

# The Allocation of Talent and U.S. Economic Growth

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## Abstract

In 1960, 94 percent of doctors were white men, as were 96 percent of lawyers and 86 percent of managers. By 2008, these numbers had fallen to 63, 61, and 57 percent, respectively. Given that innate talent for these professions is unlikely to differ between men and women or between blacks and whites, the allocation of talent in 1960 suggests that a substantial pool of innately talented black men, black women, and white women were not pursuing their comparative advantage. This paper estimates the contribution to U.S. economic growth from the changing occupational allocation of white women, black men, and black women between 1960 and 2008. We find that the contribution is significant: 17 to 20 percent of growth over this period might be explained simply by the improved allocation of talent within the United States.

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## 1. Introduction

Fifty years ago, there were stark differences in the occupational distribution of white men versus women and blacks. Virtually all doctors and lawyers in 1960 were white men (94 and 96 percent, respectively). In contrast, 58 percent of white women were employed as nurses, teachers, sales clerks, secretaries, and food preparers; 54 percent of black men were employed as freight handlers, drivers, machine operators, and janitors. Only 2 percent of women and blacks worked in high skilled occupations (defined as executives, managers, architects, engineers, computer scientists, mathematicians, natural scientists, doctors, and lawyers). The number for white men, in contrast, was 18 percent.

The segregation of white men versus women and blacks across occupations in 1960 is quite remarkable. If we believe that every person has talent drawn from a similar distribution of ability, the fact that there were virtually no black or female doctors and lawyers in 1960 suggests that many blacks and women with a comparative advantage in high skill occupations were somehow prevented from realizing that advantage in 1960. Conversely, some of the white male doctors and lawyers in 1960 may not have had a comparative advantage in medicine or in the legal profession despite their having sorted into these occupations.

This has changed over the last 50 years. By 2007, only 63 percent of doctors and 61 percent of lawyers were white men. Similarly, the share of women and blacks in skilled occupations increased from 2 percent in 1960 to 15 percent for women and 11 percent for black men by 2007. These shrinking occupational gaps suggest that the forces that resulted in blacks and women not becoming doctors and lawyers have diminished over the last 50 years.

This paper measures the effect of the change in the occupational distribution of white men versus blacks and women since 1960 on aggregate productivity growth in the United States. At the heart of our approach is a canonical model of occupational choice driven by comparative advantage. We assume that every person is born with a range of talents across all possible occupations. In a frictionless world, each person chooses the occupation where she earns the highest return. We depart

from the standard model of occupational choice in that we allow for frictions. To generate the gaps in the occupational distribution between groups, we do not need to take a stand on exactly what these frictions represent. For example, they can take the form of occupation-specific discrimination in the labor market. Or they can be interpreted as barriers to human capital accumulation (e.g. lack of quality of schools or discrimination in admission to professional degree programs). The frictions could even be “culture” or “norms” about what are acceptable professions for blacks and women. Finally, declining fertility may have made it easier for women to work outside the home. All these forces will generate differences in the occupational distribution and gaps in the wages of white men vs. others.

To measure the effect of these occupational choices on aggregate productivity, we *do* need to take a stand on whether there is misallocation in the labor market or simply different (though possibly distorted) human capital investments. For example, if the main friction is that blacks had worse access to quality schooling in 1960, the gaps in the occupational distribution in 1960 reflect differences in the marginal product of labor between blacks and whites in high-skilled occupations. With this interpretation, all workers are paid their marginal product and the allocation of labor across occupations, *conditional on these marginal products*, is efficient. In this case, gains in aggregate productivity are closely related to changes in the familiar wage gaps for blacks and women vs. white males.

If the main friction is instead labor market discrimination faced by blacks and women in high skilled occupations, then the occupational segregation of blacks and women vs. white males requires gaps in the marginal product of blacks and women (respectively) across occupations. Because the marginal product is no longer equal to the wage for all workers, the wage gaps are no longer so informative about the impact of frictions on aggregate productivity. Instead, a key parameter is the dispersion of a person’s ability across occupations at birth. To infer this parameter from the data, we use the result that if the distribution of ability follows an extreme value distribution, the distribution of the maximum of a person’s ability also obeys that distribution. Thus, we can back out the dispersion of ability from the observed distribution of wages within an occupation. This allows us to recover the gaps in

marginal products implied by the gaps in the occupational distribution, and thus measure the effects of frictions on aggregate productivity.

We do not know which of the two interpretations of the frictions is correct, so we provide estimates for both polar models. Our main finding is that 17 to 20 percent of the growth in aggregate productivity in the United States between 1960 and 2008 was due to the declining frictions in occupational choice for women and blacks. Our point estimate is somewhat sensitive to parameter values for the model where we interpret the frictions as marginal product gaps, but not for the model where we interpret the frictions as differences in marginal products between groups. We also show that aggregate productivity growth would have been negative in the 1970s in the absence of the decline in labor market frictions facing women and blacks. Additionally, we show that the convergence in occupational choices was more pronounced in the U.S. South. As a result, we estimate, 25% of the income convergence of the South to the Northeast between 1960 and 1980 was due to declining labor market frictions in the South vs. the North. For our aggregate productivity results, most of the productivity gains are driven by declining frictions for white women. This is not surprising given that white women comprise a much larger fraction of the population than do black men and black women. However, for our regional convergence results, most of the convergence is driven by declining frictions for black men and women. Finally, our results indicate that almost all of the convergence in the educational attainment of women relative to white men from 1960 to 2008, and about a third of the convergence of schooling of black men relative to white men over the same time period, can be explained by the decline in the occupational frictions facing each group.

## **2. Related Literature**

Under construction.

### 3. Basic Setup of the Model

The economy consists of a continuum of people working in  $N$  possible occupations. Importantly, one of these occupations is “work at home,” which includes raising kids. Each person possesses an idiosyncratic ability in each occupation — some people are good economists while others are good nurses. The basic economic allocation to be determined in this economy is how to match workers with occupations.

#### 3.1. People

The economy consists of a unit measure of people. These people are divided into a total of  $G$  distinct groups, such as “black men” and “white women.” The parameter  $q_g$  denotes the fraction of the population that is part of group  $g$ .

Each person has 2 units of time. The first unit can be used either for leisure or for schooling,  $s$ , while the second unit of time is solely used for working (think of the first unit as occurring when young and the second when middle-aged). A person with consumption  $c$  and leisure time  $1 - s$  gets utility

$$U = c^\beta(1 - s). \tag{1}$$

$\beta$  parameterizes the tradeoff between consumption and schooling: a lower  $\beta$  thus implies that future wages must rise more steeply with schooling in equilibrium.

Each person chooses to work in one of the  $N$  occupations. The home sector is considered one of the  $N$  occupations. These occupations differ in several ways, one of which is how useful schooling is in generating human capital. This will give rise to differences in schooling by occupation, which will ultimately lead to differences in wages across occupations. A person’s human capital is produced by combining goods  $e$  and schooling time  $s$ . In particular, the production function for human capital in occupation  $i$  is<sup>1</sup>

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<sup>1</sup>We can allow for a multiplicative group-specific term in the human capital equation defined by (2). This can represent differences in the ability of different groups to accumulate human capital for a given amount of time investment and a given amount of educational spending. This term is

$$h(e, s; i) = s^{\phi_i} e^{\eta}. \quad (2)$$

Finally, people are part of different groups, such as race and gender, denoted  $g$ . A person in occupation  $i$  and group  $g$  is paid a wage equal to  $\delta_{ig}w_i$ . That is,  $w_i$  denotes the wage per efficiency unit of labor paid by the firm. What a worker in group  $g$  receives, however, is  $\delta_{ig}w_i$ .

The parameters  $\delta_{ig}$  are the occupational frictions in the model and ultimately a key force behind the changing occupational distribution over time. They are exogenously given in this economy and can be interpreted in several different ways. We will explicitly distinguish between two of these interpretations, as “Model 1” and “Model 2.” In Model 1,  $\delta_{ig}$  denotes the efficiency units of labor supplied by a person in group  $g$ . This could reflect, for example, differences in human capital across groups because of discrimination in schooling. In Model 2, in contrast,  $\delta_{ig}$  functions like a “discrimination tax” on members of group  $g$  in occupation  $i$ : firms pay  $w_i$  to such workers, but the workers only receive the fraction  $\delta_{ig}$  of the wage payment. We will elaborate on possible interpretations of Model 1 versus Model 2 below.

Let  $\epsilon$  denote a worker’s idiosyncratic talent in his or her chosen occupation, to be discussed further below. An individual’s consumption is then given by

$$c = \delta w \epsilon h(e, s) - e. \quad (3)$$

That is, consumption equals labor income less expenditures on education. Labor income is the product of the wage received per efficiency unit of labor, the idiosyncratic talent (to be discussed in more detail below), and the individual’s human capital  $h$ .

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isomorphic to the occupational  $\delta$ ’s that we define below. As a result, we cannot tell how much of the group differences in occupational frictions we identify are actually resulting from frictions in human capital attainment versus frictions in the labor market itself. For expositional simplicity, we set the multiplicative frictions in the human capital equation to one for all groups.

### 3.2. Occupational Skills

In setting up the occupational choice problem, we follow McFadden (1974) and, more recently, Eaton and Kortum (2002). The latter used extreme value theory, and in particular the Fréchet distribution, to make an  $N$ -country trade model of comparative advantage tractable. We apply their insight to the problem of choosing among  $N$  occupations.

Each person gets an iid skill draw  $\epsilon_i$  for each of the  $N$  occupations. These draws come from a Fréchet distribution:

$$F_i(\epsilon) = \exp(-T_i \epsilon^{-\theta}). \quad (4)$$

The parameter  $\theta$  governs the dispersion of skills, with a higher value of  $\theta$  corresponding to *smaller* dispersion. As in Eaton and Kortum (2002), for tractability we assume that  $\theta$  is common across occupations. The parameter  $T_i$ , however, can differ across occupations: talent is easy to come by in some occupations and scarce in others. As we discuss below, there is an observational equivalence between  $T_i$  and a production technology parameter that varies across occupations. We will therefore end up normalizing  $T_i = 1$  without any loss of generality. Notice that our iid assumption means that we do not allow any correlation of  $\epsilon$  across occupations for a given individual.

### 3.3. Aggregation

To simplify aggregation, we assume the  $N$  occupations combine in a CES fashion to produce a single aggregate output  $Y$  according to

$$Y = \left( \sum_{i=1}^N (A_i H_i)^\rho \right)^{1/\rho} \quad (5)$$

where  $H_i$  denotes the total efficiency units of labor employed in occupation  $i$  and  $A_i$  is the exogenously-given productivity of the occupation. As mentioned above, the  $A_i$  and  $T_i$  (the mean-shift parameter of the Fréchet distribution) are observationally

equivalent in our setup, so we will set  $T_i = 1$  from now on without loss of generality.

The total efficiency units of labor in each occupation differ according to whether we are studying Model 1 or Model 2. In Model 1, recall, the  $\delta_{ig}$  parameters affect the efficiency units of labor explicitly, so we have

$$H_i = \sum_{g=1}^G \delta_{ig} q_g \int h_{ijg} \epsilon_{ijg} dj. \quad (6)$$

To understand this equation, start from the right. First, we integrate over all people  $j$  in group  $g$ , adding up their efficiency units, which are the product of their human capital and their idiosyncratic ability. Next, group  $g$ 's efficiency units get reduced by the factor  $\delta_{ig}$  in occupation  $i$ , and there are  $q_g$  people belonging to group  $g$ . Finally, we add up across all the groups.

In Model 2, recall, the occupational frictions function only as discrimination taxes and do not affect productivity. Hence, the aggregation over efficiency units is just like in the previous equation, but without the  $\delta_{ig}$ :

$$H_i = \sum_{g=1}^G q_g \int h_{ijg} \epsilon_{ijg} dj. \quad (7)$$

What are some possible forces that would act like  $\delta_{ig}$  in Model 1 versus Model 2? In Model 1, groups enter the labor market with different efficiency units to supply a given occupation. But all workers are paid their marginal product in all occupations. Thus Model 1 could be called the Human Capital Model, as differences in wages and occupational choices reflect differences in skills brought to the labor market. Groups might obtain different amounts of human capital for a variety of reasons, such as discrimination at school or in access to quality schools. The latter might stem from differences in public school quality or discrimination in admissions to schools. Even discrimination in the labor market in a previous generation could, indirectly, lower the human capital of the current generation. Parental liquidity constraints could affect their children's health and nutrition. And parents denied access to a profession may be less helpful at guiding their children to that



profession. More benignly, some people might choose to acquire less human capital for their market occupation because they plan to work only part-time or for fewer years, say to help raise children. Any differences in innate ability between men and women in different occupations (e.g. in physical strength or childbearing) would contribute to the  $\delta_{ig}$ 's in Model 1.

In Model 2, in contrast, all groups arrive in the labor market with the same distribution of efficiency units as do white men. This is true within and across occupations. In Model 2, however, the  $\delta_{ig}$ 's drive wedges between the marginal product of workers and the wages they receive. As they are group-occupation specific, these wedges can generate not only wage gaps across groups but also inequality in occupational choices across groups. This might be called the Labor Market Discrimination Model, because two possible interpretations are discrimination by employers and discrimination by customers. Employers may be willing to sacrifice some of their pecuniary return to discriminate in how they compensate their employees. And customers may boycott sellers who employ particular groups in certain occupations. The latter might affect the pecuniary returns from hiring workers from a particular group in a certain occupation, but not the physical marginal product of workers.

The world could feature a combination of the  $\delta_{ig}$ 's in Models 1 and 2, of course. But we treat them as distinct models to underscore their divergent properties. In Model 2, there is misallocation of talent in the labor market with variation in the  $\delta_{ig}$ 's across occupations and groups. Gains in aggregate productivity can be realized simply from reallocating workers across occupations in Model 2. In Model 1, in contrast, there is no misallocation in the labor market itself. Workers are efficiently allocated to occupations *given their skills* in Model 1. But variation in the  $\delta_{ig}$ 's can arise from misallocation in schooling – and in human capital investments more generally – in Model 1. In Model 1, gains in aggregate productivity can be reaped from more and better-directed investment in human capital.

That completes the basic setup of the model. We can now define an equilibrium and then start exploring the model's implications.

### 3.4. Equilibrium

A competitive equilibrium in this economy consists of individual choices  $\{c, e, s\}$ , an occupational choice by each person, total efficiency units of labor in each occupation  $H_i$ , final output  $Y$ , and an efficiency wage  $w_i$  in each occupation such that

1. Given an occupational choice, the occupational wage  $w_i$ , and idiosyncratic ability  $\epsilon$  in that occupation, each individual chooses  $c, e, s$  to maximize utility:

$$U(\delta, w, \epsilon) = \max_{c, e, s} (1 - s)c^\beta \quad s.t. \quad c = \delta w \epsilon s^{\phi_i} e^\eta - e \quad (8)$$

2. Each individual chooses the occupation that maximizes his or her utility:  $i^* = \arg \max_i U(\delta_i, w_i, \epsilon_i)$ , taking  $\{\delta_i, w_i, \epsilon_i\}$  as given.

3. A representative firm chooses labor input in each occupation,  $H_i$ , to maximize profits:

$$\max_{\{H_i\}} \left( \sum_{i=1}^N (A_i H_i)^\rho \right)^{1/\rho} - \sum_{i=1}^N w_i H_i \quad (9)$$

4. The occupational wage  $w_i$  clears the labor market for each occupation:

$$H_i = \begin{cases} \sum_{g=1}^G \delta_{ig} q_g \int h_{ijg} \epsilon_{ijg} dj & \text{Model 1} \\ \sum_{g=1}^G q_g \int h_{ijg} \epsilon_{ijg} dj & \text{Model 2} \end{cases} \quad (10)$$

5. Total output is given by the production function in equation (5).

## 4. Solving the Model

The details of the solution are discussed in the appendix. We summarize the results in a series of propositions, based loosely on the order of the problems as presented in the definition of equilibrium.

**Proposition 1** (Individual Consumption and Schooling): *The solution to the indi-*

vidual's utility maximization problem, given an occupational choice, is

$$s_i^* = \frac{1}{1 + \frac{1-\eta}{\beta\phi_i}}$$

$$e_{ig}^*(\epsilon) = \left( \eta \delta_{ig} w_i s_i^{\phi_i} \epsilon \right)^{\frac{1}{1-\eta}}$$

$$c_{ig}^*(\epsilon) = \bar{\eta} (\delta_{ig} w_i s_i^{\phi_i} \epsilon)^{\frac{1}{1-\eta}}, \quad \bar{\eta} \equiv (1-\eta) \eta^{\frac{-\eta}{1-\eta}}$$

$$U(\delta_i, w_i, \epsilon_i) = \bar{\eta}^\beta (\tilde{\delta}_{ig} \epsilon_i)^{\frac{\beta}{1-\eta}}, \quad \tilde{\delta}_{ig} \equiv \delta_{ig} w_i s_i^{\phi_i} (1-s_i)^{\frac{1-\eta}{\beta}}.$$

This result is an intermediate one, with the key piece coming in the last line describing the equation for  $U_{ig}$ . In particular, the individual's occupational choice problem then reduces to picking the occupation that delivers the highest value of  $\tilde{\delta}_{ig} \epsilon_i$ .

**Proposition 2 (Occupational Choice):** *Let  $p_{ig}$  denote the fraction of people in group  $g$  that work in occupation  $i$  in equilibrium. Aggregating across people, the solution to the individual's occupational choice problem leads to*

$$p_{ig} = \frac{\tilde{\delta}_{ig}^\theta}{\sum_{s=1}^N \tilde{\delta}_{sg}^\theta} \quad \text{where } \tilde{\delta}_{ig} \equiv \delta_{ig} w_i s_i^{\phi_i} (1-s_i)^{\frac{1-\eta}{\beta}}. \quad (11)$$

Moreover, the equilibrium labor supply by group  $g$  to occupation  $i$  in Model 2 is

$$H_{ig} = \gamma \bar{\eta} q_g p_{ig} \cdot \frac{1}{\delta_{ig} w_i} \cdot (1-s_i)^{-1/\beta} \cdot m_g \quad \text{where } m_g \equiv \left( \sum_{s=1}^N \tilde{\delta}_{sg}^\theta \right)^{\frac{1}{\theta} \cdot \frac{1}{1-\eta}} \quad (12)$$

and  $\gamma \equiv \Gamma(1 - \frac{1}{\theta} \cdot \frac{1}{1-\eta})$  is related to the mean of the Fréchet distribution for abilities. (For Model 1, it is nearly the same equation: just multiply both sides by  $\delta_{ig}$ .)

To understand this proposition, notice that  $\tilde{\delta}_{ig}$  can be interpreted as the effective utility benefit per unit of talent in occupation  $i$  for group  $g$ . That is, it is the wage per unit of talent received by the person,  $\delta_{ig} w_i$ , multiplied by the terms reflecting human capital and leisure. Occupational choice then depends on the value of  $\tilde{\delta}_{ig}$  in an occupation relative to all other occupations.

Equation (12) gives the equilibrium efficiency units of labor supplied to occupation  $i$  by group  $g$ . The first term in this equation captures the number of people working in the occupation; the remaining terms capture the “quality” of those people. For example, the second main term,  $\frac{1}{\delta_{ig}w_i}$ , is a selection effect: a higher wage per efficiency unit of labor attracts lower ability people to the occupation, other things equal. The third term captures the fact that occupations with higher schooling will have more human capital. Finally, the last term,  $m_g$ , captures a general equilibrium misallocation effect: the average quality of workers from group  $g$  going into all occupations depends on the average post-friction wages they face.

**Proposition 3 (Occupational Wage Gaps):** *Let  $\overline{wage}_{ig}$  denote the average earnings in occupation  $i$  by group  $g$ . Its value in equilibrium is*

$$\overline{wage}_{ig} \equiv \frac{\delta_{ig}w_iH_{ig}}{q_g p_{ig}} = (1 - s_i)^{-1/\beta} \gamma \bar{\eta} m_g. \quad (13)$$

*Importantly, this implies that the occupational wage gap between any two groups is the same across all occupations. For example,*

$$\frac{\overline{wage}_{i,women}}{\overline{wage}_{i,men}} = \left( \frac{\sum_s \tilde{\delta}_{s,women}^\theta}{\sum_s \tilde{\delta}_{s,men}^\theta} \right)^{\frac{1}{\theta} \cdot \frac{1}{1-\eta}} = \frac{m_{women}}{m_{men}}. \quad (14)$$

The first equation of the proposition reveals that average earnings only differs across occupations because of the first term,  $(1 - s_i)^{-1/\beta}$ . Occupations in which schooling is especially productive (a high  $\phi_i$  and therefore a high  $s_i$ ) will have higher average earnings, and that is the only reason for earnings differences across occupations in the model. For example, occupations that face less discrimination or a better talent pool or higher efficiency do not yield higher average earnings. The reason is that each of these factors leads to lower quality (i.e. lower  $\epsilon$ ) workers entering those jobs. This composition effect exactly offsets the direct effect on earnings. This leads to the novel prediction, given in equation (14), that the earnings gap between two groups (men and women, for example, or blacks and whites) will be *constant* across occupations. We test this proposition in the empirical work that follows.

**Proposition 4 (Solving the General Equilibrium):** *The general equilibrium of the model is  $\{p_{ig}, H_i^{supply}, H_i^{demand}, w_i\}$  and  $Y$  such that*

1.  $p_{ig}$  satisfies equation (11).
2.  $H_i^{supply}$  aggregates the individual choices:

$$H_i^{supply} = \begin{cases} \gamma \bar{\eta} w_i^{\theta-1} (1-s_i)^{(\theta(1-\eta)-1)/\beta} s_i^{\theta\phi_i} \sum_g q_g \delta_{ig}^\theta m_g^{1-\theta(1-\eta)} & \text{Model 1} \\ \gamma \bar{\eta} w_i^{\theta-1} (1-s_i)^{(\theta(1-\eta)-1)/\beta} s_i^{\theta\phi_i} \sum_g q_g \delta_{ig}^{\theta-1} m_g^{1-\theta(1-\eta)} & \text{Model 2} \end{cases} \quad (15)$$

3.  $H_i^{demand}$  satisfies firm profit maximization:

$$H_i^{demand} = \left( \frac{A_i^\rho}{w_i} \right)^{\frac{1}{1-\rho}} Y \quad (16)$$

4.  $w_i$  clears each occupational labor market:  $H_i^{supply} = H_i^{demand}$ .
5. Total output is given by the production function in equation (5).

## 5. Data and Estimation

### 5.1. Data

We use data from the 1960, 1970, 1980, 1990, and 2000 Decennial Censuses as well data from the 2006-2008 American Community Surveys (ACS) for all analysis in the paper. When using the 2006-2008 ACS data, we pool all the years together and treat them as one cross section.<sup>2</sup> We make only four restrictions to the raw data when constructing our analysis samples. First, we restrict the analysis to only include white men, white women, black men and black women. These will be the four groups we analyze in the paper.<sup>3</sup> Second, we restrict the sample to include only

<sup>2</sup>A full description of how we process the data, including all the relevant code, is available at [http://faculty.chicagobooth.edu/erik.hurst/research/chad\\_data.html](http://faculty.chicagobooth.edu/erik.hurst/research/chad_data.html).

<sup>3</sup>We think an interesting extension would be to include Hispanics in the analysis. In 1960 and 1970, however, there are not enough Hispanics in the data to provide reliable estimates of occupational sorting. Such an analysis can be performed starting in 1980. We leave such an extension to future work.

individuals between the ages of 25 and 55 (inclusive). This restriction helps to focus our analysis on individuals after they finish schooling and prior to considering retirement. Third, we exclude individuals on active military duty. Finally, we exclude currently unemployed individuals. Our model is not well suited to capture transitory movements into and out of employment. Appendix Table A1 reports the sample size for each of our six cross sections, including the fraction of the sample comprised of our four groups (white men, white women, black men and black women).<sup>4</sup>

A key to our analysis is to use the Census data to create consistent and systematic measures of occupations over time. We treat the home sector as a separate occupation. Anyone in our data who is not currently employed or who is employed but usually works less than ten hours per week is considered to be working exclusively in the home sector. Those who are employed but usually work between ten and thirty hours per week are classified as being part-time workers. We allocate part-time workers half to the home sector and half to the occupation to which they are working (i.e., we split their sampling weight into these two sectors). Individuals working more than thirty hours per week are considered to be working full-time in an occupation outside of the home sector.

For our base analysis, we define the non-home occupations using the roughly 70 occupational sub-headings from the 1990 Census occupational classification system.<sup>5</sup> We use the 1990 occupation codes as the basis for our occupational definitions because the 1990 occupation codes are available in all Census and ACS years since 1960. We start our analysis in 1960, as this is the earliest year for which the 1990 occupational cross walk is available. Appendix Table A2 reports the 67 occupations we analyze in our main specification using the 1990 occupational sub-headings. Example occupations include “Executives, Administrators, and Managers”, “Engineers”, “Natural Scientists”, “Health Diagnostics”, “Health Assessment”, “Teachers, Except Postsecondary”, “Teachers, Postsecondary”, “Lawyers and Judges”, etc.

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<sup>4</sup>When computing the fraction of people working in each occupation as well as average earnings in each occupation, we weight our data using the sample weights available in the different surveys.

<sup>5</sup><http://usa.ipums.org/usa/volii/99occup.shtml>.

Appendix Table A3 gives a more detailed description of some of these occupational categories. For example, the “Health Diagnostics” occupation includes physicians, dentists, veterinarians, optometrists, and podiatrists, and the “Health Assessment and Treating” occupations include registered nurses, pharmacists, and dieticians. The way the occupations are defined ensures that each of our occupational categories has positive mass in all years of our analysis.

As seen with the examples above, there is some heterogeneity within our 67 base occupational categories. To assess the importance of such heterogeneity, we perform a robustness exercise. Specifically, we use the roughly 340 occupations that are consistently defined (using the 1990 occupation codes) in 1980, 1990, 2000, and 2006-8. The reason we start this in 1980 is that the occupational classification system is roughly similar across the Censuses and ACS starting in 1980. We perform our main analysis using the 340 detailed occupation codes for the 1980–2008 period and show that the quantitative outcomes are very similar to what we get using our 67 base occupation codes for the same period. Additionally, we show that much of our quantitative results can be generated if we use only 20 broad occupation categories as opposed to the roughly 67 occupation codes in our base analysis. The 20 occupation categories we use for this robustness analysis are shown in Appendix Table A4. The 20 broad occupation categories include the same universe of 67 occupations just aggregated to broader categories.

Our measures of earnings throughout the paper sum together the individual’s labor, business, and farm income. The earnings measures in the Census are from the prior year. Implicitly we assume that individuals who are working in a given occupation in the survey year also worked in that same occupation during the prior year which corresponds to their income report. When measuring earnings, we only focus on those individuals who worked at least 48 weeks during the prior year and who had at least 1000 dollars of earnings during the prior year (in year 2007 dollars). We define wages of the individual as individual earnings from the prior year divided by the product of the weeks worked by the individual during the prior year and the reported current usual hours worked by the individual.<sup>6</sup>

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<sup>6</sup>In some census years, weeks worked during the prior year and usual hours worked are reported as

We impute average earnings for the home sector by extrapolating the relationship between average education and average earnings for the 66 non-home occupations, and taking into account group fixed effects. Using this year-specific relationship and the actual year-specific average education and group composition of participants in the home sector, we predict the average earnings of participants in the home sector. We only use these imputed average earnings in the home sector when we weight estimates by the wage bill in each sector.

Appendix Table A1 also shows the estimated wage gap between white men and, respectively, white women, black men, and black women for our base occupational specification. To obtain these estimates, we regress log wages of the individual on group dummies, a quadratic in potential experience, a cubic in usual hours worked and occupation dummies. We estimated this regression separately for each of the years. Appendix Table A1 reports the coefficients on the group dummies from these regressions. In 1960, the conditional log differences in wages for white women, black men, and black women compared to white men are, respectively, -0.58, -0.38, and -0.88. The corresponding gaps in 2006-8 are, respectively, -0.26, -0.15, and -0.31.

## 5.2. Estimation

Our estimation treats each decade separately. For each decade, we have  $5N$  parameters of the model to be estimated. For each of the  $i = 1, \dots, N$  occupations there are  $A_i$ ,  $\phi_i$ , and  $\delta_{ig}$ , where  $g$  stands for white women, black women, or black men. The  $\delta$ 's are identified only up to a normalization, and our normalization here is  $\delta_{i,wm} = 1$ ; that is, we set the value of  $\delta$  to one for white men in all occupations.

To identify these  $5N$  parameters, we match the following  $5N$  moments in the data, decade by decade (numbers in parentheses denote the number of moments):

- ( $4N - 4$ ) The fraction of people from each group working in each occupation,  $p_{ig}$ . (We lose 4 moments since these  $p_{ig}$  sum to one for each

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categorical variables. In these instances, we use the midpoint of the range when computing the wage rate. See the full details of our data processing in the detailed online data appendix available on the author's web sites.



group.)

- (N) The average wage in each occupation.
- (3) Wage gaps between white men and each of our 3 other groups.
- (1) The average years of schooling in one occupation.

As we discuss in more detail later, the  $\delta_{ig}$  parameters are easy to identify in the data given our setup. The  $A_i$  and  $\phi_i$  parameters involve the general equilibrium solution of the model.

The remaining parameters of the model — assumed to be constant over time for parsimony — are  $\eta$ ,  $\theta$ ,  $\rho$ , and  $\beta$ . We discuss our baseline values for these parameters briefly below.

The parameter  $\eta$  denotes the elasticity of human capital with respect to education spending. Related parameters have been discussed in the literature, for example by Manuelli and Seshadri (2005) and Erosa, Koreshkova and Restuccia (2010). In our model,  $\eta$  will equal the fraction of output spent on accumulating human capital in equilibrium, separate from time spent accumulating human capital. Absent any solid evidence on this parameter, we set  $\eta = 1/4$  in our baseline and explore robustness to  $\eta = 0$  and  $\eta = 1/2$ . In general, this parameter affects the *level* of the  $\delta_{ig}$  parameters, but not much else in the results.

The parameter  $\theta$  is a key parameter that governs the dispersion of wages. Given the occupational choice model developed above, one can show that the dispersion of wages across people within an occupation-group obeys a Fréchet distribution with the shape parameter  $\theta(1 - \eta)$ : the lower is this shape parameter, the *more* wage dispersion there is within an occupation. Wage dispersion therefore depends on the dispersion of talent (governed by  $1/\theta$ ) and amplification from accumulating human capital via spending (governed by  $1/(1 - \eta)$ ). In particular, the coefficient of variation of wages within an occupation-group in our model satisfies

$$\frac{\text{Variance}}{\text{Mean}^2} = \frac{\Gamma(1 - \frac{2}{\theta(1-\eta)})}{\Gamma(1 - \frac{1}{\theta(1-\eta)})} - 1. \quad (17)$$

To estimate  $\theta(1 - \eta)$  in a given year, we first take residuals from a cross-sectional regression of log worker wages on 66 occupation dummies and 3 group dummies (one each for white women, black men, and black women). The wage is the hourly wage, and the sample includes both full-time and part-time workers. The occupation dummies capture the effect of schooling requirements ( $\phi_i$  levels) on average wages in an occupation, and the group dummies absorb the wage gaps created by frictions (the average  $\delta_{ig}$  across occupations for each group). We calculate the mean and variance across workers of the exponent of these wage residuals. We then solve equation (17) for the value of  $\theta(1 - \eta)$ . Sampling error is trivial here because there are 300-400k observations per year for 1960 and 1970 and 2-3 million per year for 1980 onward. The point estimates for  $\theta(1 - \eta)$  hover around 3. They drift down over time, from 3.25 in 1960 to 2.84 in 2006-2008, as one would expect given rising wage inequality. For our baseline model, we use the simple average of the point estimates across years, namely  $\theta(1 - \eta) = 3.11$ . We will explore robustness to setting it as low as 2 or as high as 15.

The parameter  $\rho$  governs the elasticity of substitution among our 67 occupations in aggregating up to final output. We have little information on this parameter and choose  $\rho = 2/3$  for our baseline value. In the empirical section we will explore the robustness of the estimates to different values of  $\rho$ .

The parameter  $\beta$  is the geometric weight on consumption relative to time in an individual's utility function (1). As schooling trades off time for consumption, the model implies that wages increase more steeply with schooling the lower is  $\beta$ . Workers must be more heavily compensated for sacrificing time to schooling the more they care about time relative to consumption. To be specific, the average wage of group  $g$  in occupation  $i$  is proportional to  $(1 - s_i)^{\frac{-1}{\beta}}$ . If we take a log linear approximation around average schooling  $\bar{s}$ , then  $\beta$  is inversely related to the Mincerian return to schooling across occupations (call this return  $\psi$ ):  $\beta = (\psi(1 - \bar{s}))^{-1}$ . We estimate  $\beta$  in each year in this way. We calculate  $s$  as years of schooling divided by a pre-work time endowment of 25 years, and find the Mincerian return  $\psi$  from a regression of log wages on average occupation schooling (with group dummies as controls). We set  $\beta = 0.693$ , the simple average of the estimates across years. This

method has the advantage that the model roughly matches the Mincerian return to schooling across occupations, which averages 12.7% across the six decades. For robustness we will consider a low value of  $\beta = 0.5$  and a high value of  $\beta = 0.8$ .

Finally, we need to set  $\phi_i$ , the impact of time spent in schooling on human capital for each occupation  $i$ . Recall from equation (13) that wages are increasing in schooling across occupations. And, from Proposition 1, we know that schooling increases with  $\phi_i$ . Thus we can infer from wages in each occupation the *relative* values of  $\phi_i$  across occupations. But we cannot determine their levels, as wage levels are also affected by levels of the productivity parameters  $A_i$ . This leaves us in need of one final normalization that will set the  $\phi_i$  levels. Once we have one  $\phi_i$ , we can infer all of the others from relative wages across occupations. We choose to normalize the level of  $\phi$  in the lowest wage occupation across most years, “Farm Non-Managers.” We set this  $\phi_{min}$  in each year to match the observed average schooling in this occupation in the same year:  $\phi_{min} = \frac{1-\eta}{\beta} \frac{s_{min}}{1-s_{min}}$ .

## 6. Estimates of Occupational Frictions Over Time

### 6.1. Occupational Wage Gaps

A prediction of our model is that the wage gaps between various groups should be constant across occupations due to the sorting that takes place in response to occupational frictions: an occupation that pays a high wage per unit of ability will attract less talented workers. The sorting is what makes the wage gap in a given occupation a poor measure of any friction in that single occupation. There are, however, at least three reasons why the estimated wage gaps between groups will not be exactly equal across all occupations. First, there is obviously some measurement error. Second, although we expect sorting will help offset the effect of differences in wages per ability on the average wage in an occupation, the exact offset due to sorting is a feature of the extreme value distribution. We would not get the complete offset if ability is not exactly distributed according to an extreme value distribution. Third, we focus on occupational sorting due to heterogeneity in ability, but some of the occupa-

Table 1: Occupational Wage Gaps versus  $p_{i,g}/p_{i,wm}$ 

	1960			1980			2006-8		
	ww	bm	bw	ww	bm	bw	ww	bm	bw
SD Increase in $P_i^f/P_i^{wm}$	0.006	-0.038	0.037	0.001	-0.030	-0.008	-0.008	-0.050	-0.017
Standard Error	0.021	0.015	0.037	0.011	0.010	0.015	-0.008	0.011	0.015
R-squared	<0.01	0.09	<0.01	<0.01	0.10	<0.01	<0.01	0.22	<0.01

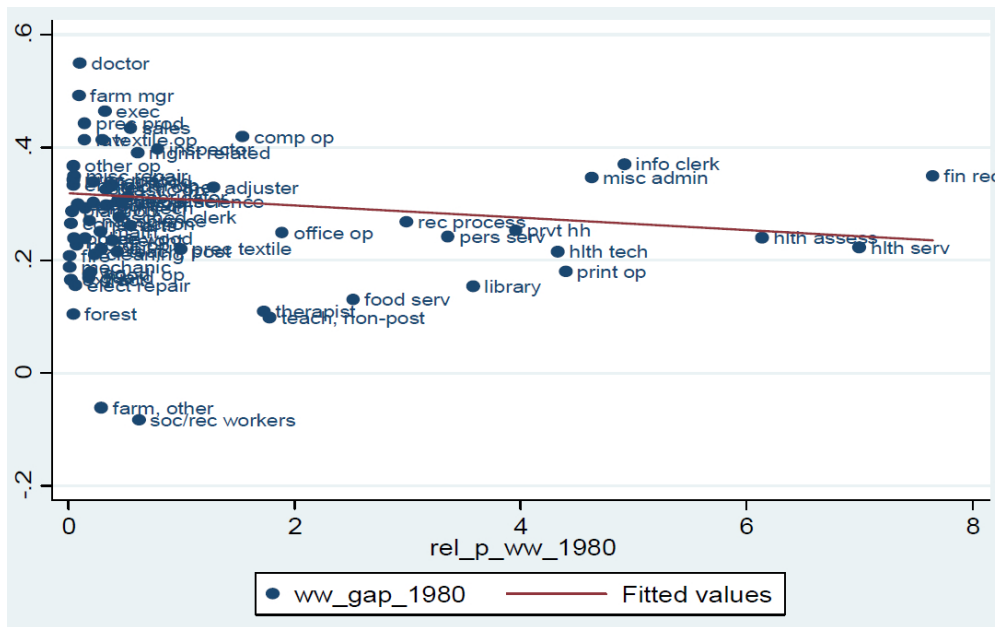
Note: The table reports the results of regressing the log of occupational wage gaps on  $p_{i,g}/p_{i,wm}$  for various groups and years. The first row reports the regression coefficient as the effect of a one standard deviation increase in the relative propensity on the log wage gap. Each column is a regression.

tional sorting might be driven by heterogeneity in tastes or preferences. High wage (per unit of ability) occupations might induce the entry of people with high disutility for an occupation rather than individuals with low ability in the occupation. All three forces will generate variation in wage gaps across occupations.

Therefore, a key test of the plausibility of our framework is to examine the variation in the wage gap with the variables measuring the frictions facing women or blacks in an occupation. Figure 1 plots the (log) occupational wage gap for white women in 1980 against  $p_{i,ww}/p_{i,wm}$ . The latter variable is the relative propensity of a white woman to work in a particular occupation, versus a white man. As an example, in 1980, a white women was 65 times more likely than a white man to work as a secretary, but only 0.14 times as likely to work as a lawyer. Given this enormous variation, the differences in the wage gaps are remarkably small. White women secretaries earned about 33 percent less than white men secretaries, while the gap was 41 percent for lawyers. Figure 1 confirms the relatively small slope more generally. Notice that, within the model, it is the relative propensity that pins down the friction facing a group (relative to white men) in that occupation. As seen from Figure 1 there is relatively little difference in occupational wage gaps across occupations but there are much bigger differences in the relative  $p_i$ 's.

Table 1 shows these slopes for other years and other groups as well. There is little correlation between the relative propensity of a group to work in an occupation and

Figure 1: Occupational Wage Gaps for White Women in 1980



Note: The figure shows that there is little correlation between the (log) occupational wage gap for white women compared to white men and the relative propensity to work in the occupation between white women and white men,  $p_{i,ww}/p_{i,wm}$ . Secretaries (with a relative propensity of 65 and a log wage gap of 0.33) are excluded from the graph to make it easier to read.

Table 2: Occupational Wage Gaps versus Log Income

Independent Variable	White women	Black Men	Black Women
	2006-8		
SD increase in ln(income <sup>i</sup> )	0.034	0.055	0.056
Standard Error	(0.009)	(0.007)	(0.013)
	<<0.18>>	<<0.48>>	<<0.21>>
	1980		
SD increase in ln(income <sup>i</sup> )	0.064	0.024	0.057
Standard Error	(0.009)	(0.007)	(0.009)
	<<0.42>>	<<0.14>>	<<0.36>>
	1960		
SD increase in ln(income <sup>i</sup> )	0.079	0.053	0.111
Standard Error	(0.014)	(0.013)	(0.025)
	<<0.35>>	<<0.19>>	<<0.26>>

Note: The table reports the results of regressing the log of occupational wage gaps on log earnings in that occupation for various groups and years. The first row reports the regression coefficient as the effect of a one standard deviation increase in log income. Each column is a regression, and the last row reports the  $R^2$  from each regression.

that group's occupational wage gap – precisely as suggested by our model.

Table 2 shows that this invariance of the wage gap to occupation characteristics does not completely hold when we look at the average income in the occupation. On average, high income occupations tend to have larger wage gaps. This suggests that the extreme value distribution might not entirely correct for high income occupations. Nonetheless, the magnitude of this correlation is small. For example, in 2006-2008, white working women had a 3.4 percentage point larger wage gap in response to a one-standard deviation increase in occupational log income.

## 6.2. Estimating Occupational Frictions: $\delta_{ig}$

Using equations (11) and (14), it is straightforward to show that the value of  $\delta_{ig}$  can be recovered directly from the data as

$$\delta_{ig} = \left( \frac{p_{ig}}{p_{i,wm}} \right)^{\frac{1}{\theta}} \left( \frac{\overline{\text{wage}}_g}{\overline{\text{wage}}_{wm}} \right)^{1-\eta}. \quad (18)$$

That is, up to exponents, the friction parameters are the product of the relative propensity of a group to work in an occupation and the overall wage gap for that group. When, say, more women work in an occupation relative to men, or when women are paid closer to what men are paid, the higher is  $\delta_{ig}$  for women in that occupation. And the closer is  $\delta_{ig}$  to 1, the smaller is the friction in that occupation.

Table 3 reports the  $\delta$  parameters that we recover in this way for white women for a subset of our baseline occupations. To compute these, we use the average wage gaps reported in Appendix Table A1 and the  $p_{i,g}/p_{i,wm}$  computed directly from each of the census waves.<sup>7</sup> First, consider the results for 1960, recalling that the  $\delta$ 's for white men are normalized to one in all occupations. White women in the “home” occupation have a  $\delta$  just over one. In contrast, the  $\delta$ 's for managers, lawyers, doctors, and college professors range from 0.27 to 0.44. The low participation of white women in these occupations in 1960 is interpreted as arising from low values of  $\delta$ . Interestingly, the  $\delta$  for teachers is appreciably less than one in 1960 for this group. While white women were 1.7 times more likely than white men to work as teachers, this propensity is more than offset by the overall wage gap in 1960, where women earned  $\exp(-.578) \approx 0.56$  times what men earned.

Contrast this with secretaries in 1960. A white woman in 1960 was 24 times more likely to work as a secretary than was a white man. The model can only explain this enormous discrepancy by assigning a  $\delta$  of 1.4 for white women secretaries. Thus the model interprets these data patterns as either white women secretaries had more human capital or there was discrimination against white *men* being secretaries. For example, if there were discriminatory norms in 1960 preventing white men from

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<sup>7</sup>As a reminder, our base specification assumes that  $\theta(1 - \eta)$  is equal to 3.11 with  $\eta$  being equal to 0.25.

Table 3: Estimated  $\delta$  for White Women

	1960	1970	1980	1990	2000	2008
Home	1.05	0.99	1.03	1.02	0.98	1.02
Executives/Admin	0.40	0.42	0.53	0.66	0.70	0.73
Lawyers	0.27	0.30	0.44	0.57	0.65	0.71
Doctors	0.30	0.33	0.40	0.53	0.62	0.70
Teachers, Post Second.	0.44	0.47	0.57	0.68	0.78	0.81
Teachers, Others	0.74	0.71	0.80	0.94	1.07	1.11
Secretaries	1.40	1.41	1.92	2.07	1.82	1.81

Note: Author's calculations based on equation (18) using baseline parameter values.

being secretaries, the model treats this as akin to a subsidy for white women in this occupation relative to white men.

Next, consider how the  $\delta$ 's change over time in Table 3. For the home occupation,  $\delta$  for white women stays right around 1.0. However, for managers, lawyers, doctors, and college professors, the  $\delta$ 's approximately doubled, rising from around 0.3 or 0.4 to around 0.7 or 0.8. School teachers (i.e., Teachers, Other) also see a substantial increase in their average  $\delta$  from 0.74 to a value exceeding one. Interestingly, the  $\delta$  for secretaries also rises, from 1.4 in 1960 to over 2.0 in 1990 before retreating to 1.8 in 2008. Remarkably, a white woman is even more likely than a white man to work as a secretary in 2008 than in 1960: the propensity rises from 24 to 26.5.

The  $\delta$ 's for black men and black women for these same select occupations are shown in Table 4. A similar overall pattern emerges, with the  $\delta$ 's being substantially less than one in general in 1960 and rising appreciably through 2008, though typically remaining below one, especially for the high-paying occupations.



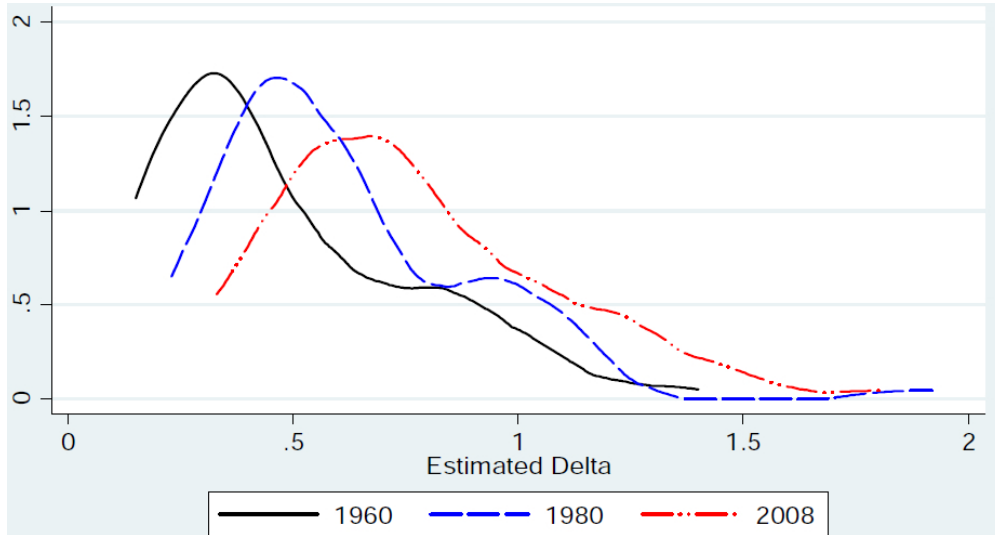
Table 4: Estimated  $\delta$  for Black Men and Women

	1960	1970	1980	1990	2000	2008
<b><u>Black Men</u></b>						
Home	0.91	0.95	1.06	1.13	1.11	1.08
Executives/Admin	0.47	0.58	0.66	0.71	0.71	0.73
Lawyers	0.48	0.52	0.58	0.64	0.63	0.65
Teachers, Others	0.72	0.75	0.77	0.82	0.87	0.89
Secretaries	0.65	0.78	0.89	1.01	0.91	0.93
<b><u>Black Women</u></b>						
Home	0.82	0.87	0.99	1.01	0.97	0.95
Executives/Admin	0.25	0.35	0.47	0.60	0.63	0.65
Lawyers	0.18	0.25	0.38	0.49	0.55	0.59
Teachers, Others	0.59	0.67	0.81	0.92	0.99	1.02
Secretaries	0.81	1.04	1.71	1.95	1.66	1.63

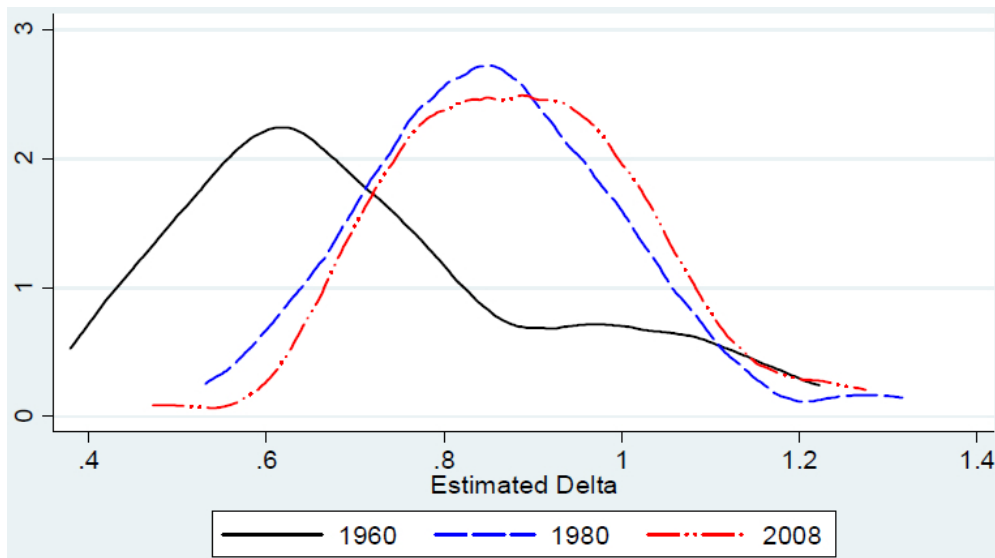
Note: Author's calculations based on equation (18) using baseline parameter values.

Figures 2, 3, and 4 provide a different perspective on the  $\delta$ 's, plotting the densities across the 67 occupations in 1960, 1980, and 2008 for white women, black men, and black women, respectively. These figures are based on the unweighted estimates of the  $\delta$ 's across occupations. Summary statistics for the mean and standard deviation of the  $\delta$ 's, weighted by the occupations share of the wage bill, are shown in Table 5.

For white women, a few main features stand out from Figures 2 and Table 5. First, as seen in Figures 2, there is the general rightward shift in the distribution of  $\delta$  over time. This means that, on average, the frictions facing white women have been declining. This can also be seen in Table 5, where the weighted average  $\delta$  for white women increases from 0.65 in 1960 to 0.83 in 2006-2008. This fact is identified from both the declining wage gaps and the more equal occupational sorting from 1960 through 2006-2008. Second, the standard deviation of the  $\delta$ 's for white women are falling over time. Both the increase in the mean and the decline in the standard deviation are essentially monotonic. When computing the productivity gains under

Figure 2: The Distribution of  $\delta$ 's for White Women

Note: The graph shows a kernel density plot, calculated using a Epanechnikov smoother.

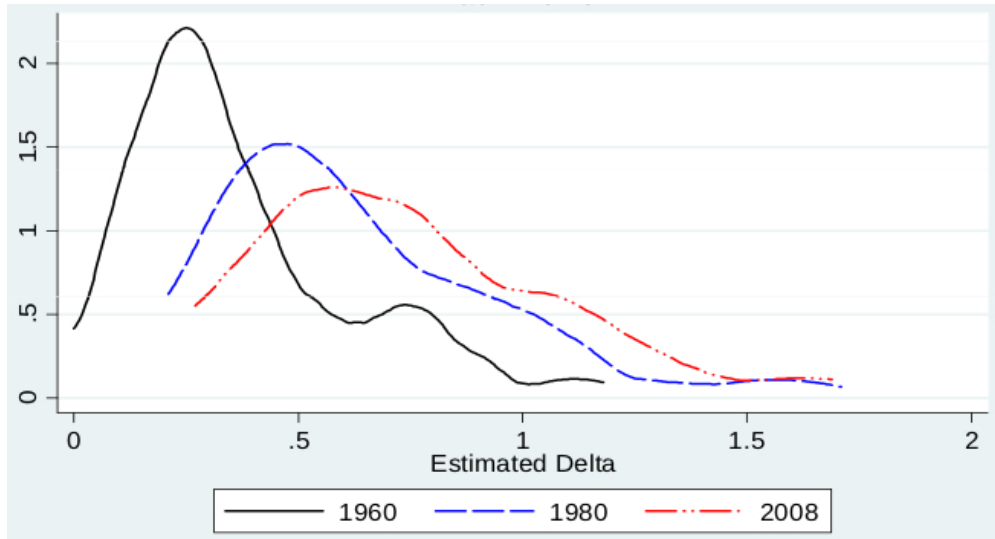
Figure 3: The Distribution of  $\delta$ 's for Black Men

Note: The graph shows a kernel density plot, calculated using a Epanechnikov smoother.

Table 5: Weighted Means and Standard Deviations of  $\delta$ , by Group, Region, and Time

	Year						Difference 2006-8 vs. 1960
	1960	1970	1980	1990	2000	2006-8	
<b>White Women</b>							
Average Delta	0.651	0.631	0.702	0.767	0.800	0.830	0.179
Standard Deviation of Delta	0.348	0.312	0.325	0.307	0.262	0.266	-0.082
Standard Deviation of Delta/Average Delta	0.534	0.494	0.463	0.401	0.328	0.320	-0.214
Average Delta: North	0.679		0.723			0.850	0.171
Average Delta: Midwest	0.637		0.693			0.831	0.194
Average Delta: South	0.639		0.697			0.827	0.188
Average Delta: West	0.672		0.712			0.842	0.170
<b>Black Men</b>							
Average Delta	0.717	0.781	0.849	0.873	0.873	0.873	0.156
Standard Deviation of Delta	0.204	0.174	0.170	0.173	0.165	0.148	-0.056
Standard Deviation of Delta/Average Delta	0.284	0.223	0.201	0.198	0.189	0.169	-0.115
Average Delta: North	0.828		0.895			0.869	0.041
Average Delta: Midwest	0.847		0.934			0.879	0.032
Average Delta: South	0.686		0.821			0.879	0.193
Average Delta: West	0.812		0.912			0.926	0.114
<b>Black Women</b>							
Average Delta	0.484	0.551	0.682	0.750	0.769	0.777	0.293
Standard Deviation of Delta	0.286	0.286	0.314	0.311	0.267	0.265	-0.021
Standard Deviation of Delta/Average Delta	0.590	0.518	0.460	0.414	0.347	0.341	-0.249
Average Delta: North	0.619		0.728			0.801	0.182
Average Delta: Midwest	0.565		0.722			0.799	0.234
Average Delta: South	0.431		0.661			0.776	0.345
Average Delta: West	0.545		0.725			0.824	0.279

Note: Author's calculations based on equation (18). To compute the mean and standard deviation of  $\delta$  across occupations for a given group in a given year, we weight the occupations by the occupation's income share out of total income across all occupations. We use standard Census classifications when defining Census regions.

Figure 4: The Distribution of  $\delta$ 's for Black Women

Note: The graph shows a kernel density plot, calculated using a Epanechnikov smoother.

Model 2 in the subsequent section, it is the standard deviation of the  $\delta$ 's relative to the mean of the  $\delta$ 's that drives misallocation. As seen from Table 5, this statistic has fallen sharply for white women over time from 0.53 in 1960 to 0.32 in 2008.

The changing distribution of  $\delta$ 's for black men and black women are shown in Figure 3 and 4. Similar to white women, there have been sharp increases in the average  $\delta$ s and sharp declines in the standard deviation of the delta between 1960 and 2008. As shown in Table 5, for black men, the mean  $\delta$  increased from 0.71 in 1960 to 0.87 in 2006-2008. The standard deviation of the  $\delta$ 's relative to the mean of the  $\delta$ s fell by 12 percentage points and 22 percentage points, respectively, for black men and black women over our sample. There is one difference in the  $\delta$  trends for black men relative to women. For black men, the trends in the  $\delta$ s are almost exclusively due to changes between 1960 and 1980. After 1980, there was essentially no movement in the average  $\delta$  for black men. Conversely, white and black women continued to experience increasing  $\delta$ 's and a declining standard deviation of their  $\delta$ 's between 1980 and 2008. These results will underlie our findings in the next section that changes in the occupational frictions facing black men did not add much to

overall U.S. growth between 1980 and 2008.

### 6.3. Changing $\delta$ 's by U.S. Region

One goal in the next section will be to assess how much of the convergence between the North and the South during the last 50 years can be explained by differential trends in occupational frictions across the regions. As seen from Table 5, for white women the  $\delta$  trends were nearly identical across the four census regions. All regions experienced increasing  $\delta$ 's that were close to our estimates for the country as a whole. For black men and black women, however, there are clear differences between regions. The average  $\delta$ 's started much lower in 1960 for blacks in the South than for blacks in other regions. For example, the average  $\delta$  in 1960 for black men in the North was 0.83 versus 0.69 in the South. By 1980, the differences in these average  $\delta$ s were smaller (0.90 versus 0.82). In summary, blacks in the South faced bigger frictions in 1960 than did blacks in other regions. And the decline in occupational frictions from 1960 to 2008 was more pronounced for blacks in the South than for blacks in other regions. Consistent with the overall trends, most of the convergence in occupation frictions for blacks across regions occurred prior to 1980.

These trends will form the basis of our model's predictions about how declines in occupational frictions for blacks contributed to income growth in the South relative to the north during the 1960-1980 period.

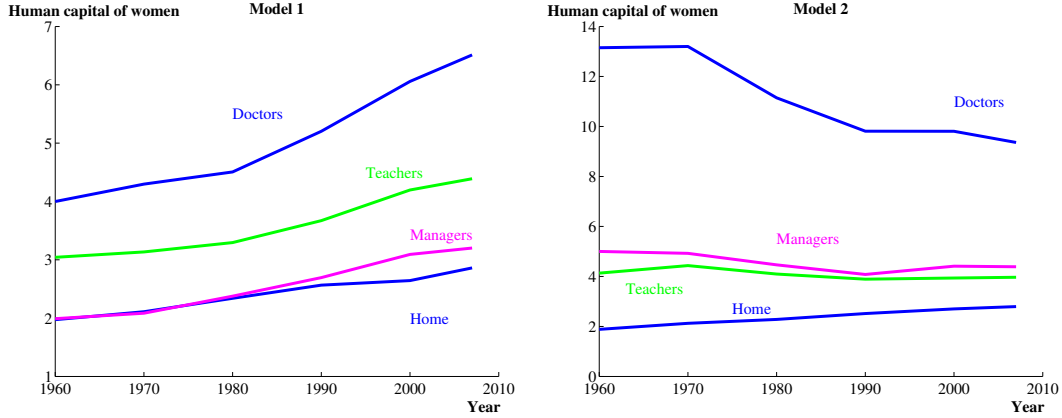
### 6.4. Human Capital of Workers by Occupation

Using equation (12), the amount of human capital per worker — including innate ability — for group  $g$  in occupation  $i$  in Model 2 is given by

$$\frac{H_{ig}}{q_g p_{ig}} = \gamma \bar{\eta} \cdot \frac{1}{\delta_{ig} w_i} \cdot (1 - s_i)^{-1/\beta} \cdot m_g. \quad (19)$$

The amount of human capital per worker for a group *relative* to white men in

Figure 5: Human Capital per Worker for White Women



Note: Average quality (human capital and innate ability) in various occupations for white women, in both Model 1 and Model 2. Computed using equation (19).

Model 2 is therefore

$$\frac{H_{ig}/q_g p_{ig}}{H_{i,wm}/q_{wm} p_{i,wm}} = \frac{1}{\delta_{ig}} \cdot \frac{\overline{\text{wage}}_g}{\overline{\text{wage}}_{wm}} \cdot [a] \quad (20)$$

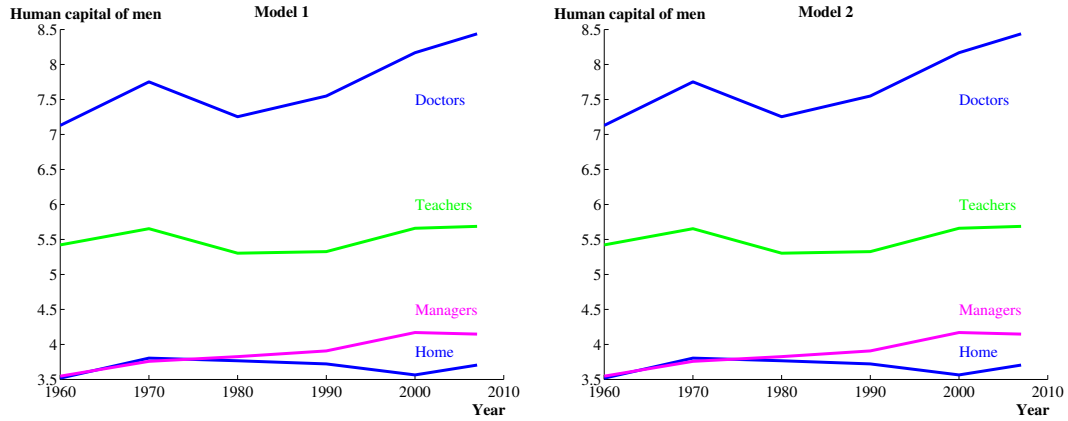
That is, relative quality in an occupation is simply the wage gap divided by the occupational frictions.

For Model 1, the expressions are similar: we simply multiply both sides of each equation by  $\delta_{ig}$ . Equation (20) then implies that the amount of human capital per worker for a group relative to white men is *the same across all occupations* in Model 1. In particular, relative quality is precisely equal to the wage gap.

Figure 5 shows the average amount of human capital per worker for white women for select occupations, in Model 1 and Model 2. These measures are shaped by several forces. First, there is the general rise in human capital over time. These forces are especially apparent for Model 1.

Second, however, are the substantial selection effects that occur as the  $\delta$ 's change, evident in Model 2. For example, human capital per worker among women doctors has declined over time: in 1960, only the most able women became doctors, according to the model, while in 2008 far less able women have entered this pro-

Figure 6: Human Capital per Worker for White Men



Note: Average quality (human capital and innate ability) in various occupations for white men, in both Model 1 and Model 2. Computed using equation (19).

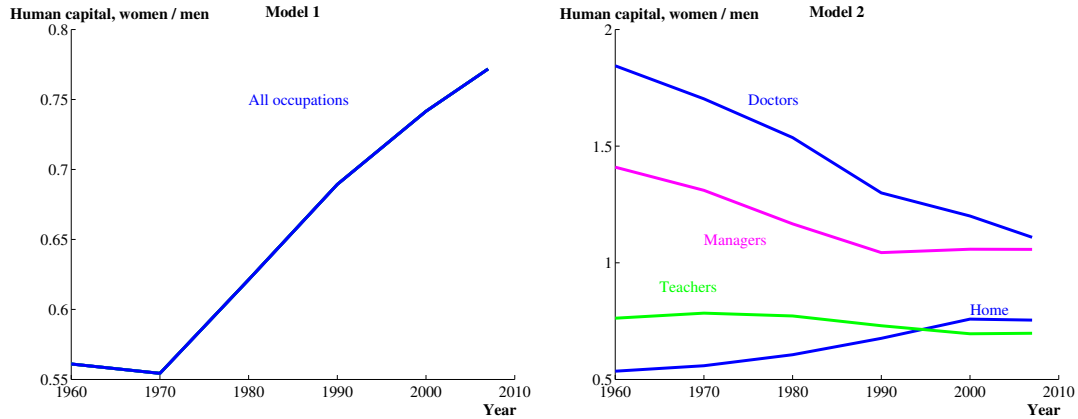
profession, lowering the overall human capital per doctor among women. For teachers and managers, this same selection effect is roughly offset by the general rise in educational attainment, leading to a relatively stable amount of human capital per female worker.

For white men, the results are shown in Figure 6. Importantly, notice that the average quality of white men is the same in Model 1 and Model 2; this reflects our normalization that  $\delta_i = 1$  for white men. In both models, the general rise in human capital per worker is dominant. But the selection effects are also apparent in some occupations, such as school teachers, for example.

Figure 7 shows the relative amounts of human capital between white women and white men for these same occupations, as in equation (20). For Model 1, as mentioned above, relative qualities are equated for all occupations. The graph shows that the relative quality of women to men in each occupation rose substantially between 1960 and 2008, from 0.56 to 0.77.

Model 2 presents a very different view of the data. Relative qualities are not the same across occupations, as shown in the right panel. In 1960, human capital per worker was substantially higher for women relative to men for doctors and managers. Only the most talented women overcame frictions to become doctors and

Figure 7: Relative Human Capital per Worker, White Women vs. White Men



Note: Relative quality (human capital and innate ability) in various occupations for white women versus white men, in both Model 1 and Model 2. Computed using equation (20).

managers in 1960, and some lesser talented white men entered these professions instead. According to Model 2, this difference in quality has faded substantially over time due to declining frictions, but remains present even in 2008.

Of course, the real world could reflect forces in both Models 1 and 2. To this end, independent information on quality trends for occupation-groups could be quite helpful in discriminating between the models empirically (or quantifying their relative contribution).

## 7. Productivity Gains from Changing Frictions

Given our model and the parameter values, we can now answer one of the key questions of the paper: how much of overall earnings growth between 1960 and 2008 can be explained by the changing  $\delta$  frictions?

In answering this question, the first thing to note is that output growth in our model is a weighted average of earnings growth in the market sector and in the home sector. Earnings growth in the market sector can be measured as real earnings growth in the census data. Deflating by the NIPA Personal Consumption Deflator,



real earnings in the census data grew by 1.32 percent per year between 1960 and 2007. As for the home sector, we impute the wage by extrapolating the relationship between average education and average earnings in the market sectors to average education in the home sector. (See the discussion in section 5.1. for additional details.) Taking a weighted average of the imputed wage in the home sector and the wage in the census data, we estimate that output (as defined by our model) grew by 1.45 percent per year between 1960 and 2008. Note that this is lower than standard output growth measures because it is calculated solely from wages; for example, it omits employee benefits.

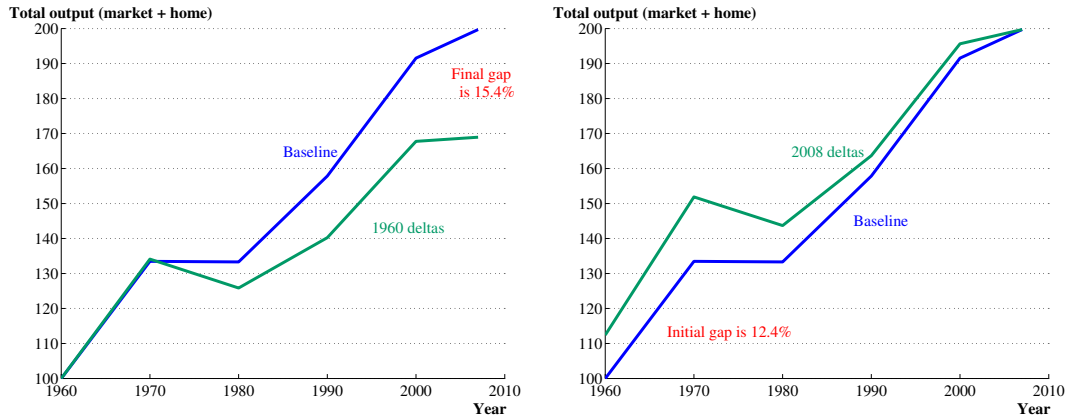
How much of this growth is accounted for by changing  $\delta$ 's? One way to answer this question would be to hold the  $A$ 's (productivity parameters by occupation),  $\phi$ 's (schooling parameters by occupation), and  $q$ 's (group shares of the working population) constant over time and let the  $\delta$ 's change. But at which year's value should we hold the  $A$ 's,  $\phi$ 's, and  $q$ 's constant? We follow the standard approach in macroeconomics and use *chaining* to answer our question. That is, we compute growth between 1960 and 1970 allowing the  $\delta$ 's to change but holding the other parameters at their 1960 value. Then we do the same thing holding the other parameters at their 1970 value. We take the geometric average of these two estimates of the growth that results from changing  $\delta$ 's. We do the same for other decadal comparisons (1970 to 1980 and so on) and cumulate the growth to arrive at an estimate for our entire 47 year sample from 1960–2007.

When the frictions are interpreted as differences in marginal products across groups (Model 1), this calculation indicates that the change in occupational frictions contributed an average of 0.298 percentage points to growth per year. This would explain 20.2 percent of overall earnings growth over the last half century.

Could these Model 1 gains be inferred from the declining wage gaps alone? To a close approximation, yes. The faster wage growth for blacks and white women contributed 0.32 percentage points per year to overall wage growth from 1960 to 2007. This is indeed similar to our estimate of the productivity gain from changing  $\delta$ 's. The reason is that Model 1 assumes all workers are paid their marginal product.

If we instead interpret the frictions as gaps in marginal products across occupa-

Figure 8: Counterfactuals in Model 1



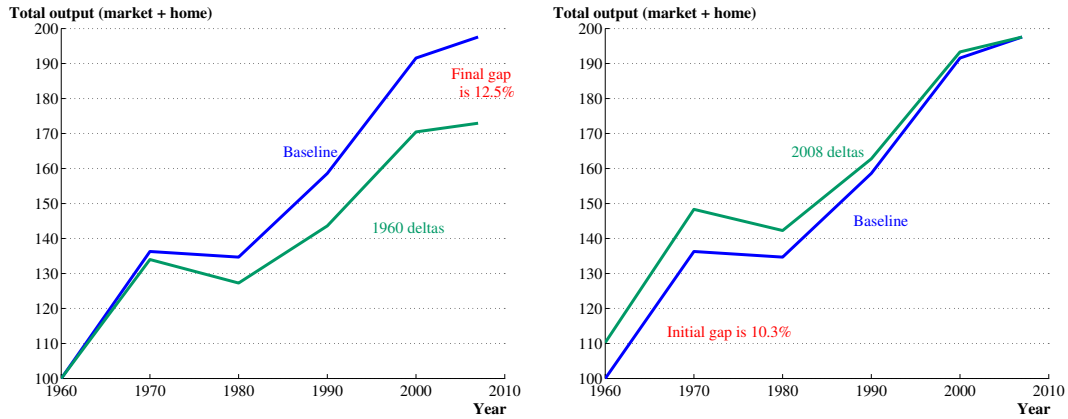
Note: The left panel shows the counterfactual path of output in the model if the  $\delta$ 's were kept at their 1960 values in every period. The right panel shows the counterfactual where the  $\delta$ 's are kept at their 2008 values.

tions (Model 2), the wage gaps no longer provide an accurate estimate of the effect of changing frictions on aggregate productivity. Here, chain-weighted growth from changing  $\delta$ 's is 0.241 percent per year. According to Model 2, the changing frictions account for 16.7 percent of the cumulative earnings growth from 1960 to 2008.

An alternative calculation is to hold the  $\delta$ 's constant and calculate the hypothetical growth rate due to the change in the  $A$ 's,  $\phi$ 's, and  $q$ 's. Figure 8 plots the results of this calculation for Model 1. The left panel considers the case when the occupational frictions are held constant at 1960 levels; the right panel presents the case when the  $\delta$ 's are kept at 2008 levels. Holding the  $\delta$ 's fixed at their 1960 level, output in 2008 would be 15.4 percent lower than it actually was. Holding the  $\delta$ 's fixed at their 2008 level, output in 1960 would be 12.4 percent higher than in the data. Figure 9 presents similar estimates, this time for Model 2. Here, holding the  $\delta$ 's fixed at their 1960 level would result in output being 12.5 lower in 2008. Holding the  $\delta$ 's fixed at their 2008 level, output in 1960 would be 10.3 percent higher. Note that both models predict output growth would have been negative in the 1970s in the absence of the reduction in  $\delta$ s for blacks and women in that decade.

Tables 6 and 7 probe the robustness of our estimates to different parameter

Figure 9: Counterfactuals in Model 2



Note: The left panel shows the counterfactual path of output in the model if the  $\delta$ 's were kept at their 1960 values in every period. The right panel shows the counterfactual where the  $\delta$ 's are kept at their 2008 values.

choices for Model 1 and 2, respectively. The first row checks sensitivity to the assumed value of the elasticity of substitution ( $\rho$ ) between the different occupations; the second row considers different values of the elasticity of human capital with respect to goods invested in human capital ( $\eta$ ), the third row allows for different values of the degree of comparative advantage ( $\theta$ ); and the fourth row reports sensitivity to different estimates of the weight placed on time vs. goods in utility ( $\beta$ ).

As Table 6 shows, the productivity effect of changing frictions is remarkably robust to altering the parameter values when the frictions are interpreted as productivity differences between groups (Model 1). This is because, again, wage gaps are almost a sufficient statistic for the productivity effect of the occupational frictions in Model 1. And the wage gaps are the same regardless of our parameter values – we choose the  $\delta$ 's to fit the wage gaps conditional on the assumed values of  $\rho$ ,  $\eta$ ,  $\theta$ , and  $\beta$ .

In contrast, Table 7 shows that the estimates are more sensitive to the parameter values for Model 2. The share of earnings growth explained by changing  $\delta$ 's ranges from 12.7 percent when the occupations are almost Leontief ( $\rho = -90$ ) to 19.1 percent when they are almost perfect substitutes ( $\rho = 0.95$ ). This compares to

16.7 percent with our baseline value of  $\rho = 2/3$ . In Row 2 the gains are lower than baseline when goods are not inputs to human capital accumulation (15.0 percent gains when  $\eta = 0$ ), and higher than baseline when goods are more important to human capital accumulation (17.9 percent gains when  $\eta = 0.5$ ). Row 3 indicates that the estimates are also sensitive to the degree of comparative advantage. The share of earnings growth explained by changing frictions ranges from 8.8 percent when the dispersion of skills is low and occupations are complementary ( $\theta(1 - \eta) = 15$  and  $\rho = -90$ ) to 20.1 percent when the degree of comparative advantage is high ( $\theta(1 - \eta) = 15$  and  $\rho = 2/3$ ).

For Model 2, like for Model 1, we re-estimate the  $\delta$ 's for any set of parameter values to ensure that the model matches observed wage gaps by year. Yet in Model 2 this does not tightly pin down the productivity gains from changing  $\delta$ 's. The reason is that wage gaps are due to high labor market discrimination in Model 2, not low average productivity for women and blacks. There are no direct effects on aggregate output of low *average*  $\delta$ 's for a given group. Instead, in Model 2 the damage to aggregate productivity is done by the *dispersion* in the  $\delta$ 's across occupations for a given group. It is this dispersion that leads talented women and blacks to be misallocated away from the professions in which they have a comparative advantage, dragging down output per worker.

How much of the productivity gains reflect changes in the occupational frictions facing women vs. those facing blacks? Tables 8 and 9 answer this question for Models 1 and 2 with baseline parameter values. The second column presents the overall wage growth for each time period and the third column replicates the estimates (already shown in Figures 8 and 9) of setting the  $\delta$ 's of all the groups to their level at the end of each period (1960–1980, 1980–2008, and 1960–2008 for Rows 1, 2, and 3). Take Model 1. Almost three-quarters (15.1/20.2) of the total gains from reduced occupational frictions over the last fifty years can be explained by the change in the frictions faced by white women (row 3). Falling frictions faced by black men account for less than 10 percent of the overall gains, with the remainder (almost 16 percent) due to the lower frictions for black women. We would naturally expect the share of white women to be larger than that of blacks because the share of white

Table 6: Robustness Results: Percent of Growth Explained in Model 1

<b>Base: <math>\rho = 2/3</math></b>	<b><math>\rho = -90</math></b>	<b><math>\rho = -1</math></b>	<b><math>\rho = 1/3</math></b>	<b><math>\rho = 0.95</math></b>
20.2%	19.5%	19.7%	20.0%	20.8%
<hr/>				
	<b>Base: <math>\eta = 0.25</math></b>	<b><math>\eta = 0</math></b>	<b><math>\eta = 0.5</math></b>	
	20.2%	20.3%	20.1%	
<hr/>				
	<b><math>\theta(1-\eta) = 3.11</math></b>	<b><math>\theta(1-\eta) = 2</math></b>	<b><math>\theta(1-\eta) = 15</math></b>	<b><math>\theta(1-\eta) = 15</math> with <math>\rho = -90</math></b>
	20.2%	19.2%	21.5%	21.2%
<hr/>				
	<b>Base: <math>\beta = 0.693</math></b>	<b><math>\beta = 0.5</math></b>	<b><math>\beta = 0.8</math></b>	
	20.2%	20.2%	20.2%	
<hr/>				

Note: Entries in the table represent the share of earnings growth that is explained by the changing  $\delta$ 's using the chaining approach. Each entry changes one of the parameter values relative to our baseline case.

Table 7: Robustness Results: Percent of Growth Explained in Model 2

<b>Base: <math>\rho = 2/3</math></b>	<b><math>\rho = -90</math></b>	<b><math>\rho = -1</math></b>	<b><math>\rho = 1/3</math></b>	<b><math>\rho = 0.95</math></b>
16.7%	12.7%	13.9%	15.4%	19.1%

<b>Base: <math>\eta = 0.25</math></b>	<b><math>\eta = 0</math></b>	<b><math>\eta = 0.5</math></b>
16.7%	15.0%	17.9%

<b><math>\theta(1-\eta) = 3.11</math></b>	<b><math>\eta = 0.25</math></b>		
	<b><math>\theta(1-\eta) = 2</math></b>	<b><math>\theta(1-\eta) = 15</math></b>	<b><math>\theta(1-\eta) = 15</math> with <math>\rho = -90</math></b>
16.7%	20.1%	9.6%	8.8%

<b>Base: <math>\beta = 0.693</math></b>	<b><math>\beta = 0.5</math></b>	<b><math>\beta = 0.8</math></b>
16.7%	16.7%	16.7%

Note: Entries in the table represent the share of earnings growth that is explained by the changing  $\delta$ 's using the chaining approach. Each entry changes one of the parameter values relative to our baseline case.

Table 8: Contribution of Each Group to Total Earnings Growth, Model 1

Year	Base Model Growth	Percent of Growth Explained			
		Setting All $\delta$ 's to End Levels	Setting WW $\delta$ 's to End Levels	Setting BM $\delta$ 's to End Levels	Setting BW $\delta$ 's to End Levels
1960-1980	33.3 percent	19.6%	11.2%	3.4%	5.1%
1980-2008	49.8 percent	20.7%	17.9%	0.9%	1.9%
1960-2008	99.7 percent	20.2%	15.1%	1.9%	3.2%

Note:

women in the labor force is larger than that of blacks. But white women account for less than 50 percent of the non-white-male labor force, so the fact that almost a three-quarters of the overall gains are associated with white women suggests their frictions have fallen farther than those for blacks.

The share of gains associated with falling frictions for white women vs. blacks differs across the time periods. Again, consider Model 1. Blacks accounted for a larger share of the gains in the 1960s and 1970s than in later decades. From 1960 to 1980, reduced frictions for blacks account for 43 percent  $((5.1 + 3.4)/19.6)$  of the overall gains from reduced frictions. From 1980 to 2008, reduced frictions for blacks account for only 13.5 percent of the overall gains. This timing might link the gains for blacks to the Civil Rights movement of the 1960s and laws passed in response.

What was the consequence of shifting occupational frictions for the wage growth of different groups? Tables 10 and 11 try to answer this question. The first column presents the actual growth of real wages for the different groups from 1960 to 2008. Real wages increased by 77 percent for white men, 126 percent for white women, 143 percent for black men, and almost 200 percent for black women. For brevity, consider the predictions of Model 1. In the absence of the change in occupational frictions, the Models say real wages for white men would have been 6 percent

Table 9: Contribution of Each Group to Total Earnings Growth, Model 2

Year	Base Model Growth	Percent of Growth Explained			
		Setting All $\delta$ 's to End Levels	Setting WW $\delta$ 's to End Levels	Setting BM $\delta$ 's to End Levels	Setting BW $\delta$ 's to End Levels
1960-1980	34.7 percent	17.7%	12.3%	2.1%	3.3%
1980-2008	46.7 percent	15.8%	13.5%	0.6%	1.7%
1960-2008	97.6 percent	16.7%	13.0%	1.3%	2.4%

Note:

higher. Put differently, real income of white men declined due to the changing opportunities for blacks and women. But at the aggregate level, this loss is swamped by the resulting income gains for blacks and women. Almost 42 percent of the income gain for white women was due to the change in occupational frictions. For black men, 45 percent of their earnings growth was due to increased opportunities, according to Model 1.

Tables 12 and 13 look at the regional dimension of the decline in frictions confronting blacks and women. Here, we assume that workers are immobile across regions. With this assumption, a decline in occupational frictions in the South relative to the North will increase average income in the South relative to the North. From 1960 to 2008, incomes in the South increased by 10 percent relative to incomes in the Northeast. According to Model 1, about 7 percentage points of this income convergence was due to reduced occupational frictions facing blacks and women in the South relative to the Northeast – with the bulk of the effect due to reduced  $\delta$ 's for blacks in the South. This effect appears stronger in the first two decades after 1960, when incomes converged by 20 percentage points between the two regions. This is consistent with the hypothesis that lower occupational frictions in the South raised output per worker in the South.

From 1980 to 2008, we see a reversal of the North-South income convergence,



Table 10: Group Changes in Wages, Model 1

Year	Base Model Growth	Percent Explained By Changing $\delta$ 's	
		Model 1 ( $\eta = 0.25$ )	Model 1 ( $\eta = 0$ )
White Men	77.0 percent	-5.8%	-5.2%
White Women	126.3 percent	41.6%	41.1%
Black Men	143.0 percent	44.9%	44.7%
Black Women	198.1 percent	58.4%	58.0%

Note:

Table 11: Group Changes in Wages, Model 2

Year	Base Model Growth	Percent Explained By Changing $\delta$ 's	
		Model 2 ( $\eta = 0.25$ )	Model 2 ( $\eta = 0$ )
White Men	77.0 percent	-6.8%	-6.1%
White Women	126.3 percent	42.8%	42.6%
Black Men	143.0 percent	44.5%	44.3%
Black Women	198.1 percent	59.3%	59.0%

Note:

Table 12: Contributions to Northeast - South Convergence, Model 1

Year	Base Model Convergence in P. Points	Percentage Points of Growth Explained	
		Setting All $\delta$ 's to End Levels	Setting BM and BW $\delta$ 's to End Levels
1960-1980	20.7	5.0	3.7
1980-2008	-16.5	1.5	1.9
1960-2008	10.0	7.1	5.6

Note:

perhaps driven by the reverse migration of blacks to the U.S. South. In turn, the reverse migration is exactly what one would expect to see if workers are responding to the improved labor market outcomes in the South by relocating to the South. In a long run with higher labor mobility, the main effect of declining occupational frictions for blacks in the South relative to the North might be to increase the number of blacks living in the South relative to the North. Persistent income gaps might reflect skill differences between regions. Of course, to the extent mobility is costly even in the long run, frictions can contribute to wage gap differences across regions even in the long run.

Finally, we try to address how the changing  $\delta$ 's might have affected the educational attainment of blacks and women relative to white males. As we report in Table 14, gaps in average years of schooling narrowed from 1960 to 2008 for all three groups vs. white males: by 0.41 years for white women, 1.81 years for black men, and 1.55 years for black women. As the  $\delta$ 's rose faster in occupations with above-average schooling, the models predict n women and blacks will have increased their representation in high-schooling occupations in response.<sup>8</sup> How much did this

<sup>8</sup>In both Models 1 and 2, schooling varies only across occupations. As both models are calibrated to fit occupational choices by year, they have the same predictions for education.

Table 13: Contributions to Northeast - South Convergence, Model 2

Year	Base Model Convergence in P. Points	Percentage Points of Growth Explained	
		Setting All $\delta$ 's to End Levels	Setting BM and BW $\delta$ 's to End Levels
1960-1980	16.9	2.3	2.5
1980-2008	-17.8	0.3	1.2
1960-2008	1.6	2.8	3.7

Note:

contribute to the shrinking education gaps? For white women, our model predicts that all of the schooling convergence can be “explained” by the decline in the frictions they faced (as the model predicts a 0.81 year increase in white womens years of education relative to white men). For black men, the falling frictions might have narrowed the schooling gap with white men by 0.71 years, about one-third of the convergence observed in the data. For black women, we estimate declining distortions could explain almost 80 percent (1.23/1.55) of their catch-up in schooling.

## 8. Conclusion

Under construction

## A Derivations and Proofs

The propositions in the paper summarize the key results from the model. This appendix shows how to derive the results.

### Proof of Proposition 1. Individual Consumption and Schooling

Table 14: Education Predictions, All Households (Age 25–55)

	1960 Level	2008 Level	Change 2008 - 1960	Change Relative to WM	Model Prediction
White Men	11.11	13.47	2.35		
White Women	10.98	13.75	2.77	<b>0.41</b>	<b>0.81</b>
Black Men	8.56	12.73	4.17	<b>1.81</b>	<b>0.71</b>
Black Women	9.24	13.15	3.90	<b>1.55</b>	<b>1.23</b>

Note:

Proposition 1 comes directly from the first order conditions for the individual's optimization problem.

### Proof of Proposition 2. Occupational Choice

As given in Proposition 1, the individual's utility from choosing a particular occupation is  $U(\delta_i, w_i, \epsilon_i) = \bar{\eta}^\beta (\tilde{\delta}_{ig} \epsilon_i)^{\frac{\beta}{1-\eta}}$ , where  $\tilde{\delta}_{ig} \equiv \delta_{ig} w_i s_i^{\phi_i} (1 - s_i)^{\frac{1-\eta}{\beta}}$ . The solution to the individual's problem, then, involves picking the occupation with the largest value of  $\tilde{\delta}_{ig} \epsilon_i$ . To keep the notation simple, we will suppress the  $g$  subscript in what follows.

Let  $p_i$  denote the probability that the individual chooses occupation  $i$ . Then

$$\begin{aligned}
 p_i &= \Pr[\tilde{\delta}_i \epsilon_i > \tilde{\delta}_s \epsilon_s] \quad \forall i \neq s \\
 &= \Pr[\epsilon_s < \tilde{\delta}_i \epsilon_i / \tilde{\delta}_s] \quad \forall s \neq i \\
 &= \prod_{s \neq i} F_s(\tilde{\delta}_i \epsilon_i / \tilde{\delta}_s)
 \end{aligned} \tag{21}$$

if  $\epsilon_i$  is known for certain. Since it is not, we must also integrate over the probability

distribution for  $\epsilon_i$ :

$$p_i = \int \Pi_{s \neq i} F_s(\tilde{\delta}_i \epsilon_i / \tilde{\delta}_s) f_i(\epsilon_i) d\epsilon_i, \quad (22)$$

where  $f_i(\epsilon) = \theta T_i \epsilon^{-(1+\theta)} \exp\{-T_i \epsilon^{-\theta}\}$  is the pdf of the Fréchet distribution. Substituting in for the distribution and pdf, additional algebra leads to

$$\begin{aligned} p_i &= \int \theta T_i \left( \Pi_{s \neq i} \exp\{-T_s (\tilde{\delta}_i \epsilon_i / \tilde{\delta}_s)^{-\theta}\} \right) \epsilon_i^{-(1+\theta)} \exp\{-T_i \epsilon_i^{-\theta}\} d\epsilon_i \\ &= \int \theta T_i \epsilon_i^{-(1+\theta)} \exp\left\{-\sum_{s=1}^N T_s \left(\frac{\tilde{\delta}_i}{\tilde{\delta}_s}\right)^{-\theta} \epsilon_i^{-\theta}\right\} d\epsilon_i. \end{aligned} \quad (23)$$

Now, define  $\bar{T}_i \equiv -\sum_{s=1}^N T_s \left(\frac{\tilde{\delta}_i}{\tilde{\delta}_s}\right)^{-\theta}$ . Then the probability simplifies considerably:

$$\begin{aligned} p_i &= \frac{T_i}{\bar{T}_i} \int \theta \bar{T}_i \epsilon_i^{-(1+\theta)} \exp\{-\bar{T}_i \epsilon_i^{-\theta}\} d\epsilon_i \\ &= \frac{T_i}{\bar{T}_i} \int d\bar{F}_i(\epsilon_i) \\ &= \frac{T_i}{\bar{T}_i} \\ &= \frac{T_i \tilde{\delta}_i^\theta}{\sum_s T_s \tilde{\delta}_s^\theta} \end{aligned} \quad (24)$$

where  $\bar{F}_i(\epsilon)$  is the cdf of a Fréchet distribution with parameters  $\bar{T}_i$  and  $\theta$ . The first main result in Proposition 2 then comes from our normalization that  $T_i = 1$  for all  $i$ .

Total efficiency units of labor supplied to occupation  $i$  by group  $g$  are

$$H_{ig} = q_g p_{ig} \cdot \mathbb{E}[h_i \epsilon_i \mid \text{Person chooses } i].$$

Recall that  $h(e, s) = s^{\phi_i} e^\eta$ . Using the results from Proposition 1, it is straightforward to show that

$$h_i \epsilon_i = \tilde{h}_i (\tilde{\delta}_i w_i)^{\frac{\eta}{1-\eta}} \epsilon_i^{\frac{1}{1-\eta}},$$

where  $\tilde{h}_i \equiv \eta^{\eta/(1-\eta)} s_i^{\frac{\phi_i}{1-\eta}}$ . Therefore,

$$H_{ig} = q_g p_{ig} \tilde{h}_i (\tilde{\delta}_i w_i)^{\frac{\eta}{1-\eta}} \cdot \mathbb{E}\left[\epsilon_i^{\frac{1}{1-\eta}} \mid \text{Person chooses } i\right]. \quad (25)$$

To calculate this last conditional expectation, we use the extreme value magic of the Fréchet distribution. Let  $y_i \equiv \tilde{\delta}_i \epsilon_i$  denote the key occupational choice term. Then

$$y^* \equiv \max_i \{y_i\} = \max_i \{\delta_i \epsilon_i\} = \delta^* \epsilon^*.$$

Since  $y_i$  is the thing we are maximizing, it inherits the extreme value distribution:

$$\begin{aligned} \Pr[y^* < z] &= \prod_{i=1}^N \Pr[y_i < z] \\ &= \prod_{i=1}^N \Pr[\tilde{\delta}_i \epsilon_i < z] \\ &= \prod_{i=1}^N \Pr[\epsilon_i < z/\tilde{\delta}_i] \\ &= \prod_{i=1}^N \exp\left\{-T_i \left(\frac{z}{\tilde{\delta}_i}\right)^{-\theta}\right\} \\ &= \exp\left\{-\sum_{i=1}^N T_i \tilde{\delta}_i^\theta \cdot z^{-\theta}\right\} \\ &= \exp\{-\bar{T} z^{-\theta}\}. \end{aligned} \tag{26}$$

That is, the extreme value also has a Fréchet distribution, with a mean-shift parameter given by  $\bar{T} \equiv \sum_s T_s \tilde{\delta}_s^\theta$ .

Straightforward algebra then reveals that the distribution of  $\epsilon^*$ , the ability of people in their chosen occupation, is also Fréchet:

$$G(x) \equiv \Pr[\epsilon^* < x] = \exp\{-T^* x^{-\theta}\} \tag{27}$$

where  $T^* \equiv \sum_{i=1}^N T_i \left(\tilde{\delta}_i/\delta^*\right)^\theta$ .

Finally, one can then calculate the statistic we needed above back in equation (25): the expected value of the chosen occupation's ability raised to some power. In particular, let  $i$  denote the occupation that the individual chooses, and let  $\alpha$  be some positive exponent. Then,

$$\begin{aligned} \mathbb{E}[\epsilon_i^\alpha] &= \int_0^\infty \epsilon^\alpha dG(\epsilon) \\ &= \int_0^\infty \theta T^* \epsilon^{-(1+\theta)+\alpha} e^{-T^* \epsilon^{-\theta}} d\epsilon \end{aligned} \tag{28}$$

Recall that the “Gamma function” is  $\Gamma(\alpha) \equiv \int_0^\infty x^{\alpha-1} e^{-x} dx$ . Using the change-of-variable  $x = T^* \epsilon^{-\theta}$ , one can show that

$$\begin{aligned} \mathbb{E}[\epsilon_i^\lambda] &= T^{*\lambda/\theta} \int_0^\infty x^{-\lambda/\theta} e^{-x} dx \\ &= T^{*\lambda/\theta} \Gamma(1 - \lambda/\theta). \end{aligned} \quad (29)$$

Applying this result to our model, we have

$$\begin{aligned} \mathbb{E} \left[ \epsilon_i^{\frac{1}{1-\eta}} \mid \text{Person chooses } i \right] &= T^{*\frac{1}{\theta} \cdot \frac{1}{1-\eta}} \Gamma \left( \frac{1}{\theta} \cdot \frac{1}{1-\eta} \right) \\ &= p_{ig}^{-\frac{1}{\theta} \cdot \frac{1}{1-\eta}} \Gamma \left( \frac{1}{\theta} \cdot \frac{1}{1-\eta} \right). \end{aligned} \quad (30)$$

Substituting this expression into (25) and rearranging leads to the last result of the proposition.

### **Proof of Proposition 3. Occupational Wage Gaps**

The proof of this proposition is straightforward given the results of Proposition 2.

## **B Data Appendix**

### **References**

- Eaton, Jonathan and Samuel S. Kortum, “Technology, Geography, and Trade,” *Econometrica*, 2002, 70 (5), 1741–1779.
- Erosa, Andres, Tatyana Koreshkova, and Diego Restuccia, “How Important Is Human Capital? A Quantitative Theory Assessment of World Income Inequality,” *Review of Economic Studies*, October 2010, 77 (4), 1421–1449.
- Manuelli, Rodolfo and Ananth Seshadri, “Human Capital and the Wealth of Nations,” March 2005. University of Wisconsin working paper.

Table 15: Sample Statistics By Census Year

	1960	1970	1980	1990	2000	2006-8
Sample Size	641,686	694,419	4,057,685	4,711,405	5,216,431	3,147,547
Share of White Men in Sample	0.432	0.433	0.435	0.435	0.431	0.431
Share of White Women in Sample	0.475	0.468	0.459	0.447	0.437	0.431
Share of Black Men in Sample	0.042	0.044	0.047	0.054	0.061	0.065
Share of Black Women in Sample	0.052	0.055	0.059	0.065	0.071	0.074
Relative Wage Gap: White Women	-0.578	-0.590	-0.476	-0.372	-0.299	-0.259
Relative Wage Gap: Black Men	-0.379	-0.289	-0.215	-0.158	-0.142	-0.150
Relative Wage Gap: Black Women	-0.875	-0.705	-0.479	-0.363	-0.317	-0.313

Note:

McFadden, Daniel, "Conditional Logit Analysis of Qualitative Choice Behavior," in P. Zarembka, ed., *Frontiers of Econometrics*, New York, NY: Academic Press, 1974, pp. 105–142.



Table 16: Occupation Categories for our Base Occupational Specification

Home Sector	Police
Executives, Administrative, and Managerial	Guards
Management Related	Food Preparation and Service
Architects	Health Service
Engineers	Cleaning and Building Service
Math and Computer Science	Personal Service
Natural Science	Farm Managers
Health Diagnosing	Farm Non-Managers
Health Assessment	Related Agriculture
Therapists	Forest, Logging, Fishers, and Hunters
Teachers, Postsecondary	Vehicle Mechanic
Teachers, Non-Postsecondary	Electronic Repairer
Librarians and Curators	Misc. Repairer
Social Scientists and Urban Planners	Construction Trade
Social, Recreation, and Religious Workers	Extractive
Lawyers and Judges	Precision Production, Supervisor
Arts and Athletes	Precision Metal
Health Technicians	Precision Wood
Engineering Technicians	Precision Textile
Science Technicians	Precision Other
Technicians, Other	Precision Food
Sales, All	Plant and System Operator
Secretaries	Metal and Plastic Machine Operator
Information Clerks	Metal and Plastic Processing Operator
Records Processing, Non-Financial	Woodworking Machine Operator
Records Processing, Financial	Textile Machine Operator
Office Machine Operator	Printing Machine Operator
Computer and Communication Equipment Operator	Machine Operator, Other
Mail Distribution	Fabricators
Scheduling and Distributing Clerks	Production Inspectors
Adjusters and Investigators	Motor Vehicle Operator
Misc. Administrative Support	Non Motor Vehicle Operator
Private Household Occupations	Freight, Stock, and Material Handlers
Firefighting	

Note:

Table 17: Examples of Occupations within Our Base Occupational Categories

**Management Related Occupations**

Accountants and Auditors  
 Underwriters  
 Other Financial Officers  
 Management Analysts  
 Personnel, Training, and Labor Relations Specialists  
 Purchasing Agents and Buyers  
 Construction Inspectors  
 Management Related Occupations, N.E.C.

**Health Diagnosing Occupations**

Physicians  
 Dentists  
 Veterinarians  
 Optometrists  
 Podiatrists  
 Health Diagnosing Practitioners, N.E.C.

**Personal Service Occupations**

Supervisors, personal service occupations  
 Barbers  
 Hairdressers and Cosmetologists  
 Attendants, amusement and recreation facilities  
 Guides  
 Ushers  
 Public Transportation Attendants  
 Baggage Porters  
 Welfare Service Aides  
 Family Child Care Providers  
 Early Childhood Teacher Assistants  
 Child Care Workers, N.E.C.

Note:

Table 18: Occupation Categories for our Broad Occupation Classification

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Home Sector	Sales, All
Executives, Administrative, and Managerial	Administrative Support, Clerks, and Record Keepers
Management Related	Fire, Police, and Guards
Architects, Engineers, Math, and Computer Science	Private Household and Food, Cleaning, and Personal Services
Natural and Social Scientists, Recreation, Religious, Arts, and Athletes	Farm, Related Agriculture, Logging, Forest, Fishing, Hunters and Extraction
Doctors and Lawyers	Mechanics and Construction
Nurses, Therapists, and Other Health Services	Precision Manufacturing
Teachers, Postsecondary	Manufacturing Operators
Teachers, Non-Postsecondary and Librarians	Fabricators, Inspectors, and Material Handlers
Health and Science Technicians	Vehicle Operators

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Note: