# The Role of Second Menu Costs -Two-Product Firms and the Real Effects of Nominal Shocks* 

Preliminary - please do not circulate!

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#### Abstract

Midrigan (2008) shows that if two-product firms have increasing returns in their re-pricing technology, then nominal shocks can have real effects - a result contradicting the findings of Golosov-Lucas (2007) in a model of single-product firms. This paper builds a standard menu cost model with two-product firms where the cost of changing the second price (the "second menu cost") can be anything between zero (as in Midrigan) and the first menu cost (as in Golosov-Lucas), and shows that under reasonably calibrated leptokurtic shocks the real effect of nominal shocks is decreasing fast with the increasing relative size of the second menu cost.

To identify the size of the second menu cost, we use the distributional characteristics (frequency, size, kurtosis) of micro-level price changes during the natural experiment provided by a series of large value-added tax increases in Hungary. We find that without further channels, the real effects of monetary policy in these models are still limited.


Keywords: Menu Cost, Value-Added Tax, Store-Level Pricing, Real Effects of Nominal Shocks

JEL Classification: E30

## 1 Introduction

This paper uses store-level consumer price data collected in a period of frequent nominal (value-added tax) shocks in Hungary to investigate the real effects of monetary shocks under menu costs. Despite the large number of studies, there is still no consensus on this issue. A recent paper by Golosov-Lucas

[^0](2007) - calibrated to match micro data features in the US - argues that in an environment where firms have to pay menu cost to change their prices and are hit by idiosyncratic productivity shocks, monetary shocks perfectly pass through to the aggregate price level and hence do not affect real output. The reason, shortly, is selection: firms that adjust their prices after a monetary shock are exactly those whose desired adjustment is the largest, and this makes up for the non-adjustment of other stores.

On the other hand, Midrigan (2008) shows that if we calibrate the model to match the entire distribution of store-level price change sizes, nominal shocks do have real effects. Midrigan discusses two empirical regularities that the Golosov-Lucas model is unable to match: the large proportion of small price changes, and the leptokurtic distribution of price change sizes. In the GolosovLucas model, homogenous menu costs prevent stores from changing their prices by small amounts: the additional profit from doing so would not exceed the marginal cost of it (i.e. the menu cost). The Golosov-Lucas model is also unable to match the leptokurtic shape of the price change size distribution.

In order to make the model well equipped to account for these two additional empirical observations, Midrigan modifies it in two respects: first he observes that stores generally sell more than one product, and assumes that there is increasing returns in the price change technology within these multi-product stores. (This assumption is supported by the empirical observation of withinstore synchronization of price changes, see for example Levy et al (1997) and Lach-Tsiddon (2007).) He then assumes that once the price of one of the goods changes, the store can change any of its other goods' prices for free. Second, he assumes that the firms' idiosyncratic productivity shocks are leptokurtic, which then leads to the leptokurtic distribution of price change sizes. In such an environment Midrigan shows that monetary shocks do have effect on real output.

In this paper we develop a model which incorporates both the Golosov-Lucas and the Midrigan models as special cases. We do this by assuming multi-product firms, in which the cost of changing the price of the second good (given that the price of the other good has changed) can be anything between zero (as in Midrigan) and the first menu cost (as in Golosov-Lucas). As we demonstrate, a positive second menu cost does not necessarily eliminate small price changes, and therefore the model will still be able to match the empirical price change size distributions. We then investigate how the magnitude of the second menu cost affects the real effect of monetary shocks, i.e. the different results of the Golosov-Lucas and Midrigan models.

In the empirical part, we use observations on the price change size distributions from subsequent episodes of Value-Added Tax (VAT) changes in Hungary to identify the magnitude of the second menu cost. As VAT-changes induce unprecedentedly large fraction of stores to adjust their prices (in Hungary, 50-67\% of stores adjusted in the month of the VAT-changes, see Gabriel-Reiff (2007)), we think that these episodes are extremely well suited to address questions related to the distribution of price change sizes. Therefore we calibrate our model parameters - i.e. the first and second menu costs and the distribution of id-
iosyncratic productivity shocks - to match the (kurtosis of the) price change size distributions at months of VAT-changes. Based on the calibrated parameters, we find that monetary shocks that have similar size to the Hungarian VAT-changes have relatively small real output effects.

The paper is organized as follows. Section 2 presents the model. Section 3 discusses how the second menu cost affects the real effects of monetary shocks. Section 4 presents the calibration results, and Section 5 concludes.

## 2 The Model

Our starting point is a standard two-product menu cost model with idiosyncratic productivity shocks, as in Midrigan (2008). In this model each firm produces two different products, and firms have to pay fixed costs to change the price of the products. However, the price change technology exhibits increasing returns to scale: once the price of any products is changed, it is possible to change the price of the other product for free.

We generalize this model by assuming that the cost of changing the price of the second product (the "second menu cost") is anywhere between zero and the cost of changing the price of the first product (the "first menu cost"). Therefore the Golosov-Lucas and the Midrigan models become special cases of our model: in the former, the second menu cost is equal to the first, while in the latter it is zero.

### 2.1 Consumers

The representative consumer consumes a Dixit-Stiglitz aggregate $C$ of individual goods $C^{1}(i)$ and $C^{2}(i)$, holds money balances $M / P$, and supplies labor $L$ in such a way that maximizes the present value of its expected utility. (Our notational convention is that first and second products $(j=1,2)$ are in superscripts, time $(t)$ is in subscripts, and firm identifiers $(i=1, \ldots, n)$ are in brackets.) Then the representative consumer's problem is

$$
\begin{equation*}
\max _{\left\{C_{t}^{1}(i), C_{t}^{2}(i), L_{t}, M_{t}\right\}} E \sum_{t=0}^{\infty} \beta^{t}\left(\log \left[C_{t} \cdot\left(\frac{M_{t}}{P_{t}}\right)^{\nu}\right]-\frac{\mu}{1+\psi} L_{t}^{1+\psi}\right) \tag{1}
\end{equation*}
$$

where the aggregate consumption $C_{t}$ is determined by the CES-aggregator

$$
C_{t}=\left[\sum_{i=1}^{n} n^{-\frac{1}{\theta}}\left(\frac{1}{2} C_{t}^{1}(i)^{\frac{\gamma-1}{\gamma}}+\frac{1}{2} C_{t}^{2}(i)^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1} \frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}
$$

where $\gamma$ is the within-firm elasticity of substitution, $\theta$ is the between-firm elasticity of substitution, with $\gamma \geq \theta$. We assume that the number of firms $(n)$ is large, so that firm $i$ 's consumption goods $C^{1}(i)$ and $C^{2}(i)$ are only infinitesimal parts of the consumer's utility $C$.

The consumers budget constraint in each time period is

$$
\begin{equation*}
\sum_{i=1}^{n}\left(P_{t}^{1}(i) C_{t}^{1}(i)+P_{t}^{2}(i) C_{t}^{2}(i)\right)+\sum_{h_{t+1}} B_{t+1}\left(h_{t+1}\right)+M_{t+1}=R_{t} B_{t}+M_{t}+\tilde{w}_{t} L+\tilde{\Pi}_{t}+T_{t} \tag{2}
\end{equation*}
$$

where $P_{t}^{1}(i)$ and $P_{t}^{2}(i)$ are nominal prices of the products of firm $i, B_{t}$ is a nominal Arrow-security with state-dependent gross return $R_{t}, M_{t}$ is the nominal money balance, $\tilde{w}_{t}$ is the nominal wage, $\tilde{\Pi}_{t}$ is the nominal profits the representative consumer earns from ownership in firms, $T_{t}$ is a lump-sum transfer, and $h_{t+1}$ is the history of events realized up to time $t+1$ (history dependence is suppressed for the sake of notational simplicity).

The firm-level and the aggregate prices are the CES-averages of individual prices:

$$
P_{t}(i)=\left(\frac{1}{2} P_{t}^{1}(i)^{1-\gamma}+\frac{1}{2} P_{t}^{2}(i)^{1-\gamma}\right)^{\frac{1}{1-\gamma}}, \quad P_{t}=\left[\sum_{i=1}^{n} \frac{P_{t}(i)^{1-\theta}}{n}\right]^{\frac{1}{1-\theta}}
$$

implying that the aggregate expenditure is $P_{t} C_{t}$.
The solution of the representative consumer's problem is the consumer's demand function:

$$
\begin{equation*}
C_{t}^{j}(i)=\frac{1}{n}\left(\frac{P_{t}^{j}(i)}{P_{t}(i)}\right)^{-\gamma}\left(\frac{P_{t}(i)}{P_{t}}\right)^{-\theta} C_{t} \tag{3}
\end{equation*}
$$

The Euler-equation of the representative consumer implies that the stochastic discount factor $\frac{1}{R_{t+1}}$ is given by

$$
\begin{equation*}
\frac{1}{R_{t+1}}=\beta \frac{P_{t} C_{t}}{P_{t+1} C_{t+1}} \tag{4}
\end{equation*}
$$

the labor supply equation is

$$
\begin{equation*}
\mu L_{t}^{\psi} C_{t}=\frac{\tilde{w}_{t}}{P_{t}} \tag{5}
\end{equation*}
$$

and the money demand equation is

$$
\begin{equation*}
\frac{M_{t}}{P_{t}}=\nu C_{t} \frac{i_{t}+1}{i_{t}} \tag{6}
\end{equation*}
$$

where $i_{t}$ is the nominal interest rate.

### 2.2 Firms

Firms maximize the present value of their expected future profits, net of menu costs they have to pay whenever they adjust their prices. Their key decision is therefore whether to adjust prices or not. The menu costs are assumed to be proportional to the firms' revenues.

The net present value of future expected profits is

$$
\begin{equation*}
\max E \sum_{t=0}^{\infty} \frac{1}{\prod_{q=0}^{t} R_{q}} \tilde{\Pi}_{t}(i) \tag{7}
\end{equation*}
$$

where the periodic profit function is

$$
\begin{equation*}
\tilde{\Pi}_{t}(i)=\sum_{j=1}^{2}\left(P_{t}^{j}(i) Y_{t}^{j}(i)-\tilde{w}_{t} L_{t}^{j}(i)\right) \tag{8}
\end{equation*}
$$

In the profit function, revenues are determined by demand from the consumers, described in equation (3). The costs are derived from the firms' linear technology, which uses labor as a single input:

$$
\begin{equation*}
Y_{t}^{j}(i)=Z_{t} A_{t}^{j}(i) L_{t}^{j}(i) \tag{9}
\end{equation*}
$$

where $Z_{t}$ is an aggregate productivity shock, and $A_{t}^{j}(i)$ are idiosyncratic productivity shocks $(j=1,2)$. We assume that the growth rate of the aggregate productivity shock, $g_{Z}$ is constant, ${ }^{1}$ and the log of idiosyncratic shocks $A_{t}^{j}(i)$ follow independent $\operatorname{AR}(1)$ processes with zero-mean i.i.d. innovations $\varepsilon_{t}^{j}(i)(j=$ $1,2)$ :

$$
\begin{equation*}
\ln A_{t}^{j}(i)=\rho_{A} \ln A_{t-1}^{j}(i)+\varepsilon_{t}^{j}(i) \tag{10}
\end{equation*}
$$

where the standard deviation of the i.i.d. innovations is $\sigma_{\varepsilon}$.
The individual labor demands are therefore

$$
\begin{equation*}
L_{t}^{j}(i)=\frac{Y_{t}^{j}(i)}{Z_{t} A_{t}^{j}(i)} \tag{11}
\end{equation*}
$$

Substituting the representative consumer's demand function (equation (3)) and the firms labor demand function (equation (11)) to the periodic profit function, and using the goods market clearing condition $C_{t}^{j}(i)=Y_{t}^{j}(i)$, we have
$\tilde{\Pi}_{t}(i)=\sum_{j=1}^{2}\left[\left(\frac{P_{t}^{j}(i)}{P_{t}(i)}\right)^{(1-\gamma)}\left(\frac{P_{t}(i)}{P_{t}}\right)^{(1-\theta)} \frac{P_{t} Y_{t}}{n}-\tilde{w}_{t} \frac{Y_{t}}{n Z_{t}}\left(\frac{P_{t}^{j}(i)}{P_{t}(i)}\right)^{-\gamma}\left(\frac{P_{t}(i)}{P_{t}}\right)^{-\theta}\right]$.
We assume that the central bank follows a nominal income targeting rule by holding the growth rate of the nominal aggregate output $P_{t} Y_{t}$ constant. Then normalizing this profit function with the per-firm nominal GDP $\frac{P_{t} Y_{t}}{n}$, we obtain a stationary "normalized" profit function

$$
\Pi_{t}=\frac{\tilde{\Pi}_{t} n}{P_{t} Y_{t}}
$$

[^1]Let us denote the product-level relative prices $\frac{P_{t}^{j}(i)}{P_{t}}$ by $p_{t}^{j}(i)$, the firm-level relative prices $\frac{P_{t}(i)}{P_{t}}$ by $p_{t}(i)$, the normalized wage $\frac{\tilde{w}_{t}}{P_{t} Y_{t}}$ by $w_{t}$, and the sectoral cost factor $w(t) \frac{Y_{t}}{n Z_{t}}$ by $\zeta_{t}$. The normalized profit function becomes

$$
\begin{aligned}
& \Pi_{t}\left[p_{t}^{1}(i), p_{t}^{2}(i), A_{t}^{1}(i), A_{t}^{2}(i), \zeta_{t}\right] \\
= & p_{t}(i)^{(\gamma-\theta)} \sum_{j=1}^{2}\left(p_{t}^{j}(i)^{(1-\gamma)}-\zeta_{t} p_{t}^{j}(i)^{-\gamma} A_{t}^{j}(i)^{-1}\right)-\phi_{1}-\phi_{2},
\end{aligned}
$$

where the firms has to pay the first menu cost $\phi_{1}$ if at least one price is changing, and the second menu cost $\phi_{2}$ if both prices change.

For each firm $i,\left[A_{t}^{1}(i), A_{t}^{2}(i)\right]$ and $\left[p_{t-1}^{1}(i), p_{t-1}^{2}(i), \zeta_{t}, \pi_{t}, \Gamma_{t}\right]$ are the exogenous and endogenous state variables, respectively, with $\pi_{t}$ being the inflation rate, and $\Gamma_{t}$ being the distribution of relative prices. For notational convenience, we express the set of state variables as $\left[p_{t-1}^{1}(i), p_{t-1}^{2}(i), \Omega_{t}\right]$, where $\Omega_{t}$ is the vector of state variables other than the two relative prices.

Given these state variables, the value of the firm is determined by the maximum it can achieve by changing both prices $\left(V^{C C}\right)$, by changing the price of good 1 only $\left(V^{C N}\right)$, by changing the price of good 2 only $\left(V^{N C}\right)$, and by not changing any of the prices $\left(C^{N N}\right)$ :
$V\left(p_{-1}, \Omega\right)=\max _{\{C C, C N, N C, N N\}}\left(V^{C C}\left(p_{-1}, \Omega\right), V^{C N}\left(p_{-1}, \Omega\right), V^{N C}\left(p_{-1}, \Omega\right), V^{N N}\left(p_{-1}, \Omega\right)\right)$.
The value functions on the right-hand side of this equation are given by

$$
\begin{align*}
V^{C C}\left(p_{-1}^{1}, p_{-1}^{2}, \Omega\right) & =\max _{\left\{p^{1}, p^{2}\right\}}\left[\Pi\left(p^{1}, p^{2}, A^{1}, A^{2}, \zeta\right)+\beta E V\left(p^{1}, p^{2}, \Omega\right)\right],  \tag{13}\\
V^{C N}\left(p_{-1}^{1}, p_{-1}^{2}, \Omega\right) & \left.=\max _{p^{1}}\left[\Pi\left(p^{1}, \frac{p_{-1}^{2}}{1+\pi}, A^{1}, A^{2}, \zeta\right)+\beta E V\left(p^{1}, \frac{p_{-1}^{2}}{1+\pi}, \Omega\right)\right] 14\right) \\
V^{N C}\left(p_{-1}^{1}, p_{-1}^{2}, \Omega\right) & \left.=\max _{p^{2}}\left[\Pi\left(\frac{p_{-1}^{1}}{1+\pi}, p^{2}, A^{1}, A^{2}, \zeta\right)+\beta E V\left(\frac{p_{-1}^{1}}{1+\pi}, p^{2}, \Omega\right)_{1}\right] 5 .\right) \\
V^{N C}\left(p_{-1}^{1}, p_{-1}^{2}, \Omega\right) & =\Pi\left(\frac{p_{-1}^{1}}{1+\pi}, \frac{p_{-1}^{2}}{1+\pi}, A^{1}, A^{2}, \zeta\right)+\beta E V\left(\frac{p_{-1}^{1}}{1+\pi}, \frac{p_{-1}^{2}}{1+\pi},(\Omega 1 \phi)\right.
\end{align*}
$$

### 2.3 Equilibrium

The equilibrium is a collection of allocations $\left\{C_{t}^{j}(i), P_{t}^{j}(i), M_{t}, L_{t}^{j}(i), B_{t}, \tilde{w}_{t}, Y_{t}^{j}(i)\right\}$ such that:

1. The representative consumer chooses $C_{t}^{j}(i), M_{t}, L_{t}$ to optimize (1) under its budget constraint (2), taking prices $P_{t}^{J}(i)$, interest rates $R_{t}$ and wages $\tilde{w}_{t}$ as given,
2. The firms set prices $P_{t}^{j}(i)$ to maximize the value function (12) given the current exogenous state variables, the law of motion of the idiosyncratic productivity shocks, and having correct beliefs about the endogenous state variables,
3. The central bank sets $i_{t}$ and $M_{t}$ to keep the growth rate of the nominal output, $g_{P Y}$ constant,
4. Goods markets clear: $C_{t}^{j}(i)=Y_{t}^{j}(i)$,
5. Assets are in zero net supply: $B_{t}=0$,
6. Labor markets clear: the representative consumer's labor supply in equation (5) equals the sum of aggregate labor demands in (11).

### 2.4 Computing the steady-state

In our model, we assume that the central bank keeps a constant nominal output growth $g_{P Y}$, and that the aggregate productivity $Z_{t}$ is growing at a constant rate $g_{Z}$. These two assumptions imply that the steady-state inflation is $\pi=g_{P Y}-g_{Z}$.

The other two endogenous state variables, the aggregate cost factor $(\zeta)$ and the steady-state distribution of relative prices $(\Gamma)$ are determined simultaneously. To solve for these, we apply an iterative procedure:

1. We choose an arbitrary value for the aggregate cost factor $\zeta$. In the first step, we start from the flexible price equilibrium value of $\zeta$.
2. We solve the value function (12) under the steady-state inflation rate $g_{P Y}-g_{Z}$ and the chosen $\zeta$, with value function iteration.
3. With the resulting value and policy functions, we simulate an artificial data set and obtain the corresponding relative price distribution.
4. In the artificial data set, we compute the resulting aggregate cost factor $\zeta$ (nothing ensures that this $\zeta$ is the same that we have chosen in the first step).
5. If the resulting $\zeta$ is different from the initial one, then we start again this procedure. We do this until the resulting $\zeta$ is equal to the initial value we have chosen.

## 3 The effect of the second menu cost on the real effects of nominal shocks

In this section we investigate the real effect of nominal shocks in the model. For the sake of simplicity, we examine the inflation effect of a transitory $1 \%$ nominal shock.

### 3.1 Modelling the real effect of nominal shocks

We model the nominal shock by a one-time, $1 \%$-point increase in the $g_{P Y}$ growth rate of the nominal output. To compute the inflation (and output) effect of this nominal shock, we use the shooting algorithm, which is an iterative procedure in the resulting inflation path. The time horizon $(T)$ in which we study the inflation effect is long enough to reach again the steady-state. (Therefore, our notational convention is that the nominal shock hits at $t=1$, and its effect vanishes after $t=T$.)

The shooting algorithm consists of the following steps:

1. We choose an initial inflation path $\left\{\pi_{1}, \pi_{2}, \ldots, \pi_{T}\right\}$ as the provisional inflation effect of the nominal shock. Again, we start with the flexible-price case, i.e. when there is immediate and full pass-through.
2. We calculate the resulting path of the aggregate cost factor: $\left\{\zeta_{1}, \zeta_{2}, \ldots, \zeta_{T}\right\}$. As we defined $\zeta=\frac{Y_{t}}{n Z_{t}}$, it follows that $g_{\zeta}=g_{Y}-g_{Z}=g_{P Y}-\pi-g_{Z}$, which is easy to compute given our assumptions and the provisional inflation path.
3. As we reach the steady state after $T$ periods, the $(T+1)$-st period's value function will be the steady-state value function: $V_{T+1}\left(p^{1}, p^{2}, \Omega\right)=V^{S S}$. Knowing this, we can use equations (12)-(16) to compute the $T$-th period's value function, and with backward induction, we can also calculate all the value functions back to the 1 -st period, i.e. when the nominal shock hit.
4. With all the value and policy functions, we can simulate an artificial data set and calculate the resulting inflation path.
5. If the resulting inflation path is different from the one we started from, we update the inflation path and start over the iteration.

### 3.2 A calibration exercise

Golosov-Lucas (2007) and Midrigan (2008) have different assumptions about the magnitude of the menu cost of changing the second price. The former assumes that this second menu cost is the same as the first, while in the latter it is zero.

In this subsection we make a series of calibrations of the theoretical model, where the second menu cost increases gradually. Initially, we do the calibration when $\phi_{2} / \phi_{1}=0$ (as in Midrigan), and at subsequent calibrations we continuously increase this ratio until we reach $\phi_{2} / \phi_{1}=1$ (as in Golosov-Lucas). In order to be able to compare the real effects of nominal shocks across calibrations, we will keep the monthly inflation rates, and the average frequency and absolute size of price changes fixed throughout the calibrations, by changing parameters $\phi_{1}$ and $\sigma_{\varepsilon}$. Based on the results reported in Midrigan (2008) and Golosov-Lucas (2007), we expect the real effect of nominal shocks to be decreasing in the $\phi_{2} / \phi_{1}$ ratio.

According to Table 1, the other model parameters (other than the menu costs and shock standard deviations) and calibration targets are similar in the two papers: they both calibrate the model to a monthly inflation rate of approximately $0.2 \%$, a monthly price change frequency of $22-24 \%$, and to an average absolute size of approximately $12 \% .^{2}$ Out of the deep parameters of the model, the discount factors $(\beta)$ and the idiosyncratic shock persistences $\left(\rho_{A}\right)$ are almost identical, the only difference being in the elasticity parameters $(\theta, \gamma)$.

Table 1: Model parameters and some calibrated moments in Golosov-Lucas (2007) and Midrigan (2008)

| Model parameters | Midrigan | Golosov-Lucas | Our choice |
| :--- | :---: | :---: | :---: |
| Between-firm elasticity $(\theta)$ | 3 | 7 | 5 |
| Within-firm elasticity $(\gamma)$ | 11.5 | 7 | 11 |
| Discount factor $(\beta)$ | 0.997 | 0.9966 | 0.9966 |
| First menu cost $\left(\phi_{1}\right)$ | 0.0111 | 0.0025 | to calibrate |
| Second menu cost $\left(\phi_{2}\right)$ | 0 | 0.0025 | to calibrate |
| Idios. shock stdev. $\left(\sigma_{\varepsilon}\right)$ | 0.065 | 0.105 | to calibrate |
| Idios. shock persistence $\left(\rho_{A}\right)$ | 0.473 | 0.45 | 0.5 |
| Beta distr. 1st param. $\alpha_{1}$ | 0.046 | n.a. | 0.05 |
| Beta distr. 2nd param. $\left(\alpha_{2}\right)$ | 1.057 | n.a. | 1 |
| Calibrated values | Midrigan | Golosov-Lucas | Our choice |
| Monthly inflation $(\pi)$ | 0.002 | 0.0021 | 0.002 |
| Frequency of price change | 0.24 | 0.219 | 0.23 |
| Average size of price change | 0.12 | $0.095^{*}$ | 0.12 |

The fourth column of Table 1 shows our choices for the model calibrations under different $\phi_{2} / \phi_{1}$ ratios. We wanted to be as close to both the GolosovLucas and the Midrigan-calibrations as possible. Therefore we calibrate the model to a monthly inflation rate of $0.2 \%$, a monthly frequency of $23 \%$ and to an average absolute size of $12 \%$. Also, we have fixed some model parameters in advance to stay close to both papers' parametrization: the elasticity of substitution are $\theta=5$ and $\gamma=11$, the discount factor is $\beta=0.9966=0.96^{1 / 12}$, and the shock persistence is $\rho_{A}=0.5$. Regarding the distribution of the idiosyncratic shock innovations, we follow Midrigan and assume beta distribution with parameters $\alpha_{1}=0.05$ and $\alpha_{2}=1$. The two menu cost parameters $\left(\phi_{1}, \phi_{2}\right)$ and the idiosyncratic shock standard deviation $\left(\sigma_{\varepsilon}\right)$ are calibrated to match the average frequency and size figures just discussed.

Table 2 contains the inflation effect of a $1 \%$-point nominal shocks for different ratios of the second and the first menu cost. We find that if the second menu cost is zero, then we have substantial real effect of nominal shocks in the shortrun. This is consistent with the findings of Midrigan. However, if we increase the magnitude of the second menu cost, the real effect of the nominal shocks

[^2]quickly decreases: if $\phi_{2} / \phi_{1}>0.3$, then the inflation effect is approximately $80 \%$ and $90 \%$ after 1 and 2 months, respectively, implying that the real effect after 1 month is merely $20 \%$, and it is smaller than $10 \%$ after 2 months.

Table 2: Inflation effect of a 1\%-point nominal shock under different second menu costs

|  | Calibrated |  | Inflation effect after |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\phi_{2} / \phi_{1}$ | frequency | size | $\mathbf{1} \mathbf{m t h}$ | $\mathbf{2} \mathbf{~ m t h}$ | $\mathbf{3} \mathbf{\text { mth}}$ | $\mathbf{4} \mathbf{m t h}$ |
| $0(\mathrm{M})$ | 0.2287 | 0.1201 | $0.42 \%$ | $0.66 \%$ | $0.75 \%$ | $0.81 \%$ |
| 0.2 | 0.2311 | 0.1197 | $0.63 \%$ | $0.67 \%$ | $0.77 \%$ | $0.80 \%$ |
| 0.3 | 0.2323 | 0.1199 | $0.78 \%$ | $0.86 \%$ | $0.88 \%$ | $0.95 \%$ |
| 0.5 | 0.2300 | 0.1194 | $0.81 \%$ | $0.91 \%$ | $0.96 \%$ | $1.00 \%$ |
| $1(\mathrm{GL})$ | 0.2320 | 0.1198 | $0.77 \%$ | $0.86 \%$ | $0.92 \%$ | $0.99 \%$ |
| Targets | 0.2300 | 0.1200 |  |  |  |  |

Therefore we can conclude that the size of the second menu cost affects the real effect of the nominal shocks. This naturally leads to the question of whether we could identify the size of this second menu cost.

## 4 Calibration of the second menu cost

### 4.1 Simple facts from VAT-changes

To calibrate the model and the second menu cost parameter, we use store-level data from Hungary in 2002-2006, when the Hungarian authorities implemented a series of Value-Added Tax (VAT) changes: within a period of 32 months, the initial VAT-rates of $5 \%, 12 \%$ and $25 \%$ were changed three times.

1. In January 2004, the middle rate was increased to $15 \%$.
2. In January 2006, the top rate was cut to $20 \%$.
3. In September 2006, the middle rate was again increased to $20 \%$.

Gabriel-Reiff (2007) investigates the store-level responses for the various VAT-changes in detail. The main conclusions are the following:

1. The frequency of price changes increased dramatically. For example, in January 2004 and September 2006 the frequency of price changes of the affected items were $59.0 \%$ and $66.3 \%$, much higher than in any other months (fluctuating between 17-29\% in our sample).
2. The average absolute size of price changes decreased significantly. In January 2004 and September 2006 the average absolute sizes among the affected items were $9.6 \%$ and $10.8 \%$, clearly smaller than in any other months (the average absolute size in regular months is $13.3 \%$, the range being $11.2 \%-14.9 \%$ ).
3. The extra price changes (i.e. those which probably would not have been made in the absence of VAT-change) were highly concentrated around the exact size of the VAT-shock. For example, Figure 1 depicts the distribution of price changes (conditional on not being zero) in January 2003 and September 2006 for the products affected by the September 2006 VAT-increase of 5 percentage points. Most of the additional price changes in September 2006 (relative to January 2003) have a size of $3-7$ percent, i.e. close to the size of the shocks. This indicates that the desired price change of these price-changing stores would have been close to zero, if there were no VAT-change in this month.

Figure 1: Size distributions in January 2003 and September 2006


In what follows, we use empirical evidence from the September 2006 VATchange to calibrate the theoretical model. The reason is that this was the biggest VAT-shock affecting the highest number of products in our sample. We will consider evidence from other VAT-changes in future versions of the paper.

### 4.2 Calibration

## [PRELIMINARY]

In this subsection we calibrate the model to match the frequency and average absolute size of price changes in regular months, and also the frequency in a month with a $5 \%$ nominal shock. This is similar to what we have done in section 3 , the difference being that now we also calibrate to the frequency response after the nominal shock. Also, instead of the US figures now we calibrate to match the statistics calculated from Hungarian data: the frequency of price changes is $20.3 \%$, the average absolute size of price changes is $13.3 \%$, and the frequency of price changes when the shock hits is $66.3 \%$.

To be able to hit the extra statistics we are calibrating for, by changing the $\alpha_{1}$ parameter we also calibrate the shape of the idiosyncratic shock distribution. The intuition is the following: in the previous calibration, the Midrigan-model (i.e. when $\phi_{2} / \phi_{1}=0$ ) implied a very modest frequency response - from $23 \%$ to approximately $35 \%$ - for the $5 \%$ nominal shock. This is much smaller than what we see in the data. The reason is that in the previous calibration, the (first) menu cost is relatively high, which prevents most of the stores from adjusting even when a $5 \%$ nominal shock hits. Therefore in order to generate a much higher frequency response for the nominal shock, we have to decrease the (first) menu cost. But then the frequency of price changes in regular times would increase. One way to avoid this is changing the shape of the distribution of the idiosyncratic productivity innovations: if they are more concentrated around zero, the frequency of price changes will decrease. This can be achieved by decreasing the parameter $\alpha_{1}$ of the shock-generating beta-distribution.

Table 3 contains the results of this calibration. The first three columns indicate that we can hit the empirical values quite well. In the last three columns there are the calibrated model parameters. Note the calibrated shape parameter $\left(\alpha_{1}\right)$ : for small $\phi_{2} / \phi_{1}$ ratios, it is smaller than 0.05 in the previous section, indicating more leptokurtic idiosyncratic productivity innovations, which is necessary to compensate for the frequency-increasing smaller first menu cost. For higher $\phi_{2} / \phi_{1}$ ratios, the first menu cost is already small enough not to prevent the big frequency response for the nominal shock, and we do not have so leptokurtic shock innovations to achieve the same frequency of price changes in regular months.

Table 3: Calibrated model parameters and moments to match the frequency and size effect of nominal shocks

|  | Moments |  |  |  | Calibrated |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Matched |  |  | Non-matched |  | parameters |  |  |
| $\phi_{2} / \phi_{1}$ | $f r^{N T}$ | $d p^{N T}$ | $f r^{T}$ | $k u r t^{N T}$ | $k u r t^{T}$ | $\phi_{1}$ | $\sigma_{\varepsilon}$ | $\alpha_{1}$ |
| 0.05 | 0.202 | 0.133 | 0.681 | 3.570 | 11.390 | 0.057 | 0.078 | 0.046 |
| 0.1 | 0.204 | 0.132 | 0.679 | 3.193 | 10.125 | 0.053 | 0.075 | 0.055 |
| 0.17 | 0.203 | 0.133 | 0.668 | 2.858 | 8.999 | 0.051 | 0.073 | 0.065 |
| 0.2 | 0.203 | 0.134 | 0.651 | 2.685 | 8.296 | 0.051 | 0.073 | 0.070 |
| 0.3 | 0.202 | 0.133 | 0.656 | 2.471 | 8.178 | 0.046 | 0.071 | 0.078 |
| Targets | 0.203 | 0.133 | 0.663 | 3.970 | 8.770 |  |  |  |

Using the different model calibrations in Table 3, in Figures 2-5 in the Appendix we depict the same size distribution histograms in the model as we saw in Figure 1 for the data. Several features emerge: first, the size distribution in the VAT-months (right panels) are quite similar to each other, although these distributions become less concentrated as the $\phi_{2} / \phi_{1}$ ratio increases. Second, the regular months' size distributions (left panels) differ considerably "in the middle". This is because for higher $\phi_{2} / \phi_{1}$ ratios, the very small price changes become much less frequent. However, for small values of $\phi_{2} / \phi_{1}$, they are very
much concentrated around the small price changes, something we do not observe empirically. So based on simple visual inspection, we can generate the empirical size distribution when the $\phi_{2} / \phi_{1}$ ratio is around 0.1-0.2

One possibility to describe the shape of a distribution is kurtosis (this is what Midrigan (2008)) uses. Therefore in column 4-5 of Table 3 we can see the simulated kurtosis values (as "non-matched moments"), as well as the empirical values of these kurtosis. ${ }^{3}$ These figures indicate that the calibrated kurtosis values in the VAT-months are approximately equal to the observed kurtosis when the $\phi_{2} / \phi_{1}$ ratio is around 0.17 (however, the kurtosis of the regular months' size distributions are non hit in this case).

Of course, matching the kurtosis of the size distribution does not do a perfect job in matching the whole size distribution. The exploration of possibilities to match the whole distributions as well as possible is subject of future research.

Finally, we investigate the real effect of nominal shock in the calibrated model (where the parameters were calibrated as in Table 3, for $\phi_{2} / \phi_{1}=0.17$ ). Model simulations indicate that the inflation effect of a $5 \%$-point nominal shock is $4.8 \%$ in the first month, which means that the real effect is only $0.2 \%$. This result may be driven by the large frequency effect that we calibrated for. Investigating the real effect of smaller nominal shocks (and the ratio of the output volatility and shock volatility) is subject of future research.

## 5 Conclusion

In this paper we set up a standard menu cost model with multi-product firms that contained the model of Midrigan (2008) and Golosov-Lucas (2007) as special cases. In the model we allowed the cost of changing the stores' second goods' prices to be anything between zero (as in Midrigan) and the first menu cost (as in Golosov-Lucas).

We then showed that the second menu cost is important for the real effects of nominal shocks: when the second menu cost is small, nominal shocks have relatively larger real effects than in case of large second menu cost. These results are in line with the results of Midrigan and Golosov-Lucas.

Next we calibrated our model to match simple facts (e.g. frequency, size) from a series of Value-Added Tax shocks in Hungary. To identify the second menu cost, we used the kurtosis of the distribution of the price change sizes. We found that the second menu cost is a non-trivial fraction of the first menu cost, approximately $16-17 \%$. In this case, however, a $5 \%$ nominal shock has very little effect on real output.

[^3]
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## 6 Appendix: Figures

Figure 2: Size distributions in the model for $\phi_{2} / \phi_{1}=0.05$


Figure 3: Size distributions in the model for $\phi_{2} / \phi_{1}=0.10$


Figure 4: Size distributions in the model for $\phi_{2} / \phi_{1}=0.20$


Figure 5: Size distributions in the model for $\phi_{2} / \phi_{1}=0.30$



[^0]:    *All errors are ours. The views expressed are those of the authors and do not necessarily reflect the views of the National Bank of Hungary.
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[^1]:    ${ }^{1}$ In a more general version, we could let $g_{Z}$ to follow an $\operatorname{AR}(1)$ process around its mean $\mu_{g Z}$.

[^2]:    ${ }^{2}$ Golosov-Lucas calibrate the average size of price increases, which is generally smaller than the average size of price changes. In empirical studies, an average increase of $9.5 \%$ is generally consistent with an average absolute size of $10-13 \%$.

[^3]:    ${ }^{3}$ When calculating the kurtosis from empirical data, we followed Midrigan's approach and eliminated all price changes that were bigger than $100 \%$ in absolute value. The way we treat these outliers matters a lot for the calculated kurtosis figures.

