How Bad was Lehman Shock?: Estimating a DSGE model with Firm and Bank Balance Sheets in a Data-Rich Environment*

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June, 2011
(Preliminary Version)

Abstract

Recent financial crisis in the U.S. which was precipitated by so-called 'Lehman Shock' has clearly exemplified that a deterioration of the balance sheet condition, especially those of the financial sector, can cause a deep and long-lasting recession of the economy. In modeling Lehman Shock, this paper embeds both corporate sector and banking sector balance sheets to the stylized DSGE model. We follow Bernanke, Gertler, and Gilchrist (1999) in embedding the corporate sector balance sheet, while we follow Gertler and Kiyotaki (2010) in embedding the banking sector balance sheet to the model. In our empirical analysis, we focus on the identification and estimation of the banking sector net worth shock, which is regarded as a proxy for Lehman Shock in this paper. In order to assess the impact of Lehman Shock reliably, we estimate the model using Data-Rich estimation method proposed by Boivin and Giannoni (2006). According to our preliminary estimation results, Lehman Shock turned out to be the worst banking sector net worth shock in past 25 years. However, the shock seems to have been successfully countered by TARP and the recessionary effect directly caused by Lehman Shock seems to be over.

JEL Classification: E32, E37, G01, G21, C32, C53

Key Words: DSGE; business cycle; Lehman Shock; financial friction; agency cost; data-rich estimation; measurement error; MCMC; Bayesian estimation.

*Preliminary and incomplete. We would like to acknowledge Stéphane Adjemian, Kosuke Aoki, Rochelle Edge, Shigeru Fujita, Ippei Fujiwara, Kohei Fukawa, Naohisa Hirakata, Shigeru Iwata, Michel Juillard, Rhys Mendes, Yoshiyasu Ono, Takashi Sakuma, Toshitaka Sekine, Nao Sudo, Kozo Ueda, and Toshiaki Watanabe for their valuable comments. We would like to especially acknowledge Yasuharu Iwata for his valuable update on the latest literature. The views expressed in this paper are those of the authors and do not necessarily reflect those of Economic and Social Research Institute (ESRI) or Cabinet Office, Government of Japan. All remaining errors are the responsibility of the authors.

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"We are in the midst of a once-in-a century credit tsunami." – Alan Greenspan (Testimony made at the House of Representatives, October 23, 2008)

1 Introduction

1.1 Focus of this paper

In this paper, our focus is to quantify and assess the impact of Lehman Shock. In particular, we ask how large was the magnitude of Lehman Shock and we also ask how large was the impact of Lehman Shock to the economy. Taking into account that Lehman Shock mainly affected the balance sheet conditions of the financial intermediaries directly, but not by much for the corporate sector's balance sheet – at least as a direct effect –, we regard Lehman Shock as a shock that occurred in the financial sector. Specifically, we assume Lehman Shock to be an aggregate net worth shock that affected banking sector's balance sheet condition.

In order to quantify and assess the impact of Lehman Shock as a banking sector net worth shock, we need a model that explicitly incorporates firm and bank balance sheets. We need bank balance sheet so that we can actually model a shock that affects bank's balance sheet condition and we need corporate balance sheet so that we can model a shock that affects corporate balance sheet condition separately from a shock that affects bank's balance sheet condition. In this paper, we construct a DSGE model with firm and bank balance sheets that allows the presence of corporate and banking sector net worth shocks at the same time. We utilize this banking sector net worth shock in a hope to capture the impact of Lehman Shock. After constructing the model with both corporate and bank balance sheets, we then estimate the model. In our empirical analysis, identification and reliable estimation of the bank net worth shock will be the main agenda. In order to quantify and assess the impact of Lehman Shock reliably, identification and reliable estimation of the bank net worth shock is crucial for our purpose.

Literature of financial friction model such as Bernanke, Gertler, and Gilchrist (1999) (hereafter, BGG) have emphasized the financial acceleration mechanism in the past. However, the merit of financial friction model should not be confined to this aspect. As we see it, the virtue of financial friction model is that it explicitly models the balance sheet conditions of the firm or financial intermediary that it allows us to incorporate the aggregate shock to the balance sheet conditions. This shock (or we can call it as financial shock) is intrinsically different from other macroeconomic shocks and can be an important factor in accounting for the business cycle of the economy. In this paper, we pursue this aspect of the financial friction model, try to identify financial shocks, and empirically investigate the impact of Lehman Shock.

1.2 Contributions of this paper

There have been models with financial friction that explicitly take into account corporate balance sheet condition, such as BGG or Kiyotaki and Moore (1997). Yet, these models are not effective in capturing the effect of Lehman Shock which we regard it as a shock that affected the bank's balance sheet. There have been models with financial friction that explicitly take into account financial intermediary's balance sheet condition. These models include Meh and Moran (2010), Gertler and Karadi (2010), and Gertler and Kiyotaki (2010). Yet, this is still not good in capturing the impact of Lehman Shock accurately, because the bank balance sheet in reality can be affected by corporate net worth shock as well, albeit indirectly. In order to purely capture Lehman Shock, we need to model both firm and bank balance sheet at the same time and clearly separate the impact from corporate net worth shock and bank net worth shock. Unfortunately, however, there are only few papers that model firm and bank balance sheets at the same time.[1]

Our theoretical contribution in this paper is that we model firm and bank balance sheets simultaneously by juxtaposing two canonical models. On the one hand, we adopt BGG which is canonical for modeling firm's balance sheet condition. On the other hand, we adopt Gertler and Kiyotaki (2010) and Gertler and Karadi (2010) which are canonical for modeling bank's balance sheet condition.[2] By combining two canonical models, we believe this will be the best approach in assessing the impact of Lehman Shock, at least given the currently available menu of financial friction models.

Another novel feature of this paper is the adoption of Data-Rich estimation method proposed by Boivin and Giannoni (2006). The idea of Data-Rich estimation is to extract the common factors from massive panel of data and to match those to the observable variables in the model. A merit of this approach is that by utilizing multiple time series information for each observable variable, we can expect an improved efficiency in estimating the parameters and structural shocks in the model. Since the focus of this paper is to assess the impact of Lehman Shock, efficient and reliable estimation of the structural shocks, especially that of bank net worth shock, is crucial.[3]

1.3 Organization of this paper

Organization of this paper is as follows. Section 2 describes how we juxtapose the essences of BGG, Gertler and Kiyotaki (2010), and Gertler and Karadi (2010) and synthesize it with the stylized DSGE model. Section 3 describes the idea and procedures of Data-Rich estimation method. Section 4 describes the preliminary settings for Bayesian estimation of the model and also describes the data we have used in this paper. Section 5 reports the empirical results of our paper and assess the impact of Lehman Shock. Section 6 concludes the paper with some remarks.

[1] One of the few notable exceptions is Hirakata, Sudo, and Ueda (2010). They assume information asymmetry between entrepreneur and financial intermediary, as well as between financial intermediary and investor. Following BGG, they impose costly-state-verification problem for both information asymmetries. In our paper, while we impose a costly-state-verification problem between entrepreneur and financial intermediary following BGG, we impose a moral hazard/costly enforcement problem between financial intermediary and depositor following Gertler and Kiyotaki (2010).

[2] Indeed, Gertler and Kiyotaki (2010), in their paper, claim their model to be cannonical in modeling bank's balance sheet condition.

[3] Recently, Boivin, Giannoni, and Stevanovic (2010) have conducted a Data-Rich estimation of the financial friction model. However, the financial friction model they have adopted is BGG and does not allow for the financial intermediary's balance sheet. In our paper, since our focus is on Lehman Shock, inclusion of financial intermediary's balance sheet is indispensable.

2 Model Description

In this section, we will describe how we embed corporate sector and banking sector to the, otherwise, stylized DSGE model. In embedding the corporate sector, we closely follow BGG (1999). In embedding the banking sector, we closely follow the structure proposed by Gertler and Karadi (2010) and Gertler and Kiyotaki (2010). In a sense, this paper combines the essence of BGG (1999), Gertler and Karadi (2010), and Gertler and Kiyotaki (2010) in modeling the corporate sector and banking sector balance sheets in a DSGE framework.

2.1 Household Sector

Following the idea of Gertler and Karadi (2010) and Gertler and Kiyotaki (2010), we construct the household sector in a way such that representative agent approach is valid. There are continuum of members in the household where the total population measures to one. Within the household, there are fractions of f^E entrepreneurs, f^F financial intermediaries (or "bankers" for short and we use this terminology interchangeably), and $1 - f^E - f^F$ workers. Entrepreneurs engage in a business where they produce intermediate goods and transfer the net worth back to the household when they exit from the business. Now, each financial intermediary manages a bank where it accepts the deposits from the household sector and lend to entrepreneurs. When financial intermediaries exit from their business, they also transfer their net worth back to the household sector. Finally, remaining fraction of the members of the household become workers. Workers supply labor input to earn wage and they transfer their wage earnings to the household each period. Within the household, each member shares the risk perfectly.

The representative household derives utility from final goods consumption and disutility from supplying aggregate labor inputs. The representative household maximizes their expected discounted sum of utility over time and their objective function is specified as follow;

$$E_t \sum_{i=0}^{\infty} \beta^i \chi_{t+i}^c \left[\frac{(c_{t+i} - hC_{t+i-1})^{1-\sigma^c}}{1-\sigma^c} - \chi_{t+i}^L \frac{(l_{t+i})^{1+\sigma^L}}{1+\sigma^L} \right] \tag{1}$$

where parameter β stands for the discount rate, parameter h stands for habit persistence coefficient, parameter σ^c stands for the inverse of long-run intertemporal elasticity of substitution, c_t stands for final goods consumption, and C_{t-1} represents external habit formation from last period *á la* Abel (1990) which is exogenously given to the household at period t, but $c_t = C_t$ in equilibrium. Turning to the labor disutility, parameter σ^L stands for the inverse of Frisch labor supply elasticity and l_t stands for the supply of aggregate labor by workers which is determined as a result individual workers' labor supply decisions and, thus, it is exogenous to the representative household's decision. Now, there are two structural shocks embedded in the function; χ_t^c and χ_t^L. Structural shock, χ_t^c, represents an intertemporal preference shock to the household's current consumption and labor supply against those in the future, while χ_t^L represents labor disutility shock relative to consumption utility. Both χ_t^c and χ_t^L follow AR(1) stochastic process.[4]

[4] The specification of intertemporal preference shock here follows Smets and Wouters (2003). Recently, the importance of preference shock has been emphasized by the real business cycle literature as well. See, for instance, Wen (2007).

Next, turning to the budget constraint of the representative household, they make a deposit, b_t, at period t and earn real interest rate, R_t/π_{t+1}, next period where R_t stands for risk-free gross nominal interest rate at period t and π_{t+1} stands for gross inflation rate at period $t+1$. In addition, the household pays lump sum tax of τ_t to the government. Now, they receive a lump-sum transfer of wage incomes from workers which is expressed as $\int_0^1 w_t(x)l_t(x)dx$, where $w_t(x)$ and $l_t(x)$ stand for real wage and labor supply by individual worker x, respectively.[5] The amount of wage income transfer is determined as a result of individual workers' decision and, thus, is exogenously given to the representative household. Finally, the household earns the combined dividend of Ξ_t^{div} from retailers, earns the net transfer of Ξ_t^E from entrepreneurs, and the net transfer of Ξ_t^F from bankers each period. Thus, the representative household's budget constraint at period t can be expressed as, in real terms, as follow,

$$c_t + b_t = \frac{R_{t-1}}{\pi_t} b_{t-1} - \tau_t + \int_0^1 w_t(x)l_t(x)dx + \Xi_t^{div} + \Xi_t^E + \Xi_t^F. \tag{2}$$

2.1.1 Consumption and Deposit Decision

Maximizing the household's objective function (1) with respect to c_t and b_t subject to the budget constraint (2) yields the following first-order conditions;

$$\zeta_t^H = \chi_t^c \left(c_t - hc_{t-1}\right)^{-\sigma^c} \quad \text{and} \tag{3}$$

$$\zeta_t^H = \beta E_t \zeta_{t+1}^H \frac{R_t}{\pi_{t+1}}. \tag{4}$$

Here, ζ_t^H stands for Lagrangian multiplier associated with the budget constraint (2) and can be interpreted as the shadow price of additional final goods at period t. Eq. (3) is the first-order condition of consumption decision which equates the marginal utility of consumption to the shadow price of the final goods. Note that we have used the property of external habit formation and the equilibrium condition such that $c_{t-1} = C_{t-1}$ to derive eq. (3). Eq. (4) is the first-order condition of deposit decision. It should be noted that intertemporal preference shocks, χ_t^c and χ_{t+1}^c, affect the deposit decision by the household via Lagrangian multipliers, ζ_t^H and ζ_{t+1}^H, in equilibrium. For instance, if the ratio of intertemporal preference shocks, χ_{t+1}^c/χ_t^c, is expected to be temporarily less than one, the household will temporarily put lower weight on the future marginal utility of consumption. Consequently, the household will decide to consume more and deposit less in the current period. Thus, the ratio of intertemporal preference shocks play an important role in deposit decision by the household.

2.1.2 Wage Setting Decision by Workers

Next, we describe the individual worker's decision and explain how the aggregate labor supply and aggregate wage index are formulated. Following Erceg, Henderson, and Levin (2000)

[5] Here, the real wage set by worker x is defined as

$$w_t(x) \equiv \frac{W_t(x)}{P_t}$$

where $W_t(x)$ stands for the nominal wage set by worker x and P_t stands for the price index of final goods. The formulation of $W_t(x)$ and P_t will be described later in this section.

(hereafter, EHL), there is a continuum of workers indexed by $x \in [0,1]$.[6] Each worker supplies differentiated labor input, $l_t(x)$, monopolistically and sells this service to the labor union who is perfectly competitive. The labor union transforms labor services to an aggregate labor input, l_t, using the following Dixit and Stiglitz (1971) type aggregator function,

$$l_t = \left[\int_0^1 l_t(x)^{\frac{1}{1+\psi^w}} dx\right]^{1+\psi^w} \tag{5}$$

where parameter ψ^w can be interpreted as wage markup. The factor demand function for $l_t(x)$ is given by

$$l_t(x) = \left(\frac{W_t(x)}{W_t}\right)^{\frac{-(1+\psi^w)}{\psi^w}} l_t \tag{6}$$

where $W_t(x)$ stands for the nominal wage set by worker x and W_t stands for the aggregate nominal wage index which is given as

$$W_t = \left[\int_0^1 W_t(x)^{\frac{-1}{\psi^w}} dx\right]^{-\psi^w}. \tag{7}$$

Following EHL, each worker sets his nominal wage according to a variant of Calvo (1983) - Yun (1996) style sticky price setting where, for any given period t, fraction θ^w of the entire workers cannot freely adjust the wages at their discretion. Further, following the treatment of Smets and Wouters (2003, 2007) and CEE, we allow for the partial indexation of nominal wage to past inflation by the workers who did not receive a 'signal' of wage revision. Specifically, for fraction θ^w of workers at period t, the partial indexation of the nominal wage is given as follow,

$$W_t(x) = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\iota^w} W_{t-1}(x) \tag{8}$$

where parameter $\iota^w \in [0,1]$ controls the degree of nominal wage indexation to past inflation rate.

Now, each worker shares exactly the same objective function with the representative household as given in eq. (1).[7] Under this setting, for $(1-\theta^w)$ fraction of workers who received a 'signal' of wage revision at period t, they will maximize their objective function by setting the nominal wage, $\widetilde{W}_t$, such that

$$E_t \sum_{i=0}^{\infty} \beta^i (\theta^w)^i \left[\frac{\widetilde{W}_t}{P_{t+i}} \left(\frac{P_{t-1+i}}{P_{t-1}}\right)^{\iota^w} \chi^c_{t+i} \left(c_{t+i} - hc_{t+i-1}\right)^{-\sigma^c} - (1+\psi^w)\chi^c_{t+i}\chi^L_{t+i}\left(l_{t+i}(x)\right)^{\sigma^L}\right] l_{t+i}(x) = 0. \tag{9}$$

[6] Here, the indexation of workers from 0 to 1 is auxiliary – i.e., it simplifies the calculation of aggregate index. Relative to the population of household members, total number of workers indexed by x amounts to the fraction $(1 - f^E - f^F)$ of total population of the household members and it is constant over time.

[7] Although each worker share exactly the same objective function with the representative household, however, this does not mean that each worker will chose his own consumption level, $c_t(x)$, independent of the representative household. Due to the perfect risk-sharing assumed in the model, the consumption level for each member of the household is dictated by the representative household's decision given in eq. (3) and not by the individual worker x's decision. Rather, each worker will act as a faithful agent of the representative household in maximizing the objective function eq. (1) by chosing the amount of individual labor supply, $l_t(x)$, (or, equivalently, chosing nominal wage $W_t(x)$) while taking the amount of consumption, c_t, as given. Finally, all the revenues from selling labor services will be transfered to the representative household.

Here, notice that intertemporal preference shock, labor disutility shock, and marginal utility of consumption are commonly given to each worker. From the definition of aggregate wage index, the law of motion of the aggregate wage index can be shown to be as follow,

$$W_t^{-1/\psi^w} = \theta^w \left[W_{t-1} \left(\frac{W_{t-1}}{W_{t-2}} \right)^{\iota^w} \right]^{-1/\psi^w} + (1-\theta^w) \widetilde{W}_t^{-1/\psi^w}. \tag{10}$$

Finally, the real wage index in the economy is defined as

$$w_t \equiv \frac{W_t}{P_t}. \tag{11}$$

2.2 Entrepreneurial Sector

2.2.1 Entrance and Exit of Entrepreneurs

Following BGG, there is a continuum of entrepreneurs indexed by $j \in [0,1]$ where each entrepreneur is risk-neutral and has a finite expected horizon. As in BGG, these assumptions will ensure that each entrepreneur will not accumulate enough net worth to self-finance their new capital. Following CMR, each entrepreneur faces an exogenous time-varying stochastic survival rate of γ_{t+1}^E from period t to $t+1$ which is common across all entrepreneurs.[8] We assume that the stochastic process of γ_t^E is uncorrelated with any other shocks in the economy and has its mean equal to γ^E – i.e., $E[\gamma_t^E] = \gamma^E$.

Between period t and $t+1$, after $1-\gamma_{t+1}^E$ fraction of entrepreneurs have exited from the business, exactly the same amount of new entrepreneurs will enter the business so that the population of entrepreneurs in the economy remains the same (i.e., fraction f^E of the total members of the household) from period t to $t+1$. Each entering entrepreneur receives a 'start-up' transfer from the household and the total 'start-up' transfer from the household will be equal to the constant fraction of aggregate net worth available in the entrepreneurial sector – i.e., $\xi^E n_t^E$.[9] For $1-\gamma_{t+1}^E$ fraction of entrepreneurs who happened to exit the business, they will first sell off the capital they purchased last period and retire all of their debts before maturity. And then, they will transfer their remaining net worth back to the household. The total amount of transfers from exiting entrepreneurs to the household will be $(1-\gamma_{t+1}^E)n_t^E$. Accordingly, net transfer, Ξ_{t+1}^E, that the household receives from entrepreneurs at period $t+1$ is $(1-\gamma_{t+1}^E-\xi^E)n_t^E$.

2.2.2 Individual Entrepreneur's Problem

Turning to individual entrepreneur's problem, each entrepreneur produces homogenous intermediate goods, $y_t(j)$, and they are perfectly competitive when selling their products to retailers. Each entrepreneur uses capital inputs and labor inputs and has a constant-return-to-scale technology in producing intermediate goods. The production function for the intermediate goods is given by

$$y_t(j) = \omega_t(j) A_t k_t(j)^\alpha l_t(j)^{1-\alpha}, \tag{12}$$

[8] CMR interprets this stochastic survival rate, γ_{t+1}^E, as reduced form way to capture shocks unrelated to preference or technology in the economy. They name 'asset price bubble' and 'irrational exuberance' for such examples.

[9] So each entrepreneur entering at period $t+1$ receives the amount of $\frac{\xi^E n_t^E}{1-\gamma_{t+1}^E}$ as a 'start-up' from the household.

where $k_t(j)$ stands for capital inputs and $l_t(j)$ stands for labor inputs by an entrepreneur j at period t. It should be noted that the total factor productivity shock, A_t, is common across all entrepreneurs. Also, the capital share parameter, α, is common across entrepreneurs as well. Following Carlestrom and Fuerst (1997) and BGG, we assume each entrepreneur is subject to an idiosyncratic shock, $\omega_t(j)$, which affects the total factor productivity of intermediate goods, $y_t(j)$. An idiosyncratic shock, $\omega_t(j)$, is a private information to entrepreneur j and assumed to be i.i.d. shock with mean equal to one – i.e., $E[\omega_t(j)] = 1$.

The balance sheet statement of each entrepreneur at the end of period t can be expressed as

$$q_t k_{t+1}(j) = b_t^E(j) + n_t^E(j) \tag{13}$$

where q_t stands for the real price of capital, $k_{t+1}(j)$ stands for the capital which will be used for production in period $t+1$ but purchased at the end of period t, $b_t^E(j)$ stands for the real debt issued at period t and $n_t^E(j)$ stands for the net worth at period t. Basically, left-hand side of eq. (13) represents the total asset of the entrepreneur and right-hand side represents the liability and net worth of the entrepreneur at the end of period t. As can be seen from this balance sheet equation, capital is partially financed by issuing the debt. With the assumption of risk-neutrality and finite planning horizon, net worth (or internal financing) itself is never enough in financing the cost of capital purchase and, therefore, each entrepreneur will rely on external financing in equilibrium.

The income statement for entrepreneur j is specified as follow

$$n_t^E(j) = p_t^{mc}(j) y_t(j) - w_t l_t(j) - \frac{R_{t-1}^E(j)}{\pi_t} b_{t-1}^E(j) + q_t(1-\delta) k_t(j) \tag{14}$$

where $p_t^{mc}(j)$ stands for the real price of intermediate goods j (which is also equal to the marginal cost of producing intermediate goods j due to perfect competition), $R_{t-1}^E(j)/\pi_t$ stands for the real rate of borrowing cost, and parameter δ stands for capital depreciation rate. Each entrepreneur is a price-taker in the labor market, financial market, and capital market that real wage, real rate of borrowing cost, and real price of capital are exogenously given to each entrepreneur. At the beginning of period t, each entrepreneur will utilize capital, $k_t(j)$, and labor input, $l_t(j)$, to produce the intermediate goods, $y_t(j)$, according to the production function (12). Then, they will sell off the intermediate goods to retailers in a perfectly competitive manner and earn the revenue, $p_t^{mc}(j) y_t(j)$. After earning the revenue, each entrepreneur will pay the labor cost and also repay the debt. Finally, each entrepreneur will sell off a depreciated capital at the capital market. The net income after these activities are captured by n_t^E and will be a net worth for the entrepreneur j at the end of period t. Given this net worth, each entrepreneur will plan for the next period and decide how much capital to purchase and how much debt to issue at the end of period t which appears in the balance sheet equation (13).

For each entrepreneur entering period t, they will maximize their discounted cash flow by choosing capital inputs, labor inputs, and debt issuance subject to eq. (12), (13), and (14). The first order conditions for each entrepreneur j are given by

$$w_t = (1-\alpha) \frac{p_t^{mc}(j) y_t(j)}{l_t(j)} \text{ and} \tag{15}$$

$$E_t\left[\gamma_{t+1}^E \frac{R_t^E(j)}{\pi_{t+1}}\right] = E_t\left[\gamma_{t+1}^E \frac{\alpha p_{t+1}^{mc}(j) y_{t+1}(j)/k_{t+1}(j) + (1-\delta) q_{t+1}}{q_t}\right]. \tag{16}$$

Eq. (15) equates marginal cost of labor to marginal product of labor and, thus, can be thought of as labor demand function by entrepreneur j. Eq. (16) equates the expected marginal cost of capital financed by debt to the expected marginal return of capital financed by debt and can be thought of as capital demand function by entrepreneur j. Since stochastic survival rate, γ_{t+1}^E, is uncorrelated to any other shocks in the economy, eq. (16) can be further rearranged as

$$E_t\left[\frac{R_t^E(j)}{\pi_{t+1}}\right]=E_t\left[\frac{\alpha p_{t+1}^{mc}(j)y_{t+1}(j)/k_{t+1}(j)+(1-\delta)q_{t+1}}{q_t}\right] \tag{17}$$

which is the standard result in BGG. Thus, under the assumption of risk-neutrality, introduction of stochastic survival rate will not alter the capital demand equation for any entrepreneur j compared to the case with constant survival rate as in BGG.

2.2.3 Debt Contract

Each period, entrepreneur j issues a debt and engages in a debt contract with an arbitrary chosen financial intermediary m where m is an indexed number uniformly distributed from 0 to 1. Debt contract is for one period only and if entrepreneur j needs to issue a debt again next period, another arbitrary financial intermediary m' will be chosen next period. Following BGG, idiosyncratic total factor productivity shock, $\omega_t(j)$, is private information of entrepreneur j that there exists information asymmetry between entrepreneur j and financial intermediary m. Due to costly state verification, financial intermediary m cannot observe entrepreneur j's output costlessly, but need to incur a fixed monitoring cost to observe it. Entrepreneur j, after observing the project outcome, will decide whether to repay the debt or default at the beginning of period t. If the entrepreneur decides to repay, financial intermediary will receive repayment of $R_{t-1}^E(j)/\pi_t$ for each unit of credits outstanding, regardless of the realization of idiosyncratic shock, $\omega_t(j)$. On the other hand, if the entrepreneur decides to default, the financial intermediary will pay a fixed monitoring cost to observe $y_t(j)$ and seize the project outcome from the entrepreneur.

Under this problem set up, BGG shows that the optimal debt contract to require the external finance premium, $s_t(j)$, to depend on the entrepreneur's overall balance sheet condition. Specifically, they show that the external finance premium to be a function of the leverage ratio and increasing with respect to the ratio. The reduced form function can be characterized by

$$s_t(j)=s\left(\frac{q_t k_{t+1}(j)}{n_t^E(j)}\right) \tag{18}$$

where $s'(\cdot)>0$ and $s(1)=0$. Thus, discounting the external finance premium from the borrowing rate $R_t^E(j)$, the expected risk-adjusted nominal return for financial intermediary m from the debt contract from period t to $t+1$ can be expressed as

$$E_t R_{t+1}^F(m)=\frac{R_t^E(j)}{s_t(j)}. \tag{19}$$

Finally, for estimation purpose, we follow Christensen and Dib's (2008) specification of the external finance premium and it will be as follow

$$s_t(j)=\left(\frac{q_t k_{t+1}(j)}{n_t^E(j)}\right)^{\varphi} \tag{20}$$

where parameter $\varphi > 0$ can be interpreted as the elasticity of external finance premium with respect to the leverage ratio.

2.2.4 Aggregation

As shown by Carlestrom and Fuerst (1997) and BGG, the assumption of constant-return-to-scale production technology and risk-neutrality will render marginal product of labor, marginal product of capital, marginal cost, and leverage ratio to be equal across all solvent entrepreneurs in equilibrium.[10] Further, since bankruptcy cost is constant-return-to-scale and leverage ratio are equal for all entrepreneur j, the external finance premium will be equal across all solvent entrepreneurs in equilibrium – i.e., $s_t = s_t(j)$ for all j. This property will make aggregation very simple which renders eq. (15), eq. (17), and eq. (18) to hold in aggregate level as well. In particular, it is important to note that, because eq. (17) holds in aggregate level, the nominal borrowing rates across all solvent entrepreneurs become equal – i.e., $R_t^E = R_t^E(j)$ for all j. Consequently, because $R_t^E = R_t^E(j)$ and $s_t = s_t(j)$ for all j, the expected risk-adjusted nominal return for banker m (as in eq. (19)) becomes equal across all bankers – i.e.,

$$E_t\left[R_{t+1}^F(m)\right] = \frac{R_t^E}{s_t} \text{ for all } m. \tag{21}$$

Next, we derive the law of motion of the aggregate net worth of entrepreneurial sector.[11] Aggregating over income statement eq. (14) and taking into account the entrance and exit of entrepreneurs from period t to $t+1$, we obtain the following aggregate net worth transition equation

$$n_{t+1}^E = \gamma_{t+1}^E\left[r_{t+1}^k q_t k_{t+1} - \frac{R_t^E}{\pi_{t+1}} b_t^E\right] + \xi^E n_t^E \tag{22}$$

where r_{t+1}^k stands for realized gross return from capital investment at period $t+1$ and is defined as

$$r_{t+1}^k \equiv \frac{\alpha p_{t+1}^{mc}\overline{y}_{t+1}/k_{t+1} + (1-\delta)q_{t+1}}{q_t}. \tag{23}$$

Here, following the notation of BGG, $\overline{y}_{t+1}$ stands for the average of project outcomes, $y_{t+1}(j)$, across all entrepreneurs. Thus, idiosyncratic factor stemming from $\omega_t(j)$ is averaged away and r_{t+1}^k only reflects the aggregate factors in the economy. Using entrepreneur's balance sheet eq. (13), the aggregate net worth transition eq. (22) can be rearranged as

$$n_{t+1}^E = \gamma_{t+1}^E\left[\left(r_{t+1}^k - \frac{R_t^E}{\pi_{t+1}}\right) q_t k_{t+1} + \frac{R_t^E}{\pi_{t+1}} n_t^E\right] + \xi^E n_t^E. \tag{24}$$

Notice how the realization of r_{t+1}^k can affect the aggregate net worth next period. Ex-ante, by the rational expectation equilibrium condition (17), the expected return from capital investment and borrowing cost are equalized. Ex-post, however, realized return from capital investment can exceed or fall below the borrowing cost depending on the realizations of the

[10] As analyzed in Covas (2006), when production technology is decreasing-return-to-scale, leverage ratio will not be equal across the entrepreneurs. In such a case, heterogeneity across the entrepreneurs and distribution of leverage ratio must be explicityly taken into account when solving for the general equilibrium.

[11] As for notation, aggregate variable is expressed by suppressing the argument j. For instance, variable n_t^E, where argument j is suppressed, stands for the aggregate net worth of entrepreneurial sector instead of entrepreneur j's net worth.

aggregate shocks and it affects the evolution of the aggregate net worth. This is a case where forecast error has an actual effect on the economy and it is important to model this forecast error explicitly when conducting empirical analysis.[12]

Another factor that affects the evolution of the aggregate net worth is the realization of stochastic survival rate γ_{t+1}^E. At the micro-level, γ_{t+1}^E has an interpretation of stochastic survival rate of entrepreneur j from period t to $t+1$. At the aggregate level, as it turns out, γ_{t+1}^E can be interpreted as an exogenous shock to the aggregate net worth in the entrepreneurial sector. In our paper, especially when we move on to the empirical analysis, we emphasize the interpretation of γ_{t+1}^E at the aggregate level and will often interpret it as an aggregate entrepreneurial net worth shock.

2.3 Banking Sector

2.3.1 Entrance and Exit of Bankers

Following Gertler and Karadi (2010) as well as Gertler and Kiyotaki (2010), there is a continuum of bankers indexed by $m \in [0, 1]$ where each banker is risk-neutral and has a finite horizon. Rather than assuming constant survival rate, however, we assume that each banker faces exogenous time-varying stochastic survival rate of γ_{t+1}^F from period t to $t+1$ which is common to all bankers. By the same token as in entrepreneurial sector, the stochastic process of γ_t^F is uncorrelated with any other shocks in the economy and has it mean equal to γ^F – i.e., $E[\gamma_t^F] = \gamma^F$.

After $1 - \gamma_{t+1}^F$ fraction of bankers have exited between period t and $t+1$, exactly the same number of new bankers will enter the banking business from the household, so that the population of bankers in the economy remains the same. In other words, fraction f^F of the total members of the household are in banking business each period. Each banker entering the baking business will receive a 'start-up' transfer from the household, while each banker exiting the business will transfer his net worth back to the household. In aggregate, 'start up' transfer is assumed to be the constant fraction of aggregate net worth available in the banking sector ($\xi^F n_t^F$) and the aggregate transfer from the exiting bankers is equal to $\gamma_{t+1}^F n_t^F$. Thus, net transfer from the banking sector to the household, Ξ_{t+1}^F, is equal to $(1 - \gamma_{t+1}^F - \xi^F) n_t^F$.

2.3.2 Individual Banker's Problem

We now describe the individual banker's problem. The treatment here basically follows that of Gertler and Karadi (2010) and perfect inter-bank market version of Gertler and Kiyotaki (2010). The balance sheet equation of the individual banker m is given by

$$b_t^E(m) = n_t^F(m) + b_t^F(m) \tag{25}$$

where $b_t^E(m)$ stands for the asset of banker m which is lent out to an arbitrarily chosen entrepreneur j at period t, $n_t^F(m)$ stands for the net worth of banker m, and $b_t^F(m)$ stands for the liability of banker m which is also a deposit made by the household at period t.

[12] Thus, we will be adopting Sims (2002) method when solving the DSGE model instead of Blanchard and Kahn's (1980) solution method. Sims' (2002) solution method is explained in the appendix of this paper.

By lending out $b_t^E(m)$ to an entrepreneur at period t, banker m can expect to earn a gross return rate of $E_t R_{t+1}^F(m)/\pi_{t+1}$ in real terms next period. By receiving deposits $b_t^F(m)$ from household at period t, banker m pledges to pay the deposit rate of R_t/π_{t+1} in real terms next period. Notice that the deposit rate is common for all bankers. As a result of the banking business, the net worth transition for banker m at period $t+1$ is given by

$$n_{t+1}^F(m) = r_{t+1}^F(m) b_t^E(m) - r_{t+1} b_t^F(m) \tag{26}$$

where $r_{t+1}^F(m) \equiv R_{t+1}^F(m)/\pi_{t+1}$ and $r_{t+1} \equiv R_t/\pi_{t+1}$. Using the balance sheet equation (25), the above net worth transition equation can be reformulated as follow

$$n_{t+1}^F(m) = \left(r_{t+1}^F(m) - r_{t+1}\right) b_t^E(m) + r_{t+1} n_t^F(m). \tag{27}$$

As shown by Gertler and Kiyotaki (2010), with the agency cost present between banker m and depositor, the expected spread (or risk premium) between $r_{t+1}^F(m)$ and real deposit rate r_{t+1} becomes strictly positive – i.e., $E_t\left[r_{t+1}^F(m) - r_{t+1}\right] > 0$. However, of course, whether the net worth of banker m increases or decreases next period depends on the realization of $r_{t+1}^F(m)$. The agency problem between banker m and depositor will be described shortly.

Given the above net worth transition equation, risk-neutral banker m will maximize the net worth accumulation by maximizing the following objective function with respect to bank lending $b_t^E(m)$,

$$V_t^F(m) = E_t \sum_{i=0}^{\infty} \beta^i (1-\gamma_{t+1}^F) \gamma_{t+1,t+1+i}^F \left[\left(r_{t+1+i}^F(m) - r_{t+1+i}\right) b_{t+i}^E(m) + r_{t+1+i} n_{t+i}^F(m)\right] \tag{28}$$

where $\gamma_{t+1,\ t+1+i}^F \equiv \prod_{j=0}^{i} \gamma_{t+1+j}^F$. Now, since the expected spread between risk-adjusted bank lending rate and deposit rate is strictly positive, it is in the interest on banker m to lend out infinite amount to an entrepreneur by accepting infinite amount of deposits from the depositor.

In order to avoid the infinite risk-taking by the banker, Gertler and Karadi (2010) and Gertler and Kiyotaki (2010) impose a moral hazard/costly enforcement problem between the banker and depositor. Each period, the banker has a technology to divert fraction λ of his asset holding to the household and exit from the banking business. However, by doing so, the banker is forced to file bankruptcy and fraction $(1-\lambda)$ of his asset will be seized by the depositors. Thus, in order for the banker to continue business and depositors to safely deposit their funds to the banker, the following incentive constraint must be met each period,

$$V_t^F(m) \geq \lambda b_t^E(m). \tag{29}$$

In other words, the net present value of the banking business needs to always exceed the reservation value retained by the banker. To see how this constraint binds, consider the case where the banker increases the asset enormously. Then, the reservation value by the banker (right-hand side of inequality (29)) will exceed the net present value of the banking business (left-hand side of inequality (29)) that the banker will decide to divert the assets to the household. As a stakeholder, the depositors will not allow this reckless behavior by the banker and ask the banker to keep his asset, $b_t^E(m)$, low enough (or, equivalently, by not supplying the deposits beyond the incentive constraint) so that the incentive for the banker to remain in business is met.

Now, assuming that the incentive constraint (29) to be binding each period and by maximizing the objective function (28) subject to the constraint (29), Gertler and Kiyotaki (2010) shows that the value function of the banker can be expressed as follow

$$V_t^F(m) = \nu_t b_t^E(m) + \eta_t n_t^F(m) \tag{30}$$

where

$$\nu_t \equiv E_t\left[\left(1-\gamma_{t+1}^F\right)\beta\left(r_{t+1}^F(m)-r_{t+1}\right)+\beta\gamma_{t+1}^F\frac{b_{t+1}^E(m)}{b_t^E(m)}\nu_{t+1}\right] \text{ and} \tag{31}$$

$$\eta_t \equiv E_t\left[\left(1-\gamma_{t+1}^F\right)+\beta\gamma_{t+1}^F\frac{n_{t+1}^F(m)}{n_t^F(m)}\eta_{t+1}\right]. \tag{32}$$

Now, from incentive constraint (29) and the value function (30), it follows that

$$\frac{b_t^E(m)}{n_t^F(m)} \leq \frac{\eta_t}{\lambda-\nu_t} \equiv \phi_t \tag{33}$$

which states that the leverage ratio of banker m cannot exceed the (time-varying) threshold ϕ_t.[13] By the assumption that incentive constraint to bind every period, in equilibrium, the asset and the net worth by banker m have a following relationship

$$b_t^E(m) = \phi_t n_t^F(m). \tag{34}$$

2.3.3 Aggregation

Gertler and Karadi (2010) and Gertler and Kiyotaki (2010) show that time-varying threshold ϕ_t does not depend on banker-specific factors and is common across all bankers. Consequently, from eq. (34), aggregate asset and net worth in banking sector can be expressed as

$$b_t^E = \phi_t n_t^F \tag{35}$$

where $b_t^E \equiv \int_0^1 b_t^E(m)dm$ and $n_t^F \equiv \int_0^1 n_t^F(m)dm$. Now, from individual banker's net worth transition eq. (27) and taking into account entrance and exit of bankers, the aggregate net worth transition equation of banking sector is given by

$$n_{t+1}^F = \gamma_{t+1}^F\left[\left(\overline{r}_{t+1}^F - r_{t+1}\right)b_t^E + r_{t+1}n_t^F\right] + \xi^F n_t^F \tag{36}$$

where $\overline{r}_{t+1}^F$ stands for the average of realized risk-adjusted returns, $r_{t+1}^F(m)$, across all bankers. From the optimal debt contract specified in eq. (19) and using the aggregate condition in eq. (21), $\overline{r}_{t+1}^F$ is related to the borrowing rate, external finance premium, and inflation rate as follow

$$\overline{r}_{t+1}^F = \frac{R_t^E}{\pi_{t+1}s_t}. \tag{37}$$

[13] Notice the similarity to the Basel Regulation which requires the banks to keep their capital-asset ratio above certain rate. Since the capital-asset ratio is just an inverse of leverage ratio, eq. (33) also implies that banker m needs keep his capital-asset ratio above the threshold $1/\phi_t$ – i.e.,

$$\frac{n_t^F(m)}{b_t^E(m)} \geq \frac{1}{\phi_t}.$$

The difference between Basel Regulation and Gertler and Kiyotaki's (2010) incentive constraint is that the former is a time-invariant requirement while the latter is a time-variant requirement of the capital-asset ratio.

As can be seen from the above equation, idiosyncratic factor pertaining to banker m is averaged away and, thus, realization of risk-adjusted return of banking sector (i.e., $\overline{r}^F_{t+1}$) only depends on aggregate factors in the economy. Now, by using eq. (35), the aggregate net worth transition equation becomes

$$n^F_{t+1} = \gamma^F_{t+1}\left[\left(\overline{r}^F_{t+1} - r_{t+1}\right)\phi_t + r_{t+1}\right]n^F_t + \xi^F n^F_t. \tag{38}$$

(**Nishi Note:** Need more explanation here. What is net worth shock in banking sector? By construction of this paper, there is no price for corporate debt. Consequently, real asset of banking sector is not affected by the reduction in the market price of corporate bond. This is not a realistic setup, but we try to capture this kind of shock by the net worth shock in this paper. In the future, we may try to incorporate the market price of corporate bond. Now, the shock to asset-side of balance sheet may be intrinsically different from net worth shock which a shock to the right-hand side of balance sheet.)

2.4 Capital Production Sector

We now turn to a capital producer's problem. Capital producers are identical, perfectly competitive, and risk neutral. They purchase i^k_t units of final goods from the retailer, convert them to i^k_t units of capital goods, and combine them with existing capital stock, $(1-\delta)k_t$, to produce new capital stock, k_{t+1}. Capital producers will, then, sell off new capital stock to entrepreneurs in a perfectly competitive manner. Capital producers have linear production technology in converting final goods to capital goods. However, following Smets and Wouters (2003) and CEE, when they change the production capacity of capital goods from previous period, they will incur quadratic investment adjustment cost. Given this set up, the profit function for each capital producer at period t can be expressed as follows,

$$E_t \sum_{i=0}^{\infty} \beta^i \left\{ q_{t+i} i^k_{t+i} - \frac{1}{A^k_{t+i}} \left[i^k_{t+i} + \frac{\kappa}{2}\left(\frac{i^k_{t+i}}{i^k_{t+i-1}} - 1\right)^2 i^k_{t+i} \right] \right\} \tag{39}$$

where A^k_t stands for investment-specific technology shock common across all capital producers and parameter κ stands for investment adjustment cost parameter. Each capital producer will maximize the expected discounted cash flow with respect to i^k_t. The first order condition is given by

$$q_t = \frac{1}{A^k_t}\left[1 + \kappa\left(\frac{i^k_t}{i^k_{t-1}} - 1\right)\frac{i^k_t}{i^k_{t-1}} + \frac{\kappa}{2}\left(\frac{i^k_t}{i^k_{t-1}} - 1\right)^2\right] - \beta\frac{\kappa}{A^k_{t+1}}\left(\frac{i^k_{t+1}}{i^k_t} - 1\right)\left(\frac{i^k_{t+1}}{i^k_t}\right)^2. \tag{40}$$

Finally, aggregate capital accumulation equation is given by

$$k_{t+1} = i^k_t + (1-\delta)k_t. \tag{41}$$

2.5 Retailing Sector

Here, we describe the optimal price setting behavior of the continuum of retailers indexed by $z \in [0,1]$. Each retailer purchase intermediate goods from the entrepreneur at perfectly competitive price and resale them monopolistically in the retail market. The demand function for retail goods sold by sold by retailer z is given by

$$y_t(z) = \left(\frac{p_t(z)}{P_t}\right)^{-\epsilon} Y_t, \tag{42}$$

where Y_t stands for CES-aggregated final goods *á la* Dixit and Stiglitz (1971), $p_t(z)$ stands for nominal price of retail goods $y_t(z)$, P_t stands for aggregate price index of final goods Y_t, and parameter ϵ stands for the price elasticity of retail goods $y_t(z)$. Specifically, aggregated final goods, Y_t, and the aggregate price index, P_t, are, respectively, given as follows;

$$Y_t \equiv \left[\int_0^1 y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz\right]^{\frac{\epsilon}{\epsilon-1}} \text{ and}$$

$$P_t \equiv \left[\int_0^1 p_t(z)^{\frac{1}{1-\epsilon}} dz\right]^{1-\epsilon}.$$

We assume Calvo (1983) - Yun (1996) type sticky price setting for the retailer where, for any given period t, fraction θ^p of the entire retailers cannot freely revise their prices. Further, following the treatment of Smets and Wouters (2003, 2007) and CEE in modeling inflation persistence, we allow for the partial indexation by the retailers who were not able to revise their prices at period t. Specifically, θ^p fraction of the retailers who did not receive a 'signal of price change' will partially index their nominal prices to lagged inflation rate of price index as follow,

$$p_t(z) = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\iota^p} p_{t-1}(z) \tag{43}$$

where parameter $\iota^p \in [0,1]$ controls for the magnitude of price indexation to past inflation rate.

Under this setting, for $(1-\theta^p)$ fraction of the retailers who received a 'price changing signal' at period t, they will maximize their expected discounted sum of profits by setting the nominal price, $\tilde{p}_t$, such that

$$E_t \sum_{i=0}^{\infty} \beta^i (\theta^p)^i \left[\frac{\widetilde{p}_t}{P_{t+i}} \left(\frac{P_{t-1+i}}{P_{t-1}}\right)^{\iota^p} - \left(\frac{\epsilon}{\epsilon-1}\right) p^{mc}_{t+i}\right] y_{t+i}(z) = 0. \tag{44}$$

From the definition of aggregate price index, the law of motion of P_t can be shown to be as follow,

$$(P_t)^{1-\epsilon} = \theta^p \left[P_{t-1}\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\iota^p}\right]^{1-\epsilon} + (1-\theta^p)\tilde{p}_t^{1-\epsilon}. \tag{45}$$

2.6 The Rest of the Economy

In closing the model, we describe the rest of the model structure here. Since the model is already quite large in size, we will keep the rest of the model structure as simple as possible.

The central bank is assumed to follow a standard Taylor-type monetary policy rule so that the nominal interest rate is adjusted in response to the movement in inflation gap and output gap with some interest rate smoothing by the central bank. In a log-deviation form, the monetary policy rule is specified as follow,

$$\hat{R}_t = \rho^R \hat{R}_{t-1} + (1-\rho^R)\left[\mu^\pi \hat{\pi}_t + \mu^y \hat{Y}_t\right] + \varepsilon_t^R \tag{46}$$

where parameter ρ^R controls the magnitude of interest smoothing, parameter μ^π stands for Taylor coefficient in response to inflation gap (i.e., deviation of inflation rate from the inflation target), parameter μ^y stands for Taylor coefficient in response to output gap, and ε_t^R stands for *i.i.d.* monetary policy shock. In our context, monetary policy shock can be regarded as an unexpected deviation of the nominal interest rate *vis-á-vis* Taylor rule at period t.

The government budget constraint is simply specified as

$$g_t = \tau_t. \tag{47}$$

The government expenditure, g_t, is financed solely by lump-sum tax, τ_t, which appears in the representative household's budget constraint eq. (2). The government is assumed to operate on a balanced budget every period without running any deficit or surplus. In our model, we simply assume that the government expenditure to follow stochastic AR(1) process.

Finally, the market clearing condition for final goods is given as follow,

$$Y_t = c_t + i_t^k + g_t. \tag{48}$$

2.7 Structural Shocks in the Model

There are 8 structural shocks in the model, each of them having a specific economic interpretation. Here, we specify the stochastic process of each shock (in a log-linearized form) and annotate its economic interpretation as follows.

$$\begin{aligned}
\text{Total factor productivity shock} \quad &: \quad \hat{A}_t = \rho^A \hat{A}_{t-1} + \varepsilon_t^A \\
\text{Intertemporal preference shock} \quad &: \quad \hat{\chi}_t^c = \rho^c \hat{\chi}_{t-1}^c + \varepsilon_t^c \\
\text{Labor disutility shock} \quad &: \quad \hat{\chi}_t^L = \rho^L \hat{\chi}_{t-1}^L + \varepsilon_t^L \\
\text{Investment-specific technology shock} \quad &: \quad \hat{A}_t^K = \rho^K \hat{A}_{t-1}^K + \varepsilon_t^K \\
\text{Government expenditure shock} \quad &: \quad \hat{g}_t = \rho^G \hat{g}_{t-1} + \varepsilon_t^G \\
\text{Monetary policy shock} \quad &: \quad \varepsilon_t^R \\
\text{Entrepreneurial net worth shock} \quad &: \quad \hat{\gamma}_t^E = \rho^E \hat{\gamma}_{t-1}^E + \varepsilon_t^E \\
\text{Banking sector net worth shock} \quad &: \quad \hat{\gamma}_t^F = \rho^F \hat{\gamma}_{t-1}^F + \varepsilon_t^F.
\end{aligned}$$

Except for monetary policy shock, all of the structural shocks are assumed to follow AR(1) stochastic processes where parameter ρ's stand for the AR(1) coefficients for respective structural shocks and random variable ε_t's stand for i.i.d. normally distributed disturbance terms for respective structural shocks specified above (including monetary policy shock). Notice that above 8 structural shocks are all aggregate shocks in the economy, commonly affecting all members in the household.

In our paper, $\hat{\gamma}_t^E$ and $\hat{\gamma}_t^F$ deserve special attention. As noted before, at micro-level, both structural shocks have an interpretation of stochastic survival rate for entrepreneurs and bankers, respectively. However, at aggregate level, both structural shocks can be interpreted as aggregate net worth shock for entrepreneurial sector and banking sector, respectively. In the empirical analysis conducted in this paper, we will emphasize the interpretation of $\hat{\gamma}_t^E$ and $\hat{\gamma}_t^F$ at aggregate level since we are interested in how the aggregate movements of entrepreneurial and banking sector net worth are caused and how those aggregate movements affect other macroeconomic variables, such as average bank lending rate, average corporate borrowing rate, business fixed investment and GDP of the economy. Further, when identifying Lehman Shock which was clearly an aggregate shock at the financial sector, we hope to capture the impact of the shock via the banking sector net worth shock – i.e., $\hat{\gamma}_t^F$. If indeed $\hat{\gamma}_t^F$ is a good proxy for Lehman Shock, the estimated (or smoothed) $\hat{\gamma}_t^F$ should decline significantly at 2008Q3. The estimation result will be reported at Section 5 of this paper.

3 Data-Rich Estimation Method

In this section, we explain the Data-Rich estimation method proposed by Boivin and Giannoni (2006) and describe how we apply this Data-Rich estimation method to our DSGE model. The idea of Data-Rich estimation is to extract the common factor F_t from *massive panel* of macroeconomic and financial time series data X_t and to match the state variable S_t of the model to the extracted common factor F_t. A virtue of this approach is that even if the definition or the concept of a state variable S_t and observed data X_t are slightly detached, one can estimate the model by matching state variables to the common factors extracted from large panel data (i.e., data-rich environment), one can expect improved efficiency in estimating the parameters and structural shocks of the model.

Following Boivin and Giannoni (2006) and Kryshko (2010), general framework of a Data-Rich DSGE model is described. Recently, the estimation method of dynamic factor models are rapidly developed and applied for many fields of macroeconomics and finance. A Data-Rich DSGE model is also applied as one type of dynamic factor models. We firstly explain dynamic factor models and then turn to a data-rich DSGE model.

3.1 Dynamic Factor Model

Dynamic Factor Models, which are a statistical model estimating common factors of business cycles, are proposed by Sargent and Sims (1977) and empirically applied by Stock and Watson (1989) who extract one unobserved common factor of business fluctuation from many macroeconomic time series using Kalman filter. And also Stock and Watson(2002a,b) developed *approximate* Dynamic Factor Models using principal component analysis, extracting several common factors from more than one hundred macroeconomic time series and verifying that these factors include useful information on forecasting of macroeconomic time series.

The Dynamic Factor Models are represented by state space models composed from following three linear equations. Let F_t denote the $N \times 1$ vector of unobserved common factor, and X_t denote the $J \times 1$ vector of massive panel of macroeconomic and financial data. Note that $J >> N$.

$$X_t = \Lambda F_t + e_t, \tag{3.1}$$

$$F_t = \mathbf{G} F_{t-1} + \varepsilon_t, \quad \text{where} \ \ \varepsilon_t \sim iid\ N(0, \mathbf{Q}), \tag{3.2}$$

$$e_t = \Psi\ e_{t-1} + \nu_t, \quad \text{where} \ \ \nu_t \sim iid\ N(0, \mathbf{R}), \tag{3.3}$$

where Γ is $J \times N$ matrix of factor loadings, e_t is the idiosyncratic errors which is allowed to be serially correlated as equation (3.3). $\mathbf{G}$ is $N \times N$ matrix and common factor F_t is following AR process (3.2). Matrices, Ψ, $\mathbf{Q}$ and $\mathbf{R}$ are assumed to be diagonal in an *exact* dynamic factor model as Stock and Watson (2005). (3.1) is a measurement equation, and (3.2) is a transition equation. A state space model is composed from the two equations. Stock and Watson (1989) estimated state space model (3.1), (3.2), (3.3) using Maximum Likelihood Estimation (MLE) with Kalman Filter, since the model is based on linear and Gaussian model.

The advantage of the model is to extract common component ΓF_t and idiosyncratic component e_t from massive panel of macroeconomic and financial data X_t. Meanwhile, it is difficult to make an interpretation of factor F_t economically, since above equations are statistically estimated like reduced models and the parameters are not derived from economic foundation like structural models. A Data-Rich DSGE model, however, holds the advantage of dynamic factor model and overcomes the drawback.

3.2 Data-Rich DSGE Model

DSGE models are also known to be state space models and estimated using Kalman filter as well as the dynamic factor model. So we can apply the framework of the dynamic factor model to a DSGE model. But the big difference between a dynamic factor model and a DSGE model is the meaning of their parameters. The parameters of the DSGE model are based on from foundation of rational expectation theory in which economic agents solve intertemporal optimization problem. In particular, its core parameters are referred to as *deep parameters* which govern the rational behaviors of economic agents. The law of motion around steady state of the model solved from log-linear approximation is written by (3.4) and (3.5).

$$z_t = \mathbf{D}(\boldsymbol{\theta})\ S_t, \tag{3.4}$$

$$S_t = \mathbf{G}(\boldsymbol{\theta}) S_{t-1} + \mathbf{H}(\boldsymbol{\theta})\varepsilon_t, \quad \text{where } \varepsilon_t \sim iidN(0, \mathbf{Q}(\boldsymbol{\theta})), \tag{3.5}$$

z_t is a vector of non-predetermined endogenous variables (so-called jump variables) and S_t is a vector of predetermined endogenous variables and exogenous variables. ε_t is a vector of exogenous shocks. θ is deep parameters and $G(\theta)$ is matrix of parameter derived from deep parameter θ. (3.4) is a policy function of jump variables z_t for state variables S_t.

Let $\bar{S}_t$ denote all model variables including endogenous and exogenous variables, and we replace it by state vector S_t as following.

$$\bar{S}_t = \begin{bmatrix} z_t \\ S_t \end{bmatrix} = \begin{bmatrix} D(\theta) \\ I \end{bmatrix} S_t,$$

Representing common factor F_t from state variables S_t, we can get equation (3.6).

$$F_t = F\bar{S}_t = F \begin{bmatrix} D(\theta) \\ I \end{bmatrix} S_t, \tag{3.6}$$

And we get (3.7) by substituting (3.6) into (3.1).

$$X_t = \Lambda(\theta) S_{t-1} + e_t, \tag{3.7}$$

In a DSGE model, state space model are composed from (3.5) and (3.7). If we recognize the presence of the idiosyncratic components e_t, i.e., $e_t \neq 0$, e_t is regard as measurement errors serially correlated and we can extended the model as (3.5), (3.7) and (3.3) like the dynamic factor model (3.1), (3.2) and (3.3). In a regular DSGE model, there is one-to-one relation between data indicator X_t and state variables (or model concept) S_t, so that matrix $\Lambda(\theta)$ is $N \times N$ Identity matrix. But the drawback of this type of DSGE model is hardly to estimate (3.7) with measurement errors e_t neither to identify between exogenous shocks ε_t and measurement errors e_t. On the other hand, in a Data-Rich DSGE model there is many-to-many relation between X_t and S_t, so that matrix $\Lambda(\theta)$ is $J \times N$. $(J >> N)$ This type is obviously complicated but can grasp theoretical gap between data indicator X_t and model concept $\bar{S}_t$ from three equation (3.5), (3.7) and (3.3).

In the Data-Rich DSGE model, data indicator X_t is divided into two type, the first type is *sensor series* which corresponds to only one variable of model concepts $\bar{S}_t$. Another type is *information series* which is not directly relation with specific model concepts $\bar{S}_t$ but hold useful information on the law of motion in the DSGE model. Classifying the two type of data indicator, we rewrite (3.7) as (3.8).

$$\begin{bmatrix} X_{\text{sensor},t} \\ X_{\text{info},t} \end{bmatrix} = \begin{bmatrix} \Lambda(\theta)_{\text{sensor},t} \\ \Lambda(\theta)_{\text{info},t} \end{bmatrix} S_t + e_t, \tag{3.8}$$

Alternatively, representing (3.8) as (3.9), the framework of a Data-Rich DSGE model might be more understandable. As can be seen from the second row of matrix $\Lambda(\theta)$: $[\lambda_{y2}\ 0\ \cdots\ 0]$, sensor series of data is directly relation with only one model concept S_t. And in order to specify the magnitude of each model concept, the value of λ of just one variable of sensor series is unity as the first row of matrix $\Lambda(\theta)$: $[1\ 0\ \cdots\ 0]$. Meanwhile, the parameters of information series can be represented by linear combination of all model variable such as $\lambda_{11}\ \ \lambda_{12}\ \ \cdots\ \ \lambda_{1n}$ in matrix Λ.

$$
\underbrace{\begin{bmatrix} \text{Output Gap seires } \# 1 \\ \text{Output Gap seires } \# 2 \\ \vdots \\ \text{Output Gap seires } \# n_y \\ \text{inflation series } \# 1 \\ \text{inflation series } \# 2 \\ \vdots \\ \text{inflation series } \# n_\pi \\ \vdots \\ -\,-\,-\,-\,-\,-\,- \\ \text{information series } \# 1 \\ \vdots \\ \text{information series } \# n_i \end{bmatrix}}_{X_t} = \underbrace{\begin{bmatrix} 1 & 0 & \cdots & 0 \\ \lambda_{y2} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_{yn} & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ 0 & \lambda_{\pi 2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \lambda_{\pi n} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ & & & \\ \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_{ni1} & \lambda_{ni2} & \cdots & \lambda_{nin} \end{bmatrix}}_{\Lambda(\theta)} \underbrace{\begin{bmatrix} x_t \\ \pi_t \\ \vdots \\ S_{last,t} \end{bmatrix}}_{S_t} + e_t,
$$

4 Preliminary Settings and Data Description

4.1 Observed Variables and Measurement Errors

There are 11 observed variables in our empirical analysis and they are output (y_t), consumption (c_t), investment (i_t^k), inflation (π_t), real wage (w_t), labor input (l_t), nominal interest rate (R_t), nominal corporate borrowing rate (R_t^E), external finance premium (s_t), corporate leverage ratio ($q_t k_t / n_t^E$), and bank leverage ratio (b_t^E / n_t^F). Among 11 observed variables, 7 of them are macroeconomic variables which are commonly observed in empirical DSGE literature (see, for instance, Smets and Wouters (2003, 2007)). In addition to these macroeconomic variables, taking advantage of the financial structure of our model, we render 4 financial variables – R_t^E, s_t, $q_t k_t / n_t^E$, and b_t^E / n_t^F – observable in our estimation. In order to identify financial shocks such as entrepreneurial net worth shock and banking sector net worth shock, it is crucial to match the financial variables in the model to the actual financial data. Now, all of the observed variables are transformed into log-deviation from steady state and matched with the actual data. (Detrending and transformation method of the actual data will be explained shortly.)

Following Boivin and Giannoni (2006), we allow for the existence of measurement errors for all the observed variables. The reason for allowing measurement is twofold. First, most of the macroeconomic data, including NIPA, are estimated or aggregated statistics based on sample survey data or micro data. As such, estimation error or aggregation error are inherently present in the macroeconomic statistics. Second, the definition of the actual data may be detached or different from the model's concept of those variables. In order to subdue the effect from this possible discrepancy between the definition of the data and the concept of model variables, we allow for the measurement errors in our estimation. On the contrary, if the estimated observed variables are reasonably close with the actual data, then, by construction, the magnitude of the measurement error will be ignorable.

Measurement error for each observed variable is allowed for serial correlation and is specified as follows.

$$\begin{aligned} e_{obs,t} &= \delta^{obs} e_{obs,t-1} + u_{obs,t} \ \text{ for each } obs \in \left\{y, c, i^k, \pi, w, l, R, R^E, s, lev^E, lev^F\right\} \\ & \quad \text{where } u_{obs,t} \overset{\text{i.i.d.}}{\sim} N(0, \sigma^2_{obs}) \end{aligned}$$

Here, lev^E stands for corporate leverage ratio and lev^F stands for banking sector leverage ratio.

4.2 Calibration and Priors

We calibrate the subset of the structural parameters in the model that are not identifiable (i.e., the parameters that are only used to pin down the steady states) or are difficult to identify from the observed data. Calibrated parameters with their descriptions are reported in Table 1. Although most of the calibrated parameters are self-evident from the table, some of them need some explanations here. We assume discount factor $\beta = 0.995$ so as to make the steady state real interest rate to be 2% (annual rate). We assume the profit margin of the retailers to be 10% in steady state and, thus, set elasticity of substitution of intermediate goods as $\epsilon = 11$. We have no reliable information regarding the new entry rate of entrepreneurs (i.e., ξ^E) and will simply set it equal to the calibration for new banker's entry rate by Gertler and Kiyotaki (2010). The rest of the calibrated parameter values are borrowed from Smets and Wouters (2003), Christensen and Dib (2008), and Gertler and Kiyotaki (2010).

Regarding the steady states, most of them are pinned down by equilibrium conditions of the model, but some others need to be calibrated. For the steady state value of external finance premium, we follow the calibration of Christensen and Dib (2008). For the steady state corporate borrowing rate (real, quarterly rate), we calculate the historical average of the yields of Moody's Baa-rated corporate bonds and set it as the steady state rate. By the same token, we calculate the historical average of the non-farm, non-financial business leverage ratio based on Flow of Funds and set it as the steady state of corporate leverage ratio. Finally, the government expenditure to output ratio in steady state is set be 0.2 which is borrowed from Gertler and Kiyotaki's (2010) calibration.

Next, as a preamble for Bayesian estimation, we set prior distributions for the parameters that will be estimated in this paper. The settings of priors are reported in Table 2. For structural parameters, we reflect our prior beliefs for each parameter. In particular, parameter φ is quite important for our purpose, since this parameter controls the sensitivity of external finance premium with respect to corporate leverage ratio. We set 0.05 for the prior mean of this parameter following the calibration of BGG. For AR(1) persistence parameters for structural shocks, since we do not have a good prior belief regarding these parameters, we set prior mean equal to 0.5 for all of them. For standard errors of structural shocks, again, we have no good prior belief, except for monetary policy shock (where a change of policy rate for more than 25 basis point is rare). Thus, we set prior mean equal to 1% for each standard error, except for monetary policy shock. By the same token, we set prior mean equal to 1% for most of the measurement errors, except for the data related with interest rates. Finally, notice that we do not set priors for AR(1) persistence parameters for measurement errors. This is because we use OLS in estimating these parameters in data-rich estimation routine.

4.3 Data Description

Our data set consists of 21 quarterly macroeconomic and financial time series data to estimate the data-rich DSGE model. We employ 11 data series for the estimation of Case A, and combine additional 10 data series with Case A data for the estimation of Case B. Details on our data set and data arrangements are provided in Data Appendix.

For the same reasons of Boivin and Giannoni (2006) and Kryshko (2010), we estimate the model starting in 1985Q2 and ending in 2010Q2, accounting for the instability of monetary policy regime especially around the end of the 1970's and early 1980's,[14] and avoiding the issue of the "Great moderation" since mid-1980's.[15] Additionally, our estimation period is subject to the restriction of the financial data availability.[16]

In the estimation of Case A, we employ 11 data series which are grouped into two categories: seven data series used by Smets and Wouters (2007) for the estimation of their model, and four financial data series.

Seven data series are output, consumption, investment, inflation, real wage, labor input, and nominal interest rate. Output is real GDP. Consumption and investment are normalized respectively to personal consumption expenditures and fixed private domestic investment.[17] The labor input corresponds to hours worked per person.[18] The real wage is normalized with the hourly compensation for the nonfarm business sector, divided by the GDP deflator. We express all these series as percentage deviations from respective trends consistently with model concepts, taking the natural logarithm, extracting the linear trend by an OLS regression, and multiplying the resulting detrended series by 100. Inflation indicator is computed as the first difference of the natural logarithm of the GDP deflator, and multiplied by 400 to transform into the annualized percentages. The nominal interest rate is the effective Federal funds rate. Both inflation and the interest rate are detrended via Hodrick-Prescott filter (penalty parameter is 1600), accounting for time-varying targeting inflation rate.

In order to identify financial shocks in our model, we employ four financial data series: leverage ratios of banking sector and non-farm non-financial corporate sector, entrepreneur's nominal borrowing rate, and charge-off rate for banks. Leverage ratios are calculated as total asset over net worth. We take natural logarithm for both leverage ratios, then, demean for entrepreneur's leverage ratio, while banking sector leverage ratio is detrended via Hodrick-Prescott filter, taking into a consideration with Basel capital accord revision. Entrepreneur's nominal borrowing rate is the yield on Moody's Baa-rated corporate bonds, which is also detrended via Hodrick-Prescott filter for the same reason of inflation and the interest rate. To measure the model concept of the external financial premium, we employ the charge-off rates for all banks credit and issuer loans, measured as an annualized percentage of uncollectible loans. We regard the charge-off rate for banks as a proxy of the model concept for the external financial premium. The charge-off rate is demeaned to be consistent with our model concept.

[14] See Clarida, Gali and Gertler (2000) and Boivin (2006)

[15] See Stock and Watson (2002a), Kim and Nelson (1999), and McConnell and Perez-Quiros (2000).

[16] The financial data of charge-off rate for banks is available only from 1985Q1.

[17] Following Smets and Wouters (2007), the nominal series for consumption and investment are deflated with the GDP deflator.

[18] Average hours of nonfarm business sector are multiplied with civilian employment to account for the limited coverage of the nonfarm business sector, compared to GDP, as in Smets and Wouters (2007).

For the estimation of Case B, which corresponds to data-rich environment, we employ additional 10 data series. Following Boivin and Giannoni (2006), additional consumption data is real personal consumption expenditures for nondurable goods, and additional investment data is gross private domestic investment. Additional two indicators for inflation consist of price deflator of private consumption expenditure (PCE) and core CPI index excluding food and energy, and additional two indicators for labor input consist of civilian labor force and non-farm corporate employees. Furthermore, we employ four additional financial data. Additional two indicators for banking sector leverage ratio consist of the core capital leverage ratio and the domestically chartered commercial banks' leverage ratio. The core capital leverage ratio represents tier 1 (core) capital as a percent of average total assets.[19] We use the reciprocal number of the core capital leverage ratio, so as to match with the asset/net worth leverage ratio in Case A, then taking natural logarithm and detrending via Hodrick Prescott filter. Finally, additional two indicators for external financial premium consist of charge-off rate on all loans and leases of all commercial banks and on all loans of all commercial banks. We also express additional data series as percentage deviations from trends using similar detrending methods described in Case A.

5 Estimation Results

We report our estimation results in this section.[20] The model has been estimated based on Case A data set which is a data set for standard Bayesian estimation and Case B data set which is a data set for data-rich estimation. We compare the estimation results from both data sets and report the differences and similarities, paying special attention with regard to the banking sector net worth shock. For this version of our draft, our estimation results are based on 100,000 MH samplings for both Case A and Case B data set. In the future revision of the paper, we plan to increase the sampling size and include the estimation results from Case C data set.

5.1 Posterior Means and Estimated IRF's

Table 3 reports the posterior means from Case A and Case B data sets. Since the number of estimated parameters are massive, we confine our attention to the posterior means of the selected parameters. Among the structural parameters, one of the more important ones is φ which is the key parameter in linking corporate leverage ratio to external finance premium. The posterior mean was 0.041 based on Case B which is slightly higher than that of Case A. It is notable that the posterior mean based on Case B turned out to be remarkably close to the estimation result reported in Christensen and Dib (2008) which was 0.042. Turning to the structural parameters governing the nominal rigidities of price and wage (i.e., ι^p, ι^w, θ^p, and θ^w), the results were reasonably close to the estimates reported in Smets and Wouters (2007).

Another posterior estimate that deserve particular attention is, of course, the standard deviation of banking sector net worth shock. The posterior mean of the standard deviation

[19] Tier 1 capital consists largely of equity.

[20] We adopt Sims (2002) method in solving for our DSGE model described in Section 2 and all estimation procedures are implemented using GAUSS.

was 1.060 under Case A and 0.903 under Case B – volatility of the shock turned out to be slightly smaller under data-rich estimation. Further, the posterior standard error of the standard deviation of shock was 0.075 under Case A and 0.058 under Case B, possibly a sign that the banking sector net worth shock has been estimated more accurately and efficiently under data-rich estimation. Similar tendency can be observed for other standard deviation estimates of structural shocks, albeit some exceptions.

Based on Case A estimation, impulse response functions of entrepreneur net worth shock and banking sector net worth shock are depicted in Figure 1. Impulse response functions (IRF) are based on one standard deviation positive shock of net worth shocks. Upper panel of Figure 1 shows the IRF's for observed variables in the model. Lower panel shows the IRF's for selected unobserved variables. In the figure, blue line depicts the IRF's of entrepreneur net worth shock and red line depicts the IRF's of banking sector net worth shock.

The aim of Figure 1 is to exemplify how we identify (or distinguish) two financial shocks – i.e., entrepreneur net worth shock and banking sector net worth shock. For observed macroeconomic variables, both shocks reveal qualitatively same IRF patterns. For instance, positive entrepreneur and banking sector net worth shocks both contributes to an increase in output, although quantitative magnitudes are different. For the sake of identification of shocks, qualitatively equivalent patterns of IRF are problematic. If, for instance, only macroeconomic variables were observable, but not for financial variables, there will be an empirical difficulty in identifying two financial shocks, let alone an accurate estimates of financial shocks, because observed macroeconomic variables will basically respond in a same direction with respect to both net worth shocks.[21] Here, in our estimation, four financial variables – corporate borrowing rate, external finance premium, corporate leverage, and bank leverage – are made observable in a hope to identify two financial shocks.

As it turns out, the key financial variable that distinguishes two financial shocks is the bank leverage ratio. Patterns of IRF's of other three financial variables are qualitatively similar, while for bank leverage ratio, the patterns are qualitatively different for two net worth shocks. In response to entrepreneur net worth shock, the bank leverage ratio decreases for brief period, but then rises and stays significantly positive thereafter. On the contrary, in response to banking sector net worth shock, bank leverage decreases, stays negative longer than the case of entrepreneur net worth shock, and asymptotes back more or less to zero. Thus, taking advantage of this qualitative difference in the IRF of bank leverage is crucial for estimation purpose. Indeed, in data-rich estimation in our paper, we supply two additional bank leverage data series in addition to the one used in Case A in order to assure the identification of banking sector net worth shock.

(**Nishi Note:** More explanation on bank leverage IRF. Refer to bank lending IRF and banking sector net worth. Then to bank lending - deposit rate spread.)

Figure 2 compares the IRF's based on Case A and Case B estimation results. Upper panel is for entrepreneur net worth shock and lower panel is for banking sector net worth shock. The IRF's are more or less similar for both Case A and Case B, although there seem to be a tendency for Case A to underestimate the magnitude of output, investment, and corporate borrowing rate responses to both net worth shocks compared to those under Case B. These

[21] Nishiyama (2009) demonstrates this problem. He estimates a DSGE model with two financial shocks only by using macroeconomic data, but was not able to identify financial shocks. However, by supplying enough financial data, he successfully identifies two financial shocks.

differences in the magnitude of IRF's may be attributable to the difference in the posterior mean of φ where Case A is lower than Case B, implying that external finance premium is relatively insensitive vis-á-vis the change in corporate leverage ratio under Case A. This leads to a smaller change of corporate borrowing rate which, in turn, makes the response of investment and output smaller as well under Case A. Now, one notable observation from Figure 3 is that while nominal interest rate (or, equivalently, deposit rate) is rising in response to both net worth shocks, corporate borrowing rate is actually decreasing. This implies that the total spread (i.e., R_t^E/R_t) actually shrinks in response to both positive net worth shocks where the size of shrink can be as much as 40 basis points for entrepreneur net worth shock (one standard deviation) and as much as 15 basis points for banking sector net worth shock.

5.2 Smoothed Observable Variables

The results of smoothed observable variables are shown in Figure 3. Upper panel shows the smoothed results for Case A and lower panel shows the results for Case B. Notice that there are 21 smoothed results for Case B, since total of 21 data series were supplied in data-rich estimation. Additional data series on top of Case A is indicated by the number after the name of observables. For instance, the name 'Consumption 2' indicates that this is an additional observation on top of Case A's consumption data.

By taking a look at the smoothed results of Case B, we notice that some additional data series to be informative while some others are not. Consumption 2, Investment 2, Inflation 3, Labor 2, Bank Leverage 2, and Bank Leverage 3 look informative and seem to share common factors with the data observed in Case A.[22] Unfortunately, however, other additional data such as Inflation 2 and External Premium 2 do not seem to be informative at all.

Now, in order to see how data-rich smoothing compares with Case A smoothing results, see Figure 4. As can be seen, smoothing results from Case A and B are more or less close with each other, especially for macroeconomic variables. Now, turning to financial variables, we notice some discrepancies between Case A and Case B smoothing in external finance premium and corporate leverage throughout the sample period. We also notice a significant discrepancies in all financial variables especially in the period of Lehman Shock.

Another important comparison is the accuracy and efficiency of smoothing between Case A and Case B. By comparing the 90% credible interval of smoothed variables, we notice some efficiency gain under Case B, especially for consumption, nominal interest rate, external finance premium, corporate leverage, and bank leverage. This efficiency gain especially for financial variables are encouraging sign for us, since our focus of this paper is to estimate the financial shocks accurately and efficiently. Perhaps this efficiency gain has been brought thanks to the additional financial information in our data-rich estimation.

5.3 Estimated Structural Shocks

We now turn to the estimation results of the structural shocks which is shown in Figure 5. First thing we notice is a sharp negative spike for banking sector net worth shock (ε_t^F) observed in 2008Q3 and 2008Q4 which coincides with the timing of Lehman Shock. Since our aim was to capture Lehman Shock by banking sector net worth shock in the model, this

[22] For the specific descriptions of the additional data in Case B, see Data Appendix.

is an encouraging sign for our purpose. Further, by looking at the estimation result for entrepreneur net worth shock, we do not observe any conspicuous spike at 2008Q3 or Q4. This is can be considered as an evidence that the banking sector net worth shock is distinguished from entrepreneur net worth shock and clearly identified. Further, identification of banking sector net worth shock seems to be attained in both Case A and Case B.

Once a hurdle of identification is cleared, next comes how accurately and efficiently we can estimate the banking sector net worth shock. Let us see how data-rich estimation can serve this purpose. As can be observed from Figure 5, although the estimates from both Case A and Case B are quite similar, we notice some discrepancies in the estimates of the structural shocks here and there. Especially for banking sector net worth shock at the timing of Lehman Shock, we observe that Case B estimate to be considerably larger in magnitude compared to Case A. Also, regarding the efficiency of the estimates of structural shocks, we observe some slight efficiency gain in both entrepreneur and banking sector net worth shock. Unfortunately, efficiency gain for other structural shocks are too subtle or not noticeable.[23]

While the results in Figure 5 focused on the *i.i.d.* portion of the structural shock, it is also important to smooth the state of shock. Figure 6 shows the smoothed state of $\hat{\gamma}_t^F$. Notice that since the posterior mean of ρ^F is estimated to be quite low (0.090 in Case A and 0.191 in Case B), the overall impression of smoothed $\hat{\gamma}_t^F$ is not that different from *i.i.d.* bank net worth shock. As it is clear from Figure 6, we observe a sharp negative dip of $\hat{\gamma}_t^F$ around the period of Lehman Shock. The magnitude of dip is largest throughout the entire sample period (1985Q2 to 2010Q2) meaning that the impact of Lehman Shock as a banking sector net worth shock has been the worst in past 25 years, much more than those during S&L Crisis era. (**Nishi Note:** Write something about TARP. In other words, the effect of TARP to banking sector net worth. A sharp rise in banking sector net worth shock right after Lehman Shock must be attributed to TARP.)

Now, we note some differences between Case A and Case B smoothing of banking sector net worth shock. The first thing we note is the difference in the impact of Lehman Shock between Case A and Case B. While the estimate of Lehman Shock seems remarkable under Case B, it seems somehow underrated under Case A. Further, taking a look at 90% credible interval for both cases, while Case B's upper bound credible interval positions well below zero, that of Case A positions above zero.[24] From the episode of Lehman Shock that we are familiar with, it seems to be the case that the estimate of Lehman Shock under Case B is more reliable than that of Case A, thanks to the virtue of data-rich estimation. Finally, let us not forget the efficiency gain under Case B. In other words, 90% credible interval under Case B is much tighter than that of Case A, again thanks to the merit of data-rich estimation.

5.4 Historical Decompositions

Next, based on the estimates of structural shocks in the model, we conduct historical decomposition exercise for selected observable variables. The aim of the historical decomposition

[23] Indeed, it seems to be the case that efficiency is lost for TFP shock under Case B.

[24] Or using the classical statistics terminology, we cannot reject the null hypothesis such that Lehman Shock was innocuous ($H_o : \hat{\gamma}^F_{2008Q3} = 0$) under Case A, but under Case B, we can reject this hypothesis. Of course, since our results are based on Bayesian estimation, the above statement is a misusage of terminology.

exercise here is to demonstrate how the differences in the estimates of structural shocks may lead to a different accounting of the historical movement of the endogenous variables. Now, recall the result of Figure 5 which reports the estimates of structural shocks. There, we have noted that the estimates of the structural shocks under Case A and Case B are more or less similar – i.e., differences are subtle. Yet, as it turns out, this subtle differences in the estimates of structural shocks will lead to a remarkably different accounting of historical movement. Let us turn to each historical decomposition one after another.

Figure 7 shows the historical decomposition results for output. The result under Case A is shown at the upper panel and the result under Case B is shown at the lower panel. Let us compare the results from Case A and Case B and focus on the differences. We notice that the historical accounting of in early 90's (the period right after S&L crisis) to be quite different. Under Case A, it emphasize the role of corporate net worth shock in accounting for a dip in output, while Case B result play down the role of corporate net worth shock, yet emphasizing the role of labor supply shock and investment specific technology shock in accounting for a dip. Turning to the period of Lehman Shock, Case A and Case B seem to disagree on the magnitude of the investment specific technology shock.[25]

Figure 8, which shows the historical decomposition of investment, tells a similar story with Figure 7. Case A emphasize the role of corporate net worth shock during the early 90's, while Case B de-emphasize it. Case A and Case B also disagree on the magnitude of investment specific technology shock during and after Lehman Shock (although they agree on the qualitative direction). Indeed, since investment is an important factor in GDP, no wonder the results of Figure 7 and 8 to be similar.

Figure 9 shows the historical decomposition of corporate borrowing rate. The movement of corporate borrowing rate is affected by variety of structural shocks as can be seen from the figure. One noticeable difference between Case A and Case B is that while Case A emphasize the importance of corporate net worth shock, on the contrary, Case B seem to emphasize the importance of bank net worth shock.

Figure 10 shows the historical decomposition of corporate leverage. Here, before we point out the differences of accounting by Case A and Case B, we would like to direct reader's attention to the discrepancy between smoothed and observed corporate leverage. Compared to other observables, the discrepancy seems to be relatively large (or, equivalently, measurement error for corporate leverage is relatively large). The reason behind this result is that since external finance premium and corporate leverage is linked tightly by the optimal debt contract eq. (20), there is no room for either external finance premium or corporate leverage to deviate from each other due to theoretical restriction. Indeed, in a log-linearized form, the theory requires linear relationship between two variables. However, reality is that, as can be seen from Figure 3, external finance premium data and corporate leverage data do not necessarily move in tandem. As a result, there is no other way but to rely heavily on measurement errors in keeping the internal consistency within the model, which makes the

[25] Both Case A and Case B emphasize the importance of negative labor supply shock in accounting for the sharp decrease in output right after Lehman Shock. This is a puzzling account. By the construction of our model, labor supply shock (or labor disutility shock) is a shock to the household, not to the corporate sector who is on the labor demand side. Taking for the face value, this account implies that the household suddenly started to prefer leisure after Lehman Shock and decided to cut back on the labor supply to the corporate sector. Knowing the aftermath of Lehman Shock, we should admit that this story is hard to buy.

discrepancies of smoothed and observed variables to be large. Perhaps, this is a shortcoming of BGG model and needs to be amended if we are to further account for the reality in the financial market.

Now, turning back to the result of historical decomposition in Figure 10, we notice that Case A to emphasize the role of corporate net worth shock more than that of Case B. Somehow, in order to keep balance, Case A exploits government expenditure shock and labor supply shock in countering the magnitude of corporate net worth shock. This countering phenomenon cannot be observed in Case B which makes more natural sense in interpreting the movement of corporate leverage ratio.

Finally, we turn to Figure 11 which shows the historical decomposition of bank leverage. We notice a tendency of Case B to rely heavily on investment specific technology shock, especially in the latter half of 90's, which is quite different from the accounting pattern observed in Case A. Other than that, Case A and Case B seem to have similar accounting of the movement in bank leverage, at least qualitatively.

As we have seen thus far, the results of historical decomposition can be very different depending on the estimates of structural shocks. Even if the differences of the estimates of the structural shocks (*i.i.d.* portion) are subtle, the results of the historical decomposition can be drastically different, especially when the AR(1) persistence parameters of the structural shocks are large. In order to obtain a reliable account of the historical movements of the variables, reliable estimates of the structural shocks (and also parameters) are crucial and, in this sense, data-rich estimation can be useful. In our empirical analysis, Case B data set was estimated using data-rich estimation. On the account that Case B revealed more reasonable smoothing of Lehman Shock and more efficient estimates for the net worth shocks than Case A, it is natural to regard the results of historical decomposition under Case B to be more reliable than those under Case A.

One final word regarding historical decomposition exercise. Historical decomposition exercise is becoming prevalent in empirical DSGE literature. However, as we demonstrated in this paper, accounting of historical movement in endogenous variables is sensitive to the estimates of structural shocks. In order to conduct an accurate and reliable historical decomposition, an accurate and reliable estimate of structural shocks is crucial. For this reason, Boivin and Giannoni's (2006) data-rich estimation method which potentially facilitates an accurate and reliable estimation of structural shocks is recommended, especially when conducting historical decomposition exercise.

5.5 So How Bad was Lehman Shock?

In closing this section, we now return to our original question; "how bad was Lehman Shock?" Perhaps, this question can be posed in two ways in our context. One way to pose is how bad was the magnitude of Lehman Shock as a bank net worth shock and another way to pose is how bad was the effect of Lehman Shock to the economy. We answer to each question in turn.

In answering the first question, smoothed results of bank net worth shock (i.e., $\hat{\gamma}_t^F$) in Figure 6 is indicative. Identifying the sharp dip of smoothed bank net worth shock in 2008Q3 and Q4 as Lehman Shock, it is clear from the figure that the magnitude of the shock was worst throughout the entire sample period (i.e., worst in 25 years), especially under Case B's data-rich estimation. Although we do observe some dip in bank net worth shock around 1990 – a period during S&L crisis – the magnitude of the shock in 1990 is much smaller than those in 2008Q3 and Q4. This, perhaps, can be interpreted that the magnitude of Lehman Shock as a bank net worth shock has been much worse than those during S&L crisis.

Next, we ask how bad was the effect of Lehman Shock to the economy. In answering this question, Figure 12, which shows the historical contribution of bank net worth shock to other endogenous variables, may be indicative. In Figure 12, historical contributions of bank net worth shock to output, investment, corporate borrowing rate, and bank leverage are shown. The results under Case A are shown in blue and Case B are shown in red.

Looking at the period of 2008Q3 and Q4, we notice that bank net worth shock to affect the bank leverage adversely and thereby affecting corporate borrowing rate adversely (i.e., increase in borrowing rate). The effect of bank net worth shock to both bank leverage and corporate borrowing rate seem to have been the worst in 25 years. Turning to output and investment, we can infer a similar story. Perhaps, due to a rise in corporate borrowing rate, investment is suppressed considerably – as much as 10% decline from steady state is attributable to bank net worth shock under Case B – and thereby output is restrained by nearly 2% due to adverse bank net worth shock. Again, the magnitude of the effect from bank net worth shock to output and investment have been the worst in 25 years. Interpreting the bank net worth shock in 2008Q3 and Q4 as Lehman Shock, we can say that the adverse effect of Lehman Shock to the economy has been the worst in 25 years, much worse than those during S&L crisis, as a bank net worth shock. Now, it is worth to note that the estimated effects of Lehman Shock to other variables under Case A are all smaller compared to those under Case B. However, based on our empirical evidence in this section, we believe the estimates to be underestimated in Case A and regard the estimates of Case B to be more reliable.

Finally, an interesting question we may ask is whether the effect of Lehman Shock is over or not. Based on our estimates reported in Figure 12, the results suggest that the direct effect from Lehman Shock to, say, output seems to be over. In other words, after a sharp decline of output during 2008 to 2009 due to Lehman Shock, output starts to recover quickly and reaches zero as of 2010Q2, meaning that the direct effect of Lehman Shock to output has vanished as of 2010Q2. This quick recovery can be attributed to a sharp decline in bank leverage right after Lehman Shock. Perhaps, thanks to relatively prompt implementation of TARP by the U.S. government and also thanks to aggressive credit easing by FRB, consecutive positive bank net worth shocks have been created right after the financial crisis as can be seen from Figure 6. Indeed, the magnitude of positive shock has been so large that it was nearly the largest positive shock to the banking sector's net worth in 25 years. Due to this large positive bank net worth shock right after the financial crisis, the negative effect from Lehman Shock has been successfully countered, slashing bank leverage back to comfort zone. Restoring the health of balance sheet condition in banking sector, corporate borrowing rate calmed down and, consequently, investment has been restored and output has been recovered. At the time this paper is written, as far as the direct effect of Lehman Shock is concerned, it seems to be that the effect is all gone from the economy. Or putting it differently, based on the shock estimates in this paper, we can say that the U.S. economy is already at post-Lehman Shock era.

Having said that, let us hasten to note this is not say that the U.S. current recession is over. On the contrary, as can be seen from the historical decomposition of output in Figure 7, smoothed result suggests that the output gap is still at deep negative zone as of 2010Q2. Due to a sizable negative effect from labor supply shock, entrepreneur net worth, and, to some certain extent, TFP shock, it is expected that the current U.S. recession to linger for substantial amount of period.[26] Yet, we still claim that the recessionary effect directly stemming from Lehman Shock is over.

6 Concluding Remarks

In this paper, we have embedded corporate sector and banking sector balance sheets to the stylized DSGE model. In embedding the corporate sector balance sheet, we closely followed BGG. In embedding banking sector balance sheet, we closely followed Gertler and Kiyotaki (2010) and Gertler and Karadi (2010). The theoretical aim of this paper was to juxtapose the essences of BGG, Gertler and Kiyotaki (2010), and Gertler and Karadi (2010) and to synthesize those with the stylized empirical DSGE model such as CEE and Smets and Wouters (2003, 2007). We then estimated the model using Data-Rich method proposed by Boivin and Giannoni (2006). The idea of Data-Rich estimation is to extract the common factors from massive panel of macroeconomic and financial time series data and to match those to the observable state variables in the model. A merit of this approach is that by utilizing the multiple time series information for each observable variable in the model, we can expect improved efficiency in estimating the parameters and structural shocks in the model. Throughout the paper, we focused on the estimation of bank net worth shock in the model. In particular, we focused on the identification and estimation of the magnitude of Lehman Shock – which we know to have occurred in 2008Q3 – as a bank net worth shock and we also assessed the ramifications of Lehman Shock to the economy as part of historical decomposition exercise.

A natural question one might have is why do we need both corporate *and* bank sector balance sheets in the model in estimating the impact of Lehman Shock. Our answer is that we need both type of balance sheets in order to clearly identify Lehman Shock as a bank net worth shock. If there is only corporate balance sheet, there is no way we can model bank net worth shock in the model. On the other hand, if there is only bank balance sheet in the model, a shock that might be occurring in the corporate sector in reality may be recognized as a shock in banking sector in the estimation[27] and, thus, may 'contaminate' the estimate of bank net worth shock, including Lehman Shock. In order to identify and estimate the impact of Lehman Shock, which we regard it as a shock in financial sector, not a shock in corporate sector, the presence of both corporate and banking sector balance sheets in the

[26] The assumption regarding the structural shocks in this paper is that they are independent from each other. Thus, above labor supply shock, entrepreneur net worth shock, and TFP shock are assumed to occur independent of Lehman Shock which is presumed to be a bank net worth shock in this paper. It is arguable that this independence assumption across structural shocks may be too strong. It will be interesting to allow for correlationship among structural shocks or endogeniesing correlations among these factors. However, this extension will be left for future research.

[27] For instance, a rise in corporate borrowing rate due to a deteoriation in corporate balance sheet (in reality) may be recognized as a consequence of bank balance sheet deteoriation if corporate balance sheet is not considered in the model.

model is crucial.

Another, rather crude, question may be why do we need to estimate the model in assessing the impact of Lehman Shock. Or putting it more bluntly, "Isn't it obvious from the data how large the shock was?" It is true that a sharp decline in the output after 2008Q3 or a sharp rise in bank leverage on 2008Q3 are obvious. However, what is not obvious is how much of those decline or rise can be attributed to bank net worth shock. Since many types of shock bombard the economy each period, if one cannot identify all those shocks, there is no way one can assess the impact of the specific type of shock, including Lehman Shock. For instance, according to our historical decomposition exercise, a decline in the output after 2008Q3 was a result of the combination of negative factors including labor supply shock, corporate net worth shock, bank net worth shock, and TFP shock. Bank net worth shock accounts only a portion of the decline in the output. If one attributes all of the decline of the output to bank net worth shock (i.e., Lehman Shock), based on our empirical results, that will be a misleading analysis.

Further, in order to assess the impact of Lehman Shock reliably, we need to estimate the bank net worth shock accurately and efficiently. For this purpose, we regard Data-Rich estimation method to be very useful. By utilizing available information as much as possible, not just matching one data to one endogenous variable, we can get closer to a reliable estimate of structural shocks. Since the result of historical decomposition is very sensitive to the estimate of structural shocks, an improved efficiency from Data-Rich estimation is precious.

After all these efforts, theoretically and empirically, we believe we can reach a reliable assessment of the impact of Lehman Shock and we hope we have attained our goal in this paper.

A Appendix: Sims (2002) Solution Method

A.1 Solving DSGE model

A.1.1 General Form of Linear Rational Expectation Model

A linear rational expectations model(hereafter, LRE model) proposed by Blanchard and Kahn (1980) has been the representative of LRE models [28]. Nowadays, Sims (2002), however, generalized their linear rational expectations model[29]. Blanchard and Kahn (1980) do not explicitly build one-step-ahead prediction errors of endogenous variables in LRE models by setting these errors as zero (i.e. these endogenous variables are treated as predetermined ones.), whereas Sims (2002) do explicitly build the one-step-ahead prediction errors in the models [30]. The solving methods are characterized by whether the errors are built in the model or not [31] . The method proposed by Klein (2000) based on Blanchard and Kahn (1980) is adopted by Otrok(2001) and DeJong et al (2000a,b) etc, The Sims (2002)' method is adopted by Scorfheide (2000), and Lubik and Scorfheide (2004).

Sims'LRE model can be represented as

$$\Gamma_0 \boldsymbol{s_t} = \Gamma_1 \boldsymbol{s_{t-1}} + \Psi \boldsymbol{\varepsilon_t} + \Pi \boldsymbol{\eta_t}, \tag{3.1}$$

where $\boldsymbol{s_t}$ is a vector of endogenous variables, $\boldsymbol{\varepsilon_t}$ is a vector of exogenous shock variables, and $\boldsymbol{\eta_t}$ is a vector of one-step-ahead prediction errors (or rational expectations errors), satifying $E(\eta_{t+1}) = 0$.

The vector $\boldsymbol{s_t}$ denotes the variables in the model with the more advanced subindices, as well as the conditional expectations in the model. All of them are determined in the model. The vector $\boldsymbol{\varepsilon_t}$ denotes variables which are determined outside the model such as demand shocks, supply shocks, or errors in controlling government policy variables. The vector $\boldsymbol{\eta_t}$ denotes prediction errors, which will be solved for endogenously, together with state and decision variables s_t in the model.

As mentioned above, the features of Sims (2002)' model are that conditional expectation is defined as the endogenous variables $\boldsymbol{s_t}$ and that the prediction errors $\boldsymbol{\eta_t}$ are built in the

[28] Klein(2000) take over from the form which bulids no endogenous prediction error in the DSGE model used by Blanchard and Kahn (1980).

[29] This section is following the work by Novales et al. (1999)

[30] Accoring to Sims (2002, p1-2), there are four advantages of the method as following. (1) It covers all of the linear models with endogenous prediction error. (2)The approach handles automatically situations where linear combinations of variables are predetermined, while Blanchard and Kahn (1980) require that the analyst specify which elements of endogenous variables are predetermined. (3) This approach makes an extension to continuous time. (4) Blanchard and Kahn (1980) assume that boundary conditions at infinity are given in the form of maximal rate of growth for any element of the endogenous variables. Meanwhile, this approach recognizes that in general only certain linear combinations of variables are required to grow at bounded rates and that different linear combinations may have different growth rate restrictions.

[31] According to Klein(2000, p1407), In Sims(2002) the LRE model is transformed into a triangular one using the Schur decomposition described above, and the unstable block of equations is isolated. This block is solved forward, and the endogenous prediction error process is solved for by imposing the informational restriction that the solution must be adapted to the given filtration. At this stage, no extraneous assumption (e.g. what variables are predetermined.) are invoked. all information about the solution is given in the coefficient matrices of the difference equation itself. Meanwhile, In Klein(2000) following Blanchard and Kahn (1980), the unstable block of the triangular system is solved forward without having to solve for prediction error separately. Instead, the endogenous prediction error process is solved for when solving the stable block of equations

LRE model. And if a stability condition does not hold in Eq.(3.1), the vector of endogenous variables $\boldsymbol{s_t}$ always traces unstable path which will violate the transversality conditions under arbitrary initial conditions $\boldsymbol{s_0}$ and sample realizations for ε_t. However, $\boldsymbol{s_t}$ converges to equilibrium by necessity, if the stability condition hold in the model (3.1), although the structure of the stability conditions is generally model-specific. The linear combinations of prediction errors, $\boldsymbol{\eta_i}$, which are endogenously determined in the models as explained later, contribute to the setup of the stability conditions.

Note that Sims (2002) proposed two approach to find the solution and the stable condition depending on the property of the matrix Γ_0. In general, the second method, however, is more commomly used regardless of this property. In the case that the matrix Γ_0 is invertible, the first method is applied. In the method, we can find the eigenvalues $\boldsymbol{\Lambda}$ of $\boldsymbol{\Gamma_0^{-1}\Gamma_1}(= \boldsymbol{P\Lambda P^{-1}})$ using Jordan decompostion. Then we get the recurive equilibrium law of motion which will thread out stable path consisting of the stable eigenvalues and their corresponding eigenvectors. Meanwhile, in the case that the matrix Γ_0 is not invertible,(i.e, it is singular), the second method is applied. In the method, we need to compute the generalized eigenvalues of the pair (Γ_0, Γ_1) using Schur decomposition (or QZ decompostion) as explained in the next subsection.

A.1.2 Solving DSGE model by Schur decompostion

In this section, we deal with the solving method of DSGE model by Schur decomposition (or QZ decomposition) [32]. When sampling parameters in the underling DSGE model as explained in the next section, whether the models specified by sampled parameter set traces on stable path or on unstable path, is judged by this method. Only parameter sets in which the model trace on stable path are saved and otherwise are removed from the sample.

In the LRE model, equation (3.1), explained in the last subsection such as

$$\Gamma_0 \boldsymbol{s_t} = \Gamma_1 \boldsymbol{s_{t-1}} + \Psi \boldsymbol{\varepsilon_t} + \Pi \boldsymbol{\eta_t},$$

the matrix Γ_0 and Γ_1 are decomposed by QZ decomposition as below.

$$Q'\Lambda Z' = \Gamma_0,$$

$$Q'\Omega Z' = \Gamma_1,$$

where $Q'Q = Z'Z = I$, and Q,Z are both possibly complex. Also Ω and Λ are possibly complex and upper triangular. Note that the above QZ decomposition always exists. Letting $\omega_t = Z's_t$,, and premultiplying the both side of equation (3.1), by Q, then we get

$$\Lambda \boldsymbol{\omega_t} = \Omega \boldsymbol{\omega_{t-1}} + Q\Psi \boldsymbol{\varepsilon_t} + Q\Pi \boldsymbol{\eta_t}. \tag{3.2}$$

Although QZ decomposition is not unique, the ratio of diagonal elements of Ω and Λ, $\{\omega_{ii}/\lambda_{ii}\}$ (it is reffered to generalized eigenvalue.) is generally unique. The matrix Ω, Λ is ordered with respect to the absolute value of the ratio $\{\omega_{ii}/\lambda_{ii}\}$ (or generalized eigenvalue) by ascending order. By partitioning equation (3.2) in two blocks so that the stable generalized eigenvalues corresponding to $|\omega_{ii}/\lambda_{ii}| < \bar{\xi}$ and the unatable generalized eigenvalue corresponding to $|\omega_{ii}/\lambda_{ii}| \geq \bar{\xi}$, it is rewritten as equation (3.3). The upper and the lower in

[32] This subsection follows Sims (2002).

equation (3.3) are the stable block and the unstable block, respectively. Here, $\bar{\xi}$ is the bound of maximal growth rate of endogenous variables $\boldsymbol{s_t}$, that holds the transversality condition.

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{22} \end{bmatrix} \begin{bmatrix} \omega_S(t) \\ \omega_U(t) \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{bmatrix} \begin{bmatrix} \omega_S(t-1) \\ \omega_U(t-1) \end{bmatrix} + \begin{bmatrix} Q_{1\cdot} \\ Q_{2\cdot} \end{bmatrix} \Big(\Psi \varepsilon(t) + \Pi \eta(t) \Big), \tag{3.3}$$

where $Q_{1\cdot}$ and $Q_{2\cdot}$ denote the first and the second rows of the matrix Q. For canceling out the term of expectation errors $\boldsymbol{\eta(t)}$ from equation (3.3), we premultiply Eq.(3.3) by $[I \quad -\Phi]$ and translate its stable block into the upper of equation (3.4). Note that Φ is set to satisfy a linear combination, $Q_{1\cdot}\Pi = \Phi Q_{2\cdot}\Pi$, and this linear combination of expectation errors $\boldsymbol{\eta(t)}$ is the stability condition of the DSGE model.

Meanwhile, on the unstable block (i.e. the lower) in Eq.(3.3), the last term, $Q_{2\cdot}\Pi\eta_{t+1}$, is solved forward[33], and then it becomes $Q_{2\cdot}\Pi\eta_{t+1} = \sum_{s=1}^{\infty} M^{s-1}\Omega_{22}^{-1}Q_{2\cdot}\Psi\epsilon_{t+s}$. Here, we set $M = \Omega_{22}^{-1}\Lambda_{22}$. Substituting it into equation (3.3), we get

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} - \Phi\Lambda_{22} \\ 0 & I \end{bmatrix} \begin{bmatrix} \omega_S(t) \\ \omega_U(t) \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} - \Phi\Omega_{22} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_S(t-1) \\ \omega_U(t-1) \end{bmatrix} + \begin{bmatrix} Q_{1\cdot} - \Phi Q_{2\cdot} \\ 0 \end{bmatrix} \Psi\varepsilon(t) + E_t \begin{bmatrix} 0 \\ \sum_{s=1}^{\infty} M^{s-1}\Omega_{22}^{-1}Q_{2\cdot}\Psi\varepsilon_{t+s} \end{bmatrix} \tag{3.4}$$

Here, we set $E_t(\varepsilon_{t+s}) = 0$ for $s = 1 \cdots T$ in the last term of equation (3.4) and remind that $\omega_t = Z' s_t$, then we get the recursive equilibrium law of motion such as equation (3.5).

$$\boldsymbol{s_t = \Theta_1 s_{t-1} + \Theta_0 \varepsilon_t}, \tag{3.5}$$

where

$$\boldsymbol{\Theta}_1 = Z_{\cdot 1}\Lambda_{11}^{-1}[\Omega_{11} \quad (\Omega_{12} - \Phi\Omega_{22})]Z,$$

$$\boldsymbol{\Theta}_0 = H \begin{bmatrix} Q_{1\cdot} - \Phi Q_{2\cdot} \\ 0 \end{bmatrix} \Psi,$$

$$H = Z \begin{bmatrix} \Lambda_{11}^{-1} & -\Lambda_{11}^{-1}(\Lambda_{12} - \Phi\Lambda_{22}) \\ 0 & I \end{bmatrix} Z,$$

where $Z_{\cdot 1}$ denotes the first column of matrix Z. Equation (3.5) traces the stable path converging to the equilibrium and corresponds to our target, say, the solution of the DSGE models.

From equation (3.5), we set a state space model which consists of a transition equation and a measurement equation using $\boldsymbol{\Theta_1}$ and $\boldsymbol{\Theta_0}$ as below. The transition equation (or recursive equilibrium law of motion) is given by

$$\boldsymbol{s_t = \Theta_1 s_{t-1} + \Theta_0 \varepsilon_t}, \tag{3.6a}$$

[33] This derivation is described in Sims (2002). Here, we omit it since this calculation is not used in the later part of our study.

And the measurement equation is given by

$$\boldsymbol{y_t} = \boldsymbol{A}\boldsymbol{s_t}, \tag{3.6b}$$

where, $\boldsymbol{y_t}$ is a vector of observed variables, $\boldsymbol{s_t}$ is a vector of endogenous variables. $\boldsymbol{A}$ is a $n \times k$ matrix expressing relations between observed variables $\boldsymbol{y_t}$ and unobserved variables $\boldsymbol{s_t}$.

For this state space model with Gaussian error terms, unobservable variables $\boldsymbol{s_t}$ and the likelihood of the model are obtained using Kalman filter. In the next section, the Bayesian estimation for the state space model with the recursive equilibrium law of motion is explained.

B Appendix: Data-Rich Estimation Procedure

Appendix: Data-Rich Estimation Procedure

Following Boivin and Giannoni (2006) and Kryshko (2010), the state-space representation of the Data-Rich DSGE model (3.5) (3.7) and (3.3) explained in Section3.1 is estimated by their similar method based on Bayesian estimations via Markov Chain Monte Carlo (MCMC) algorithm. However, we adopt simulation smoother developed by DeJong and Shephard (1995) in order to sample state variable S_t unlike the approach of earlier works. The first advantage of DeJong and Shephard (1995)'s method is to overcome the drawbacks that they have to set ad-hoc variance covariance matrix of state variables S_t for each DSGE model since they use Carter and Kohn (1994)'s method and we can easily extend the data-rich method to any DSGE model. [34] The second is to sample exogenous shocks ε_t directly and, owing to this, to make credible bands of shocks and historical decompositions. This is important elements of our political study, since we will verify that extending the number of data reduce the range of band of shocks and historical decompositions in a data-rich environment.

We discuss estimation method of the data-rich DSGE model (3.5) (3.7) and (3.3). For convenience, we divide parameters of the model into two types, the first type is deep parameters θ and the second type is parameters of measurement equation (3.7) and AR process of measurement errors (3.3). We collect the second type parameters as $\mathbf{\Gamma} = \{\mathbf{\Lambda}(\boldsymbol{\theta}),\ \mathbf{\Psi},\ \mathbf{R}\}$. Note that parameters $G(\theta), H(\theta), Q(\theta)$ in the transition equation (3.5) are directly derived from deep parameters θ. Generally speaking, a Bayesian estimation is implemented based on following procedure.

Step 1. Set the prior distribution $p(\boldsymbol{\theta})$, which is the distribution the researcher have in mind before observing the data.

Step 2. Convert the prior distribution to the posterior distribution $p(\boldsymbol{\theta}|X^T)$, which is the distribution conditional on the data X^T, using the Bayes theorem

$$p(\boldsymbol{\theta}|X^T) = \frac{p(X^T|\boldsymbol{\theta})\ p(\boldsymbol{\theta})}{\int p(X^T|\boldsymbol{\theta})\ p(\boldsymbol{\theta})d\boldsymbol{\theta}}. \tag{3.10}$$

where $p(X^T \mid \theta)$ is likelihood function.

Step 3. Estimate the parameters $\boldsymbol{\theta}$ using the posterior distribution.

In our model, posterior distribution (3.10) can be represented as (3.11) since parameters are divided into θ, and Γ.

$$p(\ \boldsymbol{\theta}, \Gamma \mid X^T) = \frac{p(X^T|\ \boldsymbol{\theta},\ \Gamma)\ p(\boldsymbol{\theta},\ \Gamma)}{\int p(X^T|\ \boldsymbol{\theta},\ \Gamma)\ p(\boldsymbol{\theta},\ \Gamma)d\boldsymbol{\theta}\ d\Gamma}. \tag{3.11}$$

where prior distribution is $(\boldsymbol{\theta},\ \Gamma) = p(\boldsymbol{\theta} \mid \Gamma)\ p(\Gamma)$. In order to generate posterior distribution $p(\ \boldsymbol{\theta}, \Gamma \mid X^T)$, since it is not directly tractable, we divide it into the following four conditional

[34] According to Chib (2001, p3614-3615), De Jong and Shephard (1995) provide an important alternative procedure called simulation smoother that is particularly useful if variance covariance matrix of state variables is not positive definite or if the dimension of the state vector is large.

posterior distributions and adopt *Metropolis-within-Gibbs algorithm* (it is also referred to as *component-wise Metropolis-Hasting algorithm*).

$p(\Gamma \mid \theta, X^T), \qquad p(S^T \mid \Gamma, \theta, X^T), \qquad p(\Gamma \mid S^T, \theta, X^T), \qquad p(\theta \mid \Gamma, X^T)$

Essentially, the Gibbs sampler generate draws from joint posterior distribution $p(\boldsymbol{\theta}, \Gamma \mid X^T)$ by repeating iteratively generation of draws from conditional posterior distribution $p(\Gamma \mid \theta, X^T)$ and $p(\theta \mid \Gamma, X^T)$.

In a DSGE model in a data-rich environment, the main steps of Metropolis-within-Gibbs algorithm are

Step 1. Specify initial values of parameters $\boldsymbol{\theta^{(0)}}$ and $\boldsymbol{\Gamma^{(0)}}$. And Set iteration index $g = 1$.

Step 2. Solve the DSGE model numerically at $\theta^{(g-1)}$ based on Sims(2002)' method and obtain $\mathbf{G}(\boldsymbol{\theta}^{(\mathbf{g-1})})$, $\mathbf{H}(\boldsymbol{\theta})$, and $\mathbf{Q}(\boldsymbol{\theta})$ in equation (3.5).

Step 3. Draw $\Gamma^{(g)}$ from $p(\Gamma \mid \theta^{(g-1)}, X^T)$.

(3.1) Generate unobserved state variables $S_t^{(g)}$ from $p(S^T \mid \Gamma^{(g-1)}, \theta, X^T)$ using simulation smoother by DeJong and Shephard (1995).

(3.2) Generate parameters $\Gamma^{(g)}$ from $p(\Gamma \mid S^{T(g)}, \theta^{(g-1)}, X^T)$ using the sampled draw $S^{T(g)}$.

Step 4. Draw deep parameters $\theta^{(g)}$ from $p(\theta \mid \Gamma^{(g)}, X^T)$ using Metropolis step:

(4.1) Sample from proposal density $p(\boldsymbol{\theta}|\boldsymbol{\theta}^{(g-1)})$ and, using the sampled draw $\boldsymbol{\theta}^{(\text{proposal})}$, calculate the acceptance probability q as follows.

$$q = \min\left[\frac{p(\boldsymbol{\theta}^{(\text{proposal})} \mid \Gamma^{(g)}, X^T)\; p(\boldsymbol{\theta}^{(g-1)}|\boldsymbol{\theta}^{(\text{proposal})})}{p(\boldsymbol{\theta}^{(g-1)} \mid \Gamma^{(g)}, X^T)\; p(\boldsymbol{\theta}^{(\text{proposal})}|\boldsymbol{\theta}_{g-1})}, 1\right].$$

(4.2) Accept $\boldsymbol{\theta}^{(\text{proposal})}$ with probability q and reject it with probability $1 - q$. Set $\boldsymbol{\theta}^{(g)} = \boldsymbol{\theta}^{(\text{proposal})}$ when accepted and $\boldsymbol{\theta}^{(g)} = \boldsymbol{\theta}^{(g-1)}$ when rejected.

Step 5. Set iteration index $g = g + 1$ and return to Step 2 up to $g = G$.

In Step 4 of this algorithm, it is important to make the acceptance probability q as close to one as possible especially around the mode of the posterior density $p(\boldsymbol{\theta}|\Gamma, X^T)$ because the same values are sampled consecutively if q is low. To achieve this purpose, we should choose the proposal density $p(\boldsymbol{\theta}|\boldsymbol{\theta}^{(g-1)})$ that mimics the posterior density $p(\boldsymbol{\theta}|\Gamma, X^T)$ especially around its mode. This is why we firstly run regular DSGE model estimation and compute the posterior mode of the DSGE model parameters to obtain initial value $\theta^{(0)}$ of Step 1. And then, we generate smoothed state variables $S_t^{(0)}$ using $\theta^{(0)}$ and obtain initial value $\Gamma^{(0)}$ from OLS regressions of X^t on $S_t^{(0)}$.

Following previous literature that applies the MH algorithm to DSGE models, we use so-called the *random-walk MH algorithm* as Metropolis step in Step 4, where the proposal $\boldsymbol{\theta}^{(\text{proposal})}$ is sampled from the random-walk model:

$$\boldsymbol{\theta}^{(\text{proposal})} = \boldsymbol{\theta}^{(g-1)} + \boldsymbol{\nu_t}, \quad \boldsymbol{\nu_t} \sim \text{i.i.d.N}(0, c\boldsymbol{H}),$$

where $\boldsymbol{H}$ is usually set arbitrarily or equal to the Hessian of logarithm posterior distribution $-l''^{-1}(\hat{\boldsymbol{\theta}})$, where $l(\boldsymbol{\theta}) = \ln p(\boldsymbol{\theta} \mid \Gamma, X^T)$. And c is a scalar called the adjustment coefficient, whose choice will be explained below.

The merit of using this random-walk proposal is that $p(\boldsymbol{\theta}^{(g-1)}|\boldsymbol{\theta}^{(\text{proposal})}) = p(\boldsymbol{\theta}^{(\text{proposal})}|\boldsymbol{\theta}^{(g-1)})$, so that the acceptance probability q collapses to:

$$q = \min\left[\frac{f(\boldsymbol{\theta}^{(\text{proposal})})}{f(\boldsymbol{\theta}^{(g-1)})}, 1\right],$$

which does not depend on the proposal density $p(\boldsymbol{\theta}|\boldsymbol{\theta}^{(g-1)})$. Hence, we need not find a proposal density that mimics the posterior density. We must, however, be careful for $\boldsymbol{\theta}^{(\text{proposal})}$ not to deviate from $\boldsymbol{\theta}^{(g-1)}$ so much because the acceptance probability q may be low when those deviate far from each other. This may be achieved by making c low, but $\boldsymbol{\theta}^{(\text{proposal})}$ may be sampled only from the narrow range if c is too low. In random walk sampler, the optimal acceptance rate q according to Roberts et al. (1997) and Neal and Roberts (2008) is around 25%, ranging from 0.23 for large dimensions to 0.45 for univariate case. Following the previous literature, we simply use this random-walk MH algorithm with $\boldsymbol{H} = -l''^{-1}(\hat{\boldsymbol{\theta}})$.

The form of a prior density of each parameter is given in advance by an investigator in the Bayesian inference. In general, the prior densities in the DSGE models are set up as following.

It is assumed that the exogenous shocks ϵ_t such as technology shock, preference shock or monetary shock are persistent for their past shocks and these motions follow an AR (1) process such that $u_t = \rho u_{t-1} + \varepsilon_t$ where error term ε_t is *i.i.d.* Since the coefficient ρ must be between zero and one in the AR(1) process with the stationary property, their prior densities obey *beta distributions*. The variances of the error term ε_t are set up to be based on *inverted gamma distributions*. For the other parameters of the DSGE models *normal distributions* are adopted as their prior densities.

The distinction of the prior in the DSGE models is not to use normal-gamma distributions directly like other Bayesian estimations but to build up their own prior distributions by sampling the draws of the prior distributions given above. The aim that the priors are built up by sampling is to exclude the drawn parameters which are on *unstable* path in the DSGE model or are *not* the equilibrium solution from the sampling of the priors, and to include only the draws which are on *stable* path in the DSGE model or are the equilibrium solution into the sampling of the priors

This MCMC based Bayesian estimation has many advantages over the MLE. First, we may include the prior information into the prior distribution. Our method for choosing prior distribution will be discussed below. Second, it enables us to sample not only the parameters but also the impulse-response function from its posterior distribution. Since the impulse-response function is a function of the parameters, all we have to do is to substitute the sampled parameter values into that function. Third, we may compare the DSGE models with non-nested models such as the VAR models using the posterior odds ratio, which is a usual tool for a Bayesian model comparison.

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Data Appendix

The format is: series number; transformation code; series description; unit of data and data source. The transformation codes are: 1 - demeaned; 2 - linear detrended; 3 - logarithm and demeaned; 4 - logarithm, linear detrend, and multiplied by 100; 5 - log per capita, linear detrended and multiplied by 100; 6 - detrended via HP filter; 7 - logarithm, detrended via HP filter, and multiplied by 100; 8 - first difference logarithm, detrended via HP filter, and multiplied by 400; 9- the reciprocal number, logarithm, detrended via HP filter, and multiplied 100. A $*$ indicate a series that is deflated with the GDP deflator.

No.	Code	Series description	Unit of data	Source
Case A				
1	6	Interest rate: Federal Funds Effective Rate	% per annum	Federal Reserve Bank of New York
2	5	Real gross domestic product	billion of chained 2000 dollars	Bureau of Economic Analysis
3	5*	Gross personal consumption expenditures	billion dollars	Bureau of Economic Analysis
4	5*	Gross private domestic investment - Fixed investment	billion dollars	Bureau of Economic Analysis
5	8	Price deflator: Gross domestic product	2005Q1 = 100	Bureau of Economic Analysis
6	2	Real Wage (Smets and Wouters)	1992Q3 = 0	Smets and Wouters (2007)
7	1	Hours Worked (Smets and Wouters)	1992Q3 = 0	Smets and Wouters (2007)
8	6	Moody's bond indices - corporate Baa	% per annum	Bloomberg
9	7	Commercial banks leverage ratio	total asset/net worth ratio	Federal Reserve Bank
10	3	Nonfarm nonfin corp business leverage ratio	total asset/net worth ratio	Federal Reserve Bank
11	1	Charge-off rates for all banks credit and issuer loans	% per annum	Federal Reserve Bank
Case B				
12	5*	Gross personal consumption expenditures : Nondurable Goods	billion of chained 2000 dollars	Bureau of Economic Analysis
13	5*	Gross private domestic investment	billion dollars	Bureau of Economic Analysis
14	8	Price deflator - Private consumption expenditure	billion dollars	Bureau of Economic Analysis
15	8	Core CPI excluding food and energy	1982Q1 = 100	Bureau of Labor Statistics
16	4	Civillian labor force: Employed total	Thous	Bureau of Labor Statistics
17	4	Employees, nonfarm - Total nonfarm	Thous	Bureau of Labor Statistics
18	9	Core capital leverage ratio PCA all insured institutions	core capital/total asset ratio	Federal Deposit Insurance Corporation
19	7	Domestically chartered commercial banks leverage ratio	total asset/net worth ratio	Federal Reserve Bank
20	1	Charge-off rate on all Loans and Leases all commercial banks	% per annum	Federal Reserve Bank
21	1	Charge-off rate on all loans all commercial banks	% per annum	Federal Reserve Bank

Table 1: Calibrated Parameters and Key Steady States

Calibrated Param.	Description	Value	Source
β	Discount factor	0.995	Our setting
δ	Depreciation rate	0.025	Christensen and Dib (2008)
α	Capital share	0.33	Gertler and Kiyotaki (2010)
γ^E	Survival rate of entrepreneur in steady state	0.972	Christensen and Dib (2008)
γ^F	Survival rate of banker in steady state	0.972	Gertler and Kiyotaki (2010)
λ	Bank's participation constraint parameter	0.383	Gertler and Kiyotaki (2010)
ψ^w	Wage markup	0.05	Smets and Wouters (2003)
ϵ	Elasticity Substitution of intermediate goods	11	Our setting
ξ^E	New entrepreneur entry rate	0.003	Our setting
ξ^F	New banker entry rate	0.003	Gertler and Kiyotaki (2010)
Key Steady State	Description	Value	
mc_{ss}	Steady state marginal cost	$\frac{\epsilon-1}{\epsilon}$	-
S_{ss}	Steady state external financial premium	1.0075	Christensen and Dib (2008)
rr^E_{ss}	Steady state corp. borrowing rate (real, QPR)	1.0152	From data (1980Q1-2010Q2)
rr^F_{ss}	Steady state bank lending rate (real, QPR, ex-premium)	rr^E_{ss}/S_{ss}	-
rr_{ss}	Steady state real interest	$1/\beta$	-
ν_{ss}	Steady state Nu	$\frac{(1-\gamma^F_{ss})\beta(rr^F_{ss}-rr_{ss})}{(1/\beta-\gamma^F_{ss})}$	-
η_{ss}	Steady state Eta	$\frac{1-\gamma^F_{ss}}{1-\beta\gamma^F_{ss}}$	-
Lev^F_{ss}	Steady state leverage ratio of banker	$\frac{\eta_{ss}}{\lambda-\nu_{ss}}$	-
K_{ss}/N^E_{ss}	Steady state leverage ratio of entrepreneur	1.919	From data (1980Q1-2010Q2)
K_{ss}/Y_{ss}	Steady state capital/output ratio	$\frac{\alpha mc_{ss}}{rr^E_{ss}-(1-\delta)}$	-
I_{ss}/Y_{ss}	Steady state investment/output ratio	$\delta K_{ss}/Y_{ss}$	-
G_{ss}/Y_{ss}	Steady state government expenditure/output ratio	0.2	Gertler and Kiyotaki (2010)
C_{ss}/Y_{ss}	Steady state consumption/output ratio	$1 - I_{ss}/Y_{ss} - G_{ss}/Y_{ss}$	-

Table 2: Prior Settings

Parameter	Description	Density	Prior Mean	Prior SE
Structural Parameters				
Parameter	Description	Density	Prior Mean	Prior SE
κ	Investment adjustment cost	Normal	0.600	0.050
h	Habit formation	Beta	0.500	0.200
σ^C	IES of consumption	Normal	1.500	0.250
σ^L	Inverse Frisch elasticity of labor supply	Normal	1.400	0.100
φ	Elasticity of premium to leverage ratio	Inv. Gamma	0.050	1.000
ι_P	Price indexation	Beta	0.600	0.100
ι_W	Wage indexation	Beta	0.700	0.100
θ_P	Calvo parameter for goods pricing	Beta	0.500	0.100
θ_W	Calvo parameter for wage setting	Beta	0.700	0.100
ρ_R	Moneatary policy persist. param.	Beta	0.500	0.250
μ_π	Taylor coefficient for inflation	Normal	1.600	0.100
μ_Y	Taylor coefficient for output gap	Normal	0.500	0.250
Persistence Parameters for Structural Shocks				
Parameter	Description	Density	Prior Mean	Prior SE
ρ_A	Persistent parameter for TFP shock	Beta	0.500	0.250
ρ_C	Persistent parameter for preference shock	Beta	0.500	0.250
ρ_K	Persistent parameter for investment tech. shock	Beta	0.500	0.250
ρ_E	Persistent parameter for entrepreneur net worth shock	Beta	0.500	0.250
ρ_F	Persistent parameter for banking sector net worth shock	Beta	0.500	0.250
ρ_G	Persistent parameter for government expenditure shock	Beta	0.500	0.250
ρ_L	Persistent parameter for labor supply shock	Beta	0.500	0.250
Standard Errors for Structural Shocks				
Parameter	Description	Density	Prior Mean	Prior SE
$SE(\varepsilon_A)$	SE of TFP shock	Inv. Gamma	1.000	1.000
$SE(\varepsilon_C)$	SE of preference shock	Inv. Gamma	1.000	1.000
$SE(\varepsilon_E)$	SE of entrepreneur net worth shock	Inv. Gamma	1.000	1.000
$SE(\varepsilon_F)$	SE of banking sector net worth shock	Inv. Gamma	1.000	1.000
$SE(\varepsilon_G)$	SE of government expenditure shock	Inv. Gamma	1.000	1.000
$SE(\varepsilon_K)$	SE of Investment specific technology shock	Inv. Gamma	1.000	1.000
$SE(\varepsilon_L)$	SE of labor supply shock	Inv. Gamma	1.000	1.000
$SE(\varepsilon_R)$	SE or monetary policy shock	Inv. Gamma	0.250	1.000
Persistence Parameters for Measurement Errors				
Parameter	Description	Density	Prior Mean	Prior SE
δ_R	Persist. param. of measure. err. for interest rate	Normal	0.000	1.000
δ_Y	Persist. param. of measure. err. for output gap	Normal	0.000	1.000
δ_C	Persist. param. of measure. err. for consumption	Normal	0.000	1.000
δ_K	Persist. param. of measure. err. for investment	Normal	0.000	1.000
δ_π	Persist. param. of measure. err. for inflation	Normal	0.000	1.000
δ_W	Persist. param. of measure. err. for real wage	Normal	0.000	1.000
δ_L	Persist. param. of measure. err. for labor input	Normal	0.000	1.000
δ_{RE}	Persist. param. of measure. err. for corporate borrowing rate	Normal	0.000	1.000
δ_{LevE}	Persist. param. of measure. err. for corporate leverage ratio	Normal	0.000	1.000
δ_{LevF}	Persist. param. of measure. err. for banking sector leverage ratio	Normal	0.000	1.000
δ_S	Persist. param. of measure. err. for external financial premium	Normal	0.000	1.000
Standard Errors for Measurement Errors				
Parameter	Description	Density	Prior Mean	Prior SE
$SE(u_R)$	SE of measurement err. for interest rate	Inv. Gamma	0.200	1.000
$SE(u_Y)$	SE of measurement err. for output gap	Inv. Gamma	0.200	1.000
$SE(u_C)$	SE of measurement err. for consumption	Inv. Gamma	0.200	1.000
$SE(u_K)$	SE of measurement err. for investment	Inv. Gamma	0.200	1.000
$SE(u_\pi)$	SE of measurement err. for inflation	Inv. Gamma	0.200	1.000
$SE(u_W)$	SE of measurement err. for real wage	Inv. Gamma	0.200	1.000
$SE(u_L)$	SE of measurement err. for labor input	Inv. Gamma	0.200	1.000
$SE(u_{RE})$	SE of measurement err. for corporate borrowing rate	Inv. Gamma	0.200	1.000
$SE(u_{LevE})$	SE of measurement err. for corporate leverage ratio	Inv. Gamma	0.200	1.000
$SE(u_{LevF})$	SE of measurement err. for bank leverage ratio	Inv. Gamma	0.200	1.000
$SE(u_S)$	SE of measurement err. for external financial premium	Inv. Gamma	0.200	1.000

Table 3: Posterior Mean: Case A vs. Case B

	Case A					Case B				
	Structural Parameters									
Parameter	Mean	SE	CI (low)	CI (high)	CD	Mean	SE	CI (low)	CI (high)	CD
κ	0.549	0.051	0.462	0.633	-0.708	0.520	0.051	0.435	0.602	1.460
h	0.567	0.091	0.429	0.721	0.468	0.451	0.098	0.287	0.611	-0.631
σ^C	1.538	0.028	1.496	1.589	3.902	1.530	0.026	1.486	1.571	-6.337
σ^L	1.389	0.068	1.283	1.504	-7.042	1.306	0.063	1.206	1.412	3.395
φ	0.037	0.004	0.031	0.043	5.726	0.041	0.004	0.034	0.047	-1.406
ι_P	0.604	0.105	0.437	0.778	2.300	0.518	0.109	0.336	0.689	0.285
ι_W	0.741	0.040	0.674	0.804	1.784	0.755	0.076	0.626	0.874	1.570
θ_P	0.781	0.049	0.705	0.861	-0.859	0.830	0.036	0.770	0.885	-1.168
θ_W	0.626	0.046	0.546	0.699	1.849	0.698	0.042	0.630	0.767	-1.509
ρ_R	0.673	0.073	0.566	0.793	-0.312	0.577	0.103	0.418	0.734	3.108
μ_π	1.706	0.093	1.552	1.856	5.134	1.680	0.091	1.534	1.831	-5.381
μ_Y	0.010	0.009	0.000	0.022	-2.816	0.014	0.010	0.000	0.028	0.696
	Persistence Parameters for Structural Shocks									
Parameter	Mean	SE	CI (low)	CI (high)	CD	Mean	SE	CI (low)	CI (high)	CD
ρ_A	0.991	0.006	0.983	1.000	-4.985	0.989	0.008	0.979	1.000	0.338
ρ_C	0.787	0.154	0.586	0.978	1.773	0.869	0.073	0.779	0.963	-0.496
ρ_K	0.156	0.100	0.003	0.278	1.391	0.202	0.134	0.007	0.405	2.019
ρ_E	0.078	0.043	0.004	0.138	-1.625	0.111	0.064	0.001	0.192	-0.436
ρ_F	0.090	0.061	0.003	0.175	7.718	0.191	0.061	0.092	0.292	13.133
ρ_G	0.857	0.051	0.792	0.947	1.606	0.937	0.039	0.876	0.992	-4.174
ρ_L	0.744	0.096	0.595	0.904	4.099	0.366	0.157	0.083	0.583	4.016
	Standard Errors for Structural Shocks									
Parameter	Mean	SE	CI (low)	CI (high)	CD	Mean	SE	CI (low)	CI (high)	CD
$SE(\varepsilon_A)$	0.447	0.048	0.367	0.519	-0.325	0.433	0.047	0.356	0.511	0.364
$SE(\varepsilon_C)$	0.940	0.041	0.879	1.008	-4.276	0.937	0.030	0.889	0.983	2.174
$SE(\varepsilon_E)$	0.398	0.047	0.320	0.469	-0.958	0.399	0.050	0.317	0.479	1.522
$SE(\varepsilon_F)$	1.060	0.075	0.926	1.173	-3.512	0.903	0.058	0.825	1.002	13.410
$SE(\varepsilon_G)$	1.678	0.012	1.662	1.699	7.667	1.687	0.005	1.679	1.695	1.810
$SE(\varepsilon_K)$	0.573	0.085	0.433	0.708	-2.304	0.551	0.094	0.407	0.691	-2.949
$SE(\varepsilon_L)$	2.169	0.014	2.146	2.191	5.045	2.171	0.011	2.154	2.188	-8.818
$SE(\varepsilon_R)$	0.099	0.010	0.082	0.116	-4.878	0.097	0.011	0.080	0.114	2.584
	Persistence Prameters for Measurement Errors									
Parameter	Mean	SE	CI (low)	CI (high)	CD	Mean	SE	CI (low)	CI (high)	CD
δ_R	0.874	0.060	0.787	0.981	4.057	0.875	0.059	0.784	0.973	-2.356
δ_Y	0.724	0.123	0.532	0.924	1.222	0.866	0.081	0.753	0.997	0.221
δ_C	0.870	0.105	0.727	1.000	-0.858	0.842	0.090	0.716	0.983	0.927
δ_K	0.745	0.103	0.589	0.920	-0.379	0.756	0.100	0.596	0.914	0.042
δ_π	0.164	0.143	-0.082	0.387	2.094	0.153	0.139	-0.066	0.386	0.439
δ_W	0.888	0.062	0.798	0.989	-0.068	0.932	0.047	0.869	0.999	-1.466
δ_L	0.577	0.164	0.313	0.847	6.857	0.887	0.076	0.785	1.000	-0.621
δ_{RE}	0.343	0.145	0.108	0.585	-0.406	0.413	0.135	0.191	0.631	1.996
δ_{LevE}	0.393	0.143	0.162	0.636	2.007	0.176	0.145	-0.057	0.417	-5.194
δ_{LevF}	0.859	0.085	0.736	0.977	2.831	0.904	0.048	0.835	0.981	0.000
δ_S	0.785	0.124	0.592	0.961	-3.171	0.686	0.150	0.458	0.930	-1.431
δ_{C2}	-	-	-	-	-	0.675	0.139	0.467	0.911	-0.588
δ_{K2}	-	-	-	-	-	0.898	0.054	0.816	0.985	0.901
$\delta_{\pi 2}$	-	-	-	-	-	-0.042	0.107	-0.217	0.135	1.127
$\delta_{\pi 3}$	-	-	-	-	-	0.078	0.125	-0.122	0.286	-2.262
δ_{L2}	-	-	-	-	-	0.855	0.068	0.750	0.969	-0.344
δ_{L3}	-	-	-	-	-	0.988	0.011	0.972	1.000	0.289
δ_{LevF2}	-	-	-	-	-	0.686	0.078	0.565	0.819	-1.681
δ_{LevF3}	-	-	-	-	-	0.339	0.150	0.091	0.587	-5.489
δ_{S2}	-	-	-	-	-	0.978	0.019	0.953	1.000	-1.483
δ_{S3}	-	-	-	-	-	0.805	0.111	0.645	0.977	1.275
	Standard Errors for Measurement Errors									
Parameter	Mean	SE	CI (low)	CI (high)	CD	Mean	SE	CI (low)	CI (high)	CD
$SE(u_R)$	0.358	0.055	0.269	0.448	0.000	0.382	0.056	0.289	0.470	-0.798
$SE(u_Y)$	0.614	0.080	0.482	0.739	2.990	0.681	0.081	0.551	0.812	5.040
$SE(u_C)$	0.463	0.055	0.371	0.548	0.000	0.462	0.051	0.377	0.541	5.302
$SE(u_K)$	4.132	0.549	3.239	5.017	3.318	4.388	0.578	3.423	5.324	3.900
$SE(u_\pi)$	0.856	0.077	0.731	0.981	3.179	0.854	0.077	0.727	0.974	0.990
$SE(u_W)$	0.697	0.061	0.597	0.795	-1.056	0.746	0.066	0.637	0.852	-2.603
$SE(u_L)$	0.841	0.123	0.648	1.046	1.803	0.850	0.102	0.687	1.018	0.722
$SE(u_{RE})$	1.167	0.134	0.957	1.389	-1.954	1.251	0.138	1.020	1.464	6.068
$SE(u_{LevE})$	2.166	0.206	1.825	2.502	2.834	1.839	0.182	1.541	2.135	-6.828
$SE(u_{LevF})$	2.371	0.263	1.939	2.787	-0.837	2.337	0.248	1.940	2.743	0.954
$SE(u_S)$	0.432	0.048	0.354	0.508	-1.612	0.388	0.044	0.316	0.458	1.338
$SE(u_{C2})$	-	-	-	-	-	0.795	0.074	0.678	0.917	-0.539
$SE(u_{K2})$	-	-	-	-	-	3.573	0.324	3.046	4.098	0.466
$SE(u_{\pi 2})$	-	-	-	-	-	1.704	0.129	1.493	1.912	-0.628
$SE(u_{\pi 3})$	-	-	-	-	-	0.666	0.056	0.570	0.753	-0.558
$SE(u_{L2})$	-	-	-	-	-	0.322	0.027	0.277	0.363	0.487
$SE(u_{L3})$	-	-	-	-	-	0.494	0.047	0.415	0.566	2.764
$SE(u_{LevF2})$	-	-	-	-	-	2.536	0.198	2.206	2.851	4.154
$SE(u_{LevF3})$	-	-	-	-	-	1.524	0.166	1.244	1.786	-10.436
$SE(u_{S2})$	-	-	-	-	-	0.146	0.012	0.126	0.166	0.746
$SE(u_{S3})$	-	-	-	-	-	0.276	0.026	0.234	0.317	1.136

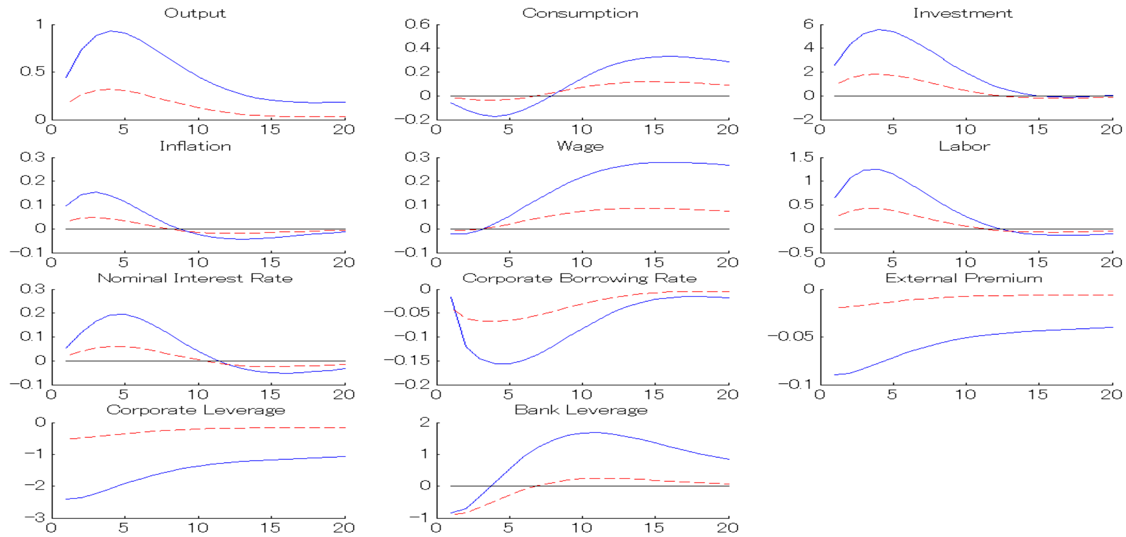
Notes

Results are based on 100,000 replications (burin-in 20,000). CI denotes 90% credible interval. CD denotes Geweke(1992)'s convergence diagnostic.

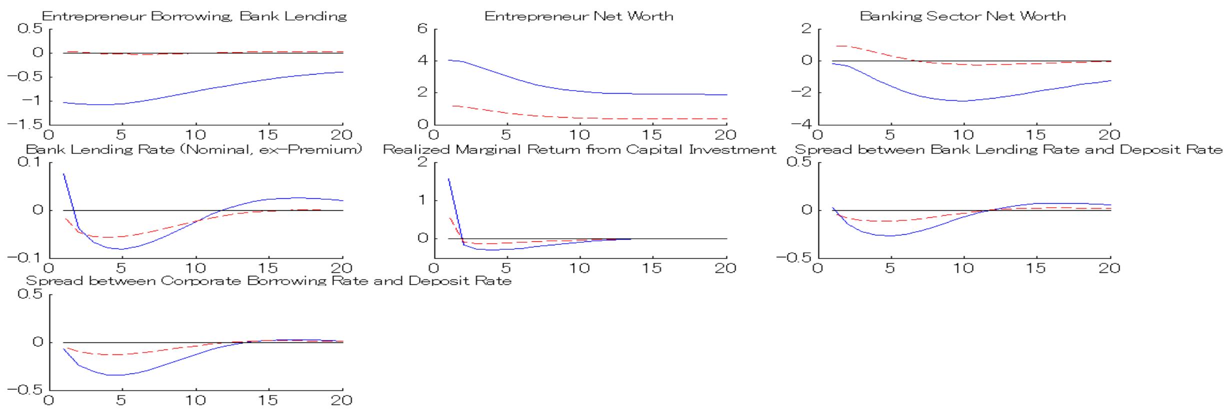
Figure 1: Impulse Response Functions (based on Case A estimation)

Entrepreneur Net Worth Shock (Blue) vs Banking Sector Net Worth Shock (Red)

Observed Variables



Unobserved Variables



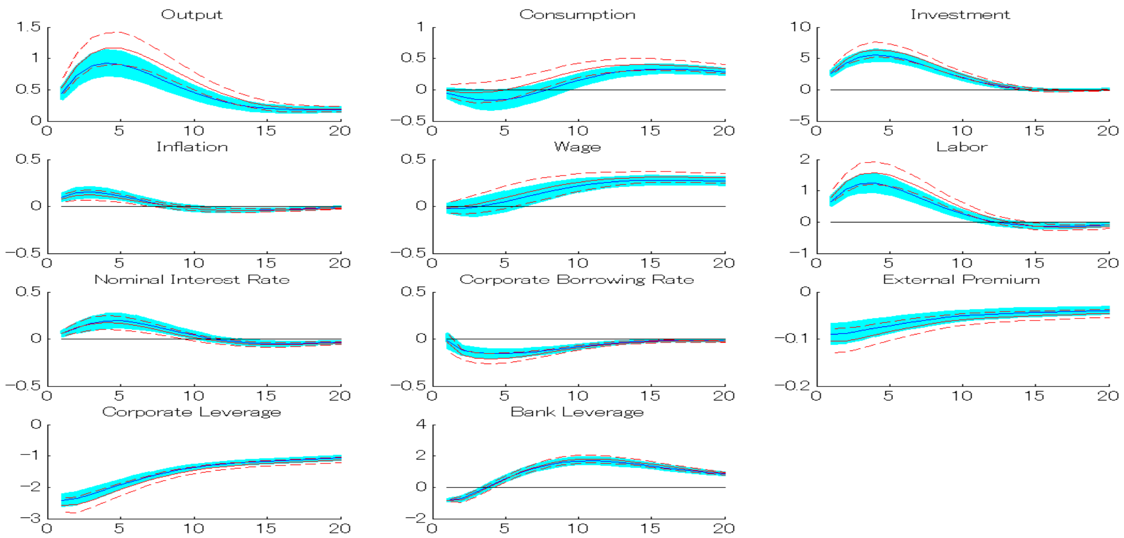
Notes

Blue solid lines: the means of impulse responses to entrepreneur net worth shock

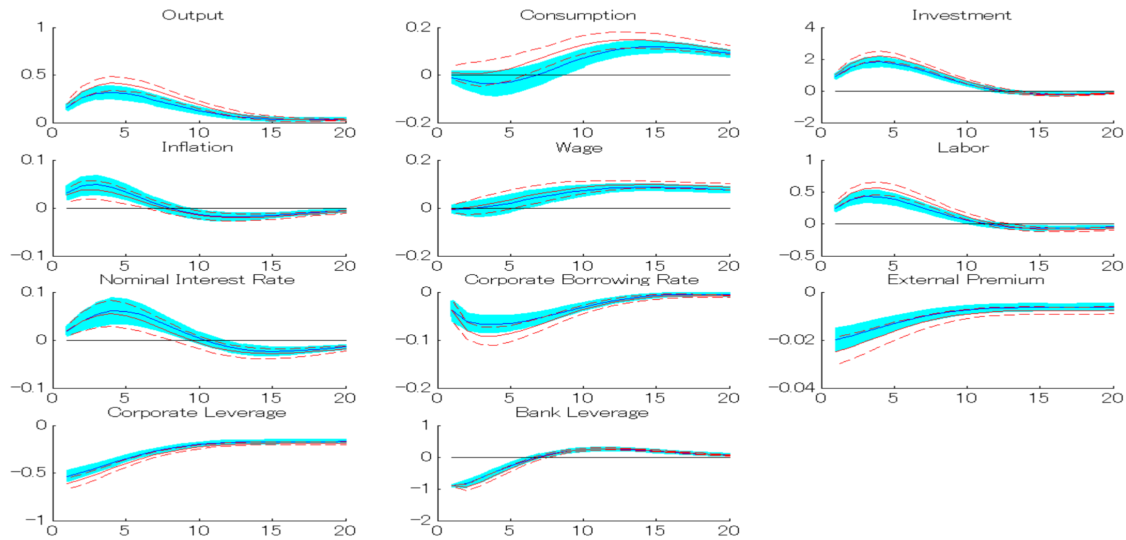
Red broken lines: the means of impulse responses to banking sector net worth shock

Figure 2: Impulse Response Functions with 90% Credible Interval:
Case A (Blue) vs Case B (Red)

Entrepreneur Net Worth Shock



Banking Sector Net Worth Shock



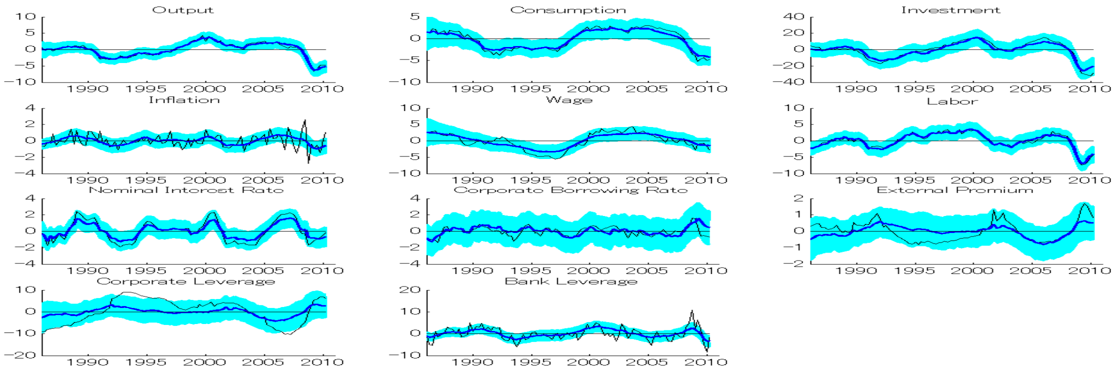
Notes

Blue lines and bands: the means and 90% credible intervals of impulse responses based on Case A estimation

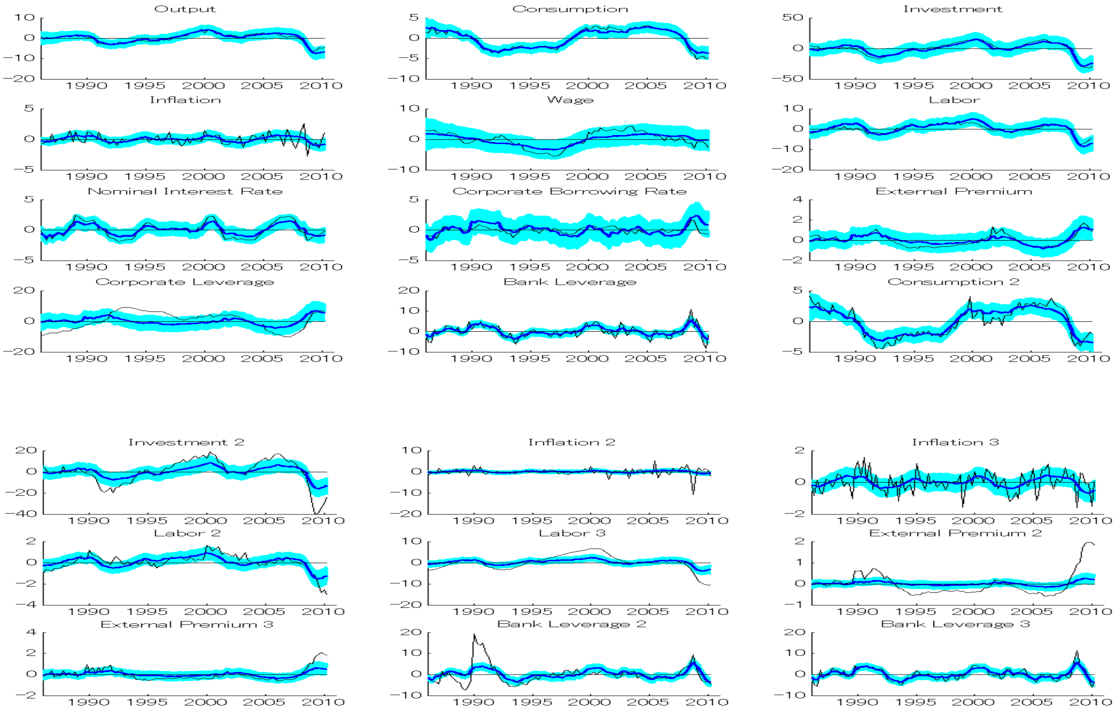
Red lines: the means (solid lines) and 90% credible intervals (upper and lower broken lines) of impulse responses based on Case B estimation

Figure 3: Smoothed Endogenous Variables with 90% Credible Interval

Case A



Case B

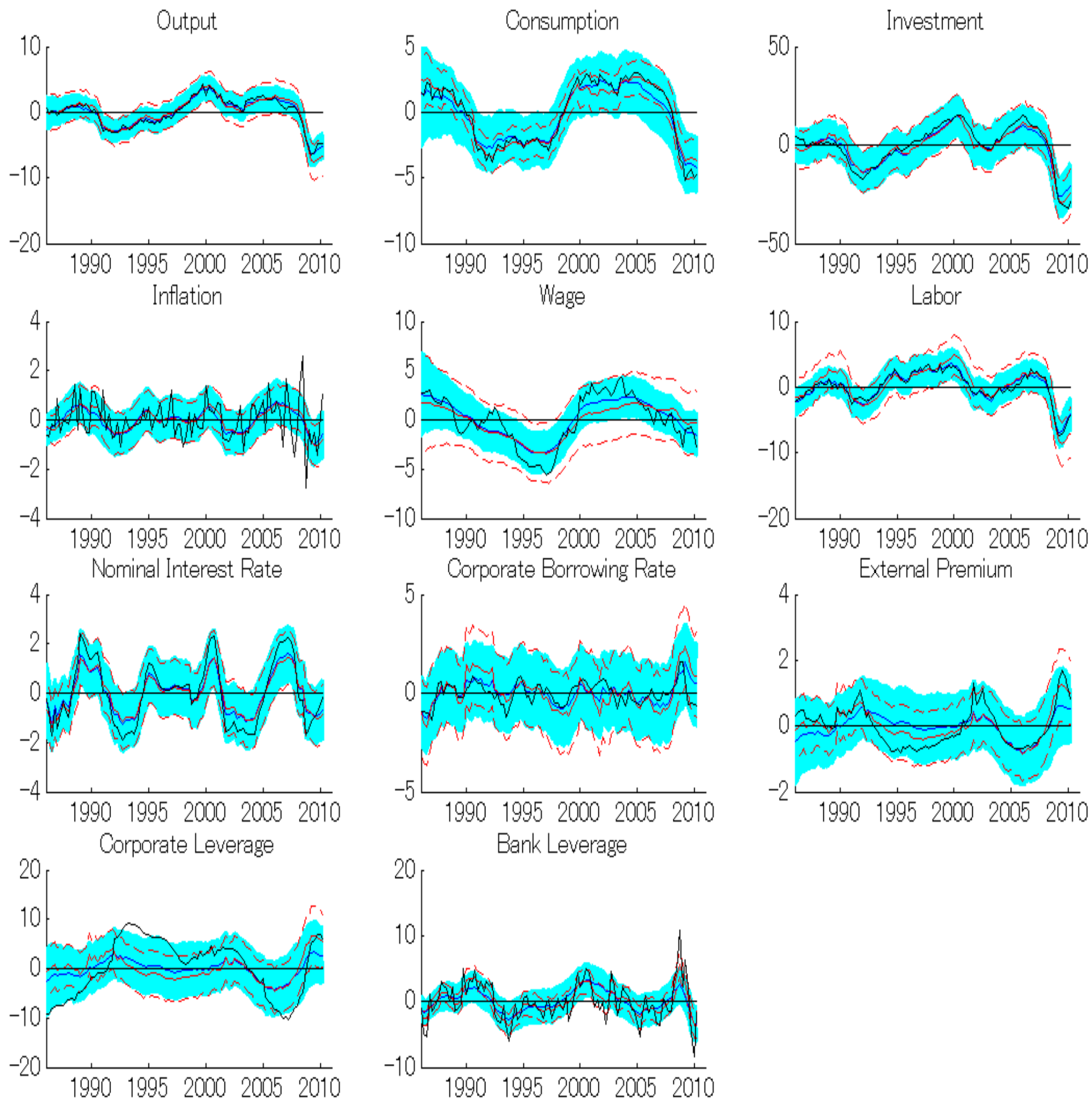


Notes

Blue lines and bands: the means and 90% credible intervals of smoothed endogenous variables

Black lines: corresponding data

Figure 4: Smoothed Endogenous Variables with 90% Credible Interval:
Case A (Blue) vs Case B (Red)



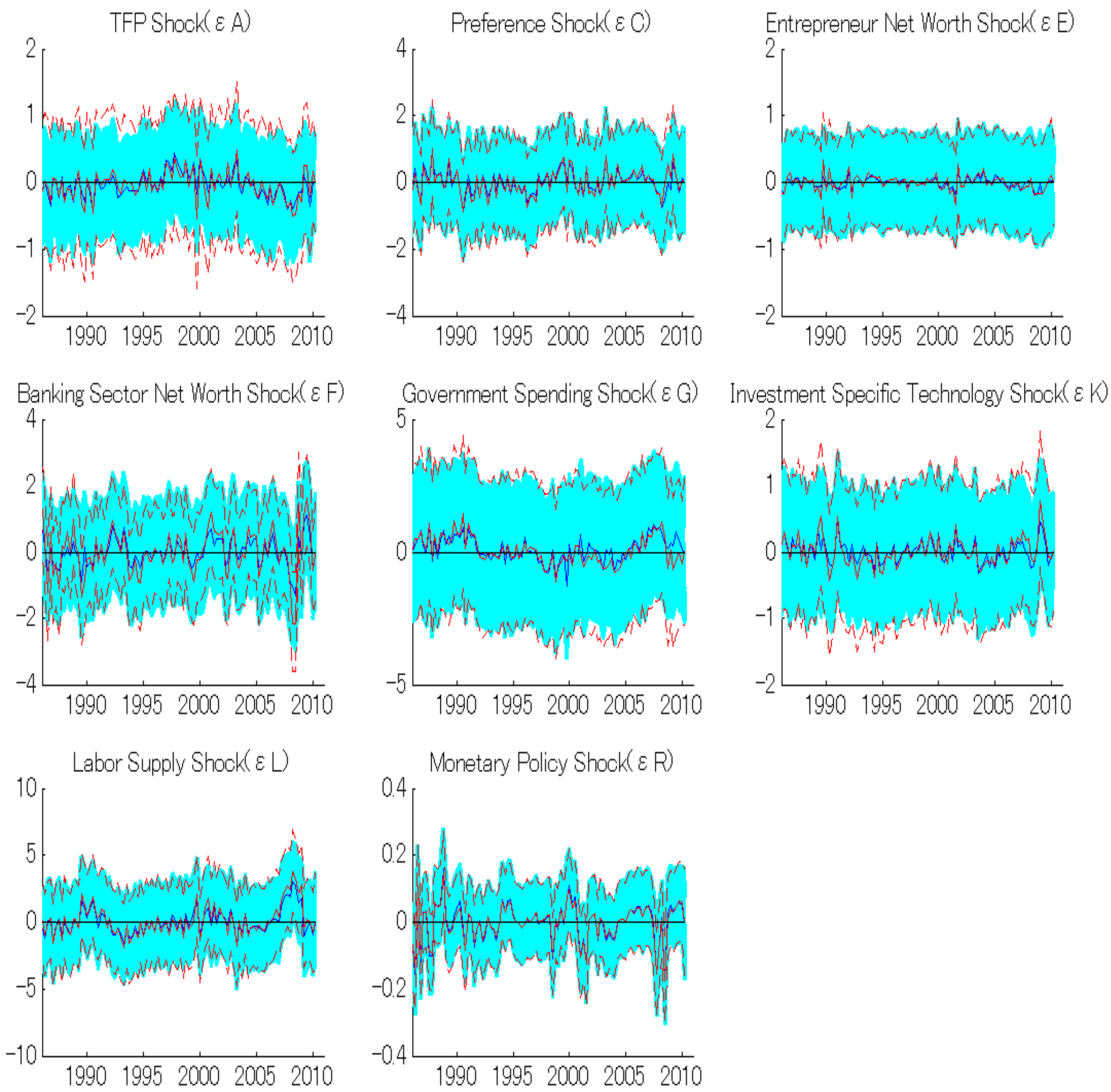
Notes

Blue lines and bands: the means and 90% credible intervals of smoothed endogenous variables based on Case A estimation

Red lines: the means (solid lines) and 90% credible intervals (upper and lower broken lines) of smoothed endogenous variables based on Case B estimation

Black lines: corresponding data

Figure 5: Estimated Shocks with 90% Credible Interval: Case A (Blue) vs Case B (Red)

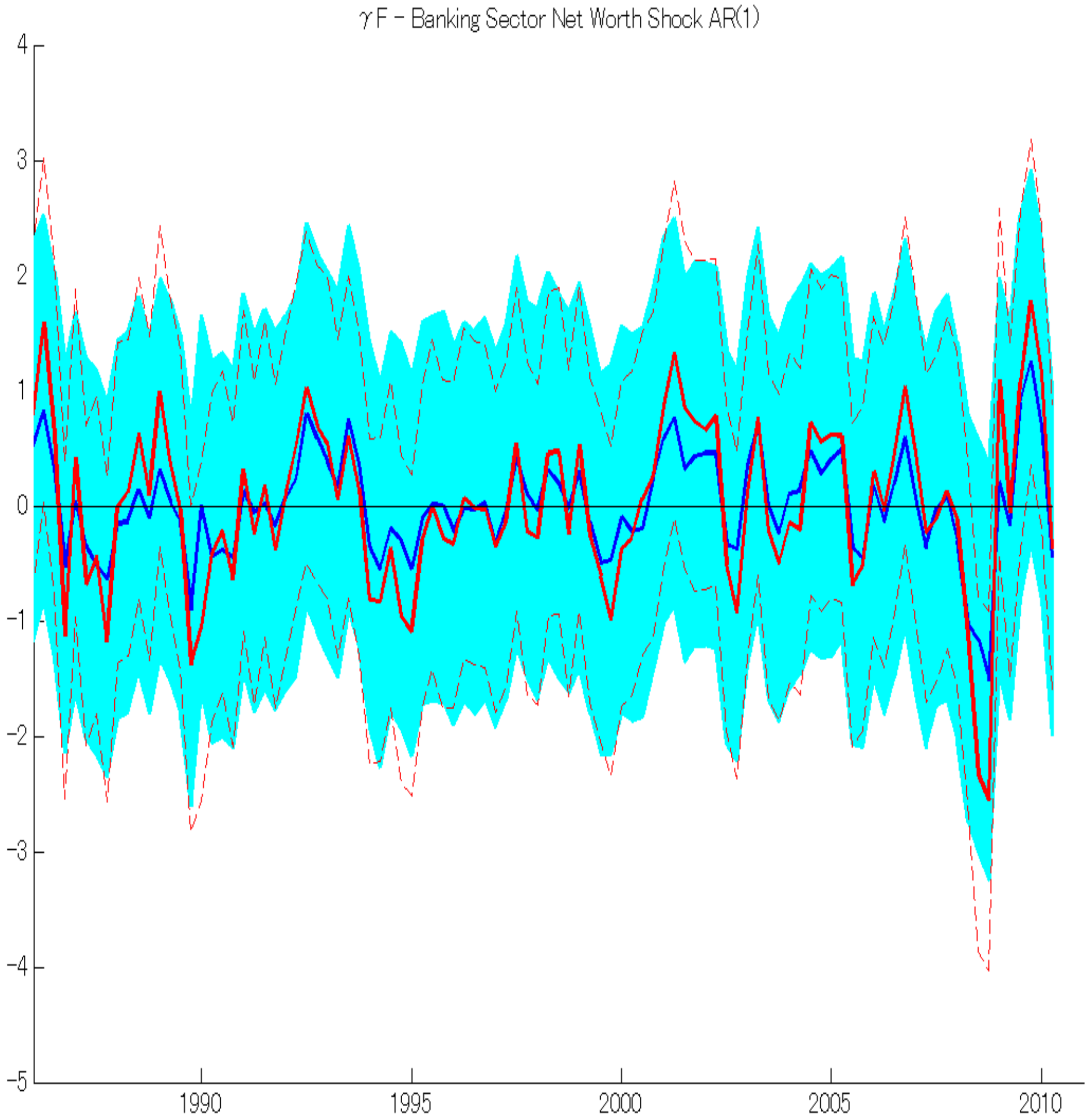


Notes

Blue lines and bands: the means and 90% credible intervals of estimated shocks based on Case A estimation

Red lines: the means (solid lines) and 90% credible intervals (upper and lower broken lines) of estimated shocks based on Case B estimation

Figure 6: Smoothed Banking Sector Net Worth Shock (γ^{F}_{t}) with 90% Credible Interval: Case A (Blue) vs Case B (Red)



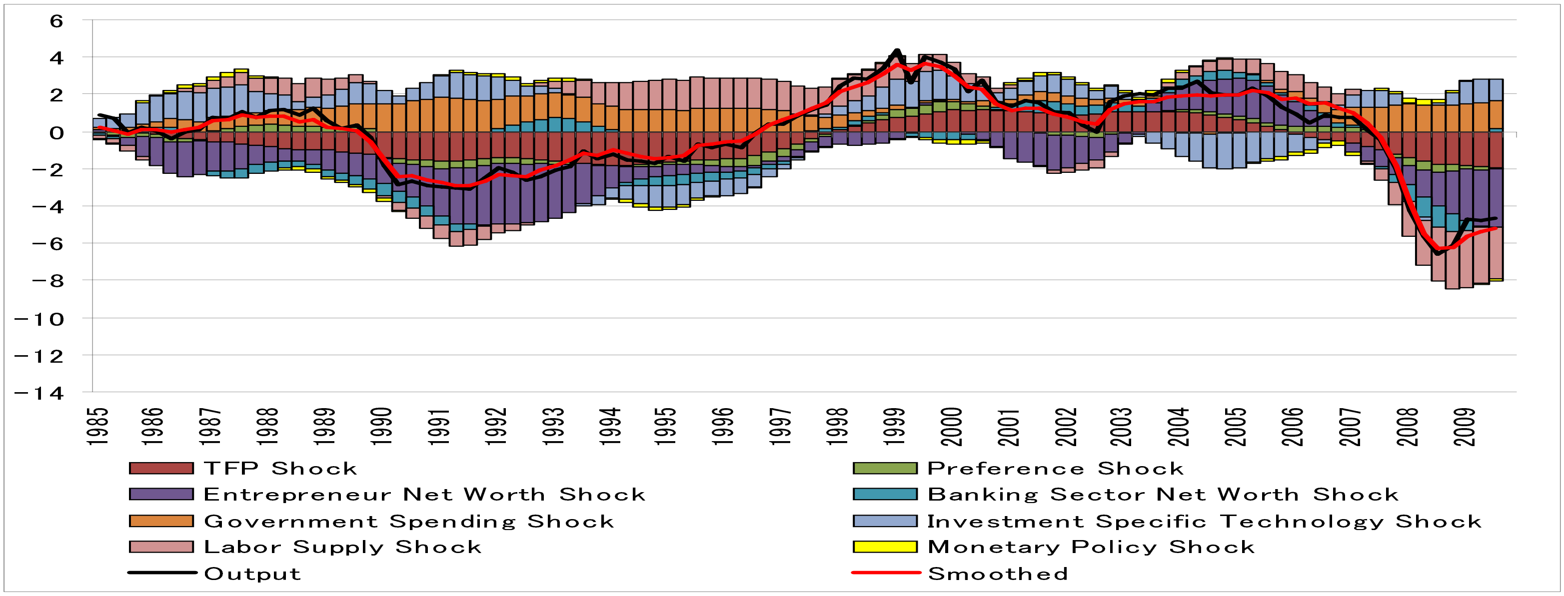
Notes

Blue line and band: the mean and 90% credible interval of smoothed banking sector net worth shock (γ^{F}_{t}) based on Case A estimation

Red lines: the mean (solid lines) and 90% credible interval (upper and lower broken lines) of smoothed banking sector net worth shock (γ^{F}_{t}) based on Case B estimation

Figure 7: Historical Decomposition - Output

Case A



Case B

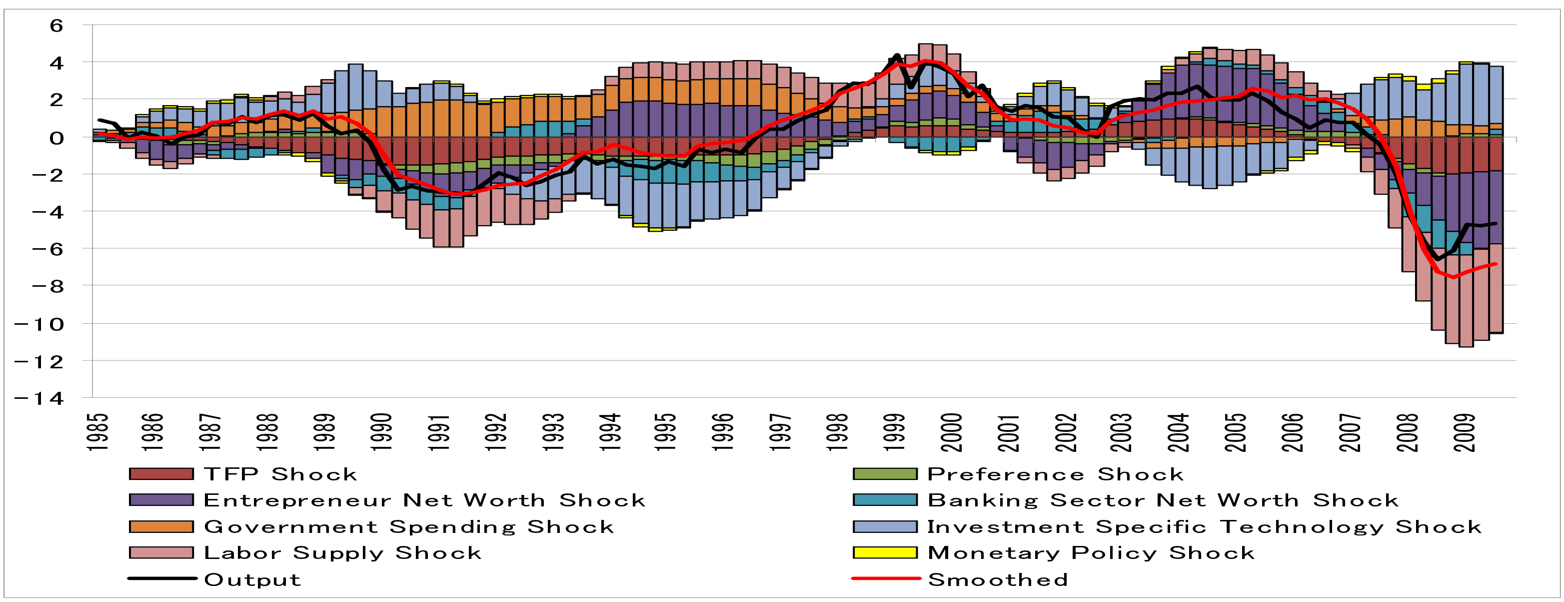
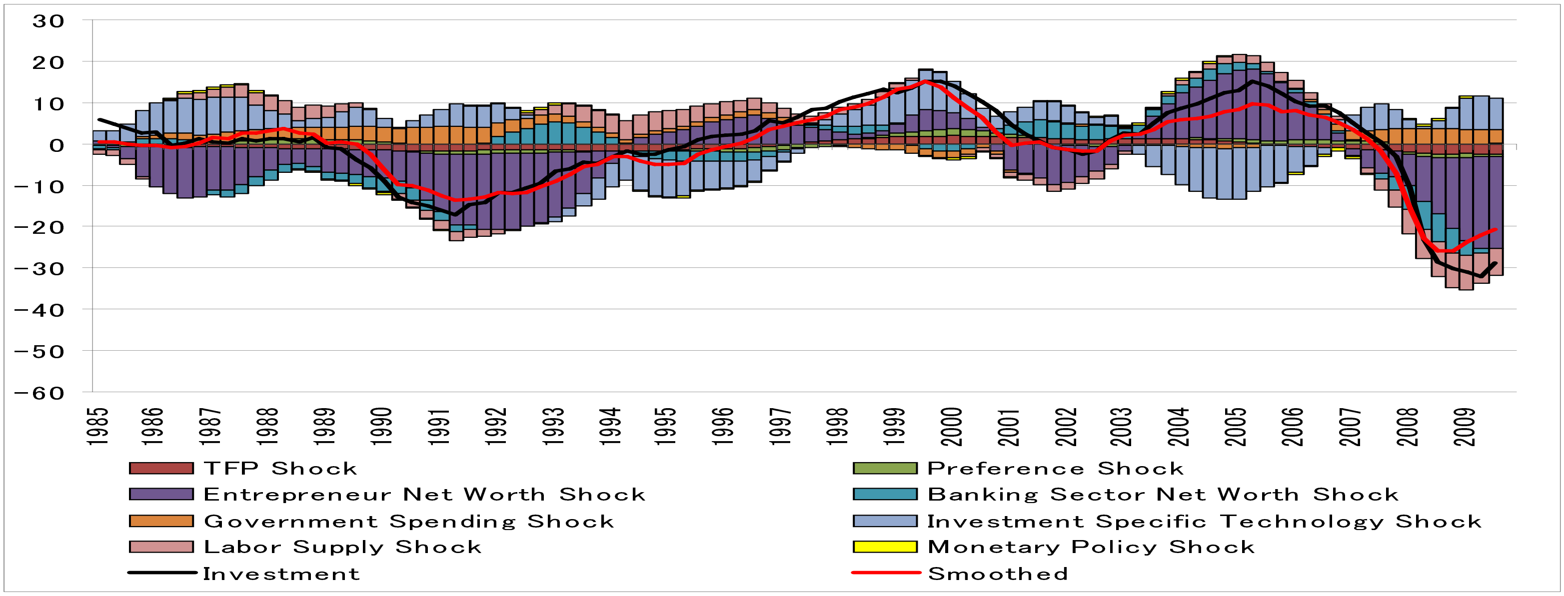


Figure 8: Historical Decomposition - Investment

Case A



Case B

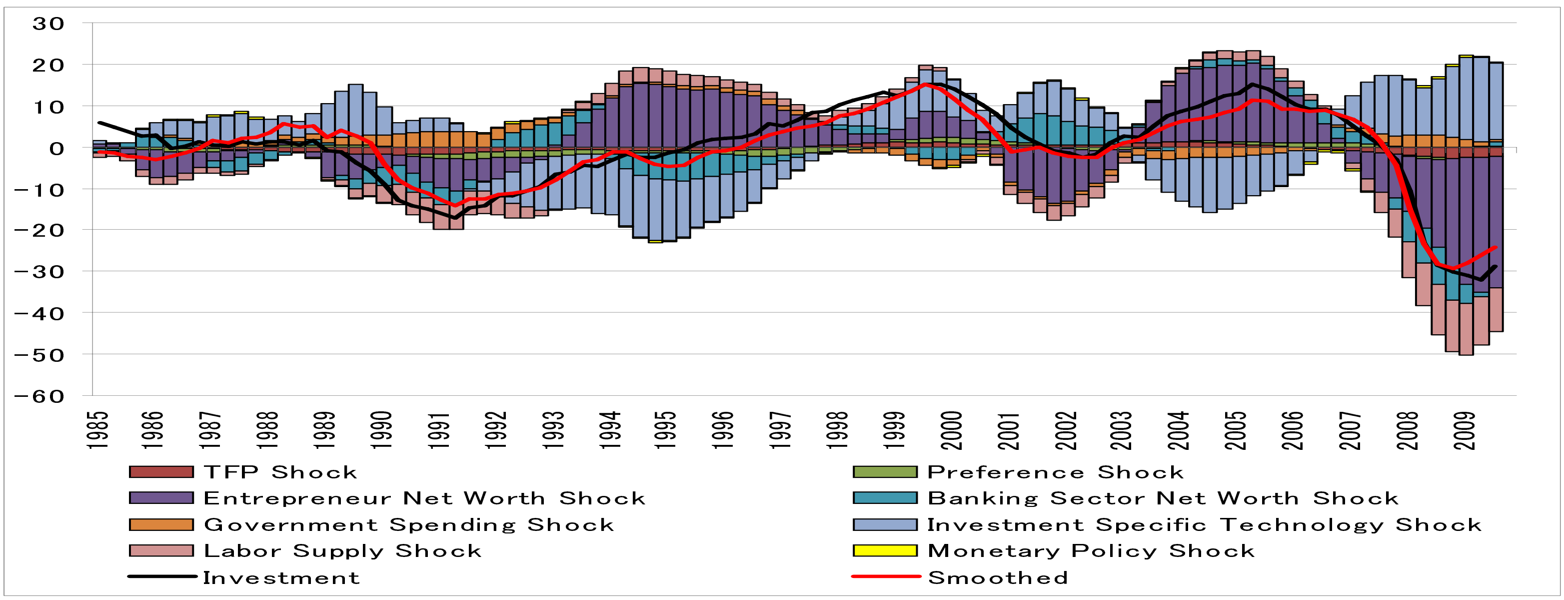
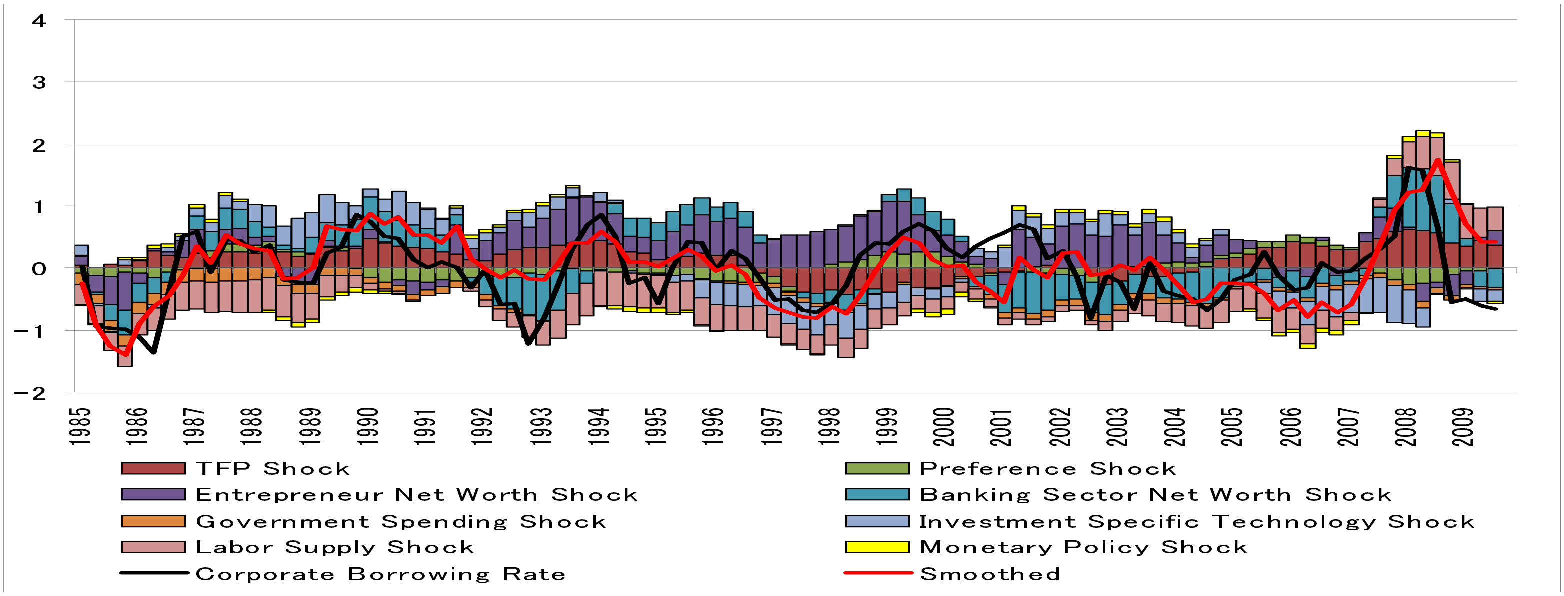


Figure 9: Historical Decomposition - Corporate Borrowing Rate

Case A



Case B

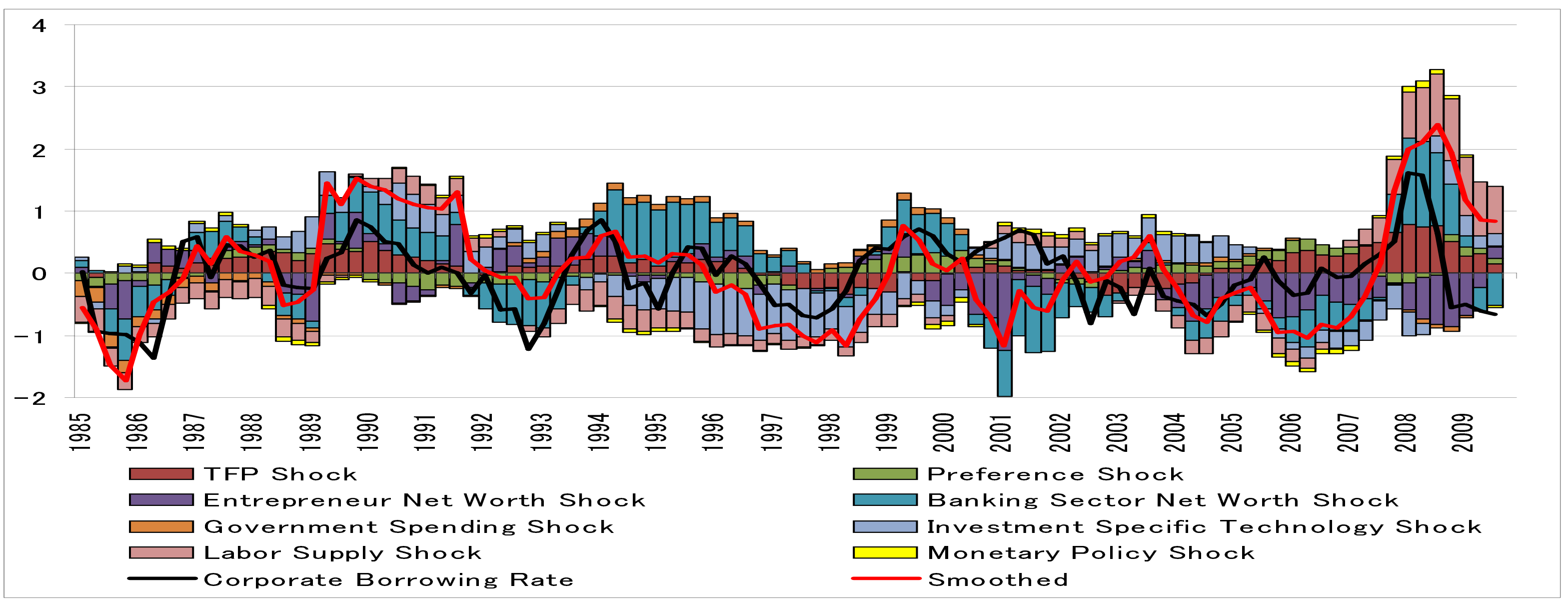
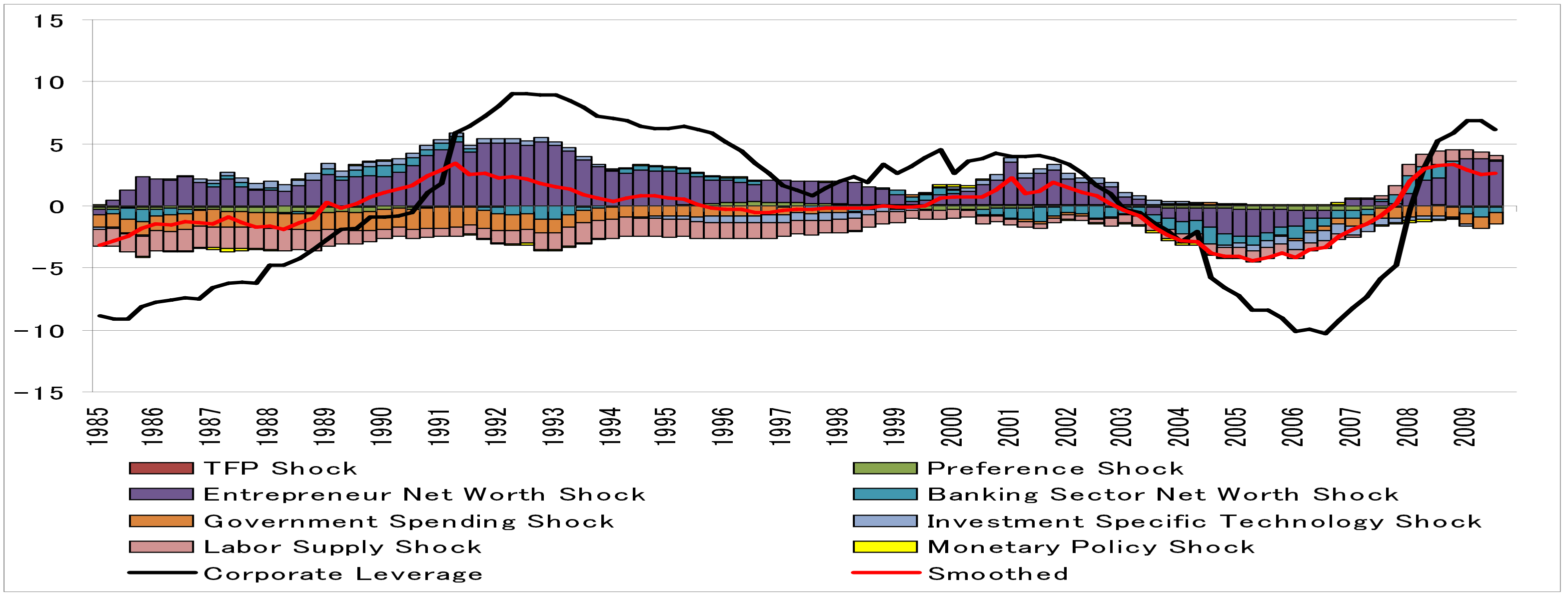


Figure 10: Historical Decomposition - Corporate Leverage

Case A



Case B

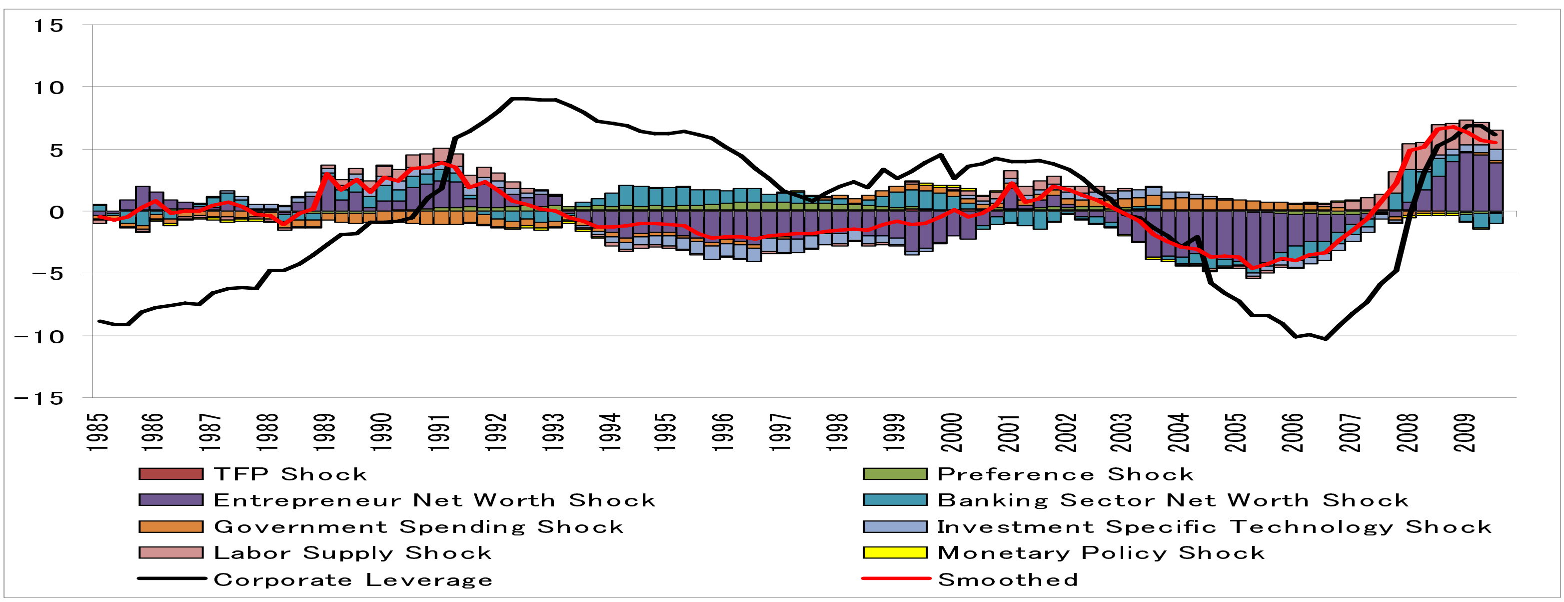
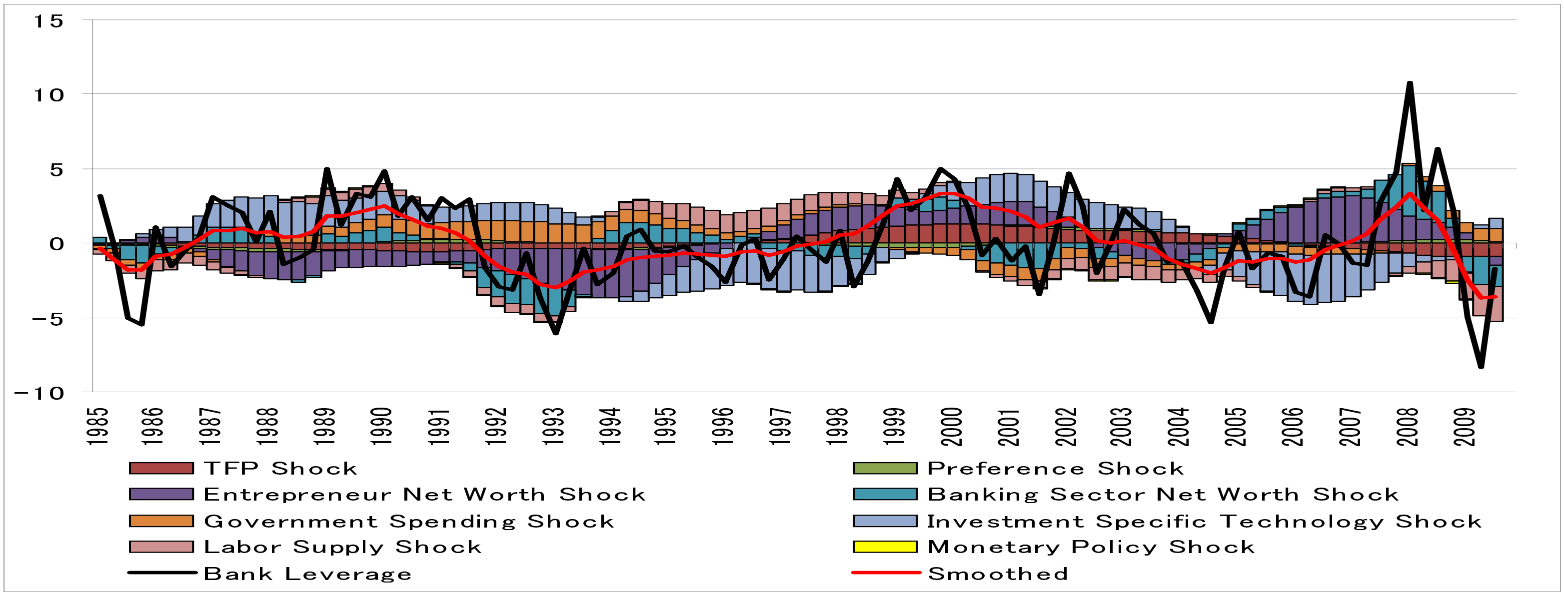


Figure 11: Historical Decomposition - Bank Leverage

Case A



Case B

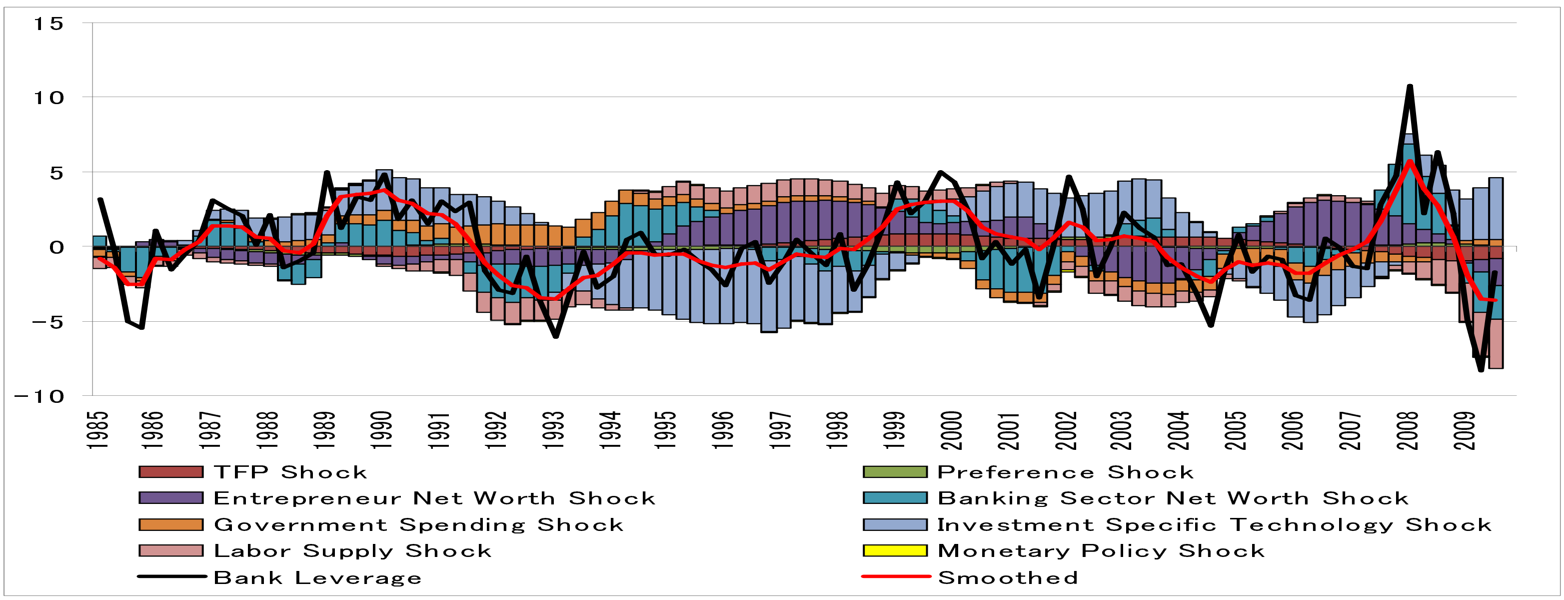
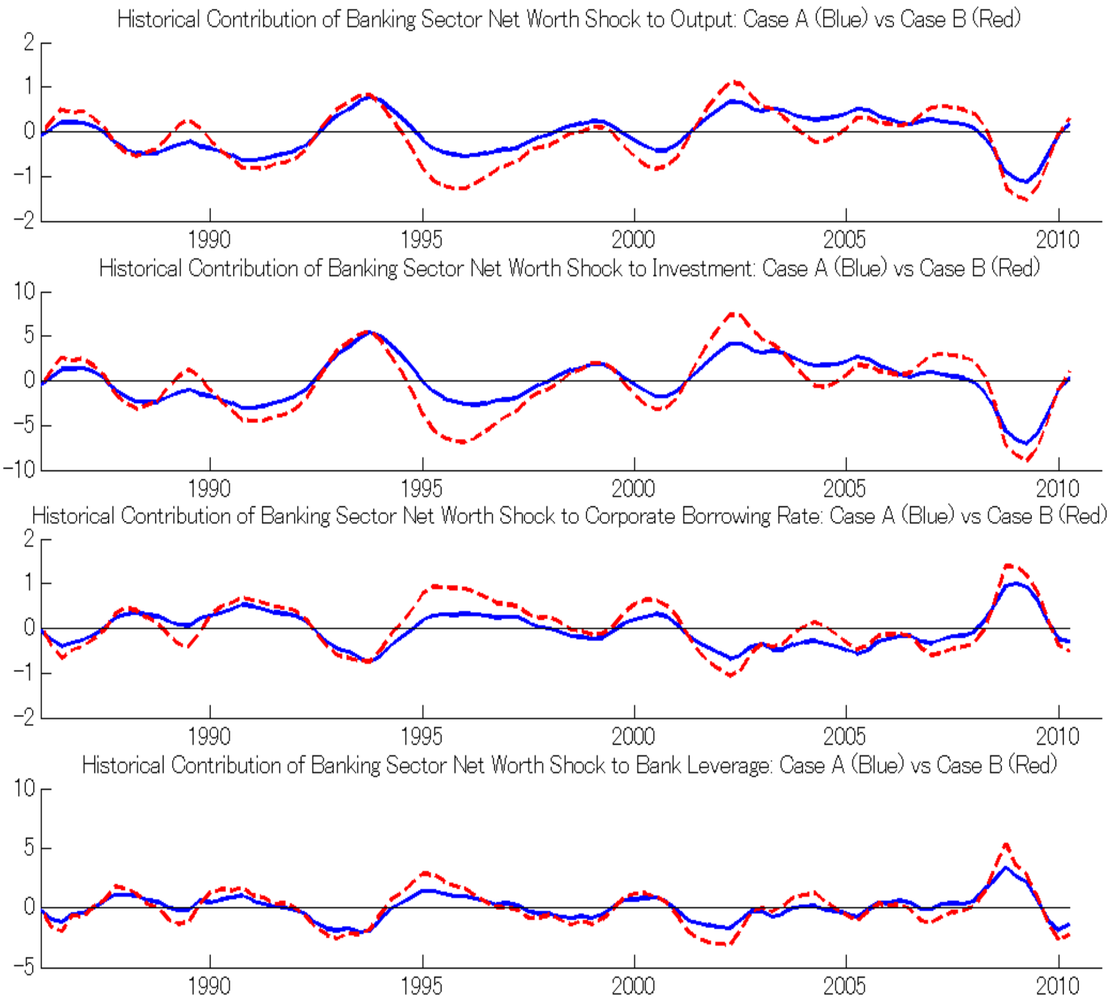


Figure 12: Historical Contribution of Banking Sector Net Worth Shock to Endogenous Variables:
Case A (Blue) vs Case B (Red)



Notes

Blue solid lines: historical contribution of banking sector net worth shock to endogenous variables based on Case A estimation

Red broken lines: historical contribution of banking sector net worth shock to endogenous variables based on Case B estimation