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Rating Systems and Procyclicality: an Evaluation in a DSGE Framework

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Abstract

This paper introduces a DSGE model that focuses on the modelling of endogenous default and a regulated, active banking sector. One purpose of the paper is to study the impacts of shocks on an economy with financial frictions. The main financial friction comes from the combination of default and bank capital requirement. We focus on studying banks' responses in terms of portfolio decisions, capital buffer, dividend distributions and debt repayment to their creditors. Another purpose of the paper is to study different impacts of the point-in-time regime and the through-the-cycle regime on the economy. The results point to the conclusion that the preference of banks for certain regimes depends on the type of shocks.

Key Words and Phrases: Procyclicality, Rating Systems, Basel, DSGE

JEL Classification Numbers: D58, E44, G21, G38

1 Introduction

This paper has two agendas. Firstly, it builds a DSGE model, with strong emphasis on modelling a regulated, active banking sector and endogenous default by agents to study the responses to different types of shocks in the presence of financial frictions. Secondly, the impacts of Basel Internal-Rating-Based regimes on the economy as well as the preference of banks for certain regimes is explored.

For the first agenda, three types of shocks are introduced: productivity, monetary and financial shocks. We see that the financial frictions amplify the effect of the productivity shock by making credit more costly and scarce. Meanwhile, monetary shock, which is modelled as an upward adjustment in interest rate, and financial shock, which is modelled as a one-off capital loss by banks, do have a real effect on the economy.

We focus on studying banks' behaviors to explore the banking channel. There are three main findings. We find that, although banks' response to adverse shocks by tightening credit conditions does amplify the economic downturn, banks also actively absorb the impact of shocks by constraining their own consumption and paying back less to their debtors even though they suffer higher default penalties by defaulting more. These imply that the impacts of adverse shocks would have been larger in case banks were reluctant to have their owners and creditors sharing the risk with the real sector.

We also find that, banks' portfolio decisions depend on the types of shock. When a productivity or a monetary shock is realised, banks tend to reconstruct their portfolio towards the consumer

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credit, while when a capital shock occurs, banks shift their portfolio towards the corporate loans. The portfolio reconstruction by banks propagates the shocks and amplifies the impacts.

Finally, we find that bank capital buffer tends to increase following an adverse productivity or monetary shock. The deleverage is mainly because bank creditors withdraw funding following the shocks. The countercyclical nature of bank capital buffer is consistent with empirical observations. These findings are relevant for assessing the effectiveness of the countercyclical capital buffer proposal as a way to dampen the banking procyclicality.

Regarding the Internal-Rating-Based regimes, we show that, compared with the through-the-cycle regime, the point-in-time regime amplifies the impact of all types of shocks by more and that it always leads to lower capital ratios for banks. Moreover, the preference of banks for the two regimes depends on the shock that occurs. The findings point to the conclusion that, when banks are affected in the second-order sense, they prefer the through-the-cycle regime whereby they help to absorb the impact of the shock. In the case that banks are directly affected by a monetary or a financial shock, they prefer the point-in-time scheme which allows banks to actively reconstruct their portfolio and passes some of the impact from the shocks on to other agents.

As there is a substantial body of work in this area, to contribute to the advancement of the research agenda, we consider it helpful to first clarify our approaches and assumptions. In the following, we discuss the main model ingredients, particularly the types of frictions. The frictions come from two sources: 1) the combination of default and capital requirement; 2) the price of liquidity.

The main friction comes from the combination of default and bank capital requirement. Default by borrowers results in loss-making for the banks. Upon the occurence of a loss, a depleted capital position will be undesirable for banks. As part of the actions to regain adequate capital positions, banks limit their credit extensions. This prompts a credit crunch and may result in a further round of default and lower output. Moreover, the Internal-Rating-Based regime may encourage banks to actively reconstruct their portfolio. If banks adopt the point-in-time regime, the capital requirement will be adjusted upward for riskier loans to pre-fund the unexpected loss. Banks then follow this to shrinking their balance sheet and shifting to other types of asset. Correspondingly, liquidity becomes more scarce, particularly for the sector that needs it the most.

We hasten to add, however, that in our view, default by itself does not amplify the business cycle. On the contrary, it could be considered as a "financial decelerator". It is essentially a loss-sharing or an insurance scheme when the bad state turns out. When time is buoyant borrowers tend to pay back more; thereby the upturn in consumption is limited. Thus default by itself acts to smooth impacts of shocks on consumption and welfare. In our view, default shall create frictions in the following context. When combined with the capital requirement, as discussed above, it leads to procyclical credit extensions and countercyclical risk premium that amplify the downturns. Besides, default leads to another inefficiency when creditors cannot differentiate between the credit-worthy or risky borrowers. Thus default by one agent imposes an externality on other agents. Such a mechanism is studied in Lin et al. [2010].

In this paper, default is modelled following Dubey et al. [2005], whereby borrowers default partially on the loans and suffer default penalties accordingly. They choose endogenous default rates that are optimal for them. Alternative ways for modelling the credit-risk friction include the collateral approach and the financial accelerator approach. Following the seminal paper by Kiyotaki and Moore [1997], a stream of literature assumes that borrowers are required to pledge a sufficient amount of collateral such that under no circumstances in the future will the borrowers have the incentive to default. To the extent that collateral is scarce, the cost of pledging the secured asset leads to inefficiency. An alternative approach, following Bernanke et al. [1999], assumes the existence of idiosyncratic default but banks are able to hedge credit risk on an aggregate level. Such an approach again leads to an external financing premium and suboptimal results.

We follow Dubey et al. [2005] because, among the three modelling approaches, this is the only approach where default is in equilibrium and that default is costly to the lenders. It does not assume away the existence of *ex post* default as in Kiyotaki and Moore [1997], nor does it assume that the creditor could hedge risk and therefore would not lose. These properties are both consistent with observations in the practices and essential for our purpose. One important feature of the 2007/08 crisis is the link between banks' loss-making from mortgage default and economic downturn. Modelling banks' losses due to default is therefore essential to capture the linkage between the financial sector and the real economy. Moreover, explicit presentations of default rates are needed because we are studying the Basel Internal-Rating-Based regime, where banks' capital requirement and risk weights on loans are functions of the default rates. The alternative modelling approaches are not appropriate for our purpose here.

More specifically, we model two types of default penalties: non-pecuniary default penalties and pecuniaries ones. Non-pecuniary penalties aim to capture borrowers' loss of reputation due to default while the pecuniaries penalties represent search costs for new loans after default. The specific functional forms of default penalties in a dynamic (infinite-horizon) model are discussed. This complements the work of Dubey et al. [2005] and Dubey and Geanakoplos [1992] as they consider default penalties in two-period models. We compare three typical functions (linear, log and quadratic forms) and conclude that the quadratic form is the most appropriate, since it allows for both time-varying consumption and procyclical repayment rates (or equivalently, countercyclical risk premium).

Bank capital ratio is formulated following the Basel risk-weighted average style. The risk weight is a function of the expected default probability of a loan. The risk weight function is constructed to approximate the real Basel risk curve with high goodness-of-fit. The introduction of both endogenous default rates and a realistic risk weight function are necessary to study the impacts of the Basel Internal-Rating-Based regime. To motivate banks' capital accumulation, in the model, banks receive utility from a capital buffer, which is defined as the difference between the effective capital ratio and the regulatory minimum capital requirement. Alternative formalisations appear in Goodhart et al. [2006] and Gerali et al. [2010], where banks suffer quadratic penalties or adjustment cost if the capital ratio deviates from a target. Our modelling approach is simpler but more intuitive. The concave utility from the capital buffer could be considered as positive reputational gains as compared with the reputation losses due to default. This set-up naturally allows for the introduction of a countercyclical capital buffer requirement, which is our next research agenda.

The second type of friction comes from the positive price of liquidity or money. By assuming cash-in-advance and letting money or liquidity have a role in transaction purposes, the price and quantity of liquidity matter for trade decisions. Other alternatives to modelling money include introducing money into the utility function or assuming nominal rigidity and sticky prices. Our preference for the cash-in-advance assumption not only comes from the interest to introduce the role of liquidity but is also based on a theoretical argument, which states that cash-in-advance and default should be studied together. In the case that cash-in-advance was not required for transactions, then everyone's I-owe-U would be sufficient for payment. However, the basic assumption behind the existence of default is that the I-owe-U is not acceptable for most transactions. By studying default one implicitly assumes the necessity of cash-in-advance(See Goodhart and Tsomocos [2011] for more discussions on the interactions among cash-in-advance, liquidity and default.). The cash-in-advance modelling is on the same lines as the working capital mechanism adopted by recent macro-financial linkage literature such as that of Jermann and Quadrini [2012].

Our research is related to the stream of DSGE literature which has a similar agenda. Regarding the study of the interactions between the banking sector and the rest of the economy, the paper that is the closest to ours is that of De Walque et al. [2010]. They build a DSGE model with endogenous default probabilities and heterogeneous banking sectors. However, our research differs on three main points. Firstly, we model money and liquidity explicitly while they adopt a real model. Secondly, we focus on the Basel internal-rating-based regime while their emphasis is on the importance of supervisory and monetary policy actions in restoring financial stability. Finally, we view default as a "financial decelerator" by itself and point out that only when combined with capital requirement does default become a friction that would amplify the adverse shocks. Dib [2010] also includes endogenous probabilities of default into the model but his focus is on the impact of market frictions on the business cycle fluctuations. Regarding the financial shock, Iacoviello [2011] studies the impact of a financial shock which is also modelled as a capital loss by banks. The magnitude of impacts due to the capital shock is larger in our model. However, the difference may lie in his modelling of a "re-distributional shock" while ours is a one-side loss by banks.

Regarding the study of the procyclicality issues from risk-sensitive capital requirement, the two papers that are closest to our approach are those of Covas and Fujita [2010] and Pariès et al. [2011]. Covas and Fujita [2010] compare the output fluctuations under the Basel I and Basel II economies with those in the non-requirement economy. To model the time-varying capital requirement they assume that capital requirement is a log-linear function of the total factor productivity. Pariès et al. [2011] also introduce risk-sensitive capital requirement into the DSGE model. In their case, the Basel risk curve is a function of the leverage ratio and is approximated around the steady state level of the leverage ratio. Our approximation of the Basel risk curve has high goodness-of-fit and is non-linear. Therefore we believe that our approach helps move these experiments closer to actual practice.

Our findings on the procyclicality of the point-in-time regime is consistent with empirical findings, such as Lowe and Segoviano Basurto [2002] and Kashyap and Stein [2004]. However, as commented by Drumond [2009], empirical research, given the lack of data, typically simulates the capital charge cyclicality during a given period of time by assuming that the Basell II rules had been in use. However, the implementation of Basel II started in EU from 2008 and in US only after 2009. Our approach to study the potential impacts of Basel rules account for banks' endogenous portfolio decisions given the new rules and thus avoid the "Lucas Critique".

Our work on predicting banks' preference for different regimes and the associated welfare implications is closely related to Catarineu-Rabell et al. [2005] and Pederzoli et al. [2010]. Catarineu-Rabell et al. [2005] study the procyclicality issues raised by the risk-based Basel capital requirement. They conclude that banks achieve the highest profit under the countercyclical rating schemes. Moreover, banks prefer the procyclical system to the neutral scheme, which results in the lowest profit for the bank. Since regulators are cautious about proposing the countercyclical regime, they predict that banks would follow a procyclical approach in their internal rating. Pederzoli et al. [2010] study the point-in-time and through-the-cycle rating schemes in a two-period model including heterogeneous banks, which allows for the assessment of the contagion issue.

The remainder of the paper proceeds as follows. Section 2 introduces the broad features of the model and the optimisation problems. We also discuss the efficiency gain of default, the endogenous interest rate margin and the modelling of capital requirement in this section. Section 3 calibrates the model and solves for the steady state. Section 4 discusses the responses to different types of shocks. Section 5 studies the impact of the Internal-Rating-Based regimes and explores the preference of banks for different schemes. Section 6 concludes.

2 The baseline model

The economy is populated by four types of agents: deposit households, borrowing households, firms and commercial banks. These are the minimum characteristics for the purpose of this paper. Since we aim to study banks' behaviour as well as the impacts of banking regulation on the real economy, we model banks explicitly. We separate the economic agents into two groups: deposit households as suppliers of fund (creditors) and borrowers consisting of borrowing households and firms. Two types of borrowers are introduced to allow for the modelling of two types of loans (consumer credit/retail loans and commercial loans). The two types of loans provide diversification opportunities for banks. Finally, we have an authority that is a combination of central bank, regulators and government which sets the interest rate, governs banks' capital adequacy and transfers the seniorage revenue to the household sector. We assume that monetary policy is operated through the discount window where banks access to funding from the central bank. An overview of the markets and participants is given in Figure 1.

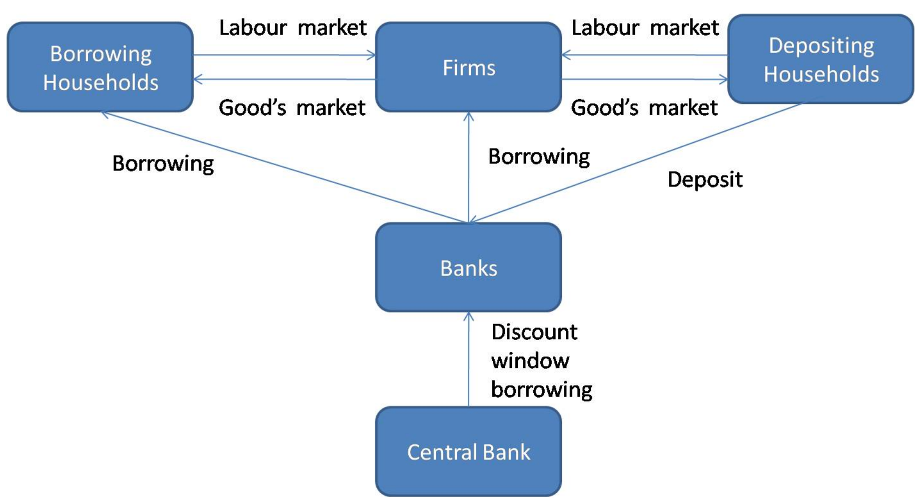


Figure 1: The model

2.1 Banks

Banks assume deposit from rich households $\frac{\mu_t^D}{1+r_t^D}$ and borrow from the central bank $\frac{\mu_t^I}{1+r_t^I}$, where r_t^D and r_t^I are the corresponding interest rates. We allow for the possibility of default on the central bank loans while we assume away the chance of default on deposit. Default on deposit has become rare due to the introduction of the deposit protection scheme. Banks may partially default on the central bank loans taken in the previous period $\frac{\mu_{t-1}^I}{\Pi_t}$ by repaying v_t^I proportion of the loan.

Consequently they suffer a disutility $U_1^D\left((1-v_t^I)\frac{\mu_{t-1}^I}{\Pi_t}\right)$, where $U_1^D()$ is a function of the amount defaulted $\left((1-v_t^I)\frac{\mu_{t-1}^I}{\Pi_t}\right)$. This could be considered as a positive reputation cost. Meanwhile, they pay a pecuniary cost $U_2^D\left((1-v_t^I)\frac{\mu_{t-1}^I}{\Pi_t}\right)$. The pecuniary cost could be considered as the search cost for additional funding due to the credit record downgrade. We assume that access remains to new financing even though banks default partially on their obligations. The same modelling approach applies for the default by borrowing households and firms. Alternatively, we may model equilibrium default as in Chatterjee et al. [2007] where defaulting agents are put into financial autarky. However, this implies that, once in default, the entire sector loses funding opportunities completely. Instead, our approach allows for partial default. We view loans to a certain type of borrowers as a pool with time-varying default rates across periods. Discussions on the functional forms of default penalties can be found in subsection 3.1.1. On the asset side, banks lend to firms m_t^F and borrowing households m_t^P. The loans are risky with repayment rates $R_t^F \leq 1$ and $R_t^P \leq 1$ on the commercial loans and the consumer loans respectively.

We denote the equity of banks by E_t, the capital adequacy ratio by k_t and the regulatory minimum capital requirement by k^0. Banks enjoy utility from the capital buffer, which is the difference between banks' capital ratio and the minimum regulatory requirement $(k_t - k^o)$. The utility from capital buffer could be considered as a positive reputation cost. The capital ratio is a risk-weighted one calculated following the Basel II style.

Banks maximise expected utility from consumption $U^{I,C}\left(c_t^I\right)$[1] plus the utility from the capital buffer $U^K\left(k_t - k^o\right)$ and net of the disutility due to the non-pecuniary default penalty $U_1^{I,D}\left((1-v_t^I)\frac{\mu_{t-1}^I}{\Pi_t}\right)$. β_I is the time discount rate for banks. We introduce the forms of utility functions in the calibration subsection (3.1.2)

$$\underset{\{c_t^I, m_t^F, m_t^P, \mu_t^D, \mu_t^I, v_t^I\}}{\text{Max}} E_0 \sum_{t=1}^{\infty} \beta_I^t \left(U^{I,C}\left(c_t^I\right) + U^K\left(k_t - k^o\right) - U_1^{I,D}\left((1-v_t^I)\frac{\mu_{t-1}^I}{\Pi_t}\right)\right)$$

Banks are subject to the following flow of fund constraint (see equation (1)). Banks' cash outflows consist of purchase for consumption, loan extensions to firms and to household borrowers, repayments to the central bank and to household depositors, and search cost due to defaulting. The banks' cash inflows are deposits from households and borrowing from the central bank, as well as repayments from firms and household borrowers.

$$\begin{aligned} & c_t^I + m_t^F + m_t^P + v_t^I\frac{\mu_{t-1}^I}{\Pi_t} + \frac{\mu_{t-1}^D}{\Pi_t} + U_2^{I,D}\left((1-v_t^I)\frac{\mu_{t-1}^I}{\Pi_t}\right) \\ \leq\ & \frac{\mu_t^D}{1+r_t^D} + \frac{\mu_t^I}{1+r_t^I} + R_t^F\frac{m_{t-1}^F}{\Pi_t}(1+r_{t-1}^F) + R_t^P\frac{m_{t-1}^P}{\Pi_t}(1+r_{t-1}^P) \quad (\lambda_t^I) \end{aligned} \tag{1}$$

Equation (2) defines banks' equity. At the beginning of period t the equity of banks equals the equity in the previous period plus profit/loss made during the past period net of the current consumption. The profit/loss from business operation is the income from loan extension net of the cost of borrowing and the pecuniary cost due to default.

[1] We consider banks as a group of professional bankers who own and operate banks and that bankers are not involved in other types of work. Therefore, consumption could be considered as dividend distributions to bank investors.

$$
\begin{aligned}
E_t = {} & \frac{E_{t-1}}{\Pi_t} - c_t^I + \frac{m_{t-1}^F}{\Pi_t}\left(R_t^F\left(1+r_{t-1}^F\right)-1\right) + \frac{m_{t-1}^P}{\Pi_t}\left(R_t^P\left(1+r_{t-1}^P\right)-1\right) \\
& - \frac{\mu_{t-1}^I}{\Pi_t\left(1+r_{t-1}^I\right)}\left(v_t^I\left(1+r_{t-1}^I\right)-1\right) - \frac{\mu_{t-1}^D}{\Pi_t\left(1+r_{t-1}^D\right)} r_{t-1}^D - U_2^{I,D}\left(\left(1-v_t^I\right)\frac{\mu_{t-1}^I}{\Pi_t}\right)
\end{aligned}
\tag{2}
$$

From equation (2), when the default rate increases, the income of banks $\frac{m_{t-1}^F}{\Pi_t}\left(R_t^F\left(1+r_{t-1}^F\right)-1\right)$ $+ \frac{m_{t-1}^P}{\Pi_t}\left(R_t^P\left(1+r_{t-1}^P\right)-1\right)$ will be lower, which puts a downward pressure on banks' equity E_t and capital ratio k_t. If lower capital adequacy ratios are undesirable, banks may react by repaying less to the central bank (i.e. v_t^I goes down), lowering its dividend distribution or consumption (i.e. c_t^I goes down) or shrinking the balance sheet (i.e. the new loan extensions m_t^F and m_t^P are smaller). In the results section we shall see these impacts. Lower new loans availability may lead to further default by the households and firms. Lower repayment to the central bank reduces government revenue, which is then transferred to the household sectors. Therefore, equation (2) presents the possible contagion and propagation effect caused by banks' desire to maintain the levels of capital ratios after loss-making.
Alternatively, banks' equity can be specified as in the following equation, which is a direct reading from the balance sheet (Figure 2). One could show that it is consistent with equation (2) . In this paper we follow equation (2) since it allows for intuitive interpretations of the optimality conditions.

$$
E_t = m_t^F + m_t^P - \frac{\mu_t^I}{1+r_t^I} - \frac{\mu_t^D}{1+r_t^D}
$$

Commercial loan	m_t^F	Deposit	$\frac{\mu_t^D}{1+r_t^D}$
Consumer loan	m_t^P	Borrowing from the central bank	$\frac{\mu_t^I}{1+r_t^I}$
		Equity	E_t

Figure 2: Balance sheet of the bank

Banks are subject to capital requirement as in the Basel style. The capital ratio formula is defined as the ratio of equity to the risk-weighted asset. w_t^F and w_t^P are risk weights for exposures towards the corporate and the household loans respectively. They are functions of the repayment rate. In subsection 2.2, we discuss the modelling of capital requirement and rating schemes in detail.

$$
k_t = \frac{E_t}{\left(w_t^F m_t^F + w_t^P m_t^P\right)} = \frac{E_t}{RWA_t} \tag{3}
$$

The optimality conditions with respect to the loan extensions m_t^F and m_t^P, the obligations μ_t^D and μ_t^I, the repayment rate v_t^I and consumption c_t^I are given by the following equations in order. The impact of default and the capital adequacy constraint on loan margins is reflected in the first-order conditions. We shall interpret these conditions and illustrate why different classes of assets have different returns. Banks' default decisions will also be discussed using the optimality conditions.

$$-\lambda_t^I-\frac{\partial U^K}{\partial k_t}\frac{w_t^F k_t}{RWA_t}+\beta_I E_t\left[\frac{\lambda_{t+1}^I R_{t+1}^F\left(1+r_t^F\right)}{\Pi_{t+1}}\right]+\beta_I E_t\left[\frac{\partial U^K}{\partial k_{t+1}}\frac{1}{RWA_{t+1}}\frac{1}{\Pi_{t+1}}\left(R_{t+1}^F\left(1+r_t^F\right)-1\right)\right]=0 \quad (4)$$

$$-\lambda_t^I-\frac{\partial U^K}{\partial k_t}\frac{w_t^P k_t}{RWA_t}+\beta_I E_t\left[\frac{\lambda_{t+1}^I R_{t+1}^P\left(1+r_t^P\right)}{\Pi_{t+1}}\right]+\beta_I E_t\left[\frac{\partial U^K}{\partial k_{t+1}}\frac{1}{RWA_{t+1}}\frac{1}{\Pi_{t+1}}\left(R_{t+1}^P\left(1+r_t^P\right)-1\right)\right]=0 \quad (5)$$

$$\lambda_t^I-\beta_I E_t\left[\frac{\lambda_{t+1}^I}{\Pi_{t+1}}\left(1+r_t^D\right)\right]-\beta_I E_t\left[\frac{\partial U^K}{\partial k_{t+1}}\frac{1}{RWA_{t+1}}\frac{1}{\Pi_{t+1}}r_t^D\right]=0 \quad (6)$$

$$\lambda_t^I-\beta_I E_t\left[\frac{\lambda_{t+1}^I}{\Pi_{t+1}}\left(1+r_t^I\right)\right]-\beta_I E_t\left[\frac{\partial U^K}{\partial k_{t+1}}\frac{1}{RWA_{t+1}}\frac{1}{\Pi_{t+1}}r_t^I\right]=0 \quad (7)$$

$$\lambda_t^I+\frac{\partial U^K}{\partial k_t}\frac{1}{RWA_t}-\frac{\partial U_1^{I,D}}{\partial\left(\left(1-v_t^I\right)\frac{\mu_{t-1}^I}{\Pi_t}\right)}-\lambda_t^I\frac{\partial U_2^{I,D}}{\partial\left(\left(1-v_t^I\right)\frac{\mu_{t-1}^I}{\Pi_t}\right)}-\frac{\partial U^K}{\partial k_t}\frac{1}{RWA_t}\frac{\partial U_2^D}{\partial\left(\left(1-v_t^I\right)\frac{\mu_{t-1}^I}{\Pi_t}\right)}=0 \quad (8)$$

$$\frac{\partial U^{I,C}}{\partial c_t^I}-\lambda_t^I-\frac{\partial U^K}{\partial k_t}\frac{1}{RWA_t}=0 \quad (9)$$

Conditions (4) and (5) relate to banks' decision on loan extensions. Take equation (4) for instance. Banks may extend one unit of loan to firms. As a results, the tightened flow of fund constraint lowers banks' utility by λ_t^I, which is the corresponding Lagrangian multiplier. Moreover, the loan extension expands the risk-weighted asset and pushes down the capital ratio. The corresponding reputation cost due to the lower capital ratio is $\frac{\partial U^K}{\partial k_t}\frac{w_t^F k_t}{RWA_t}$. In the next period, firms repay the loans. The cash inflow softens the flow of fund constraint and banks' utility rises by $\beta_I E_t\left[\frac{\lambda_{t+1}^I R_{t+1}^F(1+r_t^F)}{\Pi_{t+1}}\right]$. Meanwhile, banks profit from the loan extension. The additional profit $E_t\left[\frac{1}{\Pi_{t+1}}\left(R_{t+1}^F\left(1+r_t^F\right)-1\right)\right]$ adds to banks' equity. This leads to higher capital ratio and the additional reputation benefit for banks is measured by $\beta_I E_t\left[\frac{\partial U^K}{\partial k_{t+1}}\frac{1}{RWA_{t+1}}\frac{1}{\Pi_{t+1}}\left(R_{t+1}^F\left(1+r_t^F\right)-1\right)\right]$. Similar interpretation follows for condition (5).

Conditions (6) and (7) are related to banks' financing decisions. Condition (6) says that when banks assume one unit of deposit from households, the cash inflow adds to banks' utility by λ_t^I . In the next period, banks repay the depositor. The cash outflows reduce banks' utility by $\beta_I E_t\left[\frac{\lambda_{t+1}^I}{\Pi_{t+1}}\left(1+r_t^D\right)\right]$. Meanwhile, banks pay the funding cost r_t^D and their equity level (and capital ratio) is reduced. The corresponding utility loss is $\beta_I E_t\left[\frac{\partial U^K}{\partial k_{t+1}}\frac{1}{RWA_{t+1}}\frac{1}{\Pi_{t+1}}r_t^D\right]$. The same interpretation follows for condition (7), which is the first-order condition with respect to the borrowing from the central bank.

Condition (8) is related to the banks' default decision. By repaying one unit less, banks retain their cash flow as well as equity level by one unit, which together increase utility by $\lambda_t^I+\frac{\partial U^K}{\partial k_t}\frac{1}{RWA_t}$. However, banks suffer reputation cost and pecuniary cost due to default. The disutility is $\frac{\partial U_1^D}{\partial\left(\left(1-v_t^I\right)\frac{\mu_{t-1}^I}{\Pi_t}\right)}$.

The pecuniary cost results in cash outflow and a lower equity level. The loss of utility due to the pecuniary cost is $\left(\lambda_t^I+\frac{\partial U^K}{\partial k_t}\frac{1}{RWA_t}\right)\frac{\partial U_2^D}{\partial\left(\left(1-v_t^I\right)\frac{\mu_{t-1}^I}{\Pi_t}\right)}$.

2.1.1 Efficiency gain and risk sharing

In this subsection we discuss the role of default as a buffer against negative shocks. The possibility of default allows creditors to share risk with the owners. In this sense, default is equivalent to an insurance. To illustrate this point, we transfer equation(1) into the following two parts:

$$
\begin{aligned}
\text{cash outflow}_t &= c_t^I + m_t^F + m_t^P + v_t^I \frac{\mu_{t-1}^I}{\Pi_t} + \frac{\mu_{t-1}^D}{\Pi_t} + U_2^{I,D}\left((1-v_t^I)\frac{\mu_{t-1}^I}{\Pi_t}\right) \\
\text{cash inflow}_t &= \frac{\mu_t^D}{1+r_t^D} + \frac{\mu_t^I}{1+r_t^I} + R_t^F \frac{m_{t-1}^F}{\Pi_t}(1+r_{t-1}^F) + R_t^P \frac{m_{t-1}^P}{\Pi_t}(1+r_{t-1}^P)
\end{aligned}
$$

where a lower cash inflow can be due to lower repayment by banks' debtors or a funding shock from creditors. Assuming that the inflation rate Π_t does not change, had the default not been allowed (i.e. $v_t^I = 1$) then a lower cash inflow will reduce banks' dividend payouts or loans extensions one by one. When the possibility of default is introduced, it acts as a buffer to support the dividend distribution and credit supply when a negative shock to the cash inflow is realised[2]. On the other hand, with a positive shock in the cash inflow, banks would opt for repaying more and therefore dampen the fluctuations in dividend distribution and loan extension. In this sense, default acts as a "financial decelerator".

2.1.2 Loan margin

The default risk and capital requirement naturally determine loan margins without assuming monopolistic competition and interest rate stickiness, as in the case in the literature. Default results in risk premiums. To simplify the discussion, we assume away capital requirement for the moment. Thus from the first-order conditions (4), (5), (6) and (7) we have the connections of different interest rates in the following:

$$
E_t\left[R_{t+1}^F\right](1+r_t^F) = E_t\left[R_{t+1}^P\right](1+r_t^P) = 1 + r_t^D = 1 + r_t^I
$$

where $r_t^F - r_t^D$ and $r_t^P - r_t^D$ are the default/credit risk premium. When the expected repayment rates $E_t\left[R_{t+1}^F\right]$ and $E_t\left[R_{t+1}^P\right]$ are lower, the interest rates charged on commercial loans and consumer loans increase such that the expected effective interest rates are equal to the cost of the fund r_t^D or r_t^I.

The effect of the riskiness of loans on interest rates also works through the capital requirement channel. From banks' optimality conditions (4) and (5), besides the default risk, the difference between the commercial loan interest rate r_t^F and the consumer loan rate r_t^P comes from the risk weights imposed by the regulator. Higher risk weight adds to banks' marginal costs for the provision of funding, which leads to the higher interest rate charged.

2.2 The rating scheme

Since one purpose of the paper is to study the implications of the Internal-Rating-Based regimes on the procyclicality of capital requirement, in this subsection we model the rating schemes explicitly. We start by introducing a tractable risk weight function to mimic the Basel risk curve. Then, banks' optimisation conditions are used to illustrate the possible impact of the through-the-cycle and point-in-time regimes on the procyclicality of capital requirement.

[2] The efficiency gain of default is analysed by Lin[2012] in the context of collateral default. She points out that when the possibility of default is eliminated the equilibrium volume of the debt market becomes too low, which acts against consumption smoothing. A similar idea can be found in the works of Dubey et al. [2005] and Zame [1993] which model partial default as in this paper.

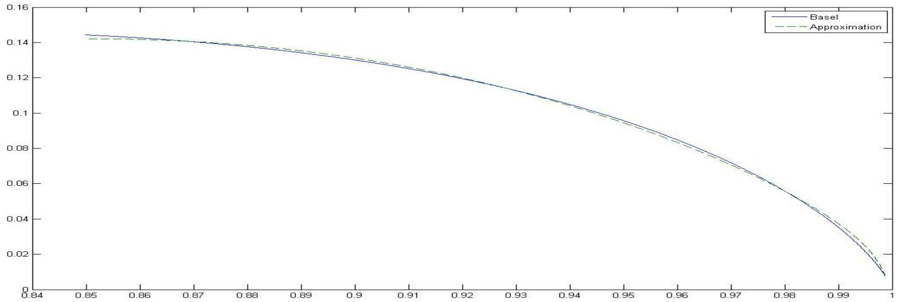


Figure 3: Commercial loans risk curve

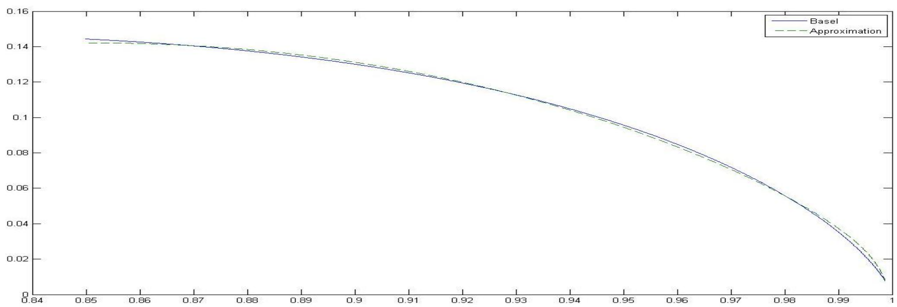


Figure 4: Retail loans risk curve

2.2.1 Risk weight function

According to the Basel proposal on the internal-rating-based capital requirement, the risk weight curve is a function of the probability of default, loss given default, correlation of an asset with the common exposure and maturity. The loss given default is generally assumed to be fixed and the asset correlation and maturity adjustment are functions of the probability of default. Therefore, the Basel risk curve is essentially a function of the probability of default. In this paper, instead of working with the probability of default directly we model the repayment rate. With a constant loss given default, the repayment rate is a linear function of the probability of default (i.e. repayment rate = 1 - loss given default × probability of default). Since the Basel risk functions are intractable for solving the model, we approximate the risk curves with a tractable functions. The goodness of fit are over 99.92% and 99.86% for the commercial loans curve and the consumer loans curve respectively. Figures 3 and 4 above plot the curves.

The risk weight for the exposure to corporate loans as a function of the repayment rate is given by $\forall E_t\left[R^F_{t+1}\right] \in [0.85, 1]$

$$w^F_t = w^F\left(E_t\left[R^F_{t+1}\right]\right) = 12.5\left[-1.8899 + 5.6894\left(E_t\left[R^F_{t+1}\right]\right)^2 - 4.2973\left(E_t\left[R^F_{t+1}\right]\right)^3 + 0.0227ln\left(1 - E_t\left[R^F_{t+1}\right]\right)\right] \tag{10}$$

The risk weight for the exposure to the household loans is given by $\forall E_t\left[R^P_{t+1}\right] \in [0.85, 1]$

$$w^P_t = w^P\left(E_t\left[R^P_{t+1}\right]\right) = 12.5\left[-0.8820 + 4.2389 * \left(E_t\left[R^P_{t+1}\right]\right)^2 - 3.2918\left(E_t\left[R^P_{t+1}\right]\right)^3 + 0.0089ln\left(1 - E_t\left[R^P_{t+1}\right]\right)\right] \tag{11}$$

Corresponding to the same probability of default, the risk weight on commercial loans is higher than the risk weight on the consumer credit, since the Basel programme assumes that commercial loans have higher correlations with the business cycle and impose a larger common exposure factor on commercial loans.

2.2.2 Rating mechanism

To study the rating regime, we follow the spirit of Pederzoli et al. [2010]. Under a TTC or a neutral rating system the risk weight would not change through the cycle. It does not depend on forecasted or current economic conditions. We set the risk weight in the neutral rating scheme to be the steady state level of w_t^F and w_t^P. A PIT rating system is one where the risk weights change according to the expectations of default probabilities, as in the risk curve functions (10) and (11). From the risk curve (Figures 3 and 4), the capital requirements are decreasing functions of the repayment rates. During boom times when the expected repayment rate goes up the risk weight drops, while in recession when the expected default rate rises the risk weight goes up. Such variations of risk weight across time have impacts on the funding cost. We illustrate this point using the optimality condition (4):

$$-\lambda_t^I - \frac{\partial U^K}{\partial k_t}\frac{w_t^F k_t}{RWA_t} + \beta_I E_t\left[\frac{\lambda_{t+1}^I\left(1+r_t^F\right)R_{t+1}^F}{\Pi_{t+1}}\right] + \beta_I E_t\left[\frac{\partial U^K}{\partial k_{t+1}}\frac{1}{RWA_{t+1}}\frac{1}{\Pi_{t+1}}\left(E_t\left[R_{t+1}^F\right]\left(1+r_t^F\right)-1\right)\right] = 0$$

Under the TTC regime, the risk weight w_t^F will not change with the dynamics of the economy. When the expected repayment rate $E_t\left[R_{t+1}^F\right]$ goes down, r_t^F increases to compensate the creditor for the credit risk. However, under the PIT regime, the risk weight w_t^F goes up and puts more pressure on the upward adjustment of r_t^F. This is an illustration assuming that other variables in the equation are fixed. We will see the movements in the impulse response functions.

2.3 Borrowing Households

Following the literature, the economic size of borrowing households σ is given by their wage shares in output. They supply labour in a competitive factor market. They choose time spent on working n_t^P taking as given the wage rate w_t^P. They borrow from banks promising to pay back μ_t^P at the consumer loan interest rate r_t^P. They endogenously choose the repayment rate v_t^P for the liability due ($\frac{\mu_{t-1}^P}{\Pi_t}$), anticipating a contemporary disutility corresponding to the amount defaulted $U_1^{B,D}\left(\left(1-v_t^P\right)\frac{\mu_{t-1}^P}{\Pi_t}\right)$ and a quadratic search cost $U_2^{B,D}\left((1-v_t^P)\frac{\mu_{t-1}^P}{\Pi_t}\right)$
They maximise the expected utility from consumption $\left(c_t^P\right)$ and leisure $\left(1-n_t^P\right)$. β^P is the time discount rate for borrowing households.

$$\underset{\{c_t^P, n_t^P, \mu_t^P, v_t^P\}}{\text{Max}} \; E_0 \sum_{t=1}^{\infty} \beta_P^t \left(U^{P,C}\left(c_t^P\right) + U^{P,L}\left(1-n_t^P\right) - U_1^{B,D}\left(\left(1-v_t^P\right)\frac{\mu_{t-1}^P}{\Pi_t}\right)\right)$$

The constraint (12) says that borrowing households repay loans, pay the search cost due to default and consume using the resources including the labour income, borrowing and the net transfer from the government. This is a cash-in-advance constraint. Following Espinoza et al. [2009] and Martinez and Tsomocos [2011], we assume that households receive s portion of their contemporary labour income while $(1-s)$ portion of the income realises in the next period. The cash-in-advance constraint provides a rationale for the role of money and necessitates liquidity provisions from banks. In this model, each period is a quarter. It is reasonable to expect that households could use part of the wage for the quarter they work to make payment. The same cash-in-advance assumption applies for the labour income of deposit households and the sales proceeds of firms.

$$v_t^P \frac{\mu_{t-1}^P}{\Pi_t} + U_2^{B,D} \left((1 - v_t^P) \frac{\mu_{t-1}^P}{\Pi_t} \right) + c_t^P \leq s w_t^P n_t^P + (1 - s) \frac{w_{t-1}^P n_{t-1}^P}{\Pi_t} + \frac{\mu_t^P}{1 + r_t^P} + T_t^P \qquad (12)$$

2.4 Deposit Households

Deposit households also supply labour in a competitive factor market. They choose the time spent on working n_t^R taking as given the wage rate w_t^R. They deposit d_t^R into banks through a one-period loan with the rate r_t^R. Due to the deposit insurance and also the rare observation of default on deposit, we assume that the deposit at the bank is safe.

Deposit households maximise the expected utility from consumption $\left(c_t^R\right)$ and leisure $\left(1 - n_t^R\right)$. β^R is the time discount rate of deposit households.

$$\underset{\{c_t^R, n_t^R, d_t^R\}}{\text{Max}} \quad E_0 \sum_{t=1}^{\infty} \beta_R^t \left(U^{R,C} \left(c_t^R\right) + U^{R,L} \left(1 - n_t^R\right) \right)$$

Equation (13) is also a cash-in-advance constraint. It says that households deposit money and consume using the income from labour, delivery of the previous deposit and the net transfer from the government. We again assume that deposit households receive s portion of their wage contemporaneously.

$$d_t^R + c_t^R \leq s w_t^R n_t^R + (1 - s) \frac{w_{t-1}^R n_{t-1}^R}{\Pi_t} + \frac{d_{t-1}^R}{\Pi_t} \left(1 + r_{t-1}^R\right) + T_t^R \quad \left(\lambda_t^R\right) \qquad (13)$$

2.5 Firm

Firms hire labour h_t^P and h_t^R and accumulate physical capital K_t. We do not separate the production sector into subsectors including entrepreneurs, wholesale firms and intermediate-good-producing firms, since we are not modelling monopolistic competition or nominal rigidities. The firms sell q_t amount of the output, with the rest becoming investment to replenish the capital K_t , for consumption c_t^F or as search cost due to default $U_2^{F,D}\left((1 - v_t^F)\frac{\mu_{t-1}^F}{\Pi_t}\right)$. τ is the depreciation rate. We assume a time lag for the realisation of part of the revenue. They borrow money μ_t^F at the commercial loan interest rate r_t^F. Firms have the option to default on the loan by repaying v_t^F proportion of the obligations. The modelling of the default penalty is as usual.

Firms maximise the expected utility, which equals to the utility from consumption c_t^F deducted by the default penalty. β^F is the time discount rate of firms.

$$\underset{\{h_t^P, h_t^R, K_t, q_t, \mu_t^F, v_t^F\}}{\text{Max}} \quad E_0 \sum_{t=1}^{\infty} \beta_F^t \left(U^F(c_t^F) - U_1^{F,D} \left((1 - v_t^F) \frac{\mu_{t-1}^F}{\Pi_t} \right) \right)$$

Firms pay labour cost and repay banks using new borrowings as well as sales proceeds. $(1 - s)$ portion of the sales proceeds come with a lag, implying goods could not be perfectly exchanged for goods. λ_t^F is the Lagrangian multiplier.

$$w_t^P h_t^P + w_t^R h_t^R + v_t^F \frac{\mu_{t-1}^F}{\Pi_t} \leq \frac{\mu_t^F}{1 + r_t^F} + s q_t + (1 - s) \frac{q_{t-1}}{\Pi_t} \quad \left(\lambda_t^F\right) \qquad (14)$$

The consumption is given in equation (15), which is the current output deducted by investment, search cost due to default, and sales.

$$c_t^F = y_t + (1-\tau)K_{t-1} - K_t - U_2^{F,D}\left((1-v_t^F)\frac{\mu_{t-1}^F}{\Pi_t}\right) - q_t \tag{15}$$

2.6 Central bank

We assume that the central bank sets a target for the policy rate and allows only for exogenous shock to the policy rate.

$$r_t^I = \left(1-\rho^I\right)\bar{r}^I + \rho^I r_{t-1}^I + \varepsilon_{I,t} \quad \varepsilon_{I,t} \sim \left(0, \sigma_I^2\right) \tag{16}$$

Alternatively, following the DSGE literature one may introduce a standard Taylor rule. We do not use this approach because we are not modelling frictions from wage or price rigidities and therefore the output here is always the potential output defined in the literature. In other words, there is no output gap in this model. The policy rate will only respond to inflation. However, given that our modelling of money is different from the literature, we are not sure which inflation rate to react to (i.e. past, current or expected) and how to set the coefficients. More research has to be done before an appropriate policy rate rule is introduced.
The central bank transfers the seniorage revenue to the households according to their labour share. The entire seniorage revenue is given by $T_t = \frac{M_{t-1}\left(R_t^I\left(1+r_{t-1}^I\right)-1\right)}{\Pi_t}$. The borrowing households receive $T^P = \sigma T_t$ and the deposit households receive $T^R = (1-\sigma)\, T_t$

2.7 Definition of equilibrium

An economy ε is a collection of utility functions for consumption and leisure, cost function of default, cost function of capital buffer, production function, risk weight functions, impatience parameters, cash-in-advance parameter, depreciation parameter, transition probabilities and the interest rate rule:

$$\varepsilon = \left(\left(U^{h,C}, U^{h,L}, U_1^{h,D}, U_2^{h,D}, \beta_h\right)_{h\in H}, U^k, F, w^F, w^P, s, \tau, \pi, r^I\right), \quad H = \{P, R, F, I\}.$$

Let agents' decisions be

$$\sigma^I = \left\{\left(c^I, m_t^F, m_t^P, \mu_t^D, \mu_t^I, v_t^I, E_t\right)\right\}_t^\infty \quad \sigma^P = \left\{\left(c_t^P, n_t^P, \mu_t^P, v_t^P\right)\right\}_t^\infty$$
$$\sigma^R = \left\{\left(c_t^R, n_t^R, d_t^R\right)\right\}_t^\infty \quad \sigma^F = \left\{\left(h_t^P, h_t^R, K_t, q_t, \mu_t^F, v_t^F\right)\right\}_t^\infty \quad \sigma^{CB} = \left\{\left(M_t\right)\right\}_t^\infty$$

Let the macroeconomic variables be

$$\kappa = \left\{\left(r_t^I, r_t^P, r_t^R, r_t^F\right), y_t, \Pi_t\right\}$$

and let the budget set of all agents be denoted by

$$B^I(\kappa) = \{\sigma^I : (1) - (11) \text{ hold}\} \quad B^P(\kappa) = \{\sigma^P : (12), (22) - (25) \text{ hold}\}$$
$$B^R(\kappa) = \{\sigma^R : (13), (26) - (29) \text{ hold}\} \quad B^F(\kappa) = \{\sigma^F : (14 - 15), (28) - (33) \text{ hold}\}$$
$$B^{CB}(\kappa) = \{\sigma^{CB} : (16) \text{ hold}\}$$

We define a monetary equilibrium with default and banks as follows:
DEFINITION: A *monetary equilibrium with default and banks* for an economy ε is a collection

$$\left(\left(\sigma^I, \sigma^P, \sigma^R, \sigma^{CB}\right), \kappa\right)$$

satisfying the following conditions:
i) The labour market, commercial loans market, consumer loans market, discount window borrowing market and goods' market clear:

$$(a)\ n_t^P = h_t^P \qquad (b)\ n_t^R = h_t^R \qquad (c)\ 1 + r_t^F = \frac{\mu_t^F}{m_t^F} \qquad (d)\ 1 + r_t^P = \frac{\mu_t^P}{m_t^P} \qquad (e)\ 1 + r_t^I = \frac{\mu_t^I}{M_t}$$
$$(f)\ c_t^R + c_t^P + c_t^F + c_t^I + K_t + U_2^{I,D} + U_2^{P,D} + U_2^{P,F} = y_t + (1-\tau)K_{t-1} \tag{17}$$

ii) All agents maximise given their budget set $\sigma^h \in \text{argmax}_{\sigma^h \in B^h(\kappa)} U^h \quad \forall h \in H$
iii) All market players are correct in their expectations of repayment rates.

$$E\left[R_{t+1}^P\right] = v_{t+1}^P \qquad E\left[R_{t+1}^F\right] = v_{t+1}^F \qquad E\left[R_{t+1}^I\right] = v_{t+1}^I$$

3 Calibration

3.1 Specific functions

3.1.1 Default penalties

We assume linear non-pecuniary penalty and quadratic pecuniary penalty: $U_1^{h,D}\left(\left(1 - v_t^h\right)\frac{\mu_{t-1}^h}{\Pi_t}\right) = \max\left[0, \theta^h\left(1 - v_t^h\right)\frac{\mu_{t-1}^h}{\Pi_t}\right]$ and $U_2^{h,D}\left(\left(1 - v_t^h\right)\frac{\mu_{t-1}^h}{\Pi_t}\right) = \max\left[0, \frac{d^h}{2}\left(\left(1 - v_t^h\right)\frac{\mu_{t-1}^h}{\Pi_t}\right)^2\right]$, with $\theta^h > 0$ and $d^h > 0$, $\forall h \in \{P, F, I\}$.
Borrowers deliver something on their promise only because of the default punishment. Default on secured debt results in the loss of the collateral. Default on unsecured debt generally brings pecuniary cost such as search cost for new loans or non-pecuniary penalties such as reputation tarnishment. For discussions on the consequences of default, refer to Dubey et al. [2005].
We decide to introduce double default penalties to capture both types of cost due to default. Moreover, such a modelling approach (i.e. a linear-form of non-pecuniary penalty and a quadratic form of pecuniary default penalties) allows for time-varying consumption levels and the positive correlation of repayment rate and consumption. We discuss this in greater detail below.
Note that having both types of penalty in linear form will result in a constant consumption level. Suppose that we allow for both a linear non-pecuniary penalty and a linear pecuniary penalty

(i.e. when the borrowing households repay $v_t^P \frac{\mu_{t-1}^P}{\Pi_t}$, the corresponding non-pecuniary penalty is $\max\left[0, \theta^P(1-v_t^P)\frac{\mu_{t-1}^P}{\Pi_t}\right]$ and the pecuniary penalty is $\max\left[0, d^P(1-v_t^P)\frac{\mu_{t-1}^P}{\Pi_t})\right]$, the first-order condition of the repayment rate v_t^P becomes $\frac{\partial U^P}{\partial c_t^P} = \frac{\theta^P}{1-d^P}$. Since the coefficients for default penalties θ^P and d^P are constant, this implies no variations in the borrowing households' intertemporal consumption. This could be interpreted as a complete consumption smoothing. However, constant level of consumption strongly violates the data. Therefore we choose not to adopt the dual-linear forms.

Another option is to impose a concave pecuniary penalty, such as $\max\left[0, d^P ln((1-v_t^P)\frac{\mu_{t-1}^P}{\Pi_t})\right]$. The first-order condition of the repayment rate v_t^P becomes $\theta^P = \frac{\partial U^P}{\partial c_t^P}\left(\frac{\mu_{t-1}^P}{\Pi_t} - \frac{d^P}{1-v_t^P}\right)$. Given a concave utility function from consumption, it implies a negative correlation between the repayment rate v_t^P and consumption c_t^P assuming that the inflation rate Π_t does not change. This again violates the data.

By using a quadratic pecuniary penalty, we avoid the above issues. The first-order condition regarding borrowing households' repayment rate becomes: $\frac{\partial U^P}{\partial c_t^P} = \theta^P + \frac{\partial U^P}{\partial c_t^P} d^P(1-v_t^P)\frac{\mu_{t-1}^P}{\Pi_t}$, which allows for a varying consumption level c_t^P and a positive correlation between consumption and the repayment rate.

3.1.2 Other functions

We define the utility from capital buffer as $U^K(k_t - k^o) = \eta\ ln(k_t - k^o)$. Instead of modelling utility from capital buffer, previous research by Catarineu-Rabell et al. [2005] assumes a quadratic penalty due to the violation of the targeted capital ratio $\frac{d}{2}\max\left[0, \bar{k} - k_t\right]^2$. Their approach requires the estimation of the targeted capital ratio, which could not be easily implied from the data. In our approach, we model the utility from the capital buffer, which is the difference between the effective capital ratio k_t and the minimum regulatory requirement k^0. k^0 is directly available from the regulatory documents.

We adopts standard log utility function for the two households' sectors and firms. Following de Walque and Pierrard [2011], we define the concave utility functions for banks as $U^I(c^I) = \frac{(1+c_t^I)^{1-\sigma}}{1-\sigma}$. We move the standard CRRA utility leftward because otherwise it would imply a very steep slope, or equivalently, a high level of marginal utility from banks' consumption.

The output is modeled using a Cobb-Douglas production function and the total factor productivity follows an autoregressive process.

$$y_t = Y\left(K_{t-1}, h_t^P, h_t^R\right) = A_t K_{t-1}^{\alpha}\left(\left(h_t^P\right)^{\sigma}\left(h_t^R\right)^{1-\sigma}\right)^{1-\alpha} \tag{18}$$

$$ln\left(A_t\right) = (1-\rho)ln\left(\bar{A}\right) + \rho ln\left(A_{t-1}\right) + \varepsilon_{A,t} \quad \varepsilon_{A,t} \sim \left(0, \sigma_A^2\right) \tag{19}$$

3.2 Parameter calibrations

We calibrate the model to data. Unless otherwise specified, the data period covers from January 1985 to May 2012 and the data are derived from the Federal Reserve System statistics. From the Federal Reserve Loan Survey, the average quarterly rate for loans of all sizes is 1.575%, which is the sum of the reported spread and the Intended Fed Fund rate. Moreover, the average majority prime rate charged by banks on short-term loans to business is 1.64%. We set the commercial loan

Table 1: Parameters

Parameter	Values	Parameter	Values
depreciation rate τ	0.025	poor households' discount rate β^P	0.973
capital share α	0.33	rich households' discount rate β^R	0.998
share of poor households σ	0.036	firms' discount rate β^F	0.987
elasticity of leisure j^P	2.997	banks' discount rate β^I	0.998
elasticity of leisure j^R	2.672	regulatory capital ratio k^o	0.04
elasticity of capital buffer σ^I	1	cash-in-advance parameter s	0.422
coefficient of capital buffer η	0.009	total factor productivity A	1
coefficient on the consumer loan non-pecuniary penalty θ^P	56.843	coefficient on the commercial loan pecuniary default penalty d^F	9.288
coefficient on the consumer loan pecuniary default penalty d^P	0.488	coefficient on banks' non-pecuniary default penalty θ^I	0.851
coefficient on the commercial loan non-pecuniary default penalty θ^F	2.760	coefficient on banks' pecuniary default penalty d^I	28.553
policy rate r^I	0.009		

Table 2: Steady state variables

Variables	Values	Variables	Values
consumer loan interest rate r^P	0.035	banks' effective capital ratio k	0.170
commercial loan interest rate r^F	0.020	risk weight on consumer loan rw^F	1.401
deposit rate r^d	0.009	risk weight on commercial loan rw^P	0.797
inflation rate Π	1.007	labour supply by the poor household $n^P = h^P$	0.252
money base to GDP m^I	0.640	labour supply by the rich household $n^R = h^R$	0.266
capital K	6.630	borrowing households' consumption c^P	0.018
output y	0.767	deposit households' consumption c^R	0.504
consumer loan repayment rate v^P	0.975	firms' consumption c^F	0.003
commercial loan repayment rate v^F	0.990	banks' consumption c^I	0.0217
banks' repayment rate v^I	0.993	output y_t	0.767

rate $r^F = 2.03\%$ to reflect the premium on loans of small size and longer maturities. The average quarterly rate on consumer loans of a 24-month time-to-maturity is 3.227%. The quarterly consumer loan rate we set does not match the data; rather it is set at a slightly higher level: $r^P = 3.55\%$. The average quaterly overnight rate is 0.9% so we set the steady state borrowing rate from the central bank at this level. We set the steady state average quarterly inflation rate at 0.7% which again matches the data. The discount factors for borrowing households, deposit households, firms and banks are $\beta^P = 0.9725$, $\beta^R = 0.9980$, $\beta^F = 0.9870$ and $\beta^I = 0.9979$.

For the repayment rate, the average quarterly charge-off rates on business loans and consumer loans are 0.93% and 2.52%. Therefore, we set the steady state repayment rates on the commercial loans and consumer loans as 99% and 97.5% respectively. Since banks rarely default we set the steady state repayment rate by banks at $v^I = 99.3\%$. Correspondingly, we have the coefficients for non-pecuniary default penalties $\theta^P = 56.84$, $\theta^F = 2.76$, $\theta^I = 0.85$ and the coefficients for the pecuniary default penalties $d^P = 0.49$, $d^F = 9.29$ and $d^I = 28.55$.

The regulatory minimum capital ratio k^o is set at 0.04, which equals to the tier-one capital ratio requirement in Basel I and Basel II. Banks generally keep higher capital ratio than k^0. From a recent paper Carlson et al. [2011], average risk-adjusted tier-one capital ratio is 16%. In our model,

k_t is set at a slightly higher level of 17%. This implies the coefficient for the utility from the capital buffer $\eta = 0.0085$. Based on the risk curve, the steady state risk weights on are 1.4 and 0.80 on the commercial loans and consumer loans respectively.

We set the depreciation rate $\tau = 0.025$ and the capital share $\alpha = 0.33$. The shares of deposit households and borrowing households are set at $1 - \sigma = 0.96$ and $\sigma = 0.04$ respectively. The elasticities on leisure are $j^P = 3.00$ and $j^R = 2.67$. The portion of wage or sales receipts which is available to use contemporaneously is $s = 42.2\%$. The steady sate consumption level by the borrowing households, deposit households, firms and banks are $c^P = 0.018$, $c^R = 0.504$, $c^F = 0.003$ and $c^I = 0.021$. The sum of the consumption is about 71.2% of the output. The capital-to-output ratio is $\frac{K}{y} = 8.64$ and investment to output ratio is around 21.6%. The steady state working hours of deposit households and borrowing households are 25.3% and 26.6%.

4 Responses to shocks

In this section, we study the real effect of a standard productivity shock, a monetary shock and a financial shock within an otherwise standard model economy with frictions. The following discussions will focus on three types of variables to show the linkages and responses. The first set includes output and its factors input (capital stock and labour supply) and prices (wages and inflation), as well as consumption. The second group relates to default as well as its impact on interest rates and risk-weights on loans. The third collection covers banks' balance sheet, which includes loan extensions, deposit, equity and capital ratio. Studying the impacts of the shocks, we are particularly interested in the behaviours of repayment rates and banks' profit [3]. Our interest in these variables is based on the concept of financial fragility following Goodhart et al. [2006] where financial fragility is defined as higher default probability and lower bank profitability.

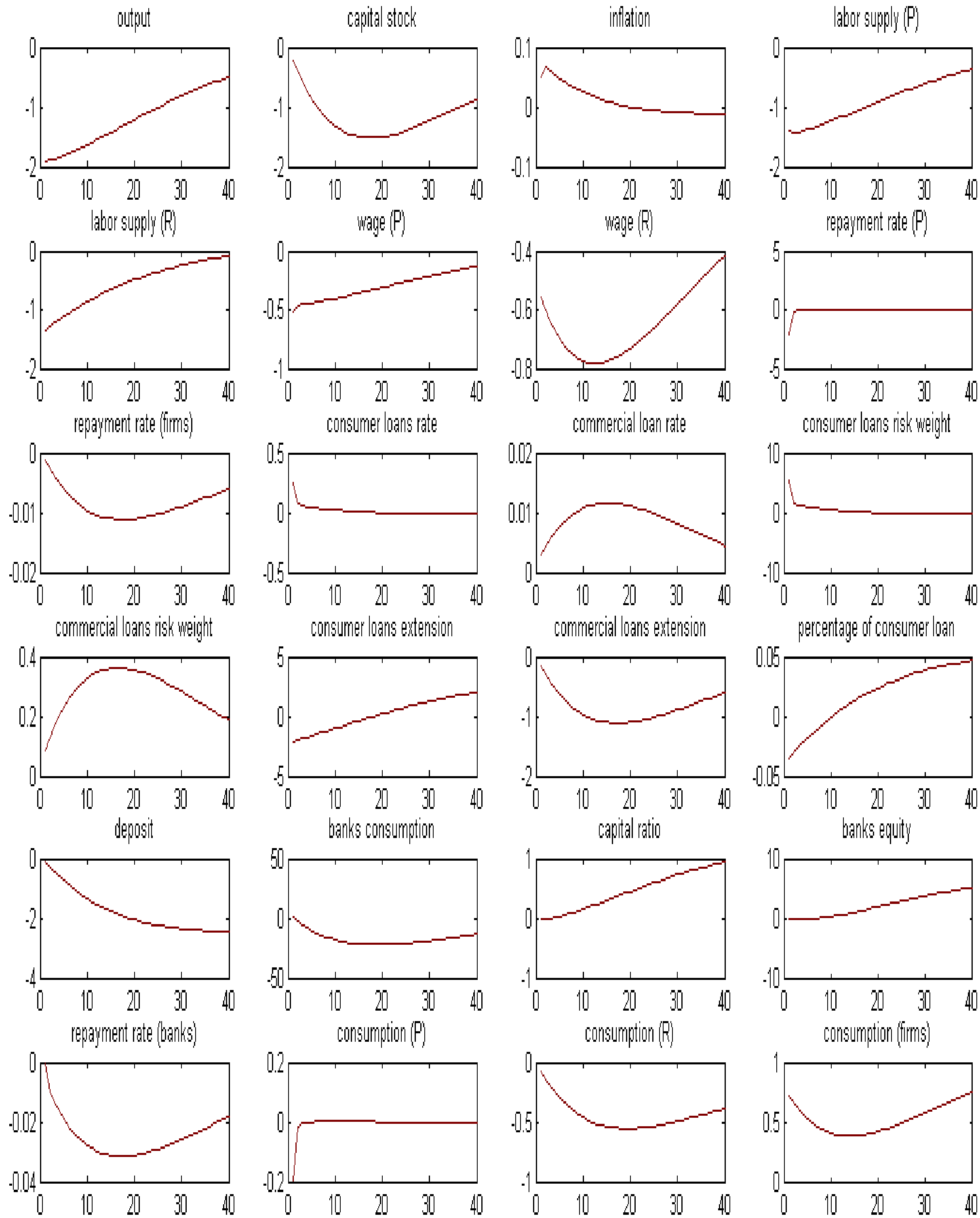
4.1 Responses to a productivity shock

Since concerns on the impact of capital requirement focus more on the downturn, where the procyclicality issues exacerbate the consequences of an adverse shock, we focus on a negative shock here. Figure (5) plots the responses to 1% negative productivity shock.

Overall, the responses are persistent. Default rates are higher and banks' consumption is lower following the negative productivity shock, which indicates a higher degree of financial fragility according to the definition by Goodhart et al. [2006]. Higher default results in lower profit for banks and a downward pressure on their capital ratio. Meanwhile, under the point-in-time system, higher default risk leads to a larger risk weight, which imposes upward pressure on the risk-weighted asset and further downward pressure on bank' capital ratio. To regain adequate capital positions, banks follow with credit contractions, portfolio reconstruction, and setting more costly financing terms, which amplify the downturn. We also observe that banks actively restrain their own consumption and repay less to the central bank as ways to attenuate the impact of the shock. We discuss the responses in detail in the following.

The first-order effect of the adverse productivity shock goes to the corporate sector and households. Since the marginal output is lower, capital stock goes down on impact and investment level is lower correspondingly. Wages are cut initially and remain persistent. The substitution effect dominates

[3] Banks' profit is measured by banks' consumption.



Note: the graphs show deviations from the steady state given a 1% negative productivity shock. The deviations are in percentage points for interest rates and repayment rates. They are in percentage for the other variables.

Figure 5: Impulse response functions (negative productivity shock)

the wealth effect and both types of households work less. One quarter after the shock, employment is about 1.4% below the long-run equilibrium levels. The loss in employment remains persistent. Lower employment and wages cause around 1.9% losses in income for each type of households in the first quarter. The cumulative reductions in income stay at around 8.9%. Output shrinks by 1.9% in the first quarter with a cumulative loss around 9.5%. Inflation rate rises on impact and remains beyond its steady state level until the 20th quarter.

Borrowing households react to the shock to income by initially defaulting more. In the first quarter after the shock their repayment rate plunges by 2%. However, the repayment rate reverts to steady state level within three quarters. Lowering the default rate directly benefits borrowing households by cutting the interest cost and default penalties. Borrowing households manage to payback banks swiftly by constraining consumption. Figure (5) shows that their consumption plunges by 0.2 % immediately following the shock. Higher default rates are also observed in commercial loans, although the magnitude and the degree of persistence are different. Firms reduce the repayment rate slightly one quarter after the shock but take longer to regain their long-run level of credit quality. We shall discuss firms' different behaviour on repayment rate later.

Regarding banks' responses, they set higher interest rates to compensate for the expected loss. Meanwhile, due to the Basel regulation, the interest rates are adjusted upward further to compensate for the unexpected losses under the assumption that the unexpected losses are larger for riskier loans. Consequently, interest rate on the consumer credit is 25 basis points higher immediately after the shock. The pattern of the commercial loans interest rate is different. It was up by smaller scales but the changes are persistent.

Regulators react to the higher delinquency rates by imposing stricter capital requirements on banks. Following the shock, the risk weight on consumer loans rises by 5.5 % in one quarter but then quickly returns to the steady state level. The risk weight on commercial loans rises by small scales on impact but has been persistent.

The higher risk weights do not lead to larger risk-weighted asset, because banks cut loan extensions to both borrowers' sectors. The commercial loan extensions show a small but steady decline, which reaches the minimum with a loss of 1.1%. The loan extensions to borrowing households are down by 2% one quarter after the shock but this quickly returns to the pre-shock level. Meanwhile, banks appear to reconstruct their portfolio by shifting to the consumer credit. Figure (5) shows the share of the consumer credit out of the total loan extensions in banks' books. The share of consumer loans drops on impact but then recovers.

Banks are able to maintain their capital ratio, which keeps increasing and remains persistent. The reasons for the higher capital ratio are four-folds. Firstly the risk-weighted asset is lower due to shrinking assets. Reconstructing loan extensions towards the consumer credit further softens the upward pressure on the risk-weighted asset. Secondly, by charging higher rates on loans, banks' profit increases. Thirdly, banks restrain their own consumption and therefore the equity level increases. Post-shock, banks' consumption (or, dividend distributions) goes down substantially. The lowest point is about 22% below the steady state level. Finally, banks default more in the central bank loans.

The third and final points raised in the above paragraph show that banks choose to have their owners and creditors share the consequences of the shock. By doing so, banks are better insulated against the knock-on effect from another round of default. Otherwise, banks could have used the resources to support the steady state level of consumption and repayment rate while cutting credit extensions and turning the credit terms even more costly. The alternatives would have amplified the downturn in the economy and led to further output contraction and deterioration in financial stability. These, ultimately, would imply further and possibly larger losses by banks. Facing the trade-off, banks rationally choose to reduce their own consumption and repayment rates to the central bank, instead of adopting further credit tightening.

Finally, we discuss reactions by deposit households and firms. Deposit households observe a sustained reduced level of consumption post-shock due to the smaller income from labour and deposit. This is intuitive. The somewhat surprising observation is that firms enjoy more consumption post the negative productivity shock while their repayment rate is persistently lower than the steady state level, as shown in figure (5). This is in contrast to borrowing household's reactions where they pay back more even though their consumption falls below the steady state level. To understand this, it is helpful to refer to equations (14) and (15). Firms could have sold more of their output and repaid more to banks. However, given that the aggregate demand in the economy is low, firms have to cut the goods' price substantially to increase sales. On net it is more beneficial from the firms' respective to increase consumption and continue the lower payments to banks. The persistently lower repayment rate on the part of commercial loans explains why banks switch loans extensions from commercial loans to consumer credit.

4.2 Responses to monetary shock

In this section we study the responses to a positive shock of 6.25 basis points in the policy rate[4]. We again see lower banks' consumption and higher default rates in the economy. Therefore, positive monetary shock potentially has a destabilising effect on the financial system. For all variables, the directional effect of the monetary shock is similar to the productivity shock, although the patterns are different.

The upward adjustment of the policy rate by 6.25 basis points translates almost one-to-one into the rates on commercial loans and consumer loans. In the face of such shocks, both borrowers reduce their repayment rates to banks. Borrowing households repay 5 basis points less and firms reduce the repayment rate by 0.125 basis point. As under the productivity shock, here the reaction by the consumer loans repayment rate is temporary while the reaction by firms' repayment rate is persistent. Correspondingly, the risk weights on loans are higher. The consumer loans risk weight rises by 1% and commercial loans risk weight rises by 5 basis point. Again the responses on the commercial loans risk weight is persistent.

Higher risk weight leads to larger risk-weighted asset. Meanwhile, lower repayment rates result in lower bank capital. Thus we observe lower bank capital ratio immediately following the shock. However, the downturn is reversed at a relatively fast pace. Starting from the 10th quarter, banks accumulate more equity and sustain a higher capital ratio than the long-run equilibrium level. To achieve these, banks constraint their consumption. Banks' consumption is cut by 0.4% in the first quarter and arrives at the minimum level of 0.9% in the 14th quarter. To restrain cash outflow, banks also repay less to the central bank. Meanwhile, banks cut funding to both types of borrowers. Loans to the corporate sector are around 3 basis points lower in the first quarter after the shock. The downturn is persistent. The consumer credit extension is down by 5 basis points initially.

[4] Since the central bank generally adjusts the interest rate by 25 basis points per time, we shock the quarterly rate by 6.25 basis points.

However, starting from the 10th quarter, the borrowing households start to obtain more loans than the steady state level. Overall, banks appear to reconstruct the portfolio towards the consumer loan.

Higher financing cost leads to lower investment, with capital stock persistently below the steady state level post the shock. Wages and labour supply are also both lower. The initial downturn in output is 6 basis points with cumulative loss of output at 1.53%. Inflation rises by 7.5 basis points at the maximum initially and returns steadily to the long-run equilibrium level. Due to the lower labour income and higher interest cost, the borrowing households consume less. The responses are on a small scale, and temporary. Compared with the borrowing household, consumption by the deposit households goes down by more and is persistent. It is the case, however, that the deposit households fail to benefit from the higher interest rate since they deposit less.

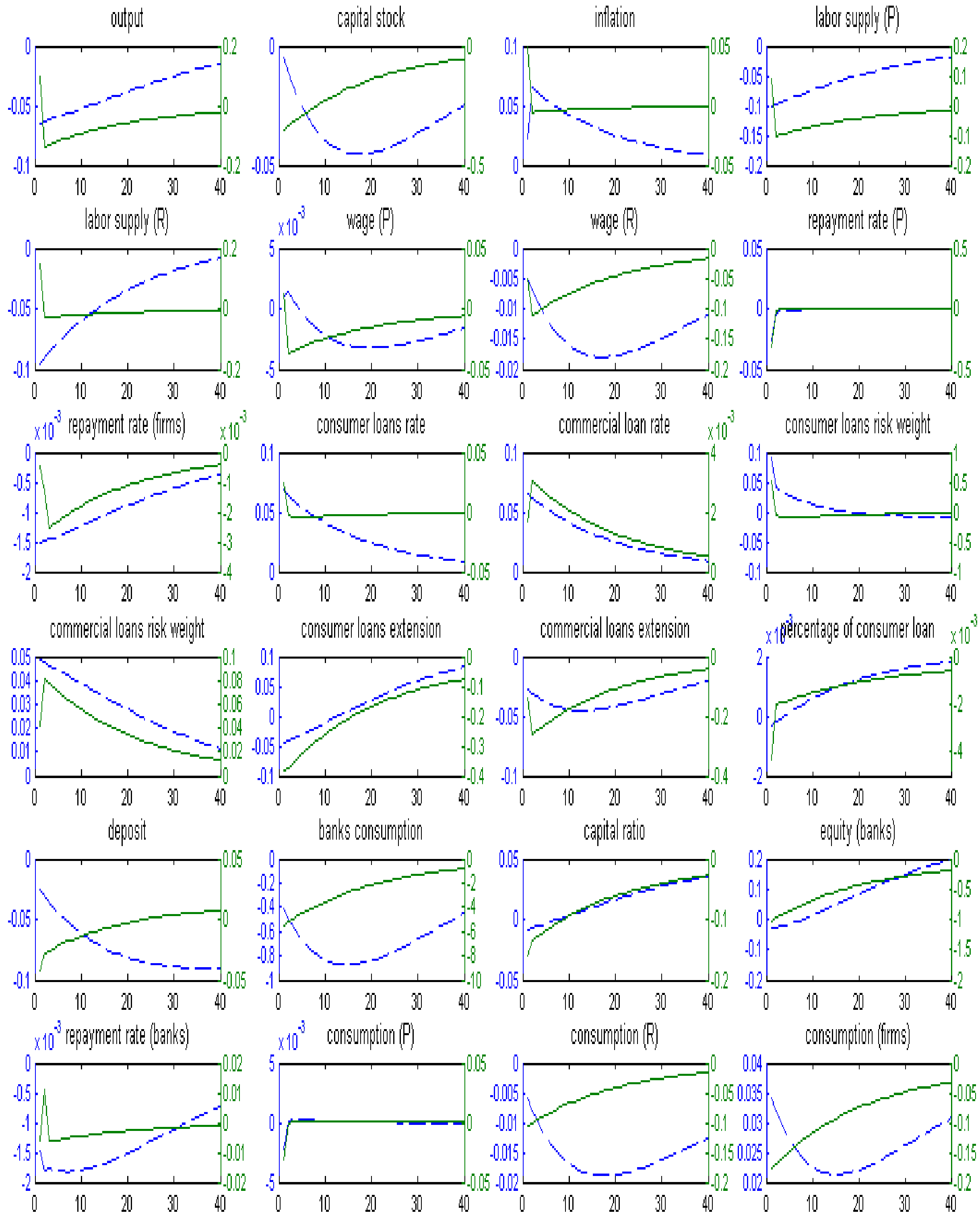
4.3 Responses to financial shock

In this section, we study the impact of a shock originating from the banking sector. To the extent that banks are under capital constraint and that liquidity matters for production and trade, a loss by banks would have a real impact on the economy. This highlights the necessity to include the banking sector in macroeconomic modelling. Figure (5) plots the responses to a 1% negative shock in banks' equity. To do this, we simultaneously adjust equations (1), (2) and (17) by introducing an one-off shock to the bank capital E^{shock}, which equals to 1% of the steady state level bank equity. Overall, we observe that a loss of 1% equity by banks leads to an accumulative loss of 2.37% in output. Meanwhile, banks appear to cut the funding to the households sector more aggressively and restructure the portfolio towards the corporate sector post the shock. Another way in which the impact of the financial shock differs from the cases of the productivity shock and the monetary shock is that, with the financial shock, all economic agents consume less while in the other two cases firms benefit from the shock.

The first-order effect of the shock goes to banks. Figure (6) shows that banks' equity level drops by 1% in the first quarter and recovers gradually from then on. By the 40th quarter banks have almost re-accumulated all capital. Banks' capital ratio is down by 0.15% after the shock and again steadily adjusts back to the pre-shock level. Banks manage to regain capital and boost the capital ratio by tightening credit extensions. They reduce loan extensions to both borrowing households and firms. Consumer loans extension is cut by 0.4% in the first quarter following the shock. The downturn in the consumer loans extension remains persistent. Loans provided to the corporate sector drop on impact and then adjust upward gradually. Overall, it turns out that banks cut consumer loans more aggressively than commercial loans. Differing from the case of the productivity shock, banks reconstruct their portfolio towards commercial loans, as shown in Figure (6). Here, the bank suffer a direct capital loss. Banks chooses to cut the consumer loan extensions, which attract higher risk weight, more aggressive with the aim to increase the capital ratio.

Lower credit extensions from banks act as a funding shock and result in higher default rates by both types of borrowers. Repayment rate on the consumer loan is down by 0.4 % but recovers to the steady state level within three quarters. On the side of firms, the initial downturn of repayment rate is on a smaller scale, although this has persisted.

Higher default rate leads to more costly financing. Banks charge higher rates to compensate for both expected and unexpected losses. Immediately after the shock, the cost of consumer loans is



Note: the graphs show deviations from the steady state given a 25 basis points positive monetary shock and 1% negative capital shock. The deviations are in percentage points for interest rates and repayment rates. They are in percentage for the other variables. The dash line and the vertical axis on the left are corresponding to the monetary shock. The solid line and the vertical axis on the right are corresponding to the financial shock.

Figure 6: Impulse response function (negative financial shock)

2.5 basis points higher. It returns to the steady state levels again within a short time frame of three quarters. The commercial loans rate rises on impact and remain persistent.

The wave of higher default affects banks' portfolio decisions. Risk weights rise correspondingly under a PIT system. The risk weight on consumer loans is up by 0.5 % in the first quarter, which then returns to the steady-state level within three quarters. The upward movement in the commercial loans risk weight is persistent but at a smaller scale. As a result, banks may further cut credit extensions to both sectors to restrain the upward movement in the risk-weighted asset.

To maintain the capital ratio, banks cut their consumption by 5% in the first quarter after the shock and would not consume more until the equity position is recovered. Generally, banks default more on their loan obligations as well, although they increase the repayment rate initially.

Higher financing cost leads to smaller accumulation of capital stock by firms, which drops by 0.4% in the first quarter and again is persistent. Immediately following the financing shock, both households' sectors decide to work more. labour supply by borrowing households is about 0.1 % higher in quarter one, while deposit households supply about 0.2% more labour. However, wages for the two households become lower and persisted at lower levels such that the two households decide to reduce their labour supply. Deposit households return to the steady state level of labour supply swiftly while borrowing households work less than the long-run equilibrium level. Correspondingly, output goes down by 0.18% in the third quarter. The cumulative loss in output is about 2.37%. Meanwhile, consumption by all agents is lower. Borrowing households' consumption is down by 2.5 basis points in the first quarter although it soon returns to the previous level. Post-shock, deposit households lose 0.1% of consumption in the first quarter and firms lose around 0.2%. Both the borrowing households and the deposit households see their consumption levels persistently below the steady state levels until the 40th quarter.

5 The rating scheme

Another purpose of this paper is to study the potential procyclical effect of the Basel Internal-Rating-Based system. In subsection 5.1, we present the results with the aim to highlight the differences in responses to shocks, if any, under the point-in-time (PIT) and the through-the-cycle (TTC) regimes when the economy faces different types of shock. To predict the choice of banks regarding the two systems, we calculate banks' expected utility. Our results show that banks' preference varies with different shocks. Banks tend to adopt the PIT system if they are directly impacted by shocks while they prefer the TTC system if they are not directly affected by the shock. We explain banks' choice in subsection 5.2.

5.1 The impacts of rating schemes

In this section, we compare the different responses of variables under the two internal-rating-based schemes: point-in-time and through-the-cycle. Figure (7) shows the differences in the responses of variables under the two schemes upon a realisation of negative productivity shock. Figure (8) shows the results with the financial shock and the monetary shock. The curve represents the values under the PIT scheme minus that under the TTC scheme. Generally, as expected, the point-in-time scheme amplifies the responses of variables in the downturn, while the through-the-cycle scheme plays the role of a “buffer”.

We found that banks sustain lower capital ratios under the PIT scheme given any of the three types of shock introduced. Those negative shocks lead to higher default rates and greater risk-weighted asset, which results into downward pressure in the capital ratio. However, with the TTC scheme, both the risk weight and capital ratio remain unchanged. Thus banks would be worse off in terms of utility from the capital buffer if they choose the PIT scheme over the TTC scheme.

Another finding is that banks reconstruct their portfolio more actively under the PIT scheme. Firstly, banks switch from commercial loans to consumer loans more actively. Compared with the TTC scheme, under the PIT scheme consumer loans extensions increase more while the commercial loans extensions decrease on a larger scale. Figure (7) shows that the percentage of consumer loans in banks' portfolio is higher under the PIT scheme and the difference is persistent. Secondly, banks reduce the credit extensions to a larger degree under the PIT scheme. Figure (7) shows that the amount of asset extensions is lower under the PIT scheme. The restructuring in loan extensions reinforces the movements in default rates, risk weights and capital ratio. Thus, compared with the TTC scheme, the PIT system tends to amplify shocks.

Under the productivity shock, the active portfolio restructuring associated with the PIT scheme benefits banks' owners and creditors initially, because it allows banks to retain more financial resources for the purpose of consumption and debt fulfillment. Meanwhile, banks' profit is higher due to the higher interest rates charged on the loans. However, the PIT system results in more default by the corporate sector and higher risk-weighted asset, which impose stronger pressure in the downturn of the capital ratio. To support the capital ratio, banks finally turn to lower consumption and lower repayment rate under the PIT scheme. Most of the results follow for the other two shocks. Employment and output both drop further under the PIT scheme while the inflation rate rises further. Under the PIT scheme, borrowing households enjoy more consumption initially because they receive more loans from banks. However, the direction is reversed later on due to the lower labour income and higher repayment to banks. Deposit households are able to consume more initially because they deposit less under the PIT scheme. This direction is also reversed later on due to the smaller income from wages and interest. Firms consume more under the PIT scheme. Again, the reason could be attributed to the higher inflation rate from which firms benefit. The results apply to all three types of shocks considered.

5.2 Banks' choice

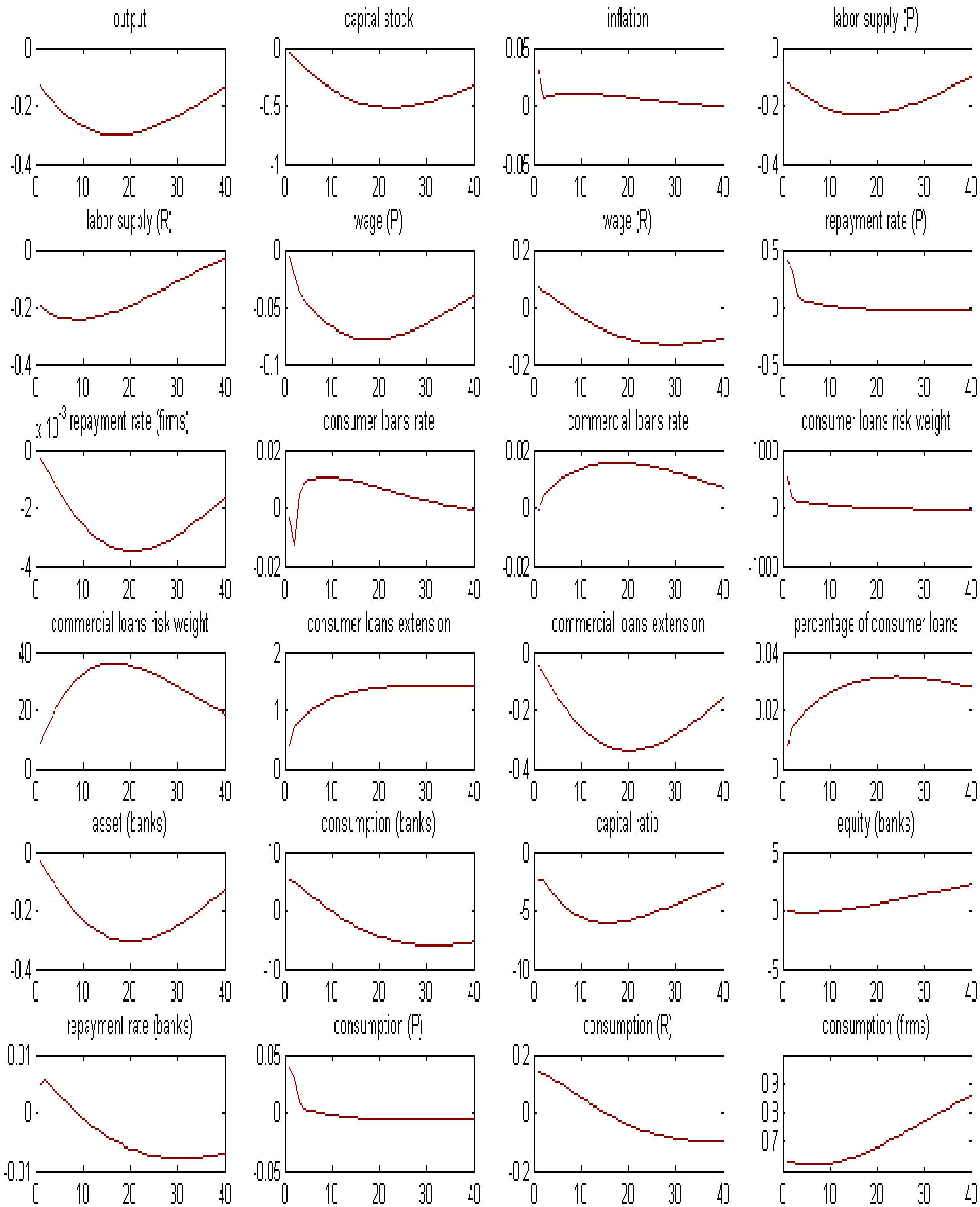
Banks' preference regarding the two schemes depends on their utility. Banks' overall utility is approximated by the sum of utility in the first 1000 periods [5]. The same approach follows for estimating overall utilities of other agents.

$$U^I = E_0 \sum_{t=1}^{T=1000} \beta_I^t \left(U^I \left(c_t^I\right) + U^K \left(k_t - k^o\right) - U_1^D \left((1 - v_t^I) \frac{\mu_{t-1}^I}{\Pi_t} \right) \right)$$

Given a productivity shock, banks' utility under the two regimes are $U^{pit} = -1663.58$ and $U^{ttc} = -1663.57$ respectively. Therefore, banks would prefer the through-the-cycle scheme if a negative productivity shock realises. The decompositions (see Table (3)) show that banks are better off in all dimensions (i.e. consumption, capital buffer and default penalties) under the TTC scheme. The

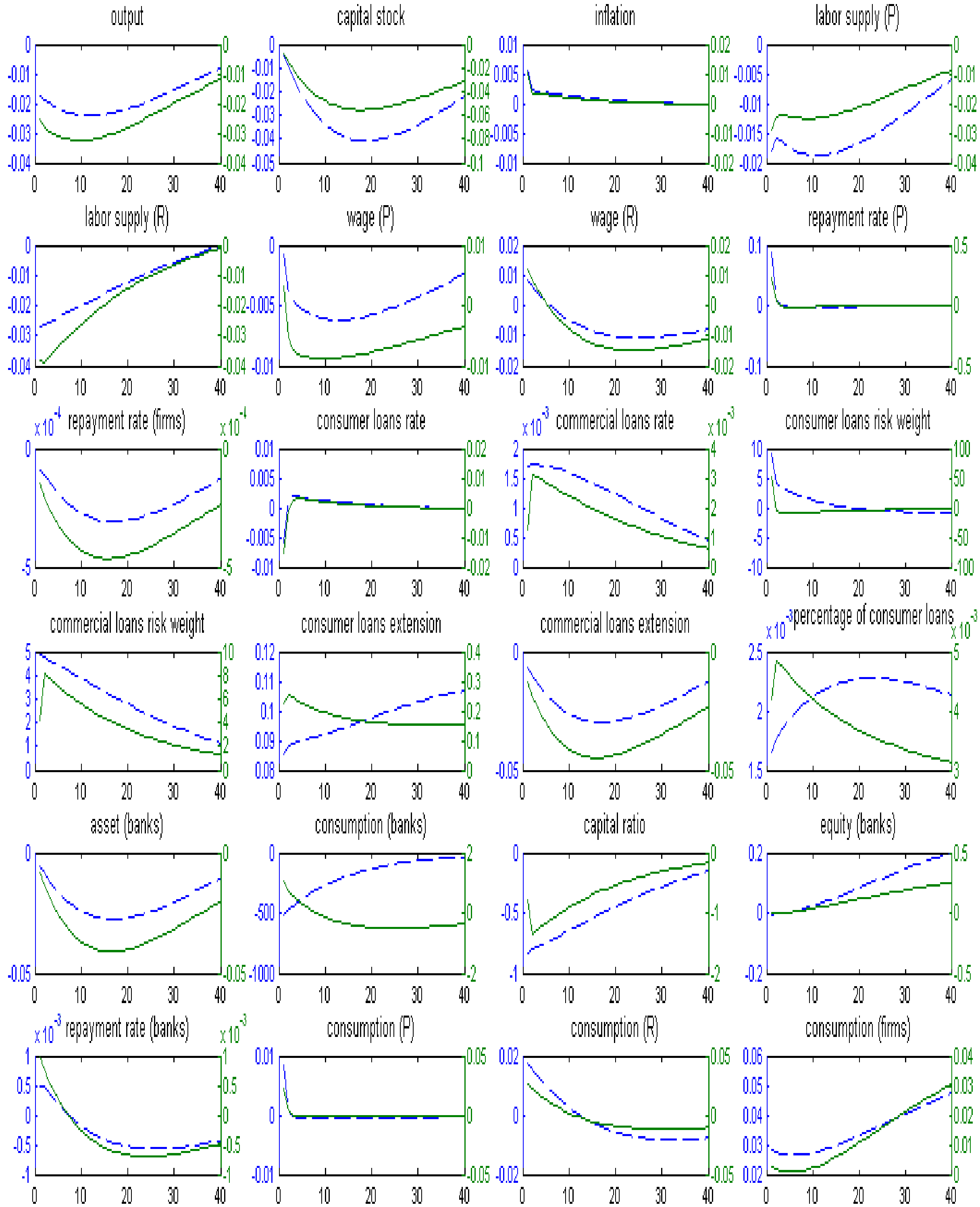
[5]The 1000 periods is chosen for computation purposes. Any time periods before which all variables have returned to the steady state levels are applicable in this case.

Figure 7: Comparison of responses to the PIT and TTC schemes (productivity shock)



Note: the graphs show the differences in responses under the point-in-time scheme and the through-the-cycle scheme given a one % negative productivity shock. The curve represents the values under the PIT scheme minus those under the TTC scheme. They are in basis point.

Figure 8: Comparison of responses to the PIT and TTC schemes (monetary and financial shock)



Note: the graphs show the differences in responses under the point-in-time scheme and the through-the-cycle scheme given a one % negative financial shock. The curve represents the values under the PIT scheme minus those under the TTC scheme. They are in basis points. The dash line and the vertical axis on the left are corresponding to the monetary shock. The solid line and the vertical axis on the right are corresponding to the financial shock.

Table 3: Welfare

	banks	borrowing households	deposit households	firms
Productivity shock				
utility_all	-0.0061	1.2060e-04	5.0316e-04	0.0051
utility_consumption	-0.0060	-3.5204e-06	5.7971e-04	0.0051
utility_capital buffer/leisure	-2.4388e-05	1.2413e-04	-7.6554e-05	
disutility_default penalty	1.0347e-07	3.2466e-09		-2.3626e-06

	banks	borrowing households	deposit households	firms
Financial Shock				
utility_all	0.0016	1.6282e-05	-4.8574e-05	2.0047e-04
utility_consumption	0.0017	-2.62434-07	-3.3421e-05	2.0128e-4
utility_capital buffer/leisure	-1.3398e-04	1.6544e-05	-1.5153e-05	
disutility_default penalty	-1.4783e-07	-3.6480e-11		8.0847e-07

	banks	borrowing households	deposit households	firms
Monetary Shock				
utility_all	6.0874e-04	1.0844e-5	1.3826e-06	2.8675e-04
utility_consumption	6.6837e-04	-2.4254e-7	1.1342e-05	2.8709e-04
utility_capital buffer/leisure	-5.9683e-05	1.1087e-5	-9.9584e-6	
disutility_default penalty	-5.8543e-08	1.3008e-9		3.4368e-07

Note: the tables report the difference of each agent's overall utilities under the two schemes (PIT minus TTC). We report the comparison in the overall utilities. We also decompose the overall utility into utility from consumption, capital buffer, leisure and disutility from default penalty and then compare utilities under the two schemes against these detailed dimensions.

results are slightly different if an adverse financial shock realises. Banks would be better off under the PIT scheme. They enjoy higher utility from consumption and suffer smaller disutility due to default, although the utility from capital buffer is lower in the case of the PIT scheme. The same results apply to the comparison under an adverse monetary shock where banks would prefer the PIT system. They are better off in terms of consumption and default penalties but worse off regarding the capital buffer.

We see that banks' preference depends on the type of shock. Upon the realisation of a negative productivity shock which affects the real economy directly, banks would prefer the TTC scheme. One the other hand, when a shock, such as a financial or a capital shock, which has first order effect on banks is realises, banks tend to adopt the PIT scheme.

When the productivity shock occurs, banks are affected in a second-order effect, i.e. through the higher default rate by the now relatively poorer borrowers instead of being affected directly. As discussed, default could be considered as a "risk-sharing" mechanism. In this sense, banks are sharing the impact of the shock with the households and firms. If banks had chosen a PIT scheme, according to Figure (5), this would have led to tightened credit conditions and a further round of default by the corporate sector. Thus in turn would have resulted in greater losses by banks and even lower capital ratio and repayment rate to banks' owners. As shown in Table (3), banks would have been worse off in terms of consumption and default penalties if they chose the PIT scheme post the productivity shock. Thus, instead of amplifying the downturn, the better strategy for banks is to absorb the shock by adopting the TTC scheme.

When the financial shock realises, banks are affected in a first-order effect. With lower capital levels, banks naturally reduce credit extensions, cut their own consumption and default more on the central bank loan. Tightened credit conditions lead to more default by borrowers. Facing the higher default, banks actively reconstruct the portfolio and shrink the balance sheet by adopting the PIT scheme. These actions provide banks with more resources for consumption and debt repayment. Meanwhile, banks are able to charge higher rates on the loans, which again add to banks' financial resources. By retaining resources, banks limit the downturn in their consumption and debt repayment under the PIT scheme. Although the active portfolio restructuring amplifies the impact of the shocks on the economy and may lead to a further round of default by borrowers and loss-making by banks, these are effects of smaller magnitude. As shown in Table (3), banks would have been worse off had they chosen a TTC scheme following the financial shock.

The welfare of the households and firms under different schemes is discussed in the following. We found that agents' preferences for the two schemes vary with the shock. Moreover, depending on the shock, banks' preference regarding the two schemes may or may not be consistent with other agents' best choice.

When a productivity shock realises, all these agents would prefer the PIT scheme over the TTC scheme. Under the PIT scheme, although borrowing households enjoy less utility from consumption and suffer higher default penalties, the higher utility from leisure has a dominate effect. Deposit households enjoy more utility from consumption under the PIT scheme albeit obtaining lower utility from leisure. Regarding firms, under the PIT scheme, they enjoy more utility from consumption and face less disutility from defaulting. The preferences of households and firms are not consistent with banks' choice.

When the financial shock occurs, borrowing households would also be better off under the PIT scheme. Although the utility from consumption is lower, they enjoy higher utility from leisure and smaller disutility due to default. The deposit households are better off under the TTC scheme, since they consume more and enjoy more leisure. Under the PIT scheme, firms enjoy higher utility from consumption but face larger utility costs from default penalties. Overall, firms are better off with the PIT scheme. Banks' choice is therefore consistent with borrowing households' and firms' preference.

All agents' choices are the same when a positive shock in the interest rate realises. They all prefer the point-in-time scheme. Under this scheme, borrowing households enjoy more from leisure, which has a dominant effect over the higher disutility from default and lower utility from consumption. Deposit households' gain from utility in consumption outweighs the loss in utility from leisure under the PIT scheme. Firms are better off in terms of consumption but worse off with respect to the default penalty. The former effect dominates the second, thus firms prefer the PIT scheme. Banks' choice is consistent with all other agents' preference.

6 Conclusion

In this paper we build a DSGE model with two representative households' sectors (borrowers and depositors), firms and banks. Our approach is different from the literature in the following aspects: 1) we model money using a cash-in-advance constraint instead of nominal rigidity as in the New-Keynesian framework. The upshot of this approach is that liquidity directly affects the efficiency of trade and production. In essence, the price of liquidity acts as a wedge between the marginal utility of the sales side and buy side. Monetary policy and banks' channeling of funds affect the price of liquidity, which then translates into variations in trade and output; 2) we model endogenous default by borrowers, particularly by the borrowing households and the corporate sector. The default probability, or in our case the repayment rate, is connected with the risk weight on banks' asset through the real Basel risk curve. Therefore, this allows us to study the impact of the internal-rating-based regime in a way that is closest to reality as possible.

One purpose of our work is to show the amplification and propagation effect due to the frictions, i.e. money, default and capital requirement. We demonstrate the the effect is significant given a realisation of negative productivity shock. We also show that financial shock and monetary shock have real impact. Our second purpose concerns the procyclicality issue of the internal-rating-based regimes and the prediction of banks' preference. We show that the point-in-time scheme indeed amplifies the fluctuations of the variables while the through-the-cycle scheme mitigates it. We show that banks' preference depends on the type of shock. With the negative productivity shock banks are better off with the TTC scheme while the reverse is true with a negative financial shock or a positive monetary shock. We explain banks' choice as a "loss balancing" mechanism. When banks are directly affected by the shock, such as the financial shock and the monetary shock, they tend to adopt the PIT scheme under which other agents in the economy share the impact of the shock to a larger degree. In case banks are affect by shocks in a second-order sense, they tend to follow the TTC scheme where banks help to absorb the impact of shocks. We also show the corresponding welfare for other agents. Banks' choice may or may not be consistent with the interests of other agents.

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7 Appendix

7.1 Formula for the Basel risk weight

The Basel capital requirement for an given exposure depends on its probability of default, loss given default, asset correlation and maturity. The function of the risk curve for exposure to the corporate sector is:

$$
\begin{aligned}
&\text{capital requirement} = LGD \times \left[N\left(\frac{N^{-1}(PD) + \sqrt{Corr} \times N^{-1}(0.9999)}{\sqrt{1 - Corr}}\right) - PD\right] \\
&\times \left(\frac{1 + (M - 2.5) \times b}{1 - 1.5 \times b}\right)
\end{aligned}
\tag{20}
$$

where $N(x)$ denotes the cumulative distribution function for a standard normal random variable; $N^{-1}(x)$ is the inverse cumulative distribution function for a standard normal random variable; LGD is the loss given default and PD is the probability of default; $Corr$ is the correlation factor, which is linked with PD, and b is the adjustment factor for the maturity of loan. $Corr$ and b are given by [6]

$$
Corr(PD) = 0.12 \times \left(\frac{1 - exp^{-50PD}}{1 - exp^{-50}}\right) + 0.24 \times \left[1 - \left(\frac{1 - exp^{-50PD}}{1 - exp^{-50}}\right)\right]
\tag{21}
$$

$$
b(PD) = (0.11852 - 0.05478 \times log(PD))^2
\tag{22}
$$

From equations (20), (21) and (22), the capital requirement is a function of LGD, PD and M. However, the introduction of the actual Basel formula would be intractable from a computational point of view when solving the optimisation problem. We therefore proxy the Basel risk weight requirement with a single function. Assuming $LGD = 50\%$ and $M = 0.25$, we express the capital requirement as a function of the single variable PD.

$$
\begin{aligned}
&\text{capital requirement} = 0.5 \times \left(\frac{1 + (0.25 - 2.5) \times (0.11852 - 0.05478 \times log(PD))^2}{1 - 1.5 \times (0.11852 - 0.05478 \times log(PD))^2}\right) \\
&\times \left[N\left(\frac{N^{-1}(PD) + \sqrt{0.12 \times \left(1 - \frac{exp^{-50PD}}{1 - exp^{-50}}\right) + 0.24 \times \left[1 - \left(1 - \frac{exp^{-50PD}}{1 - exp^{-50}}\right)\right]} \times N^{-1}(0.9999)}{\sqrt{1 - 0.12 \times \left(1 - \frac{exp^{-50PD}}{1 - exp^{-50}}\right) + 0.24 \times \left[1 - \left(1 - \frac{exp^{-50PD}}{1 - exp^{-50}}\right)\right]}} - PD\right)\right]
\end{aligned}
$$

Since the repayment rate in our model is in essential $R_t = 1 - PD \times LGD$, we replace PD by $PD = \frac{1 - R_t}{0.5}$. Therefore, the capital requirement becomes a function of R_t. For $R_t \in [0, 1]$, we simulate the value for the capital requirement. Finally, we approximate the capital requirement with a simple function of the repayment rate R_t.

We follow similar approach for the exposure towards to corporate sector.

7.2 Bank and firm's stochastic discount rate

We model firms and banks as bankers and entrepreneurs, who are the sole owners of the institutions and their work is solely on management. This is not an optimal modelling approach. Practically, firms maximise their value, which is the present value of the future dividends, to the owner. However, in our model there are two households sectors. It is not clear which households' discount factor to

[6] For simplicity, we assume away the size adjustment in the asset correlation function.

use. Moreover, using the stochastic discount factor of the owner in firms' and banks' optimisation problem causes problems in motivating financial flow among the agents.
For instance, if banks maximise their value to the owner (the depositor), then the first order condition for deposit taking (μ_t^D) (equation (6)) becomes $\lambda_t^I - \beta_I E_t\left[\frac{\partial U^K}{\partial k_{t+1}} \frac{1}{RWA_{t+1}} \frac{1}{\Pi_{t+1}} r_t^D\right] = \beta_R E_t\left[\frac{\lambda_{t+1}^I}{\Pi_{t+1}}\left(1+r_t^D\right)\right]$, which can not agree with the depositor's first order condition, which is $\frac{1}{c_t^R} = \beta_R E_t\left[\frac{1}{\Pi_{t+1} c_{t+1}^R}(1+r_t^D)\right]$.
In such a case, the deposit market collapses because it is costly for banks to assume deposit due the to capital requirement. The same reason follows for the conflict between the optimality conditions of loan extensions had we assumed that firms and banks value the cash flow using the depositor's stochastic discount factor.

7.3 Optimality conditions

The first order conditions for borrowing households' labour supply n_t^P, borrowing μ_t^P and repayment rate v_t^P are given by

$$U^{P,L}\left(n_t^P\right) = \beta_P E_t\left[(1-s)\frac{w_t^P}{\Pi_{t+1}}\frac{\partial U^{P,C}}{\partial c_{t+1}^P}\right] + s\frac{w_t^P}{c_t^P}\frac{\partial U^{P,C}}{\partial c_t^P} \tag{23}$$

$$\frac{1}{(1+r_t^P)}\frac{\partial U^{P,C}}{\partial c_t^P} = \beta_P E_t\left[\frac{1}{\Pi_{t+1}}\frac{\partial U^{P,C}}{\partial c_{t+1}^P}\right] \tag{24}$$

$$\frac{\partial U^{P,C}}{\partial c_t^P} = \frac{\partial U_1^{P,D}}{\partial\left((1-v_t^P)\frac{\mu_{t-1}^P}{\Pi_t}\right)} + \frac{\partial U^{P,C}}{\partial c_t^P}\frac{\partial U_2^{P,D}}{\partial\left((1-v_t^P)\frac{\mu_{t-1}^P}{\Pi_t}\right)} \tag{25}$$

The optimality conditions for deposit households' labour supply n_t^R and borrowing d_t^R are given by

$$U^{R,L}\left(n_t^R\right) = \beta_R E_t\left[(1-s)\frac{w_t^R}{\Pi_{t+1}}\frac{\partial U^{R,C}}{\partial c_{t+1}^R}\right] + s\frac{w_t^R}{c_t^R}\frac{\partial U^{R,C}}{\partial c_t^R} \tag{26}$$

$$\frac{\partial U^{R,C}}{\partial c_t^R} = \beta_R E_t\left[\frac{\partial U^{R,C}}{\partial c_{t+1}^R}\frac{(1+r_t^R)}{\Pi_{t+1}}\right] \tag{27}$$

The first order conditions with respect to labour input h_t^P and h_t^R, capital K_t, goods sale q_t, repayment rate v_t^F and borrowing μ_t^F are given by

$$w_t^R \lambda_t^F = \frac{\partial U^{F,C}}{\partial c_t^F} \frac{\partial y_t}{\partial h_t^R} \tag{28}$$

$$w_t^P \lambda_t^F = \frac{\partial U^{F,C}}{\partial c_t^F} \frac{\partial y_t}{\partial h_t^P} \tag{29}$$

$$\frac{\partial U^{F,C}}{\partial c_t^F} = \beta_F \frac{\partial U^{F,C}}{\partial c_{t+1}^F} \left(\frac{\partial y_{t+1}}{\partial K_t} + 1 - \tau \right) \tag{30}$$

$$\frac{\partial U^{F,C}}{\partial c_t^F} = s\lambda_t^F + (1-s)\,\beta_F \frac{\lambda_{t+1}^F}{\Pi_{t+1}} \tag{31}$$

$$\lambda_t^F = \frac{\partial U_1^{F,D}}{\partial \left((1-v_t^F) \frac{\mu_{t-1}^F}{\Pi_t} \right)} + \frac{\partial U^{F,D}}{\partial c_t^F} (1 - v_{t-1}^F) \frac{\mu_{t-1}^F}{\Pi_t} \tag{32}$$

$$\frac{\lambda_t^F}{1 + r_t^F} = \beta_F E \left[\frac{\lambda_{t+1}^F}{\Pi_{t+1}} \right] \tag{33}$$