Entrepreneurs and Cities: Complexity, Thickness and Balance

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November 13, 2008
Version: May 20, 2010

* We are grateful to Gilles Duranton, Pierre Picard, Issi Romem and seminar participants at Baruch College, HEC, Paris School of Economics, and University of Toronto for helpful comments. We thank the Social Sciences and Humanities Research Council of Canada, the UBC Centre for Real Estate and Urban Economics, the Rotman School of Management at the University of Toronto, and the Fisher Center for Real Estate and Urban Economics at UC Berkeley for financial support.

Abstract

It is well established that the thickness of local markets can enhance entrepreneurial activity (Vernon (1960)). It has been more recently established that because they carry out so many different tasks, a balance of skills may be beneficial to entrepreneurs (Lazear (2004, 2005)). This paper unifies these approaches to agglomeration and entrepreneurship. The paper's model of multidimensional task completion generates several interesting results. First, agglomeration economies arising from market thickness are reflected in shorter completion times. Second, complex projects that are infeasible in small cities may be feasible in large cities, where adaptation costs and completion times are lower. Third, it may be possible for less balanced entrepreneurs to manage successfully in large cities by substituting local market thickness for a balance of skills. Fourth, the Lazear result on the balance of entrepreneurs is shown to be related to Jacobs' (1969) classic result on urban diversity (city balance). Both are special cases of a more general sort of balance.

I. Introduction

It is well established that the thickness of local markets can foster entrepreneurial activity (Vernon (1960), Jacobs (1969)). It has been more recently established that because they carry out so many different tasks, a balance of skills may be beneficial to entrepreneurs (Lazear (2004, 2005)).[1] This paper unifies these approaches to agglomeration and entrepreneurship. Our analysis develops microfoundations for the value of thick markets and entrepreneurial balance, and shows the two concepts to be closely related. The Lazear result on balanced skills is shown to be related to Jacobs' classic result on the benefits of urban diversity, or balanced cities. Both are special cases of a more general sort of balance.

The paper begins by specifying a model of multidimensional projects involving many distinct tasks. Tasks require heterogeneous local inputs; a task is differentiated by the address of the local input that it requires. When local inputs do not exactly match task requirements, then inputs must be adapted. As noted above, we focus on the temporal aspect of this problem -- adaptation takes time. As distance in the characteristic space between task requirements and local resources increases, more adaptation time is required. The *ex ante* complexity of a project is defined by the number of tasks that it contains.

Each project is managed by an entrepreneur. Entrepreneurs differ in their ability to adapt available resources to task needs in a timely fashion. We consider both horizontal and vertical differentiation in entrepreneurial ability. In the former case, some entrepreneurs can effect more rapid adaptation for any task. In the latter case, entrepreneurs are in an aggregate sense equally adaptive, but differ in the tasks that they can adapt most rapidly. Entrepreneurs do not know the precise input requirements of tasks *ex ante*. They form expectations about adaptation time, and these expectations in turn inform the choice of project and task location.

The project's outcome is influenced by the city in which it takes place. In thicker urban markets less adaptation is required for any given task because the expected quality of the match between available resources and task requirements is higher. This is

[1] For analysis of thick markets and innovation, see also Duranton and Puga (2001), Helsley and Strange (2002), and Strange et al (2006) among others.

conventional. What is unconventional is that in this setting thickness translates into lower expected time to completion.

The model generates several interesting results. First, expected completion time is larger for more complex projects, *ceteris paribus*. Second, agglomeration economies are reflected in shorter completion times: in a thicker local input market, the expected value of the largest order statistic of adaptation distance is smaller, resulting in higher expected project values. This temporal city size effect suggests a new dimension on which to search for evidence of the benefits of agglomeration: projects, especially highly complex projects, may be completed more quickly in large cities. Third, complex projects that are infeasible in small cities may be feasible in large cities, where adaptation costs and completion times are lower. This can lead to an urban hierarchy based on complexity, an interesting contrast to Christaller (1933). Fourth, it may be possible for lower ability entrepreneurs to manage successfully in large cities where adaptation costs are lower. Thus, thick local input markets may be a substitute for the entrepreneurial balance introduced by Lazear (2004, 2005) and examined further empirically by Wagner (2003, 2006), Silva (2007), Astebro et al (2008), and Astebro and Thompson (2009).

Finally, we show that the Lazear result on the balance of entrepreneurs is related to Jacobs' (1969) classic result on urban diversity (city balance).[2] Both are special cases of a more general sort of balance. One important implication of this is that unbalanced entrepreneurs may perform better than balanced entrepreneurs if their skills are complementary to the resources that are present in an unbalanced city. Similarly, unbalanced cities may perform better than balanced cities if the entrepreneurial population is unbalanced in a complementary way.

The paper contributes to several lines of research on agglomeration and entrepreneurship. First, the identification of the completion-time agglomeration economy is new to the microfoundations literature. There is a substantial body of work showing that agglomeration economies can manifest themselves in many sorts of productivity. These include the productivity of labor (wages), the productivity of land (rent), and the

[2] See also Vernon (1960) and Chinitz (1961) on the general importance of diversity.

shifting of the production function more generally.[3] None of this work contains formal analysis of the impact of agglomeration on the speed with which activities are completed, another dimension of productivity.[4] Second, the paper contributes to the substantial literature that has documented the spatial concentration of entrepreneurship and considered its foundations. For instance, Rosenthal and Strange (2003, 2005, 2009) document the spatial concentration of entrepreneurship within cities, while Figueirido et al (2002), Acs and Armington (2006), Glaeser (2007), and Glaeser and Kerr (2008) document the between city concentration of entrepreneurship. The analysis here suggests a new channel that helps to explain the observed relationships. Third, we contribute as well to the literature on the background of the entrepreneur. Lazear (2003, 2005) emphasizes that a background -- either educational or in business -- that contributes to balance leads to entrepreneurial success. Klepper and Buenstorf (2009) show that successful entrepreneurs in the tire industry tended to have backgrounds with successful firms in that industry. Hvide (2009) shows that a background in a large firm is associated with later entrepreneurial success. Our model emphasizes the interaction between the characteristics and background of the entrepreneur and the location in which the entrepreneurial activity takes place.

The remainder of the paper is organized as follows. Section II lays out the primitives of the model. Section III establishes a relationship between input market thickness and the viability of complex entrepreneurial projects. Section IV examines the role of balanced skills, and Section V considers balance in the thickness of local input markets. Section VI discusses extensions and concludes.

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[3] See Combes et al (2008), Lee (2008), Rosenthal and Strange (2008), and Bacolod et al (2009) for recent work on the relationship of agglomeration to wage. The relationship of agglomeration to rent is considered in Rauch (1993) and Dekle and Eaton (1999). For a recent treatment of agglomeration and productivity, see Henderson (2003)

[4] The closest is Duranton and Puga (2001), who establish the existence of a nursery city effect in a general equilibrium system of cities. This effect depends on the ability of young firms to learn, specifically to identify an ideal prototype, in a concentrated environment.

II. Model

A. Overview

The model considers an entrepreneur who chooses whether to initiate a multi-task project in a metropolitan area. The value of the project depends on the time required to complete its constituent tasks. Completion times, in turn, depend on the availability and characteristics of local resources and the task-specific abilities of the entrepreneur.

B. Entrepreneurs, projects and tasks

There are an arbitrarily large number of potential entrepreneurs. Each is randomly endowed with an idea for a project that may be possible to realize locally. We are thus assuming that the locations of potential entrepreneurs are fixed, and that the decision to become active depends on local economic conditions. The successful realization of a project requires completing $N > 1$ tasks. For example, if the project is the introduction of a new video game, the tasks might include conceptualization, financing, graphic arts, software development, video and audio capture and editing, production, and marketing. The number of tasks, N, characterizes the complexity of the project. We assume that projects are spatially indivisible in the sense that all tasks in a given project are performed in the same metropolitan area or region. We also assume that tasks are performed simultaneously. Potential entrepreneurs become active, that is, they initiate their projects, if the project provides an expected payoff that is greater than an exogenous outside option. We discuss spatial divisibility and sequenced projects in the Conclusion.

Each task requires a specialized input. All specialized inputs must be acquired locally. Let y_i, $i = 1,2,..,N$, describe the characteristic or ability of the local input that would be best suited to the completion of task i. Continuing with the video game example, the needs of this project in the "video and audio capture and editing" task would likely vary with the specifics of the project -- the genre of the game, details of the underlying software, the number of platforms it is to be produced for, and so on. Formally, we assume that y_i is an address on the unit circle. We also assume that y_i is unknown when the entrepreneur decides whether to initiate the project. The nature and source of this uncertainty is discussed below.

C. Payoffs

Let $C_i \geq 0$ represent the outlay or resource cost for task i, and $C = \sum_{i=1}^{N} C_i$ represent total cost for the project. C_i includes the cost of hiring a local input. Let $R_i \geq 0$ denote the revenue from task i, and $R = \sum_{i=1}^{N} R_i$ represent total revenue for the project. Assume that all costs are paid out of project revenue, and that no revenue is received until all tasks are complete.[5] Let t_i be completion time for task i, and $T = \max_i\{t_i\}$ be completion time for the longest task. Then the value of the project (at time 0) is

$$\pi = e^{-rT}(R - C), \tag{II.1}$$

where $r > 0$ is the discount rate. In this framework, the critical path for the project contains only the longest duration task. For this reason, we will refer to this as the critical task in the analysis that follows. Note that $\partial\pi/\partial T = -re^{-rT}(R - C) < 0$. So long as total revenues exceed total costs, project value is a decreasing function of completion time for the critical task.

Assume that the entrepreneur has logarithmic preferences $U(\pi) = \ln \pi$. Then, from (II.1) the entrepreneur's payoff is

$$U(\pi) = \ln(R - C) - rT. \tag{II.2}$$

Note that the payoff to the entrepreneur is linear in completion time for the critical task. As noted above, the entrepreneur becomes active if $E[U(\pi)]$ is at least as large as some reservation payoff level, U^0. Since we are assuming that entrepreneurs are immobile, and are exogenously endowed with ideas, the key determinant of the level of entrepreneurial

[5] These timing assumptions are made for convenience. One could easily include interim revenues and costs, non-specialized input choice (including effort), explicit initiation dates and durations for each task, and interim financing, for example.

activity at a particular location is the feasibility of the projects that arise from the local population.

D. Entrepreneurship and the adaptation of local inputs

Lazear (2004, 2005) argues that it is important for an entrepreneur to have "balanced" skills to "be sufficiently well-versed in a variety of fields to judge the quality of applicants," or "know enough about a field to hire specialists intelligently;" and to "bring together many different resources," or be able to "combine talents and manage those of others." Lazear's analysis focuses on labor market choice, and in particular on the choice between specialist and generalist occupations. He offers relatively little formal characterization of the role of the entrepreneur or the underlying entrepreneurial process.

The quotes given above suggest several alternatives. The entrepreneur's role may be fundamentally about the evaluation of specialized skills. To hire a good accountant, it is useful to know some accounting, to hire a good engineer, it is useful to know some engineering, and so on. To formalize this, one might consider a model where entrepreneurial ability reduces the noise around signals of unobservable input quality, for example.

Alternatively, the entrepreneur's role may be fundamentally about the management of specialized skills. Of course, there is no universally accepted theory of what "management" is, at least in a formal sense. However, it seems reasonable to assert that, in the present context, the process involves the manager or entrepreneur combining her skills with the characteristics of the specialized inputs to achieve a more desirable outcome. [6]

Our model formalizes the second, managerial interpretation of the entrepreneurial process. Specifically, we view the entrepreneur as an agent who uses her own skills to

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[6] Our conception of the role of an entrepreneur is related to Becker and Murphy's (1992) model of coordination and multi-task production. They note (p. 1144): "An important function of entrepreneurs is to coordinate different types of labor and capital: economists like John Bates Clark [1899] believed that this is their main function. Economic systems that encourage entrepreneurship would have lower costs of coordination, and presumably a more widespread division of labor among workers and firms." Our model could be interpreted as an examination of the impacts of thick markets on the costs of coordination.

adapt the best available local input to better meet the needs of a particular task. This type of management could involve giving direction (do this), instruction (here's how to do this), or facilitating communication with others (ask this other person). For any task, all of these activities could be enhanced by education or experience. This interpretation of entrepreneurship seems consistent with Lazear's "jack-of-all-trades" idea.

The local economy contains M specialized inputs. Each local input has a particular skill or ability x_j, $j = 1,2,...,M$, where x_j is also an address on the unit circle. We assume that specialized inputs are not congestible; each input can be assigned to more than one task without impacting its effectiveness. We also assume that local inputs are evenly spaced on the unit circle. Under these conditions M characterizes the thickness of the local input market. Later in the paper we allow the thickness of local input markets to vary by task.

The entrepreneur has task-specific managerial or adaptive skills. The ability of the entrepreneur to "manage" task i, that is, to adapt available resources to meet the needs of task i, is $b_i > 0$, $i = 1,2,...,N$. As discussed above, we assume that completion time for task i depends on the ability of the entrepreneur and on the amount of adaption that is required. Formally, let $d_i = \text{Min}_j|x_j - y_i|$ be the distance in the characteristic space between the best available local input and the needs of task i. We refer to d_i as the adaptation distance for task i. Completion time for task i is given by

$$t_i = d_i/b_i. \tag{II.3}$$

There are two key features of this specification. The first is that completion time is lower the higher is the skill of the entrepreneur, $\partial t_i/\partial b_i < 0$. This seems quite natural, and is almost a definition of task-specific entrepreneurial ability. Second, completion time is lower the closer is the match in the characteristic space between the skill embodied in the best available local input and skill that the task requires, $\partial t_i/\partial d_i > 0$. In other words, for given entrepreneurial ability, completion time is shorter when less adaptation of local inputs is required.

III. Complexity, thickness and entrepreneurial activity

A. The weakest link

In this section we assume that the entrepreneur's skills are balanced in the sense of being equally skilled at all tasks: $b_i = b > 0$ for all i. The consequences of unbalanced skills will be considered in the next section. With balanced skills, completion time for the critical task is completely determined by the worst of the best matches between available local inputs and task needs. From (II.3), completion time for the critical task is

$$T = \max_i\{t_i\} = (1/b) \max_i d_i, \tag{III.1}$$

and so, from (II.2), the entrepreneur's payoff is

$$U(\pi) = \ln(R - C) - (r/b) \max_i d_i. \tag{III.2}$$

There is a weakest link element to entrepreneurial payoff in this case -- project value is determined by the maximum adaptation distance, or by the weakest link in the chain of task and local input matches. Through this feature, the model bears some resemblance to the "O-ring" model of production presented in Kremer (1993) and explored further in the context of entrepreneurship by Astebro et al (2008).

B. Uncertainty

As noted above, we assume that the particular tasks needed to complete a project are unknown when the entrepreneur decides whether or not to initiate. This may be the result of intrinsic uncertainty about task requirements, or uncertainty about the physical or economic environment, or, perhaps most likely, a result of problems that arise after a project has been initiated. Consider, for example, a project that involves demolition of a building as one of its tasks. The need for special environmental remediation skills will generally not be known until that task in the project is underway. This means that the adaption distance d_i in (III.1) is a random, *ex ante*. To determine whether the entrepreneur chooses to initiate the project, we must calculate the expected value of the maximum adaption distance over all tasks in the project, that is, $E[\max_i d_i]$.

C. The distribution of adaptation distance

Assume that the y's are independent draws from a uniform distribution on the unit circle. With M evenly spaced local resources, the distance between adjacent resources is 1/M. The adaptation distance d_i must be smaller than the midpoint of the arc between any two resources, that is, $d_i < 1/(2M)$. For $d < 1/2$, there are two values of y on the unit circle satisfying $d_i = d$. Thus, the probability density of d_i equals 2 for $0 < d < 1/2$ and 0 otherwise. The density of $d_i = d$, conditional on $y \in (x-1/(2M), x+(1/2M))$, is $\Pr\{d_i = d, y \in (x-1/(2M), x+(1/2M)\}/\Pr\{y \in (x-1/(2M), x+(1/2M)\} = 2/(1/M) = 2M$. Thus, the density function of d_i is $f(d) = 2M$ for $0 < d < 1/(2M)$ and 0 otherwise. The associated distribution function is $F(d) = 2Md$ for $0 < d < 1/(2M)$ and 0 otherwise.

D. Completion time

The probability that the largest of N realizations of d_i takes on a value not larger than d is $F(d)^N$. This is the distribution of d_N, the largest order statistic of d_i. The density of d_N is $g(d) \equiv NF(d)^{N-1}f(d)$. Using the results given above, this can be written $g(d) = N(2Md)^{N-1}2M = 2^N M^N N d^{N-1}$, for $0 < d < 1/(2M)$, and 0 otherwise. The expected value of d_N, or the expected maximum adaptation distance, is thus

$$E[d_N] = 2^N M^N N \int_0^{1/(2M)} z^N dz = \frac{1}{2M}\frac{N}{N+1} \qquad \text{(III.3)}$$

The comparative statics of the expected maximum adaption distance play a key role in the analysis that follows. First, note that $E[d_N]$ is increasing in N,[7]

[7] Both N and M are integers. Our use of calculus is an approximation.

$$\frac{\partial E[d_N]}{\partial N} = \frac{1}{2M}\frac{1}{(N+1)^2} > 0. \tag{III.4}$$

Thus, as the number of tasks in a project increases, the expected value of the maximum adaptation distance rises, or the expected quality of the worst of the best matches between available resources and task needs declines.

Second, $E[d_N]$ is decreasing in M,

$$\frac{\partial E[d_N]}{\partial M} = \frac{-1}{2M^2}\frac{N}{(N+1)} < 0. \tag{III.5}$$

Thus, the expected value of the maximum adaptation distance decreases as the thickness of the local input market rises. This reflects the basic thick market matching benefit that has been developed elsewhere in the literature (Helsley and Strange (1990, 2002)).

From (III.1), these are also the comparative statics of expected completion time for the critical task: E[T] rises with project complexity, N, and declines with input market thickness, M. Intuitively, the time required to adapt local resources is expected to be larger for a more complex project, but smaller in a thick local market, where the match between task needs and available resources is better. For the record, with balanced entrepreneurial ability,

$$E[T] = \frac{1}{2Mb}\frac{N}{N+1}. \tag{III.6}$$

This analysis is summarized in the following proposition:

Proposition 1 (Completion Time, Complexity and Thickness): Expected completion time rises with the complexity of the project, and decreases with the thickness of the local input market.

Proposition 1 identifies a new aspect of the agglomeration-productivity relationship: agglomeration lowers completion times. This temporal agglomeration economy can be added to the long list of productivity enhancements that have been associated with agglomeration. Despite its novelty, there is suggestive evidence in the literature consistent with a temporal dimension of agglomeration economies. Specifically, the vast body of evidence establishing the innovativeness of cities speaks to agglomeration's temporal advantages. This literature shows more patenting in cities (Carlino et al (2007)), the spatial localization of patenting (Jaffe et al (1993)), and the spatial localization of new product introductions (Audretsch and Feldman (1996)).[8] These sorts of innovative activities are quite properly modeled as races, contests where the first innovator reaps disproportionate rewards. Thus, the empirical results on innovation and agglomeration mean that innovators complete their innovative activities more rapidly when agglomerated.

E. Project value and feasibility

From (III.2) and (III.3), expected payoff for an entrepreneur with balanced skills is

$$E[U(\pi)] = \ln(R - C) - \frac{r}{2Mb}\frac{N}{N+1}. \tag{III.7}$$

The comparative statics of $E[U(\pi)]$ follow directly from the results given above:

$$\frac{\partial E[U(\pi)]}{\partial N} = \frac{-r}{2Mb}\frac{1}{(N+1)^2} < 0, \tag{III.8}$$

$$\frac{\partial E[U(\pi)]}{\partial M} = \frac{r}{2M^2 b}\frac{N}{(N+1)} > 0. \tag{III.9}$$

[8] See Audretsch and Feldman (2004) for a survey.

Since expected completion time is longer for more complex projects, the payoff to the entrepreneur is decreasing in complexity, N. Since expected completion time is shorter in thicker local input markets, the payoff to the entrepreneur is increasing in market thickness, M.

The entrepreneur proceeds with a project if $E[U(\Pi)] \geq U^0$, or if

$$E[U(\pi)] = \ln(R - C) - \frac{r}{2Mb}\frac{N}{N+1} \geq U^0. \tag{III.10}$$

As noted above, entrepreneurial activity in this framework is determined by the feasibility of projects that arise locally. (III.10) implicitly defines a locus N(M), relating the complexity of feasible projects to the thickness of the local input market for a given level of entrepreneurial ability. By the implicit function theorem, $dN/dM = N(N + 1)/M > 0$, and $d^2N/dM^2 = 2N^2(N + 1)/M^2 > 0$. Thus, the complexity-market thickness locus is upward sloping and convex in (M,N) space, as shown in Figure 1.

N(M) describes the complexity of the marginally feasible project, for any level of input market thickness. Projects above the N(M) locus are too complex for the local input market. For these projects, the amount of input adaption that is anticipated for the critical task makes expected completion time so long that the project is uneconomical. As the input market becomes thicker (moving to the right in the figure), the expected maximum adaptation distance decreases, as does the amount of adaptation required, and thus the expected completion time. If $\ln(R - C) - U^0 \geq 0$, then there is a level of local input market thickness at which even the most complex project becomes feasible in this model.

These results are summarized in the following proposition:

Proposition 2 (Feasibility, Complexity and Thickness): The complexity of feasible projects rises with the thickness of the local input market. For any level of local input market thickness, there exists a critical level of complexity such that less complex projects are feasible at that location, while more complex projects are not.

An increase in entrepreneurial ability, b, pivots the N(M) locus upward. Thus, with a higher level of entrepreneurial ability, some projects that were previously too complex for the local input market become feasible. Alternatively, with a lower level of entrepreneurial ability, projects of a given complexity require a thicker local input market to satisfy the feasibility requirement. In this sense, thick markets can be a substitute for the ability of the entrepreneur. The relationship between entrepreneurial ability and market thickness is discussed in detail in the next section.

The result that complexity increases with city size suggests a kind of urban hierarchy, where larger cities and industry clusters contain more complex activities. This complexity-based hierarchy is obviously quite different than the classic internal economies-of-scale based central place theory offered by Christaller (1933).[9] The positive correlations between city size and education (Berry and Glaeser (2005)) or skills (Bacolod et al (2009)) and between agglomeration and innovation (as discussed above) are at least loosely consistent with our urban hierarchy result.

E. Extensions

One could consider other types of complexity and thickness effects in this model. For example, one could imagine an extension in which more complex projects generate higher revenue, causing R to increase with N directly, or involve higher costs, causing C to increase with N directly. Similarly, one could consider a model in which competition or congestion is stronger in thicker markets, causing R to decrease and C to increase with M. Any of these changes would alter the feasibility locus. Some combinations of these changes would, of course, have ambiguous impacts on N(M).

The various cases are detailed in Table 1. The table relates the slope of N(M) to the behavior of net revenue R – C with respect to complexity (N, in the rows) and thickness (M, in the columns). In the body of the table, subscripts represent partial derivatives: $R_N = \partial R/\partial N$, and so on. From (III.7), the slope of the N(M) locus in the general case is

[9] See also the shopping based central place theory model of Eaton and Lipsey (1982).

$$\frac{dN}{dM} = -\frac{\left(\frac{\partial R/\partial M - \partial C/\partial M}{R - C}\right) - r\frac{\partial E[T]}{\partial M}}{\left(\frac{\partial R/\partial N - \partial C/\partial N}{R - C}\right) - r\frac{\partial E[T]}{\partial N}}, \tag{III.11}$$

where E[T] is given by (III.6). As noted above, $\partial E[T]/\partial N > 0$, while $\partial E[T]/\partial M < 0$.

The basic complexity-market thickness effect described in Proposition 2 corresponds to $dN/dM > 0$. More complex projects are feasible only in thick markets. The table makes clear that the result depends on assumptions about the relationship of R and C to N and M. The feasibility relationship between N and M at the heart of Proposition 2 can change when additional effects are considered. For example, if net revenue increases strongly with complexity (the first or third elements of the first row of the table), then one does not need a thicker input market in order to satisfy the feasibility requirement. In fact, in these cases, there is a minimum level of complexity at which a project is feasible in a market of given thickness (the feasible region lies above the downward sloping N(M)). As the market gets thicker, this minimum level declines, so in a larger city less complex projects are feasible; stated differently, in this case, small cities can support only complex projects. As the table shows, a negatively sloped feasibility locus can arise in other circumstances as well. Given the tendency for the economies most skilled workers to be found in large cities (Bacolod et al, 2008), this case does not seem to be an apt description of the spatial division of activities.

In any case, the key implication of the hierarchy result in Proposition 2 is based on one force that makes thick markets suitable locations for complex projects. Even if there are other forces at work, the complexity-thickness relationship continues to operate.

IV. The balance of skills

Suppose now that the ability of the entrepreneur to adapt local inputs varies by task within the project: $b_i \neq b_j$ for $i,j \in \{1,2,\ldots,N\}$. We will refer to this as the case of an entrepreneur with "unbalanced skills." With unbalanced skills, it is no longer true that completion time for the critical task is determined solely by the worst of the best matches between available local inputs and task needs. The task-specific skill of the entrepreneur matters as well.

Completion time for task i is given by (II.3). Since $0 \leq d_i \leq 1/(2M)$, it must be the case that $0 \leq t_i \leq 1/(2Mb_i)$. On this support, the probability that $t_i \leq t$ is

$$\Pr\{d_i/b_i \leq t\} = F(b_i t), \tag{IV.1}$$

where $F(\cdot)$ is the distribution of the adaptation distance d_i. Thus, the distribution of t_i is

$$G_i(t) = \begin{cases} 2Mb_i t & 0 \leq t \leq 1/(2Mb_i) \\ 1 & \text{otherwise} \end{cases} \tag{IV.2}$$

The distribution of completion time for the critical task is

$$H(t) \equiv \Pr\{T = \max_i\{t_i\} \leq t\} = \prod_{i=1}^{N} G_i(t). \tag{IV.3}$$

Index abilities so that $b_1 \geq b_2 \geq \ldots \geq b_N$, and let $\tau_i = 1/(2Mb_i)$, $i = 1,2,\ldots,N$, with $\tau_0 = 0$. Then, using (IV.2) and (IV.3), we can write the distribution of completion time for the critical task as

$$H(t) = \begin{cases} \dfrac{t^{N+1-i}}{\prod_{j=i}^{N} \tau_j} & \tau_{i-1} \leq t \leq \tau_{i+1},\ i = 1,2,\ldots,N \\ 1 & \text{otherwise} \end{cases}. \tag{IV.4}$$

Expected completion time for the critical task becomes

$$
\begin{aligned}
E[T] = & \sum_{i=1}^{N} \int_{\tau_{i-1}}^{\tau_i} \frac{(N+1-i)t^{N-i}}{\prod_{j=i}^{N} \tau_j} \\
= & \sum_{i=1}^{N} \frac{N+1-i}{N+2-i} \frac{1}{\prod_{j=i}^{N} \tau_j} \left(\tau_i{}^{N+2-i} - \tau_{i-1}{}^{N+2-i} \right). \qquad \text{(IV.5)} \\
= & \frac{1}{2}\tau_N + \sum_{i=1}^{N-1} \frac{1}{(N-i+1)(N-i+2)} \frac{\tau_i{}^{N+1-i}}{\prod_{j=i+1}^{N} \tau_j}
\end{aligned}
$$

The first expression in (IV.5) follows directly from (IV.4). The third expression is derived from the second by collecting terms.[10]

In what follows, we will focus on projects with N = 2 tasks; the general case is discussed in the Appendix. For N = 2, using $\tau_i = 1/(2Mb_i)$, (IV.5) gives

$$E[T] = \frac{3b_1{}^2 + b_2{}^2}{12b_1{}^2 b_2 M} \ . \qquad \text{(IV.6)}$$

With unbalanced skills, expected completion time for the critical task decreases with market thickness M, and with the task-specific abilities, b_1 and b_2. For the record,

$$\frac{\partial E[T]}{\partial M} = -\frac{3b_1{}^2 + b_2{}^2}{12b_1{}^2 b_2 M} < 0, \qquad \text{(IV.7)}$$

$$\frac{\partial E[T]}{\partial b_1} = -\frac{b_2}{6b_1{}^3 M} < 0, \qquad \text{(IV.8)}$$

[10] As a check of the algebra, setting $b_i = b$, and thus $\tau_i = 1/(2Mb)$, for all i > 0, all of the expressions in (IV.5) give E[T] = (1/2Mb)(N/(N + 1)), as in (III.6).

$$\frac{\partial E[T]}{\partial b_2} = \frac{-3b_1^2 + b_2^2}{12b_1^2 b_2^2 M} < 0, \tag{IV.9}$$

where the last inequality follows from $b_1 \geq b_2$.

To examine the impact of unbalanced entrepreneurial skills, it is useful to compare entrepreneurs with a fixed aggregate ability β that is divided over the tasks of a project in different ways. Specifically, we now examine whether there is a particular distribution of skills that minimizes completion time for the critical task, subject to the constraint that $\sum_{i=1}^{N} b_i = \beta$. For the N = 2 case, the first-order conditions for this problem require that the partial derivatives in (IV.8) and (IV.9) be equal, and that the constraint be satisfied. A bit of algebra then gives $b_1 = b_2 = \beta/2$.[11] Thus, expected completion time for the critical task is minimized when the skills of the entrepreneur are balanced.

This analysis is summarized in the following proposition:

Proposition 3 (Completion Time and Balanced Skills): Expected completion time is minimized when entrepreneurial skills are balanced.

This model can be seen as providing a microfoundation for Lazear's jack-of-all-trades model of entrepreneurial labor choice. In Lazear's case, the advantage of balance arises from the assumption that output for an entrepreneurial firm is limited by weakest of the skills of an entrepreneur. The mechanism in this model is quite different.
Intuitively, since the entrepreneur does not know *ex ante* which of the project's tasks will be critical, there is an expected advantage associated with a balanced distribution of skills. Indeed, with perfectly balanced skills (and with balanced input markets, a maintained hypothesis thus far which we will shortly relax) expected completion time for every task is the same. Then, any movement away from balanced skills increases expected completion time for some task, and thus for the critical task. For a specialized entrepreneur, there is a risk that the task that will require the greatest input adaptation will

[11] The constrained objective, $(\beta^2 - 2\beta b_1 + 4b_1^2)/(12(\beta - b_1)b_1^2M)$, is strictly convex at $b_1 = \beta/2$.

be one with which he has little ability or experience. Thus, expected completion time for the critical task will be higher for an entrepreneur with more specialized, or less balanced, skills.

The relationship between completion time and the balance of skills has interesting implications for the location of entrepreneurial activity. First, note that the marginal impact of input market thickness is greater when entrepreneurial skills are unbalanced. To see this, differentiate the expression for E[T] from (IV.6):

$$\partial E[T]/\partial M = (-1/M)E[T] < 0. \tag{IV.10}$$

Since E[T] is minimized when skills are balanced by Proposition 3, (IV.10) implies that the decrease in expected completion time is smallest when skills are balanced. This in turn implies that the difference in expected payoff between an unbalanced and a balanced entrepreneur grows smaller as the thickness of the local input market increases, and that the complexity-market thickness locus in Figure 1 is steeper for an entrepreneur with unbalanced skills.

This analysis is summarized in the following proposition:

Proposition 4 (Balance and Thickness): The benefit at the margin of locating in a thicker market is larger for an entrepreneur with unbalanced skills.

This result, coupled with our prior results on agglomeration and completion time, is consistent with the broad literature showing a positive relationship between agglomeration and entrepreneurial activity. The city-level dimensions of the relationship are considered by Figueirido et al (2002), Acs and Armington (2006), Glaeser (2007) and Glaeser and Kerr (2008). The neighborhood dimensions are considered by Rosenthal and Strange (2003, 2005, 2009). The analysis here suggests a new channel that helps to explain the observed relationships. Of course, there are many other effects also at work, such as the spinoff mechanisms set out by Sorenson and Audia (2000) and by Klepper (2007).

V. The balance of cities

Following Lazear (2004, 2005), the analysis thus far has focused on the balance of entrepreneurs. There is another sense in which balance may be relevant: the balance of cities and the input markets that they contain. Jacobs (1969) argues that diverse cities are more likely to generate certain entrepreneurial activities than are specialized cities. This idea also appears in Vernon's (1960) work on external economies and in Chinitz's (1961) comparative analysis of New York and Pittsburgh. There is considerable econometric support for the view that diversity can be conducive to growth (see the review in Rosenthal and Strange (2004)).[12] We will therefore now explore the relationship between the balance of a city and the balance of an entrepreneur.

It is again helpful to simplify by supposing that $N = 2$. As before, a balanced entrepreneur has the same ability for all tasks, which we label b. However, we now allow the thickness of input markets to vary by task: there are M_1 local inputs available to be matched with the needs of task 1, and M_2 local inputs available for task 2. If $M_1 \neq M_2$, we say that the city, or more precisely, its input markets, are unbalanced. It has already been established that balanced entrepreneurs' completion times are shorter than are the completion times of unbalanced entrepreneurs. When one admits the possibility that cities are unbalanced, then this result must be qualified: balanced entrepreneurs are fastest in balanced cities.

In unbalanced cities, a more nuanced relationship emerges. As before, choose the index set so that $M_1 \geq M_2$. The market is thus (weakly) thicker for task 1 than for task 2. Following the argument from section IV, the range of possible completion times for task is now $[0, 1/(2bM_i)]$. Expected completion time is given by (IV.5), with $\tau_i = 1/(2bM_i)$:

$$E[T] = \frac{3M_1^2 + M_2^2}{12bM_1^2M_2} \tag{V.1}$$

[12] In addition to considering diversity, Chinitz also looked posited a negative relationship between local industrial concentration and local entrepreneurship and growth. See Rosenthal and Strange (2009) and Glaeser et al (2009) for more recent econometric treatments of this issue.

E[T] is now decreasing in balanced ability, b, and the task-specific market thickness measures, M_1 and M_2.

To consider the effects of the balance of a city on expected completion time, suppose that there exists a fixed level of thickness μ that can divided across tasks: $M_1 + M_2 = \mu$. When $M_1 = M_2 = \mu/2$, the city is as balanced as it can be in the sense that equal resources are available to both of the project's tasks. The city is diverse, in the usage of Jacobs (1969) and her followers. Proceeding as before, the first-order conditions for a minimum of (V.1) subject to $M_1 + M_2 = \mu$ imply $M_1 = M_2 = \mu/2$. Put as a proposition, we have:

Proposition 5 (Completion Time and Balanced Cities): With balanced entrepreneurs, expected completion time is minimized when cities are also balanced.

Proposition 5 is a traditional diversity-is-valuable result in the spirit of Jacobs (1969). This proof, however, exposes an implicit assumption in the Jacobs argument. The result is that a less diverse city results in longer completion times <u>when entrepreneurs are balanced</u>. This leaves open the question of what the effect would be of urban diversity when the entrepreneurs are not balanced.

To consider this issue, we now suppose that neither the entrepreneurs nor the city are necessarily balanced. As above, choose the index set such that $b_1M_1 \geq b_2M_2$. Following exactly the logic that underlies Propositions 3 and 5, we obtain the following:

Proposition 6 (Completion Times and General Balance): Expected completion times are minimized when the product of entrepreneurial skill and input market thickness are equal for all tasks: $b_iM_i = b_jM_j$ for all i and j.

Proposition 6 implies that there is a general sort of balance capturing both the balance of entrepreneurs and of cities, and it is this sort of balance that produces efficient entrepreneurial activity. In other words, Lazear's (2004, 2005) result on the benefits of entrepreneurial balance and Jacobs' (1969) analysis of the benefits of urban diversity are special cases of a more general sort of balance or diversity.

This finding has direct implications for the relationship of entrepreneur and city balance. Suppose that we have $M_1 > M_2$. Proposition 6 implies that completion times are now minimized for an entrepreneur who is unbalanced in a particular way: $b_1 = (M_2/M_1)b_2$. Since $M_2 < M_1$, this means that $b_1 < b_2$. In this situation, an unbalanced entrepreneur is optimal in the sense of being the best fit for the particular market thickness patterns. This result could be obtained in reverse by taking the entrepreneur's abilities as given and maximizing the allocation of a total amount of thickness across project tasks. Either way, the result makes it clear that balance is valuable to the extent that it complements the economic environment in which an entrepreneur operates. Returning to the video game example from earlier in the paper, in an environment that is rich in programmers but poor in graphic designers, an entrepreneur would do well to have the graphic design skills that are complementary to the local environment. Equivalently, the most successful entrepreneurs in a programmer-rich environment are likely to be those with complementary abilities, such as marketing and management (i.e., Bill Gates or Steve Jobs).

VI. Conclusions

This paper has analyzed a model unifying the notions of entrepreneurial balance and market thickness. Balance is shown to improve the management of entrepreneurial activities by allowing more rapid completion of projects. This effect is particularly important for complex, multi-dimensional projects. Market thickness can have a similar effect, with projects that are not viable in a small city able to generate profits in a large city. There is, thus, a natural hierarchy of cities by the complexity of the activities that they contain. Thicker markets can also allow less balanced entrepreneurs to operate profitably. Finally, the balance of entrepreneurs is shown to be closely related to the more familiar idea of urban diversity.

Several extensions of the analysis are worth discussing. First, Proposition 2's hierarchy result is built on the assumption that the geography of entrepreneurship is driven by feasibility. The nature of a location's entrepreneurial activity (i.e., complex projects or simple ones; balanced entrepreneurs or unbalanced ones) depends the extent to which the location's characteristics enhance entrepreneurial profitability, and thus feasibility. We believe this approach to be consistent with evidence of entrepreneurial fixity, such as Sorenson-Audia (2000) and Klepper (2007) on spinoffs. An alternate approach would be to suppose that entrepreneurs are mobile, choosing the most profitable locations. In this setup, the allocation of entrepreneurs to locations would be governed by a bid-rent process, with the entrepreneurs who benefit the most from thickness outbidding those who benefit less. This would tend to result in more complex projects and less balanced entrepreneurs being willing to pay more for thicker markets. This is consistent with Proposition 2's hierarchy result in that there will be a complexity - thickness relationship. It is inconsistent to the extent that less thick markets would tend to host less complex projects that would not justify paying the costs associated with thick markets. So there would not be a strict hierarchy in this case.

Second, the model is built on the assumption that all of the project's tasks take place in one city. This again seems to us to be the correct reading of the entrepreneurial spin-offs literature, as discussed above. However, later in the life cycle of a firm, there may be opportunities to geographically decentralize, with different tasks taking place in different locations. In such a situation, the paper's complexity hierarchy will be slightly modified. Instead of thicker markets containing the most complex projects, they would contain the most complex tasks of a given project.

Second, the model is also built on the assumption that all of the project's tasks are carried out simultaneously. This means that the project's critical path *ex post* is the task that takes the longest to complete. Suppose instead that there is some sequencing of project tasks, with groups of tasks organized into phases and all of a phase's tasks being required before the initiation of the next phase. In this case, the project's critical path is defined by the slowest task in each of its stages. The weakest-link results discussed above will continue to hold, although in modified form. A more substantial difference would arise if it were possible that during the completion of early tasks, learning took place in such a way that it might not be worth ultimately completing the project. In this case, there would be completion options as in Bar-Ilan and Strange (1998). In such a situation, the presence of options would produce convex objective functions, which would tend to increase the viability of unbalanced entrepreneurs.

Appendix

At several points in the paper we have considered special cases of the problem of minimizing E[T] in (IV.5) subject to a linear constraint. In Section IV, where input markets are balanced, but skills are not, $\tau_i = 1/(2Mb_i)$, the constraint was $\sum_{i=1}^{N} b_i = \beta$, and the minimum occurred at $b_1 = b_2 = \beta/2$. In Section V, where skills are balanced, but input markets are not, $\tau_i = 1/(2M_i b)$, the constraint was $\sum_{i=1}^{N} M_i = \mu$, and the minimum occurred at $M_1 = M_2 = \mu/2$. Both of these are instances of the problem of minimizing E[T] subject to $\sum_{i=1}^{N} \tau_i = T > 0$. The purpose of this appendix is to argue that the balanced outcome $\tau_i = T/N$ is a solution to the more general problem. With appropriate substitutions, this generalizes Propositions 3 and 5 to projects with $N > 2$ tasks.

The first-order conditions for the general problem require $\partial E[T]/\partial\tau_i = \partial E[T]/\partial\tau_j$ for all i and j, that this common value equal λ, the multiplier on the constraint $\sum_{i=1}^{N} \tau_i = T$, and that the constraint be satisfied. From the last expression in (IV.5), the derivatives in question are

$$\frac{\partial E[T]}{\partial \tau_1} = \frac{\tau_1^{N-1}}{(N+1)\prod_{j=2}^{N} \tau_j} \tag{A.1}$$

$$\frac{\partial E[T]}{\partial \tau_i} = \frac{\tau_i^{N-i}}{(N+2-i)\prod_{j=1+i}^{N} \tau_j} - \sum_{k=1}^{i=1} \frac{\tau_k^{N+1-k}}{\tau_i (N+1-k)(N+2-k)\prod_{j=k+1}^{N} \tau_j}, \quad 1 < i < N \tag{A.2}$$

$$\frac{\partial E[T]}{\partial \tau_N} = \frac{1}{2} - \sum_{i=1}^{N-1} \frac{\tau_i^{N+1-i}}{\tau_N (N+1-i)(N+2-i)\prod_{j=i+1}^{N} \tau_j} \tag{A.3}$$

Evaluating each of these expressions at a common value of $\tau_i = \tau$, we find $\partial E[T]/\partial\tau_i = \partial E[T]/\partial\tau_j = 1/(N + 1)$ for all i,j. The constraint then gives $\tau_i = T/N$. Thus, $\tau_i = T/N$ is an extremum of the problem. The second-order conditions for a minimum require that the determinants of the bordered principal minors of the Hessian matrix of second partial derivatives of the Lagrangian be negative at $\tau_i = T/N$.[13] Note that the idea behind the proof, namely that any movement away from balance increases expected completion time for at least one task, and therefore expected completion time for the longest task, applies to a project with any number of tasks.

[13] We presented the second-order condition for the N = 2 case in the text. For N = 3, the determinant of the second principal minor at $\tau_i = T/3$ is $-9/(2T)$; the determinant of the third principal minor at $\tau_i = T/3$ is $-243/(16T^2)$. For the N = 4 case, the determinant of the second principal minor at $\tau_i = T/4$ equals $-32/(5T)$, while the determinants of the third and fourth principal minors equal $-768/(25T^2)$ and $-16384/(125T^3)$, respectively. Thus, the second-order conditions are satisfied in these cases as well.

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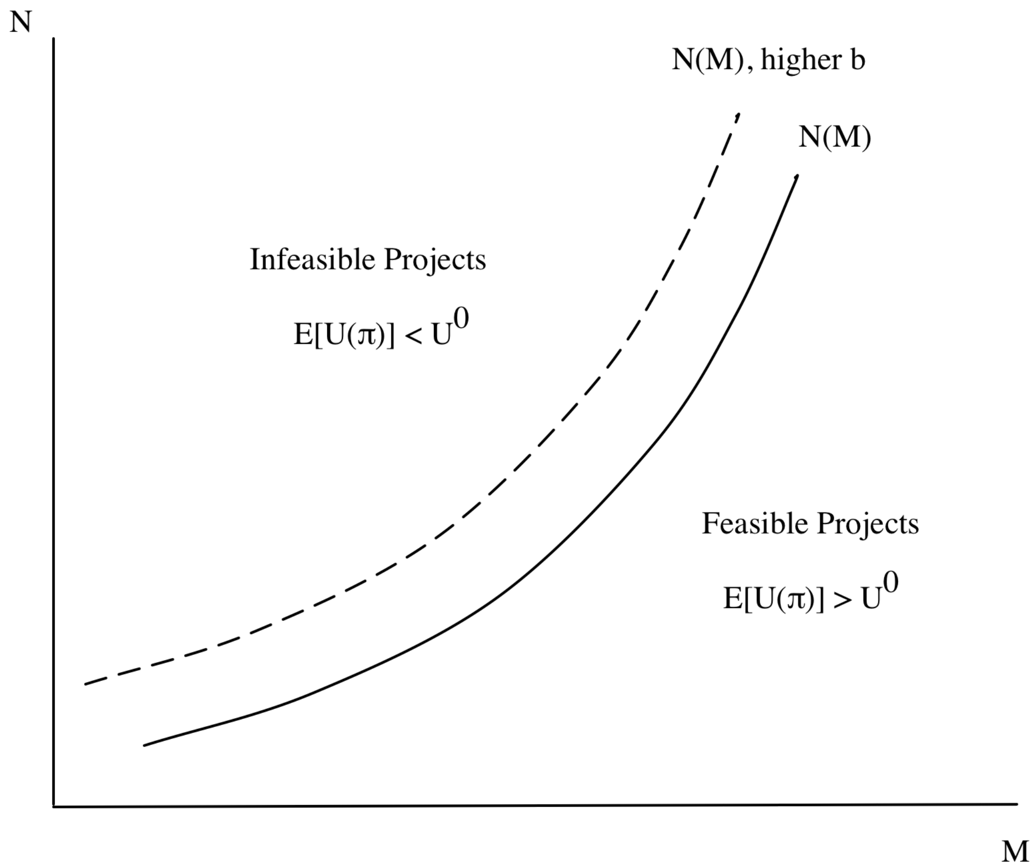

Figure 1: The Complexity-Market Thickness Locus

	Net revenue increases with market thickness (e.g., local demand) $\partial R/\partial M - \partial C/\partial M > 0$	Net revenue decreases with market thickness (e.g., opportunity cost of entrepreneurial skill; competition or congestion in input markets) $\partial R/\partial M - \partial C/\partial M < 0$	Net revenue is invariant to market thickness $\partial R/\partial M - \partial C/\partial M = 0$
Net revenue increases with complexity (e.g., output market effects) $\partial R/\partial N - \partial C/\partial N > 0$	$dN/dM > 0$ (< 0) if $(R_N - C_N)/(R - C) < rE[T]_N$ ($> rE[T]_N$)	$dN/dM > 0$ (< 0) if $(R_M - C_M)/(R - C) < rE[T]_M$ ($> rE[T]_M$) and $(R_N - C_N)/(R - C) < rE[T]_N$ ($> rE[T]_N$) or $(R_M - C_M)/(R - C) > rE[T]_M$ ($< rE[T]$) and $(R_N - C_N)/(R - C) > rE[T]_N$ ($< rE[T]_N$)	$dN/dM > 0$ (< 0) if $(R_N - C_N)/(R - C) < rE[T]_N$ ($> rE[T]_N$)
Net revenue decreases with complexity (e.g., monitoring costs) $\partial R/\partial N - \partial C/\partial N < 0$	$dN/dM > 0$	$dN/dM > 0$ (< 0) if $(R_M - C_M)/(R - C) > rE[T]_M$ ($<rE[T]_M$)	$dN/dM > 0$
Net revenue is invariant to complexity $\partial R/\partial N - \partial C/\partial N = 0$	$dN/dM > 0$	$dN/dM > 0$ (< 0) if $(R_M - C_M)/(R - C) > rE[T]_M$ ($< rE[T]_M$)	$dN/dM > 0$

Table 1: Other Complexity and Market Thickness Effects