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by Eleonora Granziera and Sharon Kozicki

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## Abstract

We investigate whether expectations that are not fully rational have the potential to explain the evolution of house prices and the price-to-rent ratio in the United States. First, a Lucas type asset-pricing model solved under rational expectations is used to derive a fundamental value for house prices and the price-rent ratio. Although the model can explain the sample average of the price-rent ratio, it does not generate the volatility and persistence observed in the data. Then, we consider an intrinsic bubble model and two models of extrapolative expectations developed by Lansing (2006, 2010) in applications to stock prices: one that features a constant extrapolation parameter and one in which the extrapolation coefficient depends on the dividend growth process. We show that these last two models are equally good at matching sample moments of the data. However, a counterfactual experiment shows that only the extrapolative expectation model with time-varying extrapolation coefficient is consistent with the run up in house prices observed over the 2000-2006 period and the subsequent sharp downturn.

*JEL classification: E3, E65, R21*

*Bank classification: Asset pricing; Domestic demand and components; Economic models*

## Résumé

Les auteures tentent de déterminer si l'évolution des prix des maisons et du ratio de ces prix aux loyers aux États-Unis peut s'expliquer par le fait que les anticipations ne soient pas entièrement rationnelles. Elles résolvent d'abord un modèle d'évaluation des actifs à la Lucas sous l'hypothèse de rationalité des anticipations afin d'obtenir une estimation du niveau fondamental des prix des maisons et du ratio prix/loyers. Même si le modèle parvient à restituer la moyenne empirique du ratio, il ne génère pas la volatilité et la persistance observées dans les données. Les auteures considèrent ensuite un modèle de bulle intrinsèque ainsi que deux modèles dotés d'anticipations extrapolatives mis au point par Lansing (2006 et 2010) pour l'analyse des cours en bourse : dans le premier, le coefficient d'extrapolation est constant, et dans le second, il dépend du processus de croissance des dividendes. Ces deux modèles réussissent pareillement à reproduire les moments empiriques. Toutefois, une simulation contrefactuelle montre que seul le modèle à coefficient d'extrapolation variable dans le temps cadre avec l'envolée des prix des maisons survenue entre 2000 et 2006 et leur forte chute subséquente.

*Classification JEL : E3, E65, R21*

*Classification de la Banque : Évaluation des actifs; Demande intérieure et composantes; Modèles économiques*

# 1 Introduction

Fluctuations in house prices can have a strong impact on real economic activity. Because housing is typically the most important component of household wealth, changes in house prices affect household wealth and expenditure. Moreover movements in house prices can impact the real side of the economy through their effect on the financial system: the rapid rise and subsequent collapse in US residential housing prices is widely considered as one of the major determinants of the financial crisis of 2007-2009, which has in turn led to a deep recession and a protracted decline in employment. In light of these considerations it is important to identify the determinants of house prices dynamics.

In this paper we investigate whether not fully rational expectations can explain the recent evolution in the price to rent ratio and house prices in the United States. We apply, to the housing market, a simple Lucas tree model in which households own an asset (house) that can be rented out in exchange for an exogenous stream of dividends (rents) used for consumption. Houses are treated merely as a financial asset and agents are viewed as real estate investors; from their perspective rents are analogous in cash flow terms to dividends that stock market investors receive from holding stocks. The choice of such a stylized model is justified by the fact that this framework allows us to clearly isolate the contribution of expectations alone from other mechanisms that could affect house prices.<sup>1</sup> Also, Lucas tree type models or simple present value models have been used in the finance and real estate literature to characterize house price movements.<sup>2</sup>

We explore the ability of four variants of this model to match the data. All models under consideration adopt the same stochastic structure of the dividend growth process and the same preferences but they differ in the way agents form their expectations on prices. We view the model solved under rational expectations as the benchmark. As alternatives, we consider a model that includes a rational bubble component and two models that feature extrapolative expectations. These models that depart from full rationality have been developed for the analysis of the stock market to generate momentum and volatility, characteristics common also to the housing market.

Solving the model under rational expectations and assuming an autoregressive process

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<sup>1</sup>Recent macroeconomic studies are assessing the role of non-fully rational expectations in conjunction with other factors for the dynamics of housing prices. For example, see Adam, Kuang and Marcet (2011) for an open economy asset pricing model, Burnside, Eichenbaum and Rebelo (2011) and Peterson (2012) for matching models.

<sup>2</sup>See Han (2011), Hott (2009), Piazzesi Schneider and Tuzel (2007), Poterba (1984).

for the growth rate of dividends we obtain a fundamental solution for the price-rent ratio that matches the sample average over the sample 1987-2011. However the price-to-rent ratio series exhibits high volatility and persistence which are not accounted for in the fundamental solution. Motivated by claims both in the media and in academic circles that the recent housing boom might in fact have been a bubble,<sup>3</sup> we relax the assumption of rational expectations and allow for a rational bubble solution<sup>4</sup> of the model as in Froot and Obstfeld (1991). This bubble, driven exclusively by the growth rate of rents, generates persistent deviations from present-value prices.

Then, we abandon the assumption of rational expectations and we explore the implications of alternative expectation formation mechanisms. In particular we follow the approach developed in Lansing (2006, 2010) for the study of the stock market and assume that agents form expectations in an extrapolative fashion so that their conditional expectations of future values are based on past realizations of the variable to forecast. The assumption of extrapolative behavior is supported by numerous microfunded studies: lab experiments which show that observed beliefs are well described by extrapolative or ‘trend following’ expectations (De Bondt 1993; Hey, 1994) and field data analysis that document how extrapolation of the most recent price increase can determine asset allocation choices (Benartzi 2001, Vissing-Jørgensen 2004). More recently, Piazzesi and Schneider (2009), using data on expectations from the Michigan Survey of Consumers, study household beliefs during the recent US housing boom and provide evidence that expectations of future increases in prices strengthen with the increase in prices, consistent with the extrapolative behaviour analyzed in this study.

In the models that assume extrapolative behavior of the agents, expectations are related to past realizations through an extrapolation coefficient which defines the weight agents put on past observations to generate their expectations. This expectation mechanism introduces persistence in the equilibrium price-rent ratio and amplifies the fluctuations of the price-rent ratio around the mean. We consider a case in which the extrapolation parameter is fixed and one in which the extrapolation parameter is an increasing function of the actual realization of the rent growth process. The second model is consistent with the Vissing-Jørgensen (2004) findings that the effect of past returns on expectations is stronger if past returns are positive.

We compare the predictions from the models along many dimensions, through a simulation exercise that explores the ability of the models to match some moments of the data and

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<sup>3</sup>See for example Burnside, Eichenbaum and Rebelo (2011), Galí (2011), Nneji, Brooks and Ward (2011).

<sup>4</sup>Rational bubble solutions are obtained by not imposing the transversality condition in the present value formula for prices.

through a historical counterfactual experiment which checks whether the models can replicate the dynamics of house prices in the sample. From the simulation exercise it emerges that the rational bubble solution of the model generates a price-rent series that increases without bound and an average growth rate of prices that is too high when compared to the data. Instead, extrapolative expectations allow the asset pricing model to successfully match the volatility, persistence and positive skewness of the data, regardless of whether the coefficient is constant or time varying.

However, the historical counterfactual exercise, conducted by feeding the actual rent growth process into the solution of the models reveals that the price to rent ratio and house prices predicted by the model solved under extrapolative expectations with a constant extrapolation parameter fail to replicate the evolution of the price-rent ratio and prices of the past 25 years. In contrast, a model with time-varying weight can account for both the surge and drop in house prices and price-rent ratio observed over the sample. This is because, differently from all the other models considered, the model with a time-varying parameter implies a positive correlation between the growth rate of rents and the price-rent ratio.

We emphasize that although we show that the near rational bubble model described in Lansing (2010) is consistent with some key characteristics of the housing market, we do not claim that the data is *only* consistent with this model. Other factors, alone or in conjunction with non fully rational expectations, might play a role in determining the evolution of house prices and price-rent series. In particular many studies identify plausible drivers to the recent house price boom: low real rates (Adam, Kuang and Marcet 2011), financial liberalization (Favilukis, Ludvigson and Nieuwerburgh 2010) and low elasticity of housing supply (Glaeser, Gyourko and Saiz, 2008).<sup>5</sup> The quantitative performance of the simple model adopted in this paper is even more surprising considering that it abstracts from these factors.

Therefore, the main contribution of our paper is to show that extrapolative expectations embedded in a simple asset-pricing model where rents are the only driving force of house prices can account for the evolution of the actual price-to-rent ratio and price series.

The paper is organized as follows: Section 2 describes the basic model and derives the fundamental prices and price-rent ratio under rational expectations as well as solutions for the models that feature alternative expectation formation mechanisms. Section 3 reports simulation results from each model. Section 4 provides a counterfactual historical experiment. Section 5 concludes.

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<sup>5</sup>The first two studies focus on the episode of house price boom-bust of 2000-2009, and the last paper focuses on the heterogeneity across US geographical markets rather than aggregate data.

## 2 The Model

We treat houses as liquid financial assets that deliver an exogenous stream of consumption (rents) and abstract from the function of houses as stores of value or collaterals and from financing decisions.<sup>6</sup> We use a Lucas tree type model<sup>7</sup> with a risky asset to obtain a fundamental value for the house price and price-dividend ratio ( $p_t/d_t$ ). We think of the dividend as rent, the stream of consumption and services that is derived from owning<sup>8</sup> (and renting out) a house; we will use the terms dividends and rents interchangeably. In the Lucas model, which is an endowment economy, the representative agent chooses sequences of consumption and equity (shares of the house) to maximize the expected present value of her lifetime utility. In particular the risk-averse representative agent solves the following intertemporal utility maximization problem:

$$\max_{c_t, s_t} \hat{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

s.t.

$$c_t + p_t s_t = (p_t + d_t) s_{t-1} \quad \text{with } c_t, s_t > 0$$

where  $c_t$  is consumption in period  $t$ ,  $s_t$  is the equity share purchased at time  $t$ ,  $d_t$  is the stochastic dividend paid by the share in period  $t$ ,  $p_t$  is the price of the share in period  $t$  and  $\beta$  is the discount factor.  $\hat{E}_0$  denotes the agent's subjective expectations at time zero.

This maximization problem yields the well-know first-order condition:

$$p_t = \beta \hat{E}_t \left[ \frac{U'(c_{t+1})}{U'(c_t)} (p_{t+1} + d_{t+1}) \right]. \quad (1)$$

Because there is no technology to store dividends, and houses are available in fixed supply, for simplicity  $s_t = 1$ , so consumption will equal to the dividend<sup>9</sup> in each period (or  $c_t = d_t \forall t$ ). Substituting this equilibrium condition in (1) and assuming a CRRA utility function

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<sup>6</sup>See Brumm, Kubler, Grill and Schmedders (2011) for implications of collateral requirements in a Lucas' tree model.

<sup>7</sup>See Lucas (1978).

<sup>8</sup>Note that renting a house does not provide utility in this model; this is model of home-owners only. See Davis and Martin (2005) or Piazzesi, Schneider and Tuzel (2007) for a consumption-based asset pricing model where housing services enter the utility function.

<sup>9</sup>For studies where the equivalence between consumption and dividend is broken, see for example Cecchetti, Lam and Mark (1993). Lansing (2010) also provides implications of separating consumption from dividends in a Lucas tree model with autoregressive dividend growth and CRRA utility function.



the price-dividend ratio can be rewritten as:

$$y_t \equiv \frac{p_t}{d_t} = \hat{E}_t \left[ \beta \exp((1 - \alpha)x_{t+1}) \left( \frac{p_{t+1}}{d_{t+1}} + 1 \right) \right] \quad (2)$$

where  $\alpha$  is the coefficient of relative risk aversion and  $x_t$  is defined as the growth rate of dividends:  $x_t \equiv \log(d_t/d_{t-1})$ .

Last, to solve the model it is necessary to specify a stochastic process for the rate of growth of dividends which is assumed to be a stationary autoregressive process of order one<sup>10</sup> with mean  $\bar{x}$  and variance  $\sigma^2 = \sigma_\varepsilon^2/(1 - \rho^2)$ :

$$x_t \equiv \bar{x} + \rho(x_{t-1} - \bar{x}) + \varepsilon_t \quad |\rho| < 1, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \quad (3)$$

When bringing the model to the data we use the rent series available from the National Accounts Table and divide it by the housing stock, owned and occupied dwellings, to obtain the rent matured by each owned and occupied house.

## 2.1 Fundamental Solution

In this section we present the fundamental solution and its implications for the price-rent ratio and real house prices. Solving the model under rational expectations (so that  $\hat{E}_t$  is the mathematical expectation operator  $E_t$ ) and following the approach in Lansing (2010), an approximate<sup>11</sup> analytical solution for the fundamental price-dividend ratio is obtained as a function of the structural parameters of the economy, i.e. the coefficient of relative risk aversion,  $\alpha$ , the discount factor,  $\beta$ , and the parameters governing the stochastic growth rate of the exogenous process for the dividends:

$$y_t^f = \frac{p_t}{d_t} = \exp(a_0 + a_1 \rho(x_t - \bar{x}) + \frac{1}{2} a_1^2 \sigma_\varepsilon^2) \quad (4)$$

where

$$a_1 = \frac{1 - \alpha}{1 - \rho \beta \exp \left[ (1 - \alpha) \bar{x} + \frac{1}{2} a_1^2 \sigma_\varepsilon^2 \right]} \quad a_0 = \log \left[ \frac{\beta \exp \left( (1 - \alpha) \bar{x} \right)}{1 - \beta \exp \left[ (1 - \alpha) \bar{x} + \frac{1}{2} a_1^2 \sigma_\varepsilon^2 \right]} \right]$$

<sup>10</sup>The lag length was chosen empirically using an AIC selection criterion for the actual series of growth rate of real imputed rents over the sample under analysis.

<sup>11</sup>See Burnside (1998) for an exact analytical solution.

as long as  $1 > \beta \exp \left[ (1 - \alpha) \bar{x} + \frac{1}{2} a_1^2 \sigma_\varepsilon^2 \right]$ . Then the rational expectation solution to this model implies that the price-dividend ratio is time varying<sup>12</sup> and it depends on the deviation of the current realization of dividend growth from its mean. The fundamental price-rent ratio can be obtained once we assign values to the parameters in (4). Given the frequency of our data we interpret the length of each period as being a quarter. Table 1 summarizes the calibration: the parameters  $\bar{x}$ ,  $\rho$  and  $\sigma_\varepsilon$  are estimated from the process for the growth rate of the rent series in (3),  $\beta$  is set to 0.9902 a common value for the discount factor for data at the quarterly frequency and the value of  $\alpha$  is chosen such that the price-dividend ratio implied by equation (4) matches the sample average of the price-rent ratio for the sample 1987Q1-2011Q4.

Table 1: Calibration

Parameter	Description	Calibrated to:	Value
$\bar{x}$	mean growth rate of dividends	mean growth rate of rents	0.0047
$\rho$	autocorrelation of dividends growth rate	autocorrelation of rents growth rate	0.3623
$\sigma_\varepsilon$	sd errors of dividends growth process	sd residuals of rents growth process	0.0053
$\alpha$	relative risk aversion	match mean of $p_t/d_t$	2.5
$\beta$	discount factor	match real rate of 4%	0.9902

Note: Calibration for the asset prices model described in Section 1, equation (4). The parameters  $\bar{x}$ ,  $\rho$  and  $\sigma_\varepsilon$  are the sample mean, autocorrelation and variance of the residuals for the rent series over the sample 1987Q1-2011Q4, the discount factor  $\beta$  is set to 0.9902 for quarterly data consistently with the literature and the value of  $\alpha$  is chosen such that the price-dividend ratio implied by equation (4) matches the sample average of the price-rent ratio.

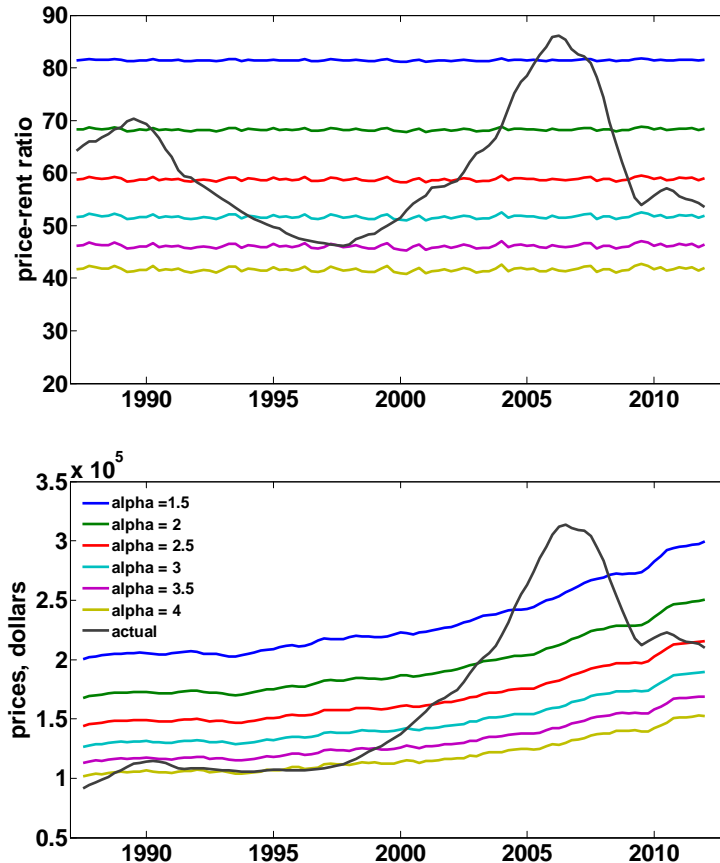
We simulate 100 observations of the model from equation (4) and from the conditions for  $a_0$  and  $a_1$ , given the parameterization<sup>13</sup> in Table 1. In this exercise the only free parameter is the coefficient of relative risk aversion. Figure 1 plots the actual price to rent ratio (upper panel) and actual house prices (lower panel) for the United States and the simulated data from the model for various values of the coefficient of risk aversion. In computing the actual price-dividend ratio, the price series is based on the Case and Shiller Composite 10 house

<sup>12</sup>A constant price-dividend ratio would arise in the case of no autocorrelation in the dividend growth process ( $\rho = 0$ ) or in the case of  $a_1 = 0$ , ( $a_1$  would be close to zero in the case of logarithmic utility, when  $\alpha$  tends to one).

<sup>13</sup>Note that there are three possible values of  $a_1$  that satisfy the non-linear equation. However, given the parameterization in Table 1 only one of these values satisfies the inequality  $1 > \beta \exp \left[ (1 - \alpha) \bar{x} + \frac{1}{2} a_1^2 \sigma_\varepsilon^2 \right]$ . We pick this value to simulate the model.

prices index while the dividend series is obtained as the average rent of the owned and occupied housing stock. Both series are deflated using the PCE deflator.<sup>14</sup>

Figure 1. Simulated and Actual Prices and Price-Rent Ratio



Note: this figure shows the actual and simulated price-rent ratio (upper panel) and prices (lower panel) for the US over the sample 1987Q1-2011Q4. The simulated data are generated for various values of the coefficient of risk aversion from equation (4) and (3) under the calibration in Table 1. Actuals are in black.

The model matches the sample average of the price-rent ratio when the coefficient of risk aversion equals 2.5; higher (lower) values of the coefficient of risk aversion imply simulated data lower (higher) than the sample average. In order to generate the higher variance observed in the data, a higher coefficient of risk aversion would be required but as a consequence the model would fail in matching the first moment.<sup>15</sup> Figure 1 also shows that the actual price-rent ratio exhibits strong persistence and it fluctuates substantially throughout the sample while from equation (4) the model delivers the prediction that the price-dividend

<sup>14</sup>A comprehensive description of the data is provided in the Appendix.

<sup>15</sup>In simulations a coefficient of risk aversion of 20 would imply a mean of 10 and a standard deviation of 0.62 while the standard deviation for the actual data is 11.

ratio should be fairly stable across time around the unconditional mean despite the autocorrelation in the dividend growth process. To increase the persistence implied by this model it would be necessary to observe a more persistent growth rate process for the rents.<sup>16</sup> Similarly, although the fundamental model can capture the upward trend in real prices, it cannot generate large deviations from its trend and therefore it cannot replicate the housing boom of the years 2000-2006.

## 2.2 A Rational Bubble Solution

We have seen in the previous section that a rational expectation model with a sensible parametrization can match the first sample moment of the price-dividend ratio, but it cannot generate the large and persistent fluctuations observed in the data. We are interested in identifying models that can generate these features of the data. High volatility and persistence in the price-dividend ratio are key characteristics of the stock market which have been extensively documented in the literature. Therefore we apply models developed for the analysis of the stock market to the housing market. In particular, we consider models in which volatility and persistence are amplified by allowing deviations from full-rationality. Because the surge in prices from 2000 to 2005 has been referred to as a housing bubble, both in the media and in the macro literature,<sup>17</sup> in this section we consider a rational bubble model first developed in Froot and Obstfeld (1991), subsequently generalized by Lansing (2010) to allow for CRRA utility function and autoregressive growth rate of dividends. Then, in section 2.3 and 2.4 we will relax the assumption of rational expectations and consider two extrapolative expectations models developed by Lansing (2006, 2010) for the analysis of the stock market.

To recover the rational bubble model, note that the fundamental solution (4) is a particular solution to the stochastic difference equation described by the Euler equation. In the first step towards the fundamental solution the law of iterated expectations is applied to the Euler equation (2). This leads to the present value equation:

$$y_t = E_t[\beta \exp((1 - \alpha)x_{t+1}) + \beta^2 \exp((1 - \alpha)(x_{t+1} + x_{t+2})) + \dots \\ \dots + \beta^j \exp((1 - \alpha)(x_{t+1} + \dots + x_{t+j})) E_{t+j}(y_{t+j} + 1)].$$

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<sup>16</sup>Simulations show that a coefficient of autocorrelation of 0.8 for the dividend growth process would imply an autocorrelation of 0.75 for the price-rent ratio while in the data the autocorrelation is 0.99.

<sup>17</sup>See for example Burnside, Eichenbaum and Rebelo (2011), Galí (2011), Nneji, Brooks and Ward (2011).

The second step consists of imposing the transversality condition

$$\lim_{j \rightarrow \infty} \beta^j E_t [\exp((1 - \alpha)(x_{t+1} + \dots + x_{t+j})) y_{t+j}] = 0$$

to the above present-value equation. But equation (2) admits solutions other than the fundamental solution (4). These solutions satisfy the no arbitrage condition only from period  $t$  to  $t+1$  but they do not satisfy the transversality condition: the rationale behind this assumption is that agents are still forward looking but they lack the infinite-horizon foresight required by the transversality condition. Then the price-dividend ratio can be decomposed into two components: the fundamental solution ( $y_t^f$ ), which is the discounted value of expected future dividends, and the rational bubble ( $y_t^b$ ):

$$y_t^{rb} = y_t^f + y_t^b. \quad (5)$$

The rational bubble component satisfies the period-by-period condition:

$$y_t^b = E_t (\beta \exp((1 - \alpha)x_{t+1}) y_{t+1}^b).$$

Therefore the rational bubble considered in this paper is intrinsic<sup>18</sup> as it does not depend on time or any other variable extraneous to the fundamental solution. For the case of autocorrelated dividends and CRRA utility function, the solution to the rational bubble component can be expressed as:

$$y_t^b = y_{t-1}^b \exp(\lambda_0 + (\lambda_1 - (1 - \alpha))(x_t - \bar{x}) + (\lambda_2 + (1 - \alpha))(x_{t-1} - \bar{x})) \quad y_0^b > 0 \quad (6)$$

together with the equilibrium conditions

$$\lambda_2 = -(\rho\lambda_1 + (1 - \alpha)),$$

and

$$\frac{1}{2} (\lambda_1)^2 \sigma_\varepsilon^2 + (1 - \alpha) \bar{x} + \log(\beta) + \lambda_0 = 0$$

The second condition is a quadratic equation that admits two solutions for  $\lambda_1$ . Given the calibration in Table 1, one solution will be negative and one positive. These solutions will be associated with values of  $\lambda_0$  of the same sign. The rational bubble solution with negative

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<sup>18</sup>The definition of intrinsic bubble is provided in Froot and Obstfeld (1991).

drift ( $\lambda_0 < 0$ ) will eventually shrink the bubble component to zero, while the solution with positive drift ( $\lambda_0 > 0$ ) implies that the price-dividend ratio will grow unboundedly. In order to pin down the values of the constants  $\lambda_0$ ,  $\lambda_1$ ,  $\lambda_2$  in the simulation exercise we also impose the additional restriction  $\lambda_0 = (\lambda_1 + \lambda_2)\bar{x}$ . This restriction allows us to interpret the rational bubble component (6) as a generalized version of the intrinsic rational bubble solution of Froot and Obstfeld (1991).

### 2.3 Extrapolative Expectations

Extrapolative expectations arise when agents form conditional expectations of future variables based on their past observations, therefore *extrapolating* future behavior from past behavior. Many studies in the behavioral finance literature confirm the presence of extrapolative behavior: Graham and Harvey (2001) use survey data to document that after periods of negative market returns agents reduce their forecasts of future risk premia. A study by Vissing-Jørgensen (2004) finds evidence of extrapolation in survey data about beliefs of stock market investors; in particular, she documents that investors who experienced high past portfolio returns expect higher future returns. We apply, to the housing market, two models of extrapolative expectations developed for the stock market by Lansing (2006, 2010).<sup>19</sup>

To derive the solution of the model under extrapolative expectations it is convenient to rewrite the equilibrium condition (2) in terms of the variable  $z_t \equiv \beta \exp((1 - \alpha)x_t)(y_t + 1)$ :

$$z_t = \beta \exp((1 - \alpha)x_t) \left( \hat{E}_t z_{t+1} + 1 \right) \quad (7)$$

that is, the Euler equation is expressed in terms of a composite variable, function of the future price-dividend ratio and the future realization of the growth rate of the dividends. We consider the simple expectation rule:<sup>20</sup>

$$\hat{E}_t [z_{t+1}] = H z_{t-1} \quad H > 0 \quad (8)$$

where  $H$  is a positive extrapolation coefficient that measures the weight agents put on the

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<sup>19</sup>Given the similarities in the framework, derivations in this paper follow Lansing (2006, 2010).

<sup>20</sup>Note the distinction between subjective expectations, denoted by  $\hat{E}_t$ , and the rational expectations, denoted by mathematical expectation operator  $E_t$ .

last observation in order to form conditional expectations of the forecast variable.<sup>21</sup> Note that as in the previous models, agents are assumed to know the process for the growth rate of dividends.<sup>22</sup> However they lack the knowledge of the law of motion of the forecast variable  $z_t$  as well as of the mapping between  $x_t$  and  $z_t$ . Therefore agents form their forecasts based on a perceived law of motion (PLM) that does not nest the actual law of motion (ALM). The forecast rule (8) can be obtained when agents use a geometric random walk<sup>23</sup> as their PLM for the variable  $z_t$ . Note however that conditional on the forecast for  $z_{t+1}$  the ALM is consistent with the Euler equation.

Replacing the expectation in (7) and applying the definitional relation for  $z_t$  the solution<sup>24</sup> for the extrapolative model with fixed extrapolation coefficient is:

$$y_t^{ee} = E_t [z_{t+1}] = (y_{t-1}^{ee} + 1) \beta H \exp((1 - \alpha) x_{t-1}) \quad (9)$$

Compared to the fundamental solution this model includes an extra free parameter  $H$  which can be used to match the variance of the price-dividend ratio. From (9) it also follows that, as in the case of the rational bubble model, the extrapolative expectation model includes an additional state variable,  $y_{t-1}^{ee}$ , therefore the rational bubble component and the extrapolative expectations mechanism can generate in the simple asset pricing model a higher persistence than the fundamental solution.

## 2.4 A Near Rational Bubble Solution

A key finding in the Vissing-Jørgensen (2004) study is not only that investors who experienced high past portfolio returns expect higher future returns but also that positive past returns have a stronger effect on expectations. In this section we outline a model consistent with this finding.

We follow the approach in Lansing (2010) and assume agents form expectations for the

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<sup>21</sup>The expectation at time  $t+1$  depends on the past realization (at  $t-1$ ) rather than the current realization (at  $t$ ). This ensures that expectations and actual realization are not determined simultaneously.

<sup>22</sup>Fuster, Hebert and Laibson (2011) consider a Lucas' tree model where agents estimate the dividend growth rate process.

<sup>23</sup>Lansing (2006) shows that lock-in of extrapolative expectations can occur if agents are concerned about minimizing forecast errors. This is "*because the (atomistic) representative agent fails to internalize the influence of his own forecast on the equilibrium law of motion of the forecast variable*", page 318.

<sup>24</sup>The solution of this model in the case of iid dividend growth rate is provided in Lansing (2006); the model considered in this section is a straightforward modification to accommodate for time dependence in the rent growth rate.

composite variable  $z_t$  in the following fashion:

$$\hat{E}_t [z_{t+1}] = \exp(b(1 + \rho)(x_t - \bar{x}) + \frac{1}{2}b^2\sigma_\varepsilon^2)z_{t-1}. \quad (10)$$

This can be seen as a model of extrapolative expectations with a time-varying weight put on past observations:

$$\hat{E}_t [z_{t+1}] = H_t z_{t-1} \quad (11)$$

where  $H_t \equiv \exp(b(1 + \rho)(x_t - \bar{x}) + \frac{1}{2}b^2\sigma_\varepsilon^2)$ . The conditional expectation in (10) can be derived by iterating forward the following perceived law of motion:

$$z_t = z_{t-1} \exp(b(x_t - \bar{x})) \quad z_0 > 0, \quad (12)$$

and then taking the conditional expectation at time  $t$ . The weight agents assign to the past observations depends on the parameter  $b$  and on the deviation of the rent growth rate from its mean. A near rational bubble solution to the model can be obtained by plugging the conditional expectation (10) into the equilibrium condition (7) and applying the definitional relation  $y_t \equiv \beta^{-1} \exp((1 - \alpha)x_t)^{-1} z_t - 1$ :

$$y_t^{nr} = E_t [z_{t+1}] = (y_{t-1}^{nr} + 1) \beta \exp\left(b(1 + \rho)(x_t - \bar{x}) + (1 - \alpha)x_{t-1} + \frac{1}{2}b^2\sigma_\varepsilon^2\right) \quad (13)$$

so that the price-dividend ratio is a function of its past values and of the current and past realizations of the dividend growth process. The solution (13) presents similarities with the bubble component solution (6). But while both solutions introduce persistence in the model with respect to the fundamental solution only the intrinsic bubble model allows for mean reversion of the price-dividend ratio.

The subjective forecast parameter  $b$  can be selected from moments of the actual data. In particular we calibrate  $b$  to match the covariance between  $\Delta(\log z_t)$  and  $x_t$  from the perceived law of motion (12):

$$b = \frac{\text{cov}[(\Delta \log z_t), x_t]}{\text{Var}(x_t)}$$

where  $\Delta \log z_t \equiv \log(z_t/z_{t-1})$ . Subsequently the near rational restricted perceptions equilibrium value for the parameter  $b$  is derived as the fixed point from the non-linear map:

$$b = \frac{(1 - \rho)m}{1 - \rho k}$$



where  $k$  and  $m$  are defined as:

$$k = \beta \exp \left( (1 - \alpha) \bar{x} + \frac{1}{2} b^2 \sigma_\varepsilon^2 \right)$$

$$m = (1 - \alpha) + b(1 + \rho) \beta \exp \left( (1 - \alpha) \bar{x} + \frac{1}{2} b^2 \sigma_\varepsilon^2 \right).$$

Relative to the calibration in Table 1, the additional parameter values are:<sup>25</sup>  $b = 4.54$ ,  $m = 4.58$  and  $k = 0.98$ . As the subjective forecast parameter  $b$  is positive, the perceived law of motion (12) implies that the weight put on past observations is (non-linearly) increasing in the growth rate of the dividend process.

### 3 Model Simulations

To compare the quantitative performance of the models described in section 2.1 through 2.4 we compute descriptive statistics from the observed and simulated data for the following variables: the price-rent ratio ( $y_t$ ),<sup>26</sup> the growth rate in the price-rent ratio ( $\log(y_t/y_{t-1})$ ), real net returns ( $r_t$ )<sup>27</sup> and growth rate of real house prices ( $\log(p_t/p_{t-1})$ ). Table 2 presents several statistics computed from the actual and simulated data: mean, standard deviation, skewness, kurtosis and autocorrelation with the first lag. Figure (2) also plots 2000 simulations from the models and the simulated growth rate of rents.

The fundamental solution fails in predicting the sample moments of order two and higher of the price-rent ratio. By construction, it matches the mean<sup>28</sup> but the standard deviation and the autocorrelation implied by this model are too low compared to the data.

Moments computed for the rational bubble solution for  $\lambda_0 < 0$  coincide with the fundamental solution as the bubble component quickly converges to zero, so the statistics for fundamental and rational bubble with negative drift are identical and are not reported in Table 2.

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<sup>25</sup>Note that there are three values of  $b$  that satisfy the non-linear mapping. As in Lansing (2010) we select the value of  $b$  that guarantees  $0 \leq k(b) < 1$  so that  $\Delta \log(z_t)$  is stationary.

<sup>26</sup>The statistics for the actual price dividend ratio are computed after running an ADF test. In the data the unit-root test rejects the null hypothesis at the 10 percent significance level but it fails to reject at 5 percent.

<sup>27</sup>Net returns are defined as  $(p_t + d_t)/p_{t-1} - 1$ .

<sup>28</sup>This is because the coefficient of risk aversion was picked to match the sample average of the price-rent ratio, see Table 1.

Table 2: Model Simulations: Unconditional Moments

Statistics	Actual	Simulated Data				
		Fundamental	Rational Bubble ( $\lambda_0 > 0$ )	Extrapolative (A) (B)		Near Rational
<i>y<sub>t</sub> = p<sub>t</sub>/d<sub>t</sub></i>						
mean	61	59	-	59	61	62
sd	11.08	0.28	-	3.91	11.11	13.38
skewness	0.69	0.01	-	0.26	0.70	0.77
kurtosis	2.54	3.00	-	2.98	3.70	3.78
autocorrelation	0.99	0.35	-	0.99	0.99	0.98
<i>log(y<sub>t</sub>/y<sub>t-1</sub>)</i>						
mean	-0.002	0.000	0.016	0.000	0.000	-0.000
sd	0.022	0.005	0.025	0.008	0.023	0.032
skewness	-0.657	-0.002	0.006	-0.005	-0.058	0.022
kurtosis	3.634	2.942	2.964	2.992	2.991	2.954
autocorrelation	0.876	-0.319	0.112	0.344	0.344	0.108
<i>r<sub>t</sub></i>						
mean	2.03%	2.18%	2.21%	2.21%	2.20%	2.24%
sd	0.023	0.005	0.031	0.008	0.021	0.039
skewness	-0.79	0.011	0.096	0.002	0.044	0.128
kurtosis	3.40	2.994	2.984	2.956	2.989	2.992
autocorrelation	0.93	0.503	0.157	-0.213	0.119	0.145
<i>log(p<sub>t</sub>/p<sub>t-1</sub>)</i>						
mean	0.3%	0.47%	2.14%	0.47%	0.47%	0.47%
sd	0.023	0.005	0.030	0.008	0.021	0.038
skewness	-0.70	-0.004	0.005	-0.014	-0.013	0.020
kurtosis	3.37	2.994	2.972	2.966	2.995	2.958
autocorrelation	0.93	0.493	0.158	-0.246	0.099	0.137

Note: descriptive statistics for the actual observations over the sample 1987Q1-2011Q4 and data simulated from the models described in Section 2.1 through 2.4. The column ‘Fundamental’ refers to the variables generated under the fundamental solution in Section 2.1, ‘Rational Bubble’ to the rational bubble model with  $\lambda_0 > 0$  from Section 2.2, ‘Extrapolative’ to the extrapolative expectation model from Section 2.3 and ‘Near Rational’ to the near rational model from Section 2.4. Models are simulated under the parameterization in Table 1. Additional parameter values are as follows: for the fundamental solution  $a_0 = 4.074$ ,  $a_1 = -2.33$ , for the rational bubble model  $\lambda_1 = 3.21$  and  $\lambda_0 = 0.0168$ , for the near rational model  $b = 4.53$ , for the ‘Extrapolative’ model column (A)  $H = 0.9999$ , for column (B)  $\alpha = 5$  and  $H = 1.012$ . Net returns  $r_t$  are defined as  $(p_t + d_t)/p_{t-1} - 1$ . Statistics are computed over 19500 simulations after discarding the first 500 draws.

As it emerges from Figure (2) the rational bubble model with  $\lambda_0 > 0$  implies an explosive series for the price-rent ratio so the moments are not computed for data simulated from this model.

The simulation exercise reports two columns for the extrapolative expectations model: for comparison with the other models in column (A) the coefficient of relative risk aversion is fixed at the value calibrated in Table 1 and therefore the extrapolation parameter is fixed

at  $H=0.9999$  to match the mean of the price dividend ratio.<sup>29</sup> In column (B),  $\alpha$  and  $H$  are chosen simultaneously to match both the first and second moment of the price-rent ratio. Twisting the expectations from rational to extrapolative is enough for the model to generate the same persistence as in the data. However parameterization in column (A) and (B) differ in their ability to match the other moments. From the extrapolative expectation model with  $\alpha = 2.5$  we obtain a higher standard deviation and skewness than from the fundamental solution although much lower than in the data. Instead, the model in column (B), where the coefficient of relative risk aversion is set equal to five, delivers standard deviation and skewness similar to the actual moments. Note that while the first result is obtained by construction through the appropriate choice of  $H$  and  $\alpha$ , the result regarding skewness is not imposed.

Finally, the near rational model also does a good job in replicating the moments of the price-rent ratio as it accounts for the high volatility and persistence observed in the data. Moreover it can generate positive skewness, although, as the extrapolative expectations model, it delivers excess kurtosis not present in the data.

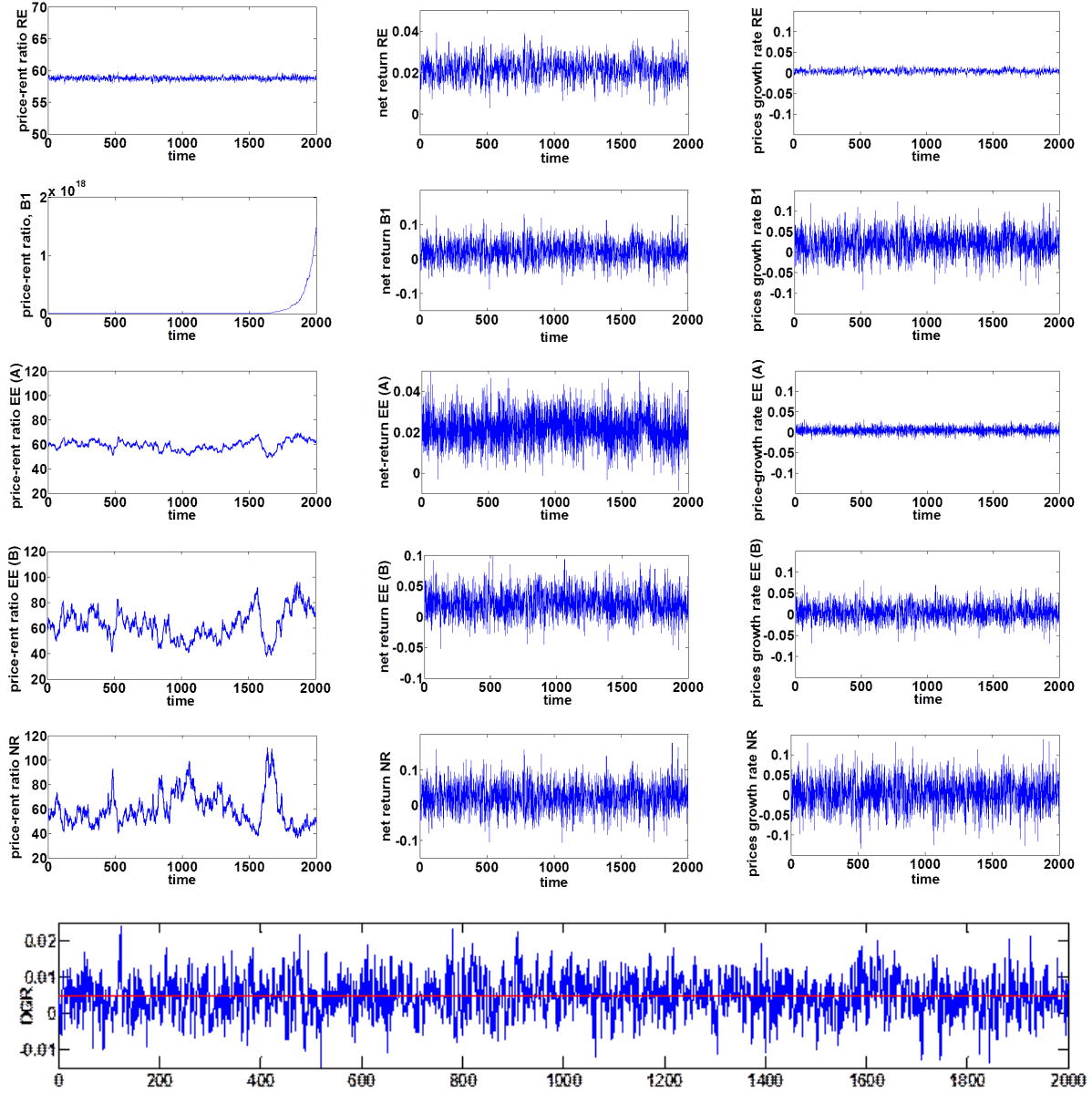
Results are mixed in terms of the other variables performance across models; the rational bubble with positive drift, the extrapolative expectation (B) and the near rational model generate higher standard deviation than the fundamental solution and extrapolative model (A) for all variables. They also generate positive, although small autocorrelation for the growth rate of the price-rent ratio, returns and growth rate of prices; however, the fundamental solution can generate higher autocorrelation than the other models in the net returns and in the growth rate of prices. Last, all models can match the first moment of net returns, growth rate of the price-rent ratio and growth rate of prices, except for the rational bubble model which predicts a much higher growth rate than the data for the last two variables. However, none of the models is able to capture the negative skewness, excess kurtosis, and high autocorrelation of the growth rate of the price-rent ratio, prices and returns.

Overall, the extrapolative expectations (B) and the near rational bubble are the models that can better replicate the moments of the price-rent ratio and they can also match the mean and variance of the other variables.

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<sup>29</sup>We mentioned in Section 2.3 that the model featuring extrapolative expectations includes an extra parameter  $H$  that can be used to match the volatility, together with the sample average, of the price-rent ratio.

Figure 2. Model Simulations: Price-Rent Ratio, Net Returns and Growth Rate of Prices



Note: simulated price-rent ratio, net returns, and house prices growth rate obtained from the fundamental (RE), rational bubble (B1), extrapolative expectation EE (A) and EE (B) and near rational bubble (NR) models described in Section 2.1 through 2.4. The lower panel shows the realization of the dividend growth rate process  $x_t$  used for simulation. The rational bubble simulation refers to the rational bubble model with  $\lambda_0 > 0$ . Models are simulated under the parameterization given in Table 1. Additional parameter values are as follows: for the fundamental solution  $a_0 = 4.074$ ,  $a_1 = -2.33$ , for the rational bubble model  $\lambda_1 = 3.21$  and  $\lambda_0 = 0.0168$ , for the extrapolative expectation model (A)  $\alpha = 2.5$  and  $H = 0.99999$ , for extrapolative expectation model (B)  $\alpha = 5$  and  $H = 1.012$ , for the near rational model  $b = 4.53$ . Net returns  $r_t$  are defined as  $(p_t + d_t) / p_{t-1} - 1$ .

Figure 2 highlights one major difference between the two models: for a short sequence of realizations of the dividend growth rate above the mean (see lower panel) the price-rent ratio generated by the extrapolative model and the near rational bubble model exhibit opposite

behavior.<sup>30</sup> For example, the price to rent ratio decreases (increases) considerably while the one generated from the near rational bubble model increases (decreases) substantially. This is because in the simulations the correlation between the growth rate of dividends and the price-rent ratio is positive for the near rational bubble model while it is negative for the other models. This will have very different implications in the historical counterfactual experiment conducted in the next section.

## 4 A Counterfactual Experiment

In the previous section we showed that both an extrapolative expectations model and a near rational bubble model can match the moments of the price-rent ratio equally well. But is this enough to determine the success of the models? And how can we distinguish between the two models if they provide with the same predictions for the moments? While the finance literature has focused on evaluating the models on their ability to match some moments of actual series, we investigate the ability of the models to replicate a sequence of realization of the data in the sample under consideration rather than just the mean, variance and autocorrelation of the data. In particular we conduct a counterfactual historical simulation exercise for the price-rent ratio and the price series by feeding the exogenous dividend process into the models. Because the results from the previous section determine that the extrapolative expectation model (B) and the near rational bubble model are the best performing models, we focus our attention to these two models only. However for completeness Figure 3 shows qualitatively the performance of all models: the fundamental model, the rational bubble with positive<sup>31</sup> drift, the models of extrapolative expectations for both calibrations and the near rational bubble model, by plotting the actual and simulated price-rent ratio (upper panel) and prices (lower panel) over the sample 1987Q1-2011Q4. The calibration of the coefficient of risk aversion and discount factor is the same as in Table 1.

Note that while the fundamental model is fully characterized by the growth rate of dividends and the calibrated parameters, the rational bubble model, the extrapolative expectations models (A) and (B) and the near rational model require the additional choice of the initialization value for the price-rent series. This value is chosen arbitrarily such that the rational bubble model takes the actual value observed in 1987Q1 while the extrapolative ex-

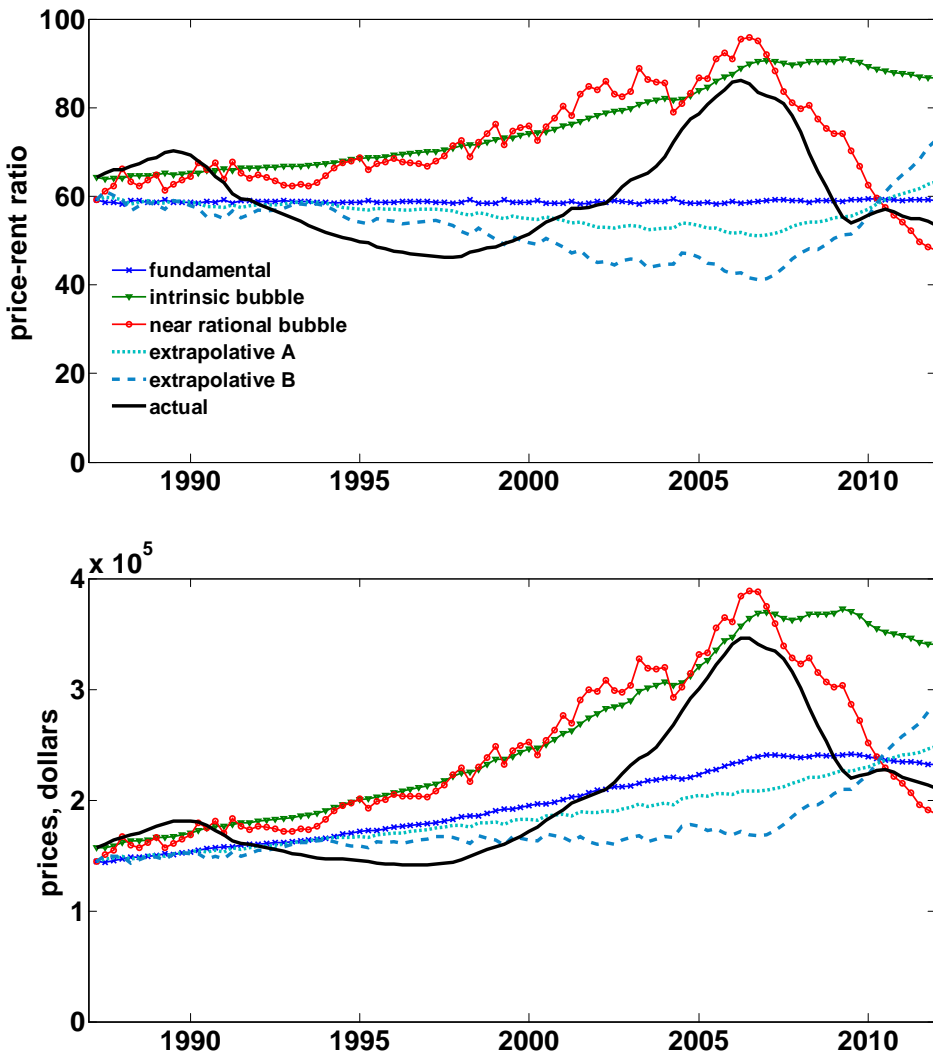
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<sup>30</sup>This behavior is most visible around observation 1600 in the simulated sample.

<sup>31</sup>In the rational bubble model with negative drift the bubble component converges to zero after the first three observations so the predictions of this model coincide with the predictions of the fundamental model.

peptations and the near rational models are initialized to the first counterfactual observation of the fundamental model.

Figure 3. Actual and Counterfactual Price-Rent Ratio and Price Series



Note: actual price-rent ratio (upper panel), actual real prices (lower panel) and counterfactual data over the sample 1987Q1-2011Q4. The blue line with cross marker refers to the fundamental solution, the green line with triangular marker to the rational bubble solution with  $\lambda_0 > 0$ , the light blue dotted line to the extrapolative expectation model A, the blue dashed line to the extrapolative expectation model B, and the red line with round marker to the near rational model. Actuals are in black. Counterfactual data are generated from the models described in Section 2.1 through 2.4 by plugging in the actual rents, growth rate of rents and parameters as calibrated in Table 1. Additional parameter values are as follows: for the fundamental solution  $a_0 = 4.074$ ,  $a_1 = -2.33$ , for the rational bubble model  $\lambda_1 = 3.21$  and  $\lambda_0 = 0.0168$ , for the near rational bubble model  $b = 4.53$ , for the extrapolative expectations model (A)  $H = 0.9999$ , (B)  $\alpha = 5$  and  $H = 1.012$ .

The rationale behind this choice is the following: for the rational bubble model we make use of equation (5) which states that the actuals are the sum of the fundamental and bubble solution, while for the extrapolative expectation and near rational bubble models

we want to highlight the difference in the evolution of these alternative series with respect to the fundamental counterfactual series. The counterfactual series from both extrapolative expectation models seem to evolve in opposite direction with respect to the data: when the price-rent ratio and prices increase sharply in the 2000-2005 period the counterfactual series drop. Similarly, when the actual series plunge at the end of the sample the counterfactual data rise. Surprisingly, the model where  $H$  and  $\alpha$  are chosen to match the mean and variance of the price-rent ratio is more at odds with the data as it predicts an accentuated decrease in price-rent ratio and prices over the years 2000-2005 and a larger surge in the last part of the sample. This behavior of the counterfactual series is due to the negative correlation between the growth rate of rents and prices that the extrapolative models generate.

The near rational bubble model can correctly replicate not only the surge in the price-rent ratio and in the prices, like the rational bubble model can do, but also the sharp downturn of the period 2006-2009. It should be noted that the counterfactual price to rent series for the near rational bubble model is generally higher than the data, implying that the mean of the predicted price-rent ratio is above the sample average of the actuals. This again is a consequence of the initialization choice.<sup>32</sup>

Next we report some quantitative measures of performance of the models: the in-sample Root Mean Squared Error (RMSE) the Mean Correct Forecast Direction (MCFD) and the correlation between the observations and counterfactual data from the different models, all shown in Table 3. Looking at the RMSE, which is a measure of how much on average the model is accurate in predicting the actual series, the extrapolative expectation model (B) is the worst for the price-dividend ratio while the near rational bubble model is the second best in predicting the price-rent ratio, but it is the worst for the net-return and the growth rate of prices. Interestingly, the rational bubble displays the lowest RMSE for net returns and it does a very good job in predicting the growth rate of prices. In interpreting these results we should keep in mind that the RMSE is sensitive to the initialization choice: in a robustness check we compute a RMSE of 17 for the price-rent ratio from the near rational model initialized at the actual data. The RMSE would decrease to 11.4 if the model was initialized to the value that allows the mean of the counterfactual series to match the sample average. For the extrapolative model (B) the RMSE for the price-rent ratio would fall to 16 if the first value was the same as the actual data. The RMSE for the other variables would

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<sup>32</sup>To match the sample average the first value for the counterfactual price-rent ratio should be fixed at 44 and the corresponding price at 108270 dollars. The evolution of both the price-rent ratio and prices series would be essentially unaltered, but there would be a downward level shift.

remain essentially unchanged.

The RMSE is a symmetric quadratic function which penalizes equally positive or negative prediction errors of the same magnitude. However for financial assets where positive profits may arise when the sign forecast is correct, it might be important to consider a loss function like MCFD which penalizes cases in which the model predicts a change in the opposite direction than the data regardless of the size of the prediction error. The MCFD loss function is defined as:

$$MCFD = T^{-1} \sum_{t=1}^T \mathbf{1} \left( \text{sign}(f_t) \cdot \text{sign}(\hat{f}_t) > 0 \right)$$

where  $\mathbf{1}(\cdot)$  is an indicator function that takes the value of one if the actual variable  $f_t$  and the counterfactual variable  $\hat{f}_t$  have the same sign. In contrast to the RMSE, the MCFD (and the correlation also) shown in Table 3 are basically unaffected by the initialization values chosen for the models.

Table 3: Models Performance

	Fundamental	Rational Bubble	Extrapolative		Near Rational
			(A)	(B)	
<i>RMSE</i>					
price-rent	11	19	13	23	15
net-return	0.027	0.026	0.030	0.039	0.041
prices growth rate	0.027	0.028	0.029	0.038	0.040
<i>MCFD</i>					
price-rent growth rate	0.485	0.535	0.424	0.373	0.586
net-return	0.838	0.838	0.838	0.818	0.727
prices growth rate	0.535	0.525	0.505	0.434	0.596
<i>CORRELATION</i>					
price-rent	-0.031	0.401	-0.487	-0.462	0.552
net-return	0.123	0.265	-0.092	-0.198	0.251
prices growth rate	0.153	0.292	-0.081	-0.205	0.261

Note: Root Mean Squared Error (RMSE), Mean Correct Forecast Directions (MCFD) and Correlation between the actual and counterfactual variables obtained for the asset price models described in Section 2.1 through 2.4. The column ‘Fundamental’ refers to the variables generated under the fundamental solution in Section 2.1, ‘Rational Bubble’ to the rational bubble model with  $\lambda_0 > 0$  from Section 2.2, ‘Extrapolative (A)’ to the extrapolative expectation model from Section 2.3 with  $\alpha = 2.5$  and  $H = 0.9999$ , ‘Extrapolative (B)’ to the same model with  $\alpha = 5$  and  $H = 1.012$  and ‘Near Rational’ to the near rational model from Section 2.4. The counterfactual observations are generated feeding into the models the actual real rents series ( $d_t$ ) and the actual growth rate ( $x_t$ ). The parameters  $\bar{x}$ ,  $\rho$  and  $\sigma_\varepsilon^2$  and  $\beta$  are given in Table 1. Additional parameter values are as follows: for the fundamental solution  $a_0 = 4.074$ ,  $a_1 = -2.33$ , for the rational bubble model with positive drift  $\lambda_1 = 3.21$  and  $\lambda_0 = 0.0168$ , for the near rational model  $b = 4.53$ . Net returns  $r_t$  are defined as  $(p_t + d_t) / p_{t-1}$ .

The near rational bubble is the one that more frequently predicts correctly the direction of change in both the price-rent ratio (58.6%) and prices (59.6%) delivering a better trading strategy than the other models. In contrast, a trader would be wrong about 60% of the time



when betting on the direction of change of house prices using the extrapolative model (B). However in identifying the correct direction of change in the net returns the near rational bubble model does worse than the other models which are correct more than 80% of the times. Finally Table 3 reports the correlation between the actual and the counterfactual series. The rational and near rational bubble models predictions exhibit similar positive correlation for the net-return and growth rate of prices. However the near rational model has a much stronger correlation with the price-rent series than the rational bubble model. Finally the extrapolative expectations model for both parameterizations implies a negative correlation between the actual and counterfactual data, the correlation being particularly strong in the case of the price-rent ratio.

Overall the counterfactual experiment conducted in this section suggests that even though the extrapolative expectation and the near rational bubble model are equally good in predicting the moments of the variables, the near rational bubble model is more successful in mimicking the evolution of the actual price-rent ratio and price series.

## 5 Conclusion

By treating houses simply as a financial asset, this paper uses a Lucas type tree model to explore the extent to which expectations can affect the evolution of house prices and the price-rent ratio in the United States. The model, when solved under rational expectations, does not generate persistent and substantial deviations from the mean nor explain the protracted surge and subsequent downturn of house prices of the last decade.

Therefore, we consider three models alternative to the fundamental that differ only in the mechanism agents use to form their expectations: one of intrinsic bubble formation and two models of extrapolative expectations. The intrinsic bubble model closely replicates the house price boom observed in the data, but not its subsequent plunge. Moreover, the model entails the theoretical unappealing property of generating a price-dividend ratio that grows unboundedly. We relax the assumption of rational expectations and solve the model by allowing the agents to form their expectations in an extrapolative fashion, by basing their conditional expectations on past realizations of the data. We distinguish among a model of extrapolative expectations with a constant extrapolation parameter and one with a time-varying weight increasing in the dividend growth process.

In a simulation exercise, both models of extrapolative expectations can match the mean, volatility, skewness and persistence of the price-dividend ratio. Therefore, we conduct a

counterfactual historical exercise which plays a critical role in highlighting additional implications of the models. In particular the exercise shows that only the predictions from the model with the time varying extrapolation parameter are consistent with the evolution of the actual data: the model captures both the substantial build up in prices of the years 2000-2005 and the sudden, sharp drop of the last part of the sample. Moreover, computation of the mean correct forecast direction on house prices shows that the near rational model delivers a better trading strategy than the other models. Therefore, we conclude that the simple model adopted in this paper performs surprisingly well, given that it abstracts from other factors that are identified as plausible drivers of the housing market.

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## 6 Appendix: Data Sources and Transformations

**House Prices:** Case and Shiller Composite 10-city house prices index seasonally unadjusted. The Case-Shiller is a constant-quality home price index constructed using repeated sales. To convert the index into prices we use the November 2011 mean price level from the National Association of Realtors. The house price series is then seasonally adjusted using the U.S. Census Bureau's X12 seasonal adjustment method and subsequently converted into quarterly data by averaging the monthly observations.

**Rents:** Imputed rental of owner-occupied nonfarm housing seasonally adjusted, series DOWNRC, from NIPA Table 2.4.5U Personal Consumption Expenditures by Type of Product. The imputed rents series available from the National Accounts is constructed as the paid rent adjusted by a coefficient of quality to take into account the higher quality of owner-occupied dwellings. This series is divided by the housing stock of owned and occupied dwellings from the American Housing Survey from US Census, available bi-annually. Bi-annual housing stock data are converted to quarterly by interpolating in the following way: for every year  $t$  available we compute the average occupants per dwelling  $w_t^A = POP_t^A / HS_t^A$  where POP is the Total Population and HS is the housing stock, owned and occupied dwellings; here the superscript indicates the frequency of the data (A=annual, Q=quarterly). Then we assume the weights are constant over the previous year and set  $w_t^Q = w_{t-1}^Q = \dots = w_{t-3}^Q = w_t^A$ ; finally the housing stock is constructed as  $HS_{t-j}^Q = w_{t-j}^Q POP_{t-j}^Q$  for  $j = 0, \dots, 3$  and  $\forall t$ . For years when the housing stock is not available the occupants per dwelling are computed as the average of the previous and following year.

**Price Indexes:** Rents and house prices are deflated by the personal consumption expenditures deflator: chain-type price index less food and energy, from Bureau of Economic Analysis, seasonally adjusted.