# FACTOR MODEL FORECASTS OF EXCHANGE RATES

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## ABSTRACT

We construct factors from a cross section of exchange rates and use the idiosyncratic deviations from the factors to forecast. In a stylized data generating process, we show that such forecasts can be effective even if there is essentially no serial correlation in the univariate exchange rate processes. We apply the technique to a panel of bilateral U.S. dollar rates against 17 OECD countries. We forecast using factors, and using factors combined with any of fundamentals suggested by Taylor rule, monetary and purchasing power parity (PPP) models. For long horizon (8 and 12 quarter) forecasts, we tend to improve on the forecast of a "no change" benchmark in the late (1999-2007) but not early (1987-1998) parts of our sample.

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# 1. INTRODUCTION

In predictions of floating exchange rates between countries with roughly similar inflation rates, a random walk model works very well. The random walk forecast is one in which the (log) level of the nominal exchange rate is predicted to stay at the current log level; equivalently, the forecast is one of "no change" in the exchange rate. This forecast works well at various horizons, from one day to three years. It does well in the following sense: the out of sample mean squared (or mean absolute) error in predicting exchange rate movements generally is about the same, and often smaller, than that of models that use "fundamentals" data on variables such as money, output, inflation, productivity, and interest rates. Classic references are Meese and Rogoff (1983a,b); a recent update is Cheung, Chinn and Pascual (2005).

Whether or not this stylized regularity is bad news for economic theory is unclear. Some economists think the regularity is very bad news. Bacchetta and van Wincoop (2006,p552) describe it as "...the major weakness of international macroeconomics." On the other hand, Engel and West (2005) argue that a near random walk is expected under certain conditions.[1]

Whether or not one thinks the empirical finding of near random walk behavior is bad news for economic theory, it is of interest to try to tease out connections (if any) between a given exchange rate and other data. A small literature has used panel data techniques to forecast exchange rates, finding relatively good success (Mark and Sul (2001), Rapach and Wohar (2004), Groen (2005), Engel et al. (2007)). A very large literature has found that factor models do a good job forecasting basic macro variables.[2] The present paper predicts exchange rates, via factor models, in the context of panel data estimation, and compares the predictions to those of a random walk via root mean squared prediction error.

The panel consists of quarterly data on 17 bilateral US dollar exchange rates with OECD countries, 1973-2007. We construct factors from the exchange rates. We take the literature on predicting exchange rates to suggest that the exchange rate series *themselves* have information that is hard

to extract from observable fundamentals. This information might be hard to extract because standard measures of fundamentals (e.g., money supplies and output) are error ridden, or because we simply lack any direct measures of non-standard fundamentals such as risk premia or noise trading.[3]

We compare four different forecasting models to a benchmark model that makes a "no change" forecast–the random walk model. One of our four models uses factors but no other variables to forecast. The other three use factors along with some measures of observable fundamentals. The three measures of observable fundamentals are: (1)those of a "Taylor rule" model; (2)those of a monetary model; (3)deviations from purchasing power parity (PPP). Our measure of forecasting performance is root mean squared prediction error (root MSPE).

On balance, these models have lower MSPE than does a random walk model for long (8 and 12) quarter horizon predictions over the late part of our forecasting sample (1999-2007). These differences, however, are usually not significant at conventional levels. Predictions that span the entire two decades (1987-2007) or the early part (1987-1998) of our forecast sample generally have higher MSPE than does a no change forecast. (Different samples involve different currencies, because of the introduction of the Euro in 1999.) The basic factor model, and the factor model supplemented by PPP fundamentals, do best. We recognize that the good performance in the recent period may be ephemeral. But we are hopeful that our approach will prove useful in other datasets.

We close this introduction with some cautions. First, we make no attempt to justify or defend the use of out of sample analysis. We and others have found such analysis useful and informative. But we recognize that some economists might disagree. Second, judgment (sometimes rather arbitrary) has been used at various stages, so we're not (yet) proposing a completely replicable strategy. Third, our exercise is not "true" out of sample. For example, revised rather than real time data are used in some specifications. We use revised data because we are using out of sample analysis as a model evaluation tool, and the models presume that the best available data are used. More importantly, perhaps, our

exercise is not true out of sample because we have relied on research that has already examined exchange rates during parts of our forecasting sample. The most pertinent reference is Engel et al. (2007), which used similar data, spanning 1973-2005 (vs. 1973-2007).

Section 2 presents a stylized model that illustrates analytically why our approach might predict well. Section 3 describes our empirical models, section 4 our data and forecast evaluation techniques. Section 5 presents empirical results. Section 6 concludes. Some appendices available on request present detailed empirical results omitted from the paper to save space.

## 2. WHY A FACTOR MODEL MAY FORECAST WELL

In this section, we present a factor model and a simple data generating process that motivates its use.

Our basic presumption is that the deviation of the exchange rate from a measure of central tendency will help predict subsequent movements in the exchange rate. Algebraically, let

(2.1) $s_{it}$ = log of exchange rate in country $i$ in period $t$,

$z_{it}$ = measure of central tendency defined below.

For concreteness, we note that $s_{it}$ is measured as log(foreign currency units/U.S. dollar), though that is not relevant to the present discussion. Algebraically, our basic presumption is that for a horizon $h$, $s_{it+h}$-$s_{it}$ can be predicted by $z_{it}$-$s_{it}$, maybe using two different measures of $z$ in a single regression.

Many papers have relied on the same presumption (that $s_{it+h}$-$s_{it}$ can be predicted by $z_{it}$-$s_{it}$). For example, Mark (1995) sets $z_{it}$ in accordance with the "monetary model", so that $z_{it}$ depends on money supplies and output levels; Molodstova and Papell (2008) set $z_{it}$ in accordance with a "Taylor rule" model, so that $z_{it}$ depends on the exchange rate, inflation rates, output gaps and parameters of monetary policy rules; Engel et al. (2008) set $z_{it}$ in accordance with PPP, so that $z_{it}$ depends on price levels. Some

papers (see references above) have used these specifications of $z_{it}$ in the context of panel data. Our twist is to construct one measure of $z_{it}$ from factors estimated from the panel of exchange rates.

To exposit the idea, consider the following example. Suppose that the *i*'th exchange rate follows the process

(2.2) $s_{it} = F_{it} + v_{it}$.

Here, $F_{it}$ is the effect the factor has on currency *i*; in a one factor model, for example $F_{it} = \delta_i f_{1t}$ where $f_{1t}$ is the factor and $\delta_i$ is the factor loading for currency *i*. The idiosyncratic shock $v_{it}$ is uncorrelated with $F_{it}$. For simplicity, make as well some further assumptions not required in our empirical work, namely, that $F_{it}$ follows a random walk and that $v_{it}$ is i.i.d.:

(2.3) $F_{it} = F_{it-1} + \varepsilon_{it}$, $\varepsilon_{it} \sim$ i.i.d. $(0, \sigma_\varepsilon^2)$, $v_{it} \sim$ i.i.d. $(0, \sigma_v^2)$, $E\varepsilon_{it}v_{is}=0$ all *t*, *s*.

An *i* subscript is omitted from the variances $\sigma_\varepsilon^2$ and $\sigma_v^2$ for notational simplicity.

Then $\Delta s_{it} = \varepsilon_{it} + v_{it} - v_{it-1}$ and the univariate process followed by $\Delta s_{it}$ is clearly an MA(1), say

(2.4) $\Delta s_{it} = \eta_{it} + \theta\eta_{it-1}$, $E\eta_{it}^2 \equiv \sigma_\eta^2$, $|\theta|<1$.

Here, $\eta_{it}$ is the Wold innovation in $\Delta s_{it}$. The variance of $\eta_{it}$ and the value of $\theta$ can be computed in straightforward fashion from the values of $\sigma_\varepsilon^2$ and $\sigma_v^2$.[4]

Let us compare population forecasts of $\Delta s_{it+1}$ using the factor model (2.2), the MA(1) model (2.4), and a random walk model. Let "MSPE" denote "mean squared prediction error." Unless otherwise stated, in this section MSPE refers to a population rather than sample quantity. (This contrasts to the discussion of our empirical work below, in which MSPE refers to a sample quantity.) To forecast using (2.2), observe that $\Delta s_{it+1} = \Delta F_{it+1} + \Delta v_{it+1} = \varepsilon_{it+1} + v_{it+1} - v_{it} \Rightarrow E_t\Delta s_{it+1} = -v_{it} \equiv F_{it} - s_{it} \Rightarrow$

(2.5) forecast error from factor model $= \varepsilon_{it+1} + v_{it+1}$, $\text{MSPE}_{factor} = \sigma_\varepsilon^2 + \sigma_v^2$.

The MSPE from the univariate model (2.4) is of course $\sigma_\eta^2$. With a little bit of algebra, and using the formula for $\sigma_\eta^2$ given in the previous footnote, it may be shown that $\sigma_\varepsilon^2+\sigma_v^2 < \sigma_\eta^2$. Hence the factor model has a lower MSPE than the MA(1) model.

But the relevant issue is whether the improvement (i.e., the fall) in the MSPE is notable, for a plausible data generating process. A plausible DGP would be one in which there is very little serial correlation in $\Delta s_{it}$. Put differently, if the DGP is such that the MSPE from the MA(1) model is essentially the same as that from a random walk model, is it still possible that the MSPE from the factor model is substantially smaller than that of the random walk?

In our empirical work, we use Theil's U-statistic to compare (sample) MSPEs relative to that of a random walk. These are square roots of the ratios of (sample MSPE alternative forecast / sample MSPE forecast of no change). The forecast error of the random walk model is the actual change in $\Delta s_{it} = \varepsilon_{it} + v_{it} - v_{it-1}$; the corresponding population MSPE is $\sigma_\varepsilon^2+2\sigma_v^2$. In the context of the present section (population rather than MSPEs), define population U-statistics as

(2.6) $U_{factor} = [\ (\sigma_\varepsilon^2+\sigma_v^2)/\ (\sigma_\varepsilon^2+2\sigma_v^2)\ ]^{1/2}$, $U_{MA} = [\ \sigma_\eta^2/(\sigma_\varepsilon^2+2\sigma_v^2)\ ]^{1/2}$.

For select values of the first order autocorrelation of $\Delta s_{it}$ (which is approximately the MA parameter $\theta$ introduced in (2.4)), these are as follows:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (2.7a) | $\text{corr}(\Delta s_{it}, \Delta s_{it-1})$ | -0.01 | -0.02 | -0.03 | -0.04 | -0.05 | -0.10 |
| (2.7b) | $U_{MA}$ | 0.99995 | 0.9998 | 0.9996 | 0.9992 | 0.9987 | 0.9949 |
| (2.7c) | $U_{factor}$ | 0.995 | 0.990 | 0.985 | 0.980 | 0.975 | 0.949 |

The factor model improves on the moving average model by an order of magnitude. For example, when the first order autocorrelation of $\Delta s_{it}$ is -0.10, the population root MSPE for the MA model is only .5% lower than for the random walk (because .9949 is about .5% smaller than 1), while the population root MSPE for the factor model is about 5% lower than the random walk.[5]

We further note that the population values for $U_{MA}$ in (2.7b) are so near 1 that in practice, for any of the values of the autocorrelations, one would not be surprised if sampling error in estimation of an MA(1) model led to sample U-statistics above 1. Hence we view the figures in the table as consistent with the well-established finding that no univariate model predicts better than a random walk. But clearly the factor model has the potential to predict better, even accepting the point that in practice the best univariate model is a random walk.

In the simple DGP consisting of (2.2) and (2.3), whether a factor model will have lower MSPE than a model that uses information not only on exchange rates but also on fundamentals such as prices and output depends on whether the additional variables help pin down $v_{it}$. To allow for this possibility, our empirical work combine factors with observable fundamentals, as discussed in the next section.

## 3. EMPIRICAL MODELS

We use models with one, two or three factors. We will use the three factor model for illustration. The one and two factor models are analogous. In the three factor model, we first estimate a set of three factors and factor loadings from the exchange rates. For currency $i$, $i$=1, ...,17, the model is

$$(3.1) \quad s_{it} = \text{constant} + \delta_{1i} f_{1t} + \delta_{2i} f_{2t} + \delta_{3i} f_{3t} + v_{it}$$
$$\equiv \text{constant} + F_{it} + v_{it}.$$

The factors (the $f$'s) are unobserved I(1) variables. Here and throughout, we do not attempt to test for unit roots in the factors or any other variable for that matter. See Bai (2004) on estimation of factor models with unit root data.

Let $F_{it} = \delta_{1i} f_{1t} + \delta_{2i} f_{2t} + \delta_{3i} f_{3t}$. We aim to use (estimates of) $F_{it}$ to forecast $s_{it}$. In contrast to much work with factors, the factors are *not* constructed from a set of additional variables. For example, in Groen's (2006) work on exchange rates, factors are constructed from data on real activity and prices,

and take the place of traditional measures of real and nominal activity. We take the literature on predicting exchange rates to suggest that the exchange rate series *themselves* may have low frequency information on common trends that is hard to extract from observable fundamentals. This low frequency information might be buried in standard, but noisy, measures of fundamentals such as relative money supplies and relative outputs. Or this information might be embedded in non-standard measures of fundamentals that are sufficiently persistent that they function in part to drive common trends; examples are persistent risk premia or persistent noise trading. Put differently, we use factors to parsimoniously capture co-movements of exchange rates that are not well-captured by observable fundamentals. Crudely, we posit that a weighted average of the log levels of exchange rates represents a central tendency for the log level a given exchange rate, and use this weighted average to help forecast.

Mechanics are as follows. We assume that the factors component soak up a common unit root component in the $s$'s. That is, we assume that $F_{it}$-$s_{it}$ is stationary, and may be useful in predicting (stationary) future changes in $s_{it}$. We do not attempt to test stationarity of $v_{it}$; our selection of number of factors was based on presumed limitations of a panel of cross-section dimension 17.[6] The factors $f_{1t}$, $f_{2t}$ and $f_{3t}$ are uncorrelated by construction. We normalize the $f$'s to have mean zero and unit variance.

So this first stage produces a time series for $\hat{f}_{1t}$, $\hat{f}_{2t}$ and $\hat{f}_{3t}$ and factor loadings, $\hat{\delta}_{1i}$, $i$=1,...,17; $\hat{\delta}_{2i}$, $i$=1,...,17; $\hat{\delta}_{3i}$, $i$=1,...,17. In our simplest specification, the measure of central tendency $z_{it}$ that was introduced in (2.1) is

$$(3.3) \qquad z_{it} = \hat{\delta}_{1i}\hat{f}_{1t} + \hat{\delta}_{2i}\hat{f}_{2t} + \hat{\delta}_{3i}\hat{f}_{3t} \equiv \hat{F}_{it}.$$

In this simplest specification, with $\hat{F}_{it} \equiv \hat{\delta}_{1i}\hat{f}_{1t} + \hat{\delta}_{2i}\hat{f}_{2t} + \hat{\delta}_{3i}\hat{f}_{3t}$, for quarterly horizons $h$=1, 4, 8, and 12 we use standard panel data estimator (least squares with dummy variable) to estimate and forecast. For example, with a horizon of $h$=4 quarters, we estimate

$$(3.4) \qquad s_{it+4}\text{-}s_{it} = \alpha_i + \beta(\hat{F}_{it}\text{-}s_{it}) + u_{it+4},$$

where $\alpha_i$ is a fixed effect for country $i$. We then use $\hat{\alpha}_i$ and $\hat{\beta}$ to predict (some details below). Here, and in all of our specifications, there is no "time effect" (in the jargon of panel data): the factors are dynamic versions of time effects (we hope).

Our three other specifications combine factors with observables. The other three specifications are all of the form

$$(3.5) \quad s_{it+h}-s_{it} = \alpha_i + \beta(\hat{F}_{it} - s_{it}) + \gamma(z_{it}-s_{it}) + u_{it+h}$$

for three different $z_{it}$'s. Let country 0 refer to the USA. The three $z_{it}$'s are:

(3.6) Taylor rule: $z_{it} = 1.5(\pi_{it}-\pi_{0t})+0.5(\tilde{y}_{it}-\tilde{y}_{0t})+s_{it}$; $\pi$ = inflation, $\tilde{y}$ = output gap;

$$z_{it}-s_{it} = 1.5(\pi_{it}-\pi_{0t})+0.5(\tilde{y}_{it}-\tilde{y}_{0t});$$

(3.7) monetary model: $z_{it} = (m_{it}-m_{0t})-(y_{it}-y_{0t})$; $m$ = ln(money), $y$=ln(output);

(3.8) PPP model: $z_{it} = p_{it}-p_{0t}$; $p$=ln(price level).

The Taylor rule model builds on the recently developed view that interest rates rather than money supplies are the instrument of monetary policy. Expositions may be found in Benigno and Benigno (2006), Engel and West (2006) and Mark (2008). The monetary model was for many years the workhorse of international monetary economics; see, for example the textbook exposition in Mark (2001) or the abbreviated summary in Engel and West (2005). The PPP model (3.6) presumes convergence of price levels.

## 4. DATA AND FORECASTING EVALUATION

We use quarterly, data 1973:1-2007:4, with the out of sample period beginning in 1987:1. The basic data source is International Financial Statistics, supplemented on occasion by national sources. Exchange rates are end of quarter values of the US dollar vs. the currencies of 17 OECD countries:

Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Japan, Italy, Korea, Netherlands, Norway, Spain, Sweden, Switzerland, and the United Kingdom. (See below on how we handled conversion to the Euro in 1999:1.) The price level is the CPI in the last month of quarter; output is industrial production in last month of quarter; money is M1 (with some complications); the output gap is constructed by HP detrending, computed recursively, using only data from periods prior to forecast period. Because some of the data appeared to display seasonality, we seasonally adjusted prices, output and money by taking a four quarter average of the log levels before doing any empirical work. For example the price level in country $i$ is $p_{it} = ¼[\log(\text{CPI}_{it}) + \log(\text{CPI}_{it-1}) + \log(\text{CPI}_{it-2}) + \log(\text{CPI}_{it-3})]$.

To explain the mechanics of our forecasting work, let us illustrate for the four quarter horizon ($h$=4), for the first forecast, and for the model that uses only factors but not in addition observable fundamentals. As depicted in (4.1) below, we use data from 1973:1 to 1986:4 to estimate factors and factor loadings, and construct $\hat{F}_{it}$ for $i$=1,...,17.

(4.1) ----------data used to estimate factors----------
----data used to estimate panel regression----
|_____|_____|_____|
85:4 86:4 87:4

We then use right hand side data from 1973:1 to 1985:4 to estimate panel data regression

(4.2) $s_{it+4}-s_t = \alpha_i + \beta(\hat{F}_{it}-s_{it}) + u_{it+4}$, $t$=1973:1, ..,1985:4.

We use 1986:4 data to predict the 4 quarter change in $s$:

(4.3) Prediction of $(s_{i,1987:4}-s_{i,1986:4}) = \hat{\alpha}_i + \hat{\beta}(\hat{F}_{i,1986:4}-s_{i,1986:4})$.

We then add an observation to the end of the sample, and repeat.

As is indicated by this discussion, the recursive method is used to generate predictions: observations are added to the end of the estimation sample, so that the sample size used to estimate factors

and panel data regressions grows. The direct (as opposed to iterated) method is used to make multiperiod predictions. The estimation technique is maximum likelihood, assuming normality.

An analogous setup is used for other horizons and for models with observable fundamentals.

For a given date, factors and r.h.s. variables are identical across horizons: for given $t$, the same values of $\hat{F}_{it}$-$s_{it}$ are used. However, the l.h.s. variable is different ($h$ period difference in $s_t$), and regression samples are smaller for larger $h$. This means that the regression coefficients ($\hat{\alpha}_i$, $\hat{\beta}$) and predictions vary with $h$.

For the 9 non-Euro currencies (Australia, Canada, Denmark, Japan, Korea, Norway, Sweden, Switzerland, and the United Kingdom), we report "long sample" forecasting statistics for a 1987-2007 sample. For all 17 currencies, we report "early" sample forecasting statistics for a 1987-1998 sample. For the 9 non-Euro currencies and the Euro, we report "late sample" forecasting statistics for a 1999-2007 sample. Early sample statistics involve forecasts whose forecast base begins in 1986:4 and ends in 1998:4. Towards the end of the early sample, the forecast occurs in the pre-Euro era, while the realization occurs during the Euro era. We rescaled Euro area currencies so that there was no discontinuity. See Table 1 for the exact number of forecasts for each sample and horizon, as well as a summary listing of models.

In all samples (long, early and late), we use data from all 17 countries to construct factors and panel data estimates. For post-1999 data, the left hand side variable in both factor and panel data estimation is identical for all 8 Euro area countries. But because all samples include some pre-1999 data in estimation, there are differences across countries in estimates of the factor $\hat{F}_{it}$, and of course the measures of prices, output and money used in the PPP, monetary and Taylor rule models. This means that the forecasts are different for the Euro countries. We construct a Euro forecast by simple averaging of the 8 different forecasts.

Our measure of forecast performance is root mean squared prediction error (RMSPE). (Here and

through the rest of the paper, all references to MSPE and RMSPE refer to sample rather than population values.) We compute Theil's U-statistic, the ratio of the RMSPE from each of our models to the RMSPE from a random walk model. We summarize results by reporting the median (across 17 countries) of the U-statistic, and the number of currencies for which the ratio is less than one (since a value less than one means our model had smaller RMSPE than did a random walk). Individual currency results are available on request.

A U-statistic of 1 indicates that the (sample) RMSPEs from the factor model and from the random walk are the same. As argued by Clark and West (2006, 2007), this is evidence *against* the random walk model. If, indeed, a random walk generates the data, then the factor model introduces spurious variables into the forecasting process. In finite samples, attempts to use such variables will, on average, introduce noise that inflates the variability of the forecasting error of the factor model. Hence under a random walk null, we expect sample U-statistics greater than 1, even though that null implies that population ratios of RMSPEs are 1.

We report 10 percent level one sided hypothesis tests on $H_0$: RMSPE(our model) = RMSPE(random walk) against $H_A$: RMSPE(our model) < RMSPE(random walk). (Here,"our model" refers to any one of the four models given in (3.4) or (3.6)-(3.8): factor model, factor model plus Taylor rule, factor model plus monetary or factor model plus PPP.) These hypothesis tests are conducted in accordance with Clark and West (2006), who develop a test procedure that accounts for the potential inflation of the factor model's RMSPE noted in the previous paragraph. Of course, with many currencies (17, in our early sample), it is very possible that one or more test statistics will be significant even if none in fact predict better than a random walk. We guarded against this possibility by testing $H_0$: RMSPE(our model) = RMSPE(random walk) for all currencies against $H_A$: RMSPE(our model) > RMSPE(random walk) for at least one currency, using the procedure in Hubrich and West (2010).[7] This statistic, however, rarely had a p-value less than 0.10. We therefore do not report it, to keep down the number of figures

reported.

## 5. EMPIRICAL RESULTS

For the largest sample used (1973-2007), Figure 1 plots the estimates of the three factors, while Table 2 presents the factor loadings. The factor loadings in Table 2 are organized so that the first block of six currencies (Austria, ..., Switzerland) includes ones in the one-time German mark area. The next block of four currencies (Finland, ..., Spain) are Euro area countries not included in the first block. The final block (Australia, ..., UK) lists the seven remaining countries.

The factor loadings suggest that the second factor reflects a central tendency of countries in the former German mark area. (If the factor loading on the second factor was zero for countries not in the mark area, then this second factor would literally be a weighted average of countries in the mark area.[8] The coefficients are not zero on all non-mark countries, so the second factor is only roughly an average of mark countries.) By similar logic, the first factor seems to represent an average of everybody except countries in the former German mark area. The third factor is hard to label.

Of course this breakdown is not precise. Denmark's factor loading on what we have labeled the "mark" factor is smaller than is Japan's (0.68 vs. 0.78), and its factor loading on the first factor is, in absolute value, larger than Japan's (0.70 vs. -0.55).

Tables 3 and 4 present some forecasting results. What we present in these tables is summaries of results over all currencies. We present the median U statistic across the currencies in the sample, the number of U-statistics less than 1 and the number of t-statistics greater than 1.282. (Recall that a U statistic less than 1 means that the model's had a lower MSPE than did a random walk.) Currency by currency results are available on request.

Table 3 presents results for $r$=2 factors, both for the model that uses only factors, and for the models that also include observable fundamentals. To read the table, consider the entry at the top of the

table for model = $\hat{F}_{it}$-$s_{it}$, sample = 87-07. The figure of "1.003" for "median U" and horizon "$h$=1" means that of the 9 currencies, half had U-statistics above 1.003, half had U-statistics below 1.003. The figures of "1(0)" immediately below the figure of "1.003" means that only 1 of the 9 U-statistics was below 1, and that 0 of the t-statistics rejected the null of equal MSPE at the 10% level.[9]

One's eyes (or at least our eyes) are struck by the preponderance of median U-statistics that are above 1. In the long sample, 13 of the 16 the median U-statistics are above one (the three exceptions are for $\hat{F}_{it}$-$s_{it}$ + PPP for the $h$=4, 8 and 12 quarter horizons). In the early sample, it is again the case that 13 of the 16 the medians are above one (the exceptions in this case being $\hat{F}_{it}$-$s_{it}$ + PPP for $h$=1 and 4, and $\hat{F}_{it}$-$s_{it}$ + monetary for $h$=1). In the late sample, however, 9 of the 16 medians are above 1, with the models doing consistently better than a random walk (median U<1) at 8 and 12 quarter horizons. Note in particular that in this sample, 8 of the 10 U-statistics were below 1 for $\hat{F}_{it}$-$s_{it}$ and $\hat{F}_{it}$-$s_{it}$ + monetary, and 9 or 10 were below 1 for $\hat{F}_{it}$-$s_{it}$ + PPP.

Table 4 illustrates how varying the number of factors affects the simplest model, that of $\hat{F}_{it}$-$s_{it}$; results for models that include Taylor rule, monetary or PPP fundamentals are similar. The Table indicates that for the long sample, $r$=3 performs a little better and the $r$=1 model a little worse than does the $r$=2 model presented in Table 3. In the early sample, the $r$=2 model is the worst performing; the $r$=1 model is the best performing. In the late sample, the $r$=2 and $r$=3 models perform similarly, with the $r$=1 model performing distinctly more poorly than either of the other models.

To depict visually what underlies a U-statistic of various values, let us focus on the United Kingdom, $r$=3 factors, model = $\hat{F}_{it}$-$s_{it}$, long sample. The U-statistics happen to be: 1.003 ($h$=1), 0.996 ($h$=4), 0.979 ($h$=8) and 0.969 ($h$=12). (These U-statistics, as well as other individual currency U-statistics discussed below, are not reported in any table.) A scatter plot of the recursive estimates of $\hat{F}_{it}$-$s_{it}$, and of the subsequent $h$-quarter change in the exchange rate, is in Figure 2. The values of 0.979 and 0.968 for the U-statistics for $h$=8 and $h$=12 imply a reduction in root MSPE relative to a no-change forecast of

about 2-3%. Despite the seemingly small reduction, the figures for $h$=8 and $h$=12 depict an unambiguously positive relation between the deviation from the factor $\hat{F}_{it}$-$s_{it}$ and the subsequent change in the exchange rate. On the other hand, there clearly is a lot of variation in a relation that is positive on average.

Our predictions fared especially poorly for the Japanese yen, which generally had one of the highest U-statistics in each sample and model. For example, the U-statistics for Japan, $r$=2 factors, model = $\hat{F}_{it}$-$s_{it}$, long sample were: 1.008 ($h$=1), 1.051 ($h$=4), 1.085 ($h$=8) and 1.160 ($h$=12). That the yen does not quite fit into the same mold as the other currencies in our study is perhaps suggested by the large negative weight of -0.55 for the yen on the first factor (see Table 2). In the late sample, continental currencies (Denmark, Norway, Sweden, Switzerland) and, to a lesser extent, the Euro were generally well predicted by our models. For example, the figures for the Euro for $r$=2 factors, model = $\hat{F}_{it}$-$s_{it}$, late sample were: 1.009 ($h$=1), 1.015 ($h$=4), 0.939 ($h$=8) and 0.816 ($h$=12).

Over all specifications and horizons (1, 2 and 3 factors; long, early and late samples; horizons of 1, 4, 8 and 12 quarters), only the $\hat{F}_{it}$-$s_{it}$+PPP model had median U-statistics less than 1 in over 50 percent of the forecasts.

We checked the robustness of these results in two ways. First, we estimated by principal components rather than by maximum likelihood. Overall, results were comparable, with one technique doing a little better (occasionally, a lot better) in one specification and the other doing a little better (occasionally, a lot better) in other specifications. Second, we used the British pound rather than the U.S. dollar as the base currency. Estimation was by maximum likelihood. Here, results were comparable for the early sample, somewhat worse for the long and late samples.

A detailed summary of the robustness checks is in an appendix available on request. To illustrate, let us take two lines form Table 3, and present analogous results from principal components estimation, and from estimation with the British pound as the base currency. These lines are chosen because they are

representative:

| | | $h$=1 | $h$=4 | $h$=8 | $h$=12 |
|---|---|---|---|---|---|
| (5.1a) | $\hat{F}_{it}$-$s_{it}$, early / N=17, maximum likelihood, U.S. dollar (Table 3) | 6(0) | 7(0) | 4(0) | 3(0) |
| (5.1b) | $\hat{F}_{it}$-$s_{it}$, early / N=17, principal components, U.S. dollar | 4(2) | 4(3) | 7(4) | 8(2) |
| (5.1c) | $\hat{F}_{it}$-$s_{it}$, early / N=17, maximum likelihood, British pound | 6(1) | 6(1) | 4(2) | 4(0) |
| | | | | | |
| (5.2a) | $\hat{F}_{it}$-$s_{it}$+PPP, late / N=10, maximum likelihood, U.S. dollar (Table 3) | 4(0) | 5(0) | 8(0) | 9(5) |
| (5.2b) | $\hat{F}_{it}$-$s_{it}$+PPP, late / N=10, principal components, U.S. dollar | 3(3) | 3(3) | 7(2) | 9(4) |
| (5.2c) | $\hat{F}_{it}$-$s_{it}$+PPP, late / N=10, maximum likelihood, British pound | 3(0) | 1(0) | 1(0) | 1(0) |

We see in (5.1a) and (5.1b) that in terms of the number of U-statistics less than one, principal components improves over maximum likelihood at $h$=12 (8 versus 3 U-statistics less than 1), while the converse is true at $h$=4 (4 versus 7 U-statistics less than 1). Results for the British pound in (5.1c) similarly are better at some horizons, worse at other horizons. We see in (5.2a) and (5.2b) that principal and components and the maximum likelihood generate very similar numbers, while (5.2c) illustrates that with the British pound as the base currency, results degrade for the late sample.

## 6. CONCLUSIONS

This first pass at extracting factors from the cross-section of exchange rates yielded mixed results. Results for late samples (1999-2007) were promising, at least for horizons of 8 or 12 quarters. With occasional exceptions for models that relied not only on factors but PPP fundamentals as well, other results suggested no ability to improve on a "no change," or random walk, forecast.

Late samples allow larger sample sizes for estimation of factors. While that may be part of the reason for good results for late samples, that is not a sufficient condition for good results because our robustness checks found that the when the British pound is the base currency, late samples perform worse than early samples. Indeed, it remains to be seen whether our results for late samples are spurious. In any event, the framework here can be extended in a number of ways. It would be desirable to allow different slope coefficients across currencies, to allow more flexible specification of parameters in monetary and

Taylor rule models, and to use a data dependent method of selecting the number of factors. Such extensions are priorities for future work. It would also be desirable to compare our predictions to not only a random walk model, but to other models that have been compared to the random walk in earlier studies.

## FOOTNOTES

1. The Engel and West (2005) argument is *not* that a random walk is produced by an efficient market; indeed the simple efficient markets model implies that exchange rate changes are predicted by interest rate differentials and any variables correlated with interest rate differentials. Rather, the argument relates to the behavior of an asset price that is determined by a present value model with a discount factor near 1.

2. See Stock and Watson (2006). We use "factor" to refer to a DGP driven by factors, even if the estimation technique involves principal components.

3. See Diebold et al. (1994) for another attempt, with methodology very different from ours, to predict exchange rates using a cross section of exchange rates.

4. Specifically, let $\gamma=\sigma_\varepsilon^2+2\sigma_v^2$ denote the variance of $\Delta s$. Then $\sigma_\eta^2 = 0.5[\gamma+(\gamma^2-4\sigma_v^4)^{1/2}]$, $\theta=-\sigma_v^2/\sigma_\eta^2$.

5. The figures for the MA model may appear surprisingly small. But consider, for example, when the autocorrelation is -0.1 (rightmost column in the table), and suppose we forecast using an AR(1). The population $R^2$ is $(-.1)^2 = .01$. This means that the variance of the forecast error is 99% of the variance of $\Delta s$, with implied U-statistic of about .995 (since $.995^2 \approx .99$). The MA(1) model does very slightly better: To be precise, the variance of the MA(1) forecast error expressed as a percentage is $98.9898 < 99$. In interpreting the table in the text, it may help to observe that the first order autocorrelation of $\Delta s$ pins down the ratio of $\sigma_\varepsilon^2$ to $\sigma_v^2$, which is all that is needed to compute the other figures in the table.

6. In principle, one could use techniques to determine cointegrating rank to determine the number of I(1) factors and the factor loadings. We take results such as Ho and Sorensen (1996) to indicate that the finite sample performance of such techniques is likely to be poor, when the cross-section dimension is 17.

7. Although we report ratios of RMSPEs for convenience of interpretation, the Clark and West (2006) and Hubrich and West (2008) tests work off the arithmetic difference of MSPEs, evaluating whether this difference is statistically different from zero. The null hypothesis is that the random walk generates the data. These tests begin by adjusting the MSPE difference to account for noise that is present in the alternative (the non-random walk model) under the null hypothesis. For Clark and West (2006), the standard Diebold-Mariano-West (DMW) statistic is then computed for the adjusted MSPE difference. For Hubrich and West (2009), a parametric bootstrap is executed, under the assumption of normality. We did 10,000 repetitions in this bootstrap. See West (1996, 2006) for basic theory and further discussion.

8. See Stock and Watson (2006) on algebra relating factor loadings, factors, and data used to estimate factors.

9. The fact the median U was 1.003, but only 1 U-statistic was below 1 of course means that the U-statistics were tightly clustered near 1.

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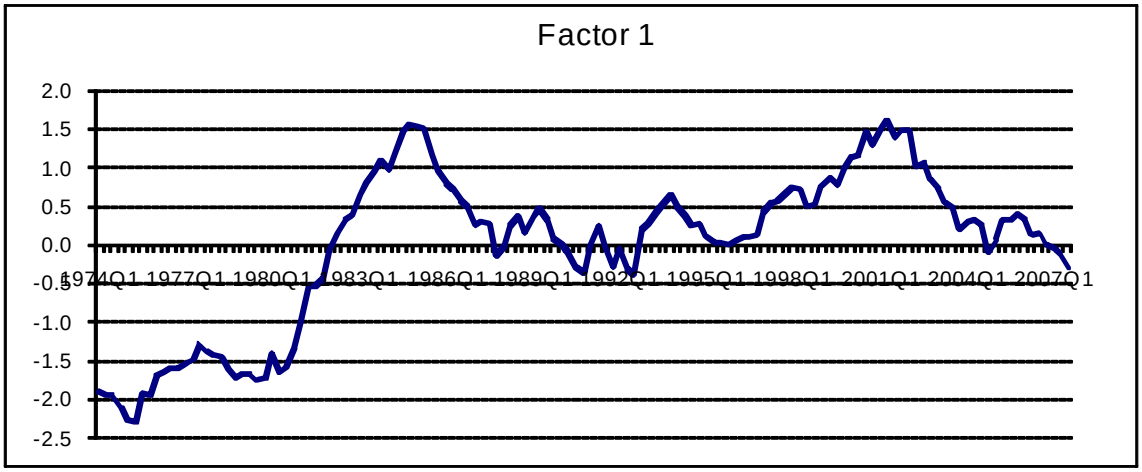
**Figure 1**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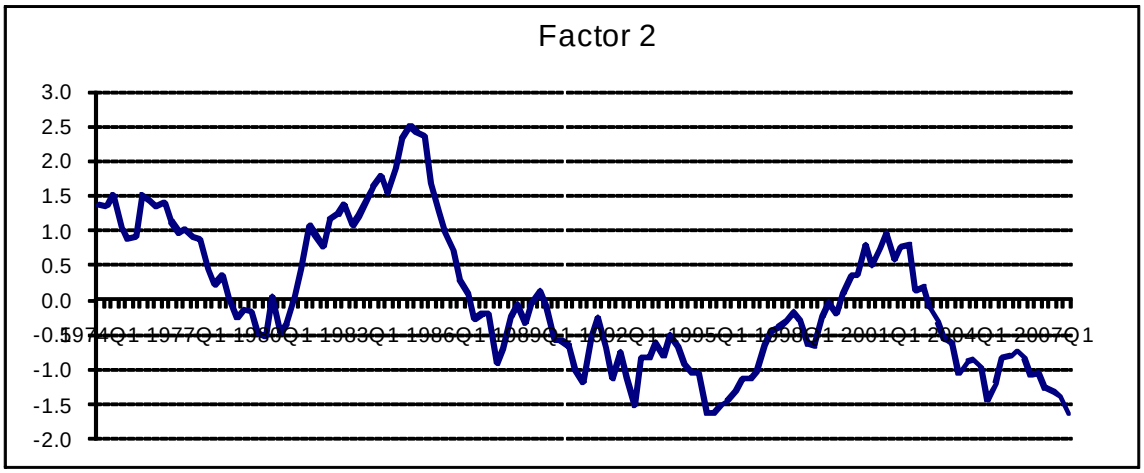
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
1974Q1 1977Q1 1980Q1 1983Q1 1986Q1 1989Q1 1992Q1 1995Q1 1998Q1 2001Q1 2004Q1 2007Q1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2.5
2.0
1.5
1.0
0.5
0.0
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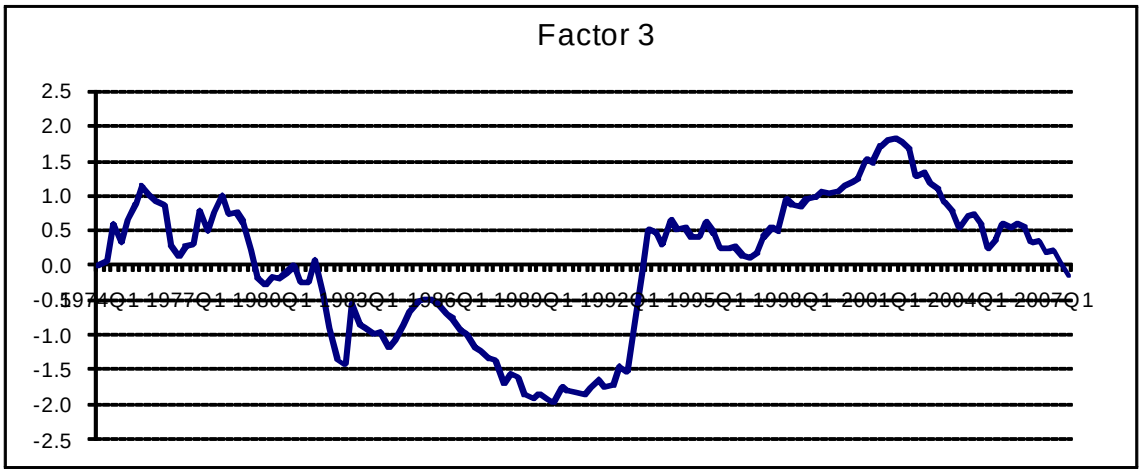
Factor 3
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1.5
1.0
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-0.5
-1.0
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-2.5
1974Q1 1977Q1 1980Q1 1983Q1 1986Q1 1989Q1 1992Q1 1995Q1 1998Q1 2001Q1 2004Q1 2007Q1

**Figure 2**

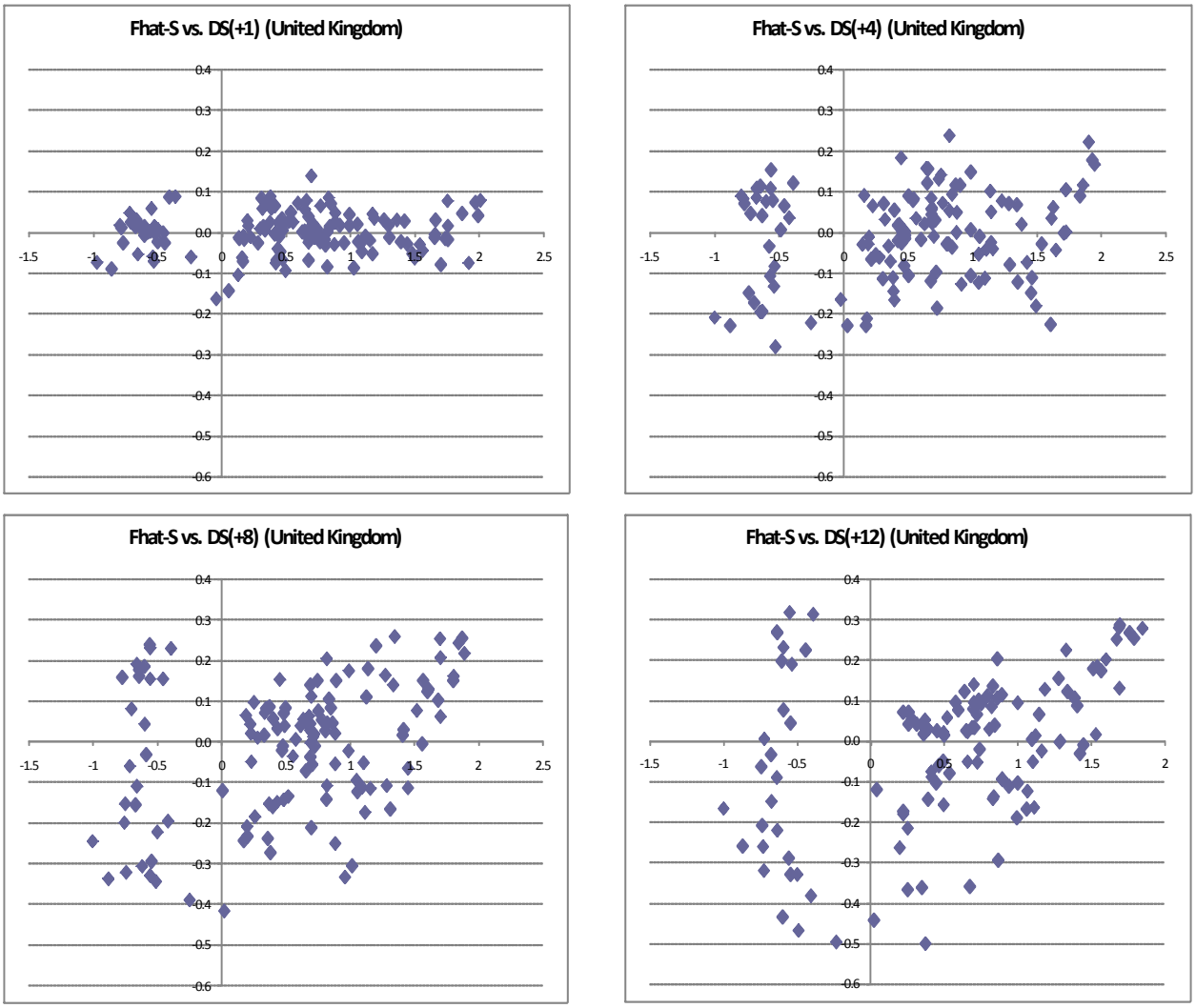
Fhat-S vs. DS(+1) (United Kingdom)
Fhat-S vs. DS(+4) (United Kingdom)
Fhat-S vs. DS(+8) (United Kingdom)
Fhat-S vs. DS(+12) (United Kingdom)

**Table 1**

**List of Currencies and Models**

A. Number of Quarterly Observations in Prediction Sample

| | Prediction Sample | Horizon $h$ | | | |
|---|---|---|---|---|---|
| | | 1 | 4 | 8 | 12 |
| Long Sample | (1986:4+$h$) - 2007:4 | 84 | 81 | 77 | 73 |
| Early Sample | (1986:4+$h$) - (1998:4+$h$) | 49 | 49 | 49 | 49 |
| Late Sample | (1999:1+$h$) - 2007:4 | 35 | 32 | 28 | 24 |

B. Currencies

| | | |
|---|---|---|
| Long sample | $N$=9 | Australia, Canada, Denmark, Japan, Korea, Norway, Sweden, Switzerland, United Kingdom |
| Early sample | $N$=17 | The long sample countries plus: Austria, Belgium, Finland, France, Germany, Italy, Netherlands and Spain |
| Late Sample | $N$=10 | The long sample countries plus the Euro |

C. Models

| | |
|---|---|
| $\hat{F}_{it}$-$s_{it}$ | $\hat{F}_{it}$ is the estimated factor component of currency $i$, estimated from 17 currencies in each sample (with identical Euro values appearing post-1998 for the 8 Euro area currencies). |
| $\hat{F}_{it}$-$s_{it}$ + Taylor | Taylor rule fundamentals (3.6) also included as a regressor |
| $\hat{F}_{it}$-$s_{it}$ + Monetary | Monetary model fundamentals (3.7) also included as a regressor |
| $\hat{F}_{it}$-$s_{it}$ + PPP | Purchasing power parity fundamentals (3.8) also included as a regressor |

Notes:

1. The sample period for estimation of models runs from 1973:1 to the forecast base. Models are estimated recursively. Factors are estimated using $N$=17 currencies, for all samples.

2. In the late sample, Euro area forecasts are made by averaging forecasts from the 8 Euro countries.

3. Long horizon forecasts are made using the direct method.

**Table 2**

**Factor Loadings, 1973-2007 Sample**

| | $\hat{\hat{\delta}}_{1i}$ | $\hat{\hat{\delta}}_{2i}$ | $\hat{\hat{\delta}}_{3i}$ |
|---|---|---|---|
| Austria | -0.06 | 1.00 | 0.02 |
| Belgium | 0.53 | 0.83 | -0.11 |
| Denmark | 0.70 | 0.68 | -0.16 |
| Germany | -0.03 | 1.00 | 0.01 |
| Netherlands | 0.08 | 1.00 | -0.02 |
| Switzerland | -0.31 | 0.93 | -0.03 |
| | | | |
| Finland | 0.88 | 0.23 | 0.34 |
| France | 0.85 | 0.50 | -0.16 |
| Italy | 0.97 | -0.14 | 0.15 |
| Spain | 0.98 | -0.08 | 0.12 |
| | | | |
| Australia | 0.87 | -0.31 | 0.16 |
| Canada | 0.80 | -0.11 | 0.29 |
| Japan | -0.55 | 0.78 | -0.16 |
| Korea | 0.84 | -0.27 | 0.28 |
| Norway | 0.95 | 0.19 | 0.12 |
| Sweden | 0.96 | -0.07 | 0.22 |
| UK | 0.84 | 0.13 | -0.01 |

Notes:

1. The fitted model is $s_{it} = \text{const.} + \hat{\hat{\delta}}_{1i}\hat{f}_{1t} + \hat{\hat{\delta}}_{2i}\hat{f}_{2t} + \hat{\hat{\delta}}_{3i}\hat{f}_{3t} + \hat{v}_{it} \equiv \hat{F}_{it} + \hat{v}_{it}$; $\hat{f}_{1t}, \hat{f}_{2t}$ and $\hat{f}_{3t}$ are estimated factors.

**Table 3**

**Two Factor (*r*=2) Results**

| Model | Sample/ No. Currencies | Statistic | --- | Horizon *h* | | --- |
|---|---|---|---|---|---|---|
| | | | 1 | 4 | 8 | 12 |
| $\hat{F}_{it}$-$s_{it}$ | long / *N*=9 | median U | 1.003 | 1.008 | 1.056 | 1.108 |
| | | #U<1 or (*t*>1.282) | 1(0) | 4(0) | 4(0) | 4(0) |
| $\hat{F}_{it}$-$s_{it}$ + Taylor | long / *N*=9 | median U | 1.010 | 1.047 | 1.089 | 1.129 |
| | | #U<1 or (*t*>1.282) | 1(0) | 0(0) | 1(0) | 4(0) |
| $\hat{F}_{it}$-$s_{it}$ + Monetary | long / *N*=9 | median U | 1.010 | 1.071 | 1.202 | 1.474 |
| | | #U<1 or (*t*>1.282) | 3(2) | 3(2) | 3(2) | 3(3) |
| $\hat{F}_{it}$-$s_{it}$ + PPP | long / *N*=9 | median U | 1.003 | 0.996 | 0.953 | 0.938 |
| | | #U<1 or (*t*>1.282) | 3(0) | 5(0) | 6(2) | 5(0) |
| $\hat{F}_{it}$-$s_{it}$ | early / *N*=17 | median U | 1.001 | 1.006 | 1.049 | 1.164 |
| | | #U<1 or (*t*>1.282) | 6(0) | 7(0) | 4(0) | 3(0) |
| $\hat{F}_{it}$-$s_{it}$ + Taylor | early / *N*=17 | median U | 1.012 | 1.048 | 1.086 | 1.156 |
| | | #U<1 or (*t*>1.282) | 1(0) | 2(0) | 1(0) | 3(0) |
| $\hat{F}_{it}$-$s_{it}$ + Monetary | early / *N*=17 | median U | 0.996 | 1.012 | 1.116 | 1.216 |
| | | #U<1 or (*t*>1.282) | 10(3) | 8(4) | 7(3) | 6(4) |
| $\hat{F}_{it}$-$s_{it}$ + PPP | early / *N*=17 | median U | 0.999 | 0.983 | 1.027 | 1.128 |
| | | #U<1 or (*t*>1.282) | 9(0) | 13(1) | 5(1) | 3(0) |
| $\hat{F}_{it}$-$s_{it}$ | late / *N*=10 | median U | 1.009 | 1.014 | 0.934 | 0.835 |
| | | #U<1 or (*t*>1.282) | 3(1) | 3(0) | 7(0) | 8(3) |
| $\hat{F}_{it}$-$s_{it}$ + Taylor | late / *N*=10 | median U | 1.010 | 1.035 | 0.979 | 0.836 |
| | | #U<1 or (*t*>1.282) | 2(1) | 2(0) | 6(0) | 8(2) |
| $\hat{F}_{it}$-$s_{it}$ + Monetary | late / *N*=10 | median U | 1.013 | 1.034 | 0.978 | 1.105 |
| | | #U<1 or (*t*>1.282) | 3(1) | 4(1) | 6(3) | 5(3) |
| $\hat{F}_{it}$-$s_{it}$ + PPP | late / *N*=10 | median U | 1.006 | 1.000 | 0.891 | 0.727 |
| | | #U<1 or (*t*>1.282) | 4(0) | 5(0) | 8(0) | 9(5) |

Notes:

1. Table 1 defines the long, early and late sample periods, lists the currencies in each sample, and describes the models.

2. The U statistic is = (RMSPE Model/RMSPE random walk); U<1 means that the model had a smaller MSPE than did a random walk model. "median U" presents the median value of this ratio across 9, 17 or 10 currencies. "#U<1" gives the number of currencies for which U<1, a number that can range from 0 to the number of currencies *N*.

3. *t* is test of $H_0$: U=1 (equality of RMSPEs) against one-sided $H_A$: U<1 (RMSPE Model is smaller), using the Clark and West (2006) procedure. The number of currencies in which this test rejected equality at the 10 percent level is given in the (t>1.282) entry.

**Table 4**

**Results for $\hat{F}_{it}$-$s_{it}$, Varying Number of Factors *r***

| No. of Factors (*r*) | Sample/ No. Currencies | Statistic | --- | Horizon *h* | | --- |
|---|---|---|---|---|---|---|
| | | | 1 | 4 | 8 | 12 |
| 1 | long / *N*=9 | median U | 1.011 | 1.037 | 1.083 | 1.154 |
| | | #U<1 or (*t*>1.282) | 1(0) | 1(0) | 1(0) | 1(0) |
| 2 | long / *N*=9 | median U | 1.003 | 1.008 | 1.056 | 1.108 |
| | | #U<1 or (*t*>1.282) | 1(0) | 4(0) | 4(0) | 4(0) |
| 3 | long / *N*=9 | median U | 1.003 | 0.996 | 0.996 | 1.038 |
| | | #U<1 or (*t*>1.282) | 3(0) | 5(0) | 5(1) | 4(0) |
| 1 | early / *N*=17 | median U | 0.996 | 0.969 | 0.995 | 1.103 |
| | | #U<1 or (*t*>1.282) | 9(3) | 10(3) | 9(1) | 3(0) |
| 2 | early / *N*=17 | median U | 1.001 | 1.006 | 1.049 | 1.164 |
| | | #U<1 or (*t*>1.282) | 6(0) | 7(0) | 4(0) | 3(0) |
| 3 | early / *N*=17 | median U | 1.000 | 0.995 | 1.000 | 1.130 |
| | | #U<1 or (*t*>1.282) | 10(1) | 9(0) | 9(0) | 3(0) |
| 1 | late / *N*=10 | median U | 1.021 | 1.081 | 1.179 | 1.294 |
| | | #U<1 or (*t*>1.282) | 1(0) | 1(0) | 1(0) | 1(1) |
| 2 | late / *N*=10 | median U | 1.009 | 1.014 | 0.934 | 0.835 |
| | | #U<1 or (*t*>1.282) | 3(1) | 3(0) | 7(0) | 8(3) |
| 3 | late / *N*=10 | median U | 1.008 | 1.020 | 0.953 | 0.822 |
| | | #U<1 or (*t*>1.282) | 3(1) | 3(0) | 8(0) | 8(1) |

Notes:

1. The results for *r*=2 in this table repeat, for convenience, those for $\hat{F}_{it}$-$s_{it}$ in Table 3.

2. See notes to Table 3.