy under PCP versus LCP comes in the comparison of condition (67) with (80). Under PCP, optimal policy trades off Home relative to Foreign output gaps with Home relative to Foreign PPI inflation. Under LCP, the tradeoff is between the real exchange rate and Home relative to Foreign CPI inflation.

First, it is helpful to consider equation (80) when the two economies are closed, so that $\nu = 2$. Using (82), under this condition, $D = 1$, and (80) reduces to $\tilde{y}_t^R + \xi\pi_t^R = 0$. Of course, when $\nu = 2$, there is no difference between PPI and CPI inflation, and so in this special case the optimal policies under LCP and PCP are identical. That is nothing more than reassuring, since the distinction between PCP and LCP should not matter when the economies are closed.

When $\nu \neq 2$, understanding these conditions is more subtle. It helps to consider the case of no home bias in preferences, so $\nu = 1$. Imagine that inflation rates were zero, so that there is no misallocation of labor within each country. Further, imagine that the world output gap is zero. There are still two possible distortions. First, relative Home to Foreign output may not be at the efficient level. Second, even if output levels are efficient, the allocation of output to Home and Foreign households may be inefficient if there are currency misalignments.

When $\nu = 1$, it follows from equations (26) and (27) that relative output levels are determined only by the terms of trade. We have $\tilde{y}_t^R = \tilde{s}_t$. On the other hand, from equation (28) when $\nu = 1$, relative consumption is misaligned when there are currency misalignments,

$\tilde{c}_t^R = \frac{1}{\sigma}\Delta_t$. Moreover, when $\nu = 1$, the deviation of the real exchange rate from its inefficient level is entirely due to the currency misalignment: $\tilde{q}_t = \Delta_t$.

Under PCP, the law of one price holds continuously, so there is no currency misalignment. In that case, $\Delta_t = 0$, and relative Home to Foreign consumption is efficient. In that case, policy can influence the terms of trade in order to achieve the optimal tradeoff between relative output gaps and relative inflation, as expressed in equation (67). Policy can control the terms of trade under PCP because the terms of trade can adjust instantaneously and completely through nominal exchange-rate adjustment. That is, $s_t = p_{Ft} - p_{Ht} = e_t + p_{Ft}^* - p_{Ht}$. While p_{Ft}^* and p_{Ht} do not adjust freely, the nominal exchange rate e_t is not sticky, so the terms of trade adjust freely.

Under LCP, the nominal exchange rate does not directly influence the consumer prices of Home to Foreign goods in either country. For example, in the Home country, $s_t = p_{Ft} - p_{Ht}$. Because prices are set in local currencies, neither p_{Ft} and p_{Ht} adjust freely to shocks. In fact, as we have seen, when $\phi = 0$, monetary policy has no control over the internal relative prices.

But under LCP, there are currency misalignments, and monetary policy can control those. Recall from (25) $\Delta_t \equiv e_t + p_{Ht}^* - p_{Ht} = e_t + p_{Ft}^* - p_{Ft}$, so the currency misalignment adjusts instantaneously with nominal exchange rate movements. Because policy cannot influence the relative output distortion (when $\nu = 1$) but can influence the relative consumption distortion, the optimal policy puts full weight on the currency misalignment. When $\nu = 1$, we can write (80) as $\frac{1}{\sigma}\Delta_t + \xi\pi_t^R = 0$. When $\Delta_t > 0$, so that the Home currency is undervalued, and $\pi_t - \pi_t^* > 0$, the implications for policy are obvious. Home monetary policy must tighten relative to Foreign. But the more interesting case to consider is when Home inflation is running high, so that $\pi_t - \pi_t^* > 0$, but the currency is overvalued, so that $\Delta_t < 0$. Then equation (80) tells us that the goals of maintaining low inflation and a correctly aligned currency are in conflict. Policies that improve the inflation situation may exacerbate the currency misalignment. Equation (80) parameterizes the tradeoff.

8. The Interest-Rate Reaction Functions

We can derive an interest rate rule that will support the optimal policies given by equations (79) and (80).

Substituting equation (80) into equations (57)-(58) and (59)-(60), and using the definitions of CPI inflation given in equations (63) and (64), we can derive:

(84) $$\pi_t^R = \frac{\beta}{1+\delta\sigma\xi}\mathrm{E}_t\pi_{t+1}^R + \frac{\nu-1}{1+\delta\sigma\xi}u_t^R,$$

Assume that u_t and u_t^* are each AR(1) processes, independently distributed, with a serial correlation coefficient given by ρ. We can solve (84) as:

(85) $$\pi_t^R = \frac{\nu-1}{1+\delta\sigma\xi-\beta\rho}u_t^R.$$ [20]

Similarly, under the optimal monetary policy, we can get a solution for "world" inflation:

(86) $$\pi_t^W = \frac{\beta}{1+\delta\sigma\xi}E_t\pi_{t+1}^W + \frac{1}{1+\delta\sigma\xi}u_t^W,$$

Under the assumption that the mark-up shocks follow AR(1) processes, we find:

(87) $$\pi_t^W = \frac{1}{1+\delta\sigma\xi-\beta\rho}u_t^W$$

Substituting these equations into the Euler equations for the Home and Foreign country, given by equations (61) and (62), making use of the consumption equations (29) and (30), we find:

(88) $$r_t^R = (\rho+(1-\rho)\sigma\xi)\pi_t^R + \frac{\sigma(\nu-1)}{D}(\mathrm{E}_t\overline{y}_{t+1}^R - \overline{y}_t^R) = (\rho+(1-\rho)\sigma\xi)\pi_t^R + \overline{rr}_t^R,$$

(89) $$r_t^W = (\rho+(1-\rho)\sigma\xi)\pi_t^W + \sigma(\mathrm{E}_t\overline{y}_{t+1}^W - \overline{y}_t^W) = (\rho+(1-\rho)\sigma\xi)\pi_t^W + \overline{rr}_t^W.$$

Here, $\overline{rr}_t$ represents the real interest rate in the efficient economy. We can use (88) and (89) to write:

(90) $$r_t = (\rho+(1-\rho)\sigma\xi)\pi_t + \overline{rr}_t$$

(91) $$r_t^* = (\rho+(1-\rho)\sigma\xi)\pi_t^* + \overline{rr}_t^*.$$

These interest-rate reaction functions are identical to the ones derived in CGG for the PCP model, except that the inflation term that appears on the right-hand-side of each equation is CPI inflation here, while it is PPI inflation in CGG.

This finding starkly highlights the difference between monetary rules expressed as "target criteria" (or targeting rules) and monetary rules expressed as interest-rate reaction functions (or instrument rules.) The optimal interest-rate reaction functions presented in equations (90) and (91) appear to give no role for using monetary policy to respond to law of one price deviations.

[20] See Appendix B for the complete solutions of the model under optimal policy.

However, the "target criteria" show the optimal tradeoff does give weight to the law of one price deviation. The key to understanding this apparent conflict is that the reaction functions, such as (90) and (91) are not only setting inflation rates. By setting Home relative to Foreign interest rates, they are also prescribing a relationship between Home and Foreign output gaps, the law of one price deviation, and Home relative to Foreign inflation rates.

If central bankers really did mechanically follow an interest rate rule, their optimal policy rules would have the nominal interest rate responding only to CPI inflation. In practice, however, central bankers set the interest rates to achieve their targets. We have shown that the optimal target criteria involves tradeoffs among the goals of achieving zero inflation, driving the output gaps to zero, and eliminating the law of one price gap.

The optimal policy indeed is not successful in eliminating the currency misalignment. Nor does policy drive inflation to zero or eliminate the output gap. Monetary policymakers do not have sufficient control over the economy to achieve the efficient outcome. Appendix B.4 displays the solutions for inflation, the output gap, and the currency misalignment under optimal policy.

10. Optimal Policy under Commitment

Although CGG do not present the optimal targeting rules under commitment, the derivation of these under PCP and LCP is straightforward, and the relationship to the rules under discretion is analogous to that found in the closed economy literature.[21]

As in the closed economy, we find in the PCP model that the optimal tradeoff under commitment has the flavor of price-level targeting. When policymakers commit to a rule at time 0, the target criteria at all dates are given by:

(92) $\tilde{y}_t + \xi(p_{Ht} - p_{H-1}) = 0$, and $\tilde{y}_t^* + \xi(p_{Ft}^* - p_{F-1}^*) = 0$.

In comparison with the optimal rules under discretion, (68), policy under commitment trades off the output gap with the deviation of the current producer price and its level the period before the policy commitment was made, $p_{Ht} - p_{H-1}$ or $p_{Ft}^* - p_{F-1}^*$.

Under LCP, the target criteria are altered in the same way. The tradeoff is between a real variable and a price level deviation under commitment, rather than between a real variable and inflation under discretion. The rules under commitment that are analogous to (79) and (80) under discretion are:

(93) $\tilde{y}_t^W + \xi(p_t^W - p_{-1}^W) = 0$,

[21] See Gali (2008).

(94) $$\frac{1}{\sigma}\tilde{q}_t + \xi(p_t^R - p_{-1}^R) = 0 .$$

11. No Mark-up Shocks

It is well known that in the PCP model, if there are no mark-up shocks and the optimal steady-state subsidies are in place, the efficient allocation is obtainable under optimal monetary policy. A policy that sets inflation of Home and Foreign prices to zero eliminates all distortions. The flexibility of the exchange rate allows for optimal terms of trade response to shocks. This holds true both under commitment and under discretion.

Under LCP, interestingly, optimal policy sets Home and Foreign CPI inflation (π_t and π_t^*) to zero in this case, but does not deliver the efficient outcome. It is easy to see that CPI inflation is zero in both countries under the optimal policies. Substitute the target criteria, (79) and (80), under discretion, into the Phillips curves, (77) and (78), assuming mark-up shocks are always zero ($u_t^R = u_t^W = 0$.) The solutions to the expectational difference equations are $\pi_t^R = \pi_t^W = 0$, which imply $\pi_t = \pi_t^* = 0$. The same conclusion is reached applying the optimal policies under commitment, (93) and (94).

If CPI inflation rates are zero, it follows from the optimal rules that $\tilde{y}_t^W = 0$ and $\tilde{q}_t = 0$: world output is at efficient levels, as is the real exchange rate. But this still does not imply that allocations are efficient. Using (76) and (82), we conclude $\Delta_t = -(\nu - 1)\tilde{s}_t$ and $\tilde{y}_t^R = -\frac{\nu(2-\nu)}{2}\tilde{s}_t$. The evolution of $\tilde{s}_t$ is governed by (73) (recalling that when $\phi = 0$, the terms of trade evolve independently of policy.)

When there is pricing to market, the relative price of Foreign to Home goods in either market is not set efficiently. Even though policymakers can achieve an efficient level of world output when there are not mark-up shocks, they cannot ensure that the mix of Home and Foreign output is optimal. The price signals under LCP do not lead to efficient allocations.

12. Conclusions

Policymakers do not in general adhere to simple interest-rate reaction functions. Instead, as Svensson (1999, 2002) has argued, they set targets for key economic variables. It has generally been believed, especially in light of CGG, that the key tradeoffs in an open economy are the same as in a closed economy. That is, policymakers should target a linear combination of inflation and the output gap. This paper shows that in fact, when our model is rich enough to allow for currency misalignments, the tradeoffs should involve not only inflation and the output gap but also the exchange rate misalignment. However, the interest-rate reaction functions rule that supports this policy has the nominal interest rate reacting only to CPI inflation.

The paper derives the policymaker's loss function when there is home bias in consumption and deviations from the law of one price. The loss function does depend on the structure of the model, of course, but not on the specific nature of price setting. Currency misalignments may arise in some approaches for reasons other than local-currency-pricing. For example, there may be nominal wage stickiness but imperfect pass-through that arises from strategic behavior by firms as in the models of Atkeson and Burstein (2007, 2008) or Corsetti, Dedola, and Leduc (2007). Future work can still make use of the loss function derived here, or at least of the steps used in deriving the loss function.

The objective of this paper is to introduce local-currency pricing into a familiar and popular framework for monetary policy analysis. But the Clarida, Gali, and Gertler (2002) model does not produce empirical outcomes that are especially plausible, even with the addition of local-currency pricing. In the model, if output gaps are eliminated, a currency misalignment is inefficient because consumption goods are misallocated between Home and Foreign households. In a richer framework, even with local currency pricing as the source of currency misalignments, there are other potential misallocations that can occur when exchange rates are out of line. For example, an overvalued currency would lead to a movement in resources away from the traded sector to the nontraded sector. Or if countries import oil from the outside, a misaligned currency affects the real cost of oil in one importing country relative to the other.

Future work should also consider the difficult issue of policymaking in this environment when there is not cooperation. A separate but related issue is a closer examination of who bears the burden of currency misalignments. If a currency is overvalued, does it hurt consumers in one country more than another, both under optimal and sub-optimal policy?

While rich models that can be estimated and analyzed numerically offer valuable insights, they seem to provide inadequate guidance to policymakers for how to react to exchange rate movements. Perhaps more basic work on simple models such as the one presented here, in concert with quantitative exploration of more detailed models, can be productive.

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Appendix A

Log-linearized Model

In this section, we present log-linear approximations to the models presented above.

We will make use of the first-order approximation $s_t^* = -s_t$, as explained in the text. That relationship is obvious in the flexible price and PCP models, but will require some explanation in the LCP case. We will postpone that explanation until later.

In this section, we present all of the equations of the log-linearized model, but we separate out those that are used in the derivation of the loss function (which do not involve price setting or wage setting) and those that are not.

Equations used for derivation of loss functions

We define the log of the deviation from the law of one price as:

(A1) $\Delta_t \equiv e_t + p_{Ht}^* - p_{Ht}$.

In the flexible-price and PCP models, $\Delta_t = 0$. In the LCP model, because $s_t^* = -s_t$, we have that the law of one price deviation is the same for both goods: $\Delta_t \equiv e_t + p_{Ft}^* - p_{Ft}$

In all three models, to a first order, $\ln(V_{Ht}) = \ln(V_{Ht}^*) = \ln(V_{Ft}) = \ln(V_{Ft}^*) = 0$. That allows us to approximate equation (23) and its foreign counterpart as:

(A2) $n_t = y_t - a_t$, and

(A3) $n_t^* = y_t^* - a_t^*$.

The market-clearing conditions, (18) and (19) are approximated as:

(A4) $y_t = \frac{\nu(2-\nu)}{2} s_t + \frac{\nu}{2} c_t + \frac{2-\nu}{2} c_t^*$,

(A5) $y_t^* = \frac{-\nu(2-\nu)}{2} s_t + \frac{\nu}{2} c_t^* + \frac{2-\nu}{2} c_t$.

The condition arising from complete markets that equates the marginal utility of nominal wealth for Home and Foreign households, equation (22), is given by:

(A6) $\sigma c_t - \sigma c_t^* = \Delta_t + (\nu - 1) s_t$.

For use later, it is helpful to use equations (A4)-(A6) to express c_t, c_t^*, s_t, in terms of y_t and y_t^* and the exogenous disturbances, a_t, a_t^*, μ_t^W, and μ_t^{*W}:

(A7) $c_t = \frac{D+\nu-1}{2D} y_t + \frac{D-(\nu-1)}{2D} y_t^* + \frac{\nu(2-\nu)}{2D}\Delta_t$

(A8) $c_t^* = \frac{D+\nu-1}{2D} y_t^* + \frac{D-(\nu-1)}{2D} y_t - \frac{\nu(2-\nu)}{2D}\Delta_t$

(A9) $s_t = \frac{\sigma}{D}(y_t - y_t^*) - \frac{(\nu-1)}{D}\Delta_t$,

where $D \equiv \sigma\nu(2-\nu) + (\nu-1)^2$.

Under a globally efficient allocation, the marginal rate of substitution between leisure and aggregate consumption should equal the marginal product of labor times the price of output relative to consumption prices. To see the derivation more cleanly, we insert the shadow real wages in the efficient allocation, $\overline{w}_t - \overline{p}_{Ht}$ and $\overline{w}_t^* - \overline{p}_{Ft}^*$ into equations (A10) and (A11) below. So, the efficient allocation would be achieved in a model with flexible wages and optimal subsidies. These equations then can be understood intuitively by looking at the wage setting equations below ((A14)-(A15), and (A16)-(A17)) assuming the optimal subsidy is in place. But, to emphasize, they do not depend on a particular model of wage setting, and are just the standard efficiency condition equating the marginal rate of substitution between leisure and aggregate consumption to the marginal rate of transformation.

(A10) $$a_t = \bar{w}_t - \bar{p}_{Ht} = \left(\frac{\sigma(1+D)}{2D} + \phi\right)\bar{y}_t + \frac{\sigma(D-1)}{2D}\bar{y}_t^* - \phi a_t,$$

(A11) $$a_t^* = \bar{w}_t^* - \bar{p}_{Ft}^* = \left(\frac{\sigma(1+D)}{2D} + \phi\right)\bar{y}_t^* + \frac{\sigma(D-1)}{2D}\bar{y}_t - \phi a_t^*.$$

Equations of wage and price setting

The real Home and Foreign product wages, from equation (38), are given by:

(A12) $$w_t - p_{Ht} = \sigma c_t + \phi n_t + \frac{2-\nu}{2}s_t + \mu_t^W,$$

(A13) $$w_t^* - p_{Ft}^* = \sigma c_t^* + \phi n_t^* - \left(\frac{2-\nu}{2}\right)s_t + \mu_t^{*W}.$$

We can express $w_t - p_{Ht}$, and $w_t^* - p_{Ft}^*$ in terms of y_t and y_t^* and the exogenous disturbances, a_t, a_t^*, μ_t^W, and μ_t^{*W}:

(A14) $$w_t - p_{Ht} = \left(\frac{\sigma(1+D)}{2D} + \phi\right)y_t + \frac{\sigma(D-1)}{2D}y_t^* + \frac{D-(\nu-1)}{2D}\Delta_t - \phi a_t + \mu_t^W,$$

(A15) $$w_t^* - p_{Ft}^* = \left(\frac{\sigma(1+D)}{2D} + \phi\right)y_t^* + \frac{\sigma(D-1)}{2D}y_t + \frac{\nu-1-D}{2D}\Delta_t - \phi a_t^* + \mu_t^{*W},$$

Flexible Prices

We can solve for the values of all the real variables under flexible prices by using equations (A2), (A3), (A7), (A8), (A9), (A14) and (A15), as well as the price-setting conditions, from (43):

(A16) $$\dot{w}_t - \dot{p}_{Ht} = a_t,$$

(A17) $$\dot{w}_t^* - \dot{p}_{Ft}^* = a_t^*.$$

PCP

Log-linearization of equations (49) and (50) gives us the familiar New Keynesian Phillips curve for an open economy:

(A18) $$\pi_{Ht} = \delta(w_t - p_{Ht} - a_t) + \beta \mathrm{E}_t \pi_{Ht+1},$$

where $\delta = (1-\theta)(1-\beta\theta)/\theta$.

We can rewrite this equation as:

$$\pi_{Ht} = \delta(\tilde{w}_t - \tilde{p}_{Ht}) + \beta \mathrm{E}_t \pi_{Ht+1},$$

or, using (A14) and (A10):

(A19) $$\pi_{Ht} = \delta\left[\left(\frac{\sigma(1+D)}{2D} + \phi\right)\tilde{y}_t + \frac{\sigma(D-1)}{2D}\tilde{y}_t^*\right] + \beta \mathrm{E}_t \pi_{Ht+1} + u_t,$$

where $u_t = \delta\mu_t^W$.

Similarly for foreign producer-price inflation, we have:

(A20) $$\pi_{Ft}^* = \delta\left[\left(\frac{\sigma(1+D)}{2D} + \phi\right)\tilde{y}_t^* + \frac{\sigma(D-1)}{2D}\tilde{y}_t\right] + \beta \mathrm{E}_t \pi_{Ft+1}^* + u_t^*.$$

LCP

Equation (A18) holds in the LCP model as well. But in the LCP model, the law of one price deviation is not zero. We have:

(A21) $\pi_{Ht} = \delta\left[\left(\frac{\sigma(1+D)}{2D}+\phi\right)\tilde{y}_t + \frac{\sigma(D-1)}{2D}\tilde{y}_t^* + \frac{D-(\nu-1)}{2D}\Delta_t\right] + \beta \mathrm{E}_t \pi_{Ht+1} + u_t$,

(A22) $\pi_{Ft}^* = \delta\left[\left(\frac{\sigma(1+D)}{2D}+\phi\right)\tilde{y}_t^* + \frac{\sigma(D-1)}{2D}\tilde{y}_t + \frac{\nu-1-D}{2D}\Delta_t\right] + \beta \mathrm{E}_t \pi_{Ft+1}^* + u_t^*$.

In addition, from (18) and (51), we derive:

(A23) $\pi_{Ht}^* = \delta(w_t - p_{Ht}^* - e_t - a_t) + \beta \mathrm{E}_t \pi_{Ht+1}^* = \delta(w_t - p_{Ht} - \Delta_t - a_t) + \beta \mathrm{E}_t \pi_{Ht+1}^*$

We can rewrite this as

(A24) $\pi_{Ht}^* = \delta\left[\left(\frac{\sigma(1+D)}{2D}+\phi\right)\tilde{y}_t + \frac{\sigma(D-1)}{2D}\tilde{y}_t^* - \left(\frac{D+\nu-1}{2D}\right)\Delta_t\right] + \beta \mathrm{E}_t \pi_{Ht+1}^* + u_t$.

Similarly, we can derive:

(A25) $\pi_{Ft} = \delta\left[\left(\frac{\sigma(1+D)}{2D}+\phi\right)\tilde{y}_t^* + \frac{\sigma(D-1)}{2D}\tilde{y}_t + \frac{D+\nu-1}{2D}\Delta_t\right] + \beta \mathrm{E}_t \pi_{Ft+1} + u_t^*$.

From (A7)-(A8) and (A14)-(A15), we see $\pi_{Ft} - \pi_{Ht} = \pi_{Ft}^* - \pi_{Ht}^*$. Assuming a symmetric initial condition, we conclude $s_t^* = -s_t$ as we noted above. That is, the relative price of Foreign to Home goods is the same in both countries. We emphasize that this is true in general for a first-order approximation.

Appendix B

B.I Derivation of Welfare Function in Clarida-Gali-Gertler model with Home Bias in preferences

The object is to rewrite the welfare function, which is defined in terms of home and foreign consumption and labor effort into terms of the squared output gap and squared inflation. We derive the joint welfare function of home and foreign households, since we will be examining cooperative monetary policy.

Most of the derivation requires only 1st-order approximations of the equations of the model, but in a few places, 2nd-order approximations are needed. If the approximation is 1st-order, I'll use the notation "$+o\left(\left\|a^2\right\|\right)$" to indicate that there are 2nd-order and higher terms left out, and if the approximation is 2nd-order, I will use "$+o\left(\left\|a^3\right\|\right)$". (*a* is notation for the log of the productivity shock)

From equation (1) in the text, the period utility of the planner is given by:

(B1) $$\upsilon_t \equiv \frac{1}{1-\sigma}(C_t^{1-\sigma} + C_t^{*1-\sigma}) - \frac{1}{1+\phi}(N_t^{1+\phi} + N_t^{*1+\phi}).$$

Take a second-order log approximation around the non-stochastic steady state. We assume allocations are efficient in steady state, so we have $C^{1-\sigma} = C^{*1-\sigma} = N^{1+\phi} = N^{*1+\phi}$. The fact that $C^{1-\sigma} = N^{1+\phi}$ follows from the fact that in steady state, $C = N$ from market clearing and symmetry, and $C^{-\sigma} = N^{\phi}$ from the condition that the marginal rate of substitution between leisure and consumption equals one in an efficient non-stochastic steady state.

We get:

(B2) $$\begin{aligned}\upsilon_t &= 2\left(\frac{1}{1-\sigma} - \frac{1}{1+\phi}\right)C^{1-\sigma} + C^{1-\sigma}\left(c_t + c_t^*\right) + \frac{1-\sigma}{2}C^{1-\sigma}\left((c_t)^2 + (c_t^*)^2\right) \\ &\quad - C^{1-\sigma}\left(n_t + n_t^*\right) - \frac{1+\phi}{2}C^{1-\sigma}\left((n_t)^2 + (n_t^*)^2\right) + o\left(\left\|a^3\right\|\right)\end{aligned}.$$

Since we can equivalently maximize an affine transformation of (B2), it is convenient to simplify that equation to get:

(B3) $$\upsilon_t = c_t + c_t^* - n_t - n_t^* + \frac{1-\sigma}{2}\left(c_t^2 + c_t^{*2}\right) - \frac{1+\phi}{2}\left(n_t^2 + n_t^{*2}\right) + o\left(\left\|a^3\right\|\right).$$

Utility is maximized when consumption and employment take on their efficient values:

(B4) $$\upsilon_t^{\max} = \overline{c}_t + \overline{c}_t^* - \overline{n}_t - \overline{n}_t^* + \frac{1-\sigma}{2}\left(\overline{c}_t^2 + \overline{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\overline{n}_t^2 + \overline{n}_t^{*2}\right) + o\left(\left\|a^3\right\|\right).$$

In general, this maximum may not be attainable because of distortions. We can write $x_t = \overline{x}_t + \tilde{x}_t$, where $\tilde{x}_t \equiv x_t - \overline{x}_t$. So, we have:

(B5) $$\begin{aligned}\upsilon_t &= \left[\overline{c}_t + \overline{c}_t^* - \overline{n}_t - \overline{n}_t^* + \frac{1-\sigma}{2}\left(\overline{c}_t^2 + \overline{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\overline{n}_t^2 + \overline{n}_t^{*2}\right)\right] \\ &\quad + \tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^* + \frac{1-\sigma}{2}\left(\tilde{c}_t^2 + \tilde{c}_t^{*2} + 2\overline{c}_t\tilde{c}_t + 2\overline{c}_t^*\tilde{c}_t^*\right) - \frac{1+\phi}{2}\left(\tilde{n}_t^2 + \tilde{n}_t^{*2} + 2\overline{n}_t\tilde{n}_t + 2\overline{n}_t^*\tilde{n}_t^*\right) \\ &\quad + o\left(\left\|a^3\right\|\right)\end{aligned}$$

or,

(B6) $$\upsilon_t - \upsilon_t^{\max} = \tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^* + \frac{1-\sigma}{2}\left(\tilde{c}_t^2 + \tilde{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\tilde{n}_t^2 + \tilde{n}_t^{*2}\right) + (1-\sigma)\left(\bar{c}_t\tilde{c}_t + \bar{c}_t^*\tilde{c}_t^*\right) - (1+\phi)\left(\bar{n}_t\tilde{n}_t + \bar{n}_t^*\tilde{n}_t^*\right) + o\left(\left\|a^3\right\|\right).$$

The object is to write (B6) as a function of squared output gaps and squared inflation if possible. We need a second-order approximation of $\tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^*$. But for the rest of the terms, since they are squares and products, the 1st-order approximations that have already been derived will be sufficient.

Recalling that $\Delta_t = 0$ in the PCP model, we can write equations (A7)-(A8) as:

(B7) $c_t = c_y y_t + (1-c_y)y_t^* + o\left(\left\|a^2\right\|\right)$,

(B8) $c_t^* = (1-c_y)y_t + c_y y_t^* + o\left(\left\|a^2\right\|\right)$,

where $c_y \equiv \dfrac{D+\nu-1}{2D}$.

It follows from (B7) and (B8) that:

(B9) $\bar{c}_t = c_y\bar{y}_t + (1-c_y)\bar{y}_t^* + o\left(\left\|a^2\right\|\right)$,

(B10) $\bar{c}_t^* = (1-c_y)\bar{y}_t + c_y\bar{y}_t^* + o\left(\left\|a^2\right\|\right)$,

(B11) $\tilde{c}_t = c_y\tilde{y}_t + (1-c_y)\tilde{y}_t^* + o\left(\left\|a^2\right\|\right)$,

(B12) $\tilde{c}_t^* = (1-c_y)\tilde{y}_t + c_y\tilde{y}_t^* + o\left(\left\|a^2\right\|\right)$.

Next, we can easily derive:

(B13) $\tilde{n}_t = \tilde{y}_t + o\left(\left\|a^2\right\|\right)$, and

(B14) $\tilde{n}_t^* = \tilde{y}_t^* + o\left(\left\|a^2\right\|\right)$.

These follow as in (A2)-(A3) because $n_t = y_t - a_t + o\left(\left\|a^2\right\|\right)$ and $\bar{n}_t = \bar{y}_t - a_t$ (and similarly in the Foreign country.)

We need expressions for $\bar{n}_t$ and $\bar{n}_t^*$. We have from (A10)-(A11):

$$a_t = \left(\frac{\sigma(1+D)}{2D} + \phi\right)\bar{y}_t + \frac{\sigma(D-1)}{2D}\bar{y}_t^* - \phi a_t,$$

$$a_t^* = \left(\frac{\sigma(1+D)}{2D} + \phi\right)\bar{y}_t^* + \frac{\sigma(D-1)}{2D}\bar{y}_t - \phi a_t^*.$$

Using $a_t = \bar{y}_t - \bar{n}_t$ and $a_t^* = \bar{y}_t^* - \bar{n}_t^*$, we can write these as

(B15) $\bar{n}_t = \dfrac{1-\sigma}{1+\phi}\left[(\nu-1)c_y + \dfrac{2-\nu}{2}\right]\bar{y}_t + \dfrac{1-\sigma}{1+\phi}\left[(1-\nu)c_y + \dfrac{\nu}{2}\right]\bar{y}_t^* + o\left(\left\|a^2\right\|\right)$, and

(B16) $\bar{n}_t^* = \dfrac{1-\sigma}{1+\phi}\left[(\nu-1)c_y + \dfrac{2-\nu}{2}\right]\bar{y}_t^* + \dfrac{1-\sigma}{1+\phi}\left[(1-\nu)c_y + \dfrac{\nu}{2}\right]\bar{y}_t + o\left(\left\|a^2\right\|\right)$.

Turning attention back to the loss function in equation (B6), we focus first on the terms $\frac{1-\sigma}{2}\left(\tilde{c}_t^2 + \tilde{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\tilde{n}_t^2 + \tilde{n}_t^{*2}\right) + (1-\sigma)\left(\bar{c}_t\tilde{c}_t + \bar{c}_t^*\tilde{c}_t^*\right) - (1+\phi)\left(\bar{n}_t\tilde{n}_t + \bar{n}_t^*\tilde{n}_t^*\right)$. As noted above, these involve only squares and cross-products of $\tilde{c}_t$, $\tilde{c}_t^*$, $\bar{c}_t$, $\bar{c}_t^*$, $\tilde{n}_t$, $\tilde{n}_t^*$, $\bar{n}_t$, and $\bar{n}_t^*$. We can substitute from equations (B9)-(B16) into this expression. It is useful provide a few lines of algebra since it is a bit messy:

$$
\begin{aligned}
&\frac{1-\sigma}{2}\left(\tilde{c}_t^2+\tilde{c}_t^{*2}\right)-\frac{1+\phi}{2}\left(\tilde{n}_t^2+\tilde{n}_t^{*2}\right)+(1-\sigma)\left(\bar{c}_t\tilde{c}_t+\bar{c}_t^*\tilde{c}_t^*\right)-(1+\phi)\left(\bar{n}_t\tilde{n}_t+\bar{n}_t^*\tilde{n}_t^*\right)\\
&=\left(\frac{1-\sigma}{2}\right)\left(2c_y^2-2c_y+1\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)-(1-\sigma)\left(2c_y^2-2c_y\right)\tilde{y}_t\tilde{y}_t^*-\left(\frac{1+\phi}{2}\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)\\
&+(1-\sigma)\left(2c_y^2-2c_y\right)\left(\bar{y}_t-\bar{y}_t^*\right)\left(\tilde{y}_t-\tilde{y}_t^*\right)+(1-\sigma)\left(\bar{y}_t\tilde{y}_t+\bar{y}_t^*\tilde{y}_t^*\right)\\
&-(1-\sigma)\left(\frac{\nu}{2}\bar{c}_t+\left(\frac{2-\nu}{2}\right)\bar{c}_t^*\right)\tilde{y}_t-(1-\sigma)\left(\frac{\nu}{2}\bar{c}_t^*+\left(\frac{2-\nu}{2}\right)\bar{c}_t\right)\tilde{y}_t^*\\
&=\left(\frac{1-\sigma}{2}\right)\left(2c_y^2-2c_y+1\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)-(1-\sigma)\left(2c_y^2-2c_y\right)\tilde{y}_t\tilde{y}_t^*-\left(\frac{1+\phi}{2}\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)\\
&+2(1-\sigma)\left(c_y^2-c_y\right)\left(\bar{y}_t-\bar{y}_t^*\right)\left(\tilde{y}_t-\tilde{y}_t^*\right)+(1-\sigma)\left(\bar{y}_t\tilde{y}_t+\bar{y}_t^*\tilde{y}_t^*\right)\\
&-(1-\sigma)\left[\left((\nu-1)c_y-\frac{\nu}{2}\right)\left(\bar{y}_t-\bar{y}_t^*\right)\left(\tilde{y}_t-\tilde{y}_t^*\right)+\bar{y}_t\tilde{y}_t+\bar{y}_t^*\tilde{y}_t^*\right]\\
&=\left(\frac{1-\sigma}{2}\right)\left(2c_y^2-2c_y+1\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)-(1-\sigma)\left(2c_y^2-2c_y\right)\tilde{y}_t\tilde{y}_t^*-\left(\frac{1+\phi}{2}\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)\\
&+(1-\sigma)\left(2(c_y^2-c_y)-(\nu-1)c_y+\frac{\nu}{2}\right)\left(\bar{y}_t-\bar{y}_t^*\right)\left(\tilde{y}_t-\tilde{y}_t^*\right)+o\left(\left\|a^3\right\|\right)
\end{aligned}
\tag{B17}
$$

Now return to the $\tilde{c}_t+\tilde{c}_t^*-\tilde{n}_t-\tilde{n}_t^*$ term in equation (B6) and do a 2nd-order approximation. Start with equation (18), dropping the k^{-1} term because it will not affect the approximation, and noting that in the PCP model, $S_t^*=S_t^{-1}$:

(B18) $$Y_t=\frac{\nu}{2}S_t^{(2-\nu)/2}C_t+\left(\frac{2-\nu}{2}\right)S_t^{\nu/2}C_t^*$$

Then use equation (22), but using the fact that $S_t^*=S_t^{-1}$ and there are no deviations from the law of one price:

(B19) $$C_t^*=C_tS_t^{\frac{1-\nu}{\sigma}}.$$

Substitute in to get:

(B20) $$Y_t=\frac{\nu}{2}S_t^{\frac{2-\nu}{2}}C_t+\left(\frac{2-\nu}{2}\right)S_t^{\frac{\nu}{2}+\frac{1-\nu}{\sigma}}C_t.$$

Solve for C_t:

(B21) $$C_t=Y_t\left(\frac{\nu}{2}S_t^{\frac{2-\nu}{2}}+\left(\frac{2-\nu}{2}\right)S_t^{\frac{\nu}{2}+\frac{1-\nu}{\sigma}}\right)^{-1}$$, or,

(B22) $$c_t=y_t-\ln\left(\frac{\nu}{2}e^{\left(\frac{2-\nu}{2}\right)s_t}+\left(\frac{2-\nu}{2}\right)e^{\left(\frac{\nu}{2}+\frac{1-\nu}{\sigma}\right)s_t}\right).$$

Take first and second derivatives, evaluated at the non-stochastic steady state:

(B23) $$\frac{\partial c_t}{\partial s_t}\Big|_{s=0}=\left(\frac{\nu-2}{2}\right)\left(\nu+\frac{1-\nu}{\sigma}\right)$$

(B24) $$\frac{\partial^2 c_t}{\partial s_t^2}\Big|_{s=0}=\left(\frac{\nu-2}{2}\right)\frac{\nu}{2}(\nu-1)^2\left(\frac{\sigma-1}{\sigma}\right)^2.$$

Then we get this 2nd-order approximation:

(B25) $$c_t = y_t + \left(\frac{\nu-2}{2}\right)\left(\nu + \frac{1-\nu}{\sigma}\right)s_t + \frac{1}{2}\left(\frac{\nu-2}{2}\right)\frac{\nu}{2}(\nu-1)^2\left(\frac{\sigma-1}{\sigma}\right)^2 s_t^2 + o\left(\left\|a^3\right\|\right).$$

Symmetrically,

(B26) $$c_t^* = y_t^* - \left(\frac{\nu-2}{2}\right)\left(\nu + \frac{1-\nu}{\sigma}\right)s_t + \frac{1}{2}\left(\frac{\nu-2}{2}\right)\frac{\nu}{2}(\nu-1)^2\left(\frac{\sigma-1}{\sigma}\right)^2 s_t^2 + o\left(\left\|a^3\right\|\right).$$

Since we are only interested in $\tilde{c}_t + \tilde{c}_t^*$, we can add these together to get:

(B27) $$c_t + c_t^* = y_t + y_t^* + \left(\frac{\nu-2}{2}\right)\frac{\nu}{2}(\nu-1)^2\left(\frac{\sigma-1}{\sigma}\right)^2 s_t^2 + o\left(\left\|a^3\right\|\right).$$

Now we can take a 1st-order approximation for s_t to substitute out for s_t^2. From equation (A9), we have:

(B28) $$s_t^2 = \frac{\sigma^2}{D^2}(y_t^* - y_t)^2 + o\left(\left\|a^3\right\|\right).$$

Substituting into equation (B28), we can write:

(B29) $$c_t + c_t^* = y_t + y_t^* + \Omega(y_t^* - y_t)^2 + o\left(\left\|a^3\right\|\right),$$

where $\Omega = \frac{\nu(\nu-2)}{4}\left(\frac{(\nu-1)(\sigma-1)}{D}\right)^2$.

Evaluating (B29) at flexible prices, we have:

(B30) $$\overline{c}_t + \overline{c}_t^* = \overline{y}_t + \overline{y}_t^* + \Omega(\overline{y}_t^* - \overline{y}_t)^2 + o\left(\left\|a^3\right\|\right).$$

It follows from the fact that $\tilde{c}_t + \tilde{c}_t^* = c_t + c_t^* - (\overline{c}_t + \overline{c}_t^*)$ that

(B31) $$\tilde{c}_t + \tilde{c}_t^* = \tilde{y}_t + \tilde{y}_t^* + \Omega(\tilde{y}_t^2 + 2\overline{y}_t\tilde{y}_t + \tilde{y}_t^{*2} + 2\overline{y}_t^*\tilde{y}_t^* - 2\overline{y}_t\tilde{y}_t^* - 2\overline{y}_t^*\tilde{y}_t - 2\tilde{y}_t\tilde{y}_t^*) + o\left(\left\|a^3\right\|\right).$$

See section B.3 below for the second-order approximations for $\tilde{n}_t$ and $\tilde{n}_t^*$:

(B32) $$\tilde{n}_t = \tilde{y}_t + \frac{\xi}{2}\sigma_{p_H t}^2 + o\left(\left\|a^3\right\|\right)$$

(B33) $$\tilde{n}_t^* = \tilde{y}_t^* + \frac{\xi}{2}\sigma_{p_F * t}^2 + o\left(\left\|a^3\right\|\right).$$

Substitute expressions (B31)-(B33) along with (B17) into the loss function (B6):

(B34) $$\begin{aligned} \upsilon_t - \upsilon_t^{\max} = & \left[\Omega + (1-\sigma)(c_y^2 - c_y)\right](\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\sigma+\phi}{2}\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) \\ & -\frac{\xi}{2}\left(\sigma_{pt}^2 + \sigma_{p*t}^2\right) + 2\Omega\left(\overline{y}_t - \overline{y}_t^*\right)\left(\tilde{y}_t - \tilde{y}_t^*\right) \\ & +(1-\sigma)\left(2(c_y^2 - c_y) - (\nu-1)c_y + \frac{\nu}{2}\right)\left(\overline{y}_t - \overline{y}_t^*\right)\left(\tilde{y}_t - \tilde{y}_t^*\right) + o\left(\left\|a^3\right\|\right) \end{aligned}$$

Some tedious algebra demonstrates that

(B35) $$2\Omega + (1-\sigma)\left(2(c_y^2 - c_y) - (\nu-1)c_y + \frac{\nu}{2}\right) = 0.$$

So, finally we can write:

(B36)
$$\begin{aligned}\upsilon_t - \upsilon_t^{\max} &= \left[\Omega + (1-\sigma)(c_y^2 - c_y)\right](\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\sigma+\phi}{2}\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) - \frac{\xi}{2}\left(\sigma_{pt}^2 + \sigma_{p^*t}^2\right) \\ &= \left(\frac{\nu(\nu-2)\sigma(1-\sigma)}{4D}\right)(\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\sigma+\phi}{2}\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) - \frac{\xi}{2}\left(\sigma_{p_H t}^2 + \sigma_{p_F^* t}^2\right) + o\left(\|a^3\|\right)\end{aligned}.$$

This expression reduces to CGG's when there is no home bias ($\gamma = 1$). To see this from their expression at the top of p. 903, multiply their utility by 2 (since they take average utility), and set their γ equal to ½ (so their country sizes are equal.

B.2 Derivation of Welfare Function under LCP with Home Bias in Preferences

The second-order approximation to welfare in terms of logs of consumption and employment of course does not change, so equation (B6) still holds. As before, we break down the derivation into two parts. We use first-order approximations to structural equations to derive an approximation to the quadratic term

$$\frac{1-\sigma}{2}\left(\tilde{c}_t^2 + \tilde{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\tilde{n}_t^2 + \tilde{n}_t^{*2}\right) + (1-\sigma)\left(\bar{c}_t\tilde{c}_t + \bar{c}_t^*\tilde{c}_t^*\right) - (1+\phi)\left(\bar{n}_t\tilde{n}_t + \bar{n}_t^*\tilde{n}_t^*\right).$$

Then we use second order approximations to the structural equations to derive an expression for $\tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^*$.

The quadratic term involves squares and cross-products of $\tilde{c}_t$, $\tilde{c}_t^*$, $\bar{c}_t$, $\bar{c}_t^*$, $\tilde{n}_t$, $\tilde{n}_t^*$, $\bar{n}_t$, and $\bar{n}_t^*$. Expressions (B9)-(B10) still gives us first-order approximations for $\bar{c}_t$ and $\bar{c}_t^*$; equations (B13)-(B14) are first-order approximations for $\tilde{n}_t$ and $\tilde{n}_t^*$; and, (B15)-(B16) are first-order approximations for $\bar{n}_t$ and $\bar{n}_t^*$. But we need to use equations (A7)-(A8) and (B11)-(B12) to derive:

(B37) $$\tilde{c}_t = c_y\tilde{y}_t + (1-c_y)\tilde{y}_t^* + \frac{\nu(2-\nu)}{2D}\Delta_t + o\left(\|a^2\|\right),$$

(B38) $$\tilde{c}_t^* = (1-c_y)\tilde{y}_t + c_y\tilde{y}_t^* - \frac{\nu(2-\nu)}{2D}\Delta_t + o\left(\|a^2\|\right).$$

With these equations, we can follow the derivation as in equation (B17). After tedious algebra, we arrive at the same result, with the addition of the terms $\frac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2}\Delta_t^2$ and $\frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{2D^2}\Delta_t(y_t - y_t^*)$. Note that the last term involves output levels, not output gaps. That is, we have:

(B39)
$$\begin{aligned}&\frac{1-\sigma}{2}\left(\tilde{c}_t^2 + \tilde{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\tilde{n}_t^2 + \tilde{n}_t^{*2}\right) + (1-\sigma)\left(\bar{c}_t\tilde{c}_t + \bar{c}_t^*\tilde{c}_t^*\right) - (1+\phi)\left(\bar{n}_t\tilde{n}_t + \bar{n}_t^*\tilde{n}_t^*\right) \\ &= \left(\frac{1-\sigma}{2}\right)\left(2c_y^2 - 2c_y + 1\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) - (1-\sigma)\left(2c_y^2 - 2c_y\right)\tilde{y}_t\tilde{y}_t^* - \left(\frac{1+\phi}{2}\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) \\ &+ (1-\sigma)\left(2(c_y^2 - c_y) - (\nu-1)c_y + \frac{\nu}{2}\right)\left(\bar{y}_t - \bar{y}_t^*\right)\left(\tilde{y}_t - \tilde{y}_t^*\right) + \frac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2}\Delta_t^2 \\ &+ \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{2D^2}\Delta_t(y_t - y_t^*) + o\left(\|a^2\|\right)\end{aligned}$$

The derivation of $\tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^*$ is similar to the PCP model. However, one tedious aspect of the derivation is that we cannot make use of the equality $S_t^* = S_t^{-1}$ that holds under PCP and flexible prices. We write out the equilibrium conditions for home output, and its foreign equivalent, from equations (18) and (19):

(B40) $$Y_t = \frac{\nu}{2} S_t^{1-(\nu/2)} C_t + \left(1 - \frac{\nu}{2}\right)(S_t^*)^{-\nu/2} C_t^*,$$

(B41) $$Y_t^* = \frac{\nu}{2} (S_t^*)^{1-(\nu/2)} C_t^* + \left(1 - \frac{\nu}{2}\right) S_t^{-\nu/2} C_t.$$

We directly take second-order approximations of these equations around the efficient non-stochastic steady state:

(B42) $$\begin{aligned} y_t + \frac{1}{2} y_t^2 = {} & \frac{\nu}{2} c_t + \left(\frac{2-\nu}{2}\right) c_t^* + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t^* \\ & + \frac{1}{2}\left\{ \frac{\nu}{2} c_t^2 + \left(\frac{2-\nu}{2}\right) c_t^{*2} + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)^2 s_t^2 + \left(\frac{\nu}{2}\right)^2 \left(\frac{2-\nu}{2}\right) s_t^{*2} \right\} \\ & + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t c_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t^* c_t^* + o\left(\|a^3\|\right) \end{aligned}$$

(B43) $$\begin{aligned} y_t^* + \frac{1}{2} y_t^{*2} = {} & \frac{\nu}{2} c_t^* + \left(\frac{2-\nu}{2}\right) c_t + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t^* - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t \\ & + \frac{1}{2}\left\{ \frac{\nu}{2} c_t^{*2} + \left(\frac{2-\nu}{2}\right) c_t^2 + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)^2 s_t^{*2} + \left(\frac{\nu}{2}\right)^2 \left(\frac{2-\nu}{2}\right) s_t^2 \right\}. \\ & + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t^* c_t^* - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t c_t + o\left(\|a^3\|\right) \end{aligned}$$

Note that in a second-order approximation, we cannot impose $s_t = -s_t^*$. However, we can impose $s_t^2 = s_t^{*2}$. Then adding (B42) and (B43) together, we find:

(B44) $$y_t + y_t^* + \frac{1}{2}(y_t^2 + y_t^{*2}) = c_t + c_t^* + \frac{1}{2}\left\{ c_t^2 + c_t^{*2} + \nu\left(\frac{2-\nu}{2}\right) s_t^2 \right\} + o\left(\|a^3\|\right).$$

Next, we can use equations (A7), (A8), and (A9) to get approximations for c_t^2, c_t^{*2}, and s_t^2. These equations are linear approximations for c_t, c_t^*, and s_t, but since we are looking to approximate the squares of these variables, that is sufficient. With some algebra, we find:

(B45) $$\begin{aligned} c_t^2 + c_t^{*2} + \nu\left(\frac{2-\nu}{2}\right) s_t^2 = {} & y_t^2 + y_t^{*2} + \nu\left(\frac{2-\nu}{2}\right)\left(\frac{(1-\nu)(1-\sigma)}{D}\right)^2 (y_t - y_t^*)^2 \\ & + \nu\left(\frac{2-\nu}{2}\right)\frac{1}{D^2}\Delta_t^2 + \nu\left(\frac{2-\nu}{2}\right)\left(\frac{2(1-\nu)(1-\sigma)}{D^2}\right)\Delta_t (y_t - y_t^*) + o\left(\|a^3\|\right) \end{aligned}.$$

Then, substituting (B45) into (B44) and rearranging, we find:

(B46) $$\begin{aligned} c_t + c_t^* = {} & y_t + y_t^* - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\left(\frac{(1-\nu)(1-\sigma)}{D}\right)^2 (y_t - y_t^*)^2 \\ & - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\frac{1}{D^2}\Delta_t^2 - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\left(\frac{2(1-\nu)(1-\sigma)}{D^2}\right)\Delta_t (y_t - y_t^*) + o\left(\|a^3\|\right) \end{aligned}.$$

Note that if set $\Delta_t = 0$ in (B46), we would arrive at the second-order approximation for $c_t + c_t^*$ from the PCP model.

Then following the derivations as in the PCP model derivation of (B31), we can write:

(B47) $$\tilde{c}_t + \tilde{c}_t^* = \tilde{y}_t + \tilde{y}_t^* + \Omega(\tilde{y}_t^2 + 2\overline{y}_t\tilde{y}_t + \tilde{y}_t^{*2} + 2\overline{y}_t^*\tilde{y}_t^* - 2\overline{y}_t\tilde{y}_t^* - 2\overline{y}_t^*\tilde{y}_t - 2\tilde{y}_t\tilde{y}_t^*) - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\frac{1}{D^2}\Delta_t^2 - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\left(\frac{2(1-\nu)(1-\sigma)}{D^2}\right)\Delta_t(y_t - y_t^*) + o\left(\left\|a^3\right\|\right)$$

As shown in section B.3, we can make the following second-order approximation:

(B48) $$\tilde{n}_t + \tilde{n}_t^* = \tilde{y}_t + \tilde{y}_t^* + \frac{\xi}{2}\left[\frac{\nu}{2}\sigma_{p_Ht}^2 + \frac{2-\nu}{2}\sigma_{p_H*t}^2 + \frac{\nu}{2}\sigma_{p_F*t}^2 + \frac{2-\nu}{2}\sigma_{p_Ft}^2\right] + o\left(\left\|a^3\right\|\right).$$

We then can substitute (B47), and (B48), along with (B39) into the loss function (B6). Notice the cancellations that occur. The cross product terms on $\Delta_t(y_t - y_t^*)$ in equations (B39) and (B47) cancel. The other cross product terms involving output gaps and efficient levels of output also cancel, just as in the PCP model, when we used equation (B35). Hence, we can write:

(B49) $$\upsilon_t - \upsilon_t^{\max} = \left(\frac{\nu(\nu-2)\sigma(1-\sigma)}{4D}\right)(\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\sigma+\phi}{2}\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) - \left(\frac{\nu(2-\nu)}{4D}\right)\Delta_t^2 - \frac{\xi}{2}\left[\frac{\nu}{2}\sigma_{p_Ht}^2 + \frac{2-\nu}{2}\sigma_{p_H*t}^2 + \frac{\nu}{2}\sigma_{p_F*t}^2 + \frac{2-\nu}{2}\sigma_{p_Ft}^2\right] + o\left(\left\|a^3\right\|\right).$$

B.3 Derivations of Price Dispersion Terms in Loss Functions

In the PCP case, we can write

(B50) $$A_tN_t = A_t\int_0^1 N_t(f)df = Y_t\int_0^1\left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} df = Y_tV_t,$$

where $V_t \equiv \int_0^1\left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} df$. Taking logs, we can write:

(B51) $a_t + n_t = y_t + v_t$.

We have $v_t \equiv \ln\left(\int_0^1 e^{-\xi\hat{p}_{Ht}(f)}df\right)$,

where we define

(B52) $\hat{p}_{Ht}(f) \equiv p_{Ht}(f) - p_{Ht}$.

Following Gali (2008), we note

(B53) $$e^{(1-\xi)\hat{p}_{Ht}(f)} = 1 + (1-\xi)\hat{p}_{Ht}(f) + \frac{(1-\xi)^2}{2}\hat{p}_{Ht}(f)^2 + o\left(\left\|a^3\right\|\right).$$

By the definition of the price index P_{Ht}, we have $\int_0^1 e^{(1-\xi)\hat{p}_{Ht}(f)}df = 1$. Hence, from (B53),

(B54) $$\int_0^1 \hat{p}_{Ht}(f)df = \frac{\xi-1}{2}\int_0^1 \hat{p}_{Ht}(f)^2 df + o\left(\left\|a^3\right\|\right).$$

We also have

(B55) $$e^{-\xi\hat{p}_{Ht}(f)} = 1 - \xi\hat{p}_{Ht}(f) + \frac{\xi^2}{2}\hat{p}_{Ht}(f)^2 + o\left(\left\|a^3\right\|\right)$$

It follows, using (B54):

(B56) $$\int_0^1 e^{-\xi\hat{p}_{Ht}(f)}df = 1 - \xi\int_0^1 \hat{p}_{Ht}(f)df + \frac{\xi^2}{2}\int_0^1 \hat{p}_{Ht}(f)^2 df + o\left(\left\|a^3\right\|\right) = 1 + \frac{\xi}{2}\int_0^1 \hat{p}_{Ht}(f)^2 df + o\left(\left\|a^3\right\|\right)$$

Note the following relationship:

(B57) $$\int_0^1 \hat{p}_{Ht}(f)^2 df = \int_0^1 (p_{Ht}(f) - \mathrm{E}_f(p_{Ht}(f))^2 df + o\left(\left\|a^3\right\|\right) = \mathrm{var}(p_{Ht}) + o\left(\left\|a^3\right\|\right).$$

Using our notation for variances, $\sigma_{p_Ht}^2 \equiv \mathrm{var}(p_{Ht})$, and taking the log of (B56) we arrive at

(B58) $v_t = \frac{\xi}{2}\sigma^2_{p_H t} + o\left(\|a^3\|\right)$.

Substituting this into equation (B51), and recalling that $\bar{y}_t = \bar{n}_t + a_t$, we arrive at equation (B32). The derivation of (B33) for the Foreign country proceeds identically.

For the LCP model, we will make use of the following second-order approximation to the equation $Y_t = C_{Ht} + C^*_{Ht}$:

(B59) $y_t = \frac{\nu}{2}c_{Ht} + \left(\frac{2-\nu}{2}\right)c^*_{Ht} + \frac{1}{2}\left(\frac{\nu}{2}\right)\left(\frac{2-\nu}{2}\right)(c^2_{Ht} + 2c_{Ht}c^*_{Ht} + c^{*2}_{Ht}) + o\left(\|a^3\|\right)$.

In the LCP model, we can write:

(B60) $A_t N_t = A_t\int_0^1 N_t(f)df = C_{Ht}\int_0^1\left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi}df + C^*_{Ht}\int_0^1\left(\frac{P^*_{Ht}(f)}{P^*_{Ht}}\right)^{-\xi}df = C_{Ht}V_{Ht} + C^*_{Ht}V^*_{Ht}$,

where the definitions of V_{Ht} and V^*_{Ht} are analogous to that of V_t in the PCP model. Taking a second-order log approximation to (B60), we have:

(B61)
$$a_t + n_t = \frac{\nu}{2}(c_{Ht} + v_{Ht}) + \left(\frac{2-\nu}{2}\right)(c^*_{Ht} + v^*_{Ht})$$
$$+\frac{1}{2}\left(\frac{\nu}{2}\right)\left(\frac{2-\nu}{2}\right)\left((c_{Ht} + v_{Ht})^2 + 2(c_{Ht} + v_{Ht})(c^*_{Ht} + v^*_{Ht}) + (c^*_{Ht} + v^*_{Ht})^2\right) + o\left(\|a^3\|\right)$$

We can follow the same steps as in the PCP model to conclude:

(B62) $v_{Ht} = \frac{\xi}{2}\sigma^2_{p_H t} + o\left(\|a^3\|\right)$

(B63) $v^*_{Ht} = \frac{\xi}{2}\sigma^2_{p_H * t} + o\left(\|a^3\|\right)$

Substituting these expressions into (B61) and cancelling higher order terms, we find:

(B64)
$$a_t + n_t = \frac{\nu}{2}(c_{Ht} + \frac{\xi}{2}\sigma^2_{p_H t}) + \left(\frac{2-\nu}{2}\right)(c^*_{Ht} + \frac{\xi}{2}\sigma^2_{p_H * t}) + \frac{1}{2}\left(\frac{\nu}{2}\right)\left(\frac{2-\nu}{2}\right)(c^2_{Ht} + 2c_{Ht}c^*_{Ht} + c^{*2}_{Ht}) + o\left(\|a^3\|\right)$$

Then using equation (B59), we can write:

(B65) $a_t + n_t = y_t + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma^2_{p_H t} + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma^2_{p_H * t}\right) + o\left(\|a^3\|\right)$.

Keeping in mind that $\bar{y}_t = \bar{n}_t + a_t$, we can write:

(B66) $\tilde{n}_t = \tilde{y}_t + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma^2_{p_H t} + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma^2_{p_H * t}\right) + o\left(\|a^3\|\right)$.

Following analogous steps for the Foreign country,

(B67) $\tilde{n}^*_t = \tilde{y}^*_t + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma^2_{p_F * t} + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma^2_{p_F t}\right) + o\left(\|a^3\|\right)$.

Adding (B66) and (B67) gives us equation (B48).

Finally, to derive the loss functions for policymakers (equation (65) for the PCP model and (69) for the LCP model), we note that the loss function is the present expected discounted value of the period loss functions derived here (equation (B36) for the PCP model and (B49) for the LCP model.) That is, the policymaker seeks to minimize $-\mathrm{E}_t\sum_{j=0}^{\infty}\beta^j(u_{t+j} - u^{\max}_{t+j})$.

Following Woodford (2003, chapter 6), we can see that, in the PCP model, if prices are adjusted according to the Calvo price mechanism given by equation (50) for P_{Ht} that

(B68) $$\sum_{j=0}^{\infty}\beta^j\sigma^2_{p_H t+j} = \frac{\theta}{(1-\beta\theta)(1-\theta)}\sum_{j=0}^{\infty}\beta^j\pi^2_{Ht+j}.$$

Analogous relationships hold for P^*_{Ft} in the PCP model, and for P_{Ht}, P^*_{Ft}, P_{Ft}, and P^*_{Ht} in the LCP model. We can then substitute this relationship into the present value loss function, $-\mathrm{E}_t\sum_{j=0}^{\infty}\beta^j(u_{t+j}-u^{\max}_{t+j})$, to derive the loss functions of the two models presented in the text.

B.4 Solutions for Endogenous Variables under Optimal Policy Rules

We assume shocks follow the processes (we have dropped the W superscript on the wage mark-up shocks):

$$a_t = \rho a_{t-1} + \varepsilon_{at}$$
$$a^*_t = \rho a^*_{t-1} + \varepsilon_{a^*t}$$
$$\mu_t = \rho\mu_{t-1} + \varepsilon_{\mu t}$$
$$\mu^*_t = \rho\mu^*_{t-1} + \varepsilon_{\mu^* t}$$

s_t is determined by :

$$s_t = \theta s_{t-1} + \frac{\theta\delta}{1-\beta\theta\rho}\left[(a_t - a^*_t) + (\mu_t - \mu^*_t)\right].$$

The solutions for the variables that appear in the loss function under optimal policy under discretion in the LCP model are:

$$\pi^R_t = \frac{\delta(\nu-1)}{1+\sigma\xi\delta-\beta\rho}(\mu_t - \mu^*_t)$$
$$\pi^W_t = \frac{\delta}{1+\sigma\xi\delta-\beta\rho}(\mu_t + \mu^*_t)$$
$$\tilde{y}^W_t = -\xi\pi^W_t$$
$$\tilde{y}^R_t = \nu(2-\nu)(s_t - (a_t - a^*_t)) - (\nu-1)\pi^R_t$$
$$\tilde{y}_t = \frac{1}{2}(\tilde{y}^W_t + \tilde{y}^R_t)$$
$$\tilde{y}^*_t = \frac{1}{2}(\tilde{y}^W_t - \tilde{y}^R_t)$$
$$\Delta_t = -\xi\sigma\pi^R_t - (\nu-1)(s_t - (a_t - a^*_t))$$
$$\pi_{Ht} = \frac{1}{2}(\pi^W_t + \pi^R_t) - \left(\frac{2-\nu}{2}\right)(s_t - s_{t-1})$$
$$\pi_{Ft} = \frac{1}{2}(\pi^W_t + \pi^R_t) + \frac{\nu}{2}(s_t - s_{t-1})$$
$$\pi^*_{Ft} = \frac{1}{2}(\pi^W_t - \pi^R_t) + \left(\frac{2-\nu}{2}\right)(s_t - s_{t-1})$$
$$\pi^*_{Ht} = \frac{1}{2}(\pi^W_t - \pi^R_t) - \frac{\nu}{2}(s_t - s_{t-1})$$