Dynamic Legislative Bargaining with Endogenous Proposers

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Abstract

We present a theory of dynamic legislative bargaining in which (1) the policy made in one period becomes the status quo for the next, and (2) every proposer is endogenously determined through an all-pay auction. We fully characterize the stationary Markov perfect equilibrium for a model with three parties, a one-dimensional policy space, single-peaked preferences and symmetric distribution of the ideal points. We show that the median party never participates actively in the contest for proposal power. Thus the model predicts that key positions with agenda control would not be occupied by politicians with moderate ideological views. We also show that the two extreme parties as proposer would propose more moderate policy than they would otherwise do in a single-period setup. This is due to the incentive to alleviate future competition costs. Overall the long-run policy choice is bounded away from the median policy provided the players are sufficiently impatient. But the generalized median voter theorem of Baron (1996) still holds if the parties are sufficiently patient.

JEL Classification Codes: C72, D72, D78.

Keywords: Legislative bargaining, all-pay contest, status quo, proposal power, median voter theorem, dynamic games, Markov perfect equilibrium.

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1 Introduction

Consider a committee with an odd number of members. The committee has to make a one-dimensional policy choice by majority rule. Each member has single-peaked policy preferences. What would be the policy choice? This is a classic question of collective decision-making.

In the median voter theorem, Black (1948) suggests that a reasonable prediction be the ideal point of the median committee member, as this is the unique policy that is unbeatable in any pairwise vote against all the other feasible policies. Romer and Rosenthal (1978), however, argue that the median policy may not be selected in practice and propose the agenda-setting model. In this model, an agenda setter, or proposer first selects a policy proposal and then all committee members vote. If a majority of the committee approves the proposal then it is chosen. Otherwise, the status quo policy, i.e., the policy that has been enacted, remains in place. In such model the agenda setter can generally bias the policy proposal toward his ideal point. Romer and Rosenthal (1979) explore data of referenda with respect to school-district budget and find evidence consistent with implications of the agenda-setting model.

Baron (1996) argues that the theory of Romer and Rosenthal (1978, 1979) may apply to some policies that are made once and for all, yet the possibility of reconsidering an approved policy may bring the median voter theorem alive. With repeated policy making, if the policy chosen in one period becomes the status quo in the next period, and in every period the agenda setter is randomly recognized from the entire committee, in the long-run the policy converges to the ideal point of the median committee member. The reason is simple: with random recognition, sooner or later the median voter would be recognized as proposer and since then the policy would move to and then stabilize at the median policy.

There is, however, a missing link in all the above theories and their variations. It has been commonly agreed that proposal power is highly valuable, as the agenda setter, whoever he is, can bias the policy towards his ideal policy.[1] So the committee members should have incentives to expend costly efforts to fight for such power. With endogenous rent-seeking behaviors, the agenda setter in such thought experiment may no longer be "randomly" recognized, as is commonly assumed in the literature. How would rent-seeking incentives and endogenous proposal power shape the one-dimensional policy choice in a legislative setup? This is one of the first key question we address in this paper.

Moreover, the status quo policy normally determines reservation values of the committee members and therefore shapes coalition formation as well as the policy choice. So a related, second question is: how would the status quo policy shape individual players' rent-seeking

[1] Knight (2005) offers evidence from field data in the U.S. Senate.

incentives? Who will fight harder for proposal power: the players with radical ideological positions or those with moderate views?

To answer these questions, we present a dynamic legislative bargaining model with three features. First, the policy choice in each period is made through the sequence of proposal making and voting. Second, the policy made in one period becomes the status quo in the next period. Third, the agenda setter in each period is endogenously determined through an all-pay contest, in which all players simultaneously expend efforts to bid for proposal power and the highest bidder wins the power. We develop methodology to analyze such dynamic games that accommodate all-pay contests, agenda-setting and committee voting.

Although we restrict attention only to models with three players, our model with endogenous proposal power implies rich policy dynamics and leads to over-looked implications. First, the median player never participates actively in the competition for proposal power. So we predict that key positions with proposal power would never be occupied by legislators with moderate ideological views. Second, the extreme players as agenda setter choose more moderate policy proposal than necessary. This is due to the incentive to alleviate future competition costs and the incentive to constrain the opposite extreme player who may be recognized as agenda setter in future periods. Third, with a sufficiently extreme initial status quo, over time the policy moves gradually toward the median policy. However, whether the generalized median voter theorem holds depends on the patience of the players. If the players are sufficiently impatient, the long-run policy choice would be bounded away from the median policy. But the generalized median voter theorem holds with sufficiently patient players.

This paper aims to bridge the legislative bargaining literature and the rent-seeking literature. The legislative bargaining literature was rooted in Romer and Rosenthal (1978, 1979) and formally proposed by Baron and Ferejohn (1989). In a typical legislative bargaining model one member in the committee is randomly selected as the agenda setter who has the authority to select a policy proposal. The entire committee then votes on the proposal by some majority voting rule. Variations of such model have been developed according to the specific legislative procedures once a proposed policy is rejected. Baron (1996) extended this analytical framework to a dynamic game in which the policy made in one period would become the status quo in the next period. The assumption of a "moving" status quo captures a common feature of many policy issues in real life, such as tax rates, education subsidy, trade policy and social security benefits. These policies, once enacted, normally stay in effect until they are reformed in a later date. Following Baron (1996) various noticeable theoretical studies have explored incentives and policy dynamics that result from the moving status quo in various policy environments and institutional contexts, for example, Kalandrakis (2004), Bernheim et al. (2006), Battaglini and Coate (2007, 2008), Anesi (2010), Diermeier and Fong (2011) and Battaglini and Palfrey (2011), to name just a few. Those models, however, all assume that the agenda setter is either exogenously given or randomly chosen. None allows the agenda setter to be endogenously determined through a political contest for proposal power. The proposed research thus makes an important step to extend this literature and would incorporate contest models into dynamic legislative bargaining games.

The rent-seeking literature follows the public choice tradition and was initiated by Tullock. Among several contest models built to analyze rent-seeking behaviors, a noticeable and tractable analytical framework is the all-pay contest model that was developed by Hillman and Riley (1987) and Baye et al. (1993, 1996) and recently generalized by Siegel (2009). In these existing models it is commonly assumed that the contest takes place once and for all, and the contest prizes are either exogenously determined or reduced-form functions of the contestants' effort inputs. In the proposed research I make a significant step to incorporate a standard all-pay contest into an infinite-horizon dynamic games in which the contest takes place every period and the players' bidding strategies depend on the status quo policy.

Yildirim (2007) was the first, and so far the only, paper that incorporates rent-seeking into legislative bargaining models. He focuses on distributive politics and assumes that the agenda setter is endogenously determined by a lottery contest due to Tullock. The agenda setter then proposes a distribution of a fixed amount of resources among all player and then everybody votes. If the proposed distribution is approved by the vote, it is enacted and the game ends. Otherwise a new contest takes place, a new agenda setter is endogenously determined and the legislative procedure repeats until some proposed distribution is approved. Our paper is different from Yildirim's pioneer work in several dimensions. First, Yildirim assumes that the policy is made once and for all whereas we assume repeated policy choice over an infinite horizon with a moving status quo. Second, Yildirim focuses on distributive policies whereas here we analyze policy dynamics of a one-dimensional policy. Third, Yildirim does not explore the effects of the status quo on rent-seeking behaviors. He mainly focuses on how the discount factor and rent-seeking technology may affect legislators' rent-seeking decisions. Here we want to understand how the status quo position and players' ideological preferences shape their incentives and behaviors in the contest for political power.

Deng and Fong (2011) is an initial attempt by me to incorporate rent-seeking models into the analytical framework of legislative bargaining. In particular, that paper presents a model of distributive politics in which legislators first expend costly efforts to bid for proposal power and then the winner is recognized as the agenda setter. Two different formal models in the rent-seeking literature are assumed to model the contest for proposal power: the all-pay auction model due to Hillmann and Riley (1987) and Baye et al. (1996)

and the lottery contest model due to Tullock. The analysis indicates that legislators with large status quo allocations have strong incentives to protect themselves from expropriation so they compete more aggressively in the contest for proposal power than those legislators with small status quo allocations. Our paper is different from DF in two ways. First, in DF the policy is made once and for all whereas in the proposed research the policy is made repeatedly with an endogenously evolving status quo. Second, DF focuses on a distributive policy whereas here a one-dimensional policy is assumed.

The rest of the paper is organized as follows. Section 2 presents our model. Section 3 discusses the benchmark case with complete myopic players. Section 4 defines a stationary Markov perfect equilibrium for the dynamic game. Section 5 constructs and characterizes one stationary equilibrium in which the median player always quits the contest for proposal power. Section 6 analyzes patterns of policy dynamics and links our results to the median voter theorem. Section 7 decomposes an agenda setter's incentives into bargaining effect and waste-reduction effect. Section 8 concludes. All proofs and technical matters are relegated to the Appendix.

2 The Model

We present a dynamic legislative bargaining model with endogenous proposers. Consider a committee with three players: L (left), M (median), and R (right). The committee must repeatedly make a one-dimensional policy choice $x \in \mathbb{R}$. Per period policy preferences of each player i are represented by utility function

$$u_i(x) = -(x - a_i)^2,$$

where a_i denotes i's ideal point. In this paper we consider the symmetric case with symmetric distribution of the ideal points, and assume $a_L = -1$, $a_M = 0$ and $a_R = 1$. All players share a common discount factor $\delta \in [0, 1)$. We call player M the median player whereas L and R extreme players. We refer to $x = 0$ as the median policy.

In each period t the policy is made through the political process with two stages: first the recognition stage and second the legislation stage. In the recognition stage all players simultaneously expend costly efforts $e_i \geq 0$ to bid for proposal power, with effort cost given by $C(e_i) = e_i$. We assume an all-pay contest so the highest bidder is recognized as the agenda setter.[2] We say a player quits the contest for proposal power if that player bids

[2] In general, the recognition probability $p_i(e_L, e_M, e_R)$ for player i depends on the profile of effort inputs by all players in the recognition stage. In an all-pay contest, the recognition probability is given by

$$p_i(e_L, e_M, e_R) = \begin{cases} 1, & \text{if } e_i > \max_{j \neq i} e_j, \\ 0, & \text{otherwise,} \end{cases}$$

0 with probability one in the recognition stage. In the legislation stage the agenda setter selects a policy proposal $y^t \in \mathbb{R}$ and then every player votes. If the proposed policy is approved by a simple majority, then it becomes the policy choice and we write $x^t = y^t$. Otherwise the status quo policy, i.e. the policy chosen in the previous period, remains in effect and we write $x^t = x^{t-1}$. The period-one status quo policy x^0 is exogenously given.

Our model is connected to classic models in the literature of legislative decision-making. With $\delta = 0$, our model degenerates into the single-period agenda-setting model of Romer and Rosenthal (1978, 1979), in which the policy is made once and for all. If proposers are instead recognized randomly with probabilities $(\overline{p}_L, \overline{p}_M, \overline{p}_R)$ such that $\overline{p}_i > 0$ for all i and $\sum_i \overline{p}_i = 1$, then our model becomes the dynamic game of Baron (1996).

3 The Benchmark Case

We first analyze the benchmark case with completely myopic players, i.e. $\delta = 0$.

Proposition 1 Suppose $\delta = 0$. (A) In any subgame perfect equilibrium, $\widetilde{f}(x, R) = -\widetilde{f}(x, L) = \min\{|x|, 1\}$ and $\widetilde{f}(x, M) = 0$. (B) There exists a subgame perfect equilibrium in which, regardless of the status quo, the median player quits the contest for political power and the extreme players actively participate with identical bidding strategy. In such equilibrium, the median player is never recognized as agenda setter whereas the extreme player is recognized randomly with probability one-half.

In the legislation stage, player M, once recognized as agenda setter, would propose his ideal point. Player R as agenda setter would select his ideal point with unanimity support if the status quo is to the right of her ideal point; he would remain the status quo if the status quo is in between his ideal point and the median policy; he would propose the mirror-image policy of the status quo if the status quo is between the ideal points of the median player and player L; he would select his own ideal point with the support of the median player if the status quo is to the left of L's ideal point.

A noticeable feature of the equilibrium we present is that the median player always quits the contest for proposal power and therefore is never recognized as agenda setter in equilibrium. With the absence of the median player in the political competition, in the recognition stage the two extreme players adopt identical bidding strategy and therefore in equilibrium each would be randomly recognized as agenda setter with one-half probability.

To understand this, consider a status quo $x \in (0, 1)$. In this case, player R as agenda setter would propose to remain the status quo; player L as agenda setter would propose

provided a tie would occur with zero probability measure.

policy $-x$ with voting support from the voting support by the median player, who is indifferent between the proposed policy and the status quo; the median player as agenda setter would propose the median policy with voting support by L. Given that the median player does not participate in the contest for political power, player R's policy utility would be $u_R(x)$ if she wins in the all-pay contest and $u_R(-x)$ if she loses proposal power to L. Therefore the additional policy utility player R gains from winning proposal power is $\pi(x) = u_R(x) - u_R(-x) = 4x$. Similarly, the additional policy utility player L gains from winning proposal power is also $\pi(x)$. We can imagine that in the recognition stage the two extreme players participate in the all-pay contest as if they face a common contest prize $\pi(x)$ and therefore they adopt identical bidding strategy that is aggressive enough just to compete away all such prize. On the other hand, the median player's policy utility would be $\frac{1}{2}u_M(x) + \frac{1}{2}u_M(-x) = -x^2$ if he quits the contest. But if the median player ever participated actively in the contest and won proposal power, he could move the policy to 0 and enjoy a policy utility of $u_M(0) = 0$. So the additional policy utility the median player can ever gain from winning proposal power is x^2. Notice that $\pi(x) > x^2$. So the extreme players both have more stakes in the contest for proposal power than the median player. As a consequence the extreme players would compete aggressively in the all-pay contest so that it is never profitable for the median player to compete.

From the above argument, we can also see that with a more extreme status quo, the extreme players have more stakes in the contest for proposal power and therefore on average more efforts are expended and wasted in the political contest.

Although the proposition above only shows the special case with symmetric distribution of the status quo, we could easily show that, for any distribution of the ideal points, there exists an equilibrium in which the median player always quits the contest for proposal power for $\delta = 0$. In the Appendix we actually present the proof for this more general version.

Most existing models in the literature of dynamic legislative bargaining assume that every player has a positive chance to be recognized as proposer, such as Baron and Ferejohn (1989), Baron (1996), Kalandrakis (2004), Battaglini and Coate (2007, 2008), Anesi (2010), Battaglini and Palfrey (2011), and Duggan and Kalandrakis (2011), to name just a few.[3] Most insights about how policy dynamics is shaped by legislative institutions have been derived based on this assumption of random recognition. But here our analysis casts doubts on the relevance and validity of such common assumption. If the agenda setter is to be endogenously determined, the median player may not actively compete for proposal power and therefore may never become an agenda setter.

[3] Rare exceptions include Diermeier and Fong (2011) and Riboni (2008), who assume a persistent proposer.

4 Equilibrium Definition

We define a stationary Markov perfect equilibrium, in which the strategy of each player is stationary and only depends on the status quo policy, i.e., the only payoff-relevant state variable. This is the standard treatment for dynamic games that involve coalition formation, e.g. in Kalandrakis (2004), Battaglini and Coate (2007, 2008), Anesi (2010), Diermeier and Fong (2011) and Duggan and Kaladrakis (2011). From now on, we drop superscript t from the notation.

A (stationary) bidding strategy of player i in the recognition stage is a cumulative distribution function G_i, where $G_i(e;x)$ is the probability for player i's bid to be smaller than or equal to constant $e \geq 0$ in a recognition stage with status quo x. For any bidding strategy profile $G = (G_L, G_M, G_R)$, let $P_j(e_i;x)$ be the probability that player j wins the all-pay contest in a recognition period with status quo x given that player i bids $e_i \geq 0$ and all the other players bid according to strategy profile G. Then for any i, $\sum_{j=1}^{3} P_j(e_i;x) = 1$, and

$$P_i(e_i;x) = \prod_{j \neq i} G_j(e_i;x). \tag{1}$$

For any distinct i, j,

$$P_j(e_i;x) = \int_{e_i}^{\infty} G_k(e_j;x)\, dG_j(e_j;x). \tag{2}$$

Let $f(x,i)$ be the (stationary) policy rule that specifies the transition of policy in any legislation stage with status quo x and proposer i. Let $U_i(x)$ denote player i's discounted sum of expected utility evaluated in any legislation stage in which policy x is enacted and call it the (dynamic) payoff function. Then

$$U_i(x) = (1-\delta)\, u_i(x) + \delta \left[\int_0^{\infty} \left(\sum_j P_j(e_i;x)\, U_i(f(x,j)) - e_i \right) dG_i(e_i;x) \right]. \tag{3}$$

In equilibrium, bidding strategies (G_L, G_M, G_R) must be best responses to each other in any contest stage. With status quo x and given the strategy profile of the others, each player i actually solves

$$\max_{e_i \geq 0} \left[\sum_j P_j(e_i, G_{-i};x)\, U_i(f(x,j)) \right] - e_i. \tag{4}$$

Moreover, policy rule f must reflect the underlying payoff maximization problem of each proposer in the legislation stage. As is standard in theory of committee voting, we assume that every player votes for the proposal whenever he is indifferent between the proposal and the status quo. Any proposer thus has incentive to select his proposal from the policies that can obtain voting support from one other player. This is because proposing a policy that is destined to be vetoed is equivalent to proposing to remain the status quo. Therefore, in any legislation stage with status quo x, $f(x, i)$ solves

$$\begin{aligned} &\max_{y \in \mathbf{R}} \quad U_i(y) \\ &s.t. \quad U_j(y) \geq U_j(x) \text{ for some } j \neq i. \end{aligned} \tag{5}$$

Now we summarize the equilibrium definition.

Definition 1 A stationary Markov perfect equilibrium is a policy rule f, a bidding strategy profile G, and a set of dynamic payoff functions (U_L, U_M, U_R), such that:

1. Given f and G, U_i satisfies equations (3) for all i.

2. Given (U_L, U_M, U_R), $f(x, i)$ solves maximization problem (5) for all i and all x.

3. Given f, G_{-i} and (U_L, U_M, U_R), $G_i(e_i^*, x)$ is increasing at e_i^* only if e_i^* solves maximization problem (4).

We say a policy rule f is continuous if $f(x, i)$ is continuous in status quo x. In other words, with a continuous policy rule, a small change in the status quo would not lead to a discrete jump of the policy choice. Similarly, we say a bidding strategy G_i is continuous if $G_i(e; x)$ is continuous in status quo x.

5 Analysis

In this section we characterize an equilibrium for the dynamic game with endogenous agenda setters. We have not yet proved the uniqueness of this equilibrium, although we are not able to find any other equilibrium. Given any bidding profile G, the recognition probability of player i is calculated as

$$p_i^*(x) = \int_0^\infty P_i(e_i; x)\, dG_i(e_i; x). \tag{6}$$

The next proposition characterizes the bidding strategy, recognition probability and policy rule in equilibrium

Proposition 2 There exists an equilibrium in which:

1. The median player quits the contest for political power regardless of the status quo and therefore is never recognized as agenda setter.

2. Both L and R adopt identical bidding strategies and in each period each of them is recognized randomly with probability one-half. In particular,

$$G_L(e;x) = G_R(e;x) = \begin{cases} \dfrac{e}{\pi(x)}, & \text{if } e \in [0, \pi(x)], \\ 1, & \text{if } e > \pi(x), \end{cases}$$

where

$$\pi(x) = U_L(f(x, L)) - U_L(f(x, R)) = U_R(f(x, R)) - U_R(f(x, L)),$$

and $p_L^*(x) = p_R^*(x) = \frac{1}{2}$ for any status quo x.

3. In case player M is recognized as agenda setter, which occurs only off the equilibrium path, the policy transitions immediately to the median policy.

4. For any status quo sufficiently close to the median, the policy rule of each extreme player is identical to that in the benchmark case. In particular, for any $x \in S \equiv [\overline{s}_L, \overline{s}_R]$, where $\overline{s}_R = -\overline{s}_L = s^* \equiv \max\{1 - 2\delta, 0\}$, $f(x, R) = -f(x, L) = |x|$.

5. For any sufficiently extreme status quo, any extreme player as agenda setter would compromise in proposal making and select a proposal that is more central than necessary. In other words, $f(x, R) < |x|$ and $|f(x, L)| < |x|$ for any status quo $x \notin S$. Specifically,

$$f(x, R) = -f(x, L) = \begin{cases} z_R^* - \dfrac{(|x| - 1)^2}{4\delta}, & |x| \in (\overline{s}_R, 1), \\ z_R^*, & |x| \in [1, \infty), \end{cases}$$

where

$$z_R^* = \begin{cases} 1 - \delta, & \delta \in \left[0, \frac{1}{2}\right], \\ \frac{1}{4\delta}, & \delta \in \left(\frac{1}{2}, 1\right), \end{cases}$$

is the policy choice closest to R's ideal point, among all policies that R would ever propose.

Existence of an equilibrium is established by construction and the proof is presented in the Appendix. The equilibrium we construct has the following properties.

First of all, like what happens in the benchmark model, the median player always quits the political contest and therefore would never control proposal power. Consider a recognition stage with some status quo $x \neq 0$. Given that the median player has no chance to propose, L's payoff would be $U_L(f(x, L))$ if he wins the contest and $U_L(f(x, R))$ if he loses to R. Therefore $\pi(x) = U_L(f(x, L)) - U_L(f(x, R))$ is the additional payoff L

gains upon winning the political contest. We thus refer to $\pi(x)$ as L's contest prize. Due to symmetry R's contest prize is also $\pi(x)$. If the median player quits the contest as he does in equilibrium, his payoff would be $\frac{1}{2}U_M(f(x,L)) + \frac{1}{2}U_M(f(x,R))$ (which equals $U_M(f(x,L))$). If the median player ever deviates to participate actively in the contest and wins, his payoff would be $U_M(0) = 0$. Since $f(x,L) < 0 < f(x,R) = -f(x,L)$, this potential contest prize for the median player is $|U_m(f(x,\ell))| < \pi(x)$. This means that the extreme players have more stakes in the contest for proposal power. As a consequence the extreme players would compete aggressively in the all-pay contest so that it is never profitable for the median player to compete. If the median player actively competes but only extends little effort, his recognition probability would be so small that his effort input would most likely be wasted. On the other hand, since the median player's potential contest prize is smaller than the extreme players', it is also not profitable for the median player to compete aggressively. These considerations explain the absence of the median player in the political contest.

This theoretical finding implies that, key political positions that come with agenda-setting power, should be most likely taken by politicians with radical ideological positions, as those with moderate views tend not to compete seriously for power.

The second noticeable feature of the equilibrium is that each extreme player as agenda setter would make a compromise and select a proposal that is more central than necessary if the status quo is sufficiently extreme. For instance, suppose the status quo is $a_R = 1$. In this case player R as agenda setter can obviously propose to remain the status quo since it is already her ideal point. However, in equilibrium player R instead selects a proposal that is closer to the median policy, i.e., $f(1,R) < 1$.

Such behavior results from two incentives. First of all, R knows that in the subsequent period with positive probability the other extreme player, L, would be recognized as agenda setter and could move the policy to the left-hand side of the median policy. By strategically positioning the policy closer to the median, R can indirectly "tie the hands of its successor" through the status quo, since L as future agenda setter will be constrained to select a more moderate left policy if he faces a more moderate status quo. Call such incentive the bargaining effect. Second, notice that the extreme players would have smaller stakes in the contest for proposal power if the status quo is closer to the median policy. Therefore a more central policy choice in one period can effectively commit the extreme players to less aggressive bidding strategies in the next period and therefore reduce waste in effort costs. Call such incentive the waste-reduction effect. Overall both bargaining effect and waste-reduction effect work in the same direction and induce an extreme player to choose a more central policy than necessary. Whereas the bargaining effect has been extensively analyzed in the literature, e.g., by Baron (1996), the waste-reduction effect has been entirely overlooked, since this can only be identified in a full-fledge dynamic model of legislative bargaining with endogenous agenda setters.

Finally, the proposition also implies that the extreme players make more compromise and propose policies closer to the median if they have a higher discount factor.

6 Policy Dynamics and the Median Voter Theorem

We introduce some useful notations to facilitate precise description of the policy dynamics in equilibrium. Let $\sigma(x, y)$ denote the equilibrium probability for policy to transition to y from status quo x. By definition, $\sigma(x, f(x, i)) = p_i^*$ for all i and $\sigma(x, y) = 0$ for all $y \notin \bigcup_i f(x, i)$. Any policy x is a steady state if $\sigma(x, x) = 1$; i.e., it persists with probability one. Proposition 2 indicates that the median policy is the unique steady state regardless of the discount factor. Let $S \subset \mathbb{R}$ denote the ergodic set in equilibrium. Then for any policy $x \in S$, $\sigma(x, y) > 0$ only if $y \in S$. In other words, no policy in the ergodic set will ever transition out of the set. By Proposition 2, the ergodic set is nonempty regardless of the discount factor since it always includes the median policy. Finally, let $\{x^t\}_t^\infty$ be any sequence of policies such that $\sigma(x^{t-1}, x^t) > 0$ for all $t \in \mathbb{N}$. This policy path traces the evolution of policy along some equilibrium path starting from an initial status quo x^0. We are ready to present patterns of policy dynamics in equilibrium. The presentation is divided into two parts, depending on values of the discount factor.

Proposition 3 Consider the equilibrium presented in Proposition 2.

1. For any $\delta \in [0, \frac{1}{2})$, any initial status quo $x^0 \in S/\{0\}$ and any equilibrium policy path $\{x^t\}_0^\infty$, $|x^t| = |x^0|$. In words, if the players are sufficiently impatient and the initial status quo x^0 is sufficiently close to the median position, then the policy oscillates between x^0 and $-x^0$ depending on which extreme player is recognized as agenda setter.

2. For any $\delta \in [0, \frac{1}{2})$, any $x^0 \notin S$, $\lim_{t\to\infty} |x^t| = s^*$ and $s^* < |x^{t-1}| < |x^t|$ for all $t \in \mathbb{N}$. In words, if the players are sufficiently impatient and the initial status quo is sufficiently extreme, then the policy gradually moves towards more central positions although the long-run policy is bounded away from the median.

3. For any discount factor $\delta \in [\frac{1}{2}, 1)$, any initial status quo $x^0 \neq 0$, and any equilibrium policy path $\{x^t\}_0^\infty$, $\lim_{t\to\infty} x^t = 0$ and $0 < |x^{t-1}| < |x^t|$ for all $t \in \mathbb{N}$. In words, unless the initial status quo is the median policy, regardless of the initial status quo the policy converges to the median policy, provided the players are sufficiently patient.

This proposition is directly implied by Proposition 2.

In a dynamic model of legislative bargaining with a moving status quo and random proposers, Baron (1996) proves a generalized median voter theorem: regardless of the discount factor and the initial status quo, the policy converges to the ideal point of the median player. The reason is simple: with random recognition, sooner or later the median player is recognized as proposer and since then the policy moves to and stays at the median policy.

In our model with endogenous proposers, the median player never actively participates in the contest for proposal power. As a consequence the median player is never recognized as proposer. Therefore, the proposer is always one of the extreme players and the median policy is never proposed and chosen unless it is the initial status quo.

For any discount factor sufficiently small, the generalized median voter theorem completely breaks down, since the extreme players as proposer, not farsighted enough, do not have strong enough incentives to make substantial compromise in their policy proposals. As a consequence the long-run policy is bounded away from the median although over time the policy choice moves towards more central positions.

On the other hand, the policy would still converge to the median, although it never reaches there, if the discount factor is sufficiently high. With sufficient patience, any extreme player as proposer has strong enough incentive to reduce intensity of future political contest and to constrain the opposite extreme player as future proposer.

7 Incentives

In this section we compare the equilibrium policy rules in various models to identify the incentives of an agenda setter. Due to symmetry, it suffices to discuss the policy rule of player R.

The difference between the policy rules in the dynamic model with endogenous agenda setters the benchmark model indicates the overall effect of intertemporal trade-offs. This effect can be decomposed into two parts. As an extreme player as agenda setter selects a policy proposal more central than necessary, he sacrifices his contemporaneous policy utility whereas reduces the intensity of future competition for proposal power (the waste-reduction effect) and also constrains the opposite extreme player as future agenda setter (the bargaining effect).

In order to identify the two effects, we consider an alternative dynamic model in which recognition probabilities are exogenously given by $p_L^*(x) = p_R^*(x) = \frac{1}{2}$ and $p_M^*(x) = 0$ for all status quo x. We call this the auxiliary model. The next proposition characterizes the unique stationary Markov perfect equilibrium in the auxiliary model.

Proposition 4 In the unique stationary Markov perfect equilibrium of the auxiliary model, $\widehat{f}(x, M) = 0$, and

$$\widehat{f}(x, R) = -\widehat{f}(x, L) = \begin{cases} |x|, & |x| \in [0, \widehat{s}], \\ \widehat{z}_R - \frac{(|x|-1)^2}{2\delta}, & |x| \in (\widehat{s}, 1), \\ \widehat{z}_R, & |x| \in [1, \infty), \end{cases}$$

where $\widehat{z}_R = 1 - \frac{1}{2}\delta$ and $\widehat{s} = 1 - \delta$. For any initial status quo $x^0 \in [-\widehat{s}, \widehat{s}] / \{0\}$ and any equilibrium policy path, $\left|x^t\right| = \left|x^0\right|$. For any $x^0 \notin [-\widehat{s}, \widehat{s}]$, $\lim_{t\to\infty} \left|x^t\right| = \widehat{s}$ and $\widehat{s} < \left|x^{t-1}\right| < \left|x^t\right|$ for all $t \in \mathbb{N}$. In words, if the initial status quo is sufficiently extreme, the policy gradually moves towards the median although it is bounded away from the median.

The extreme players in the auxiliary model have no incentive to reduce intensity of future competition, as recognition is purely random with exogenous probabilities, whereas they would still have incentive to constrain future proposers. Therefore the difference between policy rules of the single-period model and the auxiliary model identifies solely the bargaining effect, whereas the difference between policy rules of the auxiliary model and our model with endogenous proposers identifies solely the waste-reduction effect. Formally we define the bargaining effect by $\widetilde{f}(x, R) - \widehat{f}(x, R)$ and waste-reduction effect by $\widehat{f}(x, R) - f(x, R)$. The next proposition shows comparative statics of these effects.

Proposition 5 There is no bargaining effect and no waste-reduction effect if the status quo is sufficiently central, i.e., $x \in S$. Both bargaining effect and waste-reduction effect are stronger if the discount factor is higher, provided the status quo is sufficiently extreme, i.e. $x \notin S$. A more extreme status quo implies stronger bargaining effect and waste-reduction effect, provided the status quo $x \in [-1, 1] / S$.

A noticeable feature of the auxiliary model is that the policy never converges to the median policy unless the initial status quo is already there. Therefore, policy convergence to medium in the dynamic model with endogenous agenda setters for high discount factors is due to the waste-reduction effect. Without the incentive to reduce competition waste in future periods no extreme player would ever make sufficient compromise so that policy would always be bounded away from the median unless the initial status quo is already close to the median.

8 Concluding Remarks

We present a theory of dynamic legislative bargaining in which (1) the policy made in one period becomes the status quo for the next, and (2) every proposer is endogenously determined through an all-pay auction. We fully characterize the stationary Markov perfect equilibrium for a model with three parties, a one-dimensional policy space and single-peaked preferences. We show that the median party never participates actively in the contest for proposal power. Thus the model predicts that key positions with agenda control would not be occupied by politicians with moderate ideological views. We also show that the two extreme parties as proposer would propose more moderate policy than they would otherwise do in a single-period setup. This is due to the incentive to alleviate future competition costs. Overall the long-run policy choice is bounded away from the median policy provided the players are sufficiently impatient. But the generalized median voter theorem of Baron (1996) still holds if the parties are sufficiently patient.

Appendix

In this Appendix we present proofs to Propositions 1, 2, 4 and 5.

Proof of Proposition 1. This proposition is a special case of a more general statement that applies for any configuration of the ideal points such that $a_M = 0$, $a_R = 1$ and $a_L \in [-1, 0)$. (A) In any subgame perfect equilibrium, $\widetilde{f}(x, M) = 0$, $\widetilde{f}(x, R) = \min\{|x|, 1\}$, and $\widetilde{f}(x, L) = -\min\{|x|, -a_L\}$. (B) There exists a subgame perfect equilibrium in which $\widetilde{G}_M(0; x) = 1$,

$$\widetilde{G}_L(e; x) = \begin{cases} \dfrac{\widetilde{\pi}_R(x) - \widetilde{\pi}_L(x) + e}{\widetilde{\pi}_R(x)}, & \text{if } e \in [0, \widetilde{\pi}_L(x)], \\ 1, & \text{if } e > \widetilde{\pi}_L(x), \end{cases}$$

and

$$\widetilde{G}_R(e; x) = \begin{cases} \dfrac{e}{\widetilde{\pi}_L(x)}, & \text{if } e \in [0, \widetilde{\pi}_L(x)], \\ 1, & \text{if } e > \widetilde{\pi}_L(x), \end{cases}$$

where $\widetilde{\pi}_L(x) = u_L\left(\widetilde{f}(x, L)\right) - u_L\left(\widetilde{f}(x, R)\right)$ and $\widetilde{\pi}_R(x) = u_R\left(\widetilde{f}(x, R)\right) - u_R\left(\widetilde{f}(x, L)\right)$.

We proof it by backward induction.

Part 1. We first analyze the legislation stage. If player L becomes the proposer, he at least need the median player's vote in order to get his proposal passed, which implies that the median player should be indifferent with the status quo and the policy proposal. Similar argument applies if player R becomes the proposer. If player M becomes the proposer, he always can propose his ideal policy as one of the extreme players would vote for it.

Part 2. We check there is profitable deviation for each player in contest stage given $\widetilde{G}$.

Given $\widetilde{G}_M(e; x) = 1$, if $e \geq 0$, the contest degrades into a two-player all-pay auction with exogenous prize, where player R's prize is equal to $\widetilde{\pi}_R(x)$ and the player L's $\widetilde{\pi}_L(x)$. Therefore, there are no profitable deviations for both player L and R.

Given $\widetilde{G}_L(e; x)$ and $\widetilde{G}_R(e; x)$, we verify that there is also no profitable deviation for the player M. It is equivalent to prove that any positive effort input is strictly dominated by zero effort for player M given $\widetilde{G}_L(e; x)$ and $\widetilde{G}_R(e; x)$. The expected payoff of player M with no effort given $\widetilde{G}_{-M}$ is

$$u_M\left(0, \widetilde{G}_{-M}\right) = \frac{\frac{1}{2}\widetilde{\pi}_L(x)}{\widetilde{\pi}_R(x)} u_M\left(\widetilde{f}(x, L)\right) + \frac{\widetilde{\pi}_R(x) - \frac{1}{2}\widetilde{\pi}_L(x)}{\widetilde{\pi}_R(x)} u_M\left(\widetilde{f}(x, R)\right),$$

while the expected payoff of the player M with any positive effort input, e_M, no larger than

$\widetilde{\pi}_L(x)$ is

$$\begin{aligned}
u_M\left(e_M,\widetilde{G}_{-M}\right) &= \left[\frac{\frac{1}{2}\widetilde{\pi}_L(x)}{\widetilde{\pi}_R(x)}-\frac{\frac{1}{2}e_M^2}{\widetilde{\pi}_R(x)\,\widetilde{\pi}_L(x)}\right]u_M\left(\widetilde{f}(x,L)\right)-e_M \\
&\quad+\left[\frac{\widetilde{\pi}_R(x)-\frac{1}{2}\widetilde{\pi}_L(x)}{\widetilde{\pi}_R(x)}-\frac{(\widetilde{\pi}_R(x)-\widetilde{\pi}_L(x))\,e_M+\frac{1}{2}e_M^2}{\widetilde{\pi}_R(x)\,\widetilde{\pi}_L(x)}\right]u_M\left(\widetilde{f}(x,R)\right) \\
&= u_M\left(0,\widetilde{G}_{-M}\right)-\frac{\frac{1}{2}e_M^2}{\widetilde{\pi}_R(x)\,\widetilde{\pi}_L(x)}u_M\left(\widetilde{f}(x,L)\right) \\
&\quad-\frac{(\widetilde{\pi}_R(x)-\widetilde{\pi}_L(x))\,e_M+\frac{1}{2}e_M^2}{\widetilde{\pi}_R(x)\,\widetilde{\pi}_L(x)}u_M\left(\widetilde{f}(x,R)\right)-e_M \\
&\leq u_M\left(0,\widetilde{G}_{-M}\right)-\frac{(\widetilde{\pi}_R(x)-\widetilde{\pi}_L(x))\,e_M+e_M^2}{\widetilde{\pi}_R(x)\,\widetilde{\pi}_L(x)}u_M\left(\widetilde{f}(x,R)\right)-e_M \\
&= u_M\left(0,\widetilde{G}_{-M}\right)-\left[\frac{((\widetilde{\pi}_R(x)-\widetilde{\pi}_L(x))+e_M)\,u_M\left(\widetilde{f}(x,R)\right)}{\widetilde{\pi}_R(x)\,\widetilde{\pi}_L(x)}+1\right]e_M \\
&\leq u_M\left(0,\widetilde{G}_{-M}\right)-\left[\frac{\widetilde{\pi}_R(x)\,u_M\left(\widetilde{f}(x,R)\right)}{\widetilde{\pi}_R(x)\,\widetilde{\pi}_L(x)}+1\right]e_M \text{ (since } e_M\leq\widetilde{\pi}_L(x)\text{)} \\
&= u_M\left(0,\widetilde{G}_{-M}\right)-\left[\frac{u_M\left(\widetilde{f}(x,R)\right)}{\widetilde{\pi}_L(x)}+1\right]e_M \\
&= u_M\left(0,\widetilde{G}_{-M}\right)-\left[\frac{u_M\left(\widetilde{f}(x,R)\right)}{u_L\left(\widetilde{f}(x,L)\right)-u_L\left(\widetilde{f}(x,R)\right)}+1\right]e_M \\
&< u_M\left(0,\widetilde{G}_{-M}\right).
\end{aligned}$$

The last inequality is true since $u_L\left(\widetilde{f}(x,L)\right)-u_L\left(\widetilde{f}(x,R)\right)>-u_M\left(\widetilde{f}(x,R)\right)$. It is obvious that any effort input level larger than $\widetilde{\pi}_L(x)$ is strictly dominated by $\widetilde{\pi}_L(x)$ given $\widetilde{G}_{-M}$ as the recognition probability of the player M is always 1. Therefore, any positive effort input is strictly dominated by zero effort for the player M given $\widetilde{G}_L(e;x)$ and $\widetilde{G}_R(e;x)$. ■

Proof of Proposition 2. We first present the details of the equilibrium we construct:

1. The bidding strategies: $G_M(0;x)=1$, and

$$G_L(e;x)=G_R(e;x)=\begin{cases}\dfrac{e}{\pi(x)}, & \text{if } e\in[0,\pi(x)], \\ 1, & \text{if } e>\pi(x).\end{cases}$$

   where $\pi(x)=U_L(f(x,L))-U_L(f(x,R))=U_R(f(x,R))-U_R(f(x,L))$.

2. The policy rule: $f(x, M) = 0$ for any $x \in \mathbb{R}$ and

$$f(x, R) = -f(x, L) = \begin{cases} |x|, & |x| \in [0, \overline{s}_R], \\ z_R^* - \frac{(|x|-1)^2}{4\delta}, & |x| \in (\overline{s}_R, 1), \\ z_R^*, & |x| \in [1, \infty), \end{cases}$$

where

$$z_R^* = \begin{cases} 1-\delta, & \delta \in \left[0, \frac{1}{2}\right], \\ \frac{1}{4\delta}, & \delta \in \left(\frac{1}{2}, 1\right), \end{cases},$$

and $\overline{s}_R = -\overline{s}_L = s^* \equiv \max\{1-2\delta, 0\}$.

3. The dynamic payoff functions:

$$U_R(x) = U_L(-x) = \begin{cases} -(1-\delta)(x-1)^2 - 4\delta z_R^*, & x \in (-\infty, -1], \\ 4(1-\delta)x - 4\delta z_R^*, & x \in (-1, \overline{s}_L), \\ -(x-1)^2, & x \in [\overline{s}_L, 0] \\ -(x-\overline{s}_R)^2 - 4\delta z_R^*, & x \in [0, \overline{s}_R]; \\ -4\delta z_R^*, & x \in (\overline{s}_R, 1); \\ -(1-\delta)(x-1)^2 - 4\delta z_R^*, & x \in [1, \infty), \end{cases}$$

and $U_M(x) = (1-\delta) u_M(x) + \delta [U_M(f(x, L)) + U_M(f(x, R))]$.

All the properties presented in the proposition are direct implications of the above equilibrium we construct. In what follows we show that this is indeed an equilibrium following Definition 1

Part 1. Given f and G, we prove that U satisfies equation (3). Consider player L. Substitute G into equations (1) and (2) when $i = L$, and we have

$$\begin{aligned} P_L(e_L, G_{-L}; x) &= \frac{e_l}{U_L(f(x, L)) - U_L(f(x, R))}, \\ P_M(e_L, G_{-L}; x) &= 0, \\ P_R(e_L, G_{-L}; x) &= 1 - \frac{e_l}{U_L(f(x, L)) - U_L(f(x, R))}, \end{aligned}$$

As a consequence, we have

$$U_L(x) = (1-\delta) u_L(x) + U_L(f(x, R)).$$

We repeat the same procedure for player M and R, we have

$$\begin{aligned}
U_M(x) &= (1-\delta)\, u_M(x) + \delta\left[U_M(f(x,L)) + U_M(f(x,R))\right], \\
U_R(x) &= (1-\delta)\, u_R(x) + U_R(f(x,L)).
\end{aligned}$$

Therefore, it is equivalent to verify that f and U satisfies the following equation system:

$$\begin{aligned}
U_L(x) &= (1-\delta)\, u_L(x) + U_L(f(x,R)). \\
U_M(x) &= (1-\delta)\, u_M(x) + \delta\left[U_M(f(x,L)) + U_M(f(x,R))\right], \\
U_R(x) &= (1-\delta)\, u_R(x) + U_R(f(x,L)).
\end{aligned}$$

The verification is omitted here as it is just simple algebra.

Part 2. Given U, we prove that $f(x,i)$ solves maximization problem (5). $U_L(x)$ is quasi-concave, and with a plateau on interval $[-1, \overline{s}_L]$. $U_R(x)$ is quasi-concave, and with a plateau on interval $[\overline{s}_R, 1]$. $U_M(x)$ is symmetric and strict concave over $[\overline{s}_L, \overline{s}_R]$. Symmetry means $U_m(x) = U_m(-x)$, which can be easily verified by the expression of $U_M(x)$, and strict concavity can be proved by two steps.

Step 1: Let $B([\overline{s}_L, \overline{s}_R])$ be a space of bounded functions with the sup norm. Let T be a operator defined as

$$Tv(x) = (1-\delta)\, u_M(x) + \frac{\delta}{2}\left[v(f(x,L)) + v(f(x,R))\right].$$

We prove that T is a contraction because it satisfies the Blackwell's sufficient conditions for a contraction (Theorem 3.3 in Stokey et al, 1989, p54). For monotonicity, notice that for $v' \geq v$

$$\begin{aligned}
Tv(x) &= (1-\delta)\, u_M(x) + \frac{\delta}{2}\left[v(f(x,L)) + v(f(x,R))\right] \\
&\leq (1-\delta)\, u_M(x) + \frac{\delta}{2}\left[v'(f(x,L)) + v'(f(x,R))\right] \\
&= Tv'(x).
\end{aligned}$$

A similar argument follows for discounting: for $a > 0$

$$\begin{aligned}
T(v+a)(x) &= (1-\delta)\, u_M(x) + \frac{\delta}{2}\left[(v+a)(f(x,L)) + (v+a)(f(x,R))\right] \\
&= (1-\delta)\, u_M(x) + \frac{\delta}{2}\left[v(f(x,L)) + v(f(x,R))\right] + \delta a \\
&= Tv(x) + \delta a.
\end{aligned}$$

Therefore, T is a contraction.

Step 2: Following the similar argument as Theorem 4.8 in Stokey et al (1989) on page 81, we can find that $U_m(x)$ is strict concave over $[\overline{s}_L, \overline{s}_R]$.

Consider the case that player L is the proposer. If $|x| \in (\overline{s}_R, +\infty)$, $f(x, L) \in [1, \overline{s}_L]$, the policy proposal on the plateau area, and $U_M(f(x, L)) \geq U_M(x)$ as $|f(x, L)| \leq |x|$, which means that the median player will vote for the policy proposal. If $|x| \in [0, \overline{s}_R]$, $f(x, L) = -|x|$, and the median player is indifferent with the policy proposal and status quo. Therefore, $f(x, L)$ solves maximization problem (5) if $i = L$. With the same token, we can prove that $f(x, R)$ solves maximization problem (5) if $i = R$. Consider the case that player M is the proposer. The median player can always propose his ideal policy 0, and one of the extreme players will vote for it regardless of status quo x.

Part 3. Given f, G_{-i} and U, we prove that $G_i(e_i^*, x)$ is increasing at e_i^* only if e_i^* solves maximization problem (4).

Consider player L. $\left[\sum_j P_j(e_L, G_{-L}; x) U_L(f(x, j))\right] - e_L = U_L(f(x, R))$, for any $e_L \in [0, \pi(x)]$, and $\left[\sum_j P_j(e_L, G_{-L}; x) U_L(f(x, j))\right] - e_L < U_L(f(x, R))$ for any effort input level larger than $\pi(x)$, which means that any effort input level in $[0, \pi(x)]$ is the best response of player L, thus condition (3) in the equilibrium definition is satisfied for player L. With the similar argument, condition (3) in the equilibrium definition is satisfied for player R too. For player M, any positive effort input is strictly dominated by zero effort input. ■

Proof of Proposition 4. We first present the details of the equilibrium we construct for this proposition.

1. Bidding strategies: $\widehat{G}_i(0; x) = 1$ for all i.

2. Policy rule: $\widehat{f}(x, M) = 0$ and

$$\widehat{f}(x, R) = -\widehat{f}(x, L) = \begin{cases} |x|, & |x| \in [0, \widehat{s}], \\ \widehat{z}_R - \frac{(|x|-1)^2}{2\delta}, & |x| \in (\widehat{s}, 1), \\ \widehat{z}_R, & |x| \in [1, \infty), \end{cases}$$

   where $\widehat{z}_R = 1 - \frac{1}{2}\delta$ and $\widehat{s} = 1 - \delta$.

3. Dynamic payoff functions:

$$\widehat{U}_R(x) = \widehat{U}_L(-x) = \begin{cases} -(1-\delta)(x-1)^2 - 2\delta\widehat{z}_R, & x \in (-\infty, -1], \\ 4(1-\delta)x - 2\delta\widehat{z}_R, & x \in (-1, -\widehat{s}), \\ -(x-\widehat{s})^2 - 2\delta\widehat{z}_R, & x \in [-\widehat{s}, \widehat{s}], \\ -2\delta\widehat{z}_R, & x \in (\widehat{s}, 1), \\ -(1-\delta)(x-1)^2 - 2\delta\widehat{z}_R, & x \in [1, \infty), \end{cases}$$

and $\widehat{U}_M(x) = (1-\delta)\, u_M(x) + \delta \left[\widehat{U}_M\left(\widehat{f}(x,R)\right) + \widehat{U}_M\left(\widehat{f}(x,L)\right)\right].$

Now we prove that the equilibrium presented above is indeed an equilibrium according to Definition 1.

Part 1. Given $\widehat{f}$ and $\widehat{G}$, we prove that $\widehat{U}$ satisfies equation (3). We omit the analysis of verification because it is just simple algebra.

Part 2. Given $\widehat{U}$, we prove that $\widehat{f}(x,i)$ solves maximization problem (5).

$\widehat{U}_L(x)$ is quasi-concave, and with a plateau on interval $[-1,-\widehat{s}]$. $\widehat{U}_R(x)$ is quasi-concave, and with a plateau on interval $[\widehat{s},1]$. $\widehat{U}_M(x)$ is symmetric, increasing over $(-\infty,0]$, and decreasing over $[0,+\infty)$.

Consider the case that player L is the proposer. when $|x| \in (\widehat{s},+\infty)$, $\widehat{f}(x,L) \in [-1,-\widehat{s}]$, that is, the policy proposal on the plateau area, and $\widehat{U}_M\left(\widehat{f}(x,L)\right) \geq \widehat{U}_M(x)$ as $\left|\widehat{f}(x,L)\right| \leq |x|$, which means that the median player will vote for the policy proposal. When $x \in [-\widehat{s},\widehat{s}]$, $\widehat{f}(x,L) = -|x|$, and the median player is indifferent with the policy proposal and status quo. Therefore, $\widehat{f}(x,L)$ solves maximization problem (5) if $i=L$. With the same token, we can prove that $\widehat{f}(x,R)$ solves maximization problem (5) if $i=R$. Consider the case that player M is the proposer. The median player can always propose his ideal policy 0, and one of the extreme players will vote for it regardless of status quo x.

Part 3. The best bidding strategy for any player is to bid nothing as the recognition probability is exogenously determined. ∎

Proof of Proposition 5. The bargaining effect is

$$\widetilde{f}(x,R) - \widehat{f}(x,R) = \begin{cases} 0, & |x| \in [0,\widehat{s}], \\ |x| - \widehat{z}_R + \frac{(|x|-1)^2}{2\delta}, & |x| \in (\widehat{s},1), \\ 1-\widehat{z}_R, & |x| \in [1,\infty), \end{cases}$$

and the waste-reduction effect is

$$\widehat{f}(x,R) - f(x,R) = \begin{cases} 0, & |x| \in [0,\overline{s}_R], \\ |x| - z_R^* + \frac{(|x|-1)^2}{4\delta} & |x| \in (\overline{s}_R,\widehat{s}], \\ \widehat{z}_R - z_R^* - \frac{(|x|-1)^2}{4\delta}, & |x| \in (\widehat{s},1), \\ \widehat{z}_R - z_R^*, & |x| \in [1,\infty). \end{cases}$$

These are directly implied by the equilibria constructed for Propositions 1, 2 and 4. ∎

References

[1] Anesi, Vincent. 2010. "Noncooperative Foundations of Stable Sets in Voting Games." *Games and Economic Behavior*, 70: 488-493

[2] Baron, David P. 1996. "A Dynamic Theory of Collective Goods Programs." *American Political Science Review*, 90(2): 316-30.

[3] Battaglini, Marco and Stephen Coate. 2007. “Inefficiency in Legislative Policy-Making: A Dynamic Analysis.” *American Economic Review,* 97(1): 118-149.

[4] Battaglini, Marco and Stephen Coate. 2008. “A Dynamic Theory of Public Spending, Taxation and Debt.” *American Economic Review,* 98(1): 201-236.

[5] Battaglini, Marco and Thomas R. Palfrey. 2011. "The Dynamics of Redistributive Politics." Economic Theory.

[6] Baye, Michael R., Dan Kovenock, and Casper G. De Vries. 1993. "Rigging the Lobbying Process: An Application of All-Pay Auctions.” American Economic Review, 83, 289–294.

[7] Baye, Michael R., Dan Kovenock, and Casper G. de Vries, 1996, "The All-pay Auction With Complete Information," *Economic Theory*, 8, 291-305.

[8] Black, Duncan. 1948. "On the Rationale of Group Decision-making." Journal of Political Economy, 56(1): 23-34.

[9] Bernheim, Douglas, Antonio Rangel, and Luis Rayo. 2006. "The Power of the Last Word in Legislative Policy Making." *Econometrica*, 74(5): 1161-1190.

[10] Deng, Jianpeng and Pohan Fong. 2011. "Rent-seeking in distributive politics with endogenous proposers." Unpublished.

[11] Diermeier, Daniel and Pohan Fong. 2011. "Legislative Bargaining with Reconsideration." Quarterly Journal of Economics, 126(2): 947-985.

[12] Duggan, John and Tasos Kalandrakis. 2011. "Dynamic Legislative Policy Making." Mimeo, University of Rochester.

[13] Hillman, Arye L. and John G. RILEY. 1989. "Politically Contestable Rents and Transfers." Economics and Politics, 1: 17–39.

[14] Kalandrakis, Tasos. 2004. “A Three-Player Dynamic Majoritarian Bargaining Game.” *Journal of Economic Theory*, 16(2): 294-322.

[15] Knight, Brian. 2005. "Estimating the Value of Proposal Power." *American Economic Review*, 95(5): 1639-1652.

[16] Konrad, Kai. 2009. Strategy and Dynamics in Contests. New York, NY: Oxford University press.

[17] Riboni, Alessandro. 2010. "Committees as Substitutes for Commitment." *International Economic Review*, 51(1): 213-236.

[18] Romer, Thomas and Howard Rosenthal. 1978. "Political Resource Allocation, Controlled Agendas, and the Status Quo." *Public Choice*, 33(4): 27-43.

[19] Romer, Thomas and Howard Rosenthal. 1979. "Bureacrats versus Voters: Political Economy of Resrouce Allocation by Direct Democracy." *Quarterly Journal of Economics*, 93(4): 563-587.

[20] Siegel, Ron. 2009. "All-pay Contest." *Econometrica*, 77(1): 71-92.

[21] Yildirim, Huseyin. 2007. "Proposal Power and Majority Rule in Multilateral Bargaining with Costly Recognition," *Journal of Economic Theory*, 136: 167-196.