Self-Fulfilling Credit Market Freezes

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ABSTRACT

This paper develops a model of a self-fulfilling credit market freeze and uses it to study alternative governmental responses to such a crisis. We study an economy in which operating firms are interdependent, with their success depending on the ability of other operating firms to obtain financing. In such an economy, inefficient credit market freeze may arise in which banks abstain from lending to operating firms with good projects because of their self-fulfilling expectations that other banks will not be lending. Our model enables us to study the effectiveness of alternative measures for getting an economy out of an inefficient credit market freeze. In particular, we study the effectiveness of interest rate cuts, infusion of capital into financial institutions, direct lending to operating firms by the government, and infusion of capital into financial firms under lending commitment.

Key words: Credit freeze, credit crunch, credit thaw, self-fulfilling crisis, run on the economy, global game, coordination failure, bank capital, lending, strategic complementarities.
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1. Introduction

An important aspect of the economic crisis of 2008-2009 has been the "freezing" of credit to nonfinancial firms.[1] Despite government efforts to provide substantial liquidity and additional capital to the financial sector, financial firms have displayed considerable reluctance to extend loans to nonfinancial firms (as well as households). Given the central role of the financial sector in enabling real activity, financial firms' reluctance to lend to nonfinancial firms, and their election to hoard their capital, might have severe consequences for the economy. Some observers have attributed the reluctance of financial firms to lend to irrational fear, while others have attributed it to a rational assessment of the fundamentals of the economy which can be expected to make it difficult for operating firms to repay extended loans.

We show in this paper how coordination failure among financial institutions can lead to inefficient "credit markets freeze" equilibria. In such equilibria, financial institutions rationally avoid lending to nonfinancial firms (operating firms) that have projects that would be worthy if banks did not withdraw from the lending market en masse. They do so out of self-fulfilling fear, validated in equilibrium, that other financial institutions would withhold loans and that operating companies would not be able to succeed in an environment in which other operating firms fail to obtain financing.

The contribution of the paper is in analyzing the effectiveness of various government policies in getting the economy out of such self-fulfilling credit freeze equilibrium. The analysis identifies the role and potential limitations of interest rate cuts and infusion of capital into the financial sector. It analyzes less traditional forms of intervention – involving government direct intervention in lending to nonfinancial companies or provision of incentives to financial firms to lend to such companies – and discusses when they may be preferable.

Our analysis is based on the premise that operating firms, or at least a significant fraction of such firms, benefit from the success of other operating firms in the economy. Hence, the returns they will make on capital they borrow increase if other operating firms are able to obtain financing. This interdependence can be generated by multiple channels. A firm's success depends on the success of firms who use its products, of those

[1] For a description of the crisis and the events leading to it, see Brunnermeier (2009).

who supply its inputs, and of those whose employees buy its products. As a result of the interdependence, the decision of any given financial institution whether to lend to a given operating firm depends not only on the financial institution's assessment of the firm's project but also on its expectations as to whether other financial institutions will lend money to other operating firms. (Below we refer to financial institutions as banks for simplicity.)

The positive spillovers among firms can give rise to multiple equilibria. In an efficient lending equilibrium, banks expect other banks to lend to operating firms with worthy projects, and these expectations are self-fulfilling. In an inefficient credit freeze equilibrium, banks have self-fulfilling expectations that other banks will withdraw from the lending market, and they rationally avoid lending to operating firms. We use the global-games methodology, where banks observe noisy signals about the macroeconomic fundamentals – which affect the profitability of real projects – to identify when each equilibrium will occur. The result is that there is an intermediate range of fundamentals under which an inefficient credit freeze arises in equilibrium. We analyze the effect of various government policies on the probability of inefficient freeze and on the overall wealth in the economy.

A simple policy measure is interest rate reduction. During the economic crisis of 2008, the Fed and other central banks around the world slashed interest rates. In our model, interest rate cuts by the central bank make a credit market freeze less likely by reducing the payoff to banks that avoid lending and invest in government bonds. Such cuts, however, still leave a range of fundamentals where the economy ends up in an inefficient credit freeze equilibrium.

Another prominent channel of government policy works via capital infusion. Our analysis indicates that a shock to the banking system that depletes the amount of capital banks have makes an inefficient credit market freeze equilibrium more likely. This is because banks are more concerned that operating firms will not have enough capital to succeed and thus they are reluctant to lend them even the capital that they have. As a result, intervention through the infusion of capital into banks, which governments in the US, UK, and other countries did throughout the financial crisis, can be beneficial. Indeed, this measure reduces the probability of a freeze in our model, but again has limited effectiveness. The reason is that, as long as other banks are expected to avoid lending to operating firms, banks that have ample capital will still choose to park it in risk free assets rather than lend it to operating firms that are expected to fail to return it in the economic conditions that result from a credit freeze equilibrium. Hence, the Troubled Asset Relief Program (TARP), by which the US government injected capital to financial institutions, is expected in our model to make things better by raising banks' confidence in lending to operating firms, but still leaves a substantial range of fundamentals where banks fail to coordinate on lending the capital.

We then turn to examine the possibility of the government's providing loans directly to operating firms. Should the government serve as "lender of last resort" to operating firms? This has been attempted during the crisis, for example when the government injected money to General Motors and other automakers or bought commercial paper of other firms. The problem with such direct lending by the government is that it is reasonable to assume that the government does not have the same ability as banks to distinguish between operating firms with good and bad projects. Thus, while in our model, direct lending to operating firms is more effective in reducing the probability of a credit freeze – as it avoids the coordination problem among banks in lending the money – it generates some waste of resources by channeling capital to some firms with bad projects.

Comparing capital infusion to the banking system with direct lending to operating firms, our model shows that the latter is optimal only when expected fundamentals are in some intermediate range. In this range, the effectiveness that direct lending has in moving the economy out of a credit freeze dominates the negative effect associated with channeling capital to bad projects. Relating this to current debate, our model sheds light on when it is more efficient to help *Wall Street* than to help *Main Street*. The latter is associated in our model with direct lending to operating firms, and turns out to be efficient when the economy is in an intermediate state.

A key question is whether there is a government policy that can combine the benefits of capital infusion to the banking system and direct lending to operating firms. That is, a policy that achieves the reduction in credit-freeze probability obtained by direct lending while not channeling capital to bad borrowers. We identify such a mechanism in our model, which we denote as capital infusion to the banking system under lending commitment. The mechanism has the same logic as the Term Asset-Backed Securities Loan Facility (TALF) program employed by the US government later in the crisis. Here, the government provides capital to banks under the condition that they use this capital together with their own capital in lending to operating firms.

Our paper is related to the large literature on bank runs, where depositors rush to demand early withdrawal from the bank because they believe that other depositors are going to do the same. The seminal paper on bank runs is by Diamond and Dybvig (1983), and it was followed by much subsequent work on the subject (see, e.g., Allen and Gale (1998), Peck and Shell (2003), and Goldstein and Pauzner (2005)). The ideas in the bank-run literature have subsequently been applied to describe also runs by investors on currencies (Morris and Shin (1998)), financial markets (Bernardo and Welch (2004) and Morris and Shin (2004a)), and other contexts. Our paper, which builds on the analytical insights of this literature, focuses on a different context. We do not consider a run by depositors or investors on financial institutions, financial markets, or governments, but rather a run by financial institutions on the nonfinancial firms of the real economy. More importantly, our contribution is in analyzing alternative government responses that can be used in this context.

Several papers analyze policies of deposit insurance or 'lender of last resort' to prevent runs on financial institutions. These include the papers by Rochet and Vives (2004), Corsetti, Guimaraes, and Roubini (2006), and Morris and Shin (2006). The policy problem we consider here is fundamentally different. In these papers, the analysis revolves around capital infusion to an institution that might be subject to a run because it lacks capital. In our model, on the other hand, coordination failures arise among financial institutions in their decision to lend to operating firms. Hence, capital infusion to financial institutions might not be sufficient to eliminate an inefficient credit market freeze, as they might fail to coordinate on lending it. This leads to our discussion on the role of direct government intervention in lending to operating firms, and the various ways of implementing it without losing the informational advantage that banks have in lending to such firms. A recent paper by Sakovics and Steiner (2009) analyzes a related question of subsidizing agents who participate in a coordination game. While their paper focuses on who should be subsidized, ours focuses on how to subsidize.

The source of coordination failures among banks in our model is the interdependence among firms in the real economy that makes the investment in a firm profitable only if other firms are able to invest and produce. Such strategic complementarities in the macro economy were motivated in an influential paper by Cooper and John (1988), and have been used in other papers (e.g., Goldstein and Pauzner (2004)). Our paper complements this literature by showing how such complementarities can cause a credit freeze and analyze government policy in such context.

Models of strategic complementarities usually yield multiple equilibria and thus do not lend themselves naturally to policy analysis. To overcome this problem, we follow recent work on self-fulfilling crises and rely on global-games techniques. The global-games literature has been pioneered by Carlsson and van Damme (1993) and Morris and Shin (1998) and is reviewed in Morris and Shin (2003)). In particular, we build here on the model in Morris and Shin (2004b).

The recent financial crisis has generated a surge of research on many related aspects. Let us mention a few papers that are more closely related to ours. Acharya, Gale, and Yorulmazer (2009) analyze the debt rollover problem, where the fact that debt needs to be rolled over frequently reduces the debt capacity of firms with little credit risk. Diamond and Rajan (2009) argue that the possibility of future fire sales makes banks want to hoard on cash instead of extending new loans. Benmelech and Bergman (2009) analyze government policies in a model where credit traps evolve as a result of reduction in collateral value. Philippon and Schnabl (2009) analyze government policy in a model where credit does not flow because firms suffer from a debt overhang problem.

The remainder of this paper is organized as follows. Section 2 describes our framework of analysis. Section 3 provides an equilibrium analysis, identifying the conditions under which inefficient credit freeze equilibria will arise. Section 4 analyzes several governmental policies that may be used to produce a credit thaw, identifying their potential benefits and limitations. Section 5 concludes.

2. The Model

There is a continuum *[0,K]* of identical financial firms, which we call banks for simplicity. Each bank has 1 dollar of capital. Banks can choose whether to invest their capital in a risk-free asset, such as a deposit with the central bank, generating *1+r* (*>1*) dollars next period, or lend it to operating (nonfinancial) firms. Banks are risk neutral and hence make their choices so as to maximize expected payoffs.

Operating firms have access to investment projects that require investment of 1 dollar, but do not have any capital to finance them. They rely on bank lending to invest in their projects. There are two types of operating firms. Some operating firms have bad projects that always generate a gross return of 0. Others have good projects, generating a gross return of *1+R* (*>1+r*) when the macroeconomic fundamentals are strong and a sufficient number of operating firms get the required financing to invest. Specifically, the return on a good project is assumed to take the following form:

$$\begin{cases} 1+R & if \quad aL+\theta \geq b \\ 0 & if \quad aL+\theta < b \end{cases}. \tag{1}$$

Here, θ is a macroeconomic fundamental that can represent various factors, such as firms' productivity, consumers' demand, the cost of imported oil, etc. The variable L represents the mass of firms that received loans from banks to invest in their projects. In the basic model, $L = nK$, where $n \in [0,1]$, whose value is determined endogenously in the model, is the proportion of banks that decide to lend to firms. Hence, the macroeconomic fundamentals and the proportion of firms investing in their projects are together responsible for the profitability of good projects. a is a parameter capturing the importance of complementarities vs. fundamentals in making projects profitable, and b is a parameter capturing the threshold needed to become profitable.[2]

The effect of L reflects the interdependence in payoffs among operating firms in the economy. This interdependence can be due to several reasons. For example, many firms can prosper only when there are other firms in the economy that can provide them

[2] Note that we use a discontinuous return function (i.e., projects either succeed or do not succeed) for simplicity of exposition. Our results would hold in a model where the return on projects is a continuous function of θ and L.

with adequate inputs. In addition, many firms sell some or all of their output to other firms, and thus depend on the operation of other firms. Even firms that sell their output solely to individuals might suffer from declining sales if other firms are not able to employ these individuals. In sum, the success of the economy in our model requires the coordination among various operating firms and the banks that finance them. Such coordination issues in the macro economy were proposed before by other authors, e.g., by Cooper and John (1988).[3]

We assume that banks can tell the difference between firms with bad projects ("bad firms") and firms with good projects ("good firms"), and thus can choose to lend only to firms with good projects.[4] Moreover, we assume for simplicity that the mass of firms with good projects is greater than the mass of banks *K*, and thus banks are able to extract the full return *R* from lending to good firms, whose projects were successful.[5]

We assume that the fundamental θ is not publicly known. It is normally distributed around a mean of y. We consider y to be public news available to everyone about the strength of the economy. The standard deviation of θ around y is σ_θ, and we use $\tau_\theta = \frac{1}{(\sigma_\theta)^2}$ to denote the precision of the distribution of θ. Each bank *i* receives a private signal regarding the value of θ, given by $x_i = \theta + \varepsilon_i$. Here, the individual specific noise terms ε_i are independently normally distributed with mean 0 and standard deviation σ_p. We use $\tau_p = \frac{1}{(\sigma_p)^2}$ to denote the precision of banks' signals. Banks make their decisions whether to invest in the riskless asset or to lend to operating firms after observing these signals.

Because the profitability of operating firms depends on macroeconomic conditions and the availability of financing to other firms, a bank's incentive to lend to a given operating firm with a good project is higher when the economy's fundamentals are

[3] Clearly, there are some firms, who become better off when other firms are hurting. Our analysis applies to situations and firms where complementarities are the dominant force.

[4] The firms with bad projects will have an explicit role in the model later when we consider the possibility of the government extending direct loans to operating firms.

[5] We are thus able to show that a credit-freeze equilibrium may arise even when the competitive conditions enable banks to extract the full surplus from lending and are thus as favorable to lending activity as possible.

favorable and when the number of banks who are going to lend is high. While the optimal behavior of a bank usually depends on its belief regarding the behavior of other banks, there are ranges of macroeconomic fundamentals in which banks have a dominant strategy. More specifically, when the fundamental θ is above b, a bank will prefer to lend to an operating firm no matter what it believes other banks will do. This is because in this range the return on lending is guaranteed to be *1+R*. Similarly, when the fundamental is below $b - aK$, the bank will invest in a government bond even if it believes that all the other banks will lend to operating firms.

Since θ is drawn from an unbounded distribution, there are signals at which banks choose to lend to operating firms independently of their beliefs regarding other banks' behavior, as well as signals at which they choose not to lend independently of their beliefs. As to banks that receive a signal in the intermediate range, however, their optimal decision depends on their expectations about whether other banks will lend to operating firms. This calls for an equilibrium analysis to which we turn next.

3. Equilibrium Analysis

3.1. Credit Freeze

We solve the model using global-games techniques. In particular, we follow here Morris and Shin (2004b). Proposition 1 states the basic equilibrium result.

Proposition 1: *Suppose that the information in banks' signals is precise relative to prior information, so that* $\frac{\tau_\theta}{\sqrt{\tau_p}} \leq \frac{\sqrt{2\pi}}{aK}$. *Then, there is a unique Bayesian Nash Equilibrium in which all banks lend to operating firms if they observe a signal above* x^* *and withdraw from lending if they observe a signal below* x^*. *Investment projects then succeed if and only if the fundamentals are above the threshold* θ^*, *between* $b - aK$ *and* b, *which is characterized by the following equation:*

$$\theta^* = b - aK + aK\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta+\tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1+r}{1+R}\right)\right)\right), \qquad (2)$$

where $\Phi(\cdot)$ *is the cumulative distribution function for the standard normal.*

Remarks: (i) *Intuition:* The intuition behind the result of Proposition 1 can be explained as follows. Due to strategic complementarities, when banks do not know that the fundamentals are below $b - aK$ or above b, they do not have a dominant action to choose. In this case, they simply want to do what other banks do. In a model with common knowledge about the fundamental θ, this would result in multiple equilibria, as both the case where all banks lend to operating firms and the case where none of them does so can be supported by equilibrium beliefs. The assumption that banks observe slightly noisy information about θ combined with the presence of extreme regions where they have dominant actions pins down the threshold equilibrium characterized by equation (2) as the unique equilibrium here.

Intuitively, with noisy information, banks that observe a signal slightly below the upper dominance region know that the fundamental may well be higher than their signal and thus choose to lend. Knowing this, banks with even lower signals will also choose to lend. This rationale can be repeated again and again, guaranteeing a range of signals below the upper dominance region, where banks choose to lend. Similarly, due to the noisy information, there will be a range of signals above the lower dominance region, where banks will choose to invest in government bonds. The proof of equilibrium with global-game techniques demonstrates that this procedure exactly separates the real line, so that banks lend above x^* and do not lend below it, leading to success of real projects above θ^* and failure below it.

(ii) *The No-Lending Threshold:* Equation (2) characterizes the threshold fundamental θ^* below which investment projects fail. To gain some intuition for what determines this threshold, it is useful to consider the limit, as banks' private signals become infinitely precise, i.e., as τ_p approaches infinity. In this case, x^* and θ^* converge to the same value, which is given by:

$$\theta^* = b - aK + aK\frac{1+r}{1+R} \qquad (3)$$

Intuitively, a bank observing the signal θ^* is indifferent between lending to operating firms and investing in the risk-free asset under the belief that the proportion of other banks lending to operating firms is uniformly distributed between 0 and 1.[6] This implies that lending to operating firms will be profitable with probability $\left(1-\frac{b-\theta^*}{aK}\right)$, which yields the following indifference equation:

$$1+r=\left(1-\frac{b-\theta^*}{aK}\right)(1+R).$$

Rearranging this equation, we get (3).

Because banks' signals have infinitesimally small noise, the equilibrium result is that all banks lend when the fundamental is above θ^* and do not lend when the fundamental is below θ^*. Hence, below θ^*, the economy ends up in a no-lending equilibrium.

(iii) *Efficient and Inefficient No-Lending Equilibria:* When macroeconomic fundamentals are so bleak that we are below $b-aK$, the refusal of banks to lend is efficient because firms' projects will not produce payoffs exceeding the economy's riskless rate even if no banks withdraw from the lending market. When fundamentals lie between $b-aK$ and θ^*, however, the economy is in an inefficient no-lending equilibrium. In this interval, banks withdraw from lending even though, were banks all willing to lend, firms' projects would produce returns exceeding the riskless rate and the banks would be all better off relative to the no-lending equilibrium. We refer to this outcome as a *credit freeze*.

(iv) *Credit Freezes as a Coordination Failure*: When fundamentals lie tween $b-aK$ and θ^*, the credit freeze can be viewed as due to coordination failure. Here, banks do not lend to operating firms just because they fear that other banks will not lend to operating firms. The fundamentals uniquely determine banks' expectations regarding what other banks are going to do and thus (indirectly) uniquely determine whether a credit freeze will arise, but the credit freeze is still inefficient. If the banks could have concluded among themselves an enforceable agreement on how they will act, they would have agreed on a coordinated strategy of lending to firms. However, as long as the banks make their decisions separately, based on their expectations as to how other banks will act, an inefficient credit freeze equilibrium may ensue. Interestingly, this

[6] The rationale behind the uniform-distribution belief is that each bank perceives a uniform distribution on the proportion of banks getting lower signals than its own. Given that the bank observed θ^* and that other banks lend if and only if they obtained a signal above θ^*, the bank perceives a uniform distribution on *n*.

inefficiency could have been avoided if the available capital was held by one large bank (or a few large banks) instead of many small ones. Thus, from the point of view of avoiding coordination failures, having large financial institutions may be an advantage.

(v) *The 2008 Credit Crunch:* The credit crunch of 2008 was preceded by the arrival of bad economic news about macroeconomic fundamentals. For one thing, the substantial decline in housing prices considerably reduced the wealth of households, and such a reduction could have been expected to produce a subsequent decrease in consumer spending and thus the demand for firms' output. Our model indicates that the arrival of bad macroeconomic news might trigger a credit freeze that will lead to the refusal of banks to lend to firms even though the firms would still be worth financing notwithstanding the deterioration in macroeconomic fundamentals absent a self-fulfilling withdrawal of banks from the lending market. Such triggering of a credit freeze will of course further reinforce and exacerbate the effects of the deterioration in fundamentals that triggered it in the first place.

3.2. Can Reduction in Banks' Capital Trigger A Credit Freeze?

The credit crunch of 2008 was preceded by a perceived deterioration in the capital positions of financial institutions as a result of losses from real estate mortgage assets. This subsection examines whether a reduction in the banks' capital can trigger a credit freeze, even holding the fundamental θ constant.

To study this issue, let us introduce the parameter l (between 0 and 1), which denotes the proportion of capital lost by banks in the economy due to bad past investments. For simplicity of exposition, we assume that capital has been lost uniformly across banks, that is, each bank in the economy lost a fraction l of its capital. With this parameter introduced into the model, the capital of a single bank ($1-l$) does no longer suffice to finance a firm's project. Hence, each firm will have to pool resources from more than one bank. Eventually, if a fraction n of banks decide to lend the capital they have to operating firms, the total capital that will be provided as loans to such firms will be only a fraction $n(1-l)$ of K, and hence $L = n(1-l)K$.

Proposition 2 characterizes the new equilibrium results and the effect that the parameter l may have on the realization of a credit freeze.

Proposition 2: *(a) In the unique Bayesian Nash Equilibrium, investment projects succeed if and only if the fundamentals are above the threshold $\theta^*(l)$. The threshold $\theta^*(l)$ is characterized by the following equation:*

$$\theta^* = b - aK(1-l) + aK(1-l)\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta+\tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1+\mathrm{r}}{1+R}\right)\right)\right), \quad (4)$$

(b)The threshold $\theta^*(l)$ is an increasing function of the parameter l; hence, an increase in the fraction of bank capital that was lost, l, with no change in the fundamental θ, can shift the economy from an efficient lending equilibrium to an inefficient credit freeze.

Remark: The intuition behind the result of Proposition 2, which indicates that a reduction in the banking sector's capital raises the threshold, below which banks elect to withdraw from lending, is as follows. A reduction in the banking sector's capital makes each bank "less sure" that other banks will provide enough capital to operating firms to guarantee adequate return from extending loans to operating companies. Hence, such a reduction makes each bank more concerned that, in the event it provides a loan to a given operating company, the firm will nonetheless suffer from the inability of many other operating companies to obtain financing. Technically, in equilibrium, a higher fundamental θ is required to make banks indifferent between providing credit to operating companies and investing in the riskless asset, which leads to an increase in the threshold θ^* and thus in turn to a larger range of fundamentals at which an inefficient credit freeze ensues.

Thus, our results indicate that banking losses can drive the economy into a credit freeze even without any accompanying change in other macroeconomic fundamentals. What is important to stress is that such reduction in capital will make operating firms less likely to receive financing not only because of the direct effect that some capital that could have been available for loans is no longer in place but also because of the indirect effect, which our result identifies, that it might deprive operating firms even of the capital that remains in place. By influencing banks' expectations as to how many operating firms will be able to obtain financing, the disappearance of some capital can make banks more reluctant to lend the capital that still remains.

4. Government Policy

The focus of our paper is on analyzing and comparing various government policies intended to reduce the inefficiency from credit-freeze equilibria. The analysis is provided in this section.

4.1. Interest Rate Reduction

One governmental measure that is natural to examine as an instrument for addressing a credit freeze is a cut in interest rates. During the credit crisis of 2008, governments around the world have made substantial use of interest rate cuts. During 2008, in a series of moves, the Federal Reserve Board cut the federal rate considerably, bringing the Federal funds rate down from 4.25% in January to 1% in October. Similar steps have been taken by other central banks around the world. In October 2008, facing a worldwide contraction in lending, twenty one countries around the world, including the US and the UK, simultaneously cut interest rates.

Under normal market conditions, a cut in a country's interest rate can be expected to spur lending. To what extent can a cut in interest rate, however, be relied on to eliminate a coordination failure that results in an inefficient credit freeze equilibrium? As we show below, a cut in interest rate (i.e., reducing r) may – but does not have to – produce a credit thaw. The following proposition summarizes the results.

Proposition 3: *(a) For every level of bank losses l, a decrease in the interest rate r on government bonds reduces the threshold θ^*, below which a credit freeze occurs, and hence reduces the likelihood of a credit freeze.*
(b) Yet, for every $r \geq 0$ and l (between 0 and 1), there are realizations of the fundamental θ, at which an inefficient credit freeze occurs.

Remarks: (i) *The Reduction in the Likelihood of Credit Freeze*: A reduction in r makes investment in the riskless asset less attractive and thus lowers the expected return that will be necessary to induce banks to lend to operating firms, which in turn lowers the threshold θ^* above which banks will lend to such firms rather than withdraw from the lending market. It is interesting to note that the effect of the reduction in r on the decision of an individual bank is more than just the direct effect on this bank's payoff. Because the reduction in interest rate can be expected to affect other banks' decisions, it also affects the individual bank's decision through its effect on the bank's expectation concerning how other banks will act.

(ii) *The Limits of Interest Rate Cuts*: The second part of the proposition says that interest rate reductions cannot eliminate all inefficient credit freezes. Even if the government reduces r all the way to 0 (or to a very low level just above zero), θ^* will remain above $b - aK(1-l)$, which implies that inefficient credit freezes may occur in the interval between $b - aK(1-l)$ and θ^*. The intuition goes back to the coordination-failure aspect of credit freezes in our model. Even if the net return on the riskless asset is close to zero, banks will prefer to invest in it rather than lending to operating firms when they expect that other banks will all do so. Thus, while governmental reduction in interest rates can shift the threshold that triggers coordination failure and credit freezes, it cannot completely eliminate such coordination failures. This result might be thought of as similar in spirit to the well-known liquidity trap in monetary economics.

4.2. Infusion of Capital to the Banking System

During the financial crisis of 2008, governments around the world infused a large amount of capital into banks to shore up banks' capital positions, which have eroded due to losses from real estate mortgage assets and other investments. In October 2008, the US Treasury, as part of the Troubled Asset Relief Program (TARP), infused into financial firms about $250 billion in additional equity capital. During the same period, the UK invested about $90 billion in several major banks. In addition to providing additional equity capital to financial firms, the Federal Reserve Board also provided additional capital to financial intermediaries by purchasing large amounts of their commercial paper.[7]

[7] The Fed established the Commercial Paper Funding Facility in October 2008, and it purchased during the subsequent several weeks hundreds of billions of dollars worth of commercial paper from financial intermediaries such as Morgan Stanley, GMAC, and American Express.

Infusion of capital into banks is a policy measure that is natural to consider in financial crises. Infusion of capital, e.g., in the form of a lender of last resort, has been used to prevent or stop bank runs in which depositors seek to withdraw their deposits en masse from a bank. When a solvent bank faces a problem of a bank run, providing the bank with capital may ensure depositors that their money is safe and prevent a run on the bank. Infusion of capital has also been used in the case of insolvent banks when governments felt that making sure such banks can meet their obligations to depositors is necessary to prevent a contagion effect that would lead to runs by depositors on other banks.

The subject we examine using our model is different because it does not involve potential runs by depositors on banks (or financial institutions more generally). Rather, it is the banks that may "run on the economy" by not extending loans to operating firms. In our context, therefore, capital infusion will not be designed to enable banks to meet their obligations toward their creditors. Rather, in our context, capital infusion may be used to facilitate lending by banks to operating firms in two ways: first, the direct and straightforward way of providing banks with additional capital that they may use for the purposes of extending loans; and, second, the indirect effect, which our model highlights, of encouraging banks to lend to operating firms capital that they already have but that they might elect not to lend in the absence of the capital infusion to the banking sector and the shift in expectations produced by it.

To analyze governmental infusion of capital into the banking sector, let us assume that the government has or can obtain capital that would be sufficient to cover part of banks' losses. In particular, let us assume that the government has an amount $Z = \alpha l K$, enabling it to inject a proportion α of the lost capital l to all banks in the economy. If the government injects the capital, each bank will have a total capital of $1 - (1 - \alpha)l$.

Banks will again make a decision whether to lend to operating firms or invest in the riskless asset. The first option yields a gross return of $1 + R$ if firms' investment projects succeed, which happens as long as the proportion of banks lending to firms is above $\frac{b-\theta}{a(1-(1-\alpha)l)K}$, while the second one yields a certain gross return of $1+r$. To focus on capital infusion, we will assume from now that r=0, so that the government has already reduced the interest rate as much as possible. The following proposition analyzes the effect of injecting capital to the banking system.

Proposition 4: *(a) The threshold θ^*, below which a credit freeze occurs when the government covers proportion α of bank losses is implicitly determined by:*

$$\theta^* = b - aK(1-(1-\alpha)l) + aK(1-(1-\alpha)l)\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta+\tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1}{1+R}\right)\right)\right), \quad (5)$$

(b) The threshold θ^* decreases in α .Yet, for every α ≤l, there are realizations of the fundamental θ at which an inefficient credit freeze will occur.

Remarks: (i) *The Reduction in the Likelihood of Credit Freeze*: By providing capital to the banking system, the government creates externalities that make the projects of operating firms more profitable. This is because banks have more capital to lend to operating firms, and so when they decide to lend, operating firms will produce greater returns. This encourages banks to lend to operating firms, making a credit thaw more likely to occur. Importantly, the effect of capital infusion is not merely due to the fact that the government's capital flows to operating firms, but rather mostly due to the fact that the availability of this capital makes banks more likely to lend capital that they already have. This is thus the mechanism behind the effect of TARP if the underlying problem was indeed a coordination problem.

Technically, at the threshold, below which a credit freeze occurs, banks will require a lower fundamental θ to be indifferent between lending and not lending to operating firms when the government injects more capital to the banking system (α is higher). This is because a higher α implies that under a uniform distribution of banks that decide to lend, the returns from lending increase. This pushes the threshold θ^* lower and increases the likelihood of a credit thaw.

(ii) *The Limits of Capital Infusion:* Even when the government covers all the losses that banks accumulated, banks will be reluctant to lend if they believe other banks are not going to lend. Hence, this policy of the government cannot fully eliminate coordination-based credit freezes. This sharpens the difference between infusion of capital to banks in our model, where crises reflect a run of banks on operating firms, and infusion of capital in a model of a run on the bank. Because, in our model, coordination failures arise among banks in their decision to lend to operating firms, banks end up not using capital that they have for lending purposes. Hence, capital infusion might not be sufficient to eliminate an inefficient credit market freeze.

4.3. Direct Lending to Operating Firms

As explained above, the difficulty that the government faces in breaking the credit freeze by providing capital to banks is that banks might take the capital and not lend it to operating firms due to the fear that other banks will not lend. An alternative to providing capital to banks is for the government to lend directly to operating firms. This would be the truly equivalent policy to a traditional lender of last resort, as it would have the government directly providing capital to those that need it, who in our model are the operating firms. In the recent crisis, such policy took the form of lending to General Motors and the other auto makers. The government also participated in direct purchase of commercial paper of other corporations.

The problem with such policy is that the government does not have the ability that banks have to identify good firms from bad firms. Thus, providing capital to firms without using the intermediation services of banks would lead to lending to some firms that have bad projects and should not get financing.

To examine the efficiency of direct lending formally, we have to explicitly describe the bad operating firms in our model. So far, there was no need to consider them and how many of them exist, as the assumption was that banks can tell good firms from bad firms, and thus bad firms would always be avoided. If the government attempts to lend to operating firms directly, however, it will have to consider the consequences of not being able to tell good firms from bad firms.

For the formal analysis, let us denote the mass of bad (good) operating firms in the economy as B (G). Recall that G is greater than K (the mass of banks). Suppose that the government has capital at the amount of $Z = \alpha l K$ (as in Section 4.2) and it has to decide whether to inject it directly to operating firms or to the banks. When the government lends capital to operating firms, the capital is randomly allocated between good or bad firms. We denote the proportion of the capital that finds its way to bad firms as $\beta \equiv B/(B+G)$. For simplicity, we assume that the government does not know the realization of the fundamental θ (and does not get any signal about it). Initially, we will assume that the operation of firms with bad projects, while producing no returns for the lending bank, still provides a positive externality for other operating firms (as firms with bad projects do purchase inputs from other firms etc.); below we will discuss how our conclusions will change if we were to assume that such externalities flow only from the operation of firms with good projects.

We begin the analysis by comparing the likelihood of a credit freeze under direct lending to operating firms vs. under infusion of capital to banks. The result is summarized in the following proposition.

Proposition 5: *If the government lends αlK directly to operating firms, there is a credit freeze equilibrium if and only if the fundamental θ is below the threshold θ^*, which is implicitly defined by:*

$$\theta^* = b - aK(1-(1-\alpha)l) + aK(1-l)\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta+\tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1}{1+R}\right)\right)\right), \quad (6)$$

Denoting the threshold under capital injection to banks (defined in equation (5)) as θ^*_{Bank} and the one under direct lending to firms (defined in equation(6)) as θ^*_{Direct}, we get that for every α and l, $\theta^*_{Bank} > \theta^*_{Direct}$, implying that the probability of a credit freeze is higher under capital injection to banks than under direct lending to operating firms.

Remark: The intuition for why directly lending the government's capital will reduce the lending threshold more than infusing the capital into banks is simple. When the government injects capital to banks, some of this capital might remain "stuck" in the banking system as banks fail to coordinate on lending it to operating firms. When the government lends the capital directly to operating firms, banks know that it will generate the desired externalities. As a result, lending directly to operating firms more effectively increases the returns to banks from lending and encourages banks to lend, and thus is more likely to bring the economy to a credit thaw.

Focusing attention on the limit case where banks' private signals become infinitely precise, i.e., as τ_p approaches infinity, the comparison between the two cases becomes very transparent. Following (3), we can express the thresholds under the two regimes in the limit case as:

$$\theta^*_{Bank} = b - aK(1-(1-\alpha)l) + aK(1-(1-\alpha)l)\frac{1}{1+R} \quad (7)$$

$$\theta^*_{Direct} = b - aK(1-(1-\alpha)l) + aK(1-l)\frac{1}{1+R} \quad (8)$$

Equations (7) and (8) clearly reveal that $\theta^*_{Bank} > \theta^*_{Direct}$.

But, as noted above, the fact that direct lending is more likely to generate a credit thaw is not enough to make this policy measure more efficient. We now carry out a full comparison between the two measures. For a sharp comparison, we focus attention on the limit case considered above. This is easier to work with because at the limit either all banks lend or none of them does, and then we do not have to consider cases where some banks lend but projects fail and vice versa. The following proposition characterizes which policy ends up producing better results for different levels of the fundamentals.

Proposition 6: *(a) When the fundamental θ is below θ^*_{Direct} or above θ^*_{Bank}, the overall wealth in the economy is higher under injection of capital to the banking system than under direct lending to operating firms.*

(b) When the fundamental θ is between θ^*_{Direct} and θ^*_{Bank}, the comparison between the two regimes yields ambiguous results. For a sufficiently large β and/or small R the wealth is higher under injection of capital to the banking system.

(c) Ex-ante, when choosing the policy, the government should choose to inject capital to the banking system when β is sufficiently high, R is sufficiently low, and y is either sufficiently high or sufficiently low (i.e., outside an intermediate range).

Remarks: (i) *When θ is below θ^*_{Direct} or above θ^*_{Bank}*: In these circumstances, direct lending is clearly undesirable, as it does not turn a credit freeze into a thaw, but still generates the costs of lending by the government. In particular, when θ is above θ^*_{Bank}, a credit thaw is produced under both policies, but direct lending involves lending money to bad borrowers. When θ is below θ^*_{Direct}, there is a credit freeze under both policies, but direct lending involves lending to bad borrowers and also to good borrowers, whose projects fail because there is a credit freeze.

(ii) *When* θ *is between* θ^*_{Direct} *and* θ^*_{Bank}: In these circumstances, infusion of capital into the banks will fail to induce banks to lend efficiently. Direct lending by the government will accordingly have two benefits: first, it will provide financing to some operating firms with good projects; second, direct lending will induce banks to lend to operating firms. On the other hand, direct lending by the government will involve the wasteful provision of financing to firms with bad projects. If β is sufficiently large – that is, when the government's screening ability is sufficiently poor – this cost of a direct lending program may make it overall undesirable. The same is true when the return on successful good projects R is sufficiently low.

(iii) *Ex-ante choice between the two policy measures*: As noted above, the government does not know the realization of θ. Hence, it should make its decision between the two policy measures based on the characterization provided above of what will happen for different realizations of θ and on the prior distribution of θ. Clearly, based on the above, we can see that for sufficiently high β and/or low R, the government should not go with direct lending. In addition, y – the mean of the fundamentals, which can be interpreted as public news – matters for the decision. Given that direct lending may only be desirable at an intermediate range of the fundamentals, the government should not choose it when y is either too high or too low, only when it is in an intermediate range.

This result can be tied to the policy debate that came up in the recent crisis about whether the government should bail out *Wall Street* or *Main Street*. Infusing money to banks can be interpreted as helping Wall Street, while lending directly to operating firms can be interpreted as helping Main Street. Our results suggest that the latter is desirable when public news about the fundamentals of the economy is in some intermediate range, and not when it is too bad or too good.

(iv) *The Case in which only Operating Firms with Good Projects have Beneficial Spillover Effects*: Finally, we remind the reader that our analysis was conducted under the assumption that capital that is lent to bad firms still creates positive externalities to other firms even though it generates no direct return. It might be argued, however, that some bad projects create no or lower spillover benefits for other firms. To examine the consequences of this factor, let us assume that the payoffs of operating firms do not depend on the number of other firms in operation but on the number of other firms in operation with good projects. Making this assumption weakens the attractiveness of direct lending to operating firms by the government.

To see this, note that if only good firms getting capital from the government created synergies to other firms, than the equation that determined the threshold θ^*_{Direct}, below which a credit freeze occurs in a regime of direct lending, would change from equation (8) to the following (we consider the limit again, for simplicity):

$$\theta^*_{Direct} = b - aK\left(1 - \left(1 - \alpha(1-\beta)\right)l\right) + aK(1-l)\frac{1}{1+R} \qquad (9)$$

Clearly, this would increase the likelihood of a credit freeze under direct lending, making this regime overall less desirable.

4.4. Infusion of Capital to the Banking System under Lending Commitment

As we demonstrate in Proposition 6, there is a tradeoff between the two programs described in Sections 4.2 and 4.3. Under infusion of capital to the banking system (described in Section 4.2), government's capital does not get allocated to bad borrowers, but it might get stuck in the banking system as banks fail to coordinate on lending it. Under direct lending to operating firms (described in Section 4.3), government's capital is guaranteed to flow to operating firms, but it might get allocated to bad ones. The challenge is to design a plan that achieves the benefits of both these plans. We describe such a plan in this section.

As before, we assume that the government holds capital $Z = \alpha lK$. Under the proposed plan, the government allocates an amount αl to each bank under the condition that the bank lends this amount in addition to its own capital $(1-l)$ to operating firms. Essentially, under this plan, the government infuses capital to the banking system but only if banks commit to lend to operating firms. One can think about this program as a matching program, where the government matches banks' lending with its own contribution. This is very similar to the logic behind the program enacted by the government later in the crisis – the Term Asset-Backed Securities Loan Facility (TALF).

Under this plan, each bank faces the following tradeoff when deciding whether to lend or not. If the bank does not lend, it invests $(1-l)$ in the risk-free asset and receives

$(1-l)$. If it decides to lend, it can lend a total of $(1-l+\alpha l)$ and then receive $(1-l+\alpha l)(1+R)$ if real projects succeed (which happens as long as the proportion of banks lending to firms is above $\frac{b-\theta}{a(1-(1-\alpha)l)K}$) and *0* otherwise. Obviously, if banks lend, they only lend to good firms, since they have the ability to screen borrowers. The following proposition characterizes equilibrium outcomes under the proposed plan.

Proposition 7: *(a) If the government allocates αl to banks under the condition that they lend $(1-l+\alpha l)$ to operating firms, there is a credit freeze equilibrium if and only if the fundamental θ is below the threshold θ^*, which is implicitly defined by:*

$$\theta^* = b - aK(1-(1-\alpha)l) + aK(1-(1-\alpha)l)\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta+\tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1-l}{(1+R)(1-(1-\alpha)l)}\right)\right)\right), \quad (10)$$

(b) As τ_p approaches infinity, this threshold converges to θ^*_{Direct} (defined in (8)), and so the infusion of capital to the banking system under lending commitment is more efficient than both direct lending to operating firms and infusion of capital to the banking system.

Remarks: (i) *Comparison with Simple Infusion of Capital to the Banking System*: The mechanism presented here achieves better outcomes than a simple infusion of capital to the banking system. Here, the government injects the capital to banks under the condition that they lend. This provides banks a stronger incentive to lend, and hence the threshold below which a credit freeze occurs is below that in Proposition 4. Moreover, banks only lend to good firms and when there is no credit freeze (in the limit, as their information gets very precise), so the mechanism here does not entail the disadvantages that were described under direct lending by the government to operating firms.

(ii) *Comparison with Direct Lending to Operating Firms*: Thinking about the threshold below which a credit freeze occurs, there are two differences between the current mechanism and direct lending by the government to operating firms. First, under the current mechanism, banks' incentives to lend are pushed upwards by the fact that they get extra capital to profit on when they lend. This acts effectively like leverage, encouraging them to take risks and lend. This effect pushes the threshold below that achieved with direct lending by the government. Second, under the current mechanism, the government's capital is not guaranteed to flow to operating firms, as it is still channeled via banks that sometimes choose not to lend. This effect pushes the threshold above that achieved with direct lending by the government. As it turns out, as signals get very precise, these two effects exactly cancel out with each other, and so the threshold here is exactly θ^*_{Direct} that is achieved with direct lending by the government to operating firms. The current mechanism is then more efficient because it reduces the probability of a credit freeze by the same amount, but does not involve wasting capital by lending to failed projects.

(iii) *Summary*: This section shows that infusion of capital to the banking system under lending commitments – a mechanism that corresponds to what the government tried at later stages of the crisis with the TALF program – achieves the best of both worlds, and dominates both simple infusion of capital to the banking system and direct lending by the government to operating firms.

4.5. Other Mechanisms

In this section, we discuss two other mechanisms that have been mentioned in the public debate over government intervention in the recent crisis. We show that these mechanisms do not achieve as good results as the infusion of capital to the banking system under lending commitment, which was analyzed in Section 4.4.

4.5.1. Government Funds Managed by Private Firms

Consider the following mechanism. The government places the capital $Z = \alpha l K$ in a number of funds, which are managed by banks or by other private agents that have the same expertise. The managers of the funds will be paid a proportion γ on any profit that they generate on the capital invested by the fund they manage – that is, the excess of the return they generate over the riskless return. However, like hedge fund managers, they will not bear any share of the losses generated, if any, and such losses will be borne by the government.[8] The following proposition characterizes the consequences of this mechanism.

Proposition 8: *(a) If the government invests* $Z = \alpha lK$ *in funds managed by private agents, who are promised a proportion* γ *on any return they generate above 1, then (i) the funds' capital will be fully lent to operating firms with good projects, and (ii) the threshold defining whether banks will lend to operating firms will be* θ^*_{Direct}, *as characterized in equation (6).*
(b) Consider the case where τ_p *approaches infinity: The setting of government funds proposed here is less efficient than infusion of capital to the banking system under lending commitment (analyzed in Proposition 7).*

Remarks: (i) *The Decisions of the Government Funds' Managers*: The design of the mechanism ensures that the government's capital invested in the funds will be fully provided to operating firms with good projects. Because the government fully bears the losses, the managers will have no reason to avoid lending the funds given to them. Furthermore, because the managers are promised a cut of the profits, they will have an incentive to screen operating firms with good projects from operating firms with bad projects, and their dominant strategy will be to lend funds only to firms with good projects.

(ii) *The Effect on Banks' Lending Threshold*: Because the government funds program, like the direct lending program, will ensure that an amount of $Z = \alpha lK$ will be lent to operating firms, the threshold for banks' lending to operating firms will be the same as the threshold, defined in equation (6), that would result from the direct lending program.

(iii) *Comparison with Other Programs*: The government funds mechanism is more efficient than the direct-lending program, since it achieves the same lending threshold, but does not involve lending to operating firms with bad projects. While the

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[8] For a fuller discussion of the institutional details involved in implementing this mechanism, see Bebchuk (2008b). The mechanism is similar to the one proposed by Bebchuk (2008a) for the government's purchase of troubled assets through funds using governments funds and run by private agents compensated with a cut of the profits generated by the funds.

government funds program does not have this cost of the direct-lending program, it does, like the direct-lending program, provide capital to firms in circumstances in which θ *is below* θ^*_{Direct}, which are circumstances in which even funding good operating firms is inefficient (because not enough of them are being funded). Hence, as τ_p approaches infinity, this mechanism is inferior to capital infusion to banks under lending commitment. In the latter program, the lending threshold converges to θ^*_{Direct}, and no capital is being wasted on failed projects below θ^*_{Direct}.[9]

4.5.2. Government Guarantees

Another mechanism the government may use to get the banking sector out of a credit freeze is to provide banks with guarantees. In this case, the government does not provide any capital upfront, but rather just commits to cover banks' losses in case the return on their loans falls below 1.

Suppose that the government guarantees a proportion γ of a bank's losses. In this case, a lending bank will receive the return $1+R$ when projects succeed and $\gamma < 1$ when projects fail. The following proposition characterizes the consequences of using this mechanism.

Proposition 9: *(a) Suppose that the government provides a guarantee covering a proportion γ (between 0 and 1) of banks' losses, that is, the government pays γ when a bank lends and real projects fail. Then, the threshold θ^*, below which a credit freeze occurs is given by:*

$$\theta^* = b - aK(1-l) + aK(1-l)\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta+\tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1-\gamma}{1+R-\gamma}\right)\right)\right), \qquad (11)$$

which is decreasing in γ.

(b) At the limit, as τ_p approaches infinity, the threshold (denoted as $\theta^*_{Guarantees}$) is given by:

$$\theta^*_{Guarantees} = b - aK(1-l) + aK(1-l)\frac{1-\gamma}{1+R-\gamma} \qquad (12)$$

[9] Note that we assume that the fund managers bear no risk. With some risk imposed on them, they would not always lend, and then it would be possible to reach the same efficiency as under the plan in Section 4.4.

Then, when the government provides full guarantees ($\gamma = 1$), all banks lend and projects fail only when they are inefficient ($\theta < b - aK(1-l)$). Otherwise, banks do not always lend, and projects sometimes fail even though they are efficient ($\theta > b - aK(1-l)$).

Remarks: (i) *The Nature of the Mechanism:* Government guarantees reduce the threshold below which crises occur because they make it more attractive for banks to lend. Considering the case where banks' signals are very precise (consider the limit case of part (b)), the attraction in this mechanism is that the government essentially does not need to provide any capital. Above $\theta^*_{Guarantees}$, where banks lend, the government's guarantee of providing capital if loans fail is sufficient to get the economy out of a credit freeze. Hence, banks lend and loans do not fail, so the government does not need to provide the capital. Below $\theta^*_{Guarantees}$, where banks do not lend, there are no loans made, and hence no loans that fail. This implies that the government's guarantees again do not lead to any capital being spent. In sum, this mechanism leads to an improvement in the threshold below which a credit freeze occurs without any upfront cost.

(ii) *Comparing this Mechanism with Previous Ones*: It is hard to provide a sharp comparison. Such comparison depends on the extent to which the guarantees can reduce the threshold θ^*. This, in turn, depends on the level of the guarantees γ. In principle it is tempting to conclude that the government should increase γ very close to 1,[10] but this is not so easy. Essentially, while the mechanism does not lead to actual costs, its validity depends on the credibility of the government in providing the guarantees. That is, banks have to believe that the government will indeed be able to pay back a proportion γ of the losses. Hence, there is a budget constraint in the background that has to be considered. The solution is for the government to increase γ (still below 1) until this budget constraint becomes binding. A reasonable case to consider is where the maximum guarantee provided by the government is equal to its available capital $Z = \alpha lK$. The maximum that the government will have to pay is when all banks lend and fail. This will cause a liability of $\gamma(1-l)K$, implying that γ cannot exceed $\frac{\alpha l}{(1-l)}$. The following proposition

[10] There is a problem in setting $\gamma = 1$, because at that level of guarantees, banks always lend and the government will have to bail them out sometime. Setting γ very close but still below 1 ensures that banks (who have infinitesimally precise signals) never lend when projects fail.

compares the government guarantees mechanism with the infusion of capital to banks under lending commitment for this level of guarantees (assuming that $\frac{\alpha l}{(1-l)} < 1$, i.e., that the government's available capital is smaller than that in the banking sector). It turns out that the guarantees regime is not as effective in reducing the probability of a credit freeze, and thus generates an overall inferior outcome.

Proposition 10: *Suppose that the government provides a guarantee* $\gamma = \frac{\alpha l}{(1-l)} < 1$*, and that* τ_p *approaches infinity. Then the threshold under the guarantees regime* $\theta^*_{Guarantees}$ *is higher than* θ^*_{Direct}*. As a result, capital infusion to the banking system under lending commitment generates a more efficient outcome than the guarantees regime.*

5. Concluding Remarks

This paper has developed a model of credit freezes that are inefficient but arise from the rational and self-fulfilling expectations of financial institutions. In this equilibrium, banks would be collectively better off if they were all willing to extend loans to a set of operating firms, but each of them avoids doing so out of self-fulfilling expectations that others will do as well. In such circumstances, efficiency will be served by getting the economy out of the inefficient credit freeze equilibrium, and the developed model has been seen to be useful for studying and assessing government policies that can be considered for this purpose.

Our analysis has shown that infusion of capital into the financial sector might but also might not produce a credit thaw. Even with ample capital, banks will not extend loans to operating firms when they believe that their projects, even though worthy in an environment in which other such firms obtain financing, will fail in an environment in which credit to other firms is frozen. If such circumstances arise, the government will have to look beyond capital enhancement to get the economy out of the credit freeze.

An alternative or supplemental approach that may be considered would involve the government's getting more directly involved in lending to operating firms. This, however, has a cost if the government cannot identify good borrowers like banks do. We discuss when this approach dominates capital infusion to banks. Finally, we design a mechanism that combines the benefits of both approaches, where the government infuses capital to banks under the commitment that they lend this capital, as well as their own capital, to firms.

Our work has implications for the current economic crisis. With government intervention producing a substantial increase in the financial system's capital, some observers suggest that lack of expansion in the credit extended to operating firms will imply that the current economic environment has made such expansion no longer efficient. Our analysis indicates that this inference cannot be made. While banks' failure to extend additional credit may be efficient, it may also be an inefficient outcome due to coordination failure. Our paper provides a framework for examining this possibility and potential government responses to it.

Appendix

Proof of Proposition 1: The proof follows Morris and Shin (2004b). The arguments in their proof (which we don't repeat here, for brevity) establish that there can only be a threshold equilibrium, where banks lend if and only if their signal is above some common x^*. Given this result, we now characterize the threshold equilibrium and show that it is unique.

Given x^*, there is a unique threshold fundamental θ^*, at which investment projects are on the margin between failure and success. This is given by:

$$\theta^* = b - aK\left(1 - \Phi\left(\sqrt{\tau_p}(x^* - \theta^*)\right)\right)$$

Here, $\Phi\left(\sqrt{\tau_p}(x^* - \theta^*)\right)$ is the proportion of banks receiving a signal below x^* and withdrawing from lending when the fundamental is exactly θ^*.

This gives us the first equation for the two unknowns x^* and θ^*. The second equation comes from the fact that at the threshold signal x^* a bank has to be indifferent between lending to firms and investing in the risk-free asset. When bank i observes signal x_i, his posterior distribution of θ is normal with mean $\frac{\tau_\theta y + \tau_p x_i}{\tau_\theta + \tau_p}$ and precision $\tau_\theta + \tau_p$. He knows that lending to firms yields $(1 + R)$ if and only if the fundamental is above θ^*, while not lending yields $(1 + r)$ with certainty. The indifference condition is then given by:

$$\left(1 - \Phi\left(\sqrt{\tau_\theta + \tau_p}\left(\theta^* - \frac{\tau_\theta y + \tau_p x^*}{\tau_\theta + \tau_p}\right)\right)\right)(1 + R) = 1 + \mathrm{r}$$

Which can be developed as follows:

$$\theta^* - \frac{\tau_\theta y + \tau_p x^*}{\tau_\theta + \tau_p} = \frac{\Phi^{-1}\left(1 - \frac{1 + \mathrm{r}}{1 + R}\right)}{\sqrt{\tau_\theta + \tau_p}}$$

Leading to:

$$\theta^* - x^* = \frac{-\tau_\theta(\theta^* - y)}{\tau_p} + \frac{\sqrt{\tau_\theta + \tau_p}\Phi^{-1}\left(1 - \frac{1 + \mathrm{r}}{1 + R}\right)}{\tau_p}$$

Plugging this in the first equation, we get:

$$\theta^* = b - aK\left(1 - \Phi\left(\sqrt{\tau_p}\left(\frac{\tau_\theta(\theta^* - y)}{\tau_p} - \frac{\sqrt{\tau_\theta + \tau_p}\Phi^{-1}\left(1 - \frac{1+\mathrm{r}}{1+R}\right)}{\tau_p}\right)\right)\right)$$

Which yields the equation in the proposition statement:

$$\theta^* = b - aK + aK\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta + \tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1+\mathrm{r}}{1+R}\right)\right)\right)$$

The left-hand side is the 45-degree line with respect to θ^*, and the right-hand side is increasing in θ^*, and is bounded between $b - aK$ and b. A unique solution for θ^* is guaranteed when the right-hand side has a slope of less than 1 everywhere. The slope of the right-hand side is given by $aK\phi(\cdot)\frac{\tau_\theta}{\sqrt{\tau_p}}$, where $\phi(\cdot)$ is the density of the standard normal evaluated at the appropriate point. Since $\phi(\cdot) \leq \frac{1}{\sqrt{2\pi}}$, a sufficient condition for a unique solution is $\frac{\tau_\theta}{\sqrt{\tau_p}} \leq \frac{\sqrt{2\pi}}{aK}$. **QED**. ■

Proof of Proposition 2: Proving the first part of the proposition is straightforward given the proof of Proposition 1. The proof just replaces K with $K(1-l)$ to reflect the fact that when a proportion *n* of the banks lend, only $nK(1-l)$ capital makes its way to operating firms. Note that the condition for uniqueness is now $\frac{\tau_\theta}{\sqrt{\tau_p}} \leq \frac{\sqrt{2\pi}}{aK(1-l)}$, which always holds when the condition in Proposition 1 holds.

The second part is proved with the implicit function theorem. Denote

$$F(\theta^*, l) = \theta^* - b + aK(1-l) -$$
$$aK(1-l)\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta + \tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1+\mathrm{r}}{1+R}\right)\right)\right) = 0$$

Then,

$$\frac{d\theta^*}{dl} = -\frac{dF(\theta^*, l)/_{dl}}{dF(\theta^*, l)/_{d\theta^*}}$$

We know that

$$\frac{dF(\theta^*, l)}{dl} = -aK\left(1 - \Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta + \tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1 + \mathrm{r}}{1 + R}\right)\right)\right)\right) \leq 0$$

$$\frac{dF(\theta^*, l)}{d\theta^*} = 1 - aK(1 - l)\frac{\tau_\theta}{\sqrt{\tau_p}}\phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta + \tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1 + \mathrm{r}}{1 + R}\right)\right)\right)$$

$$\geq 1 - aK(1 - l)\frac{\tau_\theta}{\sqrt{\tau_p}}\frac{1}{\sqrt{2\pi}} \geq 0$$

It follows that $\frac{d\theta^*}{dl} \geq 0$. **QED**. ■

Proof of Proposition 3: Proving the first part of the proposition is again done using the implicit function theorem. Denote:

$$F(\theta^*, r) = \theta^* - b + aK(1 - l) -$$

$$aK(1 - l)\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta + \tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1 + \mathrm{r}}{1 + R}\right)\right)\right) = 0$$

Then,

$$\frac{d\theta^*}{dr} = -\frac{dF(\theta^*, r)/dr}{dF(\theta^*, r)/d\theta^*}$$

Given that $\frac{dF(\theta^*,r)}{dr} \leq 0$ and $\frac{dF(\theta^*,r)}{d\theta^*} \geq 0$, it follows that $\frac{d\theta^*}{dr} \geq 0$.

To see why the second part holds, note that, given the capital available to banks $K(1 - l)$, not lending to operating firms is efficient only when the fundamental θ is below $b - aK(1 - l)$. Since $\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta+\tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1+\mathrm{r}}{1+R}\right)\right)\right) > 0$ (unless y approaches infinity), $\theta^* > b - aK(1 - l)$. Hence, there is a range of fundamentals for which banks do not lend and projects fail, even though this is inefficient. **QED**. ■

Proof of Proposition 4: The proof is analogous to the proof of Proposition 3, and thus omitted. **QED** ■

Proof of Proposition 5: Equation (6) is based on the same principles behind the construction of equilibrium in Propositions 1 and 2. The only thing to note in equation (6) is that all the government's capital is lent and generates the positive externality. Hence, investment projects fail when the proportion n of banks that decide to lend is below $\frac{b-\theta-a\alpha lK}{a(1-l)K}$. Having established equation (6) and comparing it with (5) (using the implicit function theorem, as in Propositions 2 and 3) reveals that $\theta^*_{Bank} > \theta^*_{Direct}$. **QED** ■

Proof of Proposition 6: The overall wealth in the economy under injection of capital to the banking system is given by $(1-(1-\alpha)l)K$ when the economy is in a credit freeze, and by $(1-(1-\alpha)l)K(1+R)$ when the economy is in a credit thaw.

The overall wealth in the economy under direct lending to operating firms is given by $(1-l)K$ when the economy is in a credit freeze, and by $\big(1-l+\alpha l(1-\beta)\big)K(1+R)$ when the economy is in a credit thaw. Note that in a credit thaw, only $(1-\beta)$ of the projects financed by the government succeed, as the government cannot tell the difference between good firms and bad firms. In a credit freeze, all the projects financed by the government fail, as even the good firms cannot succeed given that too many of them do not receive financing (if this was not the case, then banks would lend, and there would not be a credit freeze).

Based on these results, we now prove the different parts of the proposition.

(a) Now, when the fundamental θ is below θ^*_{Direct}, we know that there is a credit freeze under both regimes. Then, since $(1-(1-\alpha)l)K > (1-l)K$, the wealth in the economy is higher under infusion of capital to the banking system than under direct lending to firms.

When the fundamental θ is above θ^*_{Bank}, there is a credit thaw under both regimes. Then, since $(1-(1-\alpha)l)K(1+R) > \big(1-l+\alpha l(1-\beta)\big)K(1+R)$, the wealth in the economy is again higher under capital injection to banks than under direct lending to operating firms.

(b) When the fundamental θ is between θ^*_{Direct} and θ^*_{Bank}, the economy is in a credit thaw under the regime of direct lending and in a credit freeze under the regime of injection of capital to banks. Then, there is no obvious ranking between the levels of wealth in the two regimes: Capital injection to banks yields $(1-(1-\alpha)l)K$ and direct lending yields $(1-l+\alpha l(1-\beta))K(1+R)$. Overall, a high enough β and/or a small enough R makes capital injection better than direct lending.

(c) For the choice of regime, the government should consider all possible realizations of θ, weighted by their prior probabilities, and the difference in wealth they generate between the two policy measures. Based on the results above, the expected difference between wealth under capital injection and wealth under direct lending can then be expressed as:

$$\alpha lK\Phi\left(\frac{\theta^*_{Direct}-y}{\sigma_\theta}\right) + \left(\alpha\beta l-\left(1-l+\alpha l(1-\beta)\right)R\right)K\left[\Phi\left(\frac{\theta^*_{Bank}-y}{\sigma_\theta}\right)-\Phi\left(\frac{\theta^*_{Direct}-y}{\sigma_\theta}\right)\right] + \alpha\beta lK(1+R)\left[1-\Phi\left(\frac{\theta^*_{Bank}-y}{\sigma_\theta}\right)\right]$$

The statement in (c) then follow directly based on (a) and (b). **QED**. ■

Proof of Proposition 7: (a) Using the same logic as in the proof of Proposition 1, banks lend if and only if their signal is above some common x^*, and investment projects fail if and only if the fundamental is below some threshold θ^*. This threshold is given by:

$$\theta^* = b - aK(1-(1-\alpha)l)\left(1-\Phi\left(\sqrt{\tau_p}(x^*-\theta^*)\right)\right).$$

This gives us the first equation for the two unknowns x^* and θ^*. The second equation comes from the fact that at the threshold signal x^* a bank has to be indifferent between lending to firms and investing in the risk-free asset:

$$\left(1-\Phi\left(\sqrt{\tau_\theta+\tau_p}\left(\theta^*-\frac{\tau_\theta y+\tau_p x^*}{\tau_\theta+\tau_p}\right)\right)\right)(1+R)(1-(1-\alpha)l) = 1-l,$$

which can be developed as follows:

$$\theta^* - \frac{\tau_\theta y+\tau_p x^*}{\tau_\theta+\tau_p} = \frac{\Phi^{-1}\left(1-\frac{1-l}{(1+R)(1-(1-\alpha)l)}\right)}{\sqrt{\tau_\theta+\tau_p}}$$

Leading to:

$$\theta^* - x^* = \frac{-\tau_\theta(\theta^* - y)}{\tau_p} + \frac{\sqrt{\tau_\theta + \tau_p}\Phi^{-1}\left(1 - \frac{1-l}{(1+R)(1-(1-\alpha)l)}\right)}{\tau_p}.$$

Plugging this in the first equation, we get:

$$\theta^* = b - aK(1-(1-\alpha)l)\left(1 - \Phi\left(\sqrt{\tau_p}\left(\frac{\tau_\theta(\theta^* - y)}{\tau_p} - \frac{\sqrt{\tau_\theta + \tau_p}\Phi^{-1}\left(1 - \frac{1-l}{(1+R)(1-(1-\alpha)l)}\right)}{\tau_p}\right)\right)\right),$$

which, after re-arrangement, yields the equation in the proposition statement:

$$\theta^* = b - aK(1-(1-\alpha)l)$$
$$+aK(1-(1-\alpha)l)\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta + \tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1-l}{(1+R)(1-(1-\alpha)l)}\right)\right)\right).$$

(b) As τ_p approaches infinity, θ^* above converges to

$$b - aK(1-(1-\alpha)l) + aK(1-(1-\alpha)l)\left(\frac{1-l}{(1+R)(1-(1-\alpha)l)}\right)$$
$$= b - aK(1-(1-\alpha)l) + aK\left(\frac{1-l}{1+R}\right).$$

This is exactly θ^*_{Direct} defined in equation (8). Since lending by banks occurs as frequently as when the government uses direct lending to operating firms, and since banks only lend to good firms and to successful projects. The outcome here is more efficient than under direct lending to operating firms and under capital infusion to the banking sector. **QED**. ■

Proof of Proposition 8: The proof is trivial and sketched in the text after the proposition. **QED**. ■

Proof of Proposition 9: The proof follows similar steps to those in Propositions 1, 2, and 3. **QED**. ■

Proof of Proposition 10: We need to show that:

$$b - aK(1-(1-\alpha)l) + aK(1-l)\frac{1}{1+R} < b - aK(1-l) + aK(1-l)\frac{1-\gamma}{1+R-\gamma}$$

This can be developed as follows:

$$-(1-(1-\alpha)l) + (1-l)\frac{1}{1+R} < -(1-l) + (1-l)\frac{1-\gamma}{1+R-\gamma}$$

$$-\alpha l + (1-l)\frac{1}{1+R} < (1-l)\frac{1-\gamma}{1+R-\gamma}$$

$$(1-l)\left(\frac{1+R-\gamma-(1+R)(1-\gamma)}{(1+R)(1+R-\gamma)}\right) < \alpha l$$

$$\frac{\gamma \mathrm{R}}{(1+R)(1+R-\gamma)} < \frac{\alpha l}{(1-l)}$$

Plugging in $\gamma = \frac{\alpha l}{(1-l)}$, we get:

$$\mathrm{R} < (1+R)(1+R-\gamma)$$

$$0 < (1+R)(1-\gamma) + R^2$$

which is always true. **QED**. ■

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