Inflation and Welfare with Search and Price Dispersion*

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Abstract

This paper studies the effect of inflation on welfare in an economy with consumer search and price dispersion. Consumers search harder for lower prices when facing greater price dispersion caused by higher inflation. This increased search intensifies market competition and raises welfare. The search behavior of consumers also creates welfare loss by inducing producers to post inefficiently high prices. Both effects are impacted by the consumer's real balance. I develop a general equilibrium monetary model with search frictions and incorporate the interrelationship of real balance, search, and endogenous price dispersion to study the aggregate effect of inflation on welfare. Inflation affects welfare through three channels: the real balance channel, the search channel, and the price posting channel. I calibrate the model to U.S. data and find that the welfare cost of 10% annual inflation is worth 3.23% of consumption; however, if either the real balance or the price posting channel is closed, the welfare cost significantly decreases to less than 0.2% of consumption. The price posting channel amplifies the welfare-diminishing effect of the real balance channel, and the aggregated negative effect exceeds the positive effect due to the search channel. The search cost only generates a negligible welfare loss.

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1 Introduction

This paper studies the relationship between anticipated inflation and welfare in an economy with consumer search and price dispersion. There is a long tradition of thinking that the welfare cost of inflation is intimately related to the dispersion of prices. A price system transmits necessary information for the allocation of economic resources to be efficient, but with nondegenerate price distribution this system is jammed by noise due to the indeterminacy of prices. Inefficiently high price levels exist in the economy, and they create welfare cost due to resource misallocation (as described by Friedman, 1977). As discovered by a fair number of empirical studies, price dispersion increases with inflation,[1] and so does the associated welfare loss. This is one channel through which inflation affects welfare, and the second channel is consumer search. When consumers face noisier information about prices, they invest more resources in their search for lower prices. While these resources constitute a welfare loss since they are not used in producing "real goods and services",[2] an increased search can intensify market competition, lower real prices, and increase welfare. The third channel is real balance. In an economy with inflation, the possession of real balance constitutes a welfare loss, called the cost of "economizing on currency" by Fischer and Modigliani (1978), and real balance also affects welfare by influencing other channels indirectly. The aggregate effect of inflation on welfare depends on the size of the positive effect through consumer search relative to the negative effect through price dispersion, search cost, and real balance. In order to fully understand the welfare implication of inflation and to examine the

[1] Many empirical studies document a positive relationship between inflation and relative price dispersion. For example, Parsley (1996) studies quarterly price data from 1975 to 1992 published by the American Chamber of Commerce Researchers Association and finds that higher inflation is associated with greater dispersion of relative prices; Debelle and Lamont (1997) also document a robust positive relationship using annual CPI data for U.S. cities from 1954 to 1986. Similar results are found in studies on other countries, such as Van Hoomissen (1988), Lach and Tsiddon (1992), and Tommasi (1992). There are some exceptions including Caraballo, Dabus, and Usabiaga (2006) and Caglayana, Filiztekinb, and Rauh (2008). The former article shows that the correlation of inflation and price dispersion can become unstable at very high or extreme inflation, and the latter presents a V-shaped relationship.

[2] The same issue was also discussed by Fischer and Modigliani (1978), although they were referring to the particular real effects of unanticipated inflation due to "the fixity of nominal prices." In this paper, I show that anticipated inflation also has real effects on the level of economic activity, even though economic agents are allowed to adjust their prices freely.

connections of different channels discussed above, I develop a general equilibrium monetary framework with search frictions and incorporate the interrelationship of real balance, search, and endogenous price dispersion.

Benabou (1988, 1992) and Diamond (1993) first study the connection between inflation and efficiency in a search market with price dispersion, but they abstract away from the real balance, which is a key element in the analysis since it directly affects the consumer's surplus from trade, i.e., the gain from search, and indirectly affects price dispersion. It is thus important to model explicitly the cost and benefit of holding money. Although there are various approaches in the literature to proceed, it is natural to apply a framework in which non-interest-bearing money has value and circulates due to search frictions, because consumers actually have to search for a trade in the economy with price dispersion. By attributing the sole reason for holding money to search, I can disentangle the effect of inflation on welfare through real balance and search from other exogenous factors. For the same reason, I also model the consumer's search as the main source of price dispersion, instead of menu costs or nominal rigidities. Therefore, I integrate the mechanism of price posting and nonsequential search in Burdett and Judd (1983) into the monetary framework with search frictions in Lagos and Wright (2005), which has a structure of alternating markets that makes the analysis tractable. In a recent paper, Head et al. (2010) also apply a similar approach to explain the micro-foundation of price stickiness and study the frequency and the pattern of price changes, although they model the consumer's search behavior as exogenous parameters instead of endogenous choices. In this paper, I do not discuss the pattern of price adjusting behavior, and I have to explicitly model consumer search as a choice due to its important implications on welfare.

In this model, consumers want to carry a real balance despite a positive opportunity cost because they can use it as a means of payment in the bilateral market with trading frictions, in which other kinds of payment are impossible due to anonymity or imperfect monitoring. Prices are posted by producers. Consumers sample multiple prices simultaneously and trade at the most preferable price. Their search behavior intensifies competition in the market.

If a producer posts a higher price, he gets more profit from one trade, but it is more likely for a consumer who observes his price and the price of another seller not to trade with him, since his price is probably higher. Hence, producers face this trade-off between profit per trade and expected trade volume, and this tension generates endogenous price dispersion in the model.

As discussed above, there are three channels in the model through which inflation affects welfare: the real balance channel, the price posting channel, and the search channel. The first channel has been extensively studied in the literature on the welfare cost of inflation since Bailey (1956), while the other two are new, and the aggregated effect of all three channels has never been explored. Consumers carry less real balance when inflation gets higher and they consume less; thus, welfare becomes smaller. As inflation increases, the average level of the prices posted by producers may change either way, while price dispersion keeps increasing. Meanwhile, consumers search more intensively for lower prices and still have to pay a higher search cost. Therefore, the aggregated effect of all three channels is ambiguous, and quantitative analysis becomes necessary.

I calibrate the model to match the annual monetary data of the U.S. and the degree of relative price dispersion from empirical literature. Using the economy with zero inflation as a benchmark, I find that the welfare cost of 10% annual inflation is worth 3.23% of the consumption in the benchmark economy. This finding is significantly higher than those in previous literature, such as Cooley and Hansen (1989, 1991) and Lucas (2000), who find the welfare cost of 10% inflation is worth less than 1% of consumption and is comparable to what Burstein and Hellwig (2008) find in their paper with Calvo pricing. However, if either the level of real balance or the degree of price dispersion is held constant in the model, the welfare cost significantly decreases to less than 0.2% of consumption, which is in line with the findings in previous literature.

Following Lagos and Wright (2005), there is another line of monetary literature that applies models based on search frictions to study the welfare cost of inflation, but there is no price dispersion in equilibrium. To compare with those findings, I also calibrate the model to match a target on markup, and the implied welfare cost is over 10% of consumption, which is again much higher than 4.6%, the cost reported by Lagos and Wright (2005), and 5.36%, reported by Rocheteau and Wright (2009). Even though bargaining, the source of welfare loss in those papers, does not exist in this model, the pricing mechanism of posting and search generates an even greater welfare loss. It is then important to understand the reason for market inefficiency.

By decomposing the welfare cost of inflation according to the three channels, I find that the source of inefficiency resides in the interaction of the real balance channel and the price posting channel, while the search cost only generates a negligible welfare loss. Due to the monopolistic power associated with price posting, the average price level is always higher than the efficient level and mostly increases with inflation. Hence, the existence of price dispersion amplifies the welfare-diminishing effect of the real balance channel by driving the consumption level even lower. This negative effect exceeds the positive effect due to the search channel, thus generating a big net welfare loss. I also find a nonmonotonic relationship between welfare cost and inflation in the model. Initially, the effect of the real balance channel is small since consumers still hold a fair amount of money, and the search channel dominates the other two channels. The welfare cost decreases with inflation, and the economy approaches the efficient allocation. Before it achieves efficiency, the real balance channel starts to exert a larger impact on welfare, and together with the price posting channel, these two channels become the dominant force and drive up the welfare cost significantly. This trend prevails until the inflation rate becomes really high for the economy; then the search channel strikes back and the welfare cost of inflation starts to drop.

In a recent paper, Head and Kumar (2005) develop a different framework to explain the relationship of inflation and price dispersion and they also study welfare. They apply the pricing mechanisms in Burdett and Judd (1983), but they incorporate them into the large household framework in Shi (1997). Compared to this paper, they find different relationships of inflation and price dispersion due to their modeling choices. They only focus on inflation levels that are low and moderate relative to the size of the economy, while I also study the situation in which inflation gets very high and I seek to know the magnitude of the welfare cost by a calibration exercise. My paper is also closely related to a recent work by Dutu, Julien, and King (2010). They study the welfare cost or gain of price dispersion in a model of monetary search with free entry. Transaction prices are determined by quantity posting, directed search, and auction. They find that at low levels of inflation, the welfare of an economy with dispersed prices is higher than one without dispersed prices, because price dispersion provides consumers with greater expected surplus from trade and it induces more entry thus more trades.

The remainder of the paper is organized as follows. In Section 2, I lay out the environment of the model and solve for equilibrium in Section 3. Section 4 presents the results of quantitative analysis, including calibration and a welfare analysis. Section 5 concludes the paper. Additional technical details and proofs are in the Appendix.

2 The Environment

Time is discrete. Each period is divided into two subperiods. There is a decentralized market in the first subperiod, and in the second subperiod, the market is centralized. A continuum of buyers and sellers, each with measure 1, live forever. Both buyers and sellers produce and consume in the centralized market, but they act differently in the decentralized market. Buyers want to consume but cannot produce, while sellers can produce but do not want to consume. All economic agents are assumed to be anonymous in the decentralized market, and there is imperfect monitoring technology.[3] These assumptions, as well as the lack of double coincidence of wants, make a medium of exchange, which is called money, essential. Money is storable and perfectly divisible.

I use M_t to denote the money supply in period t, and I assume it grows according to $M_{t+1} = (1+\gamma)M_t$, where M_{t+1} is the money supply in the next period $t+1$. New money is injected by lump-sum transfers, or withdrawn by lump-sum taxes if $\gamma < 0$, at the beginning

[3]For more discussions on the essentiality of money, please refer to a recent paper by Wallace (2010).

of the centralized market. For simplicity, I assume that transfer or tax goes equally to each buyer.[4]

In period t, the buyer's instantaneous utility function is

$$U_t^b(x_t, h_t, q_t) = u(q_t) + v(x_t) - h_t,$$

where q_t is the the quantity of the decentralized-market goods consumed, x_t is the quantity of the centralized-market goods consumed, and h_t is the quantity produced. The centralized-market goods are produced one-for-one using labor. The lifetime utility of a buyer is $\sum_{t=0}^{\infty} \beta^t U_t^b$. I assume that $u(0) = 0$, $u'(q) > 0$, and $u''(q) < 0$ for all q. I also assume $v'(x) > 0$ and $v''(x) < 0$ for all x, and there exists $x^* > 0$ such that $v'(x^*) = 1$. Similarly, the instantaneous utility of a seller is

$$U_t^s(x_t, h_t, q_t) = -cq_t + v(x_t) - h_t,$$

where q_t, x_t, and h_t have the same definitions as in the buyer's utility function.[5] The lifetime utility of a seller is $\sum_{t=0}^{\infty} \beta^t U_t^s$. I assume that $u'(q) = c$ holds for some $q^* > 0$.

In the centralized market, the price of the consumption good x is normalized to 1, and the relative price of money in terms of x in period t is define as ϕ_t. Hence, the price of x_t in terms of money is equal to $1/\phi_t$, and for each period, the consumption good becomes the numeraire in the economy. β is the discounting factor between today's decentralized market and tomorrow's centralized market. In this paper, I focus on the case in which $\beta < 1 + \gamma$. I assume that inflation is forecasted perfectly, and the Fisher equation holds. Hence, the nominal interest rate i is equal to $(1 + \gamma - \beta)/\beta$, and $\beta < 1 + \gamma$ implies that $i > 0$.

3 Search and Price Dispersion

In this section, I consider a particular market structure in which sellers post prices, and buyers know the price distribution but cannot observe all the prices. Burdett and Judd

[4] Alternatively, we can assume that transfer or tax goes equally to each agent or each seller, and equilibrium results stay the same.

[5] To simplify the analysis of the model, we assume that the seller's marginal cost of production is constant. The economic intuition of the results in this paper does not change with more general forms of the cost function.

(1983) study a similar search protocol in a non-monetary model of indivisible goods. In this random search environment, buyers have knowledge about the price distribution but not about an individual price or an individual seller. Hence, a buyer cannot direct his search to the seller with the lowest price, and he has to visit a seller without knowing ex ante exactly what his price is. On the other hand, buyers have the freedom to sample one or two prices, or equivalently, to visit one or two sellers.

Figure 1 presents the timeline of the events. At the beginning of the centralized market in each period, new money is injected or withdrawn by the government. Then, both sellers and buyers adjust their monetary balances, produce, and consume the centralized-market goods. After agents enter the decentralized market in the next period, each seller posts prices for the decentralized-market goods, and he is committed to producing and selling any quantity of the goods at the price posted. Every buyer then chooses to sample one or two prices and decides how much money to spend in a trade. Finally, each buyer trades with one seller. The seller produces, the buyer consumes, and then they return to the centralized market.

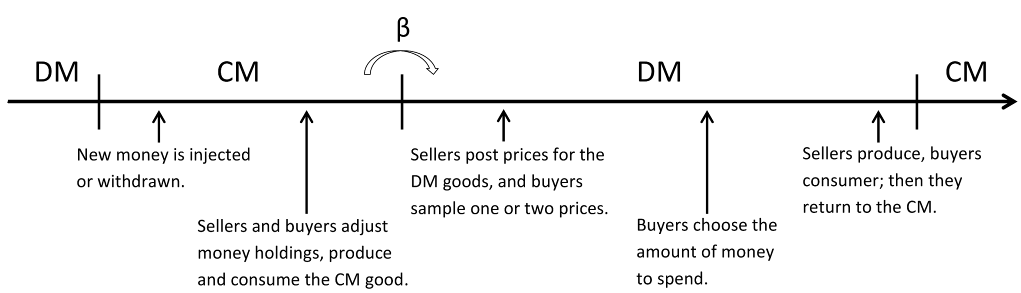

Figure 1

I allow sellers to use mixed strategies in the price posting stage. The induced price distribution in the decentralized market is denoted as F with support $\mathcal{Z}_F$. Based on the knowledge about F, buyers make their decisions on a price sampling strategy. Each buyer samples one price for free and has to pay a cost k in order to observe two prices.[6] I also

[6]To ease the presentation, we limit the maximum number of prices that a buyer can sample to be two. In fact, one can easily extend the logic of Claim 1 in Burdett and Judd (1983) and prove that in the equilibrium with price dispersion buyers do not sample more than two prices even if they are allowed to do so. The

allow buyers to randomize between the two choices and use α to denote the probability of sampling two prices; hence, a buyer samples one price with probability $1-\alpha$.[7]

I use z_t to denote the real money balance that an agent carries in period t, and $z_t = m_t\phi_t$. Starting from this point, I will focus on stationary monetary equilibrium where aggregate real variables stay constant. This implies that $\phi_t M_t = \phi_{t+1}M_{t+1}$ and $\phi_t/\phi_{t+1} = 1+\gamma$. The rate of nominal price change, i.e., the inflation or deflation rate in both the centralized and the decentralized market is equal to the money growth rate $1+\gamma$. I will suppress the time subscript and use $\hat{}$ to denote the variables of the next period. I also define $W^b(z)$ and $V^b(z)$ as the buyer's value functions in the centralized and decentralized market, respectively, and $W^s(z)$ and $V^s(z)$ as the seller's value functions. We proceed first with the buyer's optimization problem.

3.1 Buyer's Optimization

In the centralized market, the buyer's optimization problem in recursive form is given by

$$W^b(z) = \max_{x,h,\hat{z}} \left\{ v(x) - h + \beta E V^b(\hat{z}) \right\} \tag{1}$$
$$s.t.\ h + z + T = x + (1+\gamma)\hat{z}$$

where $\hat{z}$ is the buyer's real money balance of the next period, and $T = \gamma\phi M$ is the transfer payment made by the government. A buyer produces the centralized-market goods using labor as input, consumes, and adjusts the real balance of the next period. Insert the budget constraint into the value function, and (1) becomes

$$W^b(z) = z + W^b(0) \tag{2}$$

where $W^b(0) = \max_{x,\hat{z}} \left[v(x) - x + T - (1+\gamma)\hat{z} + \beta E V^b(\hat{z}) \right]$. The buyer's optimal decision on the real balance of the next period does not depend on his current money holding. This

intuition is straightforward. Given that the search cost of sampling one more price is constant while the marginal gain is decreasing, a buyer either samples just n prices or is indifferent to sampling n or $n+1$ prices, but the equilibrium with price dispersion collapses if $n > 2$.

[7] Alternatively, we can assume that each buyer can only make a discrete choice on the number of price samplings and interpret α as the proportion of buyers who observe two prices. This is slightly more complicated since we have to keep track of two different levels of money holding in equilibrium, while the intuition of the results remain unchanged.

convenient result is due to the assumption of a quasi-linear utility function in the centralized market, which yields a degenerate distribution of buyers' money holdings in the decentralized market.

The expected value function of the decentralized market, $EV^b(\hat{z})$ is given by

$$EV^b(\hat{z}) = \max_{\hat{\alpha}\in[0,1]} \left\{(1-\hat{\alpha})V^b(\hat{z};1) + \hat{\alpha}V^b(\hat{z};2)\right\}, \tag{3}$$

where $\hat{\alpha}$ represents the probability to sample two prices. $\hat{\alpha}$ can be 0 or 1 if the payoff of sampling one price is strictly higher than the payoff of sampling two or vice versa. $V^b(\hat{z};1)$ and $V^b(\hat{z};2)$ represent the values of sampling one and two prices, respectively, and they are defined as

$$V^b(\hat{z};1) = \max_{d(p;\hat{z})} \left\{ \int_{\underline{p}}^{\bar{p}} \left[u\left(\frac{d(p;\hat{z})}{p}\right) + W^b\left[\hat{z} - d(p;\hat{z})\right]\right] dF(p) \right\} \tag{4}$$

and

$$V^b(\hat{z};2) = \max_{d(p;\hat{z})} \left\{ \int_{\underline{p}}^{\bar{p}} \left[u\left(\frac{d(p;\hat{z})}{p}\right) + W^b\left[\hat{z} - d(p;\hat{z})\right]\right] d\left[1-(1-F(p))^2\right] - k \right\}. \tag{5}$$

If a buyer samples one price, he faces price distribution $F(p)$ with support $\mathcal{Z}_F = [\underline{p}, \bar{p}]$ and chooses his optimal expenditure on the decentralized-market goods $d(p;\hat{z})$, which does not depend on the buyer's price sampling strategy but relies on his money holding $\hat{z}$ and transaction price p. If a buyer samples two prices, his optimal expenditure depends on the lower price in two observations. He faces the distribution of the lower price, which is $1-(1-F(p))^2$ and pays search cost k. In both situations, a buyer still carries a real balance of $\hat{z}-d(p;\hat{z})$ after he pays for the decentralized-market goods, and $W^b\left[\hat{z} - d(p;\hat{z})\right]$ represents the continuation value of entering the following centralized market.

In order to solve for $d^*(p;z)$, the buyer's optimal expenditure rule, we apply the linearity of $W^b(z)$ from (2) and rewrite (4) and (5) as

$$V^b(\hat{z};1) = \max_{d(p;\hat{z})} \left\{ \int_{\underline{p}}^{\bar{p}} \left[u\left(\frac{d(p;\hat{z})}{p}\right) - d(p;\hat{z})\right] dF(p) \right\} + W^b(\hat{z}) \tag{6}$$

and

$$V^b(\hat{z};2) = \max_{d(p;\hat{z})} \left\{ \int_{\underline{p}}^{\bar{p}} \left[u\left(\frac{d(p;\hat{z})}{p}\right) - d(p;\hat{z})\right] d\left[1-(1-F(p))^2\right] \right\} - k + W^b(\hat{z}). \tag{7}$$

It is obvious that $d^*(p;z)$ is the solution to the following problem.

$$\max_{d\geq 0} u\left(\frac{d}{p}\right) - d$$
$$s.t.\ d \leq z$$

A buyer chooses how much money to spend on the decentralized-market goods, and he cannot spend more than what he carries. In order to explicitly characterize $d^*(p;z)$, more assumptions on the buyer's utility function in the decentralized market, $u(q)$, are required. In particular, we have the following result on $d^*(p;z)$.

Lemma 1 *If the buyer's decentralized-market utility function $u(q)$ has the CRRA form with risk aversion coefficient σ,*

(i) when $\sigma < 1$, the buyer's optimal spending rule is

$$d^*(p;z) = \begin{cases} z, & \text{if } p < \hat{p} \\ d^*(p), & \text{otherwise} \end{cases}$$

where $\hat{p}$ and $d^*(p)$ satisfy $u'(z/\hat{p}) = \hat{p}$ and $u'(d^*(p)/p) = p$, respectively, and $\partial\hat{p}/\partial z < 0$, $\partial d^*(p)/\partial p < 0$.

(ii) when $\sigma > 1$, the buyer's optimal spending rule is

$$d^*(p;z) = \begin{cases} d^*(p), & \text{if } p < \hat{p} \\ z, & \text{if } \hat{p} \leq p \leq p^R \\ 0, & \text{otherwise} \end{cases}$$

where $\hat{p}$ and $d^*(p)$ are defined similarly, p^R satisfies $u(d^*(p^R;z)/p^R) = d^*(p^R;z)$, and $\partial\hat{p}/\partial z > 0$, $\partial d^*(p)/\partial p > 0$, $\partial p^R/\partial z > 0$.

(iii) when $\sigma = 1$, the buyer's optimal spending rule is

$$d^*(p;z) = \begin{cases} \min\{\tilde{d}, z\}, & \text{if } p \leq p^R \\ 0, & \text{otherwise} \end{cases}$$

where p^R is defined similarly, and $\tilde{d}$ is a constant satisfying $u'\left(\frac{\tilde{d}}{p}\right) = p$.

The risk aversion coefficient σ characterizes the buyer's price elasticity of demand. When σ is less than one, the buyer's demand elasticity is greater than one, and the expenditure elasticity is less than one. Then, his expenditure on the decentralized-market goods decreases when he faces a higher price level. Hence, his expenditure $d^*(p;z)$ decreases as the trading price rises. A buyer cannot spend more than his monetary constraint at very low price levels even though he desires to. $\hat{p}$ is the cutoff price level at which the buyer's monetary constraint starts to unbind, and he spends less than the total amount of money carried when the price is higher than $\hat{p}$. The above intuition is reversed if σ is greater than one, in which case the buyer's optimal expenditure increases with the price, and he is constrained at higher price levels. For the remainder of this paper, I will focus on the case in which the buyer's utility function displays the CRRA form.

We then plug in the buyer's optimal expenditure rule $d^*(p;z)$, substitute $V^b(\hat{z};1)$ and $V^b(\hat{z};2)$ in (3) by (6) and (7), and insert $EV^b(\hat{z})$ into (5). The buyer's Bellman's equation in the centralized market now becomes

$$\begin{aligned} W^b(z) &= z + \max_{x,\hat{z},\hat{\alpha}} \Big\{ v(x) - x + T - (1+\gamma)\hat{z} + \beta W^b(\hat{z}) \\ &\quad + \beta(1-\hat{\alpha}) \int_{\underline{p}}^{\bar{p}} \left[u\left(\frac{d^*(p;\hat{z})}{p}\right) - d^*(p;\hat{z}) \right] dF(p) \\ &\quad + \beta\hat{\alpha} \int_{\underline{p}}^{\bar{p}} \left[u\left(\frac{d^*(p;\hat{z})}{p}\right) - d^*(p;\hat{z}) \right] d\left[1-(1-F(p))^2\right] - \beta\hat{\alpha}k \Big\}. \end{aligned} \tag{8}$$

To make the notation simpler, define $G(p;\alpha)$, the distribution of transaction price, which is the lower one of two price samplings, as

$$G(p;\alpha) = (1-\alpha)F(p) + \alpha\left[1-(1-F(p))^2\right],$$

and rewrite $W^b(z)$ as

$$\begin{aligned} W^b(z) &= z + \max_{x,\hat{z},\hat{\alpha}} \Big\{ v(x) - x + T - (1+\gamma)\hat{z} + \beta W^b(\hat{z}) \\ &\quad + \beta \int_{\underline{p}}^{\bar{p}} \left[u\left(\frac{d^*(p;\hat{z})}{p}\right) - d^*(p;\hat{z}) \right] dG(p;\hat{\alpha}) - \hat{\alpha}k \Big\}. \end{aligned} \tag{9}$$

It is obvious that the optimal decision on x does not depend on $\hat{z}$ or $\hat{\alpha}$, and it satisfies $v'(x^*) = 1$.

According to Lemma (1), $d^*(p;\hat{z})$ has different expressions based on the relationship between p and $\hat{p}$. Hence, we need to first understand the relationship between $\hat{p}$ and $\mathcal{Z}_F$ in order to characterize the buyer's optimal decision on $\hat{z}$ and $\hat{\alpha}$.

Lemma 2 *In the optimization problem in (9), when $\sigma < 1$, the buyer always chooses the real balance $\hat{z}$ such that $\hat{p} > \underline{p}$; when $\sigma > 1$, he always chooses to have $\hat{p} < \bar{p}$.*

The intuition of Lemma 2 is straightforward. Consider the case of $\sigma < 1$ as an example. When the cutoff price $\hat{p}$ is smaller than the lower limit of price distribution, the buyer's real balance does not affect the surplus from trade in the decentralized market since $d^*(p;z) = d^*(p)$. The marginal benefit of bringing more money to the decentralized market is zero, while the marginal cost is still positive. Thus, a buyer wants to reduce his real balance until there is a positive marginal gain related to the action of carrying money, which only happens when $\hat{p}$ exceeds $\underline{p}$. The intuition is analogous in the case of $\sigma > 1$.

We proceed to characterize the buyer's optimal decision. Taking $F(p)$ as given, a buyer chooses $\hat{z}$ and $\hat{\alpha}$ to solve the maximization problem. First, we consider the case of $\sigma < 1$. Knowing that $\hat{p} > \underline{p}$, we can rewrite the buyer's value function in the centralized market, which is given by (9) as the following

$$\begin{aligned} W^b(z) \;\; = \;\; & \max_{\hat{z},\hat{\alpha}} \Big\{ -(1+\gamma)\hat{z} + \beta W^b(\hat{z}) \\ & + \beta \int_{\hat{p}}^{\bar{p}} \left[u\left(\frac{d^*(p)}{p}\right) - d^*(p) \right] dG(p;\hat{\alpha}) \\ & + \beta \int_{\underline{p}}^{\hat{p}} \left[u\left(\frac{\hat{z}}{p}\right) - \hat{z} \right] dG(p;\hat{\alpha}) - \beta\hat{\alpha}k \Big\}, \end{aligned} \tag{10}$$

and we have omitted the terms unrelated to $\hat{z}$ and $\hat{\alpha}$. Therefore, the buyer's optimal real balance $\hat{z}^*$ satisfies

$$\int_{\underline{p}}^{\hat{p}} \left[u'\left(\frac{\hat{z}^*}{p}\right)\frac{1}{p} - 1 \right] \left[1 - \hat{\alpha}^* + 2\hat{\alpha}^*(1 - F(p))\right] dF(p) = i, \tag{11}$$

where i is the nominal interest rate, defined as $i = (1 + \gamma - \beta)/\beta$ via the Fisher equation. $\hat{\alpha}^*$ is the buyer's optimal price sampling strategy. We have $\hat{\alpha}^* \in (0, 1)$ if

$$\int_{\underline{p}}^{\bar{p}} \left[u\left(\frac{d^*(p;\hat{z}^*)}{p}\right) - d^*(p;\hat{z}^*) \right] (1 - 2F(p))\, dF(p) = k. \tag{12}$$

There are two possible corner solutions: $\hat{\alpha}^* = 1$ if $\int_{\underline{p}}^{\bar{p}}[u(d^*(p;\hat{z}^*)/p) - d^*(p;\hat{z}^*)](1-2F(p))dF(p) > k$; $\hat{\alpha}^* = 0$ if $\int_{\underline{p}}^{\bar{p}}[u(d^*(p;\hat{z}^*)/p) - d^*(p;\hat{z}^*)](1-2F(p))dF(p) < k$. Even though $\hat{\alpha}^*$ does not enter (12) directly, it appears in the expression of the price distribution $F(p)$ in equilibrium.

The buyer's marginal gain of holding money, which is the left hand side of (11), decreases as $\hat{z}^*$ increases. Holding everything else constant, there is less marginal gain as a buyer holds more money. From a partial equilibrium point of view, as the money growth rate increases, the nominal interest rate rises, the marginal cost of holding money gets bigger, and the buyer decides to carry less real balance. A smaller real balance makes the gain from sampling an extra price smaller; thus, the buyer is less likely to sample two prices. The price distribution $F(p)$ also affects the buyer's money holding and price sampling strategy in the following way. If $\tilde{F}(p)$ first-order stochastically dominates $F(p)$, a buyer carries less money and searches less because he faces a market with a smaller probability of sampling a low price, implying a higher price level in general.

If $\sigma > 1$, the buyer's optimal real balance $\hat{z}^*$ satisfies

$$\int_{\hat{p}}^{\bar{p}} \left[u'\left(\frac{\hat{z}^*}{p}\right)\frac{1}{p} - 1 \right] \left[1 - \hat{\alpha}^* + 2\hat{\alpha}^*(1 - F(p))\right] dF(p) = i, \tag{13}$$

and the equations for $\hat{\alpha}^*$ remain the same. The same intuition still goes through. With a bigger i, the opportunity cost of holding money is bigger. As a result, a buyer holds less money and samples two prices with a smaller probability. We also notice that in both cases of $\sigma < 1$ and $\sigma > 1$, if $F(p)$ is taken as given and both $\hat{z}^*$ and $\hat{\alpha}^*$ exist, the above conditions on $\hat{z}^*$ and $\hat{\alpha}^*$ imply a one-to-one relationship between the two variables. This observation is formally stated in the following lemma.

Lemma 3 *Taking the price distribution $F(p)$ as given and assuming that both $\hat{z}^*$ and $\hat{\alpha}^*$ exist, there is a one-to-one relationship between the buyer's optimal money holding and his optimal price sampling strategy.*

Finally, if $\sigma = 1$, the decentralized-market utility function has the log form. The buyer's optimal choice of $\hat{\alpha}$ is still governed by the same conditions. The optimal real balance is characterized by $\hat{z}^* = \min\{\tilde{d}, \tilde{z}\}$, where $\tilde{d}$ is defined in Lemma (1) by $u'\left(\tilde{d}/p\right)/p = 1$, and $\tilde{z}$ solves the following problem.

$$\max_{\tilde{z}\geq 0} \int_{\underline{p}}^{\bar{p}} \left[u\left(\frac{\tilde{z}}{p}\right) - \tilde{z} \right] dG(p; \hat{\alpha}^*) - (1+\gamma-\beta)\tilde{z}$$

This implies that $\tilde{z}$ satisfies $u'(\tilde{z}/p)/p = 1 + (1+\gamma-\beta)$, and $\tilde{z}$ does not depend on p since the utility function has the log form. Given that $1+\gamma > \beta$, $\tilde{z}$ is less than $\tilde{d}$, and $\hat{z}^* = \tilde{z}$. Therefore, if $\sigma = 1$, the buyer's optimal expenditure in the decentralized market is equal to his real balance and does not depend on the prices which he samples.

3.2 Seller's Optimization

In the centralized market, the seller's value function is

$$\begin{aligned} W^s(z) &= \max_{x,h,\hat{z}} \left[v(x) - h + \beta V^s(\hat{z}) \right] \\ &s.t.\ h + z = x + (1+\gamma)\hat{z} \end{aligned} \tag{14}$$

where $\hat{z}$ is the seller's real money balance of the next period. In the centralized market, a seller produces and consumes the centralized market goods and chooses the amount of money to bring to the next decentralized market.

Similar to the buyer's problem, the seller's optimal quantity of the centralized-market consumption x^* satisfies $v'(x^*) = 1$. We also have $W^s(z) = z + W^s(0)$, and the seller's optimal real balance $\hat{z}^*$ does not depend on z.

We then turn to the seller's value function in the decentralized market, which is

$$V^s(\hat{z}) = \max_{p \geq c} \pi(p) + W^s(\hat{z}). \tag{15}$$

$\pi(p)$ is the profit function of the seller, and it does not depend on his money holding. We insert (15) into (14), and an immediate result for the seller is $\hat{z}^* = 0$ since $1+\gamma > \beta$. The seller does not bring any money to the decentralized market because he does not want to consume, and the profit is not affected by his real balance.

In the decentralized market, a seller takes the buyer's optimal real balance, the optimal price sampling strategy, and the price distribution in the market as given. He chooses a price to maximize the following profit function

$$\pi(p) = [1 - \alpha + 2\alpha(1 - F(p))] \left(d^*(p; z) - c\frac{d^*(p; z)}{p} \right),$$

where z and α represent the buyer's choices in the same period of the seller's price posting problem, and $d^*(p; z)$ is the buyer's optimal expenditure rule on the decentralized-market goods. It happens with probability $1 - \alpha$ that this seller is the only one whom a buyer visits, and with probability α, he meets a buyer who samples two prices. In that situation, the seller can have a successful trade only if his price is lower than the other price that the buyer observes, which happens with probability $1 - F(p)$. The seller's surplus from trade is the difference between revenue and production cost.

We proceed to characterize the upper and lower limit of $F(p)$. Facing the price distribution in the decentralized market, the highest price that a seller desires to post is equal to or higher than $\bar{p}$, in which case he expects to be the only one who is visited by a buyer. Hence, this seller does not face any competition from another seller, and his profit function becomes

$$\pi(\bar{p}') = (1 - \alpha^*) \left(d^*(\bar{p}'; z^*) - c\frac{d^*(\bar{p}'; z^*)}{\bar{p}'} \right),$$

where $\bar{p}'$ stands for the seller's choice of the upper limit, and $F(\bar{p}') = 1$. The seller chooses $\bar{p}'$ to maximize his profit from trade, and each seller faces exactly the same problem which does not depend on $F(p)$. Hence, the upper limit of $F(p)$, $\bar{p}$ is determined by the following lemma.

Lemma 4 *Given the buyer's optimal expenditure rule $d^*(p; z)$, the upper limit of the price distribution $F(p)$ is given by*

(i) if $\sigma < 1$, $\bar{p} = \max\{\hat{p}, \tilde{p}\}$, where $\tilde{p}$ satisfies $\frac{d^*(\tilde{p})}{\tilde{p}} u''(\frac{d^*(\tilde{p})}{\tilde{p}}) + \tilde{p} - c = 0$, and $\hat{p}$ is defined in Lemma 1

(ii) if $\sigma \geq 1$, $\bar{p} = p^R$, as defined in Lemma 1.

If $\sigma < 1$, the buyer's price elasticity of demand is greater than one, and his expenditure in the decentralized market decreases with the price level. Thus, a seller posts $\tilde{p}$, which is less than the buyer's reservation price, to maximize his profit. This statement is true if a buyer is not bound by the monetary constraint, i.e., $d \leq z$, which happens with relatively higher prices. However, if the buyer's monetary constraint always becomes binding when inflation gets very high, the buyer's elasticity of demand becomes one, and the seller definitely posts a price as high as possible in the feasible range.

The same intuition works with the case of $\sigma = 1$ because the buyer's expenditure does not depend on the price. With $\sigma > 1$, the buyer's expenditure is bound by z at relatively higher price levels. By posting a higher price, the seller can always lower production cost and induce the buyer to spend more if his monetary constraint is not binding. Hence, the seller ends up posting the buyer's reservation price, p^R. The next lemma characterizes the price distribution in the decentralized market.

Lemma 5 *Given the buyer's choices on real balance z and price sampling strategy α, the price posting equilibrium distribution $F(p)$ in the decentralized market is uniquely characterized as*

(i) if $\alpha = 0$, $F(p)$ is concentrated at $\bar{p}$.

(ii) if $\alpha = 1$, $F(p)$ is concentrated at c.

(iii) if $\alpha \in (0, 1)$, $F(p)$ is nondegenerate and $\mathcal{Z}_F = [\underline{p}, \bar{p}]$ is connected, and for any $p \in \mathcal{Z}_F$,

$$F(p) = 1 - \frac{1-\alpha}{2\alpha}\left[\frac{d^*(\bar{p}; z)(1 - \frac{c}{\bar{p}})}{d^*(p, z)(1 - \frac{c}{p})} - 1\right],$$

where $\bar{p}$ is given in Lemma 4 and $\underline{p}$ satisfies

$$\frac{d^*(\bar{p}; z)(1 - \frac{c}{\bar{p}})}{d^*(\underline{p}, z)(1 - \frac{c}{\underline{p}})} = \frac{1+\alpha}{1-\alpha}. \tag{16}$$

If every buyer samples just one price, sellers behave like monopolist, and they all post a price as high as possible in order to capture all the surplus from trade. If every buyer samples two prices, each seller who is visited by a buyer faces the competition from another seller. The seller's price posting game becomes a Bertrand competition, and in equilibrium, the competitive price, which is equal to the marginal cost, is posted.

If a buyer samples two prices with a positive probability being less than one, a certain degree of competition is introduced among sellers. When a single seller decides which price to post, he faces the trade-off between profit per trade and expected trade volume. If the seller posts a higher price, he gets more profit from one trade, but it is more likely for the buyer who observes his price and the price of another seller not to trade with him; the result is reversed if the seller posts a lower price. This tension makes sellers indifferent among an interval of prices and generates a nondegenerate price distribution. If we focus on a symmetric equilibrium in which all sellers behave the same, each seller then plays a mixed strategy in this price posting game. He posts price p with probability $f(p)$, and $f(p) = dF(p)/dp$.

If a buyer samples two prices with a higher probability, $F(p)$ increases. The upper limit of the price distribution does not change, while the lower limit decreases. The price dispersion measured as the length of the support of $F(p)$ increases. Increased search intensifies competition among sellers; thus, it is more likely for a buyer to get a relatively low price, and in general, the average price level is lower. If a buyer brings less money to the decentralized market, $\bar{p}$ increases in general, but the effect on $F(p)$ depends on σ. If the buyer's demand elasticity is greater than one, sellers respond by increasing the highest price level in the market. Then, by the equal profit condition of sellers, the overall price level in the decentralized market rises, and $F(p)$ decreases. If a buyer has a demand elasticity which is less than one, his reservation price in a trade decreases with his real balance, and the upper limit of the price distribution also drops. As a result, the overall price level decreases, and $F(p)$ increases.

3.3 Equilibrium

In this section, I first define the symmetric stationary monetary equilibrium and then discuss its existence and properties. In the monetary equilibrium, money bears value and circulates because buyers can use it as means of payment in the decentralized market with trading frictions, and sellers may use it in exchange for the consumption goods in the centralized market. I focus on symmetric equilibrium in the sense that homogeneous buyers and sellers make identical optimal choices. A formal definition is given in the following, and I have suppressed the argument of $F(p)$ for simplicity.

Definition 1 *A symmetric stationary monetary equilibrium (SSME) is a profile* $\{F^*, z^*, \alpha^*\}$ *satisfying the following conditions:*

1. *Given the buyer's optimal real balance* z^* *and the optimal price sampling strategy* α^**, sellers post profit-maximizing prices in the decentralized market and the resulting price distribution* F^* *is determined by Lemma 5.*

2. *Given the price distribution in the decentralized market* F^* *and the buyer's optimal price sampling strategy* α^**,* z^* *represents the buyer's optimal money holding, satisfying (11).*

3. *Given the price distribution in the decentralized market* F^* *and the buyer's optimal real balance* z^**, the buyer's optimal search strategy is to sample two prices with probability* α^**, satisfying*

$$\begin{array}{ll} \alpha^* = 0, & \text{if } \int_{\underline{p}}^{\bar{p}} \left[u\left(\frac{d^*(p;z^*)}{p}\right) - d^*(p;z^*) \right] (1-2F^*)\, dF^* < k \\ \alpha^* \in (0,1), & \text{if } \int_{\underline{p}}^{\bar{p}} \left[u\left(\frac{d^*(p;z^*)}{p}\right) - d^*(p;z^*) \right] (1-2F^*)\, dF^* = k \\ \alpha^* = 1, & \text{if } \int_{\underline{p}}^{\bar{p}} \left[u\left(\frac{d^*(p;z^*)}{p}\right) - d^*(p;z^*) \right] (1-2F^*)\, dF^* > k \end{array}$$

In general, two kinds of equilibrium may potentially exist: one with a degenerate price distribution in the decentralized market and one with a nondegenerate price distribution. Proposition 1 shows that the first kind of equilibrium does not exist.

Proposition 1 *If* $1 + \gamma > \beta$*, there exists no SSME with* $\alpha^* = 0$ *or* $\alpha^* = 1$*.*

If all the buyers sample two prices in an SSME, the equilibrium price distribution becomes degenerate and concentrated at the marginal cost. Then, the gain from sampling two prices versus one is zero for the buyer, but he has to pay a positive search cost k. As a result, the buyer has an incentive to deviate from sampling two prices and simply observes one price without any cost. The equilibrium collapses.

If all the buyers sample just one price, the equilibrium price distribution again becomes degenerate and concentrated at the highest possible price posted by the seller, which is just the monopoly price since sellers are not competing against each other. When sellers post the monopoly price, they do not consider the buyer's opportunity cost of holding real balance. Hence, when a buyer decides how much money he should bring from the centralized market to the decentralized market, he always finds that the marginal cost of carrying real balance is bigger than the marginal gain because the cost due to a positive nominal interest rate is sunk at the time of a decentralized-market trade. Therefore, the buyer's optimal money holding must be zero, and it cannot be a monetary equilibrium.

I cannot analytically prove the existence and uniqueness of an SSME with a general CRRA utility function in the decentralized market, but when I actually compute the model, I always find that there exists a unique equilibrium if the net money growth rate γ is less than an upper bound $\bar{\gamma}$. The upper bound $\bar{\gamma}$ depends on the search cost k, and $\bar{\gamma}$ becomes smaller with a greater search cost. The intuition is straightforward. As γ increases, the opportunity cost of holding money gets bigger, and the buyer carries less money; hence, he gets a smaller surplus from sampling two prices instead of one. If the buyer's surplus gets even smaller than the search cost, every buyer will sample just one price, and the equilibrium collapses. If the search cost gets bigger, buyers choose not to sample two prices at a lower level of inflation. I will discuss more about equilibrium properties and the effect of inflation on welfare in the next section.

4 Quantitative Analysis

In this section, I first solve the model numerically and calibrate the parameters to match money demand and price dispersion in the data. Then, I calculate the welfare cost of inflation and compare our findings with those in previous literature. I also decompose the influence of inflation on welfare and identify the different effects through three different channels. Finally, I check the robustness of our results and seek to understand the relationship between the welfare cost of inflation and the degree of price dispersion.

4.1 Calibration

I follow the literature and consider the following functional forms for preferences and technology:

$$\begin{aligned} CM: &\quad U(x) = A\log x - h \\ DM: &\quad u(q) = B\tfrac{(q+b)^{1-\sigma}-b^{1-\sigma}}{1-\sigma} \text{ and } c(q) = cq \end{aligned}$$

where $\sigma > 0$ and $b > 0$. I include b in the decentralized-market utility function in order to make $u(0) = 0$ when $\sigma > 1$. I set $b \approx 0$ so that the relative risk aversion coefficient of $u(q)$, which is equal to $\sigma q/(q+b)$, is approximately σ. The utility function in the centralized market is standard, following the literature since Cooley and Hansen (1989). Because both the parameters A and B characterize the relative size of the centralized market trade versus the decentralized market trade, I normalize $B = 1$. Finally, I set the marginal cost of production in the decentralized market equal to one, i.e., $c = 1$ in $c(q)$, so that the cost of labor is the same in both markets.

We need to calibrate three key parameters of this model, the preference parameters σ and A, and the buyer's cost of sampling two prices k. First, I set k to target some statistics measuring the degree of price dispersion from the empirical literature. The search cost affects the buyer's price sampling strategy. If k is high, a buyer is more likely to sample one price. There is less competition in the decentralized market, and the price distribution is less dispersed. This connection between search cost and price dispersion helps us to calibrate k. Second, I calibrate A to match the money demand, $L = M/PY$, at the average nominal interest rate. The real balance M/P is proportional to the total real output Y with a factor of proportionality L, which depends on the opportunity cost of holding money. In the model, the per capita real output in the centralized market is $x^* = A$, and the per capita real output in the decentralized market is $\int_{\underline{p}}^{\bar{p}}[d^*(p; z^*)/p]dG(p; \alpha^*)$. Hence, the money demand is given by

$$L = \frac{M/P}{Y} = \frac{z^*}{2A + \int_{\underline{p}}^{\bar{p}} \frac{d^*(p;z^*)}{p} dG(p; \alpha^*)}.$$

For the same level of the nominal interest rate, if A increases, the real output in the decentralized market and the total real output get larger, and the money demand decreases. This relationship determines A. Finally, since the money demand is a function of the nominal interest rate, i.e., $L = L(i)$, I calibrate σ to fit the model-generated money demand curve to the money demand observations from real data.

The time period is one year. I chose this length of time in order to compare the results with those in previous studies, and also because of the availability of data. I use a data sample of 101 years, from 1900-2000. I use data on the nominal GDP for PY, M1 for M, and the short term (6 month) commercial paper rate for i.[8] Concerning the calibration target for search cost k, there are many empirical studies in which the magnitude of price dispersion is measured by relative price variability (RPV), defined as

$$RPV = \left[\int_{\underline{p}}^{\bar{p}} (R_i - \bar{R})^2 dF(p) \right]^{\frac{1}{2}}$$

where $R_i = \log(p_i/\bar{p})$.[9] In their paper, Debelle and Lamont (1997) find an average RPV of 0.035 at the annual inflation rate of 4.3%, and I use it as the target for k in the baseline calibration. Finally, I set β to match an annual real interest rate of 4%. This completes the baseline calibration.

In order to see how the calibration results vary with different target values, I also use another target from Parsley (1996) for price dispersion, and it gives an average RPV of 0.0923 at the annual inflation rate of 5.3%. I also consider different calibration strategies to check the robustness of the results. As a very common approach in the literature, the search cost is calibrated to match the markup in the decentralized market. I consider the value of the markup to be 30% at an annual inflation rate of 5.46%, which is about the average value in the empirical evidence discussed in Faig and Jerez (2005) and also used in Aruoba,

[8] The data is available in Craig and Rocheteau (2006). For a detailed description of the data source, please refer to Appendix 2 in their paper.

[9] There is an alternative definition of RPV, which is $RPV_{i,t} = \left[\int_{\underline{p}}^{\bar{p}} (R_{i,t} - \bar{R}_t)^2 dF(p) \right]^{\frac{1}{2}}$, where $R_{i,t} = \log(p_{i,t}/p_{i,t-1})$. Tommasi (1992) finds similar effects of inflation on price dispersion using both measures, and Parsley (1996) shows that the two measures are actually comparable. In this paper, we are not able to calculate $RPV_{i,t}$ because the model does not generate the path of each individual seller's price changes.

Waller, and Wright (2010).[10] When I present the calibration results, I report the implied markup in the aggregate economy of both markets as well.

I also consider another calibration strategy for the preference parameter σ. Instead of matching the money demand function to real monetary data, I match the estimated elasticity of money demand with respect to the nominal interest rate. Following the literature (e.g., Goldfeld and Sichel, 1990), I estimate the interest elasticity using the following equation

$$\ln z_t = \eta_0 + \eta_1 \ln i_t + \eta_2 \ln y_t + \eta_3 \ln z_{t-1} + \nu_t,$$

where z_t and z_{t-1} are real money balances, i_t is nominal interest rate, and y_t is real output. I assume that the residual ν_t follows an AR(1) process, i.e., $\nu_t = \rho\nu_{t-1} + \varepsilon_t$, and ε_t is a serially uncorrelated random error with mean zero and constant variance. I use the Cochrane-Orcutt procedure to correct the autocorrelation problem. I first estimate the interest elasticity η_1 from the data, which is -0.0806. Then, I estimate the interest elasticity again using the data series generated by the model and choose σ to match the model generated elasticity with η_1.[11]

In order to numerically solve the model, we first take the buyer's choice of α^* and z^* as given and compute the uniquely determined price distribution F^* using Lemma 5. Then, we plug F^* into (11) and solve for z^* as a function of α^*. Finally, we insert both F^* and z^* into (12) and search for the $\alpha^* \in (0, 1)$ that solves the equation.

4.2 Results

Table 1 summarizes the results of the baseline calibration. Besides the calibrated parameters, I also report the implied decentralized-market markup at the average money growth rate

[10] This data on markup is from the Annual Retail Trade Survey conducted by the U.S. Census Bureau, which is collected in the format of firm surveys. We are not claiming that the retail market is a real economy counterpart of the decentralized market in the model, but this is the best available data we can get. The average rate of annual inflation for the period covered in the sample is 5.46%.

[11] Concerning the data required for this exercise, we still need the series of real GDP and price index. The real GDPs before 1930 are from the *Historical Statistics of the United States, Colonial Times to 1970* (1970); and they are adjusted to chained 2005 dollars. From 1930 to 2000, it is from the FRED database managed by the Federal Reserve Bank of St. Louis. We use CPI data for the price index. Before 1913, it is again from the *Historical Statistics of the United States, Colonial Times to 1970* (1970), adjusted to chained CPI in 1982-84 being 100, and from 1913 to 2000, it is from the FRED database.

μ_{DM} and the overall markup of both markets μ. The markup in the decentralized market is defined as $\mu_{DM} = \int_{\underline{p}}^{\bar{p}} (p/c - 1) dF(p)$, and the markup in the centralized market is simply one. The overall markup is the average of the two weighted by the shares of total output produced in each market. In the baseline calibration, about 25% of the total real output is produced in the decentralized market.

Table 1. Baseline Calibration

β	k	A	σ	μ_{DM}	μ
0.9615	0.0043	0.4916	0.1181	9.72%	2.4%

The baseline calibration yields a value smaller than one for the preference parameter σ. In fact, this result holds in all the calibrations that I do in this paper. The reason is that σ determines the shape of the utility function in the decentralized market, hence, the shape of the money demand function generated by the model. If σ is big, the utility function shows much curvature and is far from being linear. The money demand from real data does not have much curvature, and this implies a small σ.

Figure 2. Money Demand: Model and Data

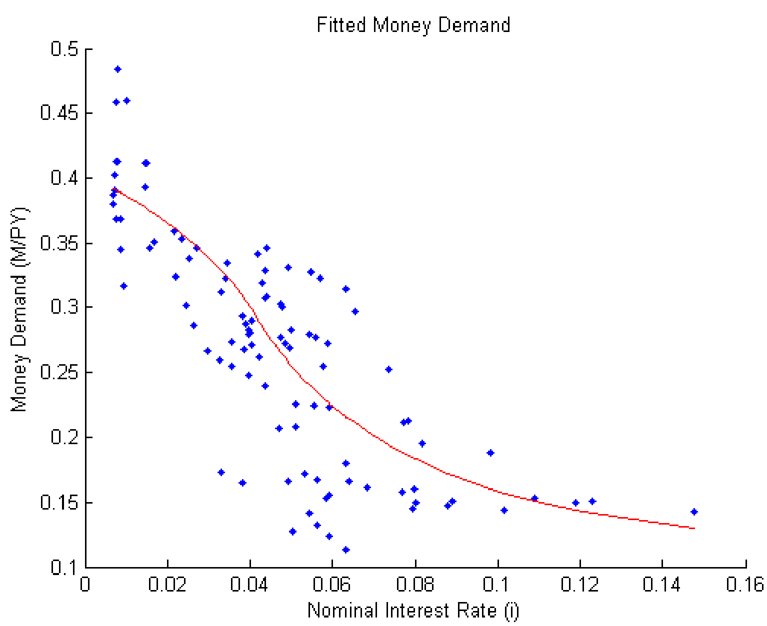

Figure 2 presents the model-generated money demand curve, which informs us of the fitness of the model. Another way to check the model's fitness is to use regressions to find the effect of inflation on price dispersion measured by RPV and to compare the coefficient of the model with what is found in the empirical papers mentioned above. I find that this model implies a coefficient of 0.2784, and the range of this coefficient found in Debelle and Lamont (1997) is from 0.115 to 0.393. The model does a good job of fitting the targets of real data on money demand and price dispersion.

Figure 3. Equilibrium Paths

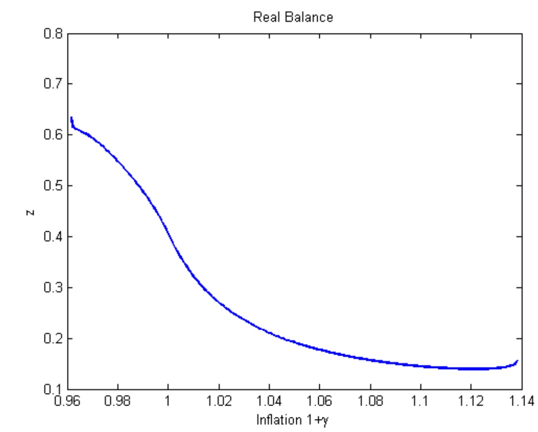

Figure 3a. Real Balance

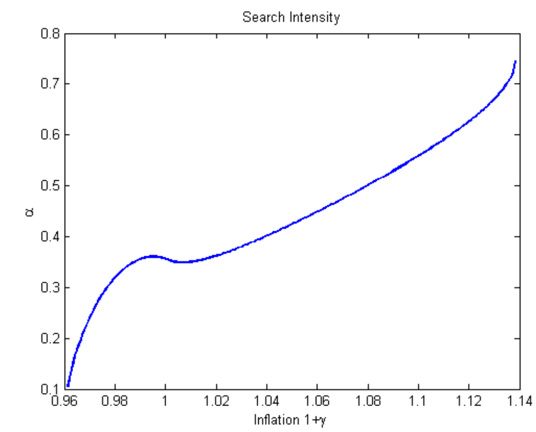

Figure 3b. Price Sampling Strategy

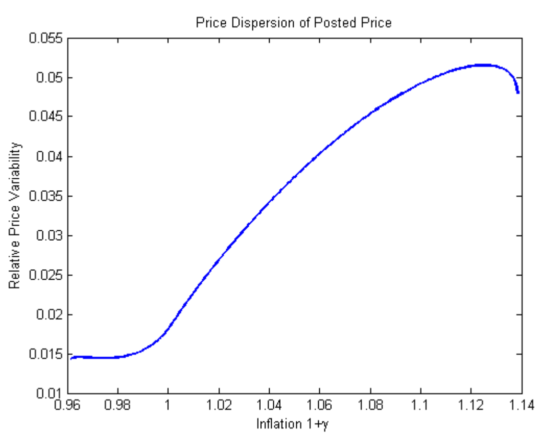

Figure 3c. Price Dispersion

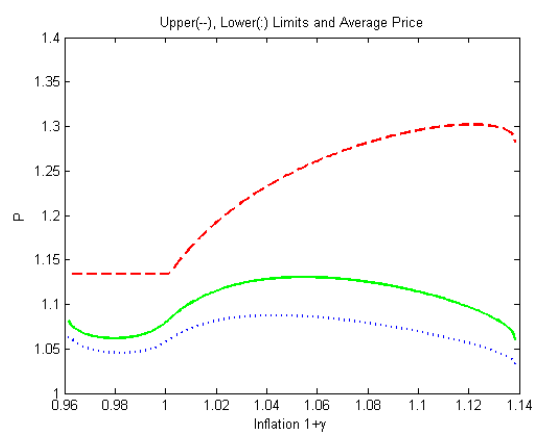

Figure 3d. Price Levels

In the baseline calibration, the model has a unique SSME with those parameters. I have always found a unique equilibrium with all the different parameter values in this paper. Figure 3 illustrates the equilibrium paths of several endogenous variables with respect to different levels of inflation: the real balance z^*, the probability of sampling two prices α^*, the degree of price dispersion measured by RPV, and the upper and lower limit and the average price level of F^*.

Figure 3a shows that as inflation increases, the buyer's optimal real balance decreases. As the opportunity cost of holding money gets bigger, a buyer tends to lower the amount of the real balance that he brings to the decentralized market so that the marginal gain of holding money increases to compensate for a larger cost. As a buyer brings less money, his surplus from trade in the decentralized market decreases, and this lowers his probability of sampling two prices. On the other hand, the degree of price dispersion generated by price posting increases, and it gives a buyer an incentive to increase the probability of sampling two prices. These two forces are put together in the general equilibrium analysis. Figure 3b shows that a buyer searches more with higher inflation, and the price dispersion effect dominates the real balance effect. In the calibrated model, the upper bound on inflation $\bar{\gamma}$ is about 14%, and there exists no equilibrium with a nondegenerate price dispersion if γ is more than $\bar{\gamma}$. As inflation approaches the upper bound, every buyer samples two prices with a very high probability. This induces intensive competition among sellers and drives the price dispersion smaller. The prices in the decentralized market are more focused on lower levels, and the buyer's marginal gain of carrying money gets bigger. As the price dispersion effect eventually exceeds the real balance effect, buyers want to bring more money even though the opportunity cost of holding it is still increasing. This is what we observe at very high levels of inflation in Figure 3a.

Because the calibration exercise generates $\sigma < 1$, the buyer's price elasticity of demand is greater than one. As a buyer brings less money with him with higher inflation, his expenditure in the decentralized market decreases and starts to be bound by his real balance. With $\sigma < 1$, a lower level of expenditure corresponds to a higher level of the upper price limit $\bar{p}$, which is not affected by the buyer's price sampling strategy. Hence, $\bar{p}$ increases with inflation, and this is a force that drives up the overall price level. On the other hand, a buyer chooses to sample two prices with a higher probability, and this drives down the overall price level. As a result of the two competing forces, the equilibrium price distribution becomes more dispersed as measured by RPV, as presented in Figure 3c. Figure 3d shows the movements of the average price level and the upper and the lower limits of the price distribution. When γ is small, a buyer still carries enough money so that he is only constrained at lower levels of prices, and at those prices he desires a consumption level higher than what he can afford. The search effect now dominates the real balance effect, and as a result, the average price level decreases, and the price distribution gets more dispersed. As inflation keeps increasing, the real balance effect becomes more significant at lower levels of z, since utility changes faster at lower levels of consumption. Hence, the real balance effect quickly dominates the search effect and drives up the average price level. Now a buyer carries even less money, and he is constrained at all the prices in the decentralized market. He continues to sample two prices with a higher probability, and this again drives down the average price, eventually the entire price distribution. If the money growth rate keeps increasing, the monetary equilibrium with nondegenerate price distribution finally collapses because the additional gain from sampling two prices cannot cover the search cost.

Table 2. Calibration: Alternative Targets

	Baseline	Target 2	Target 3	Target 4
k	0.0043	0.0328	0.0057	0.003
A	0.4916	1.0134	0.5064	0.3309
σ	0.1181	0.5326	0.2111	0.1005
μ_{DM}	9.72%	47.13%	25.64%	12.21%
μ	2.4%	8.75%	5.52%	2.95%

In Table 2, I report the calibration results using alternative targets. In the case of Target 2, I use the other price dispersion statistics from Parsley (1996). Target 3 represents the decentralized-market markup, which I use to calibrate the search cost instead of RPV. In the case of Target 4, I use interest elasticity instead of money demand to calibrate σ. We first compare column 1 with column 2. With a bigger target on price dispersion, i.e., the price distribution is more dispersed, I get a larger search cost k and a bigger A. If a buyer searches more, price dispersion gets smaller. Hence, a larger price dispersion implies a higher search cost and less competition in the decentralized market, hence, a bigger markup for sellers. As a result, more transactions occur in the centralized market, thus implying a larger A. The findings are similar if we compare the baseline calibration with the third column, which has a bigger target on markup. In the next section, I will further discuss the different targets of price dispersion and their implications on the welfare cost of inflation. Finally, when we look at column 4, we do not see a very different result from column 1; thus, it does not make a big difference to target interest elasticity rather than money demand.

4.3 Welfare Cost of Inflation

Following the literature, I study the welfare cost of increasing inflation from a stationary annual inflation rate of zero percent to τ percent. First, I use the parameters reported in the previous section to compute the welfare of the equilibrium with zero inflation, which is the sum of trade surplus in both markets. Then, I compute the welfare of the new equilibrium with a different inflation rate of τ. I ask how much agents would be willing to increase or decrease their consumption in the benchmark equilibrium with zero inflation in order to make them indifferent to the two economies.

For any τ, the equilibrium welfare is given by

$$(1-\beta)\mathcal{W}(\tau) = 2\left[v(x^*) - x^*\right] + \int_{\underline{p}}^{\bar{p}} \left[u\left(\frac{d^*(p;z^*)}{p}\right) - c\frac{d^*(p;z^*)}{p}\right] dG(p;\alpha^*) - \alpha^* k, \qquad (17)$$

where $2[v(x^*) - x^*]$ is the total trade surplus from the centralized market, and the second integral term is the surplus from trade in the decentralized market, subtracting the expected cost of sampling two prices. We can also write the equilibrium welfare at zero inflation with a reduced consumption level in both the centralized and decentralized market by a factor Δ as

$$(1-\beta)\mathcal{W}_\Delta(0) = 2\left[v(\Delta x^*) - x^*\right] + \int_{\underline{p}}^{\bar{p}} \left[u\left(\frac{\Delta d^*(p;z^*)}{p}\right) - c\frac{d^*(p;z^*)}{p}\right] dG(p;\alpha^*) - \alpha^* k.$$

I measure the welfare cost of τ percent inflation as the value Δ_0 that solves $\mathcal{W}(\tau) = \mathcal{W}_\Delta(0)$. Each economic agent would need to give up $1-\Delta_0$ percent of his consumption to be indifferent to the two economies with different inflation rates. Our results on the welfare cost below focus on $\tau = 10\%$; i.e., I consider the welfare cost of 10% inflation versus zero inflation, and I also show a graph of welfare cost at different levels of inflation in the baseline calibration.

Table 3. Welfare Cost of 10% Inflation

	Baseline	Target 2	Target 3	Target 4
$1-\Delta_0$	3.23%	8.31%	7.23%	3.25%

Table 3 shows the welfare cost of inflation under different calibration targets. In the baseline calibration, I find that the welfare cost of 10% inflation is worth 3.23% of consumption in the benchmark economy with zero inflation. We notice that the model generates a relatively higher welfare cost compared to earlier findings by Cooley and Hansen (1989,1991) and Lucas (2000), and the magnitude is slightly smaller than those reported by Lagos and Wright (2005), Craig and Rocheteau (2006), and Rocheteau and Wright (2009). However, if I recalibrate the model, in particular the search cost k, to target an average markup of 10%, which is the target for the bargaining power used in those monetary-search papers, the welfare cost of 10% inflation is as high as 13.26%.

Table 4. Shorter Sample (1959-2000)

	Baseline	Target 2	Target 3	Target 4
k	0.0089	0.033	0.0061	0.0062
A	1.1841	1.5488	0.6175	0.8702
σ	0.1861	0.5262	0.2200	0.1441
μ_{DM}	8.34%	45.14%	35.32%	9.96%
μ	1.42%	5.78%	4.79%	1.67%
$1-\Delta_0$	2.34%	5.08%	6.68%	2.69%

Table 4 reports similar experiments when I fit the model to a shorter sample from 1959 to 2000. In general, the calibration results do not change much, and the welfare cost of inflation stays in the same range. By comparing it with Table 2, we notice that both the search cost k and the centralized-market preference parameter A are relatively higher when

I use the shorter sample, while the decentralized-market preference parameter σ does not change much. Although it is now more costly to search in the decentralized market in which both trade volume and consumption level are low, agents can switch to consume more of the centralized-market goods, and their aggregate utility from consumption does not necessarily decrease. Because the share of centralized-market trade is much higher than before, the welfare cost of inflation actually becomes smaller.

Those studies with bargaining as the pricing mechanism usually get a high welfare cost of inflation due to the holdup problem. A buyer who carries money is making an investment in the decentralized-market trade, and the investment cost is the cost of holding money due to a positive nominal interest rate. He cannot get the full return on his investment, i.e., the entire surplus from trade, unless his bargaining power is one. In this model, I do not have bargaining or the holdup problem. However, a seller possesses monopolistic power because he posts a price, and he can post a price level higher than the marginal cost. When a buyer observes only one price, we cannot have an equilibrium in which a seller posts prices higher than the marginal cost, and in fact, there exists no monetary equilibrium as is proven in Proposition 1. When a buyer starts to observe two prices with a certain probability or when a proportion of buyers start to do so, a seller can mix high price postings with low prices, and the buyer is still going to carry money and trade in the decentralized market. This is because when he observes only one high price, the small surplus from a trade is compensated by the big surplus when he samples two prices and trades at a low price level. As a result, the average price in the decentralized market is always higher than the marginal cost, as shown in Figure 3d; hence, equilibrium is driven away from the efficient allocation by the seller's monopolistic power. This is the source of inefficiency in our model. As I grant buyers the freedom to sample more than one price, which could potentially yield a welfare gain, sellers are automatically given the monopolistic power to post prices higher than the efficient level, which makes all agents in the economy even worse in terms of an even higher welfare cost of inflation.

There are three main channels through which inflation affects welfare in this model.

When inflation increases, a buyer carries less money into the decentralized market; hence, there is less consumption and welfare decreases. I call this the real balance channel. Second, as a buyer carries less money, a seller responds by increasing the highest price level, since the buyer's demand elasticity is smaller than one, and this is a force that drives up the average price. A higher average price level then drives down consumption and welfare. I name this the price posting channel. Third, the price dispersion in the decentralized market increases as a seller raises the highest price level, and this induces a buyer to search more, i.e., to sample two prices more frequently. This force drives down the actual transaction price in a trade, hence, more consumption and higher welfare. I call the last one the search channel. In order to understand the aggregate and individual effects of the three channels, we first see how welfare cost changes with inflation and then decomposes the cost to identify the impact of each channel.

Figure 4. Welfare Cost of Inflation

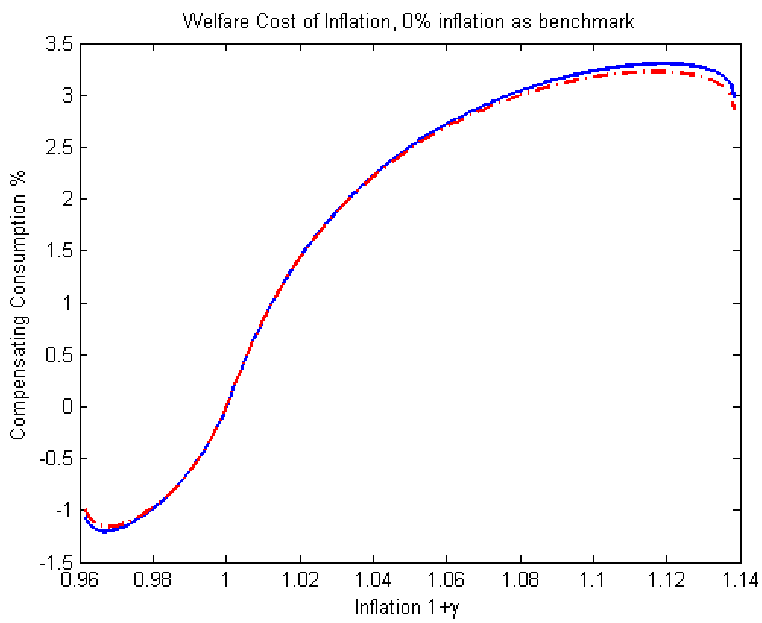

Figure 4 illustrates the welfare cost at different levels of inflation, with and without search cost, in an economy with parameters from the baseline calibration. There are three features to notice about this graph. First, it is welfare-improving to have a small deviation from the Friedman rule. Even if we run deflation in the economy and make the opportunity cost of holding real balance very close to zero, the seller's monopolistic power discussed above still functions. In that situation, a buyer carries almost as much money as possible, and he is barely subject to the monetary constraint in the decentralized market. Because he still has an incentive to search for a low price, there is still price dispersion, and a seller still posts prices higher than the marginal cost of production. If we deviate from the Friedman rule and increase inflation a little, the average price drops and welfare increases. At very low levels of inflation, the positive effect from the search channel dominates the negative effect from the real balance and the price posting channel. However, as inflation keeps increasing and the buyer's real balance keeps decreasing, the effect of inflation on welfare through the latter two channels becomes bigger. As a result, welfare cost keeps increasing, but at a decreasing rate. As a buyer samples two prices more and more frequently, in the end, the search channel dominates again and imposes a larger effect on welfare. The price dispersion in the decentralized market decreases, and the price distribution is driven toward the marginal cost. Therefore, the effect of inflation on welfare is nonmonotonic. This is the second noticeable feature about Figure 4. Finally, the solid line represents the welfare cost including the search cost, and the dashed line stands for the welfare cost excluding the search cost. Apparently, the cost of search only generates a negligible welfare loss.

In order to understand the impact of each individual channel, I proceed to decompose the welfare cost of inflation through the three channels by the following exercise. Using the parameters in the baseline calibration, I solve the equilibrium of the model at different levels of inflation. Then, I keep the equilibrium paths of two channels but hold the third channel at a constant level so that I am able to isolate the contribution of the third channel by comparing the resulting welfare cost of inflation with the original value.

I start with the real balance channel and keep α and F at their equilibrium values. First, I solve for the buyer's optimal money holding $\bar{z}$ in the equilibrium with a money growth rate very close to the Friedman rule. Using it as the benchmark value of real balance, I replace the equilibrium path of z with $\bar{z}$, and calculate the welfare cost for different levels of inflation. The dashed line in Figure 5a shows the welfare cost of inflation when I hold real balance constant, and the solid line is the original welfare cost function. The difference between the two lines represents the effect of the real balance channel on welfare cost. In particular, if a buyer carries the benchmark real balance instead of the equilibrium level, the welfare at 10% annual inflation can increase by 4.45% of consumption.

Figure 5. Welfare Cost Decomposition

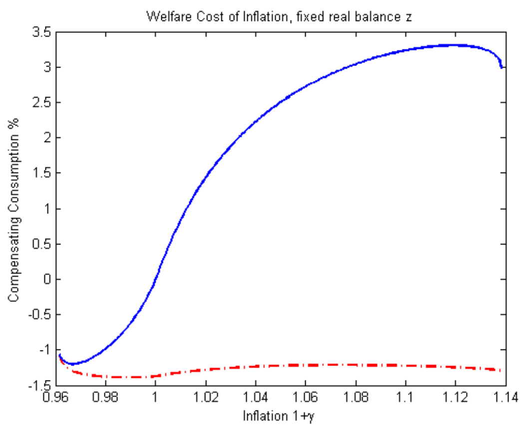

Figure 5a. Fixed Real Balance

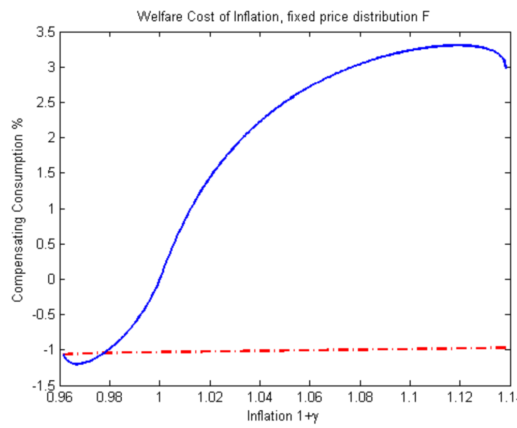

Figure 5b. Fixed Price Distribution

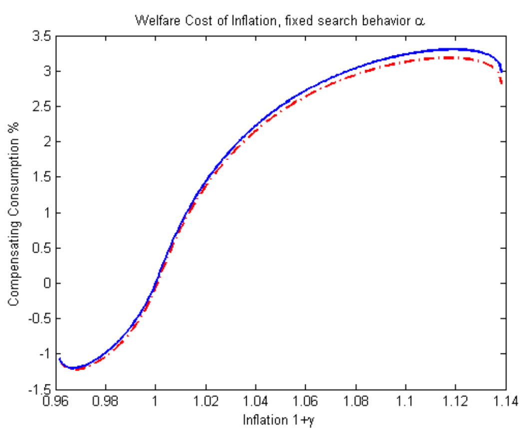

Figure 5c. Fixed Search

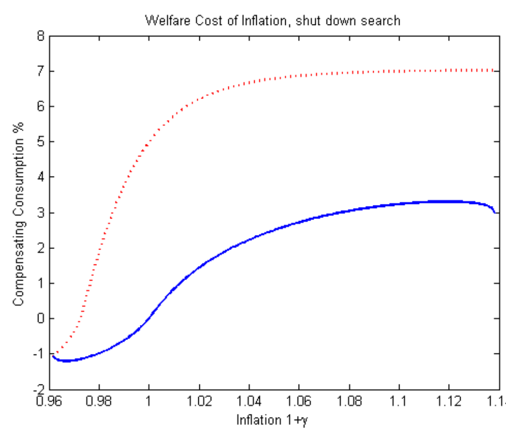

Figure 5d. Shut Down Search

Similarly, Figure 5b shows the difference of welfare cost when I hold the price distribution F constant at the benchmark value, and the welfare cost at 10% inflation can increase by

4.22% of consumption. Both the real balance and the price posting channel contribute significantly to the large welfare cost. We also notice that the welfare cost of inflation becomes negligible if either the real balance or the price distribution is held constant. In particular, the welfare cost of 10% inflation is just 0.04% of consumption when price distribution stays constant, and this result is in the range of previous estimates in Cooley and Hansen (1989,1991) and Lucas (2000). If I hold price distribution constant, the welfare cost is only 0.15% of consumption. Therefore, the price posting channel amplifies the welfare-diminishing effect of the real balance channel, and the coexistence of the two channels induces a large welfare cost.

The impact of the search channel is illustrated in Figure 5c, and we can observe a welfare gain of only 0.1% at 10% inflation due to search. However, the small number does not imply a trivial effect of the buyer's price sampling behavior on welfare. I revisit the equilibrium welfare equation (17) and notice that the price sampling probability α affects the total welfare both directly and indirectly. α directly determines $G(p;\alpha)$, but it also indirectly affects the buyer's money holding z and the price distribution F. The above exercise of holding α constant can only isolate the direct contribution of the search channel to welfare, but not the indirect contribution. Figure 5d presents the result from another exercise in which I shut down the search channel instead of holding the value of α constant. I force every buyer to sample two prices with a given probability, allow each agent to make all other choices, and resolve equilibrium at different levels of inflation. The solid line still represents the welfare cost in the original economy, and the dotted line shows the welfare cost after I shut down the search channel, in which case I find an additional welfare cost of 3.76% consumption at 10% inflation. That is the total effect that search has on the welfare of the economy.

Finally, I discuss the relationship between the target of price dispersion and the welfare cost of inflation. I recalibrate the model to two other artificial targets of price dispersion by varying RPV to be 0.06 and 0.1 at the same rate of annual inflation, 4.3%. Table 5 shows the calibration results and the welfare cost of 10% inflation for different targets of RPV.

We can see that with a bigger target on price dispersion and everything else being constant, the calibrated search cost, the markup in the decentralized market, and the welfare cost of inflation all get higher. More dispersed price distribution implies decreased search in the decentralized market; hence, the cost of search has to be higher. Then, a seller possesses more monopolistic power, and the equilibrium allocation is farther away from being efficient. In response to a bigger search cost, the output in the centralized market becomes higher, and buyers and sellers consume more centralized-market goods to compensate for smaller surplus from trade in the decentralized market. A larger RPV target also implies a bigger σ. A buyer has a smaller demand elasticity in the decentralized market, and it is more difficult to substitute the decentralized-market consumption by consumption from the other market. All these factors connect a larger RPV target to a higher welfare cost of inflation.

Table 5. Price Dispersion and Welfare Cost

RPV	0.035	0.06	0.1
k	0.0043	0.0072	0.0362
A	0.4916	0.5396	1.0373
σ	0.1181	0.3256	0.6091
μ_{DM}	9.72%	55.14%	72.75%
μ	2.40%	9.62%	11.52%
$1-\Delta_0$	3.23%	12.72%	15.40%

5 Conclusion

In this paper, I develop a general equilibrium monetary model with search frictions and incorporate the interrelationship of real balance, search, and endogenous price dispersion. I quantify the welfare cost of anticipated inflation and study different channels through which inflation affects welfare. Calibrating the model to match the annual monetary data of the U.S. and the degree of price dispersion, I find that the welfare cost of 10% annual inflation is worth 3.23% of the consumption in the benchmark economy with zero inflation, which is higher than the previous findings in the literature.

I identify three channels in the model through which inflation imposes an impact on welfare: the real balance channel, the price posting channel, and the search channel. The first channel has been extensively studied in the literature, and it usually yields a negative but small effect on welfare. It potentially improves welfare to allow buyers to sample multiple prices and to search for the most preferred terms of trade. Search intensifies competition in the market and generates a welfare gain larger than the accompanying loss due to the search cost. However, as buyers start to search for lower prices, sellers are granted the monopolistic power to post different price levels, which are even higher than the efficient level, i.e., the marginal cost. I find that due to the existence of price dispersion, the price posting channel amplifies the welfare-diminishing effect of the real balance channel by driving the consumption level even lower, and this negative effect exceeds the positive effect generated by the search channel. Therefore, the source of inefficiency in our model resides in the interaction of the real balance channel and the price posting channel. Depending on the magnitude of the negative effect relative to the positive effect at different levels of inflation, I find a nonmonotonic relationship between welfare cost and inflation.

This model endogenously generates a nondegenerate price distribution. The degree of price dispersion first increases with inflation and then decreases when inflation gets extremely high for the economy. This nonmonotonic relationship is consistent with the empirical findings discussed in the introduction of this paper. Instead of using the average market markup, a standard target in the literature, I adopt a new target on price dispersion, relative price variability, to calibrate the buyer's search cost. I find that the magnitude of the welfare cost of inflation is closely related to the value of the price dispersion target, while it is not sensitive to how I calibrate the preference parameters. A more dispersed price distribution implies less competition in the market, and hence, a bigger search cost and a higher average price level. Therefore, the output and consumption in equilibrium is driven farther away from the efficient level, and the welfare cost of inflation becomes larger.

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Appendix

Proof of Lemma 1.

First, we consider the case of $\sigma < 1$. The first order condition of the buyer's unconstrained optimization problem is $u'(d^*/p)/p - 1 = 0$. Notice that $\sigma < 1$ implies $-qu''(q)/u'(q) < 1$, so $u'(d^*/p)/p$ is a decreasing function in p and $\partial d^*(p)/\partial p < 0$. Hence, there exists $\hat{p}$ such that $d^*(\hat{p}) = z$ for $z > 0$. For $p < \hat{p}$, $d^*(p) > z$ and $u'(z/p)/p - 1 > 0$. Buyer wants to spend more, but is subject to the constraint $d^*(p; z) \leq z$. Hence, $d^*(p; z) = z$ for $p < \hat{p}$. For $p \geq \hat{p}$, $d^*(p) \leq z$, and $d^*(p; z) = d^*(p)$. It is straightforward to verify that $\partial \hat{p}/\partial z < 0$.

Second, we consider the case of $\sigma > 1$. $\sigma > 1$ implies $-qu''(q)/u'(q) > 1$, so $u'(d^*/p)/p$ is now an increasing function in p and $\partial d^*(p)/\partial p > 0$. Similarly, we can show that $\hat{p}$ exists such that $d^*(\hat{p}) = z$, but $\partial \hat{p}/\partial z > 0$. Hence, we have $d^*(p; z) = d^*(p)$ for $p < \hat{p}$, and $d^*(p; z) = z$ for $p \geq \hat{p}$. Then, we want to establish the existence of p^R and show that $\hat{p} < p^R$.

I claim that the surplus from trade at $\hat{p}$ is positive, i.e. $u(z/\hat{p}) - z > 0$. Suppose not and $u(z/\hat{p}) - z \leq 0$. $u'(z/\hat{p})/\hat{p} - 1 = 0$ holds by the definition of $\hat{p}$. For all $d < z$, we have $u'(d/\hat{p})/\hat{p} - 1 > u'(z/\hat{p})/\hat{p} - 1$ since $u''(q) < 0$, so $u'(d/\hat{p})/\hat{p} - 1 > 0$. Hence, for all $d < z$, $u(d/\hat{p}) - d < u(z/\hat{p}) - z \leq 0$. However, $u(d/\hat{p}) - d = 0$ when $d = 0$, and this is a contradiction. Therefore, $u(z/\hat{p}) - z > 0$. For $p \geq \hat{p}$, $d^*(p; z) = z$, and the buyer's surplus from trade is $u(z/p) - z$, which is a decreasing function in p. Given that $u(z/\hat{p}) - z > 0$ and $u(z/p) - z$ becomes a negative number as p approaches infinity, by intermediate value theorem there must exists p^R such that $u(z/p^R) - z = 0$ and $p^R > \hat{p}$. It is straightforward to check that $\partial p^R/\partial z > 0$.

Finally, consider the case of $\sigma = 1$, and the utility function has the log form. Because the price elasticity of demand is equal to one for log utility, the buyer's unconstrained optimal expenditure $\tilde{d}$, which is determined by the first order condition $u'(\tilde{d}/p)/p = 1$, does not depend on the price level. If $\tilde{d} \leq z$, buyer spends $\tilde{d}$, and he can only spend z if $\tilde{d} > z$. Hence, $d^*(p; z) = \min\{\tilde{d}, z\}$. Because $\min\{\tilde{d}, z\}$ does not depend on p, the buyer's surplus $u(\min\{\tilde{d}, z\}/p) - \min\{\tilde{d}, z\}$ decreases in p. Therefore, there exists p^R such that $u(\min\{\tilde{d}, z\}/p^R) - \min\{\tilde{d}, z\} = 0$, and buyer does not want to spend for $p > p^R$. ■

Proof of Lemma 2.

In this proof, I use $\hat{z}^* > 0$ to denote the buyer's optimal choice of real balance. First, we consider the case of $\sigma < 1$, and assume that $\hat{p} \leq \underline{p}$ at $\hat{z}^*$. We first consider the situation in which $\hat{p} < \underline{p}$. If $\hat{p} < \underline{p}$, $d^*(p;\hat{z}) = d^*(p)$ for all $p \in [\underline{p}, \bar{p}]$. We plug it into (10) and omit all the terms unrelated to $\hat{z}$ and $\hat{\alpha}$, and the buyer's optimization problem can be rewritten as

$$L = \max_{\hat{z},\hat{\alpha}} \left\{ -i\hat{z} + \int_{\underline{p}}^{\bar{p}} \left[u\left(\frac{d^*(p)}{p}\right) - d^*(p) \right] dG(p;\hat{\alpha}) - \hat{\alpha}k \right\}, \tag{18}$$

where $i = \frac{1+\gamma-\beta}{\beta}$ is the nominal interest rate. The first order condition with respect to $\hat{z}$ evaluated at $\hat{z}^*$ is $\partial L/\partial \hat{z}^* = -i < 0$, which is a contradiction to $\hat{z}^*$ being the optimal real balance. Then, we consider the situation of $\hat{p} = \underline{p}$. Recall that $\hat{p}$ is determined by $\hat{z}$ through $u'(\hat{z}/\hat{p}) = \hat{p}$, so $\hat{z}^*$ satisfies $\hat{p}(\hat{z}^*) = \underline{p}$. We want to solve for $\partial L/\partial \hat{z}^*$. When $\hat{z}$ approaches $\hat{z}^*$ from below, $\hat{p}(\hat{z})$ approaches $\underline{p}$ from above since $\partial \hat{p}/\partial \hat{z} < 0$, and we can rewrite (18) as

$$\begin{aligned} L &= \max_{\hat{z},\hat{\alpha}} \left\{ -i\hat{z} + \int_{\hat{p}}^{\bar{p}} \left[u\left(\frac{d^*(p)}{p}\right) - d^*(p) \right] dG(p;\hat{\alpha}) \right. \\ &\quad \left. + \int_{\underline{p}}^{\hat{p}} \left[u\left(\frac{\hat{z}}{p}\right) - \hat{z} \right] dG(p;\hat{\alpha}) - \hat{\alpha}k \right\}, \end{aligned}$$

thus $\lim_{\hat{z}\to\hat{z}^*_-} \partial L/\partial \hat{z} = \lim_{\hat{z}\to\hat{z}^*_-} \{-i + \int_{\underline{p}}^{\hat{p}} \left[u'\left(\frac{\hat{z}}{p}\right)\frac{1}{p} - 1\right] dG(p;\hat{\alpha})\} = -i < 0$. On the other hand, when $\hat{z}$ approaches $\hat{z}^*$ from above, $\hat{p}(\hat{z})$ approaches $\underline{p}$ from below, and $\lim_{\hat{z}\to\hat{z}^*_+} \partial L/\partial \hat{z} = -i < 0$. However, the fact that $\hat{z}^*$ is the optimal real balance implies $\lim_{\hat{z}\to\hat{z}^*_-} \partial L/\partial \hat{z} \geq 0$ and $\lim_{\hat{z}\to\hat{z}^*_+} \partial L/\partial \hat{z} \leq 0$. This is a contradiction. Therefore, we must have $\hat{p} > \underline{p}$ when $\sigma < 1$.

Second, we consider the case of $\sigma > 1$, and assume that $\hat{p} \geq \bar{p}$ at $\hat{z}^*$. If $\hat{p} > \bar{p}$, according to Lemma 1, $d^*(p;\hat{z}) = d^*(p)$ for all $p \in [\underline{p}, \bar{p}]$. Thus, we can similarly simplify the buyer's optimization problem to (18) and arrive at a contradiction. If $\hat{p}(\hat{z}^*) = \bar{p}$, we can get $\lim_{\hat{z}\to\hat{z}^*_-} \partial L/\partial \hat{z} = \lim_{\hat{z}\to\hat{z}^*_-} \{-i + \int_{\hat{p}}^{\bar{p}} \left[u'\left(\frac{\hat{z}}{p}\right)\frac{1}{p} - 1\right] dG(p;\hat{\alpha})\} = -i < 0$, and $\lim_{\hat{z}\to\hat{z}^*_+} \partial L/\partial \hat{z} = -i < 0$ by applying $\partial \hat{p}/\partial \hat{z} > 0$. This is again a contradiction. Therefore, $\hat{p} < \bar{p}$ must be true when $\sigma > 1$. ■

Proof of Lemma 3. We first consider the case of $\sigma < 1$. If $\hat{\alpha}^* = 1$, $\hat{z}^*$ satisfies (11) and

$$\int_{\underline{p}}^{\bar{p}} \left[u\left(\frac{d^*(p;\hat{z}^*)}{p} \right) - d^*(p;\hat{z}^*) \right] (1 - 2F(p))\, dF(p) > k.$$

Substitute $\hat{\alpha}^* = 1$ into (11), and $\hat{z}^*$ is determined by

$$\int_{\underline{p}}^{\hat{p}} [u'\left(\frac{\hat{z}^*}{p} \right) \frac{1}{p} - 1](2 - 2F(p))dF(p) = i. \tag{19}$$

It is straightforward to check that $\int_{\underline{p}}^{\hat{p}} [u'(\hat{z}/p)/p - 1](2 - 2F(p))dF(p)$ is monotonically decreasing in $\hat{z}$, thus there is a unique $\hat{z}^*$ satisfying (19).

If $\hat{\alpha}^* = 0$, $\hat{z}^*$ satisfies (11) and

$$\int_{\underline{p}}^{\bar{p}} \left[u\left(\frac{d^*(p;\hat{z}^*)}{p} \right) - d^*(p;\hat{z}^*) \right] (1 - 2F(p))\, dF(p) < k.$$

In particular, $\hat{z}^*$ is determined by

$$\int_{\underline{p}}^{\hat{p}} \left[u'\left(\frac{\hat{z}^*}{p} \right) \frac{1}{p} - 1 \right] dF(p) = i.$$

The left hand side of the equation is again monotonically decreasing in $\hat{z}$, so $\hat{\alpha}^* = 0$ determines a unique $\hat{z}^*$.

If $\hat{\alpha}^* \in (0, 1)$, $\hat{z}^*$ is determined by

$$\int_{\underline{p}}^{\hat{p}} \left[u'\left(\frac{\hat{z}^*}{p} \right) \frac{1}{p} - 1 \right] [1 - \hat{\alpha}^* + 2\hat{\alpha}^*(1 - F(p))]\, dF(p) = i. \tag{20}$$

and again the left hand side of the equation is monotonically decreasing in $\hat{z}$. Hence, a unique $\hat{z}^*$ is determined by $\hat{\alpha}^*$. Now take $\hat{z}^*$ as given, and $\hat{\alpha}^*$ must also satisfy (20). I use $H(\hat{\alpha})$ to denote the left hand side of (20) as a function of $\hat{\alpha}$, and $\partial^2 H / \partial \hat{\alpha}^2 = 0$. Hence, $H(\hat{\alpha})$ is a monotone function in $\hat{\alpha}$, and there is a unique $\hat{\alpha}^*$ satisfying (20) given $\hat{z}^*$. Therefore, $\hat{z}^*$ and $\hat{\alpha}^*$ uniquely determine each other.

We can similarly prove the statement for $\sigma > 1$ by replacing (11) with (13) for $\hat{z}^*$. ■

Proof of Lemma 4.

First, consider $\sigma < 1$. Given the cutoff $\hat{p}$ in Lemma 1, there are two cases: $\bar{p} \leq \hat{p}$ or $\bar{p} > \hat{p}$. If $\bar{p} \leq \hat{p}$, $d^*(\bar{p}; z) = z$, and $\pi(\bar{p}) = z\,(1 - \alpha^*)\,(1 - c/\bar{p})$. Seller wants to choose a price as high as possible in the feasible range, and he posts $\bar{p} = \hat{p}$. In the other case of $\bar{p} > \hat{p}$, $d^*(\bar{p}; z) = d^*(\bar{p})$ satisfying $u'(d^*(\bar{p})/\bar{p}) = \bar{p}$. Seller wants to choose $\bar{p}$ such that

$$(1 - \frac{c}{\bar{p}})\frac{\partial d^*(\bar{p})}{\partial \bar{p}} + d^*(\bar{p})\frac{c}{\bar{p}^2} = 0, \tag{21}$$

which is the first order condition of the seller's profit maximization problem. We can derive $\partial d^*(\bar{p})/\partial \bar{p}$ from $u'(d^*(\bar{p})/\bar{p}) = \bar{p}$, and insert it into (21). Hence, if $\bar{p} > \hat{p}$, $\bar{p} = \tilde{p}$ where $\tilde{p}$ is given by

$$\frac{d^*(\tilde{p})}{\tilde{p}}u''(\frac{d^*(\tilde{p})}{\tilde{p}}) + \tilde{p} - c = 0.$$

Therefore, seller wants to post the upper limit $\bar{p} = \max\{\hat{p}, \tilde{p}\}$.

Second, consider $\sigma > 1$. There are again two cases: $\bar{p} \leq \hat{p}$ or $\bar{p} > \hat{p}$. If $\bar{p} \leq \hat{p}$, $d^*(\bar{p}; z) = d^*(\bar{p})$, and seller wants to post $\bar{p} = \tilde{p}$, which is given by (21). His profit is then $\pi(\tilde{p}) = d^*(\tilde{p})\,(1 - \alpha^*)\,(1 - c/\tilde{p})$. If $\bar{p} > \hat{p}$, $d^*(\bar{p}; z) = z$, and seller wants to post a price as high as possible, i.e. $\bar{p} = p^R$. He gets profit $\pi(p^R) = z\,(1 - \alpha^*)\,(1 - c/p^R)$. We compare $\pi(\tilde{p})$ with $\pi(p^R)$, and notice that $d^*(\tilde{p}) \leq z$ and $(1 - c/\tilde{p}) < (1 - c/p^R)$. Therefore, $\pi(\tilde{p}) < \pi(p^R)$, and seller chooses to post $\bar{p} = p^R$.

Finally, if $\sigma = 1$, $d^*(p; z) = \min\{\tilde{d}, z\}$, which does not depend on p. The seller's profit function simply is

$$\pi(\bar{p}) = (1 - \alpha^*)\left(\min\{\tilde{d}, z\} - c\frac{\min\{\tilde{d}, z\}}{\bar{p}}\right),$$

and $\partial \pi(\bar{p})/\partial \bar{p} > 0$. Therefore, the seller posts $\bar{p} = p^R$. ■

Proof of Lemma 5.

If $\alpha^* = 0$, the seller's profit function is

$$\pi(p) = d^*(p; z^*) - c\frac{d^*(p; z^*)}{p}. \tag{22}$$

According to Lemma 4, there is a unique price that maximizes $\pi(p)$, so every seller posts $\bar{p}$. If a seller deviates by posting $\bar{p}' = \bar{p} + \varepsilon$, where $\varepsilon > 0$, his profit decreases since $\bar{p}$ maximizes (22) and his trade volume stays the same. Similarly, if a seller deviates to $\bar{p} - \varepsilon$, his profit drops without an increase in trade volume. Therefore, there is no incentive for any seller to deviate away from $\bar{p}$.

If $\alpha^* = 1$, it is clearly an equilibrium that every seller posts $p = c$. There is no incentive to post a price lower than c, since that yields a negative profit. On the other hand, if a seller deviates and posts $c+\varepsilon$, his profit $\pi(c+\varepsilon)$ is equal to zero since $F(c+\varepsilon) = 1$, and he loses all the buyers. Next, I want to argue that this is the only equilibrium of the seller's price posting game. If there is another $F(p)$ concentrated at $p' > c$, a seller has incentive to lower the price that he posted by ε, i.e. he wants to post $p' - \varepsilon$. In this way, he can trade with a buyer for sure even though his profit from the trade decreases a little. A discrete jump in the trading probability makes up for the infinitesimal drop of the profit, and the seller's expected profit increases. Hence, there is a profitable deviation and another degenerate $F(p)$ does not exist. If there is another nondegenerate $F(p)$, its support $\mathcal{Z}_F$ is connected. This conclusion follows directly from Lemma 1 in Burdett and Judd (1983). $\pi(p)$ must be the same for all $p \in \mathcal{Z}_F$, and in particular $\pi(p) = \pi(\bar{p}) = (2 - 2F(\bar{p}))\,(d^*(\bar{p}; z^*) - cd^*(\bar{p}; z^*)/\bar{p}) = 0$ since $F(\bar{p}) = 1$. However, for any p such that $F(p) \in (0, 1)$, $\pi(p) = (2 - 2F(p))\,(d^*(p; z^*) - cd^*(p; z^*)/p) > 0$. This is a contradiction. Therefore, $F(p)$ concentrated at c is the unique equilibrium price distribution in the seller's price posting game.

If $\alpha^* \in (0, 1)$, any $F(p)$ concentrated at $p \in [\underline{p}, \bar{p}]$ cannot be a price posting equilibrium distribution. On the one hand, seller can always increase the price, hence increase the profit, while still keeping those buyers who only sample his price. On the other hand, seller can also lower his price infinitesimally, and get a jump in the trading probability. Hence, $F(p)$ must be nondegenerate if $\alpha^* \in (0, 1)$. Again from Lemma 1 in Burdett and Judd (1983), we know $F(p)$ is continuous with connected support. For any $p \in [\underline{p}, \bar{p}]$, we must have $\pi(p) = \pi(\bar{p})$, which implies

$$\begin{aligned} & [1 - \alpha^* + 2\alpha^*(1 - F(p))] \left(d^*(p; z^*) - c\frac{d^*(p; z^*)}{p} \right) \\ = \quad & (1 - \alpha^*) \left(d^*(\bar{p}; z^*) - c\frac{d^*(\bar{p}; z^*)}{\bar{p}} \right). \end{aligned}$$

The above equation determines a unique $F(p)$ for each p. In particular, $\pi(\underline{p}) = \pi(\bar{p})$ determines $\underline{p}$, which satisfies

$$(1 + \alpha^*)\, d^*(\underline{p}; z^*)(1 - \frac{c}{\underline{p}}) = (1 - \alpha^*)\, d^*(\bar{p}; z^*)(1 - \frac{c}{\bar{p}}).$$

■

Proof of Proposition 1.

First, we consider the case of $\alpha^* = 0$. Suppose an SSME exists for an economy with $1+\gamma > \beta$. Because $\alpha^* = 0$, the equilibrium price distribution in the decentralized market, $F^*(p)$ must be concentrated at $\bar{p}$.

In the case of $\sigma < 1$, $\bar{p} = \max\{\hat{p}, \tilde{p}\}$. If $\tilde{p} \geq \hat{p}$, $d^*(\bar{p}; z^*) = d^*(\tilde{p})$. The buyer's optimal real balance z^* maximizes

$$L = -(1+\gamma-\beta)z + \beta\left[u\left(\frac{d^*(\tilde{p})}{\tilde{p}}\right) - d^*(\tilde{p})\right].$$

Immediately, we have $\partial L/\partial z^* = -(1+\gamma-\beta) < 0$, and $z^* = 0$. This contradicts the existence of a monetary equilibrium. If $\hat{p} > \tilde{p}$, $d^*(\bar{p}; z^*) = z^*$, and z^* maximizes

$$L = -(1+\gamma-\beta)z + \beta\left[u\left(\frac{z}{\hat{p}}\right) - z\right].$$

Then, $\partial L/\partial z^* = -(1+\gamma-\beta) + \beta[u'(z^*/\hat{p})/\hat{p} - 1] = -(1+\gamma-\beta) < 0$, since $u'(z^*/\hat{p}) = \hat{p}$ by Lemma 1. Hence, $z^* = 0$ and it is again a contradiction.

In the case of $\sigma \geq 1$, $\bar{p} = p^R$. The buyer's optimal real balance z^* maximizes

$$L = -(1+\gamma-\beta)z + \beta\left[u\left(\frac{d^*(p^R; z)}{p^R}\right) - d^*(p^R; z)\right],$$

which can be simplified to $L = -(1+\gamma-\beta)z$ since $u(d^*(p^R; z)/p^R) - d^*(p^R; z) = 0$ by the definition of p^R in Lemma 1. Then, $\partial L/\partial z^* = -(1+\gamma-\beta) < 0$. Therefore, there exists no SSME with $\alpha^* = 0$.

Second, we consider the case of $\alpha^* = 1$, and assume an SSME exists. According to Lemma 5, $F^*(p)$ is concentrated at c. Plugging $d^*(c; z^*)$ and $F^*(c) = 1$ into (6) and (7), we can get

$$V^b(z^*; 1) = u\left(\frac{d(c; z^*)}{c}\right) - d(c; z^*) + W^b(z^*)$$

and

$$V^b(z^*; 2) = u\left(\frac{d(c; z^*)}{c}\right) - d(c; z^*) + W^b(z^*).$$

Notice that $V^b(z^*;1) = V^b(z^*;2)$.

Insert $V^b(z^*;1)$ and $V^b(z^*;2)$ back into (8), and drop unrelated terms. The buyer's optimal price sampling strategy α^* should maximize

$$L = -(1+\gamma-\beta)z^* + \beta\left[u\left(\frac{d(c;z^*)}{c}\right) - d(c;z^*)\right] - \beta\alpha k.$$

It is then obvious that $\partial L/\partial\alpha = -\beta k < 0$, and buyer should choose $\alpha^* = 0$. This is a contradiction to our assumption $\alpha^* = 1$. Therefore, there does not exist an SSME with $\alpha^* = 1$. ■