## Repo Runs

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#### Abstract

This paper develops a model of financial institutions that borrow shortterm and invest into long-term marketable assets. Because these financial intermediaries perform maturity transformation, they are subject to potential runs. We endogenize the profits of such intermediaries and derive distinct liquidity and collateral conditions that determine whether a run can be prevented. We examine the microstructure of repo and similar markets in more detail and show that the collateral condition, and therefore the stability against runs, crucially depends on the market structure. The sale of assets can help to eliminate runs under some conditions, but because of cash-in-themarket pricing, this can become impossible in the case of a general market run.

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## 1 Introduction

This paper develops a model of financial institutions funded by short-term borrowing and holding marketable assets. We show that such institutions are subject to the threat of runs similar to those faced by commercial banks and study the conditions under which runs can occur. We argue that profits are a key stabilizing element against runs, endogenize the profits, and derive distinct liquidity and collateral conditions for such institutions. Both conditions must be violated for runs to occur, but we show that these conditions depend crucially on the microstructure of the market in which the borrowing takes place.

Our framework is general and can be used to study several types of financial institutions that use short-term borrowing as a main source of financing. Such institutions include money market mutual funds, hedge funds, off-balance sheet vehicles including asset-backed commercial paper (ABCP) conduits, and structured investment vehicles (SIVs). We apply our model to large securities dealers who use the tri-party repo market as a main source of financing. This market is particularly interesting because of the key role it played during the financial crisis of 2007-09. It played a role in the collapse of Bear Stearns, which was triggered by a run of its creditors and customers, analogous to the run of depositors on a commercial bank.<sup>2</sup> This run was surprising, however, in that Bear Stearns's borrowing was largely secured – that is, its lenders held collateral to ensure repayment even if the company itself failed. However, given the illiquidity of markets in mid-March, creditors may have lost confidence that selling the collateral would cover the funds lent. Many short-term lenders declined to renew their loans, driving Bear to the brink of default (Bernanke 2008). More generally, as noted by the Task Force on Tri-Party Repo Infrastructure (2009), "Tri-party repo arrangements were at the center of the liquidity pressures faced by securities firms at the height of the financial crisis". The creation of the primary dealer credit facility (PDCF) provided a backstop for the tri-party repo market.

We therefore first focus on the tri-party repo market and show how the

<sup>&</sup>lt;sup>2</sup>See Duffie (2010) for more details on the dynamics that can lead to the failure of a dealer bank.

settlement rules there can affect the fragility of dealers. We then compare the organization of the tri-party repo market to the bilateral repo market, which is characterized by a first-come-first-serve structure, and then extend the analysis to money market mutual funds and traditional banks, where such a constraint also plays a key role.

Our theory builds on the theory of commercial bank instability developed by Diamond and Dybvig (1983), Qi (1994), and others. In our view, there are important similarities between the fragility of commercial banking and securities trading. Our main goal is to exhibit and model these similarities, and to highlight the fundamental differences between securities dealers that borrow in the repo market against marketable securities as collateral and commercial banks that borrow unsecured deposits and hold nonmarketable loan portfolios.

In fact, as noted by Gorton and Metrick (2009), an important economic function of the tri-party repo market, and of repo markets more generally, is to perform maturity transformation. An overnight repo is a short-term liability that is backed by a longer-term asset in the form of a security. Triparty investors lend overnight repo and have access to their funds every morning, even if the securities that back the repos are not liquid. In "normal" times, maturity transformation is possible because there is a large number of tri-party lenders with largely independent needs for cash. On a given day, an individual lender may decide to "withdraw" its funds from the tri-party repo market by not rolling over the overnight loan. But in the aggregate, the amount of cash available in tri-party repos, in normal times in practice as in our model, will be stable by the law of large numbers.

The maturity transformation provided by tri-party repo contracts resembles the maturity transformation achieved by commercial banks. Banks offer demand deposit contracts that allow the depositors to obtain their funds whenever they want. Yet, banks typically hold long-term assets. The decision of a depositor not to withdraw her funds from the bank is similar to the decision of a repo lender to reinvest. The bank can provide a demand deposit contract because it knows that depositors are unlikely to all withdraw their funds at the same time, but it is nevertheless vulnerable to coordination failures. We show that the same vulnerability can arise in other arrangements performing maturity transformation. In fact, the kind of strategic complementarities that can lead to runs in our model have also been found empirically in other types of intermediaries, notably mutual funds (see Chen, Goldstein, Jiang, 2010).

An important objective of our paper is to investigate how this type of fragility depends on the market microstructure of the market under consideration. In this vein, our analysis in Section 5 shows that a particular institutional feature of the tri-party repo market, the "unwind" of repos by clearing banks, has a potentially destabilizing effect on the market. This finding lends theoretical support to the recent reform proposals by the Tri-Party Task Force of the New York Federal Reserve Bank to abolish the unwind procedure.<sup>3</sup>

Conceptually, a key contribution of our analysis is to endogenize profits of dealers and show how profits are important to reduce financial fragility. Dealers have the choice between funding securities with their own cash or with short-term debt. We derive a dynamic participation constraint under which dealers will prefer to fund their operations with short-term debt and show that this condition implies that dealers make positive profits in equilibrium. These profits can be used to forestall a run and thus serve as a systemic buffer. If current profits are insufficient to forestall a run, dealers can cut investment at the expense of future profits in order to generate further cash. Finally, a dealer can sell his assets to generate liquidity, potentially at a discount (Shleifer and Vishny, 1992). We investigate these reactions to potential runs and derive two constraints that can be interpreted as "liquidity" and "collateral" constraints and that are sufficient to prevent a run.

Our theory is based on a dynamic rational expectations model with multiple equilibria. However, unlike in conventional models of multiple equilibria, not "everything goes" in our model. The theory pins down under what conditions individual institutions are subject to potential self-fulfilling runs, and when they are immune to such expectations. Since the intermediaries in our model are heterogenous and the liquidity and collateral conditions are specific to each institution, the theory makes predictions about individual institutions, and equilibrium is consistent with observations of some institu-

<sup>&</sup>lt;sup>3</sup>See http://www.newyorkfed.org/tripartyrepo/.

tions failing and others surviving in case of changing market expectations.

Our paper is complementary to Gorton and Metrick (2009), who point out the similarity between traditional bank runs and repo market instability. In particular, they argue that Repo rates, collateral, and other features of "securitized banking", as they call it, have counterparts in commercial banking. However, Gorton and Metrick (2009) do not propose a formal model of securitized banking and thus cannot identify the determinants of profits, liquidity, and collateral vaue that are at the core of our analysis.<sup>4</sup> They document a large increase in haircuts for some repo transactions and argue that the rise in margins is akin to a run on the repo market. Their data refer to bilateral repos and do not include the tri-party repo market. Available data for the tri-party repo market, however, suggests that margins in the tri-party repo market did not increase much during the crisis, if at all. It appears that some tri-party repo investors preferred to stop financing a dealer rather than increase margins to protect themselves (see Task Force on Tri-Party Repo Infrastructure (2009) and Copeland, Martin, Walker, 2010). This is consistent with our model of expectations-driven runs in the tri-party market and in contrast to the type of margin spirals described in Brunnermeier and Pedersen (2009). The application of our model to bilateral repo markets in turn yields predictions similar to those of Brunnermeier and Pedersen (2009) and clarifies the distinction between increasing margins, which is a potentially equilibrating phenomenon, and runs, which can happen if increasing margins are insufficient to reassure investors. An important lesson of our analysis in Section 5 therefore is that the market microstructure of the shadow banking system plays an important role for the system's fragility.

The remainder of the paper proceeds as follows. Section 2 describes our model. Section 3 characterizes steady states without runs. In particular, we derive the dealers' dynamic participation constraint in this section and show that profits are positive. Section 4 studies the dealers' ability to withstand

<sup>&</sup>lt;sup>4</sup>Shleifer and Vishny's "Unstable Banking" (2010) formalizes some elements of securitized banking, but focusses mostly on the spillover of irrational investor sentiments into the securitized loan market. Rampini and Viswanathan (2010) examine a dynamic model of intermediary effects of bank capital and collateralizable assets on lending but do not examine the fragility of intermediaries' liabilities.

runs in terms of liquidity. Section 5 considers the fragility of different market microstructures and derives collateral constraints. Section 6 generalizes the liquidity constraint derived in Section 4 to the possibility of asset sales. Section 7 discusses extensions of the model in the form of market runs and liquidity provision. Section 8 concludes.

## 2 The Model

#### 2.1 Framework

We consider an economy that lasts forever and does not have an initial date. At each date t, a continuum of mass N of "young" investors is born who live for three dates. Investors are born with an endowment of 1 unit of goods, that they can invest at date t and have no endowment thereafter. Investors' preferences for the timing of consumption are unknown when born at date t. At date t + 1, investors learn their type. "Impatient" investors need cash at date t + 1, while "patient" investors do not need cash until date t + 2. The information about the investors' type and age is private, i.e. cannot be observed by the market. Ex ante, the probability of being impatient is  $\alpha$ . We assume that the fraction of impatient agents in each generation is also  $\alpha$ (the Law of Large Numbers).

The timing of the investors' needs of cash is uncertain because of "liquidity" shocks. In practice, money market investors, such as money market mutual funds, may learn about longer term investment opportunities and wish to redeploy their cash or they may need to generate cash to satisfy sudden outflows from their own investors. We do not model explicitly what investors do with their cash in the event of a liquidity shock and, for the remainder of the paper, simply assume that they value it sufficiently highly to want to withdraw it from the repo market at the given point in time.<sup>5</sup>

<sup>&</sup>lt;sup>5</sup>This assumption is as in Diamond and Dybvig (1983). As we shall show in the next section, together with a no-arbitrage assumption it implies that dealers are funded short-term. This argument is different from that of Diamond and Rajan (2001) who argue that short-term liabilities are a way to provide incentives to bankers who cannot commit to repay the proceeds of their investments to depositors. Kashyap, Rajan, and Stein (2008) also emphasize the role of short-term liabilities to provide incentives. For a critical

Their utility from getting payments  $(r_1, r_2)$  over the two-period horizon can therefore simply be described by

$$U(r_1, r_2) = \begin{cases} u_1(r_1) & \text{with prob. } \alpha \\ u_2(r_2) & \text{with prob. } 1 - \alpha \end{cases}$$
(1)

with  $u_1$  and  $u_2$  strictly increasing.<sup>6</sup>

Everybody in the economy has access to a one-period storage technology, which can be thought of as cash and returns 1 for each unit invested.

The economy is also populated by M infinitely-lived risk-neutral agents called dealers and indexed by  $i \in \{1, ..., M\}$ . Dealers have no endowments of their own but access to an investment technology, which we think of as investment in, and possibly the creation of, securities. These investments are illiquid in the sense that they cannot be liquidated instantaneously, and they are subject to decreasing returns, which we model simply by assuming that there is a limit beyond which the investment provides no returns.<sup>7</sup> Hence, investing  $I^t$  units at date t yields

$$\begin{cases}
R_i I^t & \text{if } I^t \leq \overline{I_i} \\
R_i \overline{I_i} & \text{if } I^t \geq \overline{I_i}
\end{cases}$$
(2)

with  $R_i > 1$  at date t + 2 and yields nothing at date  $t + 1.^8$  To simplify things, we assume that the return on these investments is riskless. In order to have a role for collateral in our model, we assume that the return is not verifiable. This means that investors cannot be sure that a dealer has indeed realized  $R_i I$  from his past investment. Although this is a probability 0 event, a dealer who has borrowed from investors can claim that he cannot repay the investors.

Investment returns can only be realized by the dealer who has invested in the asset, because dealers have a comparative advantage in managing their

assessment see Admati et al (2010).

<sup>&</sup>lt;sup>6</sup>We do not assume the traditional consumption-smoothing motive of the Diamond-Dybvig literature (concave  $u_t$ ), which would make little sense in our context.

<sup>&</sup>lt;sup>7</sup>All our results would continue to hold if the long-term technology required a small fixed cost per period.

<sup>&</sup>lt;sup>8</sup>The need to assume such capacity constraints (or more generally, decreasing returns) in dynamic models of liquidity provision has been pointed out by van Bommel (2006).

security portfolio. Other market participants only realize a smaller return. Investors could realize a return of  $\gamma R_i$  from these assets, with  $\gamma < 1$  and other dealers could realize  $\hat{\gamma} \in [\gamma, 1]$ .  $\gamma$  and  $\hat{\gamma}$  reflect different skills in valuing or managing the assets, possible restrictions on the outsider's portfolio composition, transactions and timing costs, and similar asymmetries.<sup>9</sup>

Dealers borrow the endowment of young investors to purchase, or invest in, securities. To make the model interesting, we must assume that the total investment capacity  $\overline{I} = \sum \overline{I_i}$  strictly exceeds the investors' amount of cash available for investment, N.<sup>10</sup> Without this assumption, there would be no competition among dealers for borrowing short-term cash from investors. Dealers could extract all the surplus from investors by simply offering to repay the storage return of 1 each period, and there would be no instabilities or runs. Instead of the condition  $\overline{I} > N$ , we assume the slightly stronger condition

$$\overline{I}_{-i} > N \tag{3}$$

where  $\overline{I}_{-i} = \sum_{j \neq i} \overline{I_j}$ . Hence no dealer is pivotal, and even if one dealer fails, there will still be competition for investor funds.

If dealer *i* in period *t* invests  $I_i^t$ , holds  $c_i^t$  in cash, borrows  $b_i^t$  from young investors, repays  $r_{1i}^t$  after one period or  $r_{2i}^t$  after two periods, impatient investors do not roll over their loans when middle-aged, but patient investors do, then the dealer's expected cash flow at any date  $\tau$  is

$$\pi_i^{\tau} = R_i I_i^{\tau-2} + c_i^{\tau-1} + b_i^{\tau} - \alpha r_{1i}^{\tau-1} b_i^{\tau-1} - (1-\alpha) r_{2i}^{\tau-2} b_i^{\tau-2} - I_i^{\tau} - c_i^{\tau}$$
(4)

The dealer's objective at each time t then is to maximize the sum of discounted expected cash flows  $\sum_{\tau=t}^{\infty} \beta^{\tau} \pi_i^{\tau}$ , where  $\beta < 1$ . In order to make the problem interesting, we assume that dealers are sufficiently patient and their long-term investment is sufficiently profitable:

$$\beta^2 R_i > 1. \tag{5}$$

 $<sup>^9 {\</sup>rm For}$  T-bills,  $\gamma$  should be very close to 1. But dealers typically also finance less liquid securities.

 $<sup>^{10}\</sup>mathrm{As}$  usual, all quantities are expressed per unit mass of investors.

Given the investors' preferences in (1), there is no scope for rescheduling the financing from investors. Hence, if  $\pi_i^t < 0$  the dealer is bankrupt, unless he is able to borrow from other dealers.

## 3 Steady-state without runs

As a benchmark, this section characterizes steady-state allocations in which in each period young investors lend their cash to dealers and withdraw their funds precisely at the time of their liquidity shocks. We assume that the Law of Large Numbers also holds at the level of the dealer: each period the realized fraction of impatient investors at each dealer is  $\alpha$ . Hence, in every period, each dealer obtains loans from young investors, and repays a fraction  $\alpha$  of middle-aged investors and all remaining old investors. Thus there is no uncertainty about dealers' cash flows, and each dealer's realized cash flow is equal to his expected cash flow (4).

Each period, dealers compete for investors' funds. Since dealers have a fixed investment capacity, they cannot make unconditional interest rate offers, but must condition their offers on the amount of funds they receive. The simplest market interaction with this feature is as follows.<sup>11</sup>

- 1. Dealers offer contracts  $(r_{1i}^t, r_{2i}^t, Q_i^t, k_i^t) \in \mathbb{R}^4_+, i = 1, ..., M.$
- 2. Investors  $j \in [0, N]$  choose a dealer *i* or none at all.

Here,  $r_{\tau i}^t$  is the (gross) interest payment offered by dealer i on  $\tau$ -period borrowing,  $Q_i^t$  the maximum borrowing for which this offer is valid, and  $k_i^t$  is the collateral posted per unit borrowed. Total borrowing by the M dealers then is  $(b_1^t, ..., b_M^t) \in \mathbb{R}^M_+$ , with  $b_i^t \leq Q_i^t$  for i = 1, ..., M and  $\sum b_i^t \leq N$ . Since investment returns are non-verifiable, the collateral posted must be sufficient to incentivize dealers to repay, i.e. to honor the repurchase leg of the repo transaction. At the time of the contract offer, the dealer owns collateral

<sup>&</sup>lt;sup>11</sup>Our analysis in this section would be unchanged if we assumed a competitive lending market, with competitive interest rates  $r_1$  and  $r_2$ . Explicit interest rate competition only becomes relevant in the later analysis of runs.

maturing one period later; hence, the dealer will prefer to repurchase the collateral instead of keeping his cash if

$$R_i k_i^t \ge r_{1i}^t \tag{6}$$

We will abstract from more complicated considerations of default and ex post bargaining,<sup>12</sup> and simply assume that collateral must satisfy (6). A steady state without a run is a collection of  $(r_{1i}, r_{2i}, k_i, b_i, I_i, c_i)$  for each dealer *i*, where  $b_i$  is borrowing,  $k_i$  collateral,  $c_i$  cash holding, and  $I_i \leq \overline{I_i}$  investment per dealer, such that no dealer would prefer another borrowing and investment policy and no investor another lending policy, given the behavior of all others.<sup>13</sup>

**Lemma 1** For each *i* with  $b_i > 0$ ,  $r_{2i} = r_{1i}^2$ .

**Proof:** Clearly,  $r_{2i} \ge r_{1i}^2$ , because otherwise investors would strictly prefer to never roll over their loans, regardless of their type. Patient middle-aged investors would withdraw their funds and then invest again with young investors. Suppose that this inequality is strict. In this case, an impatient middle-aged investor will optimally roll over the loan and at the same time borrow the amount  $r_{1i} + \varepsilon$  on the market at interest rate  $r_{1i} - 1$ . He can then claim back  $r_{2i}$  from the dealer one period later and repay his one-period loan  $(r_{1i} + \varepsilon)r_{1i}$  which is feasible and profitable if  $\varepsilon > 0$  is sufficiently small.

The proof is based on a simple no-arbitrage argument. It is different from the classical argument by Jacklin (1987) in the context of the Diamond-Dybvig (1983) model, because investors in our context do not have access to the long-term investment technology. It is also different from the argument by Qi (1994), who assumes and uses strict concavity of the investors' utility. In our market context, the no-arbitrage argument is natural and sufficient.<sup>14</sup>

 $<sup>^{12}</sup>$ See, e.g., Hart and Moore (1998) or von Thadden, Berglöf and Roland (2010).

<sup>&</sup>lt;sup>13</sup>The bound  $Q_i$  plays no role in steady state, because it only binds out of equilibrium. We therefore ignore it in the description of the steady state, where it can be thought of as being set to  $Q_i = \overline{I_i}$ .

<sup>&</sup>lt;sup>14</sup>In a market context, "early dyers" (as the Diamond-Dybvig literature calls them) do not die, and are perfectly able to transact after their liquidity shock.

Note that although Lemma 1 forces the yield curve to be flat, dealers still provide maturity transformation as long as  $r_{1i} > 1$ .

**Lemma 2**  $r_{1i} = r_{1j}$  for all dealers i, j with  $b_i, b_j > 0$ .

**Proof:** Suppose that  $r_{1i} < r_{1j}$  for some i, j with  $b_i, b_j > 0$ . Let  $\mathcal{J}_i$  be the set of all dealers k with  $r_{1k} > r_{1i}$  and  $b_k > 0$ . All  $k \in \mathcal{J}_i$  must be saturated, i.e. have  $b_k = Q_k$  (otherwise investors from i would deviate). Hence, any dealer  $k \in \mathcal{J}_i$  can deviate to  $r_{1k} - \varepsilon$  for  $0 < \varepsilon < r_{1k} - r_{1i}$  and strictly increase his profit.

By Lemma 2 the Law of One Price holds, and we can denote the single one-period interest rate quoted by all active dealers by  $r = r_1$ . Then the steady-state budget identity of dealer *i* is

$$R_{i}I_{i} + b_{i} = I_{i} + \alpha r b_{i} + (1 - \alpha)r^{2}b_{i} + \pi_{i}$$
(7)

where the left-hand side are the total inflows per period and the right-hand side total outflows.

Clearly, if  $R_i > 1$ , the higher is  $I_i$  the higher are profits.<sup>15</sup> We do not concern ourselves with showing how a steady state with  $I_i > 0$  would emerge if there were a startup period. But under our assumption (5) that dealers are sufficiently patient, it is clear that dealers have an interest in building up investment as far as possible.

We now characterize the steady states in which dealers invest by a sequence of simple observations.

#### **Lemma 3** In steady-state dealers do not hold cash: $c_i = 0$ for all *i*.

<sup>&</sup>lt;sup>15</sup>The literature has not always been clear about the distinction between investment capacity ( $\overline{I}$  in our model) and per capita borrowing (N/M). In particular, the implicit assumption that  $\overline{I} = N/M$  in Qi (1994), Bhattacharya and Padilla (1996) and Fulghieri and Rovelli (1998) is not necessary, and may even ignore interesting dynamic features. See van Bommel (2006) for an excellent discussion.

**Proof:** Since  $\beta < 1$ , and  $c_i > 0$  would not affect the dealer's budget constraint (7), each dealer does strictly better by consuming  $c_i$ .

**Lemma 4** If r > 1, total steady-state repo borrowing is maximal:  $\sum_{i=1}^{M} b_i = N$ .

**Proof:** The total supply of loanable funds is inelastically equal to N in each period if r > 1. The scarcity constraint (3) implies that there is a dealer who invests less than full capacity,  $I_i < \overline{I_i}$ . Suppose that  $\sum_{i=1}^{M} b_i < N$ . If i makes strictly positive profits, he strictly increases his profits by setting  $Q_i = \overline{I_i}$  and thus attracting more funds. If i makes zero profits, he can make strictly positive profits by reducing his interest rate marginally, setting  $Q_i = \overline{I_i}$ , and attracting the previously idle supply of funds.

**Lemma 5** If  $\pi_i > 0$ , steady-state investment of dealer *i* is maximal:  $I_i = \overline{I_i}$ .

**Proof:** Suppose the lemma is wrong. The dealer can then increase investment slightly at any date t by using his own cash. By condition (5), this yields a strict increase in discounted profits.

**Lemma 6** If there exists a dealer *i* with  $\pi_i > 0$  and  $b_i > 0$  then steady-state interest rates satisfy

$$(1-\alpha)\beta^2 r^2 + \alpha\beta r = 1 \tag{8}$$

**Proof:** For each unit of cash that dealer *i* borrows and invests at date t, he pays back  $\alpha r$  in t + 1, generates returns  $R_i$  in t + 2 and pays back  $(1 - \alpha)r^2$  in t + 2. Hence, his expected discounted profits on this one unit is  $\beta^2(R_i - (1 - \alpha)r^2) - \beta\alpha r$ . Alternatively he could invest his own cash. The discounted profits from not borrowing the one unit and rather investing his own money is  $\beta^2 R_i - 1$ . If the dealer borrows in steady state  $(b_i > 0)$  and has funds of his own  $(\pi_i > 0)$ , this cannot be strictly better, which implies  $(1 - \alpha)\beta^2r^2 + \alpha\beta r \leq 1$ .

Suppose that this inequality is strict. For an arbitrary dealer j, this means that

$$\beta^2 (R_j - (1 - \alpha)r^2) - \beta \alpha r > \beta^2 R_j - 1 \tag{9}$$

which is strictly positive by (5). Hence, all dealers strictly prefer to borrow up to the maximum. This contradicts (3), because the demand for funds would exceed supply.

We call condition (8) the dealers' "dynamic participation constraint". Basic algebra shows that its solution is  $\overline{r} = 1/\beta > 1$ . This makes sense: at the margin, dealers discount profits with the market interest rate. But it is interesting to note that  $\overline{r}$  does not depend on other supply and demand characteristics such as  $R_i$  and  $\alpha$ . Furthermore, the dynamic participation constraint implies that the marginal profit from borrowing is strictly positive. Since the profits from borrowing and from investing own funds are equal by (8), dealers make positive profits.

More formally, consider a steady state  $(r_{1i}, r_{2i}, k_i, b_i, I_i, c_i) = (\overline{r}, \overline{r}^2, k_i, b_i, \overline{I_i}, 0)$ , where  $k_i$  and  $b_i$  are free variables. In such steady states, profits are

$$\pi_i = (R_i - 1)\overline{I_i} - \left(\frac{\alpha}{\beta} + \frac{1 - \alpha}{\beta^2} - 1\right)b_i \tag{10}$$

$$\geq (R_i - 1)\overline{I_i} - \left(\frac{\alpha}{\beta} + \frac{1 - \alpha}{\beta^2} - 1\right)\overline{I_i}$$
(11)

$$= \left(R_i - \frac{\alpha}{\beta} - \frac{1 - \alpha}{\beta^2}\right)\overline{I_i}$$
(12)

Because  $\beta(R_i - (1 - \alpha)r^2) - \alpha r > 0$  for all *i* from (9) and (5), (12) is strictly positive. Hence, the assumption in Lemma 6 is consistent with its implication. We can therefore characterize steady states as follows.

#### **Proposition 1** In steady state,

- investors roll over their loans according to their liquidity needs,
- all dealers make strictly positive profits,
- $I_i = \overline{I_i}, c_i = 0, and r = \overline{r},$

• borrowing b<sub>i</sub> satisfies

$$b_i \le \frac{(1+\beta)\beta^2 R_i \overline{I_i}}{1-\alpha+\beta} \tag{13}$$

and is otherwise indeterminate,

• collateral k<sub>i</sub> satisfies

$$\frac{1}{\beta R_i} \le k_i \le \frac{(1+\beta)\beta \overline{I_i}}{(1-\alpha+\beta)b_i} \tag{14}$$

and is otherwise indeterminate.

**Proof:** Assume first that there exists a dealer with  $\pi_i > 0$  and  $b_i > 0$ . By Lemma 6  $r = \overline{r}$ . By (12) all dealers make strictly positive profits. Hence,  $I_i = \overline{I_i}$  for all *i* by Lemma 5. At rate  $\overline{r}$ , every dealer *i* is indifferent between borrowing and using his own cash  $\pi_i$  and thus finds it indeed optimal to borrow any positive amount  $b_i$ . Since  $\overline{r} > 1$  and all dealers pay the same interest, patient middle-aged investors find it indeed optimal to roll over their loans and young investors find it optimal to lend all their endowment.

The repurchase condition (6) implies the first inequality in (14). For the second inequality, note that in steady state the dealer has two types of securities to offer as collateral, those maturing at t + 1 or maturing at t + 2. Because  $r = 1/\beta$ , both dealers and investors value both types of securities identically. Hence, the maximum amount of collateral a dealer can pledge in steady state is  $\overline{I}(1 + \beta)$ , in terms of securities maturing at t + 1. The total amount of funds provided by investors per period is  $b_i [1 + (1 - \alpha)\overline{r}] =$  $b_i [1 - \alpha + \beta]/\beta$ . It follows that the maximum amount of collateral per unit borrowed that the dealer can offer is

$$\kappa_i \equiv \frac{\beta \bar{I}_i(1+\beta)}{b_i \left[1-\alpha+\beta\right]}.$$
(15)

The second inequality in (14) is the condition  $k_i \leq \kappa_i$ . Condition (13) is necessary for the two inequalities in (14) to be consistent.

Next suppose that there is no active dealer with  $\pi_i > 0$ . Hence,

$$(R_i - 1)I_i - (\alpha r + (1 - \alpha)r^2 - 1)b_i = 0$$
(16)

for all i, where r is the common interest rate by Lemma 2.

For borrowing to be positive, dealers must make non-negative marginal profits on each unit borrowed. This means that r must satisfy

$$\beta^2 (R_i - (1 - \alpha)r^2) - \beta \alpha r \ge 0 \tag{17}$$

for all *i*. It is easy to see that r > 1 in steady state, hence necessarily  $b_i \leq I_i$ for all *i*. By (16) this is equivalent to  $\alpha r + (1 - \alpha)r^2 \geq R_i$ . This however contradicts (17).

The steady states identified in Proposition 1 will serve as a benchmark for the rest of the analysis. An important and novel feature of these equilibria is that condition (8) prevents competition from driving up interest rates to levels at which dealers make zero profits. The reason why profits from short-term borrowing are positive is intuitive (but not trivial): dealers must have an incentive to use their investment opportunities on behalf of investors instead of using internal funds to reap those profits for themselves. This rationale of positive intermediation profits is different from the traditional banking argument of positive franchise values (e.g., Bhattacharya, Boot, and Thakor (1998), or Hellmann, Murdock and Stiglitz, (2000)), as it explicitly recognizes the difference between internal and external funds. Hence, the co-existence of internal and external funds and the internalization of all cash flows arising from them implies that financial intermediaries make positive profits.<sup>16</sup>

The steady states of Proposition 1 all feature maximum investment and the same interest rate  $\overline{r}$ , but dealers can differ in their short-term borrowing and the collateral they post. In fact, in steady state the exact amount of collateral, subject to constraint (14), plays no role because investors never comsume it. It is important nevertheless, because it makes sure that each period the cash changes hands as specified.

In steady state, the borrowing level  $b_i$  is only limited by the requirement that the dealer's steady state asset base must be sufficient to collateralize the borrowing. It is important to realize that in steady state dealers have no

<sup>&</sup>lt;sup>16</sup>This is different from Acharya, Myers, and Rajan (2010) where overlapping generations of bankers try to pass on the externality of debt.

incentive to change their borrowing, but that they may prefer other steady states. Hence, Proposition 1 is consistent with the notion that dealers can be "trapped" in an equilibrium with high short-term borrowing and low profits. In fact, as seen in (10), dealer profits are strictly decreasing in  $b_i$ . Therefore, to the extent that period profits act as a buffer against adverse shocks, as we show in the following sections, dealers with larger exposure to short-term borrowing will be more fragile.

## 4 Runs without asset sales

In this section, we study the stability of dealers in the face of possible runs. We analyze this problem under the assumption that behavior until date t is as in Proposition 1 and ask whether a given dealer can withstand the collective refusal of all middle-aged investors to roll over their loans and of young investors to provide fresh funds.<sup>17</sup> In the next section we will describe the specific microstructure of the tri-party repo market and other institutions that can make such collective behavior of investors optimal and thus imply that the corresponding individual expectations are self-fulfilling.

The key question is how much cash the dealer can mobilize to meet the repayment demands by middle-aged investors in such a situation. At the beginning of the period, a dealer, on the asset side of his balance sheet, holds  $R\overline{I}$  units of cash from investments at date t-2, as well as securities that will yield  $R\overline{I}$  units of cash at date t+1. The dealer holds maturing loans on the liability side of his balance sheet. In this section, we assume that the dealer cannot sell his assets.

The dealer's obligations from maturing loans in case of a run are  $(\overline{r} + (1-\alpha)\overline{r}^2)b_i$ . If there is no fresh borrowing in the run and new investment is maintained at the steady-state level  $\overline{I}$ , the run demand can be satisfied by the individual dealer if

$$(R-1)\overline{I} \ge (\overline{r} + (1-\alpha)\overline{r}^2)b_i \tag{18}$$

<sup>&</sup>lt;sup>17</sup>Note that in our infinite-horizon model, there are two sources of instability: middleaged investors may not roll over their funding and new investors may not provide fresh funds. The former corresponds to the classical Diamond-Dybvig problem, the latter arises only in fully dynamic models.

If (18) holds, a run would have no consequence whatsoever and all outof-equilibrium investor demand would be buffered by the dealer's profits. Anticipating this, investors have no reason to run. But more is possible. In the event of a run at date t, the cash position of the individual dealer who satisfies the run demand is

$$I_0 = R\overline{I} - (\overline{r} + (1 - \alpha)\overline{r}^2)b_i \tag{19}$$

Clearly, if  $I_0 < 0$  the dealer does not have the liquidity to stave off the run and is bankrupt. If  $I_0 \ge 0$ , but (18) does not hold, the dealer must adjust his borrowing or investment in order to survive the run. Since after a run in t + 1 the dealer will have  $R\overline{I}$  in cash and no debt to repay, he can resume his operations by investing  $\overline{I}$  at date t + 1 and save and invest thereafter. Whether he can attract fresh borrowing after t depends on the market, but this is immaterial for his survival.

The liquidity constraint, (20) in the following proposition, is obtained by simply writing out the condition  $I_0 \ge 0$  from (19).

**Proposition 2** In steady state, a run on dealer i can be accommodated if and only if the dealer's liquidity constraint holds, i.e. if

$$\beta^2 R_i \overline{I_i} \ge (1 - \alpha + \beta) b_i. \tag{20}$$

Condition (20) is strictly stronger than (13), in the sense that (20) can hold or fail if (13) holds. Hence, a dealer who makes positive profits in steady state may still fail in a run. The comparative statics of the liquidity constraint are simple and we collect them in the following proposition.

**Proposition 3** The liquidity constraint (20) is the tighter,

- the higher is the dealer's short-term borrowing  $b_i$ ,
- the lower is the dealer's investment capacity  $\overline{I_i}$ ,
- the lower is the dealer's productivity  $R_i$ .

Proposition 3 shows that if dealers have sufficient access to profitable investment ( $\overline{I_i}$  large), if these investment opportunities are sufficiently profitable ( $R_i$  large), or if they have sufficiently little exposure to short-term borrowing ( $b_i$  small), then dealers are more likely to be able to stave off runs individually, only by reducing their borrowing or investment temporarily. In this case, unexpected runs cannot bring down dealers out of equilibrium. If condition (20) is violated, a run would bankrupt the individual dealer if he cannot sell his illiquid assets.

# 5 Fragility

In this section, we examine different microstructures that are associated with repo markets or other money markets. We ask whether runs can occur in each of the institutional environments we consider. We focus on the tri-party repo market, but we also examine bilateral repos, money market mutual funds, and traditional bank deposits. We derive a collateral constraint for each market and show that if and only if the liquidity constraint and the collateral constraint are violated, then a run can occur for the particular market structure.

We study unanticipated runs that arise from pure coordination failures. As discussed in the previous section, in a run at date t all investors believe that i) no middle-aged investors renew their funding to dealer i, so the dealer must pay  $[\overline{r} + (1 - \alpha)\overline{r}^2] b_i$  to middle-aged and old investors, and ii) no new young investors lend to the dealer. We ask whether such beliefs can be selffulfilling in a collective deviation from the steady state.

Since the Law of One Price holds in steady state, a trivial coordination failure may induce all investors of a given dealer to switch to another dealer out of indifference. This looks like a "run", but is completely arbitrary. We will therefore assume that investors if indifferent lend to the dealer they are financing in steady state. Hence, in order for a collective deviation from the steady state to occur we impose the stronger requirement that the individual incentives to do so must be strict.

The first insight, which applies to all institutional environments considered in this section, is simple but useful to state explicitly: a run cannot occur if a dealer is liquid in the sense of Proposition 2.

**Lemma 7** If a dealer satisfies the liquidity constraint (20), there are no strict incentives to run on this dealer.

The proof is simple. In a run on this dealer, all middle-aged patient investors would be repaid in full regardless of what young investors do and without affecting the dealer's asset position. Hence, patient middle-aged and young investors are indifferent between lending to the dealer or to another one. By our assumption about the resolution of indifference, there is thus no reason to run in the first place. Intuitively, patient middle-aged investors would just "check on their money" before it is re-invested. Since the dealer has the money, such a check does not cause any real disruption, and the dealer may as well keep it until he invests in new securities.

### 5.1 The US tri-party repo market

This section briefly reviews the microstructure of the tri-party repo market and emphasize the key role played by the clearing bank.<sup>18</sup> In particular, we show that a practice called the "unwind" of repos leads to fragility in this market.

The clearing banks play many roles in the tri-party repo market. They take custody of collateral, so that a cash investor can have access to the collateral in case of a dealer default, they value the securities that serve as collateral, they make sure the specified margin is applied, they settle transaction on the repos on their books, and importantly, they provide intraday credit to dealers.

In the US tri-party repo market, new repos are organized each morning, between 8 and 10 AM. These repos are then settled in the afternoon, around 5 PM, on the books of the clearing banks. For operational simplicity, because dealers need access to their securities during the day to conduct

<sup>&</sup>lt;sup>18</sup>More details about the microstructure of the tri-party repo market can be found in Task Force (2010) and Copeland, Martin, and Walker (2010). The description of the market corresponds to the practice before the implementation of the 2010 reforms.

their business, and because some cash investors want their funds early in the day, the clearing banks "unwind" all repos in the morning. Specifically, the clearing banks send the cash from the dealers' to the investors' account and the securities from the investors' to the dealers' account. They also finance the dealers' securities during the day, extending large amounts of intraday credit. At the time when repos are settled in the evening, the cash from the overnight investors extinguishes the clearing bank's intraday loan.

From the perspective of our theory, we can model the clearing bank as an agent endowed with a large amount of cash. By assumption, the clearing bank can finance the dealer only intraday. At each date, the clearing bank finances dealers according to the following intra-period timing, which complements the timing considered in the previous section.:

- 1. The clearing bank "unwinds" the previous evening's repos. For a specific dealer i this works as follows:
  - (a) The clearing banks sends the cash amount  $b_i [\overline{r} + (1 \alpha)\overline{r}^2]$  to all investors of dealer *i*, extinguishing the investors' exposure to the dealer they have invested in.
  - (b) At the same time, the clearing bank takes possession of the assets the dealer has pledged as collateral.
  - (c) In the process, the clearing bank finances the dealers temporarily, holding the assets as collateral for its loan.
- 2.  $\overline{I}_i$  assets of a dealer mature (yielding  $R_i \overline{I}_i$  in cash), allowing the dealer to repay some of its debt to the clearing bank.
- 3. Possibly a sunspot occurs.
- 4. The dealer offers a new repo contract  $(\hat{r}_i, \hat{Q}_i, \hat{k}_i)$ .
- 5. New and patient middle-aged investors decide whether to engage in new repos with the dealer.
- 6. If the dealer is unable to repay its debt to the clearing bank, then it must declare bankruptcy. Otherwise, the dealer continues.

In this time line, we explicitly model the change of expectations that induces a run by a sunspot. This is a zero-probability event that allows investors to coordinate on a run, if such out-of-equilibrium behavior is optimal for them.<sup>19</sup> For simplicity, we assume that the clearing bank extends the intraday loan to the dealer at a zero net interest rate. Also, since runs are zero probability events the clearing banks has no reason not to unwind repos.<sup>20</sup>

In the tri-party repo market, traders choose only the interest rate applicable to the repo. The haircut for each collateral class is included in the custodial agreement between the investor, the dealer, and the clearing bank, and is not negotiated trade by trade. It is possible to change haircuts by amending the custodial agreement but this takes time. In practice, these changes appear to occur only rarely. We therefore assume that the contract offered in response to a sunspot must leave collateral unchanged from its steady state value,  $\hat{k}_i = k_i$ , from Proposition 1.<sup>21</sup>

In response to the contract offer by the dealer, individual investors must compare their payoff from investing with the dealer in question to that from investing with another dealer. The latter decision yields the common market return  $\overline{r}$ ,<sup>22</sup> the return from the former depends on what the other investors do. Table 1 shows the payoffs of the two decisions for the individual investor (rows) as a function of what the other investors do (columns), if the dealer is potentially illiquid (i.e. if the liquidity constraint (20) is violated). If the investor re-invests her funds with the dealer, the clearing bank will accept the cash, since it reduces its intraday exposure to the dealer, and give the investor assets that mature at date t + 1. These are the only assets available in case of a run since the clearing bank will not let the dealer invest in new securities

<sup>&</sup>lt;sup>19</sup>The sunspot also allows the dealer to react to the run. This adds realism to the model and makes runs more difficult to establish (because the dealer's contract offer in stage 4 can now be different from the steady-state offer  $(\bar{r}, Q_i, b_i)$ ).

<sup>&</sup>lt;sup>20</sup>In the appendix, we consider the coordination problem between the clearing bank and the investors.

<sup>&</sup>lt;sup>21</sup>Copeland, Martin, and Walker (2010) provide more details about haircuts in the triparty repo market. In particular, they document that haircuts moved very little during the crisis.

 $<sup>^{22}</sup>$ This is obvious if the investor is the only one to deviate, because then he is negligible. If all investors of the dealer in question deviate, this follows from the slack in assumption (3).

unless it obtains enough funding. Hence, in case of a run, an investor who agrees to provide financing receives securities that yield  $\gamma R_i k_i$  at date t + 1 if the dealer defaults.

	other investors		
	invest	don't	
invest	$\widehat{r}_i$	$\gamma R_i k_i$	
don't	$\overline{r}$	$\overline{r}$	

Table 1: Payoffs in tri-party repo with unwind

Hence, investors will finance the dealer (i.e., roll over their repo) in case of a run  $iff^{23}$ 

$$\overline{r} \le \gamma R_i k_i \tag{21}$$

Note that the investors' decision-making is completely dichotomous. If they anticipate a run, only collateral matters; if they anticipate no run, only interest matters. If condition (21) does not hold, the collective decision not to lend to the dealer in question is self-enforcing. In this case, the yield from the securities pledged as collateral is so low that an investor who believes that nobody will invest with dealer i would also choose not to invest. In our model, steady state collateral is not fully determinate, but clearly, if constraint (21) is violated for the maximum possible amount of collateral  $\kappa_i$ in (15), then it cannot hold in any case.

Combining the above results with those of the previous section and writing out condition (21) for  $k_i = \kappa_i$ , the maximum amount of collateral per unit borrowed, yields the following prediction about the stability of the tri-party repo market.

<sup>&</sup>lt;sup>23</sup>The weak inequality is due to the assumption that investors do not switch dealers if indifferent. If  $\overline{r} = \gamma R_i \kappa_i$ , there exists the trivial run equilibrium discussed at the beginning of this section.

**Proposition 4** In the tri-party repo market, a run on a dealer i can occur and bankrupt the dealer if and only if the dealer's liquidity constraint (20) and his collateral constraint

$$\beta^2 R_i \overline{I_i} \ge \frac{1 - \alpha + \beta}{\gamma(1 + \beta)} b_i \tag{22}$$

are both violated.

It can easily be seen that condition (22) is strictly stronger than (13), hence that there are steady states that violate (22) and others that satisfy it. Furthermore, conditions (20) and (22) are independent - neither of the two implies the other. As for the liquidity constraint derived in Proposition 2, the comparative statics of the collateral constraint for the tri-party model are simple and we collect them in the following proposition.

**Proposition 5** The collateral constraint (22) is the tighter,

- the lower is the liquidation value of collateral  $\gamma$ ,
- the higher is the dealer's short-term borrowing  $b_i$ ,
- the lower is the dealer's investment capacity  $\overline{I_i}$ ,
- the lower is the dealer's productivity  $R_i$ .

Hence, the comparative statics with respect to  $b_i$ ,  $\overline{I_i}$ , and  $R_i$  are identical for both constraints. Both constraints are relaxed if dealers have sufficient access to profitable investment ( $\overline{I_i}$  large), if these investment opportunities are sufficiently profitable ( $R_i$  large), or if they have sufficiently little exposure to short-term borrowing ( $b_i$  small). In this case, there is no reason for unexpected runs to occur on the investor side, and they cannot bring down dealers if they occur out of equilibrium. In the opposite case, a run can be a self-fulfilling prophecy and bankrupt the dealer.

#### 5.2 Tri-party repo without unwind

To highlight the importance of the unwind mechanism for the fragility of the tri-party repo market, it is interesting to consider what would happen to the game described in the previous section if there were no unwind.<sup>24</sup> This case is similar to the tri-party repo markets in continental Europe. It is also similar to the US tri-party repo market once the recommendation of the Task Force are implemented.<sup>25</sup>

When there is no unwind, the timing of events intraday is as follows:

- 1. Possibly a sunspot occurs.
- 2. The dealer offers a new repo contract  $(\hat{r}_i, \hat{Q}_i, k_i)$ .
- 3. New and patient middle-aged investors decide whether to engage in new repos with a dealer.
- 4. If the dealer is unable to repay his debt to last period's repo investors, he must declare bankruptcy. Otherwise, the dealer continues.

From Lemma 7 it is again enough to consider the case in which the dealer is illiquid after a run. The situation without the unwind facility differs in two important respets from the one with unwind. First, without the unwind, an individual investor is repaid  $\bar{r}$  if and only if the dealer can repay everybody - otherwise the dealer is bankrupt and repays everybody less than the contractual payment. Second, in contrast to the case with unwind, new and middle-aged investors are in a different situation when there is no unwind. New investors hold cash while middle-aged investors hold a repo with the dealer, until the dealer is able to repay his claim.

<sup>&</sup>lt;sup>24</sup>In this paper, we do not model why the unwind may be necessary. As described in Task Force (2010) and Copeland, Martin, and Walker (2010), the unwind makes it easier for dealers to trade their securities during the day. Collateral management technologies, as are currently used in continental Europe and are being proposed in the US, allow dealers to have access to their securities even as investors remain collateralized.

<sup>&</sup>lt;sup>25</sup>More information about the proposed change to settlement in the tri-party repo market can be found at http://www.newyorkfed.org/tripartyrepo/task\_force\_proposal.html.

In case of a run, an illiquid dealer is bankrupt. All middle-aged investors then keep their collateral and may obtain additional cash as unsecured creditors depending on the bankruptcy rules. This payment is independent of whether an individual investor has demanded to be repaid or has agreed to roll over his loan. Hence, middle-aged investors are indifferent whether to reinvest or not. Given the tie-braking rule assumed throughout this section, patient middle-aged investors therefore reinvest. This in turn induces young investors to invest with the dealer:

**Lemma 8** If middle-aged patient investors reinvest, investing is a (weakly) dominant strategy for new investors.

**Proof.** If middle-aged patient investors do not withdraw their funds, the dealer is not only liquid, but by Proposition 1 has enough assets that will mature in the future to satisfy all future claims by young agents who invest today. ■

Hence, when there is no unwind, the incentives of investors are modified so that they never have a strict incentive to run. In essence, this is because the overnight repo market is an institution that creates simultaneity: if a sufficiently large number of investors do not re-invest, there is bankruptcy and all current creditors (the middle-aged investors) are treated equally, regardless of their intention to withdraw funding. This eliminates fragility due to pure coordination failures.

**Proposition 6** In the tri-party repo market without unwind, there are no strict incentives to run on dealers.

#### 5.3 Bilateral repos

In this section, we apply our model to bilateral repos. Typically, bilateral repos have a longer term than tri-party repos. Hence, one period in our model should be thought of as representing a few weeks.<sup>26</sup> In terms of our

 $<sup>^{26}</sup>$ Also, a dealer may choose to stagger the terms of its repos, so that only a small portion of these repos are due on any given day. Because of the distribution of investor

assumptions this means that dealers can adjust the whole contract offer in response to a sunspot.

To simplify the exposition of institutional details, we consider a dealer that funds "Fed-eligible" securities; securities that can be settled using the Fedwire Securities Service<sup>®</sup>. Fedwire Securities is a delivery versus payment settlement mechanism, meaning that the transfer of the securities and the funds happen simultaneously. The settlement is triggered by the sender of securities and reserves are automatically deducted from the Fed account of the institutions receiving the securities and credited to the Fed account of the institution sending the securities.

This procedure creates a "first come first serve" constraint. In the case of a run, investors who send the securities they hold as collateral early are more likely to receive cash than investors who send their securities late. With bilateral repos, the timing is as follows:

- 1. Possibly a sunspot occurs.
- 2. The dealer offers a new repo contract  $(\hat{r}_i, \hat{Q}_i, \hat{k}_i)$ .
- 3. New and patient middle-aged investors decide whether to engage in new repos with a dealer.
- 4. Investors are repaid in the order in which they send back their collateral, until the dealer runs out of cash. From that point on, investors receive their collateral and any investor who chooses to invest receives his collateral.

The total amount of collateral available is as before. Yet, dealers can now reduce their borrowing level by changing  $\hat{Q}_i$ , which effectively allows them to increase the collateral per unit borrowed. In order to withstand the run, the dealer must at least cover the missing amount

$$m_i \equiv (\overline{r} + (1 - \alpha)\overline{r}^2)b_i - R_i\overline{I_i}$$
(23)

liquidity needs, this cannot happen in our model. He and Xiong (2010) analyze the consequences of (exogenously determined) staggered short-term debt for the stability of financial institutions.

At the time when he must pledge the collateral he has  $\overline{I_i}$  units, which will mature in t + 1. Hence, the maximum possible value of collateral per unit borrowed is

$$\overline{k}_i = \overline{I_i}/m_i. \tag{24}$$

From (23), only a fraction

$$\varphi \equiv \frac{R_i \bar{I}_i}{b_i \left[ \overline{r} + (1 - \alpha) \overline{r}^2 \right]} \in (0, 1).$$
(25)

of middle-aged investors can stop renewing their repos before the dealer becomes illiquid. With probability  $1-\varphi$ , the investor gets securities. As before, investors who are able to obtain their cash back can invest it with another dealer.

other investors			
	invest	don't	
invest	$\widehat{r}_i$	$\gamma R_i \widehat{k}_i$	
don't	$\overline{r}$	$\varphi \overline{r} + (1 - \varphi) \gamma R_i \widehat{k}_i$	

Table 2: Payoffs in bilateral repos

Table 2 gives the payoff to an individual middle-aged investor as a function of the collective behavior of all other investors. Comparing this table to Table 1 shows that the condition for a dealer to be runproof is again  $\gamma R_i \hat{k}_i \geq \overline{r}$ . However, since the dealer can increase his collateral beyond  $\kappa_i$ , this condition is less likely to be violated. Inserting  $m_i$  from (23) into (24) yields the following condition.

**Proposition 7** In bilateral repo markets, a run on a dealer i can occur and bankrupt the dealer if and only if the dealer's collateral constraint

$$\beta^2 R_i \overline{I_i} \ge \frac{1 - \alpha + \beta}{1 + \gamma \beta} b_i \tag{26}$$

is violated.

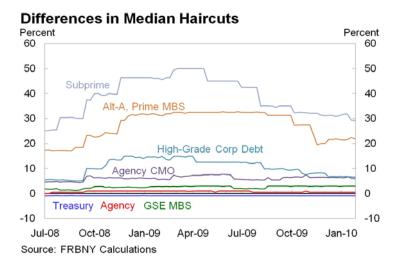


Figure 1: Differences in median haircut between bilateral and tri-party repos per asset class

As with condition (22), condition (26) is strictly stronger than (13), hence there are steady states that violate (26) and others that satisfy it. Furthermore, and differently from the tri-party case, condition (26) is strictly weaker than the liquidity constraint (20). Hence, if it is violated, (20) is violated as well. This means that (26) is necessary and sufficient for the stability of bilateral repos.

Finally, the bilateral collateral constraint is strictly weaker than the triparty constraint (22). This implies that there are dealers who are run-proof in the bilateral repo market but can fail in the tri-party market. In this sense, the tri-party market is more fragile than the bilateral market.

Our analysis of the bilateral market has assumed that collateral can adjust in response to a run and has shown that this can be achieved by reducing borrowing and is indeed optimal. This is consistent with the evidence in Gorton and Metrick (2009) of sharply rising haircuts during the crisis of 2008.<sup>27</sup> However, the behavior of haircuts was very different in the tri-party

<sup>&</sup>lt;sup>27</sup>If the price of the collateral (the loan size) is p and the market value of collateral is v, then the haircut is (v - p)/v.

and bilateral repo markets. Figure 1 provides some graphical evidence of this striking difference, taken from Copeland, Martin, and Walker (2010). In the tri-party repo market, haircuts barely moved while there were large increases in haircuts in the bilateral repo market. Lehman experienced a sudden reduction of funding in the tri-party repo market that led to its downfall with hardly any adjustment in haircuts. We are not aware of similar sudden losses of funding in the bilateral repo market. Instead, all institutions in this market saw a gradual increase in haircuts that reduced the amount of funding they could obtain (Gorton and Metrick, 2009). Our results in Sections 5.1 and 5.3 are consistent with these two different developments in the bilateral and tri-party repo markets.

### 5.4 Money market mutual funds

In this section, we adapt our model to the case of money market mutual funds (MMMFs) that can offer shares at a fixed net asset value (NAV). These funds are also known as 2a-7 funds, named after SEC rule 2a-7. MMMFs offer their investors shares that can be redeemed at a fixed price, typically \$1. Positive returns by the fund increases the number of shares, without affecting the price. If the fund loses value, however, the number of shares cannot decrease. In such a case, the fund is said to have "broken the buck" and is liquidated. Investors' shares give them a pro rata claim on the proceeds from the liquidation of the assets.

The fixed NAV makes MMMFs similar to banks since, under most circumstances, investors can obtain their funds on demand at a fixed price. However, MMMFs don't hold capital and don't have access to the discount window. MMMFs invest mainly in marketable assets. In contrast to repo investors, MMMF investors do not have a claim on a specific piece of collateral. Rather, they receive a pro-rata share of the assets in the fund if it breaks the buck.

One way to think of a MMMF in our environment would be to sets  $b_i = \bar{I}_i$ in steady-state. In this case all of the assets held by the fund are purchased with investor funds. However, this characterization does not capture the important role played by MMMFs parent institutions. MMMF are typically part of a larger financial institution that can provide discretionary financial support if the MMMF experiences difficulties. Support by parent institutions has been an important source of stability for MMMFs during the recent financial crisis and earlier episodes, as documented by Shilling, Serrao, Ernst, and Kerle (2010).

Instead, we can think of  $\bar{I}_i$  as representing the size of the whole financial institution, while the size of the MMMF is determined by  $b_i$ . The difference,  $\bar{I} - b_i$ , represent assets on which the MMMF investors have no formal claim, but that the parent institution can use to rescue the MMMF if necessary. For simplicity, we assume that the parent institution can credibly commit to use the resources at its disposal to support a troubled MMMF.<sup>28</sup> Hence, abusing terminology, we can think of the MMMF's assets as the "collateral" backing the investor's claim. The assets of the parent institution can be thought of as a "haircut".

With MMMFs, the timing is as follows:

- 1. The MMMF offers a new contract  $(\hat{r}_i, \hat{Q}_i)$ .
- 2. New and patient middle-aged investors decide whether to withdraw from the MMMF.
- 3. There is a first-come-first-serve constraint in that the first investors to withdraw can get cash until the MMMF runs out. At that time, the MMMF has broken the buck and the remaining investors get a claim on remaining assets (that mature the next period).

An MMMF is illiquid if the withdrawals it faces,  $b_i [\bar{r} + (1 - \alpha)\bar{r}^2]$ , exceed the cash available to it and its parent company,  $R_i \bar{I}_i$ . The probability that a withdrawing investor is able to obtain cash is  $\varphi$ , given by equation (25). With probability  $1 - \varphi$ , the investor is unable to withdraw early enough to obtain cash. The investor thus gets a claim on the assets of the MMMF

<sup>&</sup>lt;sup>28</sup>New SEC rules would allow a fund to "suspend convertibility", but only to avoid fire sales when a fund needs to be liquidated. In such a case, the remaining depositors have a claim on the assets of the fund, as in our setup. Hence, this type of suspension of redemption cannot prevent runs on MMMFs.

and the parent company. The value of these assets divided by the mass of investors that have received cash is given by

$$\mu \equiv \frac{\gamma R_i \bar{I}_i}{b_i \left[ \bar{r} + (1 - \alpha) \bar{r}^2 \right] - R_i \bar{I}_i}$$

Note that the denominator is equal to  $m_i$ , as defined in equation (23).

If  $\mu \geq \bar{r}$ , then investors do not have a strict incentive to run on an MMMF. Rewriting this condition we get

$$\beta^2 R_i \overline{I_i} \ge \frac{1+\beta-\alpha}{1+\gamma\beta} b_i. \tag{27}$$

This condition is the same as (26). The reason is that the MMMFs/parent company structures we consider in this section have the same balance sheet as dealers in the previous section. On the asset side they have assets maturing tomorrow and assets maturing two period from today. On the liability side, they have  $b_i$  investors and the rest can be thought of as "equity". In the case of MMMFs, which don't have capital, we associate the "equity" with the resources of the parent company. The collateral constraint considers how many assets back the claims of investors and is the same in each structure.

### 5.5 Traditional banks

The analysis for traditional banks is similar to the analysis for MMMFs. If  $b_i < \bar{I}$ , then the assets  $(\bar{I} - b_i)(1 + \beta)$  can be thought of as the equity of the bank. Like MMMF investors, bank depositors do not get a claim to a specific piece of collateral, but rather a claim on the bank's assets in case of bankruptcy. The major difference between a MMMF and a bank is that banks holds nonmarketable assets. Hence, we would think of  $\gamma$  as being very low in the case of a bank.

The timing is as follows

- 1. The bank offers a new deposit contract  $(r_i, Q_i)$ .
- 2. New and patient middle-aged investors decide whether to withdraw from the bank.

3. Investors can withdraw cash until the bank runs out. At that time, the bank is bankrupt and the remaining investors get a claim on remaining assets (that mature the next period).

The analysis and the payoff table is the same as in the case of a MMMF, as is the condition for runproofness. Hence, a version of proposition 4 also hold in that case. However, because  $\gamma$  is expected to be small for traditional banks, condition (27) is unlikely to be satisfied and the liquidity constraint (20) thus crucial for the bank's stability.

## 6 Runs and Asset Sales

In this section, we introduce the possibility of asset sales as a reaction to a run and thus generalize the analysis of Section 4. As in Section 4, we ask whether, if behavior until date t is steady state as in Proposition 1, the collective refusal to lend to the dealer can bankrupt the dealer? As pointed out by, e.g., Shleifer and Vishny (1992), Acharya and Yorulmazer (2008), and Diamond and Rajan (2009), "fire sales", i.e. asset sales under distress, can mitigate the dealer's illiquidity problem.

To investigate this possibility, consider a dealer, say i, at date t who holds assets that will yield  $R_i \overline{I_i}$  at date t+1. We assume that in response to a run, the dealer can sell these assets to other dealers at some market price p. If the dealer under distress sells an amount A of assets, this improves his current liquidity by pA and reduces his cash at date t+1 by  $R_iA$ . Generalizing (19), his cash position after the run at date t therefore is

$$I_0 = R_i \overline{I_i} + pA - (\overline{r} + (1 - \alpha)\overline{r}^2)b_i$$
(28)

Since the maximum amount of assets the dealer can sell is  $A = \overline{I_i}$ , (28) implies that the dealer can survive if and only if

$$(R_i + p)\overline{I_i} - (\overline{r} + (1 - \alpha)\overline{r}^2)b_i \ge 0$$
<sup>(29)</sup>

If p satisfies (29) the dealer will survive by selling a sufficient amount of assets, if not he will be bankrupt. Whether the dealer can raise enough cash through the asset sale depends on the cash in the market (Allen and Gale 1994), i.e. on the total amount of cash held by all other dealers. At the moment of the run, i.e. when the dealers have repaid their steady-state borrowing, received their new loans including the funds  $b_i + (1 - \alpha)\overline{r}b_i$  that have not gone to dealer *i*, but before they have invested their funds, this cash is

$$C_{i} = b_{i} + (1 - \alpha)\overline{r}b_{i} + \sum_{j \neq i} \left[R_{j}\overline{I_{j}} - (\alpha\overline{r} + (1 - \alpha)\overline{r}^{2} - 1)b_{j}\right]$$
  
$$= N + \sum_{j \neq i} R_{j}\overline{I_{j}} - (\alpha\overline{r} + (1 - \alpha)\overline{r}^{2})(N - b_{i}) + (1 - \alpha)\overline{r}b_{i}$$

By (3) and (10), this cash is clearly sufficient to cover dealer *i*'s missing amount  $m_i$  as defined in (23); intuitively, the run on the dealer simply means a redistributin of his liquidity to the other dealers.

The question is whether this cash can be mobilized to save the dealer. The benefits from mobilizing this cash are the asset returns in t + 1, the cost is the foregone investment that yields benefits in t + 2 and thereafter. The demand for cash is easily described. The dealer must raise  $m_i$ . From (29), the proceeds from the asset sale will be sufficient to cover  $m_i$  if and only if

$$p \ge \frac{m_i}{\overline{I_i}} \equiv \underline{p_i}$$

Since the assets sold by the dealer yield only  $\hat{\gamma}R_i$  to outsiders next period, the demand for these assets, hence the supply of cash, will be 0 if  $p > \beta \hat{\gamma}R_i$ . This implies the following characterization of when asset sales can save a distressed dealer.

**Proposition 8** Asset sales can save a distressed dealer if and only if  $\underline{p_i} \leq \beta \hat{\gamma} R_i$ , which means

$$\beta^2 R_i \overline{I_i} \ge \frac{1 - \alpha + \beta}{1 + \hat{\gamma}\beta} b_i \tag{30}$$

**Proof.** Suppose  $\underline{p_i} > \beta \hat{\gamma} R_i$ . Then either  $p \ge \underline{p_i}$ , in which case other dealers do not purchase the dealer's assets, or  $p < \underline{p_i}$ , in which case the price is too low to save the dealer.

Now suppose that  $\underline{p_i} \leq \beta \hat{\gamma} R$ . Consider any  $p < \beta \hat{\gamma} R$ . If all dealers  $j \neq i$  invest  $\overline{I_j}$  into their assets as in steady state they have a total of  $\sum_{j\neq i} \pi_j + (1 + (1 - \alpha)\overline{r})b_i$  in cash. Investing this cash into the distressed dealer's assets yields a return of  $\hat{\gamma} R_i/p$  next period, which is strictly preferred to consuming the cash now. This cash is sufficient to cover  $m_i$ , because of positive profits, (10), and because (13) implies

$$(1 + (1 - \alpha)\overline{r})b_i \ge (\overline{r} + (1 - \alpha)\overline{r}^2)b_i - R_i\overline{I_i}$$

Figure 1 provides a graphical illustration.

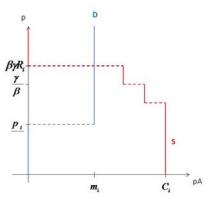


Figure 2: Demand and supply of cash in the asset market

Condition (30) is exactly the same condition as the collateral constraint for the bilateral repo market, (26), if  $\hat{\gamma} = \gamma$  and weaker if  $\hat{\gamma} > \gamma$ . If  $\hat{\gamma} = \gamma$ , investors and other dealers realize the same return from the assets. Hence, asset sales cannot loosen a dealer's liquidity constraint beyond the limitations of the collateral constraint. In contrast, if  $\hat{\gamma} > \gamma$  dealers realize a higher return from the assets than investors would. Other dealers compete to purchase the assets, raising their price up to the point where the returns are greater that the return investors would get from the assets. Condition (30) is strictly weaker than the liquidity constraint (20), confirming the possibility that sales of assets relaxes the dealer's liquidity constraint. In other words, some dealers who would go bankrupt if they could not sell their assets can survive if asset sales are possible. However, if in a distressed sale  $\hat{\gamma}$  is sufficiently small condition (30) does not provide much relief and the dealer is illiquid despite the asset sale.

#### 6.1 Interpretation

Most assets serving as collateral in the tri-party repo market are liquid, so we should expect  $\hat{\gamma}$  to be close to 1. Hence, we can interpret the result of this section as suggesting that when markets are not stressed, dealers in the triparty repo market can accommodate the demand that would arise from an idiosyncratic run. This is broadly consistent with the conventional wisdom before the financial crisis.

There are two cases, however, where we might expect  $\hat{\gamma}$  to be low in the tri-party repo market. A low  $\hat{\gamma}$  should be expected if a dealer uses less liquid collateral to back its repos. In such a case, it will be more difficult for whoever tries to liquidate the collateral to obtain a high value. Anecdotal evidence suggests that the share of less liquid collateral in the tri-party repo market had been increasing before the crisis, maybe reaching 30 percent of the collateral in that market. This would have made dealers who borrow against less liquid collateral more susceptible to runs.

A low  $\hat{\gamma}$  may also apply if the quantity of a relatively liquid asset used as collateral in tri-party repos is so large that the market may not be able to absorb all the collateral in case of a dealer default. For example, Agency MBSs are considered liquid securities, but the amount of such securities financed in tri-party is so large that the market may not have been able to absorb them without some price effect. This effect is likely to be particularly strong in times of aggregate market stress, which we discuss in the next section.

It is also worth pointing out that our model probably overstates dealer's ability to accommodate the demand for cash in a run and the ability of other dealers to purchase assets. In our model, the share of repos held by old and impatient middle aged investors is close to half of all the repos made by a dealer.<sup>29</sup>. Hence, the demand for funds in the case of a run is about twice as large as the steady state demand. Anecdotal evidence suggests that the share of repos being rolled over in the tri-party repo market is much larger, probably over 80 percent. This would mean that the run demand is five time as large as the stead-state demand, which would be more difficult for a dealer to accommodate.

Our model could be adapted to increase the share of repos rolled over every period. For example, we could consider an economy in which agents lived longer lives and assets matured after more periods. In such an economy, the share of cash and maturing assets would be a smaller share of all assets. Similarly, the share of new and withdrawing investors, which must be equal in steady state, would represent a smaller fraction of the population of all investors. Hence, the demand for funds in case of a run would be much larger than the steady state demand, compared to the economy we consider. The share of unmatured assets that can be sold, compared to the available cash, would also be greater, increasing the fire sale effect.

# 7 Extensions: Market Runs and liquidity provision

As noted above, the more dealers are in trouble, the more assets troubled dealers are trying to sell and the fewer dealers are available to buy these assets. This puts pressure on the price of assets and make it less likely that a run can be avoided. In the extreme case of a market run, all dealers are facing a run demand and only a small number may have enough liquidity to satisfy their demand and at the same time buy the assets that are put up for sale by a large number of distressed dealers. The possibility of market runs may justify liquidity provision by a lender of last resort.

 $<sup>^{29}\</sup>text{The exact share will vary depending on the parameters <math display="inline">\alpha$  and  $\beta$ 

#### 7.1 Market runs

First, we consider the conditions for the case where no dealer survives. Proposition 2 continues to apply, so a dealer will be illiquid if condition (20) is violated. The collateral constraint is slightly different in the case where no dealer survives. Indeed, in that case, investors who don't survive get a payoff of 1, rather than  $\bar{r}$ . Hence, the collateral constraint is  $1 \leq \gamma R_i k_i$ . After replacing  $k_i$  with  $\kappa_i$  we can write

$$\beta R_i \overline{I_i} \ge \frac{1 - \alpha + \beta}{\gamma(1 + \beta)} b_i. \tag{31}$$

This condition is less restrictive than (22) because investors do not have as good an outside option. If all dealers' collateral and liquidity constraints are violated, no dealer survives the market run.

If at least one dealer can survive the market run, then this dealer can attract depositors and purchase assets from other dealers. If only one dealer survives, this dealer may be able to act monopolistically. To simplify the argument, we assume here that either at least two dealers survive, or the single surviving dealer behaves competitively.

If there are no restrictions on investors switching from troubled to healthy dealers, the healthy dealers can attract all investors from troubled dealers and use these fund to purchase assets.<sup>30</sup> In this case, the same argument as in section 6 shows that there is enough cash in the market to cover the troubled dealers' need. This is because the total liquidity in the market is simply redistributed among all dealers. In this case, proposition 8 applies. Skeie (2004) obtains a similar result in the context of traditional banks.

However, it may be difficult for healthy dealers to take on all the troubled dealers' investors. On-boarding new clients can be costly and take time and there may simply be economies of scale similar to (2). When the redistribution of liquidity among dealer is not frictionless, there may not be enough cash in the market to cover the troubled dealers' need. If the supply of cash is sufficient, then proposition 8 applies. If the supply of cash in insufficient, then troubled dealers will bid down the price of assets until  $p_i = \beta \hat{\gamma} R_i$ . If

<sup>&</sup>lt;sup>30</sup>This occurs, for example, if the capacity limit  $\bar{I}_i$  applies to investment, but not the stock of assets held.

the price of the assets drops any lower, then the troubled dealer's investors generate a higher yield from the assets. In this case, the collateral constraint is

$$\beta^2 R_i \overline{I_i} \ge \frac{1 - \alpha + \beta}{1 + \widehat{\gamma}\beta} b_i. \tag{32}$$

Note that condition (32) is tighter than condition (30). This suggests that multiple equilibria may be possible. If investors expect low asset prices, more dealers will be in trouble, which increases the supply of assets in the market. Because the redistribution of liquidity among dealers is not perfect, the supply of cash in the market does not increase as much, which justifies the low price of assets. In contrast, if the price of assets is expected to be high, then fewer dealers are in trouble. This means that the supply of assets is low and the available cash is high, justifying high prices.

## 7.2 Liquidity provision

Access to a lender of last resort is a standard tool used to strengthen the banking sector in the face of financial fragility. Theoretical work has shown how access to a lender of last resort can prevent bank runs (see, for example, Allen and Gale 1998, Martin 2006, Skeie 2004). In the U.S., the broker dealers that rely on the tri-party repo market as a source of short-term funding did not have direct access to discount window. This lack of access to emergency liquidity proved destabilizing during the crisis and motivated the Federal Reserve to introduce the Primary Dealer Credit Facility (PDCF). Similar concerns about money market mutual funds, who represent an important share of investors in the tri-party repo market, motivated the creation of the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility (AMLF), and the Money Market Investor Funding Facility (MMIFF). These facilities were created under section 13.3 of the Federal Reserve Act, which allows the Federal Reserve to lend to a variety of institutions under unusual and exigent circumstances. As such, these facilities are temporary.<sup>31</sup>

The Task Force on Tri-Party Repo Infrastructure (2009) notes the need to "consider establishing an industry-sponsored utility with the ability to

<sup>&</sup>lt;sup>31</sup>The MMIFF expired on October 30, 2009. The Board of Governors approved extension of the AMLF and the PDCF through February 1, 2010.

finance the securities portfolio of a faltering or defaulted dealer and limit the associated stress on the market while their portfolio is liquidated." The model in our paper suggests that there would be benefits to the creation of a lender-of-last-resort facility for the tri-party repo market. The argument is similar to the case of banking. In case of a run, investors do not refuse to roll over their loans because they need cash, but because they are concerned about the default of the dealer and having to hold collateral that they might have to liquidate. As in Allen and Gale (1998), Martin (2006), or Skeie (2004), a lender of last resort could lend cash to the dealer taking securities as collateral. The cash could be used to pay all investors who do not roll over their loans. This would prevent the default of the dealer and allow it to manage the collateral until it matures. Knowing that the dealer will not default, investors no longer have to worry about having to hold or liquidate assets, so their incentive to run is reduced.

## 8 Conclusion

We have studied a model of short-term collateralized borrowing and the conditions under which runs can occur. Our framework resembles the dynamic bank model studied in Qi (1994), but extends that model beyond the pure theory of commercial banking. We derive a dynamic participation constraint that must hold for dealers to agree to purchase securities on behalf of investors. Under this constraint, dealers will make profits that can be mobilized to forestall runs.

A key difference between traditional banks and modern financial intermediaries is that the former mainly hold opaque assets while the latter's assets are much more liquid and marketable. We study the role of marketable assets in preventing runs on these intermediaries. Without asset sales, runs can be forestalled by mobilizing sufficient liquidity and having sufficiently valuable collateral. This gives rise to two constraints that can be interpreted as a liquidity and a collateral constraint. The liquidity constraint guarantees that the necessary resources are available at the date the run occurs. Section 4 has derived a basic version of this constraint, and Section 6 considers the more general case where dealers can sell assets. Note, however, that Section 4 becomes relevant again in the case of a market run, in which no dealer may be able to purchase other dealers' assets. The collateral constraint makes sure that investors want to continue collateralized lending instead of running for their money. As shown in Section 5, this constraint critically depends on the microstructure of the market under consideration. A run can be prevented if at least one of the constraints is satisfied, meaning that the dealer is liquid or has enough collateral.

Conceptually, our theory of intermediation differs from that of Diamond and Dybvig (1983) in several respects. Most importantly, we endogenize the liquidity of intermediaries by introducing an overlapping structure of investors that allow intermediaries to generate cash each period. Without this dynamic structure the liquidity of intermediaries would be exogenous and its determinants arbitrary. Despite the discipline imposed by dynamic steady state equilibria, the analysis yields simple explicit conditions for the fundamentals of the model that characterize the system's stability.

In Diamond and Dybvig (1983), deposit contracts are collective insurance devices for households with low elasticities of intertemporal substitution. In our framework, dealers interact with financial investors such as pension funds, money market funds and other institutions, whose preferences are probably different and less well understood. We therefore do not place restrictions on investor preferences except for monotonicity. The *raison d'être* of banking in our model therefore is not collective insurance, but rather the special ability of dealers to generate returns that other market investors do not possess.

Our framework can be used to consider a number of policy questions related to the fragility of the tri-party repo funding mechanism. For example, Lehman's demise highlighted the problem that there is no process to unwind the positions of any large bank that deals in repo should it fail. Lehman required large loans from the Federal Reserve Bank of New York to settle its repo transactions (WSJ 2009). Our framework can be used to study a liquidation agent, as suggested in the Task Force on Tri-Party Repo Infrastructure (2009), with the objective to unwind the positions of a defaulting dealer. Similarly, our analysis sheds light on the role of institutional features such as the unwind mechanism in the tri-party market or the difference between bilateral and tri-party repo lending and thus should contribute to a better understanding of the fragility of wholesale banking markets.

# 9 Appendix: Coordination problem between the clearing bank and investors

The tri-party repo market is also vulnerable to another coordination problem, this time between the clearing bank and the investors. Suppose that, in the timing described in section 5.1, just before step 1 the clearing bank comes to believe that at step 5 all investors will refuse to engage in repos with dealer *i*. In this case, the clearing bank will refuse to unwind if the loan it makes to the dealer,  $b_i [\bar{r} + (1 - \alpha)\bar{r}^2]$ , exceeds the proceeds it could obtain from the assets,  $R\bar{I}(1 + \beta\gamma)$ .<sup>32</sup> This condition can be written as

$$\beta^2 R_i \bar{I}_i \ge \frac{1+\beta-\alpha}{1+\gamma\beta} b_i. \tag{33}$$

This condition is the same as the collateral condition for bilateral repos, (26).

The flip side of this coordination problem is that investors may choose not to invest with dealer i if they believe that the clearing bank will refuse to unwind that dealer's repos the next morning.<sup>33</sup> In this case, the condition for investors to have a strict incentive to run is the same as in the case where investors believe other investors may not engage in repos.

 $<sup>^{32}\</sup>text{Here}$  we assume that the clearing bank faces the same  $\gamma$  as the investors.

<sup>&</sup>lt;sup>33</sup>Clearing banks have the contractual right not to unwind a dealer's repos. Failure to unwind the repos would almost certainly force the dealer into bankruptcy.

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