Federal Reserve Bank of Minneapolis
Research Department Staff Report

November 2009

Bailouts, Time Inconsistency and Optimal Regulation*

V. V. Chari

University of Minnesota
and Federal Reserve Bank of Minneapolis

Patrick J. Kehoe

Federal Reserve Bank of Minneapolis,
University of Minnesota,
and Princeton University

ABSTRACT

We make three points. First, ex ante efficient contracts often require ex post inefficiency. Second, the time inconsistency problem for the government is more severe than for private agents because fire sale effects give governments stronger incentives to renegotiate contracts than private agents. Third, given that the government cannot commit itself to not bailing out firms ex post, ex ante regulation of firms is desirable.

*Chari, and Kehoe thank the National Science Foundation for financial support. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Recent experience has shown that governments can, will, and perhaps should intervene during financial crises. Such interventions typically occur because governments seek to minimize the spillover effects of bankruptcy and liquidation upon the broader economy. Such interventions during financial crises alter the incentives for firms and financial intermediaries ex ante. In this paper we ask how optimal regulation should be designed to maximize ex ante welfare taking into account the temptation for the government to intervene ex post.

The theme that we explore in this paper is that, by altering private contracts, the prospect of bailouts reduces ex ante welfare. We view the prescription that governments should refrain from bailing out potentially bankrupt firms as unrealistic in practice. Benevolent governments simply do not have the power to commit themselves to such a prescription. A pragmatic approach to policy dictates that we take as given the incentives of governments to undertake bailouts and design ex ante regulation to minimize the ex ante costs of these ex post bailouts.

In thinking about bailouts by governments, a central question is why would the government find it optimal to bail out firms ex post. We argue that confronted with an ex post situation in which many firms are about to undergo costly bankruptcies, a benevolent government has a strong incentive to bail out firms. These ex post bailouts, however, may distort the ex ante incentives of managers and firms and reduce ex ante welfare. In such a situation, a government with commitment would commit itself not to undertake bailouts. If the government lacks such commitment, it will bail out firms ex post and the expectation of such bailouts will reduce ex ante welfare. In this sense, the government has a time inconsistency problem in bailout policy. We show that this time inconsistency problem creates a role for ex ante regulation. Such regulation can reduce the temptation of governments to bail out firms ex post and thereby raise ex ante welfare.

In analyzing the incentives of benevolent governments to intervene and prevent costly bankruptcies ex post, the obvious question arises, why would firms ex ante enter into contracts which impose ex post costs? More generally, why would firms design contracts that feature ex post inefficient outcomes? Here we develop a model in which the optimal contract between a firm and a manager specifies bankruptcy when outcomes are bad in order to provide proper incentives to managers to engage in effort. Bankruptcy is costly in two ways: it reduces the output of the firm and it imposes nonpecuniary costs on the manager. We think of these nonpecuniary costs as arising both from stigma-like effects on the manager's career as well as loss of private benefits from operating the firm. In the model the optimal contract is ex post inefficient in the sense that, once the manager has exerted effort, bankruptcy imposes costs on the owners of the firm and the manager.

While these ex post inefficiencies create a time inconsistency problem for the government by giving it an incentive to bailout firms ex post, they also create a time inconsistency problem for private agents by giving them an incentive to avoid costly bankruptcy by renegotiating their contracts ex post. Analyzing these incentives requires modeling the benefits and costs of both renegotiation and bailouts. The benefits are the reduction in costly bankruptcies. We assume that the costs arise from changes in the beliefs of private agents about future outcomes. In particular, if a firm ever agrees to renegotiate, private agents will believe that firm will always renegotiate in the future. Expectations of such renegotiations constrain future contracts and thereby reduce future welfare. Likewise, if a government ever bails out firms, private agents believe that the government will always bailout firms in the future. Expectations of such bailouts constrain future contracts and reduce future welfare.

In an environment without commitment, private agents and governments balance these benefits and costs in designing their interventions. For the private agents, this balance implies that ex ante optimal contracts must satisfy a *private sustainability constraint*. For the government, this balance implies that ex ante optimal contracts must, in equilibrium, satisfy a *sustainability to bailouts constraint*.

The parallel way we have modeled benefits and costs for governments and private agents leads us to ask, Given that a contract has already been designed to be privately sustainable, why would it not be sustainable to bailouts? When deciding whether to renegotiate a given contract, the private agents involved in that contract consider the benefits from eliminating bankruptcy of their firm at given prices. When the government decides to bail out firms, it takes into account the private benefits per firm in the same way that private agents do, but, in addition, it also takes into account the benefits to other firms from its intervention. These benefits arise because by bailing out firms the government can reduce the aggregate amount of assets sold in the market place and thereby raise the prices of these assets. The idea is that bankruptcy is socially costly because it forces firms to sell their assets and these *fire sales* reduce the value of assets in otherwise healthy firms. Bailouts help reduce fire sales and the resulting negative price effects that give rise to the social cost. Since governments take into account fire sale effects and private agents do not, the sustainability to bailouts constraint is tighter than the private sustainability constraint. Thus, a contract that is privately sustainable is not necessarily sustainable to bailouts. In this sense, the time inconsistency problem is for the government is more severe than it is for private agents.

The greater severity of the time inconsistency problem for the government implies that the equilibrium in an economy with bailouts has lower welfare than in an economy without bailouts. It also implies that ex-ante regulation can be desirable. Such regulation must be designed so that ex-post the government does not have an incentive to engage in bailouts. The incentive to bail out firms is large when the aggregate amount of assets in bankrupt firms is large. We show that the optimal ex-ante regulation is to impose a cap on quantity of assets used by each manager and a cap on the probability of bankruptcy. This cap on assets limits the size of individual firms and thus can be interpreted as a regulation that prevents firms from becoming too big. We refer to this regulation as a *too-big-to-fail-cap*.

The cap on the probability of bankruptcy can be implemented by a cap on the debt to value ratio of the firm. The reason is that this ratio is an increasing function of the probability of bankruptcy so that a cap on the probability of bankruptcy is equivalent to a cap on the debt to value ratio.

1. A simple economy

We begin with a simple static version of our benchmark economy. We use this version to show that, in order to provide incentives, optimal contracts often require *ex post inefficiency*, in the sense that ex post all agents can benefit by altering the terms of the contract. This feature of the model makes optimal contracts time inconsistent, in the sense that optimal contracts without commitment differ from those with commitment, and, in particular, give lower welfare.

Consider a model in which decisions are made at two stages: a first stage, called the beginning of the period, and a second stage called the end of the period. There are two types of agents, called lenders and managers both of whom are risk neutral and consume at the end of the period. There is a measure 1 of managers and a measure 1 of lenders.

The economy has two production technologies. The *storage* technology is available to all agents, which transforms one unit of endowments at the first stage into one unit of consumption goods at the second stage. The *corporate* technology specifies projects that require two inputs at the first stage: effort a of managers and an investment of 1 of goods. This technology transforms these inputs into capital goods. The capital goods then can be used to make stage two consumption goods. Effort a of managers is unobserved by lenders.

If the corporate technology is used the amount of capital goods produced in the second stage stochastically depends on the effort level a of the manager as well as an idiosyncratic exogenous shock representing the manager's current draw of ability. In particular, given effort level a and a draw of ε with probability $p_H(a)$ the high state is realized and $A_H(1+\varepsilon)$ units of capital goods are produced and with probability $p_L(a) = 1 - p_H(a)$ the low state is realized and $A_L(1+\varepsilon)$ units of capital goods are produced where $A_L < A_H$. We assume that higher effort levels increase the probability of the high state. Specifically, we assume that $p_H(a)$ is an increasing, concave function of a. We assume the distribution of ε is given by $H(\varepsilon)$ which has mean zero.

We think of the project as being undertaken by a firm. We think of managers as performing two tasks. The first task is to exert effort a and transform consumption goods from stage 1 into capital goods at stage 2. The second task is to transform capital goods stage 2 into final consumption goods.

After the manager has completed the first task and a certain amount of capital has been produced the firm can choose to continue the project under the current manager or it can declare bankruptcy. If it continues then the project produces one unit of output for every unit of capital, so that the firm's output is

$$(1) \quad Y_{ci}(\varepsilon) = A_i(1+\varepsilon) \text{ for } i \in \{H, L\}$$

where c denotes *continue*. If the firm declares bankruptcy, the manager is removed, the firm incurs a direct output loss and the manager suffers a nonpecuniary cost. The direct output loss occurs because following bankruptcy the capital $A_i(1+\varepsilon)$ is used in an inferior technology, referred to as the *traditional* technology, that yields $R \leq 1$ consumption goods for every unit of capital invested so that the value of the output of the firm in the bankruptcy state is

$$(2) \quad Y_{bi}(\varepsilon) = RA_i(1+\varepsilon)$$

where b denotes *bankruptcy*. In the event of bankruptcy the manager suffers a nonpecuniary loss $-B$. This nonpecuniary cost is supposed to represent extra costs to the manager, such as a loss in reputation or a loss in nonpecuniary benefits from being employed as a manager that are incurred from a liquidation.

Lenders are endowed with e units of a consumption good in the first stage but cannot operate the corporate technology. Managers have no endowments of goods but can operate the corporate technology. Lenders choose whether to lend to firms that operate the corporate technology or to store their endowments.

We assume that $e > 1$. Since the economy has an equal measure of managers and lenders and since the corporate technology uses 1 unit of the endowment per manager the storage technology is always active and the rate of return to lending to the corporate technology is 1.

Let $c_i(\varepsilon)$ denote the consumption of the managers in state i given the realization ε and $d_i(\varepsilon)$ the return to the investor in a project when the state is i and the idiosyncratic shock is given by ε. Let B_i denote the set of idiosyncratic shocks ε such that the firms declares bankruptcy in state $i \in \{H, L\}$ and C_i denote the complementary sent in which the project is continued.

We assume that firms, referred to as financial intermediaries, operate a continuum of projects. Given the symmetry of the expected returns across projects, financial intermediaries will choose the same effort level for all managers. The profits generated by a financial intermediary which finds it optimal to operate the corporate technology at a positive level are

$$(3) \quad \sum_i p_i(a) \left[\int_{C_i} Y_{ci}(\varepsilon) dH(\varepsilon) + \int_{B_i} Y_{bi}(\varepsilon) dH(\varepsilon) - \int [c_i(\varepsilon) + d_i(\varepsilon)] dH(\varepsilon) \right]$$

financial intermediaries compete in offering contracts to managers and lenders. These contracts must attract investment by lenders so that they must offer a return to lenders of at least one. Thus, a contract must meet the following participation constraint for lenders

$$(4) \quad \sum_i p_i(a) \left[\int d_i(\varepsilon) dH(\varepsilon) \right] \geq 1$$

The contracts must also attract managers. Let $\bar{U}$ denote the value of the best alternative contract offered to a managers. Thus, a contract must meet a participation constraint for managers

$$(5) \quad \sum_i p_i(a)\left[\int c_i(\varepsilon)dH(\varepsilon) - B\int_{B_i} dH(\varepsilon)\right] - a \geq \bar{U}.$$

Since the effort choice a of managers is unobservable a contract must satisfy an incentive constraint given by

$$(6) \quad a \in \arg\max_a \sum_i p_i(a)\left[\int c_i(\varepsilon)dH(\varepsilon) - B\int_{B_i} dH(\varepsilon)\right] - a.$$

Finally, the consumption of managers must satisfy a nonnegativity constraint

$$(7) \quad c_i(\varepsilon) \geq 0$$

A. With commitment

Suppose now that financial intermediaries and managers can commit to contracts. Under this assumption the financial intermediaries' contracting problem is to choose a recommended action a, compensation schemes $c_i(\cdot)$, $d_i(\cdot)$ and bankruptcy and continuation sets B_i and C_i to maximize profits (3) subject to (4), (5), (6), and (7).

Clearly the consumption level of a lender that lends 1 to financial intermediaries and invests the rest in the storage technology is given by

$$(8) \quad c^I = \sum_i p_i(a)\left[\int d_i(\varepsilon)dH(\varepsilon)\right] + e - 1$$

The resource constraint is

$$(9) \quad \sum_i p_i(a)\left[\int c_i(\varepsilon)dH(\varepsilon)\right] + c^I \leq$$

$$\sum_i p_i(a)\left[\int_{C_i} Y_{ci}(\varepsilon)dH(\varepsilon) + \int_{B_i} Y_{bi}(\varepsilon)dH(\varepsilon)\right] + e - 1$$

An *allocation* is a collection $a, c_i(\cdot)$, $d_i(\cdot)$, c^I, C_i, B_i. A *competitive equilibrium* is an allocation together with a minimum utility level $\bar{U}$ such that

i) the allocations $a, c_i(\cdot)$, $d_i(\cdot)$, and sets C_i, B_i solve the contracting problem.

ii) the minimum utility level $\bar{U}$ is such that firm profits are zero.

iii) the consumption of lenders satisfies (8).

iv) the resource constraint (9) holds.

Note here that $\bar{U}$ plays the role of a price and that by Walras' Law the resource constraint is implied by zero profits of financial intermediaries and the consumption of lenders (8).

Throughout we will restrict attention to environments in which the competitive equilibrium has an active corporate technology. A sufficient condition for such an equilibrium to exist is that A_H and $p'(0)$ are sufficiently large.

We turn the efficiency of a competitive equilibrium. Given a utility level of lenders $\bar{c}^I$, an allocation is *efficient* if it satisfies the following planning problem, namely to maximize the welfare of managers subject to (6), (7), (8), and

$$(10) \quad c^I \geq \bar{c}^I.$$

Proposition 1. The competitive equilibrium is efficient.

Proof: Since profits are zero in a competitive equilibrium, we can use duality to rewrite the contracting problem as one of maximizing the utility of managers subject to the constraint the firm profits be nonnegative. Substituting for the consumption of lenders from (8) into financial intermediaries' profits (3) yields the resource constraint. Clearly, the rewritten contracting problem coincides with the planning problem. *Q.E.D.*

Consider the following assumption. Let a^O be the solution to the version of the problem with publicly observed effort, namely the value of a that solves

$$(11) \quad p_H'(a)A_H - A_L = 1.$$

Assume that

$$(12) \quad p_H(a^O) < 1$$

Proposition 2. If $A_L < 1$ and (12) holds, then the competitive equilibrium with privately observed effort information has strictly lower effort level a and welfare than the competitive equilibrium with publicly observed effort.

Proof. In the competitive equilibrium with publicly observed effort it is straightforward to show that the optimal effort level solves (11) and the liquidation sets B_H and B_L are empty. The first order condition for effort in the private information economy is

$$\sum_i p_i'(a)\left[\int c_i(\varepsilon)dG(\varepsilon) - B\int_{B_i} dG(\varepsilon)\right] = 1$$

A moment's reflection makes clear that the only way to support the allocations with publicly observed effort in the economy with privately observed effort is to pay the manager an expected compensation of

$$\text{(13)} \quad \int c_H(\varepsilon)dH(\varepsilon) = A_H - A_L$$

in the high state and zero in the low state. But, since $A_L < 1$ if financial intermediaries pay managers this much and pay the lenders 1 unit then profits are negative. To establish this result substitute (1), (2), (4) with equality and (13) into the expression for firm's profits (3) and using the assumption that the expected value of ε is zero, to obtain

$$p_H(a)\left[A_H - (A_H - A_L)\right] + p_L(a)A_L - 1 = A_L - 1$$

which is negative since $A_L < 1$. *Q.E.D.*

From here onwards the term competitive equilibrium refers to competitive equilibrium with privately observed effort.

We now show that the contracting problem reduces to a simpler one under the condition that $A_L < 1$. We will show that in any competitive equilibrium the optimal contracting problem can be reduced to the following: Choose c_H, a, and ε^* to solve

$$\text{(14)} \quad \max p_H(a)c_H - p_L(a)BH(\varepsilon^*) - a$$

subject to

$$(15)\quad a \in \arg\max_{a} p_H(a)c_H - p_L(a)BH(\varepsilon^*) - a.$$

$$(16)\quad p_H(a)c_H + 1 \leq p_H(a)A_H + p_L(a)A_L\left[\int_{\varepsilon^*}^{\bar{\varepsilon}}(1+\varepsilon)dH(\varepsilon) + R\int_{\underline{\varepsilon}}^{\varepsilon^*}(1+\varepsilon)dH(\varepsilon)\right]$$

To establish this result we first note that if $A_L < 1$ the incentive constraint is always binding. Hence an optimal contract must reward the manager only in the high state and set the consumption of managers in the low state to be zero for all ε, that is, $c_L(\varepsilon) = 0$. The intuition for this result is that as long as consumption is positive in the low state, manager's incentives can be improved by shifting consumption from the low state to the high state. Since the manager cares only about expected consumption the optimum can be achieved by setting consumption in the high state to be a constant so that $c_H(\varepsilon) = c_H$.

Second, note the only role of bankruptcy is to improve incentives so that it is never optimal to declare bankruptcy in the high state. In the low state, the optimal bankruptcy rule has a cutoff form: declare bankruptcy for $\varepsilon \leq \varepsilon^*$ and continue otherwise. This result follows because the output loss from bankruptcy, $(1-R)A_L(1+\varepsilon)$, is smaller the lower is ε and the manager only cares about the probability of bankruptcy in the low state. More formally, if the optimal contract had bankruptcy for a high realization ε and continuation for a low realization of ε, then the output loss could be reduced by rearranging the set of realizations for which there is bankruptcy while maintaining the manager's incentives.

Third, in any competitive equilibrium profits are zero. Hence, we can use duality to write the optimal contracting problem as maximizing the utility of the manager subject to a nonnegativity constraint on profits. Note that we write the nonnegativity constraint on profits as (16) using the assumption that the expected value of ε is zero along with the other features of the optimal contract derived above.

We summarize this discussion in a proposition.

Proposition 3. If $A_L < 1$ the optimal contracting problem in a competitive equilibrium can be written as (14).

Next, we will say that allocations are *ex post inefficient* if $\varepsilon^* > \underline{\varepsilon}$. If this inequality holds, then clearly all agents economy can be made better off ex post by continuing the project in the states $[\underline{\varepsilon}, \varepsilon^*]$. Nonetheless, committing to ex post inefficient allocations may be desirable as a way of providing the manager with stronger incentives for providing high effort and thereby raising ex ante welfare.

We now give sufficient conditions so that the optimal allocations with commitment require ex post inefficiency. In providing these conditions, it is convenient to adopt a change of variables so that the manager can be thought of as choosing the probability of success p and incurring an effort cost $a(p)$. Formally, let $a(p)$ be the inverse of the function p_H so that $a(p) = p_H^{-1}(p)$. Consider the allocations that arise when ε^* is restricted to equal $\underline{\varepsilon}$, so that there is no ex post inefficiency (no bankruptcy). Let p_H^E denote the optimal probabilities under this restriction.

Proposition 4. If R is sufficiently close to 1 and $a''(p_H^E)$ is sufficiently small then $\varepsilon^* > \underline{\varepsilon}$. That is, supporting ex ante efficient allocations requires ex post inefficiency.

The proof of this proposition is in the appendix. The basic idea is that the incentive effects associated with bankruptcy are large when $a''(p)$ is small. To see the role of these incentive effects consider the first order condition associated with the incentive constraint, given by

$$(17) \quad c_H + BH(\varepsilon^*) = a'(p_H)$$

Consider the incentive gains from a small increase in the probability of bankruptcy resulting from an increase in ε^*, holding fixed c_H. Differentiating (17) gives

$$\frac{dp_H}{d\varepsilon^*} = \frac{Bh(\varepsilon^*)}{a''(p)}$$

Thus, when $a''(p)$ is small the incentive gains from increasing the probability of bankruptcy are large. If R is sufficiently close to 1, the resource costs of increasing the probability of bankruptcy are small. Hence, when these conditions are met, supporting efficient allocations requires a positive probability of bankruptcy.

B. Without commitment

Suppose now that the agents in this economy cannot commit to contracts. We show that equilibrium allocations without commitment give lower welfare than those with commitment.

Specifically, suppose that after the action a has been taken and the first stage investments have been made, but before the state and the realization of ε have occurred, financial intermediaries and managers can renegotiate their contracts if both parties agree. Clearly, all projects will be continued in order to avoid the output and the nonpecuniary costs of bankruptcy.

To see this result more formally, suppose now that a manager has taken an action

a and first stage investment decisions have been made, but uncertainty has not yet been realized. Consider the outcomes if the firm and the manager agree to renegotiate. We model the renegotiation as follows. The manager makes a take it or leave it offer to the firm. If the firm takes the offer that offer is implemented, while if the firm rejects the offer the existing contract is implemented. Clearly, the firm will accept any offer which yields nonnegative profits. Thus, the best take it or leave it offer is one that maximizes the manager's payoff subject to the constraint that profits are nonnegative. Since the action a has already been taken, it is optimal for the manager to set $\varepsilon^* = 0$ and avoid bankruptcy. Since financial intermediaries profits associated with an accepted offer must be nonnegative, the maximum expected consumption the manager can receive is determined by (16) with $\varepsilon^* = 0$. Hence, the maximum expected payoffs to the manager are

$$(18) \quad \hat{U}(a) = p_H(a)\hat{c}_H - a = p_H(a)A_H + p_L(a)A_L - 1 - a$$

where $\hat{c}_H$ is the consumption associated with the renegotiated contract. Under a given contract if the manager does not renegotiate then expected consumption is determined from (16) and the manager's payoffs are given by

$$(19) \quad U(a, \varepsilon^*) = p_H(a)c_H - BH(\varepsilon^*) - a$$

$$= \left[p_H(a)A_H + p_L(a)A_L\left(\int_{\varepsilon^*}^{\bar{\varepsilon}}(1+\varepsilon)dH(\varepsilon) + R\int_{\underline{\varepsilon}}^{\varepsilon^*}(1+\varepsilon)dH(\varepsilon)\right)\right] - BH(\varepsilon^*) - 1 - a$$

Since $R < 1$, clearly $\hat{U}(a) > U(a, \varepsilon^*)$ so that the payoff to renegotiating is higher than the payoff to continuing with the project if the contract specifies bankruptcy for some states in that $\varepsilon^* > \underline{\varepsilon}$.

In sum, in this static model without commitment the incentive to renegotiate is so strong that the equilibrium has no bankruptcy and, hence, no ex post inefficiency. Thus, without commitment the optimal contracting problem solves (14) subject to the additional constraint that $\varepsilon^* = \underline{\varepsilon}$. Clearly, welfare in an equilibrium without commitment is lower than that with commitment.

2. The Dynamic Contracting Model

Here we develop a dynamic contracting model without commitment. We show that this lack of commitment constrains the optimal contracts entered into by private agents, relative to an environment with commitment. Our dynamic model is an infinite repetition of a modified version of our simple model. The main point of these modifications is to allow for fire sale effects in which changes in the aggregate incidence of bankruptcy alter the prices at which assets are sold. In later sections when we turn to optimal policy these fire sale effects will play a prominent role.

A. The benchmark economy

The benchmark economy we consider is an infinitely-repeated version of a static model. Our benchmark economy has no technology to transform goods from period t to period $t+1$, so that agents cannot save across periods. The static model is a version of the simple economy with three modifications. These three modifications allows for fire sale effects. First, we assume that managers stochastically lose their ability to convert capital goods into consumption goods. Specifically, with probability α_0 the capital goods produced in stage

2 can no longer be managed by the incumbent manager and must instead be used in the traditional technology. Second, we allow for an intensive margin in the corporate technology. Specifically, rather than restricting the scale k_c of the corporate investment to be 1 we allow it vary. In particular, the amount of capital goods produced in stage 2 is

$$A_i(1+\varepsilon)g(k_c) \text{ for } i \in \{H, L\}$$

where g is an increasing concave function with $g'(0)$ finite. Third, we replace the traditional technology which previously was simply described by the constant $R < 1$ with a constant returns to scale production technology $F(k_1, k_2)$ where k_1 denotes that capital invested in this technology in stage 1 by the lenders and k_2 denotes the capital invested in this technology in stage 2. We assume that F is concave and has diminishing marginal products. We also assume that the incumbent managers are more productive in converting capital goods to consumption goods than is the traditional technology. That is, we assume that marginal product of k_2 in the traditional technology is always less than the marginal product of capital in the corporate sector. Formally, $F_2(k_1, 0) = 1$ so that $F_2(k_1, k_2) \leq 1$ for all k_1, k_2.

The capital k_2 invested in the traditional technology comes from two sources: the exogenously liquidated capital and the capital from bankrupt financial intermediaries and is given by

$$(20) \quad k_2 = \alpha_0 \left[\sum p_i \int A_i(1+\varepsilon)dH(\varepsilon)\right] g(k_c) + \alpha_1 \left[\sum p_i \int_{B_i} A_i(1+\varepsilon)dH(\varepsilon)\right] g(k_c)$$

Here competitive firms operate the traditional technology and choose k_1 and k_2 to maximize

$$F(k_1, k_2) - R_1 k_1 - R_2 k_2$$

The first order conditions are

$$(21) \quad F_1(k_1, k_2) = R_1$$

$$(22) \quad F_2(k_1, k_2) = R_2$$

The lenders in this economy choose how much of their endowment e to invest in the corporate technology, k_c at rate R_c, how much to invest in the traditional technology, k_1 at rate R_1, and how much to store, k_s at rate 1. That is, lenders solve

$$(23) \quad c_I = \max R_c k_c + R_1 k_1 + k_s$$

subject to

$$(24) \quad k_c + k_1 + k_s \leq e.$$

We will assume that all three technologies are used in equilibrium. A set of sufficient conditions is the following. First, e is sufficiently large, so that the storage technology is always used. Second, that the corporate technology is sufficiently productive in that A_H is large enough and that $p'_H(0)$ is sufficiently large, so that it is always used. Finally, that

$F_1(0, k_2) > 1$ for all $k_2 > 0$, so that the traditional technology is always used. Under these assumptions we have that

$$(25) \quad R_c = R_1 = 1$$

and we will impose this condition from now on.

The resource constraint for this economy is

$$(26) \quad \alpha_1 p_H(a) c_H + c^I \leq \alpha_1 \left[p_H(a) A_H + p_L(a) A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon) dH(\varepsilon) \right] g(k_c) + F(k_1, k_2)$$

With Commitment

To set the stage for our environment without commitment by private agents, we briefly describe the dynamic model with commitment by private agents. In our model, financial intermediaries live for only one period and financial intermediaries in any period t cannot observe the output of financial intermediaries in earlier periods. Hence, managers cannot enter into contracts that condition on their past output levels. This assumption ensures that the manager's incentive problem is static and that equilibrium in the dynamic model reduces to an infinitely-repetition of that in the static model.

Recall that in the simple economy, the incentive constraint for the manager is binding if $A_L < 1$. It is straightforward to check that the incentive constraint in the benchmark economy is binding if A_L, α_0 and $g(e)$ are sufficiently small. We will assume that the incentive constraint is binding in the benchmark economy from now on.

We now set up the contracting problem for this economy. Following the logic of Proposition 4, the contracting problem solves

$$\text{(27)} \quad \max \alpha_1 \left[p_H(a) c_H - p_L(a) B H(\varepsilon^*) \right] - a$$

subject to

$$\text{(28)} \quad a \in \arg\max_a \alpha_1 \left[p_H(a) c_H - p_L(a) B H(\varepsilon^*) \right] - a.$$

$$\text{(29)} \quad \alpha_1 p_H(a) c_H + k_c \leq \alpha_1 \left[p_H(a) A_H + p_L(a) A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon) dH(\varepsilon) \right] g(k_c) + R_2 k_2$$

where

$$\text{(30)} \quad k_2 = \alpha_0 \left[\sum p_i(a) A_i \right] g(k_c) + \alpha_1 p_L(a) A_L g(k_c) \int_{\underline{\varepsilon}}^{\varepsilon^*} (1+\varepsilon) dH(\varepsilon).$$

Recalling that in any equilibrium (25) holds, we have the following definition.

A *competitive equilibrium with commitment* is an allocation $c_H, a, \varepsilon^*, k_1, k_2, R_2$, such that

i) given R_2, the allocations solve the contracting problem (27).

ii) given R_2, k_1 and k_2 satisfy (21) and (22).

iii) the consumption of lenders satisfies (23).

iv) the resource constraints (26) and (24) hold.

Without Commitment by Private Agents

Without commitment by private agents, we require that the contracts managers and financial intermediaries enter into must be self enforcing. We say that a contract is *self-enforcing* if, after the manager has chosen the effort level, the payoff from continuing with the contract is at least as large as the payoff from deviating. In order to construct the payoff associated with a deviation, we assume that if a deviation has occurred in any period, the payoffs to the manager in all subsequent periods is given by the solution to the optimal contracting problem (27) with $\varepsilon^* = 0$. Let U^N denote the value of the contracting problem with this restriction.

Under this assumption, it should be clear that if a manager and the firm choose to deviate in some period t, they should choose a deviation that maximizes current payoffs. As in the simple economy without commitment, the best one-shot deviation is clearly to set ε^* to zero to avoid the output and nonpecuniary costs of bankruptcy.

Under the best one shot deviation the current period expected payoffs to the manager are

$$\text{(31)} \quad \hat{U}(a, k_c) = \alpha_1 p_H(a)\hat{c}_H - a = \alpha_1 \left[p_H(a)A_H + p_L(a)A_L\right] g(k_c) + R_2\hat{k}_2 - k_c - a$$

where $\hat{c}_H$ is the consumption associated with the renegotiated contract and

$$\hat{k}_2 = \alpha_0 \sum p_i(a)A_i g(k_c).$$

For some given contract a, k_c, ε^* if there is not deviation, then the manager's expected consumption is determined from (29) and the manager's payoffs are given by

$$(32)\quad U(a,\varepsilon^*,k_c) = \alpha_1 p_H(a)c_H - \alpha_1 BH(\varepsilon^*) - a$$

$$= \alpha_1 \left[p_H(a)A_H + p_L(a)A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon)dH(\varepsilon) \right] g(k_c) + R_2 k_2 - \alpha_1 BH(\varepsilon^*) - k_c - a$$

where

$$k_2 = \alpha_0 \left[\sum p_i(a)A_i \right] g(k_c) + \alpha_1 p_L(a) g(k_c) \int_{\underline{\varepsilon}}^{\varepsilon^*} (1+\varepsilon)dH(\varepsilon).$$

Given a continuation value U, we say that a contract is *privately sustainable* if

$$(33)\quad U(a,\varepsilon^*,k_c) + \frac{\beta}{1-\beta}U \geq \hat{U}(a,k_c) + \frac{\beta}{1-\beta}U^N.$$

The *optimal contracting problem without commitment* is now to maximize the manager's utility (27) subject to (28), (29), (30) and (33).

A *privately sustainable equilibrium* is an allocation $c_H, a,\ \varepsilon^*, k_1, k_2$, a price R_2, and a continuation utility U such that

i) given R_2, the allocations solve the optimal contracting problem without commitment.

ii) given R_1 and R_2, k_1 and k_2 satisfy (21) and (22).

iii) given R_c, R_1 and $k_c = 1$, the consumption of lenders satisfies (23).

iv) the continuation utility U equals $U(a,\varepsilon^*,k_c)$.

v) the resource constraints (24) and (26) hold.

One rationalization for our formalization of the optimal contracting problem without commitment is that manager and firm behavior is disciplined by trigger strategies. Under this rationalization, the optimal contracting problem finds the best trigger strategy equilibrium in a game between the manager and financial intermediaries, holding fixed the prices in a competitive equilibrium. A standard result in the game theory literature is that the best equilibrium can be supported by a trigger strategy which prescribes the worst equilibrium continuation payoff following any deviation. In our economy, the worst equilibrium is the infinite repetition of the static equilibrium without commitment. This infinite repetition has per period value U^N.

Under this rationalization, we assume that managers are infinitely-lived but all agents in future periods only observe whether or not the manager has renegotiated in the past. This assumption keeps the manager's incentive constraint static and allows us to focus on the incentives to renegotiate. Consider the following trigger strategies: if a manager ever renegotiates, then all financial intermediaries believe that the manager will always renegotiate so that bankruptcy will never be declared in the future. Since this continuation yields the worst payoffs, it follows that the best equilibrium for the game between managers and financial intermediaries, holding fixed market prices, solves the optimal contracting problem.

We emphasize that our notion of equilibrium does not depend on this rationalization. Formally, our optimal contracting problem is analogous to that in the literature on models with enforcement constraints, in that we replace the enforcement constraints by sustainability constraints.

We now turn to welfare with and without commitment. We begin by showing that the equilibrium value of R_2 is the same in the economies with and without commitment. To show this result note that in both economies $F_1(k_1, k_2) = 1$ and hence since F has constant returns to scale, this implies that $F_1(k_1/k_2, 1) = 1$ so that k_1/k_2 is the same value, say $\tilde{k}$ in both economies. Since $R_2 = F_2(k_1, k_2) = F_2(\tilde{k}, 1)$ we know R_2 is also the same in both economies. We record this result in the following lemma.

Lemma 1. The equilibrium values of R_1 and R_2 are the same in the economies with and without commitment. Furthermore, the value of $R_1 = 1$.

Since market prices are the same in the economies with and without commitment, the only difference between the associated contracting problems is the private sustainability constraint. If this constraint is binding in the contracting problem, the privately sustainable equilibrium yields lower welfare than the competitive equilibrium under commitment. The private sustainability constraint is binding if the discount factor β is not too large. We denote by $\bar{\beta}$ the critical value of the discount factor such that the the private sustainability constraint just binds at the commitment allocations. That is $\bar{\beta}$ satisfies

$$\text{(34a)} \quad U(a^c, \varepsilon^{*c}, k_c^c) + \frac{\bar{\beta}}{1-\bar{\beta}} U(a^c, \varepsilon^{*c}, k_c^c) = \hat{U}(a^c, k_c^c) + \frac{\bar{\beta}}{1-\bar{\beta}} U^N$$

where a^c, ε^{*c} denote the contract in a competitive equilibrium with commitment. Clearly, if $\beta \geq \bar{\beta}$, the commitment outcomes are privately sustainable, and if $\beta < \bar{\beta}$, the commitment outcomes are not sustainable.

3. Adding Government Policies

We now allow for the possibility of intervention by benevolent government authorities without commitment.

We begin with a bailout authority which uses lump sum taxes and transfers to alter bankruptcy decisions. After managers have chosen their actions, the bailout authority has an incentive to use taxes and transfers to reduce ex post inefficiency. In using these instruments, we assume that the bailout authority faces a trade off parallel to that faced by private agents: if the authority deviates from some given equilibrium policy, private agents believe that the bailout authority will choose future policies so as to eliminate ex post inefficiency.

These beliefs induce a government sustainability constraint which is similar to the private sustainability constraint with one important difference. This difference is that the government sustainability constraint is tighter because it takes into account fire sales effects. That is, when a bailout authority intervenes to prevent bankruptcies ex post it recognizes it recognizes that its action raise the price of liquidated assets. In contrast, the actions of individual private agents do not affect prices. In our model a rise in the price of liquidated assets raises welfare and therefore makes the government sustainability constraint tighter and hence makes the equilibrium outcomes with a bailout authority worse than without such an authority.

We then ask, Can a regulator armed with the ability to limit the terms of private contracts improve on these outcomes? We find that it can. We show that the optimal regulation imposes a cap on the size of the corporate technology, a *too-big-to-fail-cap*, and a cap on the liquidation level, a *bankruptcy cap*. Such a regulator takes into account the incentives of the bailout authority to intervene and structures the terms of private contracts so as to reduce the incentives of the bailout authority to intervene. We show that the regulator can improve upon the equilibrium outcomes with a bailout authority.

A. A Bailout Authority

Consider a bailout authority that can make transfers or levy taxes on financial intermediaries contingent on the state and the realization of the idiosyncratic shock ε. Suppose now that the bailout authority, as well as private agents, cannot commit to their future actions. The bailout authority's per period payoff is given by the sum of the consumption of all agents in the economy. The bailout authority makes its policy decision after the managers have chosen their actions but before the realization of either the state, H or L or the shocks ε. The instruments available to the bailout authority are a tax rate τ in the high state and the lump sum transfers $T_L(\varepsilon)$ in the low state. The bailout authority's budget constraint is

$$(35) \quad p_H \tau = p_L \int_{\varepsilon^*}^{\bar{\varepsilon}} T_L(\varepsilon) dH(\varepsilon).$$

We now develop the bailout authority's sustainability constraint. As in the environment without commitment by private agents, we begin by characterizing the equilibrium in which after any deviation, agents believe that all future contracts will be renegotiated and hence revert to an equilibrium with $\varepsilon^* = 0$. The reversion equilibrium has per period value U^N as before. The only subtlety to keep in mind is that, from Lemma 1, R_2 has the same value as in the static economy with commitment.

Consider the best one shot deviation for the bailout authority. It is clearly optimal for the authority to set policy so that the economy has no bankruptcy. In such a case, given some value of k_1, the sum of consumption of managers and lenders is given by

$$(36) \quad \hat{U}^G(a, k_c) = \alpha_1 \left[p_H(a) A_H + p_L(a) A_L \right] g(k_c) + F(k_1, \hat{k}_2) + e - k_c - k_1 - a$$

where

$$(37)\quad \hat{k}_2 = \alpha_0 \sum p_i(a) A_i g(k_c).$$

If the bailout authority chooses not to deviate from some given contract then the sum of consumption of managers and lenders is given by $U^G(a, \varepsilon^*, k_c)$ which equals

$$\alpha_1 \left[p_H(a) A_H + p_L(a) A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon) dH(\varepsilon) \right] g(k_c) - \alpha_1 p_L(a) B H(\varepsilon^*) + F(k_1, k_2) + e - k_c - k_1 - a$$

where

$$k_2 = \alpha_0 \left[\sum p_i(a) A_i \right] g(k_c) + \alpha_1 p_L(a) g(k_c) \int_{\underline{\varepsilon}}^{\varepsilon^*} (1+\varepsilon) dH(\varepsilon)$$

Note that the continuation payoff if the government chooses not to deviate is the same as that in (32).

Given a continuation utility U^G, we say that a contract is *sustainable to bailouts* if

$$(38)\quad U^G(a, \varepsilon^*, k_c) + \frac{\beta}{1-\beta} U^G \geq \hat{U}^G(a, k_c) + \frac{\beta}{1-\beta} U^N.$$

A policy induces a competitive equilibrium as follows. Given a policy, the budget constraint of the financial intermediary becomes

(39)

$$\alpha_1 p_H(a)c_H + k_c \leq \alpha_1 \left[p_H(a)(A_H - \tau) + p_L(a) \int_{\varepsilon^*}^{\bar{\varepsilon}} [A_L(1+\varepsilon) + T_L(\varepsilon)]\, dH(\varepsilon) \right] g(k_c) + R_2 k_2.$$

The *optimal contracting problem with a bailout policy* is to choose a contract c_H, a, k_c and ε^* to maximize the utility of the manager (27) subject to the incentive constraint for the manager (28), the private sustainability constraint (33), and the budget constraint of the financial intermediary (39) where k_2 is given by (30)

A *sustainable equilibrium with a bailout policy* consists of an allocation c_H, a, ε^*, k_1, k_2, k_c, R_2, U and a policy τ, $T_L(\varepsilon)$ such that

i) given R_2, the allocations solve the optimal contracting problem with policy

ii) given R_2, k_1 and k_2 satisfy (21) and (22)

iii) the consumption of lenders satisfies (23) with $R_c = R_1 = 1$.

iv) the resource constraints (26) and (24) hold.

v) the government's budget constraint (35) holds.

vi) the government's sustainability constraint (38).

vii) the continuation utility U equals $U(p_H, \varepsilon^*, k_c)$.

We then have the following proposition.

Proposition 5. Consider any contract $(p_H, \varepsilon^*, k_c)$ with $\varepsilon^* > \underline{\varepsilon}$ and suppose that $F(k_1, k_2)$ is strictly concave in k_2. The government sustainability constraint (38) is tighter than the private sustainability constraint (33), in the sense that if a contract satisfies (38) it also satisfies (33). Furthermore, if any contract satisfies (33) with equality, it violates (38).

Proof. From Lemma 1 it follows that the continuation utility following a deviation U^N in the private sustainability constraint is the same as it is in the government sustainability constraint. Thus, we need only show that

$$\hat{U}^G(a, k_c) - U^G(a, \varepsilon^*, k_c) > \hat{U}(a, k_c) - U(a, \varepsilon^*, k_c) \tag{40}$$

From Euler's theorem $F(k_1, k_2) = F_1 k_1 + F_2 k_2$. Since $F_1 = 1$ in any equilibrium and $F_2 = R_2$ it follows that

$$F(k_1, k_2) - k_1 = R_2 k_2 \tag{41}$$

Using (41) it follows that $U^G(a, \varepsilon^*, k_c) = U(a, \varepsilon^*, k_c) + e$. Using this result and canceling terms in (40) gives that (40) holds if and only if

$$F(k_1, \hat{k}_2) - k_1 > R_2 \hat{k}_2 \tag{42}$$

Adding $R_2 k_2$ to both sides, using Euler's theorem and rearranging terms, (42) can be written as

$$R_2(k_2 - \hat{k}_2) > F(k_1, k_2) - F(k_1, \hat{k}_2) \tag{43}$$

Since $k_2 > \hat{k}_2$ and since F is a strictly concave function of k_2, (43) must hold. This result proves that (38) is tighter than (33). *Q.E.D.*

If the production function satisfies (43) we say that the economy has fire sale effects. The key idea in the proof of Proposition 8 is that when the bailout authority contemplates a deviation it realizes that by lowering the measure of bankruptcies, it recognizes the effects of fire sales. That is, it recognizes that lowering the measure of bankruptcies raises the value R_2 of the capital that is transferred from the corporate sector to the traditional sector. In contrast, when a private firm contemplates a deviation it takes the value R_2 as given. Thus, the right side of the private sustainability constraint is lower than the right side of the sustainability to bailout constraint.

Note that if there are no fire sale effects the private sustainability constraint and the government sustainability constraint coincide. To see this suppose that F is linear in k_1 and k_2 so that it can be written as $F(k_1, k_2) = \rho_1 k_1 + \rho_2 k_2$ where ρ_1 and ρ_2 are constants. Then it is easy to show that

$$\hat{U}^G(a, k_c) - U^G(a, \varepsilon^*, k_c) = \hat{U}(a, k_c) - U(a, \varepsilon^*, k_c)$$

so that the two constraints coincide.

We use Proposition 5 to show that the sustainable equilibrium with bailouts yields lower welfare than the privately sustainable equilibrium.

Proposition 6. Suppose the discount factor β is strictly less than the threshold $\bar{\beta}$ given by (34a) at which the private sustainability constraint is binding. Any sustainable equilibrium with bailouts yields strictly lower welfare than the privately sustainable equilibrium. Furthermore, any sustainable equilibrium with bailout policy has bailouts in equilibrium, in the sense that $\tau > 0$.

Proof. Since $\beta < \bar{\beta}$, the private sustainability constraint is binding in a privately sustainable equilibrium. From Proposition 5 it follows that the privately sustainable equilibrium allocations violate the government sustainability constraint. Clearly, any sustainable equilibrium with bailout policy is a feasible allocation for the dynamic contracting problem since it satisfies the budget constraint of the financial intermediary, the incentive constraint of the manager, and the private sustainability constraint. Thus, it must yield lower welfare than the optimal allocation from the dynamic contracting problem. It follows that welfare is strictly lower in the bailout equilibrium.

We prove that any sustainable equilibrium with bailout policy has $\tau > 0$ by way of contradiction. Suppose that $\tau = 0$. Then, using Lemma 1 it follows that the solution to the dynamic contracting problem coincides with that of the privately sustainable equilibrium. This allocation violates the government sustainability constraint. Thus, any sustainable equilibrium with bailout policy must have $\tau > 0$. *Q.E.D.*

B. Can an ex ante regulator improve welfare?

Consider the situation described in the previous section in which neither the bailout authority nor the private agents can commit to their actions. We show that a regulatory authority armed with the ability the dictate the terms of the private contract, namely the compensation contract c_H^R , the scale of the corporate technology k_c^R, and the liquidation level ε^R, can improve ex ante welfare. Such a regulator must take into account the incentives of the bailout authority to intervene.

To see how a regulator can improve upon equilibrium allocations, we need to define a competitive equilibrium with regulation. We begin with an extreme form of regulation in which the regulator specifies the exact size of the firm and the exact set of states in which the firm can declare bankruptcy, and then show that less extreme regulations can achieve desired outcomes. Under the extreme form of regulation, the regulator chooses taxes, transfers and specifies the following constraints on contracts.

(44) $k_c = k^r$ and $\varepsilon^* = \varepsilon^r$.

The *optimal contracting problem with regulation* is now to choose a contract c_H and ε^* to maximize the utility of the manager (27) subject to the incentive constraint for the manager (28), the private sustainability constraint (33), the budget constraint of the financial intermediary (39) where k_2 is given by (30) and subject to the policy constraints (44).

A *sustainable equilibrium with regulation* consists of an allocation $c_H, a,\ \varepsilon^*, k_1, k_2, k_c$ R_2,U and a regulatory policy $k^r, c^r_H,\ \varepsilon^r,\ \tau,\ T_L(\varepsilon)$ is defined is defined in the same way as a sustainable equilibrium with bailout policy with one important difference. That difference, of course, is that the contracting problem now has additional constraints.

The regulator's problem is to structure policies so as to maximize the manager's welfare given that the allocations associated with a given policy must be part of a sustainable equilibrium.

Consider the *regulator's problem* given utility level e for lenders and given some continuation contract $(a^R, \varepsilon^R, k^R_c)$ is to choose $c_H, a, \varepsilon^*, k_c, k_1, k_2, k_s$ to solve

(45) $\max \alpha_1 \left[p_H(a)c_H - p_L(a)BH(\varepsilon^*)\right] - a$

subject to the manager's incentive constraint

$$(46) \quad a \in \arg\max_{a} \alpha_1 \left[p_H(a)c_H - p_L(a)BH(\varepsilon^*)\right] - a$$

the resource constraint

$$(47) \quad \alpha_1 p_H(a)c_H + c^I \leq \alpha_1 \left[p_H(a)A_H + p_L(a)A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon)dH(\varepsilon)\right] g(k_c) + F(k_1, k_2) + k_s$$

where k_2 is given by

$$(48) \quad k_2 = \alpha_0 \left[\sum p_i(a)A_i\right] g(k_c) + \alpha_1 p_L(a) g(k_c) \int_{\underline{\varepsilon}}^{\varepsilon^*} (1+\varepsilon)dH(\varepsilon)$$

voluntary savings by lenders

$$(49) \quad F_1(k_1, k_2) = 1$$

and the bailout authority's sustainability constraint

$$(50) \quad U(a, \varepsilon^*, k_c) + \frac{\beta}{1-\beta} U(a^R, \varepsilon^R, k_c^R) \geq \hat{U}^G(a, k_c) + \frac{\beta}{1-\beta} U^N$$

minimum utility level for managers

$$(51) \quad c^I \geq e$$

the stage 1 investment constraint

(52) $k_c + k_s + k_1 \leq e.$

We say that an allocation is a *regulatory equilibrium* if the action a, the cutoff level ε^*, and the scale of the corporate technology k_c that solve the regulator's problem coincide with the given continuation allocations $a^R, \varepsilon^R, k_c^R$.

Note that the voluntary savings by lenders constraint (49) arises because the regulator has no instruments that can affect the return to investment k_1 in the traditional technology.

Proposition 7. The allocations associated with a regulatory equilibrium are the same as the allocations in the best sustainable equilibrium with regulation.

Proof. Note that any sustainable equilibrium must satisfy the (46)-(49) and must satisfy (50) if it is sustainable to bailouts. Clearly, the regulatory equilibrium must maximize manager's welfare subject to these constraints. Any solution to the regulator's problem can clearly be implemented by imposing constraints of the form (44) on the contracting problem. *Q.E.D.*

Next, we have

Proposition 8. Suppose $\beta < \bar{\beta}$. The regulatory equilibrium yields higher welfare than any sustainable equilibrium with bailout policy.

Proof. The proof is by contradiction. Suppose that the bailout authority could achieve the same allocations as the regulator. Since the sustainability constraint for the government is tighter than it is for private agents, then at the regulator's allocations the private sustainability constraint in the contracting problem must be slack. Now consider the first order conditions with respect to k_c in the regulator's problem and in the contracting problem.

To derive these first order conditions for the regulator's problem we first rewrite the resource constraint (47). To do so we note from Euler's theorem that $F = F_1 k_1 + F_2 k_2$, so that using (49) we have that $F = k_1 + R_2 k_2$, where, as before, R_2 is uniquely pinned down by the condition that $F_1 = 1$. Substituting $F = k_1 + R_2 k_2$ and using that (51) and (52) hold with equality we can rewrite this constraint as

$$\alpha_1 p_H(a) c_H \leq \alpha_1 \left[p_H(a) A_H + p_L(a) A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon) dH(\varepsilon) \right] g(k_c) + R_2 k_2 - k_c$$

In the rewritten regulator's problem the first order condition for k_c is given by

$$\mu \left\{ g'(k_c) \left[Y_c + R_2 Y_l \right] - 1 \right\} = \theta \left[\hat{U}_k^G(a, k_c) - U_k(a, \varepsilon^*, k_c) \right] \tag{53}$$

where μ and θ are the multipliers on (47) and (50),

$$Y_c = \alpha_1 \left[p_H(a) A_H + p_L(a) A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon) dH(\varepsilon) \right]$$

$$Y_l = \alpha_0 \left[p_H(a) A_H + p_L(a) A_L \right] + \alpha_1 p_L(a) A_L \int_{\underline{\varepsilon}}^{\varepsilon^*} (1+\varepsilon) dH(\varepsilon)$$

Consider next the first order conditions for the dynamic contracting problem in dual form with a slack private sustainability constraint. The budget constraint for this problem is given by

$$\alpha_1 p_H(a) c_H \leq$$

$$\alpha_1 \left[p_H(a)(A_H - \tau) + p_L(a) \int_{\varepsilon^*}^{\bar{\varepsilon}} \left[A_L(1+\varepsilon) + T_L(\varepsilon) \right] dH(\varepsilon) \right] g(k_c) + R_2 k_2 - k_c$$

The first order condition for k_c for this problem is given by

$$(54) \quad \tilde{\mu} \left\{ g'(k_c) \left[Y_c(\tau, T_L) + R_2 Y_l \right] - 1 \right\} = 0$$

where

$$Y_c(\tau, T_L) = \alpha_1 \left[p_H(A_H - \tau) + p_L \int_{\varepsilon^*}^{\bar{\varepsilon}} \left[A_L(1+\varepsilon) + T_L(\varepsilon) \right] dH(\varepsilon) \right].$$

In equilibrium, the government's budget constraint implies the value of taxes equal the value of transfers so that

$$(55) \quad Y_c = Y_c(\tau, T_L).$$

Combining (54) and (55) and using that the multiplier on the budget constraint is nonzero gives that in bailout equilibrium the allocations satisfy

$$(56) \quad \left\{ g'(k_c) \left[Y_c + R_2 Y_l \right] - 1 \right\} = 0$$

The bailout allocations do not satisfy the first order condition for the regulatory equilibrium

(53) because the right side of (53) is nonzero. Hence, the allocations in the regulator's problem and the contracting problem must differ. Since the bailout allocations are feasible for the regulator's problem, the bailout equilibrium must yield lower welfare than the regulatory equilibrium. *Q.E.D.*

The idea behind this proposition is that since the bailout authority has a balanced budget, in equilibrium, the tax-transfer scheme can only indirectly influence the choice of k_c. The key idea is that the regulator has a richer set of policy instruments than does the bailout authority.

Next we show that the solution to the regulator's problem can be implemented with caps rather than exact constraints. It is straightforward to show that a too-big-to-fail cap of the form $k_c \leq k^r$ and a liquidation constraint of the form $\varepsilon^* = \varepsilon^r$, with no taxes or transfers implements the regulatory equilibrium where k^r and ε^r are the solutions to the regulator's problem. With stronger assumptions we can show that the exact liquidation constraint can be replaced by a cap on ε^*.

Theses stronger assumptions essentially bound the size of price effects from the curvature in F. We say that *price effects are bounded by* δ if for all k_1 and all $k_2, \tilde{k}_2$ with $k_2 < \tilde{k}_2$

$$F_2(k_1, k_2) - F_2(k_1, \tilde{k}_2) < \delta.$$

Proposition 9. Suppose that β and the price effects bound δ are sufficiently small. Then, the solution to the regulator's problem can be implemented with a too-big-to-fail cap of the form

$$(57) \quad k_c \leq k^r$$

and a liquidation cap of the form

$$(58) \quad \varepsilon^* \leq \varepsilon^r$$

where ε^r is the optimal level of ε^* in the solution to regulator's problem.

The proof is not contained in this draft. The idea is as follows. Consider the effect on the manager's welfare of a small increase in ε^* from ε^r, while adjusting c_H and a to satisfy the incentive constraint and the budget constraint. We claim that welfare of the manager must increase. To see this result, suppose that this change reduced the welfare of the manager. Then a small reduction in ε^* from ε^r adjusting c_H and a appropriately raises welfare and, given our assumption on H, introduces slack in the sustainability constraint. It follows that if we replace the sustainability constraint by a cap on ε^* of the form of (58), the solution to this rewritten regulator's problem attains ε^r.

4. Implementation with a cap on debt

Here we argue that the equilibrium allocations can often be implemented with financial contracts that resemble debt and equity. We use this implementation to argue that the ex-post inefficiency we identify in this paper can be interpreted as arising from bankruptcy. We then argue that the regulatory caps identified as in Proposition 9 can be realistically interpreted as caps on both the size of the firm and on its debt to value ratio.

Consider a firm operated by a manager that issues the following financial claims. The firm issues (risky) debt and equity and enters into a compensation contract with the manager. The debt promises a face value of $A_L(1+\varepsilon^*)g(k_c)$. The nature of the debt contract is that if the firm is unable to meet the face value of its debt payments, the firm is forced into bankruptcy, equity holders lose their claims and debt holders receive the liquidation value of the firm. The manager's compensation contract specifies a payment of c_H if the manager retains his managerial capability and if the firm is successful and zero otherwise. Outside equity is the residual claimant.

Suppose now that the solution to the regulator's problem satisfies

$$(59) \quad A_H(1+\underline{\varepsilon})g(k_c) - c_H \geq A_L(1+\varepsilon^*)g(k_c)$$

and

$$(60) \quad R_2 \sum p_i(a) A_i g(k_c) \geq A_L(1+\varepsilon^*)g(k_c)$$

Note that (59) guarantees that in the high state when the manager keeps the ability to manage the project, the firm can pay the face value of the debt, while (60) guarantees under the event that the manager loses the ability to manage the project, the firm can pay the face value of the debt by selling its assets.

We will develop the argument that given a cap on the size of the firm k_c, a cap on the firm's debt to value ratio is equivalent to a cap on ε^*. We start by calculating the firm's debt to value ratio under this decentralization. To do so we calculate the expected present value of debt payments. With probability α_0, the manager loses the ability to manage the firm and the debt holders receive the face value of their debt. With probability $\alpha_1(p_H(a) + p_L(a)(1 - H(\varepsilon^*)) + \alpha_0$, the firm's cash flows exceed the required debt payment. In the event of bankruptcy, debt holders receive the liquidation value of the debt. The present value of debt payments is then given by

(61)

$$\{\alpha_1 \left[p_H(a) + p_L(a)(1 - H(\varepsilon^*))\right] + \alpha_0\} A_L(1+\varepsilon^*)g(k_c) + \alpha_1 p_L(a) R_2 A_L g(k_c) \int_{\underline{\varepsilon}}^{\varepsilon^*} (1+\varepsilon) dH(\varepsilon).$$

The value of the firm is simply k_c. We now argue that if R_2 is sufficiently close to 1 then debt to value ratio is increasing in ε^*. To see this result we note that the derivative of (61) with respect to ε^* is proportional to

$$\{\alpha_1 \left[p_H(a) + p_L(a)(1 - H(\varepsilon^*))\right] + \alpha_0\} - \alpha_1(1 - R_2)p_L(a)h(\varepsilon^*)(1 + \varepsilon^*)$$

Clearly, if R_2 is close enough to 1 then the debt is increasing in ε^*.

Letting D denote the value of debt under any contract and D^r denote the value of debt (61) in a regulatory equilibrium, we summarize this discussion in a proposition.

Proposition 10. If R_2 is sufficiently close to 1 then the solution to the regulator's problem can be implemented with a too-big-to-fail cap of the form $k_c \leq k^r$ and a cap on the debt to value ratio of the form

$$\frac{D}{k_c} \leq \frac{D^r}{k^r}.$$

5. Conclusion

We have made three points in this paper. First, ex ante efficient contracts often require ex post inefficiency. Second, the time inconsistency problem for the government is more severe than for private agents because fire sale effects give governments stronger incentives to renegotiate contracts than private agents. Third, given that the government cannot commit itself to not bailing out firms ex post, ex ante regulation of firms is desirable.

6. Appendix

Proposition 4. If R is sufficiently close to 1 and $a''(p_H^E)$ is sufficiently small then $\varepsilon^* > \underline{\varepsilon}$. That is, supporting ex ante efficient allocations requires ex post inefficiency.

Proof: The proof is by contradiction. Suppose that $\varepsilon^* = \underline{\varepsilon}$. We will show that a small increase in ε^* from $\underline{\varepsilon}$ raises welfare. To show this result, we totally differentiate the budget constraint (16) and the incentive constraint (17) and evaluate these derivatives at $\varepsilon^* = \underline{\varepsilon}$. We obtain the following relationships between dc_H, dp and $d\varepsilon^*$

$$(62)\quad [A_H - A_L - c_H]\, dp - p dc_H - (1-p)A_L(1-R_2)h(\underline{\varepsilon})d\varepsilon^* = 0$$

$$(63)\quad dc_H + Bh(\underline{\varepsilon})d\varepsilon^* = a''(p)dp$$

where $p = p_H$. Substituting for dp from (63) into (62) and rearranging terms we obtain

$$(64)\quad \left(p - \frac{[A_H - A_L - c_H]}{a''(p)}\right) dc_H = \left[[A_H - A_L - c_H]\,\frac{B}{a''(p)} - (1-p)A_L(1-R_2)\right] h(\underline{\varepsilon})d\varepsilon^*.$$

The budget constraint evaluated at $\varepsilon^* = \underline{\varepsilon}$ implies that $1 - A_L = p(A_H - A_L - c_H)$, so that (64) can be rewritten as

$$(65)\quad \left(p - \frac{1-A_L}{pa''(p)}\right) dc_H = \left[\frac{1-A_L}{pa''(p)}B - (1-p)A_L(1-R_2)\right] h(\underline{\varepsilon})d\varepsilon^*$$

Totally differentiating the objective function, we obtain that the change in the utility of the manager dU is given by

$$dU = pdc_H - (1-p)Bh(\underline{\varepsilon})d\varepsilon^*$$

and from (65) we have that $dU > 0$ if and only if

$$(66) \quad dU = p\frac{\left[\frac{1-A_L}{pa''(p)}B - (1-p)A_L(1-R_2)\right]}{\left(p - \frac{1-A_L}{pa''(p)}\right)} - (1-p)B > 0.$$

Next, we show that the denominator of the first term in (66) is positive. To do so, consider the solutions to c_H and p obtained from the incentive constraint (15) and the budget constraint (16). Typically, these conditions yield multiple solutions. The solution that maximizes the manager's welfare is the largest value of p that satisfies these conditions. Substituting for c_H from (15) into (16) we obtain

$$(67) \quad pa'(p) + 1 = pA_H + (1-p)A_L.$$

At the largest value of p that satisfies (67), we must have that the derivative of the left side of (67) must be greater than the derivative of the right side of (67) so that

$$(68) \quad pa'' + a'(p) > A_H - A_L$$

Since the incentive constraint requires that $a'(p_H) = c_H$ and the budget constraint implies that $1 - A_L = p\left[A_H - A_L - c_H\right]$, (68) can be written as

$$pa'' > \frac{1 - A_L}{p}.$$

Thus, the denominator of the first term in (66) is positive.

Next we rewrite (66) as

$$dU = \left[p\frac{\left[\frac{1-A_L}{pa''(p)}\right]B}{\left(p - \frac{1-A_L}{pa''(p)}\right)} - (1-p)\right]B - p(1-p)\frac{A_L(1-R_2)}{\left(p - \frac{1-A_L}{pa''(p)}\right)} > 0$$

which, in turn can be rewritten as

$$(69) \quad dU = \left[\frac{\left[\frac{1-A_L}{pa''(p)}\right] - p(1-p)}{\left(p - \frac{1-A_L}{pa''(p)}\right)}\right]B - p(1-p)\frac{A_L(1-R_2)}{\left(p - \frac{1-A_L}{pa''(p)}\right)} > 0$$

Since $p < 1$, $p(1-p) \leq 1/4$ so that $(1-A_L)/pa''(p) - p(1-p) > 0$ if $a''(p) < 4(1-A_L)$. Thus if $a''(p)$ is sufficiently small, the first term in (69) is positive and if R_2 is sufficiently close to 1 the second term is small, so that, under these conditions, the change in utility given in (69) is positive. *Q.E.D.*

Proposition 9. For β and α_0 sufficiently small, the solution to the regulator's problem can be implemented with a too-big-to-fail cap of the form $k_c \leq k^r$ and a liquidation cap of the form

$$\varepsilon^* \leq \varepsilon^r$$

where k^r and ε^r are part of the solution to the regulator's problem.

Proof. Consider a version of the commitment problem with an extra constraint, namely $k_c = k^r$ and define ε^c, and p^c as the outcomes to the commitment problem with this extra constraint. with commitment. We claim that $\varepsilon^r \leq \varepsilon^c$. Clearly, for small enough β the government sustainability constraint must be binding in the regulator's problem. That is, there is some positive δ sufficiently small and β sufficiently small so that

$$(70)\quad \hat{U}^G(a^r, k^c) - U^G(a^r, \varepsilon^r, k^c) + \delta < \hat{U}(a^c, k^c) - U(a^c, \varepsilon^c, k^c)$$

Substituting from the government sustainability constraint we have that the left side of (70) is given by

$$(71)\quad \alpha_1(1-p^r)BH(\varepsilon^r) + \alpha_1(1-p^r)A_L \int_{\underline{\varepsilon}}^{\varepsilon^r} (1+\varepsilon)dH(\varepsilon)g(k^c) - \{F(k_1^r, k_2^r) - F(k_1^r, \hat{k}_2^r)\}$$

and the right side of (70) is given by

$$(72)\quad \alpha_1(1-p^c)BH(\varepsilon^c) + \alpha_1(1-p^c)A_L \int_{\underline{\varepsilon}}^{\varepsilon^c} (1+\varepsilon)dH(\varepsilon)g(k^c) - \{F(k_1^c, k_2^c) - F(k_1^c, \hat{k}_2^c)\}.$$

and where $\hat{k}_2 = \alpha_0 \left[p(a)(A_H - A_L) + A_L\right] g(k^r)$. Now, suppose that price effects are small enough so that

$$F(k_1^r, \hat{k}_2^r) - F(k_1^r, k_2^r) < R_2(k_2^r - \hat{k}_2^r) + \delta$$

Using the concavity of F , and substituting from (71) and (72) into (70) we obtain

$$(73) \quad \alpha_1(1-p^r)\left[BH(\varepsilon^r)+A_L(1-R_2)g(k^r)\int_{\underline{\varepsilon}}^{\varepsilon^r}(1+\varepsilon)dH(\varepsilon)\right]$$

$$<\alpha_1(1-p^c)\left[BH(\varepsilon^c)+A_L(1-R_2)g(k^r)\int_{\underline{\varepsilon}}^{\varepsilon^c}(1+\varepsilon)dH(\varepsilon)\right].$$

Rewrite (73) as

$$(74) \quad \alpha_1(1-p^r)H(\varepsilon^r)\left[B+A_L(1-R_2)g(k^r)f(\varepsilon^r)\right]$$

$$<\alpha_1(1-p^c)H(\varepsilon^c)\left[B+A_L(1-R_2)g(k^r)f(\varepsilon^c)\right].$$

where

$$f(\varepsilon^*)=\frac{\int_{\underline{\varepsilon}}^{\varepsilon^*}(1+\varepsilon)dH(\varepsilon)}{H(\varepsilon^*)}$$

Now, we claim that $f(\varepsilon^*)$ is increasing in ε^*. To see this, note that

$$f'(\varepsilon^*)=\frac{(1+\varepsilon^*)h(\varepsilon^*)H(\varepsilon^*)-\int_{\underline{\varepsilon}}^{\varepsilon^*}(1+\varepsilon)dH(\varepsilon)h(\varepsilon^*)}{[H(\varepsilon^*)]^2}$$

so that using $H(\varepsilon^*)=\int_{\underline{\varepsilon}}^{\varepsilon^*}h(\varepsilon)d\varepsilon$, we have

$$f'(\varepsilon^*)=h(\varepsilon^*)\frac{(1+\varepsilon^*)\int_{\underline{\varepsilon}}^{\varepsilon^*}h(\varepsilon)d\varepsilon-\int_{\underline{\varepsilon}}^{\varepsilon^*}(1+\varepsilon)h(\varepsilon)}{[H(\varepsilon^*)]^2}=h(\varepsilon^*)\frac{\int_{\underline{\varepsilon}}^{\varepsilon^*}(\varepsilon^*-\varepsilon)h(\varepsilon)}{[H(\varepsilon^*)]^2}>0.$$

The rest of the argument is by contradiction. Suppose $\varepsilon^r>\varepsilon^{c.}$ Then, since $f(\varepsilon)$ is increasing,

$f(\varepsilon^r) > f(\varepsilon^c)$. From (74), it follows that

$$(75) \quad (1 - p^r)H(\varepsilon^r) < (1 - p^c)H(\varepsilon^c)$$

Furthermore from (73), it follows that $(1 - p^r) < (1 - p^c)$ so that $p^r > p^c$.

Since the solution to the regulator's problem has lower welfare than the commitment allocations it follows that

$$(76) \quad \alpha_1 \left[p^r c_H^r - (1 - p^r)BH(\varepsilon^r)\right] - a(p^r)$$

$$< \alpha_1 \left[p^c c_H^c - (1 - p^c)BH(\varepsilon^c)\right] - a(p^c)$$

Multiplying (76) by $\alpha_1 B$ and adding it to this inequality gives

$$(77) \quad \alpha_1 p^r c_H^r - a^r < \alpha_1 p^c c_H^c - a^c$$

Substituting for $p^r c_H^r$ and $p^c c_H^c$ from the respective resource constraints into (77) gives

$$(78) \quad \alpha_1 \left[p_H^r A_H + p_L^r A_L \left(1 - (1 - R_2) \int_{\underline{\varepsilon}}^{\varepsilon^r} (1 + \varepsilon) dH(\varepsilon)\right)\right] g(k^c) - a^r$$

$$< \alpha_1 \left[p_H^c A_H + p_L^c A_L \left(1 - (1 - R_2) \int_{\underline{\varepsilon}}^{\varepsilon^c} (1 + \varepsilon) dH(\varepsilon)\right)\right] g(k^c) - a^c$$

Since $p^r > p^c$ and both are below first best, from the observation that $\alpha_1 \left[p_H A_H + p_L A_L\right] g(k^c) -$

$a(p)$ is increasing in p it follows that

$$(79)\quad \alpha_1\left[p_H^r A_H + p_L^r A_L\right] g(k^r) - a^r > \alpha_1\left[p_H^c A_H + p_L^c A_L\right] g(k^c) - a^c$$

Subtracting (79) from (78) and dividing by $\alpha_1\ A_L(1-R_2)g(k^c)$ we get

$$(80)\quad -p_L^r\left(\int_{\varepsilon^r}^{\bar{\varepsilon}}(1+\varepsilon)dH(\varepsilon)\right)$$

$$< -p_L^c\left(\int_{\varepsilon^c}^{\bar{\varepsilon}}(1+\varepsilon)dH(\varepsilon)\right)$$

which can be written as

$$(1-p^c)\int_{\varepsilon^c}^{\bar{\varepsilon}}(1+\varepsilon)dH(\varepsilon) < (1-p^r)\int_{\varepsilon^r}^{\bar{\varepsilon}}(1+\varepsilon)dH(\varepsilon).$$

Since $p^r > p^c$, $1-p^r < 1-p^c$ so that (**??**) can be written as

$$(1-p^r)\int_{\varepsilon^c}^{\bar{\varepsilon}}(1+\varepsilon)dH(\varepsilon) < (1-p^c)\int_{\varepsilon^c}^{\bar{\varepsilon}}(1+\varepsilon)dH(\varepsilon) < (1-p^r)\int_{\varepsilon^r}^{\bar{\varepsilon}}(1+\varepsilon)dH(\varepsilon).$$

which implies that $\varepsilon^c > \varepsilon^r$ which is a contradiction.

Since $\varepsilon^r \leq \varepsilon^c$,it follows that the regulator's problem can be implemented by a cap on ε. Q.E.D.