Procyclical Leverage and Value-at-Risk*

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Abstract

We examine the evidence on the procyclicality of the financial system and explore its microfoundations. Contrary to the classical corporate finance approach where assets are taken as given, the evidence points to equity, not assets, as being the pre-determined variable. We explore the extent to which a standard contracting model can explain the facts. Under general assumptions about the the tail of the return density, financial intermediaries' leverage is determined by a Value-at-Risk constraint which ensures a constant probability of a financial intermediarie's failure, irrespective of the risk environment. Tranquil conditions are therefore associated with balance sheet expansions.

Keywords: Financial intermediary leverage, procyclicality, collateralized borrowing
JEL codes: G21, G32

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1 Introduction

The procyclicality of financial intermediary balance sheets has been a prominent topic of debate in the wake of the financial crisis. Some cyclical variation in total lending is to be expected, even in a world with perfectly competitive markets where the conditions of the Modigliani and Miller (1958) theorem hold. There are more positive net present value (NPV) projects that need funding when the economy is strong than when the economy is weak. Therefore, we should expect balance sheets to increase during the up-swing and decline in the down-swing. The debate about procyclicality is more subtle. The question is whether the fluctuations in balance sheet size are larger than would be justified by changes in the incidence of positive NPV projects.

We examine the evidence on procyclicality and explore a possible microfoundation for such variations in balance sheets. In particular, we revisit the evidence on the balance sheet management of the five stand-alone investment banks that were the focus of interest during the financial crisis of 2008. Elsewhere (Adrian and Shin (2010)), we documented the procyclical leverage of these firms' balance sheets. Here, we delve deeper into their behavior and document the important explanatory role played by measured risks through the firms' disclosed Value-at-Risk. Based on the evidence, we explore the extent to which a standard contracting model can provide the microfoundations for procyclical leverage driven by Value-at-Risk.

Value-at-Risk is a quantile measure on the loss distribution defined as the smallest benchmark loss L such that the probability that the realized loss turns out to be larger than L is below some fixed probability p. If a bank were to manage its risk by maintaining Value-at-Risk to be no larger than its equity capital, the bank would ensure that it remains solvent with probability at least $1 - p$.

Value-at-Risk is used widely by financial intermediaries. However, in spite of the widespread (indeed ubiquitous) use of Value-at-Risk by financial institutions, the concept has remained relatively remote from the standard corporate finance discussions and the tools favored by financial economists. Nor, to our knowledge, has there been a systemic empirical investigation on whether (and if so how) banks adjust their balance sheets to manage their Value-at-Risk. Our paper bridges the gap between theory and practice by documenting the evidence and offering one possible approach to the microfoundations.

We find that the *unit* Value-at-Risk, defined as the Value-at-Risk per dollar of assets, fluctuates widely over the financial cycle in step with measures of risk such as the VIX index or the credit default swap spreads on banks and other intermediaries. However, in contrast to the wide fluctuations in the risk environment through the VaR to asset ratio, there are much more modest fluctuations in the firms' VaR to equity ratio. The difference is accounted for by the active management of leverage by intermediaries, especially the active shedding of risks through deleveraging during times of market stress. Indeed, we show that the evidence is consistent with the rule of thumb that Value-at-Risk normalized by equity is kept constant over the cycle, even at the height of the crisis. The implication is that intermediaries are shedding risks and withdrawing credit precisely when the financial system is under most stress, thereby serving to amplify the downturn.

Having documented the evidence, we then turn to an exploration of how far a standard contracting framework with moral hazard can provide microfoundations for the observed behavior. In order to keep our framework as close as possible to the existing corporate finance literature, we explore the simplest possible contracting model where a bank seeks funding from its creditors which it can channel to its customers. We find that the outcome of the contracting problem has the creditors imposing a leverage limit on the bank that implies a fixed probability of failure of the bank, irrespective of the risk environment. Since measured risk fluctuates over the cycle, imposing a constant probability of failure implies very substantial expansions and contractions of the balance sheet of the bank for any given level of bank equity. In other words, the contract implies substantial leveraging and deleveraging over the cycle.

Our modeling framework provides microfoundations to the limits of arbitrage and the fire sale literatures. Shleifer and Vishny (1992) provide an early model of fire sales, where equilibrium asset values depend on the debt capacity of the sector of the economy that invests in such assets. Shleifer and Vishny (1997) point out that the financial constraints of financial intermediaries affect equilibrium asset valuations. More recently, many studies have either implicitly or explicitly assumed a Value-at-Risk constraint in modeling the management of financial institutions. Our paper contributes to this literature by providing microfoundations for the pervasive use of Value-at-Risk type rules. Gromb and Vayanos (2002) construct a model of intermediary capital, where constraints on the intermediary can induce excessive risk taking, as intermediaries are not internalizing that their fire sales tighten margin constraints of other arbitrageurs. Gromb and Vayanos (2010) provide conditions when the presence of intermediaries stabilize or destabilize equilibrium asset prices. Brunnermeier and Pedersen (2009) introduce margin constraints that follow a Value-at-Risk rule, and stress the interaction of market and funding liquidity. Oehmke (2008 and 2009) studies the speed and the spreading of price deviations when arbitrageurs face Value-at-Risk or other risk management constraints.

The plan of the paper is as follows. We begin in the next section by reviewing the evidence on the role of Value-at-Risk as a driver in procyclical leverage of the (former) investment banks. We then explore the contracting environment and show the key comparative statics result that leverage fluctuates in response to shifts in underlying risk. A Value-at-Risk constraint is then shown to be the outcome of a contracting problem when the tail of the loss distribution is exponential. We close with some remarks on the implications of our results.

2 Value-at-Risk and Leverage

In textbook discussions of corporate financing decisions, the set of positive net present value (NPV) projects is often taken as being given, with the implication that the size of the balance sheet is fixed. Instead, attention falls on how those assets are financed. Leverage increases by substituting equity for debt, such as through an equity buy-back financed by a debt issue, as depicted by the left hand panel in Figure 1.

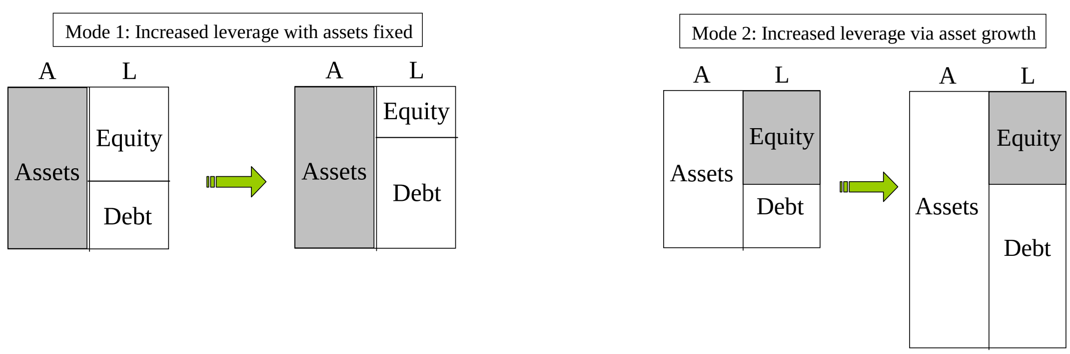

Figure 1: **Two Modes of Leveraging Up.** In the left panel, the firm keeps assets fixed but replaces equity with debt. In the right panel, the firm keeps equity fixed and increases the size of its balance sheet.

However, the left hand panel in Figure 1 turns out not to be a good description of the way that the banking sector leverage varies over the financial cycle. For US investment banks, Adrian and Shin (2010) show that leverage fluctuates through changes in the total size of the balance sheet with equity being the pre-determined variable. Hence, leverage and total assets tend to move in lock-step, as depicted in the right hand panel of Figure 1. In this paper, we present evidence on the cyclical behavior of bank leverage from the largest commercial and investement banks. In particular, we investigate the balance sheet behavior of the five largest investment banks (Goldman Sachs, Morgan Stanley, Lehman Brothers, Merrill Lynch, and Bear Stearns) and the three commercial banks with the largest trading operations (JP Morgan Chase, Citibank, and Bank of America). All eight of these institutions have been primary dealers of the Federal Reserve.[1] The three commercial bank have absorbed some of the largest formerly independent investment banks (Citibank acquired Salomon Brothers in 1998, Chase acquired JP Morgan in 2000, and Bank of America acquired Merrill Lynch in 2008). Institutions that we do not consider in the analysis are the foreign banks, some of which also own substantial trading operations in the US.

Figure 2 is the scatter plot of the quarterly change in total assets of the sector consisting of the major US banks where we plot both the changes in assets against equity, as well as changes in assets against equity. More precisely, it plots $\{(\Delta A_t, \Delta E_t)\}$ and $\{(\Delta A_t, \Delta D_t)\}$ where ΔA_t is the change in total assets of the investment bank sector at quarter t, and where ΔE_t and ΔD_t are the change in equity and change in debt of the sector, respectively.

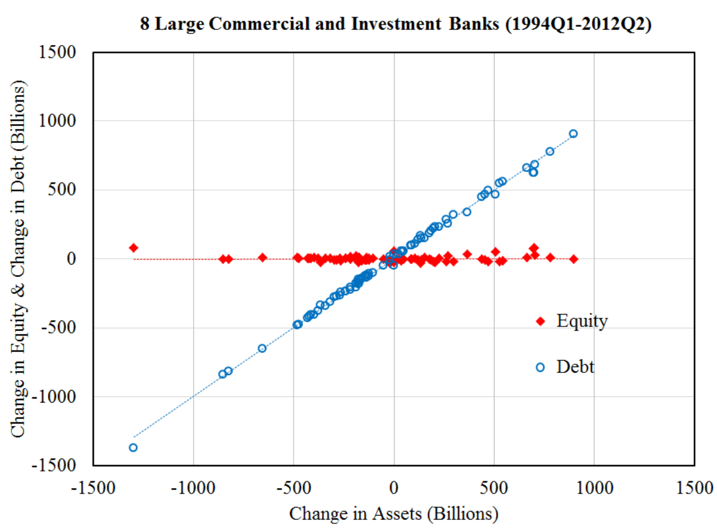

Figure 2: Scatter chart of $\{(\Delta A_t, \Delta E_t)\}$ and $\{(\Delta A_t, \Delta D_t)\}$ for changes in assets, equity and debt of US commercial banks (Bank of America, Citibank, JP Morgan) and US investment banks (Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch and Morgan Stanley) between 1994Q1 and 2012Q2 (Source: SEC 10Q filings).

We see from Figure 2 that US investment banks conform to the right hand panel of Figure 1 in the way that they manage their balance sheets. The fitted line through $\{(\Delta A_t, \Delta D_t)\}$ has slope very close to 1, meaning that the change in assets in any one

[1] see http://www.newyorkfed.org/markets/pridealers_current.html

quarter is almost all accounted for by the change in debt, while equity is virtually unchanged. The slope of the fitted line through the points $\{(\Delta A_t, \Delta E_t)\}$ is close to zero. Both features capture the picture of bank balance sheet management given by the right hand panel in Figure 1.

The equity series in the scatter chart in Figure 2 is of *book equity*, giving us the difference between the value of the bank's portfolio of claims and its liabilities. An alternative measure of equity would have been the bank's *market capitalization*, which gives the market price of its traded shares. However, market capitalization is not the same thing as the *marked-to-market value* of the book equity, which is the difference between the market value of the bank's portfolio of claims and the market value of its liabilities. For securities firms that hold primarily marketable securities that are financed via with repurchase agreements, the book equity of the firm reflects the haircut on the repos, and the haircut will have to be financed with the firm's own book equity. This book equity is the archetypal example of the marked-to-market value of book equity. We are interested in the portfolio decision of the bank - that is, how much it can lend given its equity. Thus, book equity is the correct notion of equity for us, not market capitalization. Market capitalization is the discounted value of the future free cash flows of the securities firm, and will depend on cash flows such as fee income that do not depend directly on the portfolio held by the bank. Indeed, there may only be a loose relationship between the market capitalization and the marked-to-market value of the firm's book equity, as evidenced by the strong variation of market to book values over time.

Our modeling approach is motivated by the patterns observed in Figure 2. In effect, we are asking what determines the haircut on the firm's repos. We investigate how the notion of Value-at-Risk can help to explain banks' behavior. For a bank whose assets today are A_0, suppose that its total assets next period is given by a random variable A. Then, its Value-at-Risk (VaR) represents the "approximate worse case loss" in the sense that the probability that the loss is larger than this approximate worst case loss is less than some small, pre-determined level. Formally, the bank's Value-at-Risk at confidence level c relative to some base level A_0 is smallest non-negative number V such that

$$\text{Prob}\,(A < A_0 - V) \leq 1 - c \tag{1}$$

Banks and other financial firms report their Value-at-Risk numbers routinely as part of their financial reporting in their annual reports and as part of their regulatory disclosures. In particular, disclosures on the 10K and 10Q regulatory filings to the US Securities and

Exchange Commission (SEC) are available in electronic format from Bloomberg, and we begin with some initial exploration of the data. We begin by summarizing some salient features of the VaR disclosed by the major commercial and investment banks.[2]

Figure 3 plots the asset-weighted average of the 99% VaR of the eight institutions, obtained from Bloomberg.[3] The VaRs are reported at either the 95% or 99% level, depending on the firm. For those firms for which the 95% confidence level is reported, we scale the VaR to the 99% level. We superimpose on the chart the following series unit VaR (the dollar VaR per dollar of total assets) and the equity implied volatility. Both are measures of firm risk. While the unit VaR is the firms' assessment of risk, the implied volatility is the market's assesement of equity risk. The vertical scaling is in units of the pre-2007 standard deviations, expressed as deviations from the pre-2007 mean.

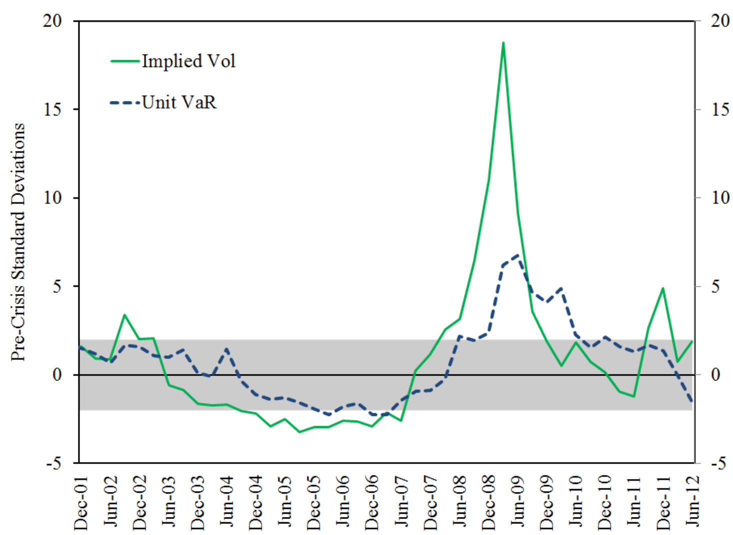

Figure 3: **Risk Measures.** The figure plots the unit VaR and the implied volatility for the eight large commercial and investment banks. Both variables are standardized relative to the pre-crisis mean and standard deviation. The measures are the lagged weighted averages of the standardized variables across the eight banks, where the weights are lagged total assets. Unit VaR is the ratio of total VaR to total assets. Implied volatilities are from Bloomberg. The grey shaded area indicates ± 2 standard deviations around zero.

Figure 3 shows how the Unit VaR compares to the implied volatility of the equity option prices as an alternative risk measure. Between 2001 and the beginning of 2007, the risk measures were stable and moved within a tight band around their mean. However,

[2] Bank of America, Citibank, JP Morgan, Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch and Morgan Stanley.

[3] The Bloomberg code is ARDR_TOTAL_VALUE_AT_RISK.

with the onset of the crisis, both measures spiked. The Unit VaR increased over five standard deviations relative to their pre crisis levels. The run-up in implied volatility was even more dramatic at "only" nearly twenty standard deviations relative to the pre crisis level. The spike in the implied volatility preceded the spike in the Unit VaR. From these series, we see that the assets held by the banks reflected the general mayhem in the markets. Notice the time lag (at around six months) in the spiking of the Unit VaR series relative to the implied volatility series. Whereas the implied volatility series spike in December 2008, the Unit VaR series peaks in June 2009. The lag can be attributed to the backward-looking nature of the VaR estimates, which are based on a window of past data.

For our exercise, we can interpret the Unit VaR series as reflecting the risk environment of the recent past which the firms have uppermost in their thinking when making decisions on how much risk exposure to take on. How did the firms respond to the hostile risk environment in managing their balance sheets? Figure 4 is revealing in this respect by plotting:

- unit VaR,
- the ratio of VaR to Equity,
- leverage (the ratio of total assets to equity)

Figure 4 plots the leverage series together with the VaR normalized by book equity. It shows that the firms reacted to the spike in measured risks by sharply reducing their leverage. While the average leverage across the banks increased slightly until the Bear Stearns crisis in March 2008, it dropped by more than five standard deviations between the second and the fourth quarters of 2008. At the same time, the VaR to Equity ratio barely changed. Thus, the five Wall Street investment banks were shedding risk exposures very dramatically over the crisis period.

Value-at-Risk turns out to be very informative in explaining leverage. Consider the so-called *Value-at-Risk (VaR) Rule*, which stipulates that the financial intermediary maintains enough equity E to cover its Value-at-Risk. The VaR rule can be stated equivalently as maintaining enough equity E so that the bank's probability of failure is kept constant, set to the confidence threshold associated with the VaR measure used by the bank. When VaR is given by V, the rule can be written as

$$E = V = v \times A \tag{2}$$

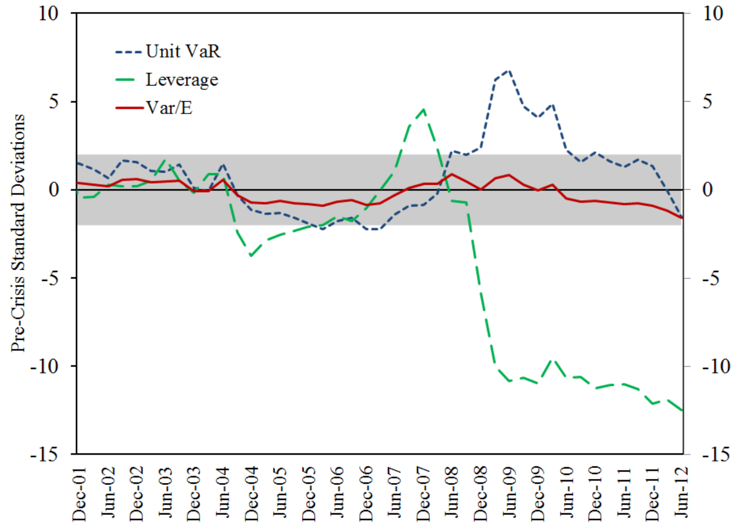

Figure 4: **Risk and Balance Sheet Adjustment.** This figure plots the unit VaR together with the intermediary balance sheet adjustment variables VaR/Equity and Leverage. All variables are standardized relative to the pre-crisis mean and standard deviation. The measures are the value weighted averages of the standardized variables across the eight banks, where the weights are lagged total assets. Unit VaR is the ratio of total VaR to total assets. VaR/Equity is the ratio of total VaR to total book equity. Leverage is the ratio of total assets to total book equity. The grey shaded area indicates ± 2 standard deviations around zero.

v is Unit VaR (Value-at-Risk per dollar of assets). Then, leverage L satisfies

$$L \equiv \frac{A}{E} = \frac{1}{v} \tag{3}$$

so that $\ln L = -\ln v$. In particular, we have the prediction

$$\ln L_t - \ln L_{t-1} = -(\ln v_t - \ln v_{t-1}) \tag{4}$$

so that the scatter chart of leverage changes against unit VaR changes should have slope -1.

Figure 5 plots log changes in leverage against log changes in Unit VaR. We can see two distinct regimes in the figure. During the financial crisis from the third quarter of 2007 through the fourth quarter of 2009, the slope of the scatter is close to -1, suggesting that a one percent increase in unit VaR is accompanied by a one percent reduction in leverage. Bearing in mind that these are annual growth rates, we can see from the horizontal scale of Figure 5 that the deleveraging was very substantial, indicating rapid balance sheet contractions. The second regime is during during normal times, when the chart does not depict any relationship between the growth of unit VaR and the growth of leverage.

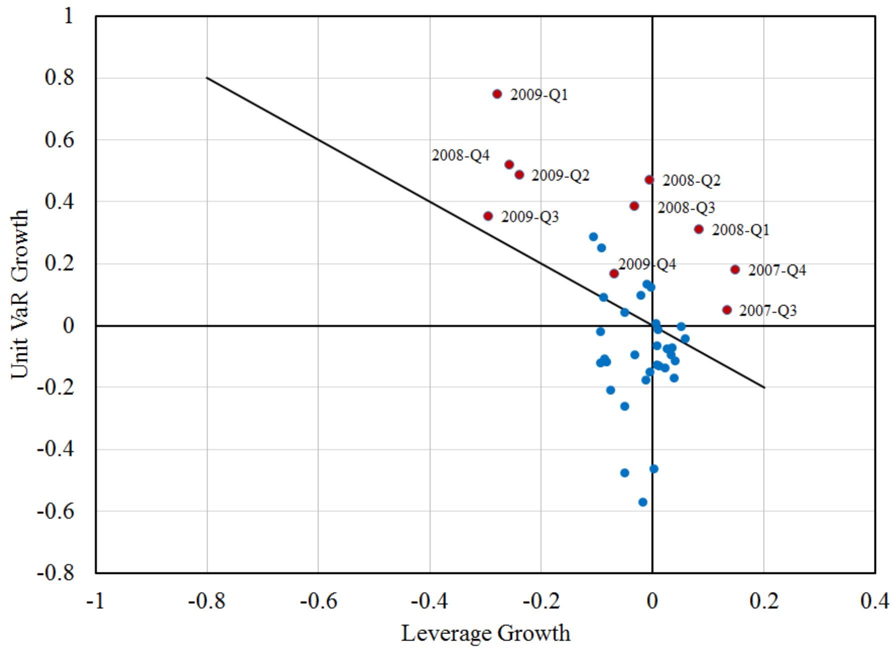

Figure 5: **Shocks to Risk and Adjustments to Leverage.** This figure plots the annual growth rate in Unit VaR against the annual growth rate in Leverage. The variables are weighted by lagged total assets. The diagonal line has slope -1. The red dots correspond to the quarters during the financial crisis.

The analysis based on annual growth rates of Figures 3–5 is further confirmed by the correlation analysis of quarterly growth rates in Table 1. We see that leverage growth is strongly negatively correlated with shocks to the risk measures (Unit VaR growth, and lagged implied volatility changes), but uncorrelated with the growth of the VaR to Equity ratio. Leveraged financial intermediaries manage their balance sheets actively so as to maintain Value-at-Risk equal to their equity in the face of rapidly changing market conditions.

	Quarterly Leverage Growth
Quarterly VaR/E growth	0.03
Quarterly Unit VaR growth	-0.33**
Quarterly CDS spread change	-0.52**
Quarterly Implied Volatility changes (lagged)	-0.31**

Table 1: Correlations. The * denotes significance at the 10% level, The ** denotes significance at the 5% level, the *** denotes significance at the 1% level. Each variable is aggregated across banks by taking the average weighted by lagged total assets.

The evidence on the balance sheet management of the financial intermediaries in our sample points to the *VaR Rule* being an important determinant of the leverage decisions of the financial intermediaries in our sample. The *Value-at-Risk* rule gives rise to procyclical leverage in the sense that leverage is high in tranquil times when *unit VaR* is low, while leverage is low in more turbulent market conditions when *unit VaR* is high. Translated in terms of risk premiums, leverage is *high* in boom times when the risk premium is *low*.

The procyclical nature of leverage is a feature that is at odds with many standard portfolio decision rules. For instance, for an investor with log utility, the leverage of the investor is monotonic in the Sharpe ratio of the risky security, so that leverage is high when the risk premium is *high* (Merton (1969)). In other words, for investors with log utility, leverage is countercyclical. Xiong (2001), He and Krishnamurthy (2009) and Brunnermeier and Sannikov (2011) are recent contributions that use the log utility formulation and hence which have the feature that leverage is countercyclical.

When financial institutions are managed with log preferences, the probability of default varies systematically with the state of the economy, as illustrated in Figure 6. A decline in the Sharpe ratio implies an increase in the default probability as institutions with log preferences increase leverage. In contrast, institutions that are managed according to the VaR rule exhibit a constant probability of default irrespective of economic conditions.

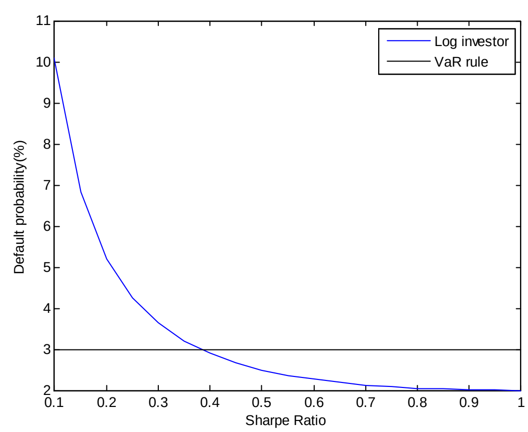

Figure 6: The figure plots the probability of failure as a function of the Sharpe ratio for two types of institutions. The blue line denotes an institution that is managed with log preferences. The black line denotes an institution that is managed according the VaR rule. The procedure is given in the appendix.

3 Contracting Framework

Having confirmed the promising nature of the Value-at-Risk rule, we turn our attention to providing possible microfoundations for such a rule. Our approach is to select the simplest possible framework that could rationalize the behavior of the intermediaries in our sample, relying only on standard building blocks. In this spirit, we will investigate how far we can provide microfoundations for the Value-at-Risk Rule in the context of a standard contracting environment. There should be no presumption that the approach developed below is the only such microfoundation. However, the spirit of the exercise is to start from very familiar building blocks, and see how far standard arguments based on these building blocks will yield observed behavior.

Our approach is to consider the contracting problem between an intermediary and uninsured wholesale creditors to the intermediary. We may think of the intermediary as a Wall Street investment bank and the creditor as another financial institution that lends to the investment bank on a collateralized basis. We build on the Holmström and Tirole (1997) model of moral hazard but focus attention on the risk choice by the borrower. The limits on leverage are seen as the constraint placed by the (uninsured) wholesale creditor on the intermediary, thereby limiting the size of balance sheet for any given level of capital of the borrower.[4]

Under natural conditions on the tail of the distribution of asset realizations, the outcome of the contracting problem between the intermediary and the wholesale creditor turns out to be equivalent to applying a Value-at-Risk rule on the intermediary's risk. In other words, the borrower must shrink or expand the balance sheet so that it remains solvent with a fixed probability, irrespective of the risk environment. Thus, when overall risks in the financial system increase after a shock, the intermediary must cut its asset exposure in order to maintain the same probability of default to additional shocks as it did before the arrival of the shock. Conversely, when the economic environment is more benign and forecast risk declines, the intermediary will expand its balance sheet in order to maintain its previous probability of default.

It is worth reiterating that there should be no presumption that the microfoundation offered here is the only way to rationalize the Value-at-Risk rule. Nevertheless, we can

[4]This is a theme that is well-known in the banking literature on minimum capital requirements that counteract the moral hazard created by deposit insurance (Michael Koehn and Anthony Santomero (1980), Daesik Kim and Santomero (1988), Jean-Charles Rochet (1992)). Gabriella Chiesa (2001), Guillaume Plantin and Rochet (2006) and Vittoria Cerasi and Rochet (2007) have further developed the arguments for regulatory capital not only in banking sector, but in the insurance sector as well.

take some comfort in the familiarity of the framework that yields the main result. The two components are, first, a standard contracting problem with moral hazard, and second, a standard risk-shifting problem.

3.1 Set up

We now describe the contracting model in more detail. There is one principal and one agent. Both the principal and agent are risk-neutral. The agent is a financial intermediary that finances its operation through collaterateralized borrowing. For ease of reference, we will simply refer to the agent as the "bank". The principal is an (uninsured) wholesale creditor to the bank. A bank is both a lender and a borrower, but it is the bank's status as the borrower that will be important here.

There are two dates—date 0 and date 1. The bank invests in assets at date 0 and receives its payoffs and repays its creditors at date 1. The bank starts with fixed equity E, and chooses the size of its balance sheet. We justify this assumption by reference to the scatter chart encountered already in Figure 2. Denote by A the market value of assets of the bank. The notional value of the assets is $(1+\bar{r})A$, so that each dollar's worth of assets acquired at date 0 promises to repay $1+\bar{r}$ dollars at date 1.

The assets are funded in a collateralized borrowing arrangement, such as a repurchase agreement. The bank sells the assets worth A for price D at date 0, and agrees to repurchase the assets at date 1 for price $\bar{D}$. Equity financing meets the gap $A-D$ between assets acquired and debt financing. Let E be the value of equity financing. The balance sheet in market values at date 0 is therefore

Assets	Liabilities
Assets A	Debt D Equity E

(5)

The notional value of the securities is $(1+\bar{r})A$, and the notional value of debt is the repurchase price $\bar{D}$. Thus, the balance sheet in notional values can be written as

Assets	Liabilities
Assets $A(1+\bar{r})$	Debt $\bar{D}$ Equity $\bar{E}$

(6)

where $\bar{E}$ is the notional value of equity that sets the two sides of the balance sheet equal. The bank has the choice between two types of assets—good securities and substandard securities. For each dollar invested at date 0, the bank can buy notional value of $1+\bar{r}$ of either security. However, for each dollar invested at date 0 the good security has expected payoff

$$1 + r_H$$

with outcome density $f_H(.)$. The bad security has expected payoff $1 + r_L$ with density $f_L(.)$. We assume that

$$r_L < 0 < r_H \tag{7}$$

so that investment in the bad security is inefficient. We assume that the bank's balance sheet is scalable in the sense that asset payoffs satisfy constant returns to scale.

Although the bad security has a lower expected return, it has higher upside risk relative to the good project in the following sense. Denote by $F_H(.)$ the cumulative distribution function associated with f_H and let $F_L(.)$ be the cdf associated with f_L. We suppose that F_H cuts F_L precisely once from below. That is, there is z^* such that $F_H(z^*) = F_L(z^*)$, and

$$(F_H(z) - F_L(z))(z - z^*) \geq 0 \tag{8}$$

for all z. The bank's initial endowment is its equity E. The bank decides on the total size of its balance sheet by taking on debt as necessary. The debt financing decision involves both the face value of debt $\bar{D}$ and its market value D. The optimal contract maximizes the bank's expected payoff by choice of A, D and $\bar{D}$ with E being the pre-determined variable.

The fact that E is the pre-determined variable in our contracting setting goes to the heart of the procyclicality of lending and is where our paper deviates from previous studies. In textbook discussions of corporate financing decisions, the set of positive net present value (NPV) projects is normally taken as being given and the size of the balance sheet is fixed and determined exogenously. The remaining focus is on the liabilities side of the balance sheet, in determining the relative mix of equity and debt. Even in a dynamic setting, if the assets of the firm evolve exogenously, the focus remains on the liabilities side of the balance sheet, and how the funding mix is determined between debt and equity. However, we have seen in our empirical section evidence suggesting that it is a intermediaries' equity, not assets, that evolves exogenously.

3.2 Optimal Contract

As noted by Merton (1974), the value of a defaultable debt claim with face value $\bar{D}$ is the price of a portfolio consisting of (i) cash of $\bar{D}$ and (ii) short position in a put option on the assets of the borrower with strike price $\bar{D}$. The net payoff of the creditor to the bank is illustrated in Figure 7.The creditor loses her entire stake D if the realized asset

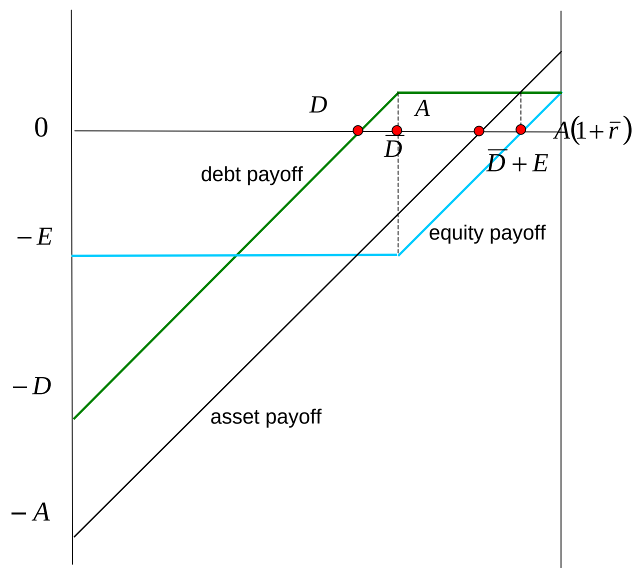

Figure 7: Net Payoffs

value of the bank's assets is zero. However, if the realized asset value of the bank is $\bar{D}$ or higher, the creditor is fully repaid. We have $\bar{D} > D$, since the positive payoff when the bank does not default should compensate for the possibility that the creditor will lose in the case of default.

The equity holder is the residual claim holder, and his payoff is illustrated as the kinked convex function in Figure 7. The sum of the equity holder's payoff and the creditor's payoff gives the payoff from the total assets of the bank.

3.3 Creditor's Participation Constraint

Denote by $\pi_H\left(\bar{D}, A\right)$ the price of the put option with strike price $\bar{D}$ on the portfolio of good securities whose current value is A. We assume that the market for assets is competitive, so that the option price satisfies constant returns to scale:

$$\pi_H\left(\bar{D}, A\right) = A\pi_H\left(\tfrac{\bar{D}}{A}, 1\right) \tag{9}$$

In other words, an option on A worth of securities with strike price $\bar{D}$ can be constructed by bundling together A options written on 1 dollar's worth of securities with strike price $\bar{D}/A$. Similarly, $\pi_L\left(\bar{D},A\right) = A\pi_L\left(\frac{\bar{D}}{A},1\right)$, for portfolios consisting of bad securities.

Define $\bar{d}$ as the ratio of the promised repurchase price at date 1 to the market value of assets of the bank at date 0

$$\bar{d} \equiv \frac{\bar{D}}{A}. \tag{10}$$

Hence $\bar{d}$ is the ratio of the notional value of debt to the market value of assets. Define:

$$\pi_H\left(\bar{d}\right) \equiv \pi_H\left(\bar{d},1\right)$$

so that $\pi_H\left(\bar{d}\right)$ is the price of the put option on one dollar's worth of the bank's asset with strike price $\bar{d}$ when the bank's portfolio consists of good assets. $\pi_L\left(\bar{d}\right)$ is defined analogously for portfolio of bad securities.

The creditor's initial investment is D, while the expected value of the creditor's claim is the portfolio consisting of (i) cash of $\bar{D}$ and (ii) short position in put option on the assets of the bank with strike price $\bar{D}$. The (gross) expected payoff of the creditor when the bank's assets are good is therefore

$$\bar{D} - A\pi_H\left(\bar{d}\right) = A\left(\bar{d} - \pi_H\left(\bar{d}\right)\right)$$

Since the creditor's initial stake is D, her net expected payoff is

$$\begin{aligned} U^C(A) &= \bar{D} - D - A\pi_H\left(\bar{d}\right) \\ &= A\left(\bar{d} - d - \pi_H\left(\bar{d}\right)\right) \end{aligned} \tag{11}$$

where $d \equiv D/A$ is the ratio of the market value of debt to the market value of assets. The participation constraint for the creditor requires that the expected payoff is large enough to recoup the initial investment D. That is,

$$\bar{d} - d - \pi_H\left(\bar{d}\right) \geq 0. \tag{IR}$$

3.4 Bank's Incentive Compatibility Constraint

The payoff of the equity holder is given by the difference between the net payoffs for the bank's assets as a whole and the creditor's net payoff, given by U^C in (11). Thus, the equity holder's payoff is

$$U^E(A) = A\left(r - \bar{d} + d + \pi\left(\bar{d}\right)\right)$$

where $r \in \{r_L, r_H\}$, and $\pi\left(\bar{d}\right) \in \left\{\pi\left(\bar{d}\right)_L, \pi\left(\bar{d}\right)_H\right\}$. The optimal contract maximizes U^E subject to the incentive compatibility constraint of the bank to hold good securities in his portfolio, and subject to the break-even constraint of the creditor. The equity holder's stake is a portfolio consisting of

- put option on the assets of the bank with strike price $\bar{D}$
- risky asset with expected payoff $A\left(r - \bar{d} + d\right)$

The expected return r and the value of the option $\pi\left(\bar{d}\right)$ depends on the bank's choice of assets. The expected payoff for the equity holder when the asset portfolio consists of the good asset is

$$A\left(r_H - \bar{d} + d + \pi_H\left(\bar{d}\right)\right) \tag{12}$$

while the expected payoff from holding bad assets is $A\left(r_L - \bar{d} + d + \pi_L\left(\bar{d}\right)\right)$, where $\pi_L\left(\bar{d}\right)$ is the value of the put option on 1 dollar's worth of the bank's assets with strike price $\bar{d}$ when the bank holds bad assets. The incentive compatibility constraint is therefore

$$\begin{aligned} r_H - r_L &\geq \pi_L\left(\bar{d}\right) - \pi_H\left(\bar{d}\right) \\ &= \Delta\pi\left(\bar{d}\right) \end{aligned} \tag{IC}$$

where $\Delta\pi\left(\bar{d}\right)$ is defined as $\pi_L\left(\bar{d}\right) - \pi_H\left(\bar{d}\right)$. The term $\Delta\pi\left(\bar{d}\right)$ is analogous to the private benefit of exerting low effort in the moral hazard model of Holmström and Tirole (1997). The bank's equity holder trades off the greater option value of holding the riskier asset against the higher expected payoff from holding the good asset. The incentive compatibility constraint requires that the option value be small relative to the difference in expected returns.

Note that the IC constraint does not make reference to the market value of debt d, but only to the face value of debt $\bar{d}$. This reflects the fact that the IC constraint is a condition on the strike price of the embedded option. In order to derive the market value of debt (and hence market leverage), we must also use the IR constraint.

Given our assumptions on the densities governing the good and bad securities, we have the following feature of our model.

Lemma 1 *$\Delta\pi(z)$ is a single-peaked function of z, and is maximized at the value of z where F_H cuts F_L from below.*

Proof. From the result in option pricing due to Breeden and Litzenberger (1978), the price of the Arrow-Debreu contingent claim that pays 1 at z and zero otherwise is given by the second derivative of the option price with respect to the strike price evaluated at z. Since both the principal and agent are risk-neutral, the state price is the probability. Thus, we have

$$\Delta\pi(z) = \int_0^z (F_L(s) - F_H(s))\, ds$$

Since F_H cuts F_L precisely once from below, $\Delta\pi(z)$ is increasing initially, is maximized at the point z^* where $F_H = F_L$, and is then decreasing. ■

3.4.1 Leverage Constraint

If the incentive compatibility constraint (IC) does not bind, then the contracting problem is trivial and the first best is attainable. We will focus on the case where the incentive compatibility constraint (IC) binds in the optimal contract. Since the value of the implicit put option held by the equity holder is increasing in the strike price $\bar{d}$, lemma 1 implies that there is an upper bound on the variable $\bar{d}$ for which the incentive constraint is satisfied. This upper bound is given by the smallest solution to the equation:

$$\Delta\pi\left(\bar{d}\right) = r_H - r_L \tag{13}$$

Denote this solution as $\bar{d}^*$. Because the IC constraint is binding, it must be the case that $\Delta\pi\left(\bar{d}^*\right)$ is increasing in $\bar{d}^*$. Again from Lemma 1, it follows that $\bar{d}^* < z^*$. Intuitively, the bank's balance sheet size is constrained by the amount of debt that it is allowed to hold by its lenders.

The quantity $\bar{d}^*$ is expressed in terms of the ratio of the repurchase price in the repo contract to the market value of assets, and so mixes notional and market values. However, we can solve for the pure debt ratio in market values by appealing to the participation constraint. The participation constraint binds in the optimal contract, so that we have:

$$d = \bar{d} - \pi_H\left(\bar{d}\right) \tag{14}$$

We can then solve for the debt to asset ratio d, which gives the ratio of the market value of debt to the market value of assets. Denoting by d^* the debt to asset ratio in the optimal contract, we have

$$d^* = \bar{d}^* - \pi_H\left(\bar{d}^*\right) \tag{15}$$

where $\bar{d}^*$ is the smallest solution to (13). The right hand side of (15) is the payoff of a creditor with a notional claim of $\bar{d}^*$. Hence, we can re-write (15) as

$$d^* = \int_0^{1+\bar{r}} \min\left\{\bar{d}^*, s\right\} f_H(s)\, ds \tag{16}$$

Clearly, d^* is increasing in $\bar{d}^*$, so that the debt ratio in market values is increasing in the notional debt ratio $\bar{d}^*$.

3.5 Balance Sheet Size

Having tied down the bank's leverage through (15), it remains to solve for the size of the bank's balance sheet. To do this, we note from (12) that the bank equity holder's expected payoff under the optimal contract is:

$$U^E(A) \equiv A\left(r_H - \bar{d}^* + d^* + \pi_H\left(\bar{d}^*\right)\right) \tag{17}$$

The expression inside the brackets is strictly positive, since the equity holder extracts the full surplus from a positive net present value relationship. Hence, the equity holder's payoff is strictly increasing in A. The equity holder maximizes the balance sheet size of the bank subject only to the leverage constraint (15). Let λ^* be the upper bound on leverage implied by d^*, defined as

$$\lambda^* \equiv \frac{1}{1-d^*} \tag{18}$$

Then, the bank chooses total balance sheet size given by:

$$A = \lambda^* E \tag{19}$$

We note the contrast between this feature of our model and the textbook discussion that either treats the asset size as fixed, or as evolving exogenously. Instead, in our model, it is *equity* that is the pre-determined variable. For given equity E, total asset size A is determined as $\lambda^* \times E$, where λ^* is the maximum leverage permitted by the creditors in the optimal contract. Thus, as λ^* fluctuates, so will the size of the bank's balance sheet. In the next section, we link λ^* to the Value at Risk, and will see that the model is giving rise to the empirical predictions that we documented in section 2.

Since the agent's payoff is increasing linearly in equity E, a very natural question is why the agent does not bring in more equity into the agency relationship, thereby magnifying the payoffs. This is an important question that deserves greater attention. However, the "pecking order" theories of corporate finance of Myers and Majluf (1984) and Jensen and Meckling (1976) shed some light on why equity may be so "sticky". In

Myers and Majluf (1984), a firm that wishes to expand its balance sheet will first tap its internal funds, and then tap debt financing. Issuing equity is a last resort. The reasoning is that the firm has better information on the value of the growth opportunities of the firm and any attempt to raise new equity financing will encounter a lemons problem. Jensen and Meckling (1976) also predict a pecking order of corporate financing sources for the reason that agency costs associated with the actions of entrenched "inside" equity holders entail a discount when issuing new equity to "outside" equity holders. The stickiness of E is intimately tied to the phenomenon of "slow-moving capital" discussed by He and Krishnamurthy (2007) and Acharya, Shin and Yorulmazer (2010).

3.6 Comparative Statics

We now explore how shifts in the volatility of assets affect the contract. Denote the volatility of assets by σ, and by $\pi_H(z,\sigma)$ the value of the put option (parameterized by σ) on one dollar's worth of the bank's assets with strike price z when the bank's assets are good. $\pi_L(z,\sigma)$ is defined analogously when the assets are bad. Both π_H and π_L are increasing in σ, since the value of the equity owner's put option is increasing in the volatility of the payoffs. We then have the following comparative statics result.

Proposition 1 *If $\Delta\pi(z,\sigma)$ is increasing in σ, then both $\bar{d}^*$ and d^* are decreasing in σ.*

We draw on two ingredients for the proof of this proposition. First, we use the binding IC constraint (IC). Second, we draw on the supposition that $\Delta\pi(z,\sigma)$ is increasing in σ. From the IC constraint, we have

$$\Delta\pi\left(\bar{d}^*(\sigma),\sigma\right) = r_H - r_L \tag{20}$$

where $\bar{d}^*(\sigma)$ is the value of $\bar{d}^*$ as a function of σ. The left hand side of (20) is increasing in $\bar{d}^*$ by Lemma 1 and by the assumption of a binding IC constraint, which implies $\bar{d}^* < z^*$. Since by assumption the left hand side of (20) is increasing in σ, it follows that $\bar{d}^*(\sigma)$ is a decreasing function of σ. Intuitively, the assumption that $\Delta\pi(z,\sigma)$ is increasing in σ amounts to saying that the benefit from moral hazard is increasing in the variance of payoffs. The finding that $\bar{d}^*$ is decreasing in σ means that the bank's constraint on leverage tightens with the riskiness of total assets.

To show that the market debt ratio d^* is decreasing in σ, we appeal to the participation constraint of the principal and the fact that the option value π_H is increasing in σ. From the participation constraint, we have

$$d^* = \bar{d}^* - \pi_H\left(\bar{d}^*\right) = \int_0^{1+r_H} \min\left\{\bar{d}^*, s\right\} f_H(s)\, ds \tag{21}$$

Since $\bar{d}^*$ is decreasing in σ, so must d^* be decreasing in σ. This proves our result.

4 Value-at-Risk

We come to our core result. For a random variable W, the *Value-at-Risk* at confidence level c relative to some base level W_0 is defined as the smallest non-negative number V such that

$$\text{Prob}\left(W < W_0 - V\right) \leq 1 - c$$

In our context, W is the realized asset value of the bank at date 1. Then the Value-at-Risk is the amount of equity capital that the bank must hold in order to stay solvent with probability c.

We now turn to the risk environment. Consider the generalized extreme value distribution, which has the cumulative distribution function:

$$G(z) = \exp\left\{-\left(1 + \xi\left(\frac{z-\theta}{\sigma}\right)\right)^{-1/\xi}\right\} \tag{22}$$

The parameter ξ can take any real number value, and the support depends on the sign of ξ. When ξ is negative, the support of the distribution is $(-\infty, \theta - \sigma/\xi)$. The general extreme value distribution has received considerable attention due to its central role in the definition of order statistics and in describing extreme outcomes. In particular, the extreme value limit theorem of Gnedenko (1948) states that the extreme values of observations $z_1, z_2, ...$ have a probability limit of the form (22). Since Value-at-Risk is inherently concerned with events in the tail of the asset distribution, the family of distributions in (22) is a natural setting for the problem we are examining. We will consider the special case of (22) where $\xi = -1$, and where we index the risk environment by means of the parameter θ.

Introduce the family of functions $\{G_L, G_H\}_\theta$ parametrized by θ where

$$G_L(z;\theta) = \exp\left\{\frac{z-\theta}{\sigma}\right\} \quad \text{and} \quad G_H(z;\theta) = \exp\left\{\frac{z-k-\theta}{\sigma}\right\} \tag{23}$$

and where k is a positive constant. We examine the case where the cdf of the risk environment have tails that are exponential in the following sense.

Condition 2 *There is $\hat{z}$ such that for all $z \in (0, \hat{z})$, we have*

$$F_L(z;\theta) = G_L(z;\theta) \text{ and } F_H(z;\theta) = G_H(z;\theta) \tag{24}$$

When $z = 0$, we have

$$F_L(0;\theta) = \int_{-\infty}^{0} G_L(s;\theta)\, ds \text{ and } F_H(0;\theta) = \int_{-\infty}^{0} G_H(s;\theta)\, ds \tag{25}$$

Let $\bar{d}^*(\theta)$ be the value of $\bar{d}^*$ in the contracting problem parameterized by θ. We then have the following feature of the optimal contract that can be characterized in terms of Value-at-Risk.

Proposition 3 *For all $\theta \in [\underline{\theta}, \bar{\theta}]$ suppose that $\bar{d}^*(\theta) < \hat{z}$. Suppose also that $r_H - r_L$ stays constant to shifts in θ. Finally, suppose that condition 2 holds. Then the probability that the bank defaults is constant over all optimal contracts parameterized by $\theta \in [\underline{\theta}, \bar{\theta}]$.*

Corollary 1 *Under the conditions of Proposition 3, the bank's Value-at-Risk is equal to its equity in the optimal contract at all $\theta \in [\underline{\theta}, \bar{\theta}]$.*

Proposition 3 and Corollary 1 are equivalent statements that are mirror images of the same feature of the optimal contract. As θ varies over the interval $[\underline{\theta}, \bar{\theta}]$, the bank will adjust the size of its balance sheet for given equity so that its Value-at-Risk is kept equal to its equity. The bank sheds assets when the environment becomes riskier and loads up on assets when the environment becomes more benign. For given equity, leverage is fully determined by the *unit* Value-at-Risk, where the unit VaR is defined as the Value-at-Risk per dollar of assets. The empirical predictions of Corollary 1 are very stark. The prediction is that the ratio of the bank's dollar Value-at-Risk to its equity is constant. This is precisely the evidence that we presented in section 2.

We prove proposition 3. As a first step, note first from (23) that for all $z \in (0, \hat{z})$,

$$\frac{F_L(z;\theta)}{F_H(z;\theta)} = \frac{G_L(z;\theta)}{G_H(z;\theta)} = e^{k/\sigma} > 1 \tag{26}$$

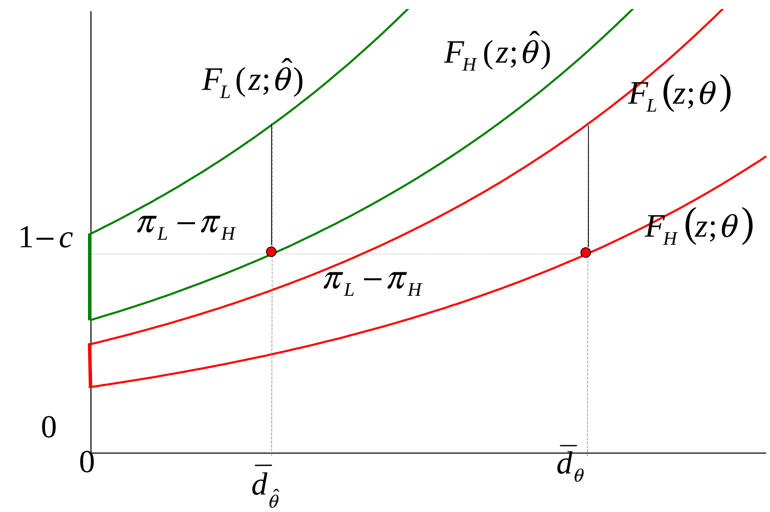

Figure 8: Value at Risk

Hence, from condition 2, we have

$$\begin{aligned} \Delta\pi(z;\theta) &= \int_0^z (F_L(s;\theta) - F_H(s;\theta))\, ds \\ &= \int_{-\infty}^z (G_L(s;\theta) - G_H(s;\theta))\, ds \qquad (27) \\ &= \left(e^{k/\sigma} - 1\right) \int_{-\infty}^z G_H(s;\theta)\, ds \qquad (28) \\ &= \sigma\left(e^{k/\sigma} - 1\right) G_H(z;\theta) \qquad (29) \end{aligned}$$

From the IC constraint, we have $\Delta\pi\left(\bar{d}^*;\theta\right) = r_H - r_L$, so that for all $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$, we have

$$\left(e^k - 1\right) G_H\left(\bar{d}^*;\theta\right) = r_H - r_L \qquad (30)$$

Therefore, from (28) and (30), we have that at every optimal contract $\bar{d}^*(\theta)$, the probability that the bank defaults is

$$G_H\left(\bar{d}^*;\theta\right) = \frac{r_H - r_L}{\sigma\left(e^{k/\sigma} - 1\right)} \qquad (31)$$

which is constant. As θ varies, the bank keeps just enough equity to meet its Value-at-Risk at a constant confidence level.

Figure 8 illustrates the case of two values of θ, with $\hat{\theta} > \theta$ where the probability of default is kept at $1-c$. In our case, the right hand side of (31) is the probability of default. Hence the probability of default is

$$1 - c = \frac{r_H - r_L}{\sigma \left(e^{k/\sigma} - 1\right)} \tag{32}$$

Our result can be given the following intuitive interpretation. The temptation payoff in the moral hazard problem is the higher option value from the riskier decision, which is given by

$$\Delta\pi\left(z;\theta\right) = \int_0^z \left(F_L\left(s;\theta\right) - F_H\left(s;\theta\right)\right) ds = (1-c)\,\sigma\left(e^{k/\sigma} - 1\right) \tag{33}$$

The exponential form of the extreme value distribution means that this temptation payoff can be written as a constant times the underlying risks (as given by equation (28)). In effect, the moral hazard increases in proportion to the underlying riskiness of the environment. The solution to the contracting problem thus stipulates maintaining sufficient equity to counteract this temptation, leading to a constant probability of default for the bank.

Technically, we see that there are two important features of the exponential form of the extreme value distribution that drive our result. First, the exponential functional form implies that the relative size of the tails associated with the good action and temptation action remains constant to shifts in the fundamental parameter θ. In other words, the ratio $F_L\left(z;\theta\right)/F_H\left(z;\theta\right)$ remains constant as θ shifts around. We see this in equation (26). Second, the exponential functional form implies that the *integral* of the cdf is the cdf itself. We see that in equation (28). We should note that the *VaR* rule also results when the coefficient ξ in the GEV distribution (22) is different from -1. However, the calculation in that case is a lot more involved, and we have only been able to show this case via simulation, not analytically.

The value at risk rule implies that the notional debt to asset ratio is a function of the state variable θ

$$\bar{d}^* = \sigma \ln\left(\frac{r_H - r_L}{\sigma\left(e^{k/\sigma} - 1\right)}\right) + k + \theta. \tag{34}$$

Using (16) which relates d^* to $\bar{d}^*$, we can show that leverage is a function of the state variable θ:

$$\frac{A}{E} = \frac{1}{1 - d^*\left(\theta\right)} \tag{35}$$

Our result implies that when overall risk in the financial system increases after a shock (e.g. a change in θ), the bank must cut its asset exposure (through deleveraging) to maintain the same probability of default to additional shocks as it did before the arrival of the shock. This is precisely the evidence that we presented in section 2 to motivate the model. When risk shoots up during the financial crisis (as measured by unit VaR or implied volatilities), banks react by deleveraging in order to maintain a constant VaR to equity ratio.

5 Concluding Remarks

In this paper, we have employed perhaps the simplest contracting model for the determination of leverage and balance sheet size for financial intermediaries, and have examined the conditions under which the Value-at-Risk rule emerges from the contracting outcome. Our framework provides one possible microeconomic foundation for the widespread use of the Value-at-Risk rule among financial institutions. Our setup sheds light on the extent to which leverage decisions are the constraints that *creditors* impose on *debtors*.

To be sure, showing that the *VaR* rule is the outcome of a contracting model says little about the desirability of the widespread adoption of such practices from the point of view of macroeconomic efficiency. Indeed, risk management tools such as Value-at-Risk that solve bilateral incentive frictions can generate spillover effects across financial institutions. The leveraging and deleveraging cycle and associated fluctuations in market risk premiums are likely to be influenced by the widespread adoption of risk management rules (Shin (2010)).

In a system context, fluctuations in leverage have far-reaching effects. To the extent that the financial system as a whole holds long-term, illiquid assets financed by short-term liabilities, any tensions resulting from a sharp increase in risk will show up somewhere in the system. Even if some institutions can adjust down their balance sheets flexibly in response to the greater stress, there will be some pinch points in the system that will be exposed by the distressed conditions. In effect, a generalized fall in the permitted leverage in the financial system can lead to a "run" on a particular institution that has funded long-lived illiquid assets by borrowing short. Developments of our techniques may be useful in richer setting with more complex intermediation relationships.

A Appendix

A.1 Probability of default for a log investor

Denote the conditional mean of continuously compounded excess returns to assets by μ, and their conditional volatility by σ. When bank managers have log preferences, leverage is

$$lev = \frac{\mu}{\sigma^2}.$$

The return to assets R_A is assumed to be conditionally Gaussian

$$R_A \sim N\left(\mu, \sigma^2\right).$$

The equity return is then $R_E = lev \cdot R_A$. The VaR of the bank is defined as

$$1 - c = \Pr\left(R_E < VaR\right)$$

Using the expressions from above

$$\begin{aligned}
1 - c &= \Pr\left(R_E < VaR\right) \\
&= \Pr\left(lev \cdot R_A < VaR\right) \\
&= \Pr\left(\frac{\mu}{\sigma^2} \cdot R_A < VaR\right) \\
&= \Pr\left(R_A < VaR\frac{\sigma^2}{\mu}\right) \\
&= \Pr\left(\frac{R_A - \mu}{\sigma} < \frac{VaR\frac{\sigma^2}{\mu} - \mu}{\sigma}\right) \\
&= N\left(\frac{VaR\frac{\sigma^2}{\mu} - \mu}{\sigma}\right) \\
&= N\left(VaR\frac{\sigma}{\mu} - \frac{\mu}{\sigma}\right).
\end{aligned}$$

In the computation of Figure 6, we assume that $VaR = -1\%$.

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