# Optimal Learning Patterns for Creativity Generation in a Field

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Creativity is a fundamental value of a free society. Creativity flourishes when individuals are enabled to pursue individual independent paths of inquiry, exploration, and creative development (Jonathan S. Feinstein 2006; Philippe Aghion, Mathias Dewatripont and Jeremy C. Stein 2005). In such an environment personal intuition and knowledge is developed in unique and creative ways that leads to cultural and economic development – the progress of civilization (Friedrich A. Hayek 1960; John Stuart Mill 1859).

Much scholarship in economics has focused on innovation as the basis for economic growth. An important tradition characterizes innovation and technological change as arising from a process in which draws are made from an underlying distribution of productivity improvements or potential new technologies (for example see Robert Evenson and Yoam Kislev 1976 and Samuel Kortum 1997). But there has been little work directed towards formal modeling of the actual creative process through which innovations arise (one example is Martin Weitzman 1998). In this paper I present a model that opens a way for proceeding on this topic.

Individuals come to be creative through a process of creative development, exploring creative interests and gathering elements – data, ideas, models, possibilities, techniques - then finding ways to combine and reconfigure these elements into new creative forms. I present a simple model of this process: Individuals choose what to learn in a field from the viewpoint of making the highest value contribution they can through novel combinations of elements. The model joins rational decision theory with formal knowledge representation. This combination has great potential for describing the rich patterns of individual exploration and learning that are the basis for creativity.

Simulation results highlight the importance of enabling individuals to define their own learning agenda based on personal intuition and information and not enforcing a standardized curriculum – there is great diversity of learning sets.

## I. The Field

### A. *Knowledge Structure and Elements*

The field, a domain of human inquiry, knowledge, creativity and innovation, is structured as a partially ordered set. This structure captures both the natural hierarchy of concepts as well as the way new elements are formed through combining two higher level concepts. In the model in this paper the field has 3 levels.[1]

The top level of the field consists of $N$ topics. A topic is relatively broad and can be, for example, a subject domain, methodology, theme, or theoretical framework. Each pair of topics can be combined defining a more focused area for inquiry and exploration. Topic intersections are denoted $e$; the intersection of topics $i$ and $j$ is denoted $e_{ij}$. There are $\binom{N}{2}$ $e$'s. Intersections of topics play an important role in the model as building blocks for creative contributions.

The fundamental definition of creativity is connecting two elements that have not previously been connected. Following this definition, creative contributions are made by linking two $e$ elements, creating an $ee$ element; the two $e$ elements must not have been

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[1]The field has in the main the structure of a lattice but is not formally a lattice. It does not have a top element from which all topics extend, though this could readily be added, in which case it would be a semi-lattice. More critically, it does not have a bottom element at which all third-level elements meet; such an element does not seem natural for a growing field of the kind described here.

linked already so the *ee* is new (we treat order as irrelevant, $e_i e_j = e_j e_i$). Two rules govern the creation of *ee* elements. One follows from the principle that creative connections link elements that are connected via a remote association or conceptual overlap (Gilles Fauconnier and Mark Turner 1998): Two *e*'s can be linked only if they share a topic in common. The second rule is based on the principle that creativity is about combining preexisting elements: The two *e*'s out of which an *ee* is created must have been employed in previous *ee* elements. Lying behind this assumption is a process through which an initial set of *e*'s are used to create an initial set of *ee*'s, which I do not model though it could be added as a precursor creative process.

Now assume two *e*'s are combined. They must share a topic, so the new *ee* is $e_i e_j = t_a t_b t_c$, where we can assume that $e_i = t_a t_b$ and $e_j = t_b t_c$. It follows that the new *ee* generates a third $e = t_a t_c$ - the new *ee* links these topics. It is this third *e* that is the novel element of the new *ee*. The valuations below reflect this: a creative contribution is more valuable if its new third element has never before been part of an *ee*. Once an *e* has been used as a third element, it becomes available as a building block. At the time of development of the field I consider some *e*'s have not yet been used in *ee* elements and only a fraction of the potential *ee*'s have been created.

I focus on creating new *ee* elements. As the field continues to develop, *ee* elements may themselves be combined, but I do not model this process.[2]

## B. Valuation

Every element in the field has a value. Further, the value of an element is the same for all individuals. This assumption is made for simplicity. The results of this paper about diversity of learning patterns are even stronger when individual values differ. The values of *e* and *ee* elements that have not yet been used or created are not known. For these elements individuals may differ in their valuation assessments as described below.

All topics have value one, an assumption made for simplicity. Each *e* element has a value that is drawn from a distribution and the random variables defining these values are independent across elements. For the simulation in this paper the distribution is log-normal with a mean of zero and a standard deviation of one. The value of an *ee* element depends on the values of the two building block *e* elements out of which it is constructed, the value of the new third *e* element that is created and a stochastic term. Specifically, the value of $ee_{ij}$ constructed out of $e_i$ and $e_j$ is:

$$v_{ij} = w_i^{\alpha} * w_j^{\alpha} * w_k^{\beta} * \varepsilon_{ij} \tag{1}$$

where $w_i$ and $w_j$ are the values of $e_i$ and $e_j$, $w_k$ is the value of the new *e* element denoted $e_k$, $\alpha$ and $\beta$ are parameters, and $\varepsilon_{ij}$ is the stochastic term. For the simulation $\varepsilon_{ij}$ is drawn from a log-normal distribution with a mean of zero and a standard deviation of one, and the $\varepsilon$'s are assumed to be independent of one another and all other random variables. Since much of the creative value of the *ee* element comes from the new third *e* it should be the case that $\beta$ is larger than $\alpha$; for the simulation $\alpha = .2$ and $\beta = .6$. If the third *e* has been used *n* times before, the value is reduced by $1 + n$.

It is assumed that individuals in the field know the values of all *ee* elements that have already been created. These are creative contributions that have been discussed and evaluated in the field. In contrast, the values of *e* elements are not directly known and individuals must form probability assessments concerning their values. For *e* elements that have been employed in the construction of at least one *ee* element individuals assess their value using Bayesian inference, working backwards from the observed values of *ee* elements. Since three *e* elements enter into each *ee* element, and different *e*'s combine in the generation of different *ee*'s this procedure must in general be done jointly over all such *e* elements.

Individuals may possess private information or intuition about the value of particular *e* elements. Private information or intuition about an element, assuming it indicates a relatively high value, may lead an individual to focus on learning *ee*'s that enable construction of new *ee*'s that contain the element, hence lead to an individually tailored learning strategy. We can thus think of such private information as generating a personal creative interest (Feinstein 2006, Chapter 2). There are two kinds of such creative interests: (i) an interest in an *e* that has already been used in the construction of at least one *ee* element, so that there is some public information about its value (but an in-

[2] In a more general framwork more than two elements could be combined to create a new contribution, possibly with grammatical rules for how elements combine. Exploring these issues is beyond the scope of this paper, but is important in developing the model further.

dividual has more information); and (ii) an interest in an *e* that has not previously been used. In this paper I focus on the second kind.[3]

## II. Learning and Creativity

An individual works for two periods in the field. In the first period he chooses a set of elements to learn. In this paper I restrict individuals to learning only *ee* elements and to learning a fixed number $K$ of such elements. When an individual chooses an *ee* he learns the *ee* element itself and, more importantly for the model presented here, its three component *e*'s.[4] In the second period the individual explores all possible pairwise recombinations of *e* elements he has learned and makes as his contribution the greatest value *ee* element he can produce that has not been created previously. In doing so he must take into account the possibility that someone else will produce the same element – I assume that in this case the overall value $\upsilon$ is divided equally among the number $R$ of individuals who create the element. Rolling back, it follows that in the first period the individual chooses the set of elements that yields the greatest expected creative potential in period 2. Formally, this is:

$$Max_{\{ee_1, ee_2 \ldots ee_K\}} E Max \left[ \frac{\upsilon_{ij}}{R_{ij}} \right] \tag{2}$$

The first maximum is taken over all subsets of *ee* elements of size $K$. The second maximum is taken over all feasible $e_i, e_j$ pairs in the given subset of *ee* elements; for a pair to be a feasible combination its two *e* elements must share a topic in common, as discussed above, and the *ee* element created must be new.[5] The expectation is taken over the posterior probability distribution defined over all relevant *e* values, including any private information the individual has, the relevant $\varepsilon$'s, and his forecast of the elements other individuals may create which influences his assessment of the $R$'s.

An equilibrium is a set of learning elements for each individual working in the field such that each individual's choice maximizes her expected creative potential given any private information she possesses and the choices made by others. I assume that individuals know the learning sets chosen by other individuals, but do not know which exact combinations, hence which new elements, others will end up creating. In the simulation below private information is restricted to *e* elements that have not been used previously in the creation of any *ee* elements. It follow from this that individuals share a common posterior probability distribution over the values of all *e* elements that have been used, since this distribution is based solely on the observed values of created *ee* elements and the probability generating process for *e* elements, both of which are public information. Finally, I assume that when two or more individuals imagine creating the same *ee* element, their $\varepsilon$ values are independent – idiosyncratic aspects of their creative processes.

In making their learning choices individuals either pursue a personal creative interest, based on private information or intuition, or puruse an interest based strictly on public information.

There are two reasons why individuals choose in general to learn different sets of elements. One is private information which leads individuals to differ in their assessment of the creative potential of certain sets of elements. The simulations reported below show how important this is, leading individuals to learn different elements from what public information would imply. The other reason is the need to differentiate from others so as to avoid creating the same element. This second reason is most relevant when individuals pursue common public information interests, for in that case two individuals who learn similar elements are more likely to generate the same combination, whereas when two individuals pursue private information interests they are more likely to create different elements even when their learning sets overlap.

## III. Simulation Results

[3] In this paper creative interests are defined at the second level of the field. As the field grows, and *ee* elements are combined to form more complex elements, one can imagine creative interests being formed at the third level as well. Assuming that the creative potential of an element in terms of its use as a building block may diverge from its value as a creative contribution private information or intuition will play a role in this case as well.

[4] An important extension is the case in which individuals do not observe all elements in a learning bundle prior to making their selection decisions. In this case some elements are revealed to them only once they select a bundle.

[5] Combinatoric formulas can be worked out for the distribution of the number of feasible pairs for a set of *e* elements given the underlying number of topics.

I explore patterns of learning and creativity via simulation. The number of topics is set at 20; there are thus 190 *e* elements. I assume 100 *e*'s have been used and 100 *ee*'s created.[6] I draw 100 *e* elements from random topic combinations, then generate 100 *ee* elements by randomly combining these, ensuring that each *e* element is used at least once. Values for all 190 *e* elements and the 100 *ee* elements are generated as described in the previous section. For convenience *ee* elements are labeled by their value ranks in the table below – element 1 is highest value. For the 100 *e*'s that have been used I generate the posterior distribution individuals use to guide their learning decisions via simulation: I run a large number of trials, for each trial draw values for the 100 *e* elements, then compute the likelihood of the observed set of *ee* values conditional on these values, which generates a posterior probability distribution for the *e* values. Note that although the original *e* values are drawn independently the posterior distribution is joint. For the 90 other *e* values I generate trial values directly.

I focus on 4 individuals working in the field. For scenarios in which individuals possess private information I use the top 4 *e* values among the set of *e*'s that have not been used previously, assign one to each individual, and assume each individual gets a signal of the value of his *e*.[7] I consider different degrees of private information based on the correlation $\rho$ between the signal and the true value: $\rho = .99, .7, .5$. I also analyze the case in which individuals do not possess private information.

I analyze the case in which each individual chooses 4 *ee* elements to learn. Individuals choose the set that maximizes their expected creative potential, given by equation (2), computing this expectation using the simulated *e* values averaging over the trials. When an individual possesses private information about a particular *e* element I generate a signal based on the true value and then for each trial a value drawn from the conditional distribution given the signal. In the simulations individuals with private information often pursue private interests, learning elements that enable them to generate new combinations creating the *e* about which they possess information. Sometimes they pursue common public interests, learning elements out of which they cannot create the *e* about which they possess information. The boundary is not sharp as in some cases a "public" interest may enable an individual to produce an *ee* creating the *e* about which he has private information.[8]

Table 1 presents results from the simulations. The table lists, for each case, for each individual whether his interest is based on private information (P), common public information (C), or can be interpreted as either (P/C, see footnote 8), the *ee* elements he learns, his expected creative potential, as well as aggregate expected social value.[9]

Strikingly, the degree of overlap of learning sets is low in all cases. For the case of only public information the lack of overlap is due to individuals differentiating themselves. The implication is that even when individuals do not possess private information or intuition they should be offered curriculum choices and can be expected to choose to learn different things.

Most strikingly, across all scenarios the overlap of learning sets for individuals who pursue private interests is zero with the public interest learning sets. The implication is that individuals who possess private intuition or information will desire to learn different things from what might be the standard curriculum. Further, this is socially desirable as expected social value is higher for cases of greater private information and pursuit of private interests.

For $\rho = .7$ and $.5$ some individuals pursue either common interests or hybrid private/common interests. In choosing whether to pursue a private or common interest individuals face a trade-off. The public information interests of highest value are sets of relatively high value *ee* elements for which there are many potential feasible new combinations. The pri-

[6]The number of *ee* elements is relatively small. This is done to keep the cominatorics, in terms of number of learning sets, tractable. In fact we would expect the number of *ee* elements to be substantially larger than the number of *e* elements.

[7]The signal and the log of the value are bivariate normal.

[8]A "private" interest is one for which a filter is applied: a set of elements is considered only if it enables *ee* elements to be created using the private information *e*. For a "public" interest no such filter is applied. As a result there will be public interests that enable *ee*'s to be created using the individual's private information *e*. In the table a P/C interest is one of these – an interest that ranks high in the no private information case but also is compatible with the individual's private information *e*. I view as strictly private interests sets of elements that rank low in the public information case.

[9]For each of the no private information and $\rho = .99$ scenarios I identified one equilibrium. For each of the other two scenarios there are a few equilibria. Results are very similar across these and for reasons of space results are shown only for the equilibrium of highest aggregate social value.

| Individual | Type | *ee* Elements | | | | Exp. Value |
|---|---|---|---|---|---|---|
| Case: $\rho = 0.99$ | | | | | | |
| 1 | P | 35 | 61 | 94 | 97 | 23.99 |
| 2 | P | 21 | 35 | 49 | 61 | 22.06 |
| 3 | P | 14 | 35 | 61 | 67 | 18.43 |
| 4 | P | 21 | 26 | 35 | 80 | 17.16 |
| Expected Social Value: | | | | | | 81.64 |
| Case: $\rho = 0.7$ | | | | | | |
| 1 | P | 35 | 61 | 94 | 97 | 18.99 |
| 2 | P | 14 | 21 | 35 | 49 | 13.11 |
| 3 | P | 14 | 35 | 61 | 68 | 13.59 |
| 4 | P/C | 1 | 4 | 45 | 89 | 13.00 |
| Expected Social Value: | | | | | | 58.69 |
| Case: $\rho = 0.5$ | | | | | | |
| 1 | C | 7 | 16 | 17 | 25 | 12.39 |
| 2 | P | 14 | 21 | 35 | 61 | 13.13 |
| 3 | C | 1 | 10 | 45 | 92 | 12.88 |
| 4 | P/C | 1 | 4 | 51 | 89 | 13.03 |
| Expected Social Value: | | | | | | 51.43 |
| Case: NoPrivateInformation | | | | | | |
| 1 | C | 1 | 10 | 12 | 41 | 12.39 |
| 2 | C | 1 | 10 | 45 | 92 | 11.85 |
| 3 | C | 1 | 4 | 51 | 89 | 12.06 |
| 4 | C | 7 | 16 | 17 | 25 | 12.99 |
| Expected Social Value: | | | | | | 49.29 |

TABLE 1—SIMULATION RESULTS: LEARNING PATTERNS AND EXPECTED CREATIVE POTENTIAL

vate information interest learning sets contain lower value *ee* elements and fewer feasible pathways creating new combinations. However, due to the private information the expected value of these combinations, specifically those creating the *e* about which the individual has private information, is high. Thus combinatoric options trade-off against high value combinations. Interestingly, none of the highest value interests are based simply on the highest ranked *ee* elements. The complexities of needing to put together *e*'s that share a common topic and generate a new *e* preclude this. The implication is that it is not a good strategy simply to learn the highest value current contributions. Optimal learning for creativity involves learning sets of elements that can fit together productively.

In current work I am building richer models. In this paper individuals search for "bridges" that enable them to create specific new *e* elements. In richer models they also explore conceptual pods – exploring sets of pods for which they imagine (have a signal) that linking elements from different pods will create high value contributions. Creative interests are thus defined at higher conceptual levels. I am also constructing models with richer knowledge structures.

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