Lipset Reconsidered: A Rational Theory of the Stability of Democracy*

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Abstract

This paper studies the endogenous emergence of political regimes, in particular democracy or oligarchy, in heterogeneous societies in which institutions do not ensure political commitments. Democracy emerges if the ruling coalition that makes political decisions represents the majority of individuals of the population, while oligarchy emerges when the political decisions are made by a minority of the population. The findings show that for any economic environment there exists a distribution of resources such that democracy is a possible political outcome within a heterogeneous society, whereas the opposite does not hold. The model also delivers results on the stability of democracy. Variations in inequality across several dimensions due to unbalanced technological change, immigration or changes in the demographic structure can reduce the scope for democracy or may even lead to its breakdown.

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1 Introduction

The importance of political institutions for economic development is one of the most intensely researched areas of the recent years. Among these, democracy has received particular attention, because democracies implement many of the institutions and policies that are thought to be beneficial for economic development, like rule of law, redistribution of incomes and social insurance, or wide-spread education. Less is known about the determinants of democracy and its stability with respect to secular changes in the economic environment or the population structure. Among the first to address this issue was Seymour Martin Lipset, who conjectured that higher levels of economic development and a more equal distribution of resources imply a higher probability for a country to *become* and to *stay* democratic.[1] While having in mind a political conflict, often about redistribution between between oligarchs and the disenfranchised people, most of the literature that studies the conditions for an endogenous transition from oligarchy or autocracy to democracy does not adequately take into account both of these factors, economic development *and* inequality. In addition, the literature usually treats democracy as an absorbing state and thereby implicitly assumes that conflicts within such political regimes are solved on the basis of "democratic rules", in the sense of an institutionalized environment in which binding political commitments are possible.

Little is known about the stability of democracy and the conditions under which it emerges from an oligarchic society with weak institutions. Importantly, an institutionalized environment cannot be taken for granted when considering the stability of democracy and its potential break-down, and political commitments made under democratic rule might not be credible to pertain when democracy disappears. From a conceptual point of view, this implies that stability of democracy should be studied in a similar environ-

[1]In his famous article, Lipset (1959) wrote:

> "Democracy is related to the state of economic development. Concretely, this means that the more well-to-do a nation, the greater the chances that it will sustain democracy. From Aristotle down to the present, men have argued that only in a wealthy society (...) the population could intelligently participate in politics. (...) A society divided between a large impoverished mass and a small favored elite would result either in oligarchy (dictatorial rule of the small upper stratum) or in tyranny (popularly based dictatorship)."

Remarkably, Lipset himself seems to have restricted most of his attention to income levels rather than inequality in the following – as has much of the earlier literature on the subject, see e.g. Diamond (1992) for an overview.

ment as the emergence of democracy from non-democratic rule. In this paper we consider democracy as an endogenous outcome of a political conflict about the distribution of income within a society in which the income generating factors are distributed unequally. The main novelty of our approach is the consideration of the role of both dimensions, the level of economic development and the distribution of resources, in an environment without exogenous institutions that ensure the possibility to make political commitments. Political decisions are made in an environment that is a priori weakly institutionalized, in the sense that no binding agreements about income redistribution can be made among the different groups of factor owners, and sub-coalitions or single groups can use their de facto power to implement their preferred redistribution scheme against the will of others. In this competition for political power, inequality across several dimensions becomes key for the determination of the politico-economic equilibrium in terms of the political structure and the ex-post allocation of incomes.

The equilibrium is characterized by a ruling coalition that is self-enforcing and winning against any other challenging coalition. The equilibrium can be a democracy if no minority of individuals in society determines political decisions, and if instead the ruling coalition represents a majority of the overall population. Equilibria where a minority dominates political decisions represent oligarchies.[2] The results provide a characterization of the levels of inequality and development, reflected by the distribution of the different factors in the population and their relative importance in the income generating process, for which democracy or oligarchy emerges in equilibrium. The model also illustrates the consequences of changes in inequality, in terms of population structure and/or factor endowments, or in the economic environment in terms of the economic importance of the different factors, for the stability of democracy. It is shown that the political influence of a group in equilibrium is non-monotonic in the de facto power of that group, and, in particular, that the likelihood of democracy to emerge is non-monotonic in the power of any group's power or size.

This paper contributes to a growing literature on endogenous political institutions. Similar to the seminal work of Acemoglu and Robinson (2001, 2006), it is the redistributive threat by part of the population that brings about a democratic equilibrium. However,

[2]The precise definition and classification of equilibria is presented in Section 3.

in addition also the level of development in terms of the economic importance of certain factors is relevant in the present paper. The model below also differs from most other papers that study the endogenous emergence of democracy, including, e.g., Acemoglu and Robinson (2001, 2006), Lizzeri and Persico (2004), Llavador and Oxoby (2005), Gradstein (2007), Cervellati, Fortunato, and Sunde (2008), in that it is not (implicitly or explicitly) assumed that the population only consists of two or three distinct groups among which coalition formation is not a problem or even an issue, and that any conflict of interest in democracies can be resolved by credible commitments concerning the policies or the coalitions that are formed. The present paper studies the emergence (and disappearance) of political regimes in an environment in which such credible commitments are not possible, even in democracy. To this end, our analysis builds on the work by Acemoglu, Egorov, and Sonin (2008), who consider the problem of coalition formation in situations where binding agreements among different groups or parties cannot be made, since no party can commit not to eliminate other parties from the ruling coalition in the future. Our model explicitly deals with the concrete problem of coalition formation among distinct groups which represent differently endowed segments of the population and struggle for redistributing factor incomes.

Apart from allowing for a realistic analysis of the stability of political regimes in heterogeneous societies, this approach delivers new insights about the necessary conditions for the emergence and stability of democracy. The main result is a novel characterization of the conditions under which democracy emerges or breaks down in the absence of some institution that ensures political commitments to be credible. Our model also provides additional implications of technological progress as the key driver of income inequality along the lines of Acemoglu (2002). We show how various forms of technological progress affect political outcomes and redistribution in equilibrium. In this respect our work is also related to Benabou (2005). Finally, the paper adds to the study of Cervellati, Fortunato, and Sunde (2007) who find that the institutional arrangements, in particular the quality of the rule of law, in democracies crucially depend on the democratization scenario. The results presented here complement their findings by studying explicitly the stability of democracy.

The paper is structured as follows. Section 2 lays out the framework, and section 3 presents the results concerning the political equilibrium. In section 4 the model is nested in a production economy, which allows us to relate the political equilibria to the economic environment in general equilibrium. In section 5 we present the main results concerning the emergence and stability of democracy. Section 6 concludes.

2 The Model

2.1 Population Structure and Production

Consider a static economy that is populated by a unit mass of individuals. These individuals live for one period and leave no bequests. By birth, individuals are endowed with labor time, physical strength and intellectual ability, all of which are supplied inelastically to the production sector and remunerated according to their marginal product. Hence, factor incomes constitute the potential disposable income that individuals can consume, and from which they may derive utility. Since consumption is the only component of utility, individuals maximize disposable incomes. While each individual has an identical endowment of labor time, $h > 0$, at his disposal, physical strength and intellectual ability are distributed unevenly in the population.[3] For simplicity, we assume the distribution of both of these characteristics to be dichotomic which means that a share $0 < \gamma < 1$ of individuals possesses one unit of physical strength, denoted by $l = 1$, whereas the complement $1 - \gamma$ possesses no physical strength at all, $l = 0$. Likewise, a share $0 < \beta < 1$ of the population possesses intellectual ability, $a = 1$, while a share $1 - \beta$ possesses no intellectual ability, $a = 0$. Since physical strength and intellectual ability are not mutually exclusive traits, the population effectively consists of four distinct groups: the twofold-privileged strong and intelligent, denoted by $\mathcal{E}$, the able weaklings, $\mathcal{A}$, the simple-minded strong, $\mathcal{L}$, and those that possess neither strength nor ability, $\mathcal{P}$.[4] Denote the set of groups by $S = \{\mathcal{E}, \mathcal{A}, \mathcal{L}, \mathcal{P}\}$, and denote the respective sizes of groups $i \in S$ as s_i with s_{MAX} being

[3]The endowment of labor time h can be normalized to 1 without loss of generality.

[4]In principle, our model society could comprise an arbitrary number of groups, and none of our main results depends on the particular population structure with four groups. As will become clear later, however, four groups constitute the simplest case for delivering the main results while retaining tractability for graphical illustrations. Increasing the number of groups would complicate the analysis without adding new insights.

the size of the largest group.[5]

According to that, the factor endowments of particular group members are given by

$$l_i = \begin{cases} 0 & \text{if } i \in \{\mathcal{A}, \mathcal{P}\} \\ 1 & \text{if } i \in \{\mathcal{L}, \mathcal{E}\} \end{cases} \tag{1}$$

and

$$a_i = \begin{cases} 0 & \text{if } i \in \{\mathcal{L}, \mathcal{P}\} \\ 1 & \text{if } i \in \{\mathcal{A}, \mathcal{E}\}\,. \end{cases} \tag{2}$$

The population structure and the respective group sizes are illustrated in Figure 1.

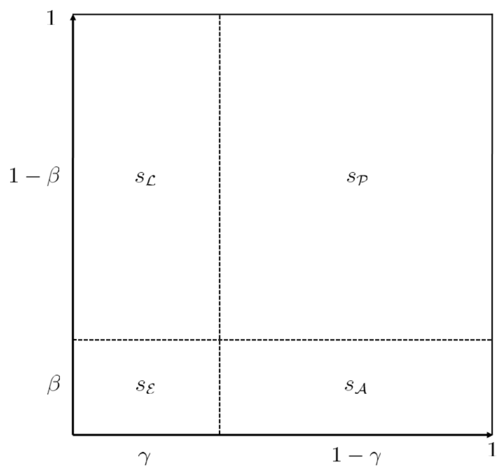

Figure 1: Population structure and respective group sizes.

Individuals supply their endowments on competitive markets to a production sector that uses labor, strength and ability as separate inputs. Per capita income y is generated by ways of a CRS production function

$$Y = Y(A, H, L, \Lambda)\ , \tag{3}$$

where $A > 0$ represents a productivity parameter or vector, reflecting the level of technology, and H, L and Λ are the aggregate levels of working hours, physical strength and ability, respectively. The marginal product of every input factor q is positive but de-

[5] Since there is no danger of confusion, individual members of groups and groups are interchangeably denoted by i.

creasing, i.e. $\partial Y/\partial q > 0$ and $\partial^2 Y/\partial q^2 < 0$. Factor prices are competitive, and $\rho = \partial Y/\partial H$ represents the price paid for one unit labor time, $w = \partial Y/\partial L$ represents the remuneration of physical strength, and $\mu = \partial Y/\partial \Lambda$ is the reward for ability. Consequently, the factor income of an individual belonging to group i is given by

$$y_i \;=\; \rho h + w l_i + \mu a_i \quad \text{with } i \in S\,. \tag{4}$$

From the unequal endowment of traits and the remuneration of these traits on competitive markets it follows that factor income is distributed unequally in the population, and individuals with higher endowments earn higher factor incomes. From the assumptions about population structure, it follows directly that members of the $\mathcal{E}$-group always earn the highest factor income per-capita, whereas the factor income of individuals in the $\mathcal{P}$-group is always the lowest, i.e.,

$$y_{\mathcal{P}} < y_{\mathcal{L}}, y_{\mathcal{A}} < y_{\mathcal{E}}\,. \tag{5}$$

Note that average income equals aggregate group income which gives

$$y \;=\; \sum_{i \in S} s_i y_i \;=\; \rho h + w\gamma + \mu\beta\,. \tag{6}$$

2.2 Redistributive Conflict, Power and Utility

The given endowment of production factors implies that factor incomes can vary considerably between different groups, which gives rise to redistributive conflicts, since the utility of individuals or of members of a certain group is not affected by the well-being of others. In consequence, a latent conflict between the different groups exists and every group tries to maximize their respective income at the expense of others.[6] All political considerations in the model are therefore reduced to the question of how the income generated by the members of society is redistributed amongst them. We assume that, in principle, *all* income can be expropriated and redistributed between groups, such that the

[6] For simplicity, and contrary to Olson (1965), we assume that no commitment problems exist within groups, i.e., single group members do not free ride on other group members. This implies that we can analyze the society as consisting of four different agents, each representing one income group. A justification for this assumption is that the collective action required in the case of intra-group conflict is transitory, and hence much easier to sustain, see Acemoglu and Robinson (2006).

feasible transfer equals per-capita income $y = \sum_{i \in S} s_i y_i$.[7] In combination with the given production structure the possibility to expropriate all factor income has the important implication that it is always beneficial to employ all available workers in the production process and redistribute their incomes afterwards, as $y_i > 0$ follows from equation (4). Since factors are supplied inelastically, there are no hold-up problems or the like through which the political game affects or distorts the production process.

Given the possibility to expropriate factor incomes we need to consider the question which group or coalition of groups actually makes political decisions and can impose its preferred redistribution scheme on the entire population, i.e., we need to elaborate on the political dimension of our model. As already mentioned before, we consider an environment where no institutions exist that would allow for binding commitments between groups. Thus, no group can make binding offers of how to redistribute income, and no group that is part of the coalition that redistributes income can commit not to exclude other members of that coalition and make political decisions autocratically later on. Given this environment we assume that it is the political power P_i of group i that describes its potential to redistribute factor incomes. To keep the conflict game simple and concentrate on the issue of coalition formation, we model the redistributive conflict as parsimoniously as possible and assume that any group or coalition Q can seize the income of group or coalition $S \backslash Q$ if $P_Q > P_{S \backslash Q}$ holds where $P_Q \equiv \sum_{j \in Q} P_j$ denotes the aggregate power of group or coalition Q.[8]

To link the economic and political environment we assume that the political power of a group or coalition is given by its aggregate income such there exists a one-to-one mapping of income into political power, $P : \mathbb{R}_{+\backslash\{0\}} \to \mathbb{R}_{+\backslash\{0\}}$, where $P_i \equiv s_i y_i$. Additionally, we assume this power mapping to be bijective such that no two groups can be equal in power, i.e. $P_i \neq P_j \ \forall \ i, j \ \in \ S$ for $i \neq j$.[9] For notational convenience, we define the least

[7] One could alternatively assume that some subsistence income, for example the factor income from time endowment, can be retained by each individual to ensure that production takes place without changing the main results.

[8] This assumption could be motivated by means of a sequential conflict game with perfect information and certain outcome where richer groups can afford more weapons, soldiers, etc., and hence overcome poorer groups in open conflict.

[9] As will turn out later this assumption is not only convenient but also plausible, since group income and – due to fixed relative group sizes – political power is affected by technological progress and other exogenous factors. Note that this assumption directly implies that the set of coalitions which are equal in power and contain at least three groups is of Lebesgue measure zero.

powerful group i_{MIN} to have power P_{MIN}, the third and second most powerful groups $\underline{i}$ and $\bar{i}$ to possess powers $\underline{P}$ and $\overline{P}$ and the most powerful group i_{MAX} to have power P_{MAX} such that $P_{MIN} < \underline{P} < \overline{P} < P_{MAX}$. From this, it follows that the most powerful group is able to make all political decisions alone if, and only if, $2\,P_{MAX} > \sum_{i \in S} P_i$ holds.

If no group has the power to rule alone, i.e., $2\,P_{MAX} \leq \sum_{i \in S} P_i$ the incentive and the ability to form a coalition become relevant. On the one hand, coalition formation is associated with making concessions to the other members of the coalition with regard to the desired redistribution scheme. Hence, forming part of a coalition is costly in terms of foregone redistribution to the other members of the coalition. On the other hand, being part of a coalition increases political power by pooling resources for a potential conflict with other groups or coalitions. The formation of coalitions is complicated by the weakly institutionalized environment, in which no binding promises concerning redistribution or coalition loyalty are possible. In the current context of a one-shot redistribution problem where every group tries to maximize its own disposable income only, no group can therefore rely on promises of others.[10]

The last important aspect of the political environment concerns the question how revenues from redistribution are shared within the ruling coalition. Here we assume that the share of transferable income seized by group $i \in S$ is determined by its relative power within the coalition that redistributes income and we denote the latter as the *effective* relative power $\tilde{p}_i$ of group i. This effective power reflects group i's ability to appropriate factor incomes. Note, however, that in order to be able to appropriate factor incomes from other groups, group i must be part of the equilibrium coalition that ultimately redistributes income. This coalition we call the ruling coalition. In other words, effective relative power is always defined conditional on the respective ruling coalition that is winning and stable against any other possible coalition. From now on, we denote the ruling coalition by RC where $RC \subseteq S$. Hence, group i's relative power is defined by

$$\tilde{p}_i = \begin{cases} \frac{P_i}{P_{RC}} & \text{if } i \in RC \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

[10] Acemoglu, Egorov, and Sonin (2008) analyzed coalition formation in equilibrium in a similar environment. Alternative settings with the possibility to commit are studied in Ray and Vohra (1997, 1999).

where $P_{RC} \equiv \sum_{j \in RC} P_j$ denotes the aggregate power of the coalition that redistributes income. To fix ideas, suppose the redistribution implies that the RC taxes away all factor incomes in the economy and then redistributes it among its members according to a simple and intuitive proportional sharing rule. Let $\widetilde{y}_i$ denote disposable income of group i, then this gives

$$\widetilde{y}_i = \tilde{p}_i y \,, \tag{8}$$

where $\tilde{p}_i$ is given as in condition (7), and y is the average factor income as in condition (6).[11] The setting implies that the indirect utility of members of group $i \in S$ depends on disposable income $\widetilde{y}_i$ and therewith on effective relative power $\tilde{p}_i$ which reads in its general form

$$u_i = u_i\left(\widetilde{y}_i(\tilde{p}_i)\right) \tag{9}$$

with $\dfrac{\partial u_i}{\partial \widetilde{y}_i} > 0$. Since factor income y_i is given by factor endowments and cannot be changed by individuals, the optimization problem amounts to maximizing $\tilde{p}_i$ in order to maximize lifetime utility, subject to the constraints imposed by the production structure and the political environment, i.e.,

$$\max_{\tilde{p}_i} u_i\left(\widetilde{y}_i(\tilde{p}_i)\right) \quad \text{subject to (4), (6), (7) and (8).} \tag{10}$$

As a direct consequence of the distribution of tax revenues, every group always prefers the coalition in which its relative power is greatest.[12] However, this does not necessarily imply a positive effect of P_i on u_i, as the latter does not monotonically increase in the former, which will become clear below.

2.3 Timing of Events

The following description of the non-cooperative ruling coalition formation and redistribution game that is played by every generation completes the timing of events. The sequence

[11] Such a rule was first used by Gamson (1961) to characterize the sharing of resources amongst coalition members and seems to be a fairly good description as several empirical studies suggest, see, e.g., Warwick and Druckman (2001) or Ansolabehere et al. (2005). Also note that this rule applies to all groups, not only members of the RC, and that it implies that there is no re-ranking of groups *within* the RC, but there might well be re-ranking in the society at large.

[12] Since utility of every individual is determined by the structure of the RC, our game is *hedonic* in the sense of Dreze and Greenberg (1980).

of events that a particular generation experiences throughout its lifetime is given by

1. Birth, realization of endowments and factor incomes.

2. Ruling coalition formation and redistribution game Γ:

   2.1 An agenda setter is randomly determined from all groups (for stage $k = 1$) or from all remaining groups (in stage $k > 1$).

   2.2 Agenda Setting: The agenda setter proposes a sub-coalition (that includes himself) to all (remaining) groups.

   2.3 Voting: The members of this sub-coalition vote sequentially in random order over the proposal (and all non-members automatically vote against the proposal); if all groups that support the proposal form a winning coalition, the game proceeds to step 2.4, otherwise to step 2.5.

   2.4 If the proposal includes all groups of the current stage k of the game, then they all form the RC and the game proceeds to step 3. If the proposal consists of a proper subset and is supported by a winning sub-coalition, all groups that are not part of this proposal are excluded by redistribution of factor incomes toward the members of the subset which causes some (arbitrarily small) costs ϵ;[13] in this case, a new stage $k + 1$ begins with step 2.1.

   2.5 A new agenda setter is determined randomly among all (remaining) groups that have not yet acted as agenda setter at the current stage of the game k, and the game proceeds to step 2.2; if all (remaining) groups have been agenda setters at the current stage k, then they all form the RC and the game proceeds to step 3.

3. Consumption of disposable income and death.

[13]This assumption allows to exclude path dependence and thereby ensures uniqueness of the equilibrium. Since $\max_{i \in S} k_i = 3$, with k_i denoting the last stage of the game during which income was redistributed to group i, we proceed without modifying equations (9) and (10) to account for the reduction of individual utility by ϵk_i, for the sake of simplicity.

3 The Political Equilibrium

We start our analysis of political equilibria with a central Lemma on the equilibrium outcome of the game described above.

Lemma 1. *In game Γ there exist subgame perfect Nash equilibria (SPNEs) in pure strategies which all lead to the same RC.*

Proof. See Appendix. □

The intuition for the equilibrium characterization of the RC is as follows. First, it must – by the nature of the game – be winning in the sense that it is powerful enough to outgun any alternative coalition that may challenge it at the current stage of the game, k. Second, every RC must be stable such that none of its proper subcoalitions will be winning and become the new RC at a subsequent stage of the game $\hat{k} > k$.[14] And third, if there exist more than one coalition which satisfy both properties the RC will be the coalition with least aggregate power, since the optimization problem $\max \tilde{p}_i$ is solved by minimizing the denominator in condition (7), i.e. $\min P_{RC}$.[15] Apart from that, we can also characterize the RC in terms of its size. Let $|RC|$ denote the cardinality of set RC.

Lemma 2. *The RC consists of at least three groups, if and only if the most powerful group is dominated by the rest of society, i.e.* $\sum_{i \in S} P_i \geq 2\, P_{MAX} \iff |RC| \geq 3$.

Proof. This proof is straightforward since we know from the proof of Lemma 1 that the RC must be a subset of all winning and stable coalitions. Due to the bijective power mapping, a coalition of two unequal groups cannot be stable, since one group always dominates the other, and therefore could always successfully propose an even smaller coalition that only contains itself at a subsequent stage of the game. Hence, $|RC| \neq 2$ always holds. Thus, it immediately follows $\sum_{i \in S} P_i \geq 2\, P_{MAX} \iff |RC| \geq 3$. □

Before we proceed, it is worth commenting briefly on the underlying concept of society, in particular concerning the possibilities and incentives for certain income groups to secede

[14] This second equilibrium property goes back to Bernheim, Peleg, and Whinston (1987) and their concept of a Perfectly Coalition-Proof Nash Equilibrium which was already studied in several other contexts, see for example Moreno and Wooders (1996) or Einy and Peleg (1995). See also Acemoglu, Egorov, and Sonin (2008) for a treatment in the context of political games.

[15] Note that this reasoning corresponds to the conceptualization in terms of the set Ω in the proof of Lemma 1, which gives a formal definition of the RC.

in order to escape taxation. In our model, it is the exploitation of political power rents that constitutes a centripetal force and prevents society from falling apart. Secessions are ruled out endogenously in equilibrium, since the groups who would be better off on their own, the net tax payers, are not powerful enough to split from the RC, whereas the RC, who would be powerful to split from the rest of society has no incentive to do so, because this would make its members worse off.[16]

Note that so far, the political equilibrium was characterized without any reference to political concepts. But the equilibrium itself can be interpreted as reflecting a particular political regime. To simplify the terminology, we first introduce a simple classification of equilibria that accounts for the two dimensions that characterize the RC: the proportion of the population that it comprises, s_{RC}, as well as the number of groups that are member of the RC.

Definition 1 (Classification of Equilibria)**.** *An equilibrium is ...*

(I) ... a Democracy (type I) if $s_{RC} \geq 0.5$ *and* $|RC| > 1$*;*

(II) ... a Democracy (type II) if $s_{RC} \geq 0.5$ *and* $|RC| = 1$*;*

(III) ... an Oligarchy if $s_{RC} < 0.5$.

According to this definition an oligarchy is a RC that, regardless of the number of groups involved, represents the minority of the population and imposes policies on the rest of society.[17] On the opposite, we call every political system a democracy when the RC represents the majority of the population.[18]

[16] Even though this result might contradict the empirical observation of an increasing number of sovereign states over the last century, it should be kept in mind that this model exclusively focuses on economic mechanisms and thereby ignores other factors like cultural identity or religion, which play a prominent role in separation processes of political entities in reality. In our model, we take the size of the polity as exogenously given, for instance due to geographical or historical reasons. For a model where state size is determined endogenously, see, e.g., Alesina and Spolaore (1997, 2003).

[17] Naturally, one might give an even more detailed definition of oligarchies, depending on which group rules. For example, an oligarchy of group $\mathcal{P}$ could be denoted as an ochlocracy (the rule of the mob), an oligarchy of group $\mathcal{A}$ or $\mathcal{L}$ as a plutocracy (the rule of the rich in the respective situation), and an oligarchy of group $\mathcal{E}$ as an aristocracy (the rule of the best along all dimensions).

[18] This positive and non-normative definition of democracy might sound strange at first, but it effectively makes no difference for the political outcome whether a homogeneous majority directly dictates the public actions (redistribution in the concrete case), or whether the same majority competes in a democratic ballot with opposing groups who *de jure* have the right to vote, but will *de facto* fail in achieving their political goals. This is in line with the famous classification of Aristotle (1943) who defined democracy as an inferior form of government where the state is ruled by the many who only pursue their private interests. Note that our pragmatic and potentially oversimplified definition of the multi-faceted concept of democracy is primarily a consequence of dealing with a one-dimensional policy space only.

Additionally, we can differentiate between two different types of democracy. A *type-I* democracy emerges if the RC represents the majority of the population as well as the majority of different income groups within society. Note that there even exists a special situation in which the RC embraces the entire population and hence all income groups. We call this the grand coalition. In this case, all groups of society are bound together by the fact that no smaller coalition is winning and stable. One might consider such a grand coalition as the purest form of democracy in which all members of society, even the small minorities, play an active role in policy determination and are actively integrated by all others. From this definition of a *type-I* democracy, one can distinguish a democracy of *type II* in which the ruling coalition comprises the majority of the population, but only a minority of groups in society, i.e., one single group in the given context.

Note that a distinction between the two types of democracies is not obvious from a normative perspective, since in both cases the majority of the population is involved in the redistribution decision. However, in a *type-II* democracy, the largest group has the power to dominate all other groups of society that are minorities and extract redistribution from them. It is this monopoly of political power within a *type-II* democracy that contradicts the typical connotation of a democracy in which different groups of society can express their will and influence public decisions. For lack of a better terminology, and since the previous literature made no such distinction, we continue to refer to those political regimes as democracies. Note, however, that incidentally the classification conceptually coincides with Lipset's distinction of democracy, popularly based dictatorship and oligarchy.[19] With this terminology in mind, we define $P_{s_{MAX}}$ as the power of the *largest* group in society and state the following propositions regarding the different types of democracy.

Proposition 1 (Democracy type I)**.** *The political regime is a democracy (type I) if the most powerful group is dominated by the rest of society and no group is dominant in size,* $2\,P_{MAX} \leq \sum_{i \in S} P_i \wedge s_{MAX} \leq 0.5 \implies s_{RC} \geq 0.5 \wedge |RC| > 1$.

Proof. The Proposition follows directly from Lemmata 1 and 2 and the application of Definition 1. □

[19]See the quote in footnote 1. The distinction of democracies of type I and II is also related to de Tocquevilles (1864) famous note on the *tyranny of the majority.* Alternatively, one might interpret a democracy of type II as a people's republic since the opinion of the mass constitutes a monopoly of power.

Proposition 2 (Democracy type II). *The political regime is a democracy (type II) if and only if society is strictly dominated by the largest group that represents the majority of the population,* $2P_{s_{MAX}} > \sum_{i\epsilon S} P_i \wedge s_{MAX} > 0.5 \iff s_{RC} \geq 0.5 \wedge |RC| = 1$.

Proof. The Proposition follows directly from Lemmata 1 and 2 and the application of Definition 1. □

Contrary to Proposition 2, which describes the necessary and sufficient condition for all possible realizations of a *type-II* democracy, Proposition 1 only states a sufficient condition for a democracy of type I to emerge in equilibrium. In fact there exist several alternative equilibrium conditions for *type-I* democracies. For completeness we derive all these conditions in Proposition 6 in the Appendix, but do not elaborate on them in order to keep the analysis as simple as possible.

4 The Politico-Economic Equilibrium

4.1 Production Environment and Factor Incomes

This section extends the previous analysis by endogenizing factor incomes with respect to the distribution of strength and ability. To illustrate the main points, we normalize the individual time endowment h to 1 and adopt a CRS specification of the production function

$$Y = (A_a \Lambda + A_l L)^{\sigma} H^{1-\sigma} , \tag{11}$$

with $0 < \sigma < 1$. Without being essential for the results, this specification provides a simple way to model redistributive conflicts along the development path by differentiating between ability-augmenting and strength-augmenting productivity parameters A_a and A_l with $A_a, A_l > 0$.[20] Assuming perfectly competitive markets, the reward for every production factor equals its marginal product. Given expressions (4) and (11), individual factor

[20]This specification of the production function is formally equivalent to the production of a homogeneous commodity in two distinct sectors, one employing exclusively ability together with time, and the other exclusively physical strength together with time. Variations in productivity parameters induce variations in income levels as well as the shares of total income going to ability and physical strength, while the time income share is constant.

income of a member of group i therefore becomes

$$y_i \;=\; (A_a\Lambda + A_l L)^{\sigma} H^{1-\sigma}\left[\frac{(1-\sigma)}{H} + \sigma\frac{(A_a a_i + A_l l_i)}{(A_a\Lambda + A_l L)}\right] \quad \text{with } i \in S\ . \tag{12}$$

For the following analysis, let us define $\lambda_i = s_i y_i / y$ as the share of total income that is produced by group i. Note that this expression also reflects the relative power of group i. Employing equations (1) and (2) and using the information contained in Figure 1 and equation (6), equation (12) can be rewritten as

$$\lambda_P \;=\; (1-\sigma)\,(1-\beta)\,(1-\gamma) \tag{13}$$

for the $\mathcal{P}$-group,

$$\lambda_{\mathcal{L}} \;=\; (1-\sigma)\,(1-\beta)\,\gamma + \sigma\frac{(1-\beta)\,\gamma A_l}{(A_a\beta + A_l\gamma)} \tag{14}$$

for the $\mathcal{L}$-group,

$$\lambda_{\mathcal{A}} \;=\; (1-\sigma)\,(1-\gamma)\,\beta + \sigma\frac{(1-\gamma)\,\beta A_a}{(A_a\beta + A_l\gamma)} \tag{15}$$

for the $\mathcal{A}$-group, and

$$\lambda_{\mathcal{E}} \;=\; (1-\sigma)\,\gamma\beta + \sigma\frac{\gamma\beta\,(A_a + A_l)}{(A_a\beta + A_l\gamma)} \tag{16}$$

for the $\mathcal{E}$-group, respectively.

On the basis of these expressions, we can now characterize different politico-economic equilibria that are implied by, and consistent with, the distribution of production factors, in particular strength and ability, in the population. The politico-economic equilibrium reflects the subgame perfect equilibrium of the game described in section 2.3 and incorporates the production and coalition formation stages.

4.2 Endogenous Democracy

In principle, all equilibria can be solved analytically, and the characterization of equilibria presented in Section 3 generally applies. But to highlight the main results as well as their intuition, it is useful to demonstrate the results by ways of parametric examples. To this end, we characterize the taxonomy of politico-economic equilibria graphically in the γ-β space, i.e., in terms of the distribution of physical strength and intellectual ability in the population. Each combination of γ and β corresponds to an economy with the corresponding distribution of endowments. As our benchmark example, we apply a production function with $A_l = A_a = 20$ and $\sigma = 0.7$, that is, the income share of mere time which is distributed equally across all individuals equals 0.3.

Figure 2 presents the corresponding allocation of politico-economic equilibria.[21] The γ-β space is decomposed into different areas of γ-β combinations that imply particular equilibrium constellations. From Lemma 1 it follows that there exists a unique equilibrium, in terms of RC and the corresponding redistribution scheme, for each single γ-β combination, i.e., everywhere in the admissible γ-β space. The corresponding characterization in terms of democracy or oligarchy follows from Propositions 1 and 2.[22]

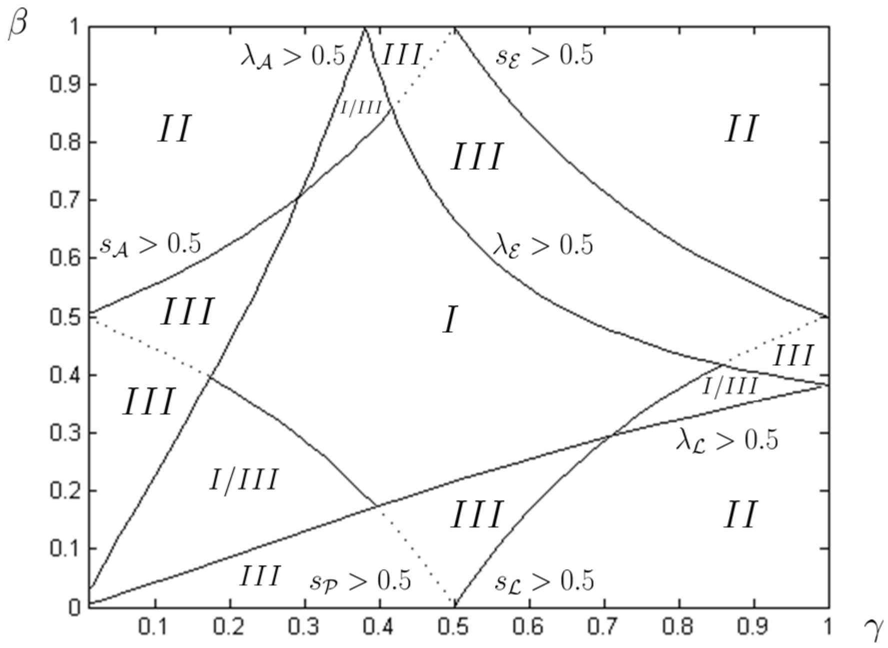

Figure 2: Political equilibria with balanced productivity levels ($A_l = 20, A_a = 20, \sigma = 0.7$).

The diamond-shaped set of lines illustrates the combinations of γ and β for which particular groups represent half of the entire population. For instance, the downward-sloping line in the South-West part of the figure corresponds to the condition for group $\mathcal{P}$ comprising half of the population, $s_{\mathcal{P}} = 0.5$. For combinations of γ and β below that

[21]In view of Lemma 2 and Figure 2, it becomes clear why a population structure with four groups is the simplest structure that allows to derive all types of equilibria, including the grand coalition, while retaining graphical tractability. Adding additional groups (or factors) would increase the number of dimensions and therefore unnecessarily complicate the analysis.

[22]In some cases this also requires the consideration of Proposition 6, see Appendix.

curve, group $\mathcal{P}$ represents the majority of the population. Similarly, for combinations of γ and β above the left-upper part of the diamond, group $\mathcal{A}$ represents the absolute majority, for combinations of γ and β above the right-upper part of the diamond, group $\mathcal{E}$ represents the absolute majority, and for combinations of γ and β below the right-lower part of the diamond, group $\mathcal{L}$ represents the absolute majority. Consequently, a *type-I* democracy in which the largest group comprises less than half of the population can only emerge in the area within the diamond.

But even in this area such a democracy is not an equilibrium outcome if a single group has the power to rule the state on its own. The condition for which this can be group $\mathcal{A}$ is given by the steep upward sloping locus from the origin that represents all γ-β combinations for which $\lambda_{\mathcal{A}} = 0.5$ holds. To the left of this line, the members of group $\mathcal{A}$ generate more than half of total income, $\lambda_{\mathcal{A}} > 0.5$, and therefore constitute the single most powerful group that can dominate in open conflict against any other group or coalition of groups. A larger endowment of ability than given by this condition – in terms of a higher value of β or combinations of γ and β above this threshold – makes the group $\mathcal{A}$ even more dominant. In this case the political equilibrium is either an oligarchy (areas III) or a *type-II* democracy (area II) depending on the respective $\gamma-\beta$ combination. The corresponding condition for group $\mathcal{L}$ to be more powerful than all others together is the flat upward sloping locus from the origin. Thus, to the right of this line, i.e., for higher values of γ, group $\mathcal{L}$ is strictly dominant and constitutes the ruling elite. Finally, the respective condition for group $\mathcal{E}$ is given by the downward sloping locus $\lambda_{\mathcal{E}} = 0.5$ in the North-Eastern region that, together with the loci $\lambda_{\mathcal{A}} = 0.5$ and $\lambda_{\mathcal{L}} = 0.5$, forms a triangular shape.[23] Since the $\mathcal{P}$-group is disadvantaged in all dimensions of endowments, it could only rule the state on its own if the group size $s_{\mathcal{P}}$ and the income share devoted to the common production factor, $1 - \sigma$, become sufficiently large. Then, the size effect compensates for disadvantages in factor endowments and a *type-II* democracy with the poor mass as the sole ruler is the equilibrium outcome.[24] We do not consider this case in Figure 2. Here, only one of the other three groups can potentially rule on its own.

The first main result that emerges from this discussion is the characterization of the

[23]The analytical expressions of all $\lambda_i = 0.5$ loci are given in the Appendix.

[24]More precisely, $s_{\mathcal{P}}(1 - \sigma) > 0.5$ must hold. Note that this inequality can only be satisfied for $\sigma < 0.5$ and $s_{\mathcal{P}} > 0.5$, but not in the current numerical example. For $\sigma < 0.5$ the $\lambda_{\mathcal{P}} = 0.5$ locus emerges in the South-Western corner of Figure 2 and the $\lambda_{\mathcal{A}} = 0.5$ locus ($\lambda_{\mathcal{L}} = 0.5$ locus) shifts up (right).

conditions, in particular of the distribution of resources in the economy, under which democracy can emerge. These conditions are summarized in terms of areas I and II in Figure 2, which represent all combinations of γ and β in which a democracy of *type I* or *type II* arises as an equilibrium. As the figure illustrates, democracy of *type I* is an equilibrium only when inequality is moderate along the two dimensions γ and β, i.e., for intermediate values. The more concentrated strength or ability or both are within a particular group, the less likely becomes a democracy of this type, as illustrated by areas II that denote *type-II* democracies in which the respective largest group rules the state on its own. For example, in the North-East area II of the figure, the elite of strong and able individuals, the members of the $\mathcal{E}$-group, dominate the political decisions, in the North-West this is true for the members of the $\mathcal{A}$-group, in the South-East it is the $\mathcal{L}$-group.

Finally, in all areas denoted by I/III, a *type-I* democracy can only emerge if additional conditions hold. In particular, in these areas a democracy emerges if, and only if, the respective largest group is part of the RC.[25] For example, in the South-West area, this refers to the $\mathcal{P}$-group. Analogous results apply for the other areas denoted by I/III in the North and East of the figure for groups $\mathcal{A}$ and $\mathcal{L}$, respectively. Note that in principle a *type-I* democracy can emerge everywhere in the $\gamma - \beta$ space whereas *type-II* democracies can by definition only occur outside the diamond-shaped area. Thus the admissible $\gamma - \beta$ space for *type-I* democracies is larger than the one for *type-II* democracies.

5 The Stability of Democracy

Having identified the conditions for the emergence of democracy, the model also delivers results on its stability with respect to two dimensions: first, it allows for an analysis of secular changes in the distribution of production factors via variations of β and γ, and second, it can be used to trace the consequences of development in terms of secular changes in the relative importance of production factors in the income generating process, i.e., variations in A_l and A_a.[26]

[25] All conditions for this to be the case are given in the Appendix, see Proposition 6.

[26] Note that within our model framework other non-economic factors that have been considered as being important for the stability of democracy by political scientists, like e.g. civic culture or democratic values, see Almond and Verba (1963) or Putnam (1993), are not taken into account.

The effects of changing the distribution of production factors for a given state of economic development, i.e., for a given combination of A_l and A_a, can already be inferred from the previous discussion of Figure 2. In particular, one can directly derive the consequences of *ceteris paribus* changes in the population structure for the politico-economic equilibrium. Applications for such an analysis are numerous. With regards to changes in β one could think for example of massive schooling programs that change the distribution of ability whereas improvements in health provision or epidemics can affect the distribution of strength γ within society. There might also be changes in the population structure that affect both dimensions simultaneously, like asymmetric population growth due to group specific birth rates caused by a quality-quantity trade-off or immigration of individuals with particular endowments of ability and strength. It is obvious that the results will depend on the status quo before the change in population structure, as well as on the distribution of the other factor. Massive increases in β will lead to an equalization of power and make democracy more likely if applied to an economy with relatively few able individuals, and hence increase the likelihood of democracy. This is particularly the case for a relatively moderate distribution of γ. If applied to an economy with an extreme distribution of γ and/or an economy where only few individuals do not have ability, β is high, however, such a policy might induce a concentration of political power, and make democracy less likely.

A different thought experiment concerns the effects of changes in the relative productivity of the different factors, reflected by A_l and A_a, on the politico-economic equilibrium and democracy in particular. Such changes might for example be caused by unbalanced technological progress like skill-biased technological change or by natural disasters. Before going to the characterization of the implications for the politico-economic equilibrium, it is worth noting that there is always scope for democracy regardless of the importance of factors, while the reverse statement does not hold true. This finding is summarized in the following propositions.

Proposition 3 (Existence)**.** *Irrespective of the productivity environment A_a and A_l, there always exist admissible $\gamma - \beta$ combinations for which ...*

1. *... a democracy of type I exists in equilibrium.*

2. *... a democracy of type II exists in equilibrium.*

Proof. See Appendix. □

Proposition 4 (Impossibility)**.** *Irrespective of the productivity environment A_a and A_l, there always exist admissible $\gamma - \beta$ combinations for which ...*

1. *... a democracy of type I does not exist in equilibrium.*

2. *... a democracy of type II does not exist in equilibrium if σ is sufficiently large* ($\sigma > 0.5$).

Proof. See Appendix. □

The results of these Propositions are particularly noteworthy from a policy perspective, since they essentially state that inequality in factor endowments, rather than the level of development in terms of technology and productivity of particular factors, is the central determinant for democracy. Democracy can be established for any productivity environment by ensuring a suitable distribution of factors or factor incomes. On the contrary, for certain (especially unequal) factor distributions, the Proposition shows that there is *no* constellation of productivity for which a *type-I* democracy can emerge in equilibrium. Hence, the model suggests that there are limits for the possibility to implement such democracies by mere technology or income transfers. These results modify Lipset's (1959) Modernization hypothesis which was introduced in the beginning in an important way.

To illustrate the implications of variations in the relative importance of factors in the income generating process, we change the baseline scenario and consider two stylized cases. The first one refers to an underdeveloped, rural society in which physical strength is much more important than ability in the production process. This we take into account by setting $A_l = 20$ and $A_a = 0.1$. The second case represents a (post-)industrialized society in which physical strength lost its relative importance and ability has become the predominant income generating factor of production. In our static model we replicate this kind of skill-biased technological change in a very simplified manner by assuming A_l to stay constant and increasing A_a to 5000.

The politico-economic equilibria for the rural society are depicted in Figure 3. Again, as in Figure 2, area I represents democracies with a *RC* of at least three groups of society. Areas I/III reflect *type-I* democracies if, and only if, the respective largest group is part of the *RC*, whereas in all areas II a *type-II* democracy occurs for sure. Finally, all areas III represent oligarchies of the respective minority that is most powerful. The most immediate result of this case is that the scope for democracy is much more limited than in the benchmark case: the area of combinations of γ and β for which democracy of *type I* can be sustained in equilibrium is much smaller. On the other hand, there is much more scope for oligarchies. In particular, democracy only emerges as outcome in societies in which ability is distributed fairly equally, i.e., intermediate values of β, whereas it can emerge for a large range of values of γ.

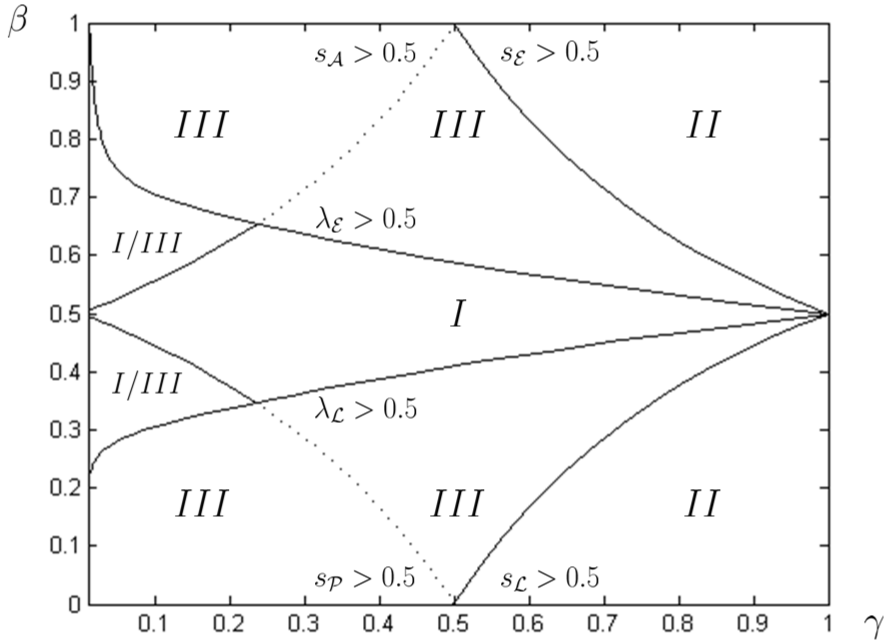

Figure 3: Political equilibria in a rural society ($A_l = 20, A_a = 0.1, \sigma = 0.7$).

A different, yet somewhat symmetric, picture emerges when considering a developed society where ability rather than physical strength is the most important factor of production, as is done in Figure 4. Again, areas I depict *type-I* democracies and areas I/III represent situations in which such democracies might occur if additional conditions are satisfied. Areas II represent *type-II* democracies and all other areas depict oligarchies of the respectively most powerful group. Here also, the scope for democracy is fairly limited, and the distribution of strength must be fairly equal, i.e. intermediate values of γ must occur, for democracy to arise. This has important implications. In an economy of this type, in which ability is by far the more important factor for production, even small variations in γ, for example due to immigration of low-skilled workers or some other asymmetric change in the demographic structure, can have far-reaching implications for the politico-economic equilibrium, up to the point that democracy becomes infeasible in equilibrium. In this respect, the model can rationalize to what extent demographic change, in particular with respect to the distribution of low-skilled and high-skilled labor, may provide a challenge for existing democracies. This way, the model can also give some guidance as to what are the likely consequences of drastic demographic changes or policies.[27]

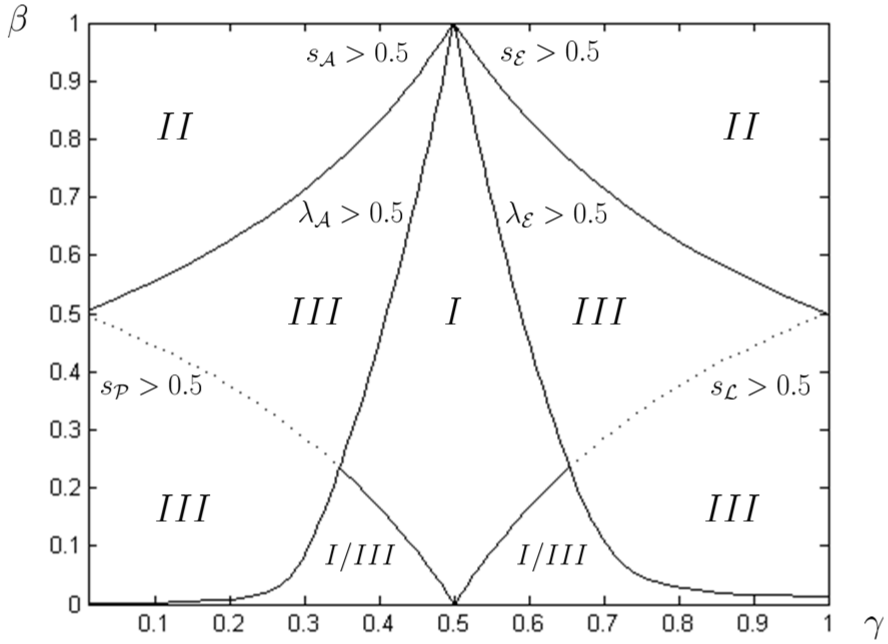

Figure 4: Political equilibria in an industrialized society (A_l = 20, A_a = 5000, σ=0.7).

Although the focus of our analysis rests on the stability of democracy the model delivers additional results on the stability of oligarchies. For the sake of completeness we state the following proposition which does not automatically follow from Proposition 4.

Proposition 5 (Oligarchic Rule). *Irrespective of the productivity environment A_a and A_l, there always exist admissible $\gamma - \beta$ combinations for which an oligarchy of group $\mathcal{E}$ exists in equilibrium.*

Proof. See Appendix. □

[27]An example would be the one-child policy conducted by the Chinese government which might not be sufficient as a regime-stabilizing measure in the long run since – despite its preserving effects on the population structure – changes in the technological environment are not taken into account.

The proof of Proposition 5 shows that the area III between the $\lambda_{\mathcal{E}} = 0.5$ and the $s_{\mathcal{E}} = 0.5$ locus within the diamond-shaped area always exists independently of the productivity environment A_a and A_l. This is quite intuitive. Since the $\mathcal{E}$-group generates the highest per-capita income, this can compensate for disadvantages in group size and facilitates minority rule.

6 Concluding Remarks

This paper has developed a model of political institutions, in which democratic or oligarchic rule emerges as equilibrium outcome of a political struggle for redistribution in an environment that is a priori weakly institutionalized in the sense that binding agreements about redistribution policies or coalitions are not possible. The results show that factual inequality along several dimensions, in terms of the distribution of factors in the economy as well as of their importance in the income generating process, is central for the emerging political institutions. Democracies can emerge only in fairly balanced economic environments whereas alternative scenarios give rise to various forms of oligarchy. This way, the model shows that the advent of democracy neither is an indispensable event in the process of development, nor necessarily marks the beginning of an era of eternal stability of democracy. The results have important implications. First, democracy might not be the automatic outcome of economic development, and even if it emerges as consequence of economic development, it might not be stable in the long run. Democracies might only be a temporary phenomenon and might fail if income inequality between the different social groups becomes too large. The model characterizes the conditions under which this is the case. Second, the model shows that the distribution of factors or incomes, respectively, rather than the level of economic development, is key for democracy to emerge. The results suggest that democratization is possible at every level of economic development if the distribution of production factors lies within a certain range.

The model presented in this paper suggests various directions for future research. Several implications of the model can be tested empirically, including the prediction that democracies are more likely to emerge in balanced economic environments, with fairly equal factor incomes. Another interesting avenue for future research would be to link the model closer to the empirical and theoretical concepts of polarization and fractionalization, as developed by Esteban and Ray (1994, 2008) and Alesina et al. (2003). Furthermore, a dynamic version of the model could be used to investigate the interdependencies of the political regime and the corresponding policies on the one hand, and endogenous technological change and the associated changes in the income distribution on the other hand. Apart from that, it would be worthwhile to analyze how the endogenous implementation of a rule of law that allows for binding commitments would affect our results.

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Appendix

Proof of Lemma 1

We first show that for any group there exists a pure strategy profile σ^* that is a $SPNE$ and leads to a unique RC.

Part I. Existence. This part of the proof follows the structure of the proof of *Lemma 1* in Acemoglu, Egorov, and Sonin (2008). First consider the preferred coalition of agenda setter $i \in S_k$ at stage $k \in \{k \in \mathbb{N}_0 : k \leq 3\}$ of game Γ where S_k is the set of all (remaining) groups whose income has not been redistributed away up to the current stage of the game, i.e., $S_0 = S$, $S_k \subset S \, \forall \, k > 0$ and $S_k \neq \varnothing \; \forall \; k$. Let $\mathscr{P}(S_k)$ denote the power set of S_k and let $\mathcal{I}_i = \{\mathcal{I} \in \mathscr{P}(S_k) : i \in \mathcal{I}\}$ be the set of all coalitions that include group i whereas $F_i = \{F \in \mathcal{I}_i : 2P_i > P_F\}$ represents the set of all coalitions in which group i is more powerful than the other coalition members at the current stage of the game. Define the generic set of winning coalitions as $W_k = \{W \in \mathscr{P}(S_k) : P_W > 0.5P_{S_k}\}$ and denote the set of stable coalitions as $E_k = \{E \in \mathscr{P}(S_k) : [\nexists\, Q \subset E : 2P_Q > P_E \wedge [P_Q \geq 2\max_{j \in Q} P_j \vee |Q| = 1]]\}$. Additionally, we define the union of the set of coalitions that are both winning and stable and the set of all (remaining) groups at the current stage of the game which is given by $R_k = [W_k \cap E_k] \cup S_k$ where the coalition that exhibits the lowest aggregate power in the set is given by

$$\Omega = \underset{X \in R_k}{\operatorname{argmin}} \, P(X) \ .$$

Then, the preferred proposal of an agenda setting group i at stage k of the game is given by

$$\Pi_{i,k} = \underset{X \in \mathcal{I}_i \cap R_k}{\operatorname{argmin}} \, P(X) \ .$$

This does not mean that there exist no other proposals which group i would support on the voting stage at a given history $\mathbb{h}_{\mathbb{k}}$ of the game.[28] Let $A_k \subseteq S_k$ be the set of all groups that have not been acting as an agenda-setter at the current stage of the game yet and let the subset $A_k^+ \subseteq A_k$ be defined as $A_k^+ = \left\{A_k^+ \in A_k : A_k^+ \in \Omega\right\}$. Now, define $\Pi_k = \bigcup_{i \in A_k} \Pi_{i,k}$ as the set of preferred proposals of all groups that have not been acting as an agenda-setter at the current stage of the game yet. Consequently, the most preferred proposal in view

[28] Since every history of the game $\mathbb{h}$ ends at a single decision node it must not be confused with the current stage of the game k.

of group i among all the proposals of groups that have not acted as agenda setter yet, can be written as

$$\Psi_{i,k} = \begin{cases} \underset{X \in \Pi_k \cap \mathcal{I}_i \cap R_k}{\operatorname{argmin}} P(X) & \text{if } \Pi_k \cap \mathcal{I}_i \cap R_k \neq \varnothing \\ \varnothing & \text{otherwise .} \end{cases}$$

For notational convenience, the power of this coalition $\Psi_{i,k}$ is defined to be infinite if it equals the empty set, i.e., $P_{\Psi_{i,k}} = \infty$ for $\Psi_{i,k} = \varnothing$. Then, the pure strategy profile for group i reads

$$\sigma^*_{i,k} = \begin{cases} \text{agenda-setting stage: } i \text{ proposes } \Pi_{i,k} \\ \text{voting stage: } i \text{ votes} \begin{cases} \textit{yes} & \text{if } \Pi_{j,k} \in \mathcal{I}_i \cap R_k \wedge \left[P_{\Pi_{j,k}} \leq P_{\Psi_{i,k}} \vee \left[i \notin \Omega \wedge A^+_k \neq \varnothing\right]\right] \\ & \text{or } \Pi_{j,k} \in F_i \wedge \left[\{i\} \neq \Omega \vee A^+_k = \varnothing\right] \\ \textit{no} & \text{otherwise .} \end{cases} \end{cases}$$

where $\Pi_{j,k}$ denotes the proposal made by group $j \in S_k$ on which groups currently vote. Now we need to prove that the pure strategy profile σ^* which is a vector of $\sigma^*_{i,k} \forall i, k$ constitutes a $SPNE$. Since we consider a finite game it is sufficient to show that there exists no one-shot deviation from $\sigma^*_{i,k}$ which is profitable for group i at any given history $\mathbb{h}$ of the game. In order to do this we need to distinguish two cases each one itself containing two sub-cases, since in this sequential game any group i is either a voter (*case A*) or an agenda setter (*case B*) at a given history of the game, and any proposed redistribution policy can either be rejected (*subcase 1*) or accepted (*subcase 2*).

Case A

Subcase A.1. Suppose that instead of voting *yes* according to $\sigma^*_{i,k}$ voter i would be better off if he voted *no*. Since the votes of the other groups do not depend on the decision of group i such a behavior could only cause a rejection of a proposal that would have been accepted otherwise if group i is pivotal for the decision outcome. In every other case such a deviation has no effect on equilibrium outcome and therefore cannot be beneficial. For this reason, let us assume that group i is pivotal for the decision outcome and that it votes *no* contrary to $\sigma^*_{i,k}$.[29]

[29]For consistency, and without loss of generality, the strategy of non-pivotal or indifferent voter is

To understand why no such deviation can be beneficial if $\Pi_{j,k} \in \mathcal{I}_i \cap R_k$ holds with $P_{\Pi_{j,k}} \leq P_{\Psi_{i,k}}$ is almost trivial since the latter condition implies that $\Pi_{j,k}$ either equals the previous or the current $\Psi_{i,k}$. Thus from the perspective of group i there exists no better proposal on which will be voted on at the given stage of the game according to σ^*. Voting *no* and thereby rejecting a proposal $\Pi_{j,k} \in \mathcal{I}_i \cap R_k$ can therefore not be beneficial for $P_{\Pi_{j,k}} \leq P_{\Psi_{i,k}}$.

Now suppose that $\Pi_{j,k} \in \mathcal{I}_i \cap R_k$ holds with $i \notin \Omega \wedge A_k^+ \neq \varnothing$. In this case, with regards to σ^* rejecting the current proposal will result in the proposal and acceptance of coalition Ω at a subsequent history of the game. Since group i is not part of this coalition it cannot benefit from voting *no* instead of *yes* in such a situation.

Next consider the case where $\Pi_{j,k} \in F_i$ holds and group i is not more powerful than all other groups, $\{i\} \neq \Omega$. Since F_i consists of all coalitions in which group i is more powerful than all other coalition members, it is clear that whenever one of those coalitions is proposed and accepted given $\{i\} \neq \Omega$, group i strictly prefers such a proposal to $\Pi_{i,k}$ as it implies $\tilde{p}_i$ to become maximal at the subsequent stage of the game. On the opposite, consider a history of the game where $\Pi_{j,k} \in F_i$ and group i is more powerful than all other groups but will not act as an agenda-setter anymore, $A_k^+ = \varnothing$. Then, the best possible proposal after a rejection of the current is $\Psi_{i,k}$. Even though voting for $\Pi_{j,k} \in F_i$ causes some additional redistribution cost ϵ for group i, these are outweighed when becoming the sole ruler at the subsequent stage of the game as we assumed ϵ to be arbitrarily small. Thus, group i strictly prefers to vote *yes* for any $\Pi_{j,k} \in F_i$ if $A_k^+ = \varnothing$. We can therefore conclude that it is not beneficial to vote *no* contrary to $\sigma_{i,k}^*$ for any group $i \in S$ at any stage of the game.

Subcase A.2. Now suppose that instead of voting *no* according to $\sigma_{i,k}^*$ group i would be better off if it voted *yes*. Again, this could only affect equilibrium outcome if group i's decision is pivotal and leads to the acceptance of a proposal that would have been rejected otherwise. Let us assume it does.

Let us first consider all cases where $\Pi_{j,k} \notin \mathcal{I}_i \cap R_k$ holds. Suppose additionally $\Pi_{j,k} \notin F_i$. In this case, it is obvious that a deviation from $\sigma_{i,k}^*$ cannot be beneficial for group i since such a decision would lead to an unstable coalition in which group i is not the most powerful group. Given this, income of group i would be redistributed away at the subsequent

assumed to be characterized by σ^* in the following.

stage of the game if such a proposal was accepted. Now suppose that $\Pi_{j,k} \notin \mathcal{I}_i \cap R_k$ holds with $\Pi_{j,k} \in F_i \wedge \{i\} = \Omega \wedge A_k^+ \neq \varnothing$. Also in this case voting *yes* instead of *no* is not beneficial for group i since it has not been acting as an agenda-setter yet and strictly prefers to propose and enforce the coalition $\Omega = \{i\}$ at a subsequent history of the game.

We next focus on all cases where $\Pi_{j,k} \notin F_i$ holds with $\Pi_{j,k} \in \mathcal{I}_i \cap R_k$ and $P_{\Pi_{j,k}} > P_{\Psi_{i,k}}$. Note that from $P_{\Pi_{j,k}} > P_{\Psi_{i,k}}$ it follows directly that $\Pi_{j,k} \neq \Psi_{i,k}$ and $\Psi_{i,k} \neq \varnothing$ must hold which rules out that $i \in \Omega \wedge A_k^+ = \varnothing$ can be true. Therefore we only need to distinguish two different cases. First consider that additionally $i \in \Omega \wedge A_k^+ \neq \varnothing$ holds true which implies $\Pi_{j,k} \neq \Omega$.[30] In this case, accepting the current proposal is not beneficial as the better proposal Ω will be made and accepted at a subsequent history of the game according to σ. Next suppose that $i \notin \Omega \wedge A_k^+ = \varnothing$ holds true instead. This implies that $\Psi_{i,k} \neq \varnothing$ will be proposed and accepted at a subsequent history of the game which generates a higher payoff for group i than the current proposal $\Pi_{j,k} \neq \Psi_{i,k}$.

Finally, consider the case where $\Pi_{j,k} \in F_i$ and $\Pi_{j,k} \in \mathcal{I}_i \cap R_k$ hold with $P_{\Pi_{j,k}} > P_{\Psi_{i,k}}$ and $i = \Omega \wedge A_k^+ \neq \varnothing$. Also in this case, group i strictly prefers to refrain from voting *yes* in order to propose and enforce Ω at a subsequent history of the game We can therefore conclude that it is not beneficial to vote *yes* contrary to $\sigma_{i,k}^*$ for any group $i \in S$ at any stage of the game.

Case B

In this case we show that group i cannot benefit from making a proposal $\pi_{i,k} \in \mathcal{I}_i$ that differs from that stipulated by $\sigma_{i,k}^*$. Again, we need to distinguish two different subcases.

Subcase B.1. Let us first assume that there exists such an alternative proposal $\pi_{i,k} \neq \Pi_{i,k}$ and that $\Pi_{i,k}$ is rejected if proposed. Then, obviously $\pi_{i,k}$ must be accepted if proposed as otherwise group i would not benefit from making this proposal.

By definition we know that $\Pi_{i,k} \in \mathcal{I}_i \cap R_k$ holds. Suppose first that $A_k^+ \neq \varnothing$ holds in addition. This implies that $j \in \Omega$ must also be true as otherwise $\Pi_{i,k}$ would not be rejected. Thus, in the given situation a rejection of $\Pi_{i,k}$ can only occur if $P_{\Pi_{i,k}} > P_{\Psi_{j,k}}$, i.e., $P_{\Pi_{i,k}} > P_\Omega$ was to hold which according to σ^* is only possible for $i \notin \Omega$. But then, there can exist no $\pi_{i,k} \in \mathcal{I}_i$ which would not also be rejected.

Now assume that $A_k^+ = \varnothing$ holds instead. In that case again, $\Pi_{i,k}$ would only be rejected if $P_{\Pi_{i,k}} > P_{\Psi_{j,k}}$ was to hold which directly rules out $\Psi_{i,k} = \varnothing$. From this it follows that

[30] Note that for $i \in \Omega \wedge A_k^+ \neq \varnothing \Leftrightarrow \Psi_{i,k} = \Omega$.

either $\Pi_{i,k} = \Psi_{i,k}$ or $\Pi_{i,k} = \Omega$ must be true according to σ^*. Thus, either the former inequality does not hold or there can exist no $\pi_{i,k} \in \mathcal{I}_i$ which would not also be rejected. For this reason no deviation from σ can be beneficial in the given subcase.

Subcase B.2. Let us now suppose that there exists an alternative proposal $\pi_{i,k} \neq \Pi_{i,k}$ and that $\Pi_{i,k}$ is accepted if proposed. Note that by the nature of the game $\pi_{i,k} \in R_k$ holds as no proposal $\pi_{i,k} \notin W_k$ can be and no proposal $\pi_{i,k} \notin E_k$ will be accepted. Furthermore, no proposal $\pi_{i,k} \notin \mathcal{I}_i$ can be made by group i. Hence $\pi_{i,k} \in \mathcal{I}_i \cap R_k$ needs to hold.

Given our assumption of a bijective power mapping $\pi_{i,k} \neq \Pi_{i,k}$ then implies $P_{\Pi_{i,k}} < P_{\pi_{i,k}}$ since $\Pi_{i,k} = \operatorname{argmin}_{X \in \mathcal{I}_i \cap R_k} P(X)$. With regards to the optimization problem (10) we can therefore conclude that it is not beneficial for group i to propose $\pi_{i,k}$ instead of $\Pi_{i,k}$ in the given subcase. □

Part II. Uniqueness. Finally, we need to show that all $SPNEs$ lead to the same RC. We do this by first emphasizing that the assumption of a bijective power mapping implies that in equilibrium different RCs cannot be equal in aggregate power. To see this suppose to the contrary that $P_M = P_Q$ holds for the two equilibrium coalitions $M, Q \in \mathscr{P}(S) \setminus \{\varnothing\}$ which are not identical, $M \neq Q$. Obviously, the bijective power mapping directly rules out $|M| = |Q| = 1$ in the given case. Additionally, a coalition of two groups can never be an equilibrium outcome, because, due to the bijective power mapping, it would not be stable as the stronger group could always propose a winning subcoalition only containing itself at a later stage of the game. Uniqueness in the case of the grand coalition comprising all four groups is trivial. Hence, we need to distinguish two cases, a case with two coalitions comprising three groups each, and a case with one coalition of three groups and another with one group only. First, suppose that each of the two coalitions comprises three groups, i.e., $|M| = |Q| = 3$. In this case, two groups $i, j \in S$ must be part of both coalitions, $i, j \in M \cap Q$. Given this, it requires the third group l also to be in both coalitions, $l \in M \cap Q$, for $P_M = P_Q$ to hold which implies $M = Q$ and thereby contradicts our former supposition. Second consider the case where a coalition M with $|M| = 3$ has the same power as some coalition Q with $|Q| = 1$, i.e., $P_M = P_Q$. Obviously, M can only be winning if it incorporates the fourth group. But then $|M| \neq 3$ holds in equilibrium which contradicts our assumption. Therefore we can conclude that in equilibrium any two coalitions M and Q can only be equal in power, $P_M = P_Q$, when they are identical, $M = Q$.

Under strategy profile σ^* the resulting RC does not depend on the moves of nature. Therefore the $SPNEs$ in our finite coalition formation and redistribution game with perfect information can only lead to different RCs if a pivotal group i is indifferent about her action at a certain decision node. Suppose first that group j is not part of the equilibrium coalition and is indifferent at a given history of the game $\mathbb{h}$. In this case, it can only be pivotal if it supports a coalition $M \neq \Omega$ with $j \in M$ that is not stable. Note that this creates nothing but some redistribution costs ϵ for group j as its income will be redistributed away in a following stage of the game. Therefore group j will always strictly prefer not to be part of any transitory coalition(s). Now suppose that the pivotal group i is part of different equilibrium coalitions and is indifferent at a given history of the game $\mathbb{h}$. This can only be the case if (at least) two actions lead to the same equilibrium payoff which requires – given the optimization problem (10) and the political power of group i – the aggregate power of (at least) two different RCs to be the same. With regards to our former reasoning this is impossible. Thus there cannot exist two different equilibrium coalitions between which any pivotal group $i \in S_k$ is indifferent at a given history of the game $\mathbb{h}$. This establishes the proof of Lemma 1. $\square$

Loci for $\lambda_i = 0.5$

$$\beta\left(\lambda_{\mathcal{P}} = 0.5\right) = \frac{\frac{1}{2} - \sigma - (1-\sigma)\gamma}{(1-\sigma)(1-\gamma)}$$

$$\beta\left(\lambda_{\mathcal{E}} = 0.5\right) = \frac{A_l\gamma[\gamma(\sigma-1)-\sigma]+A_a\left(\frac{1}{2}-\gamma\sigma\right)}{2A_a\gamma(\sigma-1)} + \frac{\sqrt{4\gamma\left[A_l^2\gamma^3(\sigma-1)^2+2A_l\gamma^2\sigma(1-\sigma)(A_l+A_a)+\gamma(A_l+A_a)[(A_l+A_a)\sigma^2+(1-\sigma)]-A_a\sigma(A_l+A_a)\right]+A_a^2}}{4A_a\gamma(1-\sigma)}$$

$$\beta\left(\lambda_{\mathcal{L}} = 0.5\right) = \frac{A_l\gamma[\gamma(1-\sigma)+\sigma]+A_a\left[\gamma(\sigma-1)+\frac{1}{2}\right]}{2A_a\gamma(1-\sigma)} + \frac{\sqrt{4\gamma\left[A_l^2\gamma^3(\sigma-1)^2+2A_l\gamma^2\left[A_l\sigma(1-\sigma)+A_a(\sigma-1)^2\right]+(A_l-A_a)^2\gamma\sigma^2+A_a\gamma[(3A_l-2A_a)\sigma-A_l+A_a]+A_a[(A_l+A_a)\sigma-A_a]\right]+A_a^2}}{4A_a\gamma(1-\sigma)}$$

$$\beta\left(\lambda_{\mathcal{A}} = 0.5\right) = \frac{A_l\gamma[\gamma(1+\sigma)-\sigma-1]+A_a\left[\sigma(\gamma-1)+\frac{1}{2}\right]}{2A_a[(\sigma-1)\gamma-\sigma+1]} + \frac{\sqrt{4\gamma\left[A_l^2\gamma^3(\sigma-1)^2+2A_l\gamma^2\left[A_a\sigma(1-\sigma)-A_l(\sigma-1)^2\right]+\gamma\left(A_l^2+4A_aA_l+A_a^2\right)\sigma^2+A_l\gamma[A_l(1-2\sigma)-A_a(1+3\sigma)]+A_a[(A_l+A_a)\sigma(1-2\sigma)+A_l]\right]+4A_a^2\left(\sigma-\frac{1}{2}\right)^2}}{4A_a[(\sigma-1)\gamma-\sigma+1]}$$

Proof of Proposition 3

Proof. **1.** The proof first shows that there always exists a combination (γ^*, β^*) for which a *type-I* democracy exists in equilibrium independently of A_a and A_l. For this to be true, it suffices that $\lambda_i(\gamma^*, \beta^*) < 0.5$ and $s_i < 0.5 \ \forall \ i \in S$ hold. Suppose that $\beta = \gamma = 0.5$, that is, all groups are equal in size, i.e. group size does not matter for political power. From condition (5) it then directly follows that $\lambda_{\mathcal{E}} > \lambda_{\mathcal{L}}, \lambda_{\mathcal{A}} > \lambda_{\mathcal{P}}$. Thus in this situation only the $\mathcal{E}$-group could possibly rule the state on its own if $\lambda_{\mathcal{E}}(\beta = \gamma = 0.5) > 0.5$ were to hold. Using eq. (16) one finds that in this case

$$\lambda_{\mathcal{E}}\left(\hat{\beta} = \hat{\gamma} = 0.5\right) = \frac{1}{4}(1+\sigma) \ < 0.5 \quad \forall \ 0 < \sigma < 1 \ .$$

Since $0 < \sigma < 1$, $\lambda_i(\beta = \gamma = 0.5) < 0.5$ always holds for every group $i \in S$, independently of the levels of productivity A_a and A_l. Additionally, $s_i < 0.5$ holds by construction. Hence, for $\gamma^* = \beta^* = 0.5$ democracy of *type I* always emerges in equilibrium, regardless of A_a and A_l. Now, note that the point described by (γ^*, β^*) always lies South-West of the $\lambda_{\mathcal{E}}(\gamma^*, \beta^*) = 1/2$ locus, such there exists a set of $\gamma - \beta$ combinations surrounding $\{\gamma^*, \beta^*\}$ for which a *type I* democracy emerges in equilibrium.

2. The proof shows that there always exists a set of $\gamma - \beta$ combinations for which a *type-II* democracy exists in equilibrium independently of A_a and A_l. For this to be true, $\lambda_i(\gamma, \beta) > 0.5$ and $s_i \geq 0.5$ must hold for every such combination. Note that the functions (13), (14), (15) and (16) share a common structure. One component of relative power of every group $i \in S$ equals $(1-\sigma) > 0$ times the respective group size, and is independent from productivity levels A_a and A_l. A second component depends on productivity through the respective factor endowments and is non-negative (and zero for group $\mathcal{P}$). This component is largest for group $\mathcal{E}$ where $(A_a + A_l) / (A_a \beta + A_l \gamma) > 1$ holds since $0 < \gamma, \beta < 1$. Hence, if $\gamma\beta > 0.5$, then

$$\lambda_{\mathcal{E}} = \gamma\beta(1-\sigma) + \gamma\beta\sigma\frac{A_a + A_l}{A_a\beta + A_l\gamma} > 0.5 \quad \forall \ A_a, A_l, \sigma > 0$$

holds, i.e., for all $s_{\mathcal{E}} = \gamma\beta > 0.5$ a *type-II* democracy occurs in equilibrium independently of A_a and A_l, which establishes the proof. This result is almost trivial, since group $\mathcal{E}$

generates the highest income per capita. Hence, it must be the most powerful group when it is the largest. □

Proof of Proposition 4

Proof. **1.** The proof shows that there always exist admissible $\gamma - \beta$ combinations for which a *type-I* democracy does not exist in equilibrium independently of A_a and A_l. This is true, if $\lambda_i > 0.5$ holds which was already shown to be the case for all $s_{\mathcal{E}} = \gamma\beta > 0.5$. Hence, the proof of Proposition 3.2 also establishes the proof of Proposition 4.1.

2. The proof shows that there always exist admissible $\gamma - \beta$ combinations for which a *type-II* democracy does not exist in equilibrium independently of A_a and A_l, if σ is sufficiently large. As already mentioned in the proof of Proposition 3.2 the only component of relative power for group $\mathcal{P}$ equals $(1 - \sigma)\, s_P$ and as such is independent of A_a and A_l. It then directly follows that a *type-II* democracy for all $\gamma - \beta$ combinations in the South-West of the $s_{\mathcal{P}} = 0.5$ locus cannot exist in equilibrium, i.e., $\lambda_{\mathcal{P}} = (1 - \sigma)\, s_P < 0.5$ always holds if $\sigma > 0.5$. □

Proof of Proposition 6

Proof. The proof shows that there always exist admissible $\gamma - \beta$ combinations for which an oligarchy of group $\mathcal{E}$ exists in equilibrium irrespective of the productivity environment A_a and A_l. This is true, if $\lambda_{\mathcal{E}}(\gamma, \beta) > 0.5$ and $s_{\mathcal{E}} < 0.5$ hold. Let us consider the $\mathcal{E}$-group. With regards to Figure 2, and Figures 3 and 4 as well, we can see that both conditions are satisfied for all $\gamma - \beta$ combinations that lie between the $\lambda_{\mathcal{E}} = 0.5$ and the $s_{\mathcal{E}} = 0.5$ locus. Setting both expressions equal and rearranging terms yields

$$\sigma \left[A_a (1 - \beta) + A_l (1 - \gamma) \right] \;=\; 0$$

which cannot be satisfied for $0 < \gamma, \beta < 1$ and $A_a, A_l, \sigma > 0$. Hence, there exists no intersection of both loci. And since the $\lambda_{\mathcal{E}} = 0.5$ locus always lies South-West of the $s_{\mathcal{E}} = 0.5$ locus, there must exist $\gamma - \beta$ combinations for which an oligarchy of group $\mathcal{E}$ exists in equilibrium independently of A_a and A_l. □

Additional Sufficient Conditions for *Type-I* Democracies

The following Lemma characterizes the groups that can be members of ruling coalitions under different constellations of political power.

Lemma 3. *Given* $|RC| \geq 3$ *then group* i *is not part of the RC if it is ...*

1. *... the most powerful group and the two middle powers are relatively equal in power, i.e.* $P_{MIN} \geq \left(\overline{P} - \underline{P}\right) \implies i_{MAX} \notin RC$*; or*

2. *... the third most powerful group, Lemma 3.1 does not apply and the two most powerful groups are relatively equal in power, i.e.*
$P_{MIN} < \left(\overline{P} - \underline{P}\right) \wedge P_{MIN} \geq \left(P_{MAX} - \overline{P}\right) \implies \underline{i} \notin RC$*; or*

3. *... the least powerful group, Lemma 3.1 and 3.2 do not apply and the two most powerful groups are not very unequal in power, i.e.* $P_{MIN} < \left(\overline{P} - \underline{P}\right) \wedge$
$P_{MIN} < \left(P_{MAX} - \overline{P}\right) \wedge \underline{P} \geq \left(P_{MAX} - \overline{P}\right) \implies i_{MIN} \notin RC$.

Proof. Given $|RC| = 3$ every group $i \in RC$ wants to exclude the most powerful group in RC in order to maximize $\tilde{p}_i$. Therefore, a coalition of the three least powerful groups is always the first-best solution. Since it is self-enforcing if $P_{MIN} \geq \left(\overline{P} - \underline{P}\right)$ holds, we can conclude that $\sum_{i \in S} P_i > 2\,P_{MAX} \wedge P_{MIN} \geq \left(\overline{P} - \underline{P}\right) \iff RC = \left\{i_{MIN}, \underline{i}, \overline{i}\right\}$.

If this condition fails the next best alternative is the exclusion of the second most powerful group. But this is not feasible under the given conditions as this would require $P_{MIN} \geq (P_{MAX} - \underline{P})$ to hold, but if this self-enforcement condition is satisfied, then $P_{MIN} \geq \left(\overline{P} - \underline{P}\right)$ is also true and exclusion of the most powerful group is feasible. Thus, the second most powerful group is *always* part of the RC under the given conditions, i.e., $\sum_{i \in S} P_i > 2\,P_{MAX} \iff \overline{i} \in RC$.

For this reason it is in fact the exclusion of the third most powerful group which represents the second-best solution. It will be realized if in the given situation the first best is not feasible and $P_{MIN} \geq \left(P_{MAX} - \overline{P}\right)$ holds. Only if this condition also fails a coalition of the three most powerful groups becomes the preferred choice which requires $\underline{P} \geq \left(P_{MAX} - \overline{P}\right)$ to be self-enforcing. □

By applying Lemmata 2 and 3 and Definition 1 we can state several other sufficient conditions for which a *type-I* democracy emerges in equilibrium.

Proposition 6. *Given* $s_{MAX} > 0.5$ *the political regime is a democracy (type I) if the* *most powerful group is strictly dominated by the rest of society and ...*

1. *... the largest group is the second most powerful group; or*

2. *... the two middle powers are relatively equal in aggregate income,* $P_{MIN} \geq \left(\overline{P} - \underline{P}\right)$*, and the largest group is not the most powerful group; or*

3. *... the two middle powers are relatively unequal in aggregate income, the two most powerful groups are relatively equal in aggregate income,* $P_{MIN} < \left(\overline{P} - \underline{P}\right) \wedge P_{MIN} \geq \left(P_{MAX} - \overline{P}\right)$*, and the largest group is not the third most powerful group; or*

4. *... the two middle powers are relatively unequal in aggregate income, the two most powerful groups are relatively but not very unequal in aggregate income,* $P_{MIN} < \left(\overline{P} - \underline{P}\right) \wedge P_{MIN} < \left(P_{MAX} - \overline{P}\right) \wedge \underline{P} \geq \left(P_{MAX} - \overline{P}\right)$*, and the largest group is not the least powerful group.*