Services and the Dynamics of Female Labor Supply

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Abstract

This paper provides a quantitative assessment of the theoretical explanation for the rise in the service share in the U.S. posited by Buera and Kaboski (2012a,b). By extending their single household model to a two-person household model, we incorporate a joint household decision on home and market production into the model, which provides a direct link between female labor supply and the growth of service economy. The calibrated analysis shows that both the BK model and the extended BK model are able to match nearly all of the growth in the service sector, and the channels emphasized in the BK model are quantitatively important. The rising scale of services, the rising demand for skill-intensive output stemming from rising incomes, skill-biased technical change, and rising female labor supply all play important quantitative roles.

1 Introduction

Over the past 50 years, the US economy has moved increasingly toward a service-based economy, with the share of services rising roughly from 60 percent to 80 percent from 1965 to 2003. Buera and Kaboski (2012a, 2012b) identify theoretical forces contributing to the growth of the service economy. These forces stem ultimately from an increase in the optimal scale of service production and a shift in demand toward more skill-intensive output, which lead to an increase in the proportion of services that are market-produced relative to home-produced. The theory is attractive in that it is *qualitatively* consistent with several observations: growth in both the relative price and quantity of services, changes in patterns of home production, and, most importantly, growth in the average scale of service establishments and the shift toward skill-intensive services.[1] This paper uses calibration to examine the extent to which such an explanation is *quantitatively* plausible.

In the Buera-Kaboski (BK, hereafter) models, specialization plays a key role in the growth of service economy. Specialized human capital is utilized more efficiently on the market, where workers may specialize in production. The increasing demand for skill-intensive services increases the returns to specialized human capital, so that workers who become skilled

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[1] The growth of the service economy actually begins around midcentury. Buera and Kaboski (2008) also focus on the late acceleration of the service economy.

earn increasingly higher wages. As the opportunity cost of their time increases, they spend less time in home production and demand increasingly more market services. In addition, specialized intermediate/capital goods give rise to more efficient, larger scale production of services on the market than at home. In this way, a rising efficient scale of services interacts with both labor supply and investment in specialized human capital.

A quantitative assessment of these forces requires combining the two theories and extending the model in important dimensions. First, we add the possibility of both sector- and skill-biased technical change. Second, we move beyond the representative household framework, introducing heterogeneity in the cost of acquiring skills. Finally, we add demographics to the model that capture the different patterns in the data and incentives of married and single (male and female) households. In married households, one spouse may specialize predominantly in home production, while the other specializes in market production, and these decisions may be linked to decisions about human capital investment as well. Indeed, at the beginning of the period in question, women worked disproportionately in home production, while men worked disproportionately in the market. Indeed, shifts in female labor supply, due to both changes in the labor supply of married woman and changes in marriage rates, are clearly linked to the growth of the service economy (see, for example, Lee and Wolpin, 2006). As Figures 1 and 2 show, the growth in service sector quantitatively mirrors the growth in female labor in services (as a percent of the total labor force), while the decline of the good sector matches the decline in male labor in goods. All four are roughly linear changes of 16 percentage points over the period in question.[2]These extensions allow us to more closely match important features of the data, but also to assess the importance of female labor supply and demographic changes to the patterns of structural change.

We calibrate both the baseline model and this extended model to the U.S. experience. That is, we choose parameter values to target key facts of the economy and labor market in 1965, as well as growth in output, schooling, the relative wage of college educated workers, the relative price of services over the period, and market labor patterns. We capture this last feature by assuming a different relative productivity in home production for men and women. We then evaluate the model's predictions for the growth in the service share and, for the latter model, female labor supply.

Remarkably, despite no free parameters, both versions of the calibrated model are able to essentially fully explain the growth in the service sector. Counterfactual analyses using the two different models allow us to highlight the quantitatively important features of the models. In the benchmark model, skill-biased technical change plays the most important role, accounting alone for over half of the growth in services. Skill-biased technical change increases the service share by increasing the relative wage and relative quantity of high-skilled workers. The higher relative wage increases the opportunity cost of home production, thereby increasing the demand for market services from high-skilled individuals. The increasing proportion of high-skilled workers increases their importance on the economy. While these channels were highlighted in the BK (2012a), skill-biased technical change was not directly part of the original BK models, but its role in the growth of services comes out of the fact that skills are specialized and therefore only productive on the market.

[2]In comparison, the relative size of the labor force that is female and working in the goods sector decreased by just 4 percentage points, while that of males in the service sector increased by just 4.5 percentage points.

Moreover, the channels that were emphasized by BK (2012a, b), the rising skill intensity of demand and the rising scale of services, are also quantitatively important, together accounting for as much growth in the service share as skill-biased technical change. Rising skill-intensity of demand due to non-homothetic preferences causes has a direct effect on the demand for services, as well as the indirect channels emphasized above for skill-biased technical change. The rising scale of services increases the costs of home production. Alone, these each account for up to roughly one-third of the growth in services.

In contrast, sector-biased technical change – the faster productivity growth in manufacturing – leads to a *fall* in the share of services by leading to more home production. This is a unique feature of the model, and it is driven by home production being relatively more intense in manufactured goods. As opposed to our model, biased productivity explanations for the growth of services assume an inelastic substitution so that higher productivity growth in the goods sector increases the growth of the service sector.[3] These models predict a rising relative price of services, but a counterfactual decline in relative real quantities. In the BK model, a unique implication is that biased productivity in manufacturing actually *reduces* the growth of the service sector, since market services economize on intermediate goods/capital relative to home production. In contrast to biased productivity models, which require counterfactually large biases, the BK calibration matches the growth in the relative price of services with productivity growth in the service sector that is roughly 0.80 percentage points lower than in goods sector, relatively comparable to productivity measurements by Jorgensen and Stiroh (2000) over this period.[4]

We therefore conclude that the channels emphasized in the benchmark model are quantitatively important. The extended model yields important additional insights. Demographics (i.e., the falling share of married couples) , play a positive but relatively small role in explaining service share growth, although they explain a substantial share of the increase in female labor supply. Still, accounting for multiperson households weakens the forces in the Buera-Kaboski model, since it allows for partial specialization. However,. the extended model alone can explain only one-third of the increase in the catch up of female (market) labor supply with male labor supply. When we introduce a declining wedge to explain the full increase in female market labor supply, the predicted growth in services increases, but the explanatory power of the other mechanisms decreases.

Moreover, the model can explain a substantial fraction of the growth in the relative supply of high-skilled labor among women. High ability women become educated and increase labor supply at the fastest rate, while the labor supply of less educated men increases most slowly. In the data, the female-male ratio of college enrollment rates has roughly doubled from 1960 to 2003 (Goldin, 2006b). Mulligan and Rubinstein (2007) explain that the increase in female labor force participation (and the relative of women) has been driven by high ability, highly educated married women entering the labor force.

The remaining paper is organized as follows. The BK model is introduced in Section 2, and calibrated and evaluated in Section 3. Section 4 extends the model, and Section 5 provides calibration, and quantitative analysis to address multi-member households. Sec-

[3] See for example, Ngai and Pissarides (2006) and Baumol (1967)

[4] In Jorgensen and Stiroh (2000), the weighted average of labor productivity growth in the goods sector is 2.07 percent vs. 1.41 percent in the service sector. The analogous TFP growth rates were 0.67 and 0.26 percent.

tion 6 quantitatively evaluates additional empirical implications of the model's mechanisms. Section 7 concludes.

2 BK Model

This section integrates the theories developed in Buera and Kaboski (2012a, 2012b) for the growth of the service economy. This presentation merely extends BK to allow for sector-specific technical change, skill-biased technical change, and time-varying efficient scale of services. In order to more easily model demographic changes, we also model heterogeneity across households in the cost education/acquiring skills.

2.1 Production

There is a continuum of manufacturing goods and services, indexed by their complexity, $z \in [0, \infty)$. Manufacturing goods are produced only on the market, but services can be produced either on the market or at home. Manufacturing goods serve as intermediate input for both home and market production of final services. Technological progress is assumed to be exogenous, sector-specific, and skilled-specific.

2.2 Technologies

Manufactured goods are produced using low and/or high skilled labor, l_m and $h_{m,}$ respectively:

$$\text{Market Goods: } G(z,t) = A_G(t) \left[A_L(z) L_G(z) + \phi(t) A_H(z) H_G(z) \right] \tag{1}$$

Here, $A_G(t)$ is a manufacturing good-specific time-varying productivity term, $\phi(t)$ is a time-varying relative productivity between high-skilled and low-skilled workers, and $A_l(z)$ and $A_h(z)$ are time invariant but z-specific productivities of low- and high-skilled labor, respectively. We choose the following functional forms:

$$\begin{aligned} A_G(t) &= e^{\gamma_G t} \\ \phi(t) &= \phi_0 e^{\gamma_h t} \\ A_l(z) &= \frac{1}{z} \\ A_h(z) &= \frac{1}{z^\lambda} \end{aligned}$$

where γ_G captures the manufacturing specific productivity growth rate and γ_h captures any skill-bias in technological change, respectively. Since z represents complexity, productivities are decreasing in z, but we assume $\lambda \in (0, 1)$, so that high-skilled work has a comparative advantage in more complex output.

Manufactured goods are used as inputs into production of services. Production of service z requires one unit of manufactured good z as an intermediate. Following Buera and Kaboski (2012b), given the intermediate, services of type z are produced with constant labor productivity up to a maximum capacity. A simple example would be a washing machine that can do a maximum number of loads of laundry per day, with a certain amount of labor required for each load. Denoting the intermediate goods into services as k_s and the (time-varying) maximum capacity as $n(t)$, the production function is:

$$S(z,t) = \begin{cases} 0 & \text{if } k_S < 1 \\ \min\{n(t), A_S(t)[A_l(z)L_S + \phi(t)A_h(z)H_S]\} & \text{if } k_S \geq 1 \end{cases}$$

The capacity $n(t)$ will reflect the efficient output scale of a productive unit at which market services will be run, which we allow to change over time. In equilibrium, this parameter $n(t)$ will also be strongly related to the number of workers per productive unit. Note that the labor requirements for service z are symmetric to those for manufactured good z, except for the sector-specific term $A_S(t) = e^{\gamma_S t}$, which grows at rate γ_S.[5]

2.3 Firm's Problem

It is assumed that both manufacturing and service firms operate at the minimum average cost curves due to free entry. Making low-skilled labor the numeraire, and denoting the relative price of high-skilled workers as $w(t)$, equilibrium prices of manufactured goods and services are:

$$p_G(z,t) = \frac{1}{A_G(t)} \min\left\{\frac{1}{A_l(z)}, \frac{w(t)}{\phi(t)A_h(z)}\right\} \tag{2}$$

$$p_S(z,t) = \frac{p_G(z,t)}{n(t)} + \frac{1}{A_S(t)} \min\left\{\frac{1}{A_l(z)}, \frac{w(t)}{\phi(t)A_h(z)}\right\}. \tag{3}$$

The competitive price of services include two terms, the cost of intermediate goods and the cost of services value-added. The $n(t)$ in the demoniator of the first term reflects the fact that intermediate goods are used at their efficient scale in market services.

The minimizations above reflect the choice between low- and high-skilled workers. Given our comparative advantage assumption, they define a threshold, $\widehat{z}(t)$:

$$\widehat{z}(t) = \left(\frac{w(t)A_l(z)}{\phi(t)A_h(z)}\right)^{\frac{1}{1-\lambda}}$$

For $z \leq \widehat{z}(t)$, firms will hire low-skilled workers. When $z > \widehat{z}(t)$, firms will hire high-skilled workers instead. The threshold $\widehat{z}$ is an increasing function of $w(t)$.

[5] The symmetry between the service and manufactured good production function can be strengthened by writing the manufacturing goods technology as:

$$G(z,t) = \begin{cases} 0 & \text{if } k_G < 1 \\ \min\{n_G(t), A_G(t)[A_l(z)L_G + \phi(t)A_h(z)H_G]\} & \text{if } k_G \geq 1 \end{cases}$$

Thus, (1) would arise as the limiting expression for large efficient scale in manfuacturing, i.e., as $n_G \to \infty$.

2.4 Households

There is a continuum of infinitesimally-lived households that hold preferences over the continuum of services, purchase market goods and services, provide labor to market and household production, and decide whether or not to home produce services and whether or not to become high-skilled.

2.5 Preferences

Preferences over the continuum of discrete and satiable wants are indexed by the service that satisfies them, z. Define the function $C(z) : \mathbb{R}^+ \to \{0,1\}$, which takes the value of 1 if a particular want is satisfied and 0 otherwise. There are two ways to satisfy wants either by procuring the service directly from the market, or by purchasing the required manufactured goods to home produce the service. Let the function, $H(z) : \mathbb{R}^+ \to \{0,1\}$, indicate whether want z is satisfied by home production. Together the consumption set is defined by the set of indicator functions, $C(z)$ and $H(z)$, mapping $\mathbb{R}^+$ into $\{0,1\}^2$. The following utility function represent those preferences over wants and the method of satisfying those wants, i.e., over indicator functions $C(z)$ and $H(z)$:

$$\tilde{u}(C,H) = \int_0^{+\infty} [H(z) + \nu(1 - H(z))]\, C(z)\, dz \tag{4}$$

where $H(z) \leq C(z)$. The parameter $\nu \in (0,1)$ indicates that home-produced service yields a greater utility.

Given that a continuum of wants are satiated sequentially, and production costs, as well as the additional costs of home production are increasing in z, the consumer's problem can be simplified by the following step functions as the choice over the restricted consumption set:

$$C(z) = \begin{cases} 1 \text{ if } z \leq \bar{z} \\ 0 \text{ if } z > \bar{z} \end{cases}$$

and

$$H(z) = \begin{cases} 1 \text{ if } z \leq \underline{z} \\ 0 \text{ if } z > \underline{z} \end{cases}$$

where $\bar{z}$ denotes the most complex want that is satisfied, and $\underline{z}$ denotes the most complex want that is home-produced.

The primitive preferences (4) can then be simplified to the preferences over the restricted consumption set as a utility function over two thresholds $\underline{z}$ and $\bar{z}$:

$$u(\underline{z}, \bar{z}) = \underline{z}(1 - \nu) + \nu \bar{z} \tag{5}$$

with $0 \leq \underline{z} \leq \bar{z}$. On the margin, there are two ways for agents to increase utility: by increasing $\bar{z}$ to satisfy a want not yet satiated or by increasing $\underline{z}$ to move a market-satisfied want into home production.

2.6 Schooling

The schooling decision involves two choices: $e \in \{l, h\}$. l denotes low-skilled, and h denotes high-skilled. In order to become specialized high-skilled workers, $e = h$, an agent must spending a fraction θ of his/her time endowment acquiring skills. The population is heterogeneous in terms of the time required to acquire specialized skills. More specifically, θ falls between 0 and 1, distributed according to the c.d.f. $F(\theta)$.

2.7 Consumer's Problem

An individual with skill e, solves:

$$\begin{aligned} & V^e(\theta; t) = \max_{0 \leq \underline{z}_e \leq \bar{z}_e} (1-\nu)\, \underline{z}_e + \nu \bar{z}_e \\ & \text{s.t.} \\ & \int_0^{\underline{z}_e} p_G(z,t)\, dz + \int_{\underline{z}_e}^{\bar{z}_e} p_S(z,t)\, dz \\ = \; & w_e \left(1 - \int_0^{\underline{z}_e} \frac{1}{A_S(t) A_l(z)} dz - \theta \mathcal{I}(e)\right) \end{aligned} \tag{6}$$

where $\mathcal{I}(e)$ is an indicator function that equals one if $e = h$ and zero otherwise. The left-hand side of the budget constraint includes expenditures on manufactured goods (as intermediates into the home production of services) and expenditures on market services. Note that home production of a single unit of service $z \in [0, \underline{z}_e]$ requires paying for an entire manufactured input, $p_G(z,t)$, rather than the $1/n(t)$ units used in market production. The right-hand side is income from market labor, which is the the unit time allocation minus home production time and schooling time. Note, that because high-skilled workers are specialized, all home production (except for a measure zero) is done with the productivity of low-skilled workers.

At an interior optimum, $\underline{z}_e$ and $\bar{z}_e$ solve the following first order conditions:

$$\mu \left[\left(1 - \frac{1}{n(t)}\right) p_G(\underline{z}_e, t) + \frac{1}{A_S(t)} \left(\frac{w_e}{A_l(\underline{z}_e)} - \min\left\{\frac{1}{A_l(\underline{z}_e)}, \frac{w}{\phi(t) A_h(\underline{z}_e)}\right\}\right)\right] \geq 1 - \nu \tag{7}$$

and

$$\mu p_s(\bar{z}_e, t) = \nu$$

where $p_S(z,t)$ has been substituted using (3), and μ denotes the marginal utility of income.

Equation (7) is the marginal condition between home producing or market purchasing a service. The benefit of market services (left-hand side) includes the goods cost savings from the efficient utilization of intermediate goods and the potential labor cost savings that comes from hiring either more productive high-skilled labor, or low-wage, low skilled labor. The cost of market services (right-hand side) is the disutility of market consumption. For any particular z, the goods cost saving will decrease as the price of the manufactured good falls, and will increase as the efficient scale of services rises. The labor cost savings of market services are higher for high-skilled workers ($w_e = w$). Thus, a shift toward high-skilled workers decreases home production time in favor of market services. Moreover, the labor cost savings is increasing in the relative wage of high-skilled workers w for high-skilled workers, but decreasing for low-skilled workers ($w_e = 1$), so that increases in the relative wage affect workers differentially.

The schooling decision depends on the time cost and the relative wage. Being high-skilled will allow workers to earn a higher wage ($w > 1$), but it will reduce the time endowment to be $1 - \theta$, so the return to becoming high-skilled drops as θ increases. There exists a threshold, $\widehat{\theta}(t)$, that equalizes that value of being high- and low-skilled $V_h(\widehat{\theta}) = V_l(\widehat{\theta})$. For $\theta < \widehat{\theta}(t)$, a household will be strictly better off being high-skilled, while for $\theta \geqslant \widehat{\theta}(t)$, a household remains low-skilled.

2.8 Equilibrium

Given $w(t)$, a household decides whether to be high-skilled and decides the levels of $\underline{z}$ and $\overline{z}$. If a household decides to be low-skilled ($\theta \geqslant \widehat{\theta}(t)$), the levels of $\underline{z}_l(t)$ and $\overline{z}_l(t)$ are independent of θ. If a household decides to be high-skilled ($\theta < \widehat{\theta}(t)$), the levels of $\underline{z}_h(\theta, t)$ and $\overline{z}_h(\theta, t)$ will increase as θ decreases. Given $w(t)$, each firm sets the prices $p_G(z, t)$ and $p_S(z, t)$ according to (2) and (3), respectively.

A competitive equilibrium consists of $w(t)$ and $\widehat{\theta}(t)$, $\underline{z}_l(t)$, $\overline{z}_l(t)$, $\underline{z}_h(\theta, t)$, $\overline{z}_h(\theta, t)$, $\widehat{z}(t)$, and the price functions $p_m(z, t)$ and $p_s(z, t)$. The model can be solved in two steps recursively. The first step is to solve for the schooling threshold ($\widehat{\theta}(t)$) and consumption thresholds ($\underline{z}_l(t)$, $\overline{z}_l(t)$, $\underline{z}_h(\theta, t)$, and $\overline{z}_h(\theta, t)$) given $w(t)$. The price functions are determined by $\widehat{z}(t)$ and $w(t)$. The second step is to solve for $w(t)$ from a market clearing condition given the schooling threshold and consumption thresholds. Then, repeat the first and second steps until the solution converges.

In a similar model, BK (2012a) show that the disaggregate model can be expressed as a more standard model over aggregate consumption of manufactured goods and services, but the preferences vary with productivity. Moreover, productivity increases that are balanced, in the sense that $A_G(t) = A_S(t)$ and $\phi(t) = 1$, productivity, have been shown to yield growth in the service sector that is qualitatively consistent with several features of the data (see BK, 2012a, 2012b). First, the growth of services is delayed. At low levels of income, growth leads to new services being consumed on the market, but old market services moving to home production as the cost of intermediates falls. This feature is least relevant for the quantitative analysis, since our analysis only covers the period of rising services. Second, and more relevant, the growth of services is driven by the growth of high-skilled services. As incomes continue to rise, demand shifts toward ever more complex output at which specialized high-skilled workers have an ever increasing comparative advantage. Market services increase as these complex services are more difficult to move into home production. In turn, the demand for high-skilled workers increases, and more agents decide to specialize. Given $F(\theta)$, the supply curve for skilled workers is upward sloping. As the relative wage increases, this increases the demand for market services among high-skilled workers, who constitute an ever increasing share of the economy. Third, since manufactured goods are produced on the market for the full range of z consumed, while only high z services are consumed on the market, market services are more intensive in high-skilled labor. Ceteris paribus, a rising relative wage w leads to increases in the relative price of services. Finally, the share of services is increasing in their efficient scale of production $n(t)$, which has trended up. This growth in scale in turn decreases labor used in home production in favor of market production, and thereby also increases the incentives for acquiring skill. The following section calibrates the relevant features to quantify these effects.

3 Calibration of the BK Model

The BK model is sufficiently different from conventional structural change models; BK (2012a) show that their model aggregates into a more conventional model with preferences over aggregate market services and manufactured goods, but that those preferences are non-homothetic and time-varying. In any case, there is no existing literature on appropriate parameter values. Although all parameters are jointly determined, we rationalize our choice of particular moments relevant to each specific parameter below.

Our calibration approach is to start by mapping the model to key features of the 1965 economy. We choose the preference parameter ν, which captures the utility of market services relative to home-produced, to match the initial share of services, 0.63. The technology parameter n determines the ratio intermediate manufacturing inputs to value-added. We choose its initial value n_0 to target this value. From input-output tables, this value is 0.12 in 1965.

We follow BK (2012a) in viewing college education as the appropriate empirical counterpart to high-skilled workers. We choose the initial relative productivity of high-skilled workers (in low complexity output), ϕ_0, so that the relative wage in the model matches the college skill premium in 1965 of 1.41. The skill premium data are taken from hourly wage data from the Current Population Survey (CPS), using male full time workers between the ages of 21 and 65. We need to also calibrate the distribution of θ, the cost of acquiring skills in the model. We assume that θ follows a Beta distribution, $\beta(a, b)$, which supports θ between 0 and 1 and assures an interior solution for the fraction of workers acquiring specialized skills. The calibrated distribution can be left-skewed or right-skewed as well as symmetric, depending on the values of a and b. We use one of these parameters to target is the fraction of workers that are college-educated in 1965, 0.22, based on workers aged 21-65 in the CPS.[6]

The other parameter in the beta distribution is chosen to match the increase in this fraction between 1965 and 2003. We target several other time trends as well. We choose the rates of technical change in each sector, γ_S and γ_G, to match growth in real GDP per capita (a 25 percent increase between 1965-2003) and the change in the relative price of services to manufacturing. which increased 50 percent over the same years. The parameter λ captures the comparative advantage of high-skilled workers in more complex output. This effectively governs the rising demand for skill that stems from rising incomes we have calibrated. The skill-biased technical change captured by γ_h has a similar effect. Together, we choose these two parameters to match both the increase in the quantity (the fraction of college-educated

[6] Details of data sources and calculations are available from the authors in an unpublished data appendix.

workers) and relative wage (college premium) between 1965 and 2003. In 2003, the skill premium target is 1.77.

Finally, we calibrate changes in n in order to capture the changing scale emphasized by BK (2012b). Changes in n translate into changes in workers per establishment in services. Once the initial efficient scale value, n_0, is calibrated, the remaining time series of n is constructed from data on the workers per service establishment.. The average service establishment is 1.64 times as many workers in 2003 as in 1965 based on County Business Patterns data.

A summary of parameters and targets is given in Table 1. Table 2 provides the actual calibrated values. (The results for the alternative models will be discussed in the later section.) As shown in the second column of Table 2, the BK model is able to hit all the data moments. The calibrated θ distribution is right-skewed (with a larger mass on the smaller values of θ). Although the rising skill premium itself leads to some growth in the relative price of services, targeting the relative prices still requires slightly lower TFP growth in the service sector is 0.014, which about two-thirds the TFP growth rate in the manufacturing sector (0.021). This is relatively comparable to productivity measurements by Jorgensen and Stiroh (2000) over this period, which is allowed for by the fact that the rising skill premium accounts for the balance of the growth in the relative price of services. A more standard model of biased productivity growth would need a (counterfactually) larger bias sectoral productivities.

The skill-biased productivity growth adds an additional roughly half a percent to high-skilled workers productivity annually, amounts to about 21 percentage points by 2003. The relative wage is 36 percentage points higher in 2003 than in 1965, so the remainder comes from the movement toward more complex goods and the comparative advantage parameter λ.

3.1 Accounting the Rising Service Share

We now analyze the model's predictions for the change in the service share over time. Note that this is purely an out of sample test, since the service share was in no way targeted by our calibration. We will focus on the predictions for the long run change between 1965 and 2003. The higher frequency dynamics of this change are not particularly interesting; the model itself is static, we do not account for business cycle factors, and the calibration assumed linear productivity trends. We simply note only that the effects occur fairly linearly with increased productivity, so the model matches the relatively stable time trends in the data quite well in this regard, with the exception of the skill premium, which declined in the 1970s before accelerating in the 1980s.[7]

The model does quite qell in reproducing this growth in the service share as shown in Table 3. In 1965, the service share in the model matches that in data (0.63) by construction (i.e., via our calibration). In 2003, the model predicts a service share of 0.80, nearly identical to the 0.79 in the data. This is our first important finding: the model is able to fully quantitatively explain the 17 percentage point increase in the share of services observed in the data. To put this change in perspective, 17 percentage points currently exceeds the total size of the manufacturing sector in 2012.

[7] The literature has typically pointed to the importance of cohort effects, specifically the baby boom, in explaining this, while assuming a constant skill bias in technical change (e.g., Katz and Murphy, 1993). These cohort effects are clearly outside the model.

We now examine which factors are most important in accounting for this increase. We have four exogenous factors that change over time, which we examine in turn. We examine their role by running counterfactuals where either the factor in question is held constant in the model (i.e., the factor is "turned off") or when the factor in question is the only factor not held constant in the model (i.e., the only factor "turned on"). We turn factors off by keeping the relevant parameters at their calibrated 1965 levels, and turn factors on by setting them at their calibrated 2003 levels.

The results are shown in Table 4. Since the calibration hits the 1965 service share for every simulation, we focus on the overall service increase explained by different simulations and how it differs from either the benchmark simulation, when we turn off factors, or how it differs from zero, when we turn on factors. Turning on factors is giving the 2003 value to the 1965 economy, while turning off factors is effectively giving the 1965 value to the 2003 economy.

The first factor is simply the increase in productivity, which pushes demand for more skill-intensive services because $\lambda > 0$. We call this the income effect. As explained by BK (2012a), this has both a direct effect, since for these more complex services, market services are cheaper than home production. It also has an indirect effect by increasing the demand for high-skilled workers. The share of services in consumption is higher for high-skilled workers, since their opportunity cost of home production is higher. Moreover, it is increasing in the skill premium, which captures this opportunity cost. Hence, higher demand for skill leads to both a higher skill premium and more high-skilled workers, both of which contribute to a higher share of services. Quantiatively, Table 4 indicates that this effect accounts for a six percentage point increase in services, or roughly one-third of the total increase in the benchmark model

The second factor is the factor emphasized by BK (2012b): the rising scale of services, n. Larger scale services lead to a larger cost differential between home and market services because the market accomodates on the manufactured inputs, which are a fixed cost. Thus, larger scale services lead to more market services. Quantitatively, this factor is also non-negligible accounting for an 8 percentage point increase in services, when it is the only factor turned on, and a 5 percentage point smaller increase when it is the only factor turned off. These amount to 45 and 32 percent of the total increase, respectively. The difference comes from the fact that cost differences are driven by scale relatively more in 1965 but skill relatively more in 2003.

These first two factors are unique to the BK model. To see how important these two factors are, we turned them both on and off together. Together, they account for 12 percent points (when both turned on) and 13 percentage points (when both turned off), or 73 and 83 percent of the total increase, respectively. Thus, it appears that these two BK-specific factors each play an important role, and together the account for the bulk of the increase in the service share.

The third factor, skill biased technical change, is also quite important. Skill-biased technical change is certainly part of other models, but our emphasis on specialized skills being specific to market production implies that skill-biased technical change leads to growth in services. The logic is the same as for the indirect channels of the income effect explained above, where the higher demand for skill leads to a higher opportunity cost of home production for high-skilled workers and a higher fraction of high-skilled workers. Skill-biased technical change accounts for a roughly 12 percentage point increase in services, larger than either of the first two factors individually. This amounts to 73 to 83 percent (the difference stems from rounding the percentage point increase) of the total increase in services.

Thus, if all factors were additivie and positive, we would already have over accounted for the increase in services.

However, the fourth factor, sector-biased technical change, works in the opposite direction. In most biased productivity models (e.g., Ngai and Pissarides, 2007), faster technical change in the manufacturing sector (coupled with an elasticity of substitution less than one) leads to a rising share of the service sector. In the BK model, however, one of the benefits of market services in the model is that they save on the cost of manufactured inputs by taking advantage of efficient scale. Biased technical change in favor of manufacturing, makes these inputs become relatively cheap. The cost savings in market services disappears leading to more purchases of manufactured goods for home production and fewer market services. In order to isolate the effect of biased technical change from overall technical change (i.e. the productivity/income effect of the first factor), we change relative productivity across the sectors without changing absolute productivity. Table 4 indeed shows that this factor works to reduce the share of services, but the strength of this factor depends strongly on the presence of others. When it is the only factor turned on, i.e., if it is turned on in 1963, the cost savings coming from market services are relatively small. Hence, it leads to a 12 percentage point decrease in the service share. However, when it is turned off in 2003, it leads to only a two percentage point decrease. This is because the other factors make the cost savings of market services in 2003 primarily skill-driven rather than goods-driven.

The results indicate that the market for skill plays an important role in the rise of services. Table 5 illustrates this more clearly by showing the role of the endogenous increase in the skill premium and the endogenous increase in schooling attainment. We do this by solving and aggregating households problems at the benchmark equilibrium prices, but keeping either the relative wage fixed, schooling decisions fixed, or both fixed. (Effectively, we model a and aggregate a partial equilibrium economy, where goods and labor markets need not clear). We learn two things. First, when both are kept fixed, the increase in the service share is only 4 percentage points, indicating that these labor market adjustments coming from the increased demand for skill are critical. Second, we see that the increase in the skill premium plays an important role in any case, but that the increase in schooling only plays an important role, when the skill premium also increases. This is because when the skill premium is high, the share of services in high-skill consumption is much higher than it is in low-skill consumption.

To summarize, the benchmark model has shown that forces of the BK model are quantitatively important and can explain the observed increase in the service share. However, much of the action in the model comes through the rising opportunity cost of home production, i.e., the rising skill premium. We now extend the model to see whether these results hold up in a model with multiple person households, where the opportunity cost may not be the skill premium because one worker can specialize in market production.

4 Extended Model

We now extend the BK model by adding a gender-specific component in home production, which generates a mechanism for household specialization. The increase in female labor supply is integrated in the process of structural change, which allows us to evaluate the model vis-a-vis its implications for female labor supply, and to assess the role of female labor supply on the quantitative predictions for the disproportionate growth of the service sector.

It is empirically interesting to disaggregate labor by gender and by the marital status. According to the Current Population Survey, in 1965, about 12 percent of the population aged 21 to 65 were single women (or widows). By 2003, single women constituted 20 percent of this population. In addition, the market work hours of single female is about 80 percent of the market work hours of their male counterparts during the same period according to the American Time Use Survey. Moreover, during the same period, the market work hours of married female relative to the market work hours of their male counterparts increased from 0.29 to 0.58, which may be in part explained by the increase of percentage of married females with high school or college education. Hence, a greater proportion of single women and the increase in skill intensity among married women could potentially explain a good portion of the increase in the service economy. On the other hand, the existence of married households themselves may weaken the impact of a rising skill premium on the demand for services, since households can specialize.

The production/technology side of the extended model is identical to the BK model presented in the previous section, so we only explain the household side of the extended model.

4.1 Households

There are three types of households in the extended model: single women, single men, and married couples.

Each type of household is again infinitesimally-lived and differ by $\theta \in [0, 1]$, where $\theta \sim F(\theta)$, but the fractions of each type of households in the overall population are exogenous to the model. We implicitly assume perfect assortative matching among spouses in married couples, which is clearly an abstraction.

Single male and single female households are identical to households in the previous section, except that they differ by gender-specific productivity of home production. There is an additional household decision to the BK model. Married couples decide schooling and labor supply decisions jointly, but may optimally choose for schooling and labor allocations between home and market production differed for the husband and wife. Based on comparative advantage, women will spend relatively more time in home production, while men supply relatively more labor to the market.

4.2 Preferences

As before, a single-person household requires one unit of services to satiate want z, but married couple households now require 2 units. Formally:

$$\widetilde{u}(C,H) = \int_0^{+\infty} [H(z) + \nu(1 - H(z))] \cdot C(z) \cdot Q \cdot dz \tag{8}$$

The additional parameter Q equals 1 if it is a single-person household's preference function and 2 if it is a married couple's preference function.

4.3 Consumer's Problem

A single household solves the following maximization problem by choosing $\underline{z}$, $\overline{z}$, and e:

$$\begin{aligned} V_{e,g}(\theta) &= \max_{\underline{z},\overline{z},e\in\{l,h\}} (1-\nu)\underline{z}_e + \nu\overline{z}_e \\ & \quad \text{s.t.} \\ \int_0^{\underline{z}_e} p_G(z,t)dz + \int_{\underline{z}_e}^{\overline{z}_e} p_S(z,t)dz &= w_e(1-\tau_{e,t})(1 - \int_0^{\underline{z}_e} \frac{1}{A_S(t)A_L(z)A_g} dz - \theta I(e)) + T(t) \end{aligned}$$

The value function V is now indexed by g, which is the gender of the individual. All terms are identical to the BK model, with the exception of the home production time which now depends on the gender- specific productivity, A_g. Thus, the productivity in home production is allowed to differ from productivity of low-skilled workers by a scalar, and this scalar differs for men and women. Quantitatively, A_f is expected to be greater than A_m so that females have a comparative advantage in home production in order to match the gender-specific differences in home-production time.[8] Given this difference, the threshold of the ability level being indifferent between becoming high-skilled and low-skilled, $\widehat{\theta}_g$, will now be gender-specific in equilibrium.

The parameter $(1 - \tau_{e,t})$ is a wedge introduced to match the gender wage gap in 1965. The wedge is modeled as a tax which is then rebated through lump sum transfers $T(t)$. That is, $\tau_{m,t} = 0$, but $\tau_{f,t} \geq 0$. We introduce this wedge for both married and single couples, but it has a larger impact for married women. As explained in Jones, Manuelli and McGrattan (2003), a decline in the gender wage gap is able to generate a larger labor supply response from married women than from single women because married women can choose to specialize in home production. For example, when there is a large gender wage gap, the labor supply of single women cannot be reduced to zero completely, but married women can choose to home-produce services by relying on their husbands' income to purchase intermediate inputs of home production. The household division of labor interacted with a declining gender wage gap could potentially explain a larger increase in married women's labor supply than in single women's labor supply. For simplicity we introduce a falling wedge, between 1965 and 2003, under the assumption that the wedge is zero in 2003. We solely focus on the tax wedge faced by low-skilled women (τ_{l0})[9]. Although this falling wedge is ad hoc, as we will

[8] In principle, one might want to allow market productivity to vary with gender as well, in order to match observed differences in market wages.

[9] It is not unreaonable to expect that high-skilled women may also face a tax wedge, τ_{h0}. However, τ_{h0}

see that it is used to match the observed increase in married women's labor supply, which we model below.

A married couple's problem is similar to a single-person household's problem, but the consumption, schooling, market labor, and home production decisions are jointly determined between a husband and wife. For simplicity, we define a threshold $\tilde{z}$ such that the wife performs all home production below $\tilde{z}$, and the husband performs all home production between $\tilde{z}$ and $\underline{z}$.[10] Using $\tilde{z}$, we define t_m and t_f as the amount of time spent in home production, and we require that these be bounded (weakly) above zero and below the available labor supply of each individual. The couple's problem is therefore:

$$\begin{aligned}
\max_{\widetilde{z}_{ee} \leq \underline{z}_{ee} \leq \overline{z}_{ee},\ e_m, e_f \in \{l,h\}} V^{e_m e_f}(\theta) &= 2(1-\nu)\underline{z}_{ee} + 2\nu \overline{z}_{ee} \\
&\quad s.t. \\
2\int_0^{\underline{z}_{ee}} p_G(z,t)dz + 2\int_{\underline{z}_{ee}}^{\overline{z}_{ee}} p_S(z,t)dz &= w_{e_m}(1 - t_m - \theta I(e_m)) + w_{e_f}(1-\tau_{e,t})(1 - t_f - \theta I(e_f)) + T(t) \\
t_m &= 2\int_{\widetilde{z}_{ee}}^{\underline{z}_{ee}} \frac{e^{-g_s t} z}{A_m} dz \geqslant 0, \\
t_f &= 2\int_0^{\widetilde{z}_{ee}} \frac{e^{-g_s t} z}{A_f} dz \geqslant 0 \\
1 - t_m - \theta I(e_m) &\geqslant 0,\ 1 - t_f - \theta I(e_f) \geqslant 0,\ 2\left(\underline{z}_{ee} - \widetilde{z}_{ee}\right) \geqslant 0 \qquad (9)
\end{aligned}$$

We denote the individual education choices of the husband and wife as e_m and e_f, respectively. There are four schooling choices: 1) both husband and wife choose to be high-skilled (hh); 2) both husband and wife choose to be low-skilled (ll); 3) only a husband chooses to be high-skilled (hl); and 4) only a wife chooses to be high-skilled (lh). If $A_f > 1$, the schooling choice (lh) will never be chosen, explained in the following proposition.

Proposition 1 *Given $A_m < A_f$, the schooling choice of lh (low-skilled husband, high-skilled wife) will always be dominated by hl (high-skilled husband, low-skilled wife).*

Proof. See Appendix B. Given a schooling choice, the Kuhn-Tucker conditions that characterize the optimum, $\underline{z}_{ee}$, $\overline{z}_{ee}$ and $\widetilde{z}_{ee}$ are

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does not play an important role in our re-calibration exercise because the majority of women are low-skilled in the initial period. If we repeat our re-calibration exercise by focusing on τ_{h0} instead of τ_{l0}, or assuming $\tau_{h0} = \tau_{l0}$, we will not be able to hit all the data moments listed in Table 1.

[10]The formulation is equivalent if we define a threshold $\tilde{z}$ such that the husband performs all home production below $\tilde{z}$, and the wife performs all home production between $\tilde{z}$ and $\underline{z}$. See Appendix A.

$$\mu(\frac{e^{-g_s t}\underline{z}_{ee}}{A_m}w_{e_m} + p_m(\underline{z}_{ee}, t) - p_s(\underline{z}_{ee}, t)) + \eta_1 \frac{e^{-g_s t}\underline{z}_{ee}}{A_m} - \eta_3 = 1 - \nu \quad (10)$$

$$\nu = \mu p_s(\overline{z}_{ee}, t)$$

$$\mu w_{e_m}\frac{e^{-g_s t}\widetilde{z}_{ee}}{A_m} - \mu w_{e_f}\frac{e^{-g_s t}\widetilde{z}_{ee}}{A_f} = \eta_3 + \eta_2 \frac{e^{-g_s t}\widetilde{z}_{ee}}{A_f} - \eta_1 \frac{e^{-g_s t}\widetilde{z}_{ee}}{A_m} \quad (11)$$

$$1 - t_m - \theta I(e_m) \geqslant 0, \eta_1 \geqslant 0, (1 - t_m - \theta I(e_m))\eta_1 = 0 \quad (12)$$

$$1 - t_f - \theta I(e_f) \geqslant 0, \eta_2 \geqslant 0, (1 - t_f - \theta I(e_f))\eta_2 = 0 \quad (13)$$

$$2\left(\underline{z}_{ee} - \widetilde{z}_{ee}\right) \geqslant 0, \eta_3 \geqslant 0, 2\left(\underline{z}_{ee} - \widetilde{z}_{ee}\right)\eta_3 = 0 \quad (14)$$

where η_1, η_2 and η_3 are the Kuhn-Tucker multipliers, described in Conditions (12), (13) and (14), respectively. μ is the marginal utility of income. Condition (11) characterizes the division of home production ($\tilde{z}$). It is driven by comparative advantage in home production, depending on the values of A_f and A_m. If $A_f > A_m$, the wife will have a comparative advantage in home production, which leads to the following proposition.

Proposition 2 *Given $A_f > A_m$ and $w > 1$, at least one spouse will fully specialize. If the husband works both at home and in the market, his wife will fully specialize in home production. If the wife works both at home and in the market, her husband will fully specialize in market production.*

Proof. See Appendix B. Without loss of generality, we assume $A_m < A_f$ for the remaining discussion. Given a married couple's value function defined in (9), both husband and wife choose to be high-skilled iff:

$$V^{hh}(\theta) \geq \max\{V^{hl}(\theta), V^{ll}\}$$

By the same token, only the husband chooses to be high-skilled iff:

$$V^{hl}(\theta) \geq \max\{V^{hh}(\theta), V^{ll}\}$$

The value of the schooling choice (ll) is independent of θ while the values of the schooling choices (hh) and (hl) are strictly decreasing in θ. If the skill premium is positive, $V^{hh}(0) > V^{hl}(0)$ and $V^{hl}(0) > V^{ll}$. Moreover, when $\theta = 1$, $V^{ll} > V^{hl}(1)$ and $V^{hl}(1) > V^{hh}(1)$. Thus, there will exist two unique thresholds, $(\widehat{\theta}_1, \widehat{\theta}_2)$, such that

$$\begin{aligned} V^{hh}(\widehat{\theta}_1) &= V^{hl}(\widehat{\theta}_1) \\ V^{hl}(\widehat{\theta}_2) &= V^{ll} \end{aligned}$$

For $\theta \in [0, \widehat{\theta}_1)$, both husband and wife choose to be high-skilled. For $\theta \in (\widehat{\theta}_1, \widehat{\theta}_2]$, only the husband chooses to be high-skilled. For $\theta \in (\widehat{\theta}_2, 1]$, both will remain low-skilled. If the wife's time constraint is not binding, $\widehat{\theta}_2$ is equal to $1 - 1/w$, which equalizes the net wages between a high-skilled and a low-skilled husband.

4.4 Equilibrium

A competitive equilibrium consists of $w(t)$, $\widehat{\theta}_m$, $\widehat{\theta}_f$, $\widehat{\theta}_1$, and $\widehat{\theta}_2$, $\widehat{z}(t)$, the price functions $p_G(z,t)$ and $p_S(z,t)$, and the consumption thresholds ($\underline{z}_l(t)$, $\overline{z}_l(t)$, $\underline{z}_h(\theta,t)$, $\overline{z}_h(\theta,t)$, $\underline{z}_{ll}(t)$, $\overline{z}_{ll}(t)$, $\underline{z}_{hl}(\theta,t)$, $\overline{z}_{hl}(\theta,t)$, $\underline{z}_{hh}(\theta,t)$, $\overline{z}_{hh}(\theta,t)$). The model can be solved in two steps recursively, in a fashion very similar to in the BK model. The derivations of a market clearing condition for high-skilled workers are provided in Appendix C.

5 Calibration of the Extended Model

In this section, we calibrate our extended model. We have added three different parameters: the relative productivity of men and women in home production, A_m and A_f, respectively, and the change in female labor supply wedge $\Delta\tau_f$ We calibrate the model using the same approach as in the benchmark BK model, but adding the following three target moments. We use A_m and A_f to match the initial relative market work hours of (1) married women to married men and (2) single women to single men. The decline in the wedge is chosen to match the increase in female's relative labor supply. A portion of this increase can be explained endogenously, but we will show that it is necessary to match the full incrase.

In addition, we change the representation of household types (married, single) to match their changing representation in the data over time. The proportion of people in married couples fell from 79 percent in 1963 to just 60 percent in 2003. Correspondingly, the proportion of single households doubled from 21 to 40 percent.

We return to Tables 1 and 2, which summarize the calibration strategy and results respectively. The extended model is given in the third column of each table. Again, we are able to hit all the data moments. The same patterns qualitatively hold with a rightward skewed θ distribution, and very similar productivity parameters. However, the comparative advantage parameter λ is about 50 percent greater in the extended model.

5.1 Accounting for Female Labor Supply and Service Growth

We now examine the models predictions for the growth in female labor supply and the value-added share of the service sector, and the roles of the different factors. We start by focusing on female labor supply, since this is the new element of the extended model, and it will provides justification for introducing the falling wedge on female labor supply.

Table 6 shows these results. We provide two different models, one in which the female labor supply wedge falls over time, and one in which it is held constant. The constant wedge model is able to explain a 9 percentage point increase in the labor supply of females relative to males. This comes directly from the mechanisms in the model, as we will see. This constitutes one-third of the actual 27 percentage point increase in the data, a substantial share, but clearly it does not explain the full growth in female labor supply. Thus, in order to fully account for the observed increase in the model it is necessary to incorporate the falling wedge. By construction, this model is able to explain the full 27 percentage point increase in the data.

We turn now to the role of the different factors. For simplicity, we only present the results for "turning off" factors in the 2003 economy. In addition to the original four factors in the benchmark model, we now have a fifth exogenous factor in the model which is the change in the fraction of the population that is married. This demographic change, shown in the bottom row of the table, is important in explaining the relative increase in female labor supply, explaining 9 percentage points alone. The scale effect and especially the income effect are relatively small in accounting for the increase in female labor supply, together accounting for about as much as the fall in the proportion married. Skill-biased technical change is the strongest positive factor accounting for 16 percentage points in the model with a constant wedge and 11 in the model with the falling wedge. However, the most important factor in explaining female labor supply is actually sector-biased technological change, which is again a negative factor, lowering relative female labor supply by 21 percentage points in the case with a constant wedge and and 13 percentage points in the case of a falling labor wedge. Females, especially married females, will tend to be less educated in these models, since they will specialize more in home production. Because of this, the opportunity cost savings from skilled work will be not as important for service decisions compared to the goods cost. Thus, the falling price of goods will has a relatively strong effect, especially in the model without the falling wedge, where female home production will be even higher and female education even lower. The wedge becomes important therefore in helping discipline the strength of the home production vs. market services margin.

Table 7 shows the results for the growth in services in the extended models with and without the falling wedge. The constant wedge model produced an increase of 17 percentage points, while the falling wedge model actually substantially overexplains the growth, yielding 20 percentage point increase. Thus it appears that the ability of the model to quantitatively explain the relative growth of the service sector is robust to the introduction of married couples and demographic changes. The higher share for the model with the larger increase in female labor supply, shows that female labor supply plays an important role in this growth, since females at home are engaged in household production of services, and demand for services therefore increases as they move to the market.

The decompositions show the importance of various factors. We compare with the results in Table 4 to see the impact of adding female labor and demographics to the model. As anticipated, the magnitudes of the income effect and scale effects are reduced substantially, since married households can specialize. The impact of turning off the two together was 13 percentage points in the benchmark BK model, whereas it is only 7 percentage points in the extended model with a constant wedge and 5 percentage points in the model with the falling wedge. The impact of skill-biased technical change also falls, though not as dramatically, falling from 12 in the benchmark to 10 percentage points in the constant wedge model and 6 in the falling wedge model. Somewhat surprisingly, although sector-biased technical change plays an important role in female labor supply, it still has a relatively small role on the share of services. Perhaps it is because these movements are concentrated among the lower income (i.e., high θ) households.

In the case of all of these factors, their impacts are lower in the model with an exogenously falling wedge. We conjecture this is because in this model the wedge drives the larger female labor supply, so female labor supply is less affected by these different factors at the margins. This wedge clearly contributes a great deal directly to the growth in the share of services. This can be seen by the facts that the 20 percentage point increase in the share of the service sector in the model with a falling wedge is both 3 percentage points higher than the model with the fixed wedge but also 7 percentage points higher than the sum of the individual factors which is just 13 percentage points.

In sum, the conclusion that the model can explain the growth in services is robust to the addition of female labor suppy and married couples, and the model itself can explain about a third of the catchup of female labor supply with male labor supply. Still, the declining proportion of married couples in the population plays a role in this, while the factors emphasized in the benchmark BK model are somewhat less important. This is especially true in a model that quantitatively matches the increase in female labor supply for exogenous reasons.

6 Conclusion

We have shown that the BK model is a quantitatively plausible explanation for the observed growth in the share of services in the United States between 1965 and 2003. In particular, the rising scale of services, rising demand for skill-intensive output stemming from income effects, and skill-biased technical change all play quantitatively important roles in the growth of services. These latter two manifest themselves largely through increases in the skill premium and fraction of the population who is high-skilled. These results are robust to extending the model to allow for married couples, specialization and gender-specific labor supply. However, in this model the falling proportion of married couples in the population and exogenous forces driving female labor supply also play quantitatively important roles.

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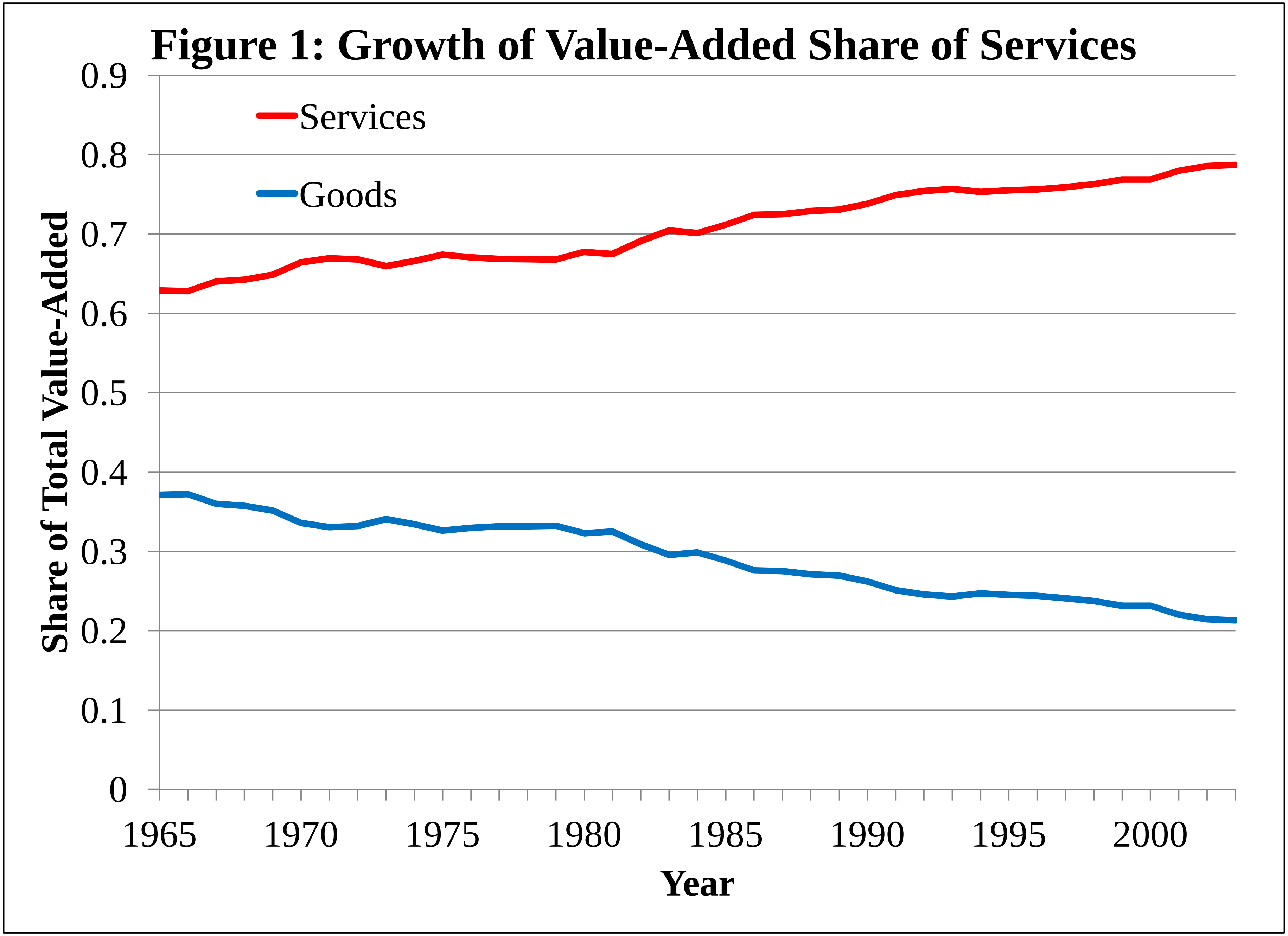
Figure 1: Growth of Value-Added Share of Services
Services
Goods
Share of Total Value-Added
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1965
1970
1975
1980
1985
1990
1995
2000
Year

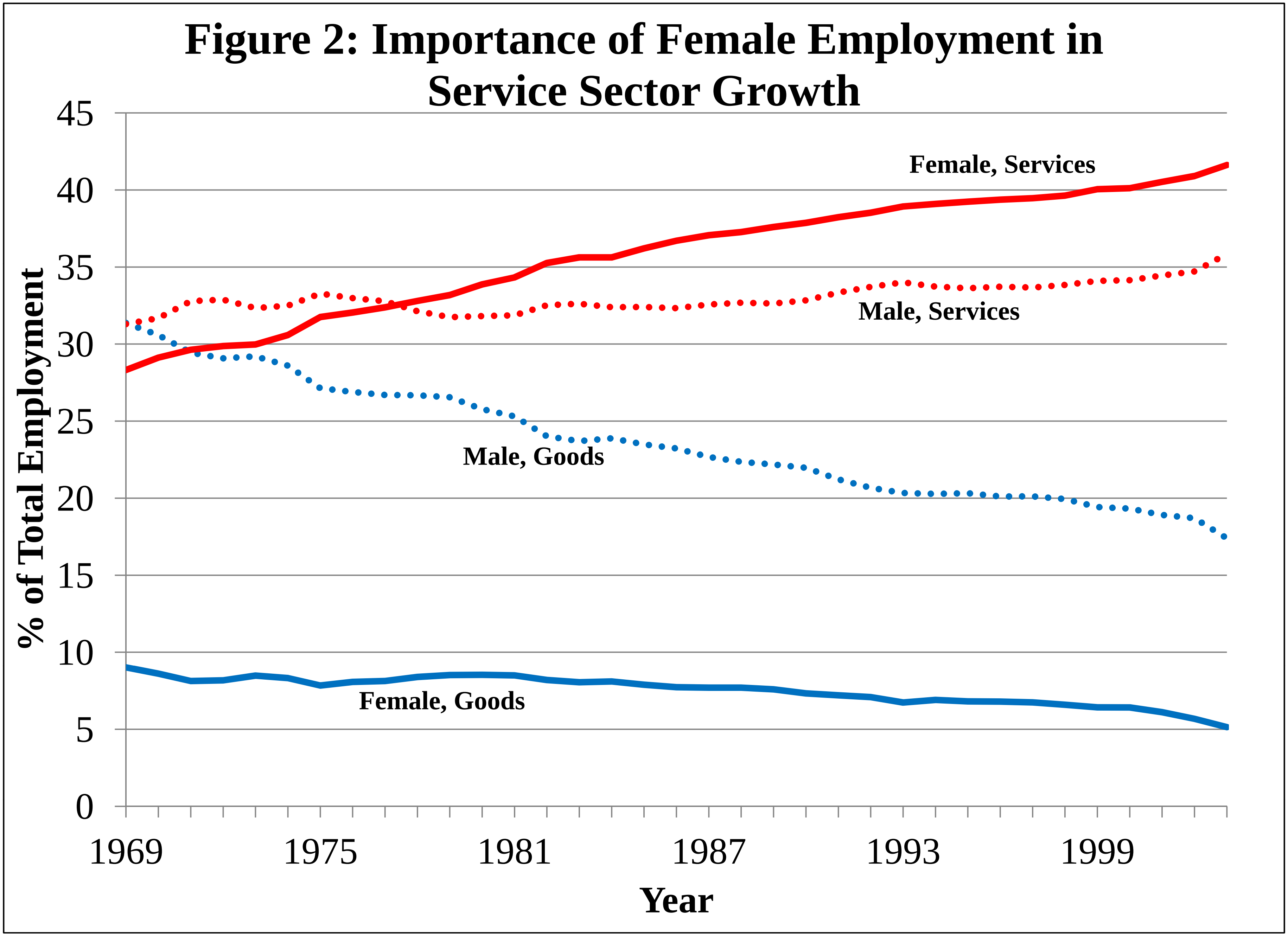
Figure 2: Importance of Female Employment in Service Sector Growth
% of Total Employment
45
40
35
30
25
20
15
10
5
0
1969
1975
1981
1987
1993
1999
Year
Female, Services
Male, Services
Male, Goods
Female, Goods

Table 1 – Calibration Strategies

	BK Model	Extended Model
Initial relative wage	A_h	A_h
Relative wage growth, high/low skilled	λ	λ
Intermediate manuf. input to the value added of the service sector	n_0	n_0
Initial service share	ν	ν
Growth in relative prices, services/manuf.	γ_S	γ_S
Real GDP per capita growth	γ_G	γ_G
Growth in the relative market work hours, high/low skilled	γ_h	γ_h
Initial fraction of high-skilled	beta: a	beta: a
Ending fraction of high-skilled	beta: b	beta: b
Initial relative market work hours, married female/married male		A_f
Initial relative market work hours, single female/single male		A_m
Growth in the relative market work hours of women to men		$\Delta\tau_{l0}$

Table 2 – Calibrated Parameters

	BK Model	Extended Model
ϕ_0	1.385	1.423
λ	0.567	0.852
n_0	8.244	8.244
ν	0.664	0.589
γ_S	0.0140	0.0120
γ_G	0.0218	0.0198
γ_h	0.0055	0.0060
beta: a	3.962	8.413
beta: b	7.368	16.339
A_f		0.613
A_m		0.451
$\Delta\tau_{l0}$		0.170

Table 3 – Benchmark Model: Service Growth

	(Current) Value-Added Service Share	
	1965	2003
Data	0.63	0.79
Benchmark Model	0.63	0.80

Note: (1): the 1965 nominal service share is matched by calibration;

Table 4 – Decomposing Service Growth: Counterfactuals

	Percentage Point Increase in Services		
Factor	Increase	Difference	Percent of Benchmark
All Factors (Benchmark)	17	—	—
No Income Effect	11	6	36%
No Scale Effect	12	5	32%
No Income or Scale	4	13	83%
No Skill-Biased Tech.	5	12	73%
No Sector-Biased Tech.	19	-2	-14%
No Factors	0	—	—
Only Income Effect	6	6	32%
Only Scale Effect	8	8	45%
Only Income and Scale	12	12	73%
Only Skill-Biased Tech.	10	10	57%
Only Sector-Biased Tech.	0	-15	-88%

Note: (1):For "Only" factors, the difference is relative to no factors.

Table 5 – Effect of Endogenous Skill Premium and Educational Choices

2003 Service Share	Skill Premium	
Educational Choices	Fixed	Endogenous
Fixed	4	8
Endogenous	4	17

Notes: The 1965 nominal service share is matched by calibration. Fixed keeps exogenously fixed at 1965 value. Service share is current value, value-added.

Table 6 – Decomposing Female Market Labor Increase in Extended Model: Counterfactuals

	Percentage Point Increase in Market Labor, Female/Male					
	Fixed Wedge			Falling Wedge		
Factor	Increase	Difference	% of Bench.	Increase	Difference	% of Bench.
All Factors (Benchmark)	9	—	—	27	—	—
No Income Effect	6	3	35%	25	2	7%
No Scale Effect	2	8	83%	22	5	20%
No Income or Scale	-1	11	114%	20	7	26%
No Skill-Biased Tech.	-7	16	172%	16	11	42%
No Sector-Biased Tech.	30	-21	-223%	40	-13	-50%
No Demographic Change	0	9	98%	20	7	27%

Table 7 – Decomposing Service Growth in Extended Model: Counterfactuals

	Percentage Point Increase in Services					
	Fixed Wedge			Falling Wedge		
Factor	Increase	Difference	% of Bench.	Increase	Difference	% of Bench.
All Factors (Benchmark)	17	—	—	20	—	—
No Income Effect	16	2	9%	19	1	5%
No Scale Effect	12	5	31%	16	4	22%
No Income or Scale	10	7	41%	15	5	27%
No Skill-Biased Tech.	7	10	61%	14	6	30%
No Sector-Biased Tech.	20	-3	-16%	20	0	0%
No Demographic Change	14	3	20%	18	2	10%

A Equivalent Formulation

In our extended model, we define a threshold $\tilde{z}$ such that the wife performs all home production below $\tilde{z}$, and the husband performs all home production between $\tilde{z}$ and $\underline{z}$. An alternative formulation is to define a threshold $\tilde{z}$ such that the husband performs all home production below $\tilde{z}$, and the wife performs all home production between $\tilde{z}$ and $\underline{z}$. In this appendix, we show that both formulations are equivalent.

The budget constraint based on the alternative formulation is

$$\begin{aligned} 2\int_0^{\underline{z}_{ee}} p_G(z,t)dz + 2\int_{\underline{z}_{ee}}^{\overline{z}_{ee}} p_S(z,t)dz &= w_{e_m}(1 - 2\int_0^{\widetilde{z}_{ee}} \frac{e^{-\gamma_S t}z}{A_m}dz - \theta I(e_m)) \\ &\quad + w_{e_f}(1 - 2\int_{\widetilde{z}_{ee}}^{\underline{z}_{ee}} \frac{e^{-\gamma_S t}z}{A_f}dz - \theta I(e_f)) \end{aligned}$$

The corresponding Kuhn-Tucker conditions that characterize the optimum, $\underline{z}_{ee}$, $\overline{z}_{ee}$ and $\widetilde{z}_{ee}$ are

$$\begin{gathered} \mu(\frac{e^{-\gamma_S t}\underline{z}_{ee}}{A_f}w_{e_f} + p_G(\underline{z}_{ee},t) - p_S(\underline{z}_{ee},t)) + \widehat{\eta}_2\frac{e^{-\gamma_S t}\underline{z}_{ee}}{A_f} = 1-\nu \\ \nu = \mu p_S(\overline{z}_{ee},t) \\ \mu w_{e_f}\frac{e^{-\gamma_S t}\widetilde{z}_{ee}}{A_f} - \mu w_{e_m}\frac{e^{-\gamma_S t}\widetilde{z}_{ee}}{A_m} = \widehat{\eta}_1\frac{e^{-\gamma_S t}\widetilde{z}_{ee}}{A_m} - \widehat{\eta}_2\frac{e^{-\gamma_S t}\widetilde{z}_{ee}}{A_f} - \widehat{\eta}_3 \\ 1 - t_m - \theta I(e_m) \geqslant 0, \widehat{\eta}_1 \geqslant 0, (1 - t_m - \theta I(e_m))\widehat{\eta}_1 = 0 \\ 1 - t_f - \theta I(e_f) \geqslant 0, \widehat{\eta}_2 \geqslant 0, (1 - t_f - \theta I(e_f))\widehat{\eta}_2 = 0 \\ 2\widetilde{z}_{ee} \geqslant 0, \widehat{\eta}_3 \geqslant 0, 2\widetilde{z}_{ee}\widehat{\eta}_3 = 0 \end{gathered}$$

Case One: both husband and wife's time constraints are not binding.
Original formulation: $\eta_1, \eta_2 = 0$; $\eta_3 > 0$; $\widetilde{z}_{ee} = \underline{z}_{ee}$
Alternative formulation: $\widehat{\eta}_1, \widehat{\eta}_2 = 0$; $\widetilde{z}_{ee} = 0$; $\widehat{\eta}_3 = 0$
Both formulations imply the same FOCs that solve for $\underline{z}_{ee}$ and $\overline{z}_{ee}$:

$$\begin{aligned} 1-\nu &= \frac{\nu}{p_S(\overline{z}_{ee},t)}(\frac{e^{-\gamma_S t}\underline{z}_{ee}}{A_f}w_{e_f} + p_G(\underline{z}_{ee},t) - p_s(\underline{z}_{ee},t)) \\ 2\int_0^{\underline{z}_{ee}} p_G(z,t)dz + 2\int_{\underline{z}_{ee}}^{\overline{z}_{ee}} p_S(z,t)dz &= w_{e_m}[1 - \theta I(e_m)] + w_{e_f}[1 - 2\int_0^{\underline{z}_{ee}} \frac{e^{-\gamma_S t}z}{A_f}dz - \theta I(e_f)] \end{aligned}$$

Case Two: only the wife's time constraint is binding
Case Two is relevant only when $A_f > A_m$.

Original formulation: $\eta_1 = 0$; $\eta_2, \eta_3 \geqslant 0$; $1 - \frac{e^{-\gamma_S t}\widetilde{z}_{ee}^2}{A_f} - \theta I(e_f) = 0$
Alternative formulation: $\widehat{\eta}_1 = 0$; $\widehat{\eta}_2, \widehat{\eta}_3 \geqslant 0; 1 - \frac{e^{-\gamma_S t}}{A_f}(\underline{z}_{ee}^2 - \widetilde{z}_{ee}^2) - \theta I(e_f) = 0$
There are two sub-cases depending on whether it is at the kink point or not.
Subcase one: $\widehat{\eta}_3, \eta_3 = 0$
Both formulations imply the same FOCs that solve for $\underline{z}_{ee}$ and $\overline{z}_{ee}$:

$$1 - \nu = \frac{\nu}{p_s(\overline{z}_{ee}, t)}\left(\frac{e^{-\gamma_S t}\underline{z}_{ee}}{A_m} w_{e_m} + p_G(\underline{z}_{ee}, t) - p_S(\underline{z}_{ee}, t)\right)$$

$$2\int_0^{\underline{z}_{ee}} p_G(z,t)dz + 2\int_{\underline{z}_{ee}}^{\overline{z}_{ee}} p_S(z,t)dz = w_{e_m}\left(1 - \frac{e^{-\gamma_S t}\underline{z}_{ee}^2 - A_f(1-\theta I(e_f))}{A_m} - \theta I(e_m)\right)$$

Subcase two: $\widehat{\eta}_3, \eta_3 > 0$

It is at the kink point, which implies that $\underline{z}_{ee} = (A_f e^{\gamma_s t}(1 - \theta I(e_f)))^{0.5}$. Both formulations imply the same budget constraint that solves for $\overline{z}_{ee}$:

$$2\int_0^{\underline{z}_{ee}} p_G(z,t)dz + 2\int_{\underline{z}_{ee}}^{\overline{z}_{ee}} p_S(z,t)dz = w_{e_m}\left(1 - \theta I(e_m)\right)$$

Case Three: only the husband's time constraint is binding

Case Three is relevant only when $A_m > A_f$. By the same token as in Case Two, both formulations are equivalent.

B Characterization of a Married Couple's Problem

Proof of Proposition 1. The budget constraint for a schooling choice, lh, is denoted as follows:

$$2\int_0^{\underline{z}_{lh}} p_G(z,t)dz + 2\int_{\underline{z}_{lh}}^{\overline{z}_{lh}} p_S(z,t)dz = (1 - 2\int_{\widetilde{z}_{lh}}^{\underline{z}_{lh}} \frac{e^{-\gamma_S t}z}{A_m}dz) + w(1 - 2\int_0^{\widetilde{z}_{lh}} \frac{e^{-\gamma_S t}z}{A_f}dz - \theta) \tag{15}$$

The budget constraint for a schooling choice, hl, is denoted as follows:

$$2\int_0^{\underline{z}_{hl}} p_G(z,t)dz + 2\int_{\underline{z}_{hl}}^{\overline{z}_{hl}} p_S(z,t)dz = w(1 - 2\int_{\widetilde{z}_{hl}}^{\underline{z}_{hl}} \frac{e^{-\gamma_S t}z}{A_m}dz - \theta) + (1 - 2\int_0^{\widetilde{z}_{hl}} \frac{e^{-\gamma_S t}z}{A_f}dz) \tag{16}$$

Assume $\underline{z}^*$, $\overline{z}^*$ and $\widetilde{z}^*$ are the optimal allocations for the schooling choice, lh. Given that the wife faces a higher opportunity cost in home production under lh, the optimal allocation should satisfy the following inequality:

$$2\int_{\widetilde{z}^*}^{\underline{z}^*} \frac{e^{-\gamma_s t} z}{A_m} > 2\int_0^{\widetilde{z}^*} \frac{e^{-\gamma_s t} z}{A_f} dz$$

$$\frac{(\underline{z}^*)^2 - (\widetilde{z}^*)^2}{A_m} > \frac{(\widetilde{z}^*)^2}{A_f} \tag{17}$$

Next, we define $\widetilde{z}^{'}$ as follows:

$$2\int_{\widetilde{z}^{'}}^{\underline{z}^*} \frac{e^{-\gamma_s t} z}{A_m} = 2\int_0^{\widetilde{z}^*} \frac{e^{-\gamma_s t} z}{A_f} dz \tag{18}$$

Equation (18) can be simplified as follows:

$$\frac{(\widetilde{z}^{'})^2}{A_m} = \frac{(\underline{z}^*)^2}{A_m} - \frac{(\widetilde{z}^*)^2}{A_f} \tag{19}$$

Next, we subtract the RHS of (15) from the RHS of (16) with $(\underline{z}^*, \widetilde{z}^*)$ for the schooling choice lh and $(\underline{z}^*, \widetilde{z}^{'})$ for the schooling choice hl, and obtain as follows:

$$\begin{aligned}
& (1 - 2\int_0^{\widetilde{z}^{'}} \frac{e^{-\gamma_s t} z}{A_f} dz) - (1 - 2\int_{\widetilde{z}^*}^{\underline{z}^*} \frac{e^{-\gamma_s t} z}{A_m} dz) \qquad (20)\\
= \; & e^{-\gamma_s t} \frac{(\underline{z}^*)^2 - (\widetilde{z}^*)^2}{A_m} - \frac{e^{-\gamma_s t} (\widetilde{z}^{'})^2}{A_f} \\
= \; & e^{-\gamma_s t} \frac{(\underline{z}^*)^2 - (\widetilde{z}^*)^2}{A_m} - e^{-\gamma_s t} \frac{A_m}{A_f} (\frac{(\underline{z}^*)^2}{A_m} - \frac{(\widetilde{z}^*)^2}{A_f}) \\
= \; & e^{-\gamma_s t} \left((\underline{z}^*)^2 - (\widetilde{z}^*)^2\right) \left(\frac{1}{A_m} - \frac{1}{A_f}\right) - e^{-\gamma_s t} (\widetilde{z}^*)^2 \left(\frac{1}{A_f} - \frac{A_m}{A_f} \frac{1}{A_f}\right) \\
> \; & e^{-\gamma_s t} (\widetilde{z}^*)^2 \frac{A_m}{A_f} \left(\frac{1}{A_m} - \frac{1}{A_f}\right) - e^{-\gamma_s t} (\widetilde{z}^*)^2 \left(\frac{1}{A_f} - \frac{A_m}{A_f} \frac{1}{A_f}\right) \\
= \; & 0
\end{aligned}$$

The line 3 of (20) follows (19), and the line 5 of (20) follows the inequality (17). The inequality (20) shows that $(\underline{z}^* + q, \overline{z}^*; q > 0)$ can be consumed if a married couple chooses to be (hl) instead of (lh). Therefore, the schooling choice (hl) always dominates the schooling choice (lh). ■

Proof of Proposition 2. According to Proposition 1, the schooling choice of (lh) will never be chosen. Then, given $A_m < A_f$ and $w > 1$, the LHS of (11) should be positive unless $\widetilde{z}_{ee}$ is equal to zero.

We first show that it is never optimal to choose $\widetilde{z}_{ee}$ to be zero. Assume that $\underline{z}^*$, $\overline{z}^*$ and $\widetilde{z}^*$ are the optimal allocations, and $\widetilde{z}^* = 0$. If $\underline{z}^* > 0$, a married couple can always improve the outcome by setting $t_m = 0$ and $t_f = 2\int_0^{\underline{z}^*} \frac{e^{-\gamma_S t} z}{A_f} dz$, so $\underline{z}^*$ has to be zero, which implies that η_1 to be zero. In order to satisfy Condition (11), η_3 has to be zero when $\widetilde{z}^* = 0$. Given that $\eta_1 = 0$ and $\eta_3 = 0$, Condition (10) can be simplified as follows:

$$\mu(\frac{e^{-\gamma_S t}\underline{z}^*}{A_m} w_{e_m} + p_m(\underline{z}^*, t) - p_s(\underline{z}^*, t) = 1 - \nu \tag{21}$$

The LHS of (21) is zero, but the RHS of (21) is positive, which leads to a contradiction. Therefore, $\underline{z}^*$ can't be zero and the LHS of (11) should be positive.

Next, we show that $t_f > t_m$. Assume $t_f \leq t_m$. Then, $\eta_2 = 0$ and $\eta_3 = 0$. It implies that the RHS of (11) will be zero or negative. It leads to a contradiction. Therefore, $t_f > t_m$.

If $t_m > 0$, then $\eta_3 = 0$. In order to let the RHS of (11) be positive, η_2 has to be positive, which implies that the wife will not work in the market. If $1 - t_f - \theta I(e_f) > 0$, then $\eta_2 = 0$. In order to let the RHS of (11) be positive, η_3 has to be positive, which implies that the husband will not work at home. ■

C The Market Clearing Condition for High-Skilled Workers

$$p_1 \int_0^1 H_m(\theta) f(\theta) d\theta + p_2 \int_0^1 H_f(\theta) f(\theta) d\theta + \tfrac{1}{2} p_3 \int_0^1 H_c(\theta) f(\theta) d\theta$$
$$= p_1 \{ \int_0^{\widehat{\theta}_m} \left(1 - \theta - \int_0^{\underline{z}(\theta)} \frac{e^{-\gamma_t z}}{A_m} dz \right) f(\theta) d\theta \} + p_2 \{ \int_0^{\widehat{\theta}_f} \left(1 - \theta - \int_0^{\underline{z}(\theta)} \frac{e^{-\gamma_t z}}{A_f} dz \right) f(\theta) d\theta \}$$
$$+ \tfrac{1}{2} p_3 \{ \int_0^{\widehat{\theta}_1} \left(2 - 2\theta - t_m(\theta) - t_f(\theta) \right) f(\theta) d\theta + \int_{\widehat{\theta}_1}^{\widehat{\theta}_2} \left(1 - \theta - t_m(\theta) \right) f(\theta) d\theta \}$$

case one: $\widehat{z} \leq \underline{z}(\theta) \leq \overline{z}(\theta)$

$$H_m(\theta) = \int_{\widehat{z}}^{\underline{z}(\theta)} \frac{e^{-\gamma_G t - \gamma_h t} z^\lambda}{A_h} dz + \frac{1}{n} \int_{\underline{z}(\theta)}^{\overline{z}(\theta)} \frac{e^{-\gamma_G t - \gamma_h t} z^\lambda}{A_h} dz + \int_{\underline{z}(\theta)}^{\overline{z}(\theta)} \frac{e^{-\gamma_S t - \gamma_h t} z^\lambda}{A_h} dz$$
$$= \frac{e^{-\gamma_G t - \gamma_h t}}{A_h(\lambda + 1)} (\underline{z}(\theta)^{\lambda + 1} - \widehat{z}^{\lambda + 1}) + \frac{1}{A_h(\lambda + 1)} (\tfrac{1}{n} e^{-\gamma_m t - \gamma_h t} + e^{-\gamma_s t - \gamma_h t}) (\overline{z}(\theta)^{\lambda + 1} - \underline{z}(\theta)^{\lambda + 1})$$

$$H_c(\theta) = 2 \int_{\widehat{z}}^{\underline{z}(\theta)} \frac{e^{-\gamma_G t - \gamma_h t} z^\lambda}{A_h} dz + 2[\frac{1}{n} \int_{\underline{z}(\theta)}^{\overline{z}(\theta)} \frac{e^{-\gamma_G t - \gamma_h t} z^\lambda}{A_h} dz + \int_{\underline{z}(\theta)}^{\overline{z}(\theta)} \frac{e^{-\gamma_S t - \gamma_h t} z^\lambda}{A_h} dz]$$
$$= \frac{2e^{-\gamma_m t - \gamma_h t}}{A_h(\lambda + 1)} (\underline{z}(\theta)^{\lambda + 1} - \widehat{z}^{\lambda + 1}) + \frac{2}{A_h(\lambda + 1)} (\tfrac{1}{n} e^{-\gamma_G t - \gamma_h t} + e^{-\gamma_S t - \gamma_h t}) (\overline{z}(\theta)^{\lambda + 1} - \underline{z}(\theta)^{\lambda + 1})$$

case two: $\underline{z}(\theta) \leq \widehat{z} \leq \overline{z}(\theta)$

$$H_m(\theta) = \frac{1}{n} \int_{\widehat{z}}^{\overline{z}(\theta)} \frac{e^{-\gamma_G t - \gamma_h t} z^\lambda}{A_h} dz + \int_{\widehat{z}}^{\overline{z}(\theta)} \frac{e^{-\gamma_S t - \gamma_h t} z^\lambda}{A_h} dz$$
$$= \frac{1}{A_h(\lambda + 1)} (\tfrac{1}{n} e^{-\gamma_G t - \gamma_h t} + e^{-\gamma_s t - \gamma_h t}) (\overline{z}(\theta)^{\lambda + 1} - \widehat{z}^{\lambda + 1})$$

$$H_c(\theta) = 2[\frac{1}{n} \int_{\widehat{z}}^{\overline{z}(\theta)} \frac{e^{-\gamma_G t - \gamma_h t} z^\lambda}{A_h} dz + \int_{\widehat{z}}^{\overline{z}(\theta)} \frac{e^{-\gamma_S t - \gamma_h t} z^\lambda}{A_h} dz]$$

$$= \frac{2}{A_h(\lambda\ +1)}(\frac{1}{n}e^{-\gamma_G t-\gamma_h t} + e^{-\gamma_S t-\gamma_h t})(\overline{z}(\theta)^{\lambda\ +1} - \widehat{z}^{\lambda\ +1})$$

$H_f(\theta)$ has the same functional form are the same as $H_m(\theta)$ except that A_m is replaced with A_f.